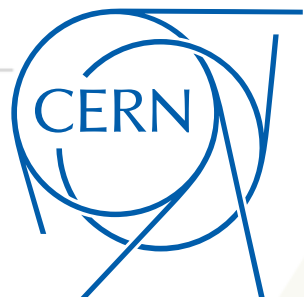


Measurement of the CKM angle γ

Mark Whitehead on behalf of LHCb

Flavour Physics with High-Luminosity Experiments
Workshop

MIAPP, Germany



Introduction

- Why are we still measuring γ ?

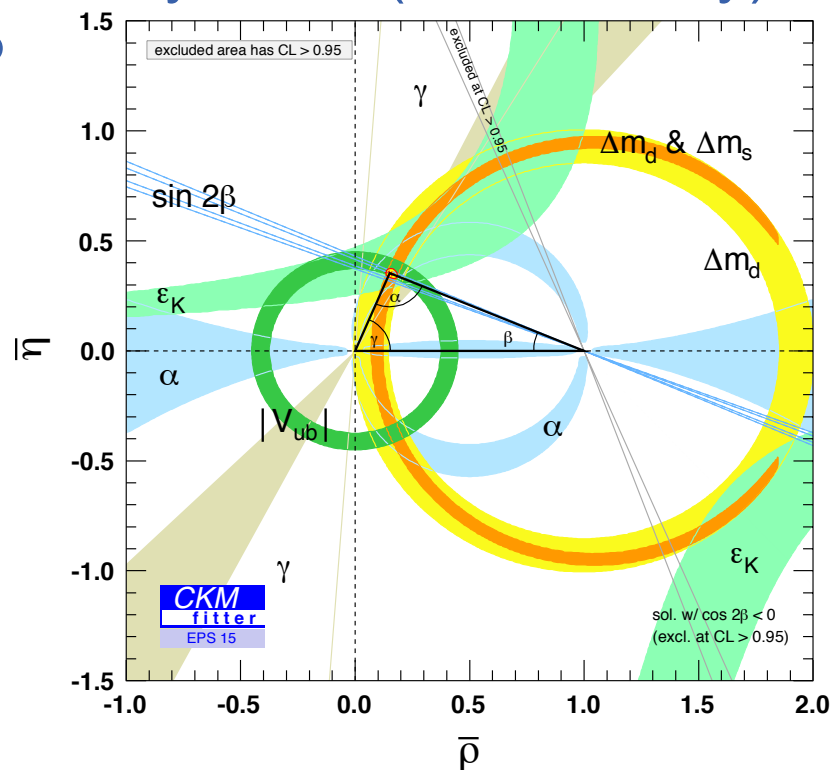
- Least well known angle of the CKM unitarity triangle
- Tree-level determination and extremely clean (theoretically)
- Loop-level access to look for NP

$$\gamma = (73.2^{+6.3}_{-7.0})^\circ \quad [\text{CKM fitter}]$$

$$\gamma = (68.3 \pm 7.5)^\circ \quad [\text{UT fit}]$$

- Whats new from LHCb?

- New decay modes
- Updates to our full Run I sample
- New combination

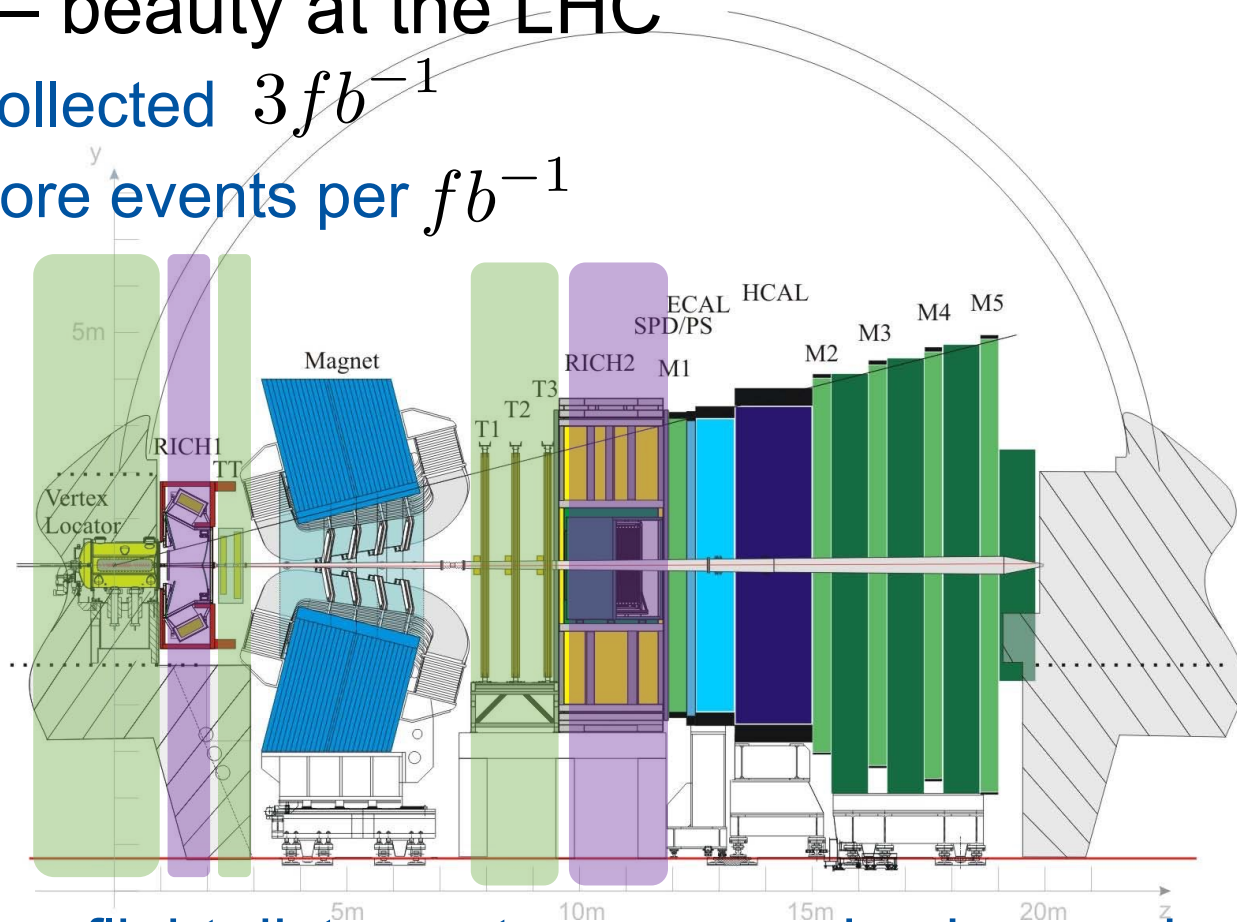


Detector

- Its all in the name – beauty at the LHC
 - During Run I we collected $3fb^{-1}$
 - Run II at 13TeV more events per fb^{-1}

Excellent tracking
and vertex
resolution

Particle ID for
 $K\pi$ separation



- Exploit B and D meson flight distance to suppress backgrounds

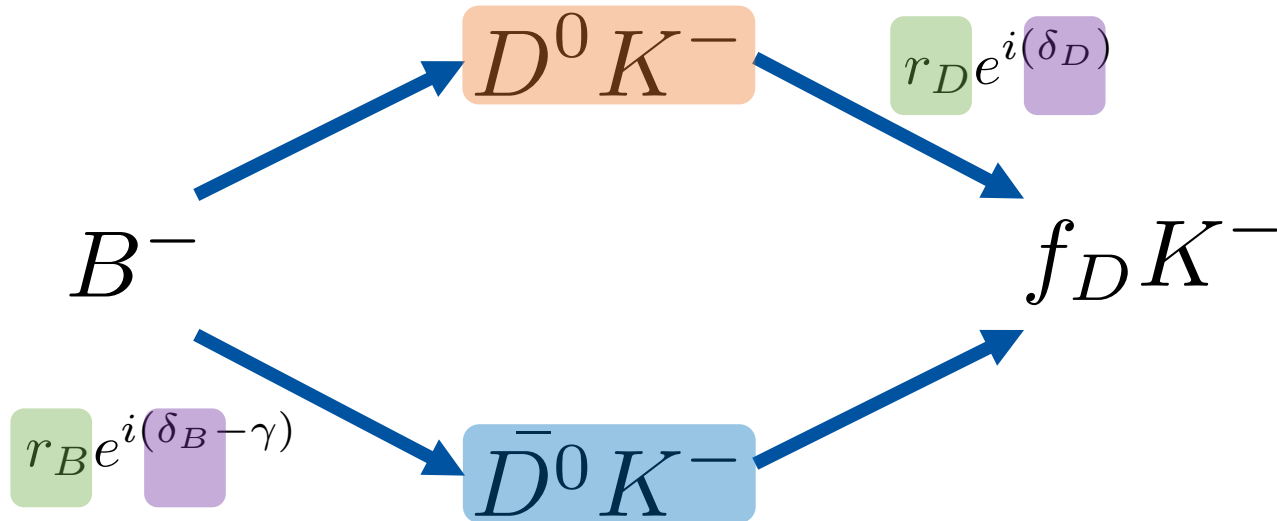
How to measure γ

- Interference

- Two amplitudes giving the same final state
- Golden example $-B^\pm \rightarrow DK^\pm$

$$b \rightarrow cW (V_{cb})$$

$$b \rightarrow uW (V_{ub})$$



- Amplitude ratios r_B, r_D
- Strong phase differences δ_B, δ_D

New results from LHCb

- Several **new** or **updated** inputs into the combination

- **Two body GLW/ADS** [M. Gronau, D. Wyler, Phys. Lett. B265 (1991) 172]
[M. Gronau, D. London, Phys. Lett. B253 (1991) 483]

$$B^{\pm} \rightarrow DK^{\pm}, D \rightarrow h^{+}h'^{-}$$

- **Four body GLW/ADS** [D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. Lett. 78 (1997) 3257, Phys. Rev. D63 (2001) 036005]

$$B^{\pm} \rightarrow DK^{\pm}, D \rightarrow h^{+}\pi^{-}\pi^{+}\pi^{-}$$

- **GGSZ** [A. Giri, Y. Grossman, A. Soffer, and J. Zupan, Phys. Rev. D68 (2003) 054018]

$$B^0 \rightarrow DK^{*0}, D \rightarrow K_S^0\pi^{+}\pi^{-}$$

- **GLW-Dalitz** [T. Gershon, Phys. Rev. D79 (2009) 051301]
[T. Gershon, M. Williams, Phys. Rev. D80 (2009) 092002]

$$B^0 \rightarrow DK^{+}\pi^{-}, D \rightarrow h^{+}h^{-}$$

New results from LHCb

- Several new or updated inputs into the combination

- Two body GLW/ADS

Update of LHCb γ combination

- Four body GLW/ADS

of $B \rightarrow DK$ like decays

- GGSZ

and all $B \rightarrow Dh$ decays

- GLW-Dalitz

$$B^0 \rightarrow DK^+ \pi^-, D \rightarrow h^+ h^-$$

- Several new or updated inputs into the combination

- Two body GLW/ADS

$$B^{\pm} \rightarrow DK^{\pm}, D \rightarrow h^{+}h'^{-}$$

- Four body GLW/ADS

$$B^{\pm} \rightarrow DK^{\pm}, D \rightarrow h^{+}\pi^{-}\pi^{+}\pi^{-}$$

- GGSZ

$$B^0 \rightarrow DK^{*0}, D \rightarrow K_S^0\pi^{+}\pi^{-}$$

- GLW-Dalitz

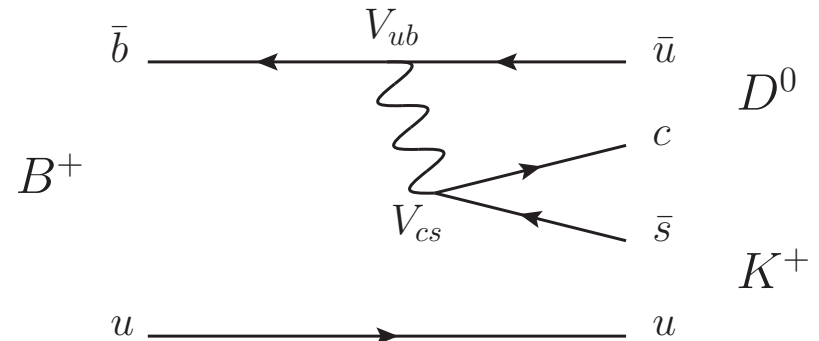
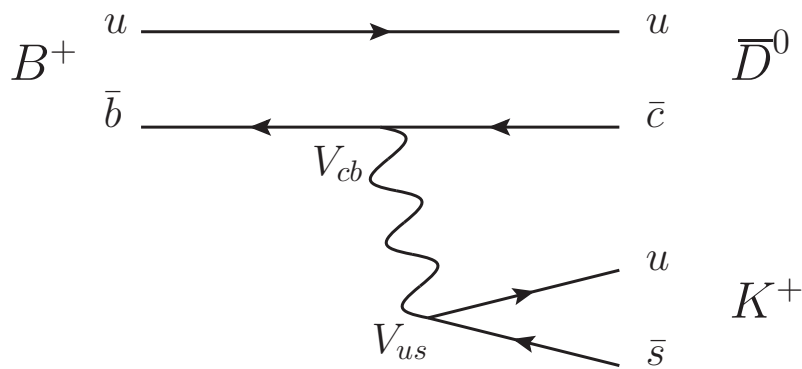
$$B^0 \rightarrow DK^{+}\pi^{-}, D \rightarrow h^{+}h^{-}$$

Two and four body GLW/ADS

- Look at $B \rightarrow Dh$ decays

- GLW: $D \rightarrow K^+ K^-, \pi^+ \pi^-, 2\pi^+ 2\pi^-$

- ADS: $D \rightarrow K^+ \pi^-, \pi^+ K^-, K^+ \pi^- \pi^+ \pi^-, \pi^+ K^- \pi^+ \pi^-$



- Share signal shape parameters between 2(4) body modes
- Constrain crossfeed between DK and $D\pi$ modes (PID)
- Use $D\pi$ as a control mode; charmless background etc

ADS favoured signals

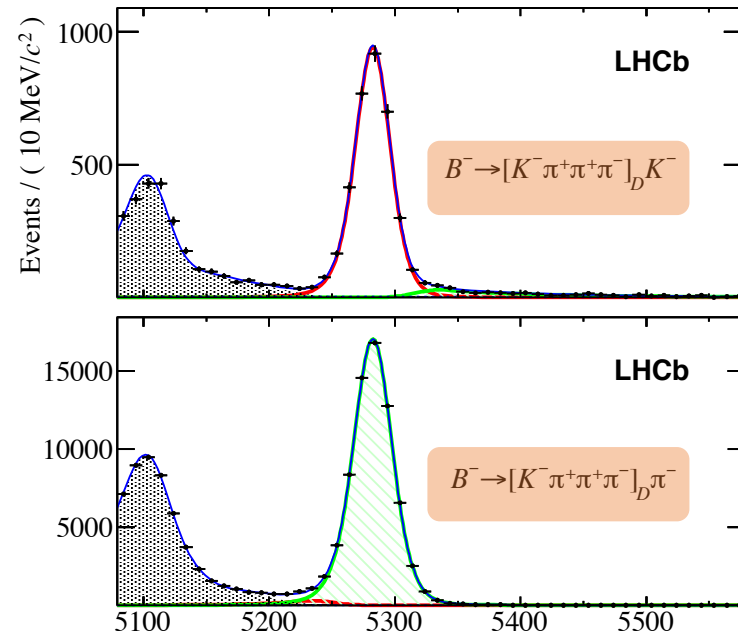
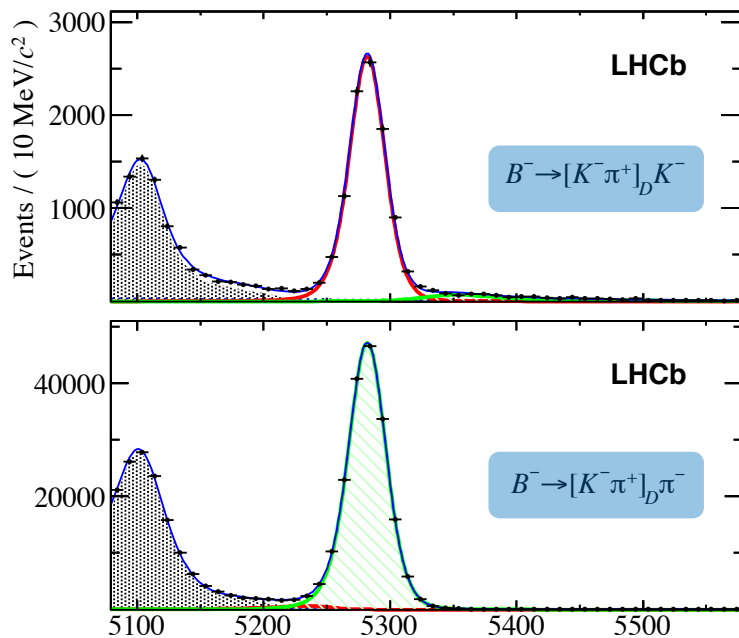
- Mass fits for the favoured modes

$$B^\pm \rightarrow [K^\pm \pi^\mp]_D \pi^\pm \quad 378,050 \pm 650$$

$$B^\pm \rightarrow [K^\pm \pi^\mp]_D K^\pm \quad 29,470 \pm 230$$

$$B^\pm \rightarrow [K^\pm \pi^\mp \pi^+ \pi^-]_D \pi^\pm \quad 142,910 \pm 390$$

$$B^\pm \rightarrow [K^\pm \pi^\mp \pi^+ \pi^-]_D K^\pm \quad 11,330 \pm 140$$

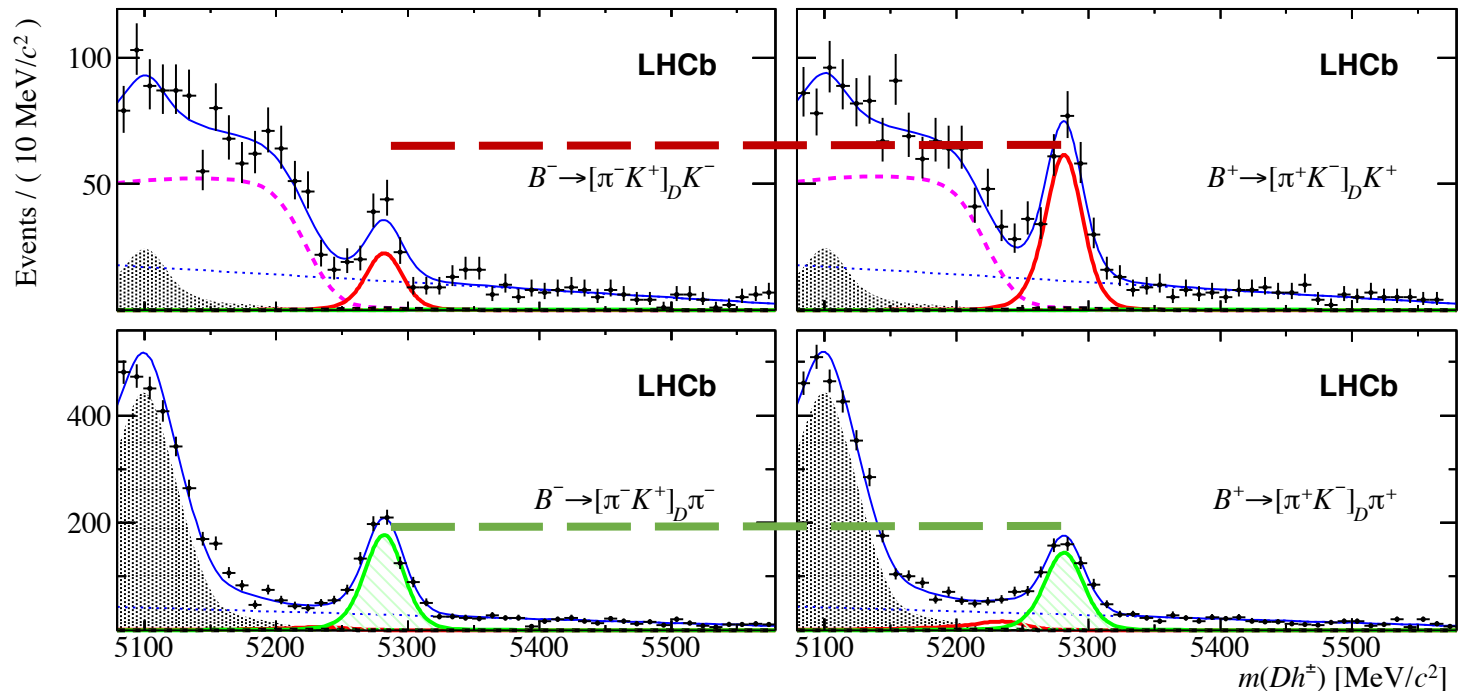


Low combinatorial background and tiny cross-feed contributions

2 body ADS suppressed signals

- Visible CP violation
 - Around 8 sigma!

$$\begin{array}{ll}
 B^\pm \rightarrow [\pi^\pm K^\mp]_D K^\pm & 553 \pm 34 \\
 B^\pm \rightarrow [\pi^\pm K^\mp]_D \pi^\pm & 1360 \pm 44
 \end{array}$$



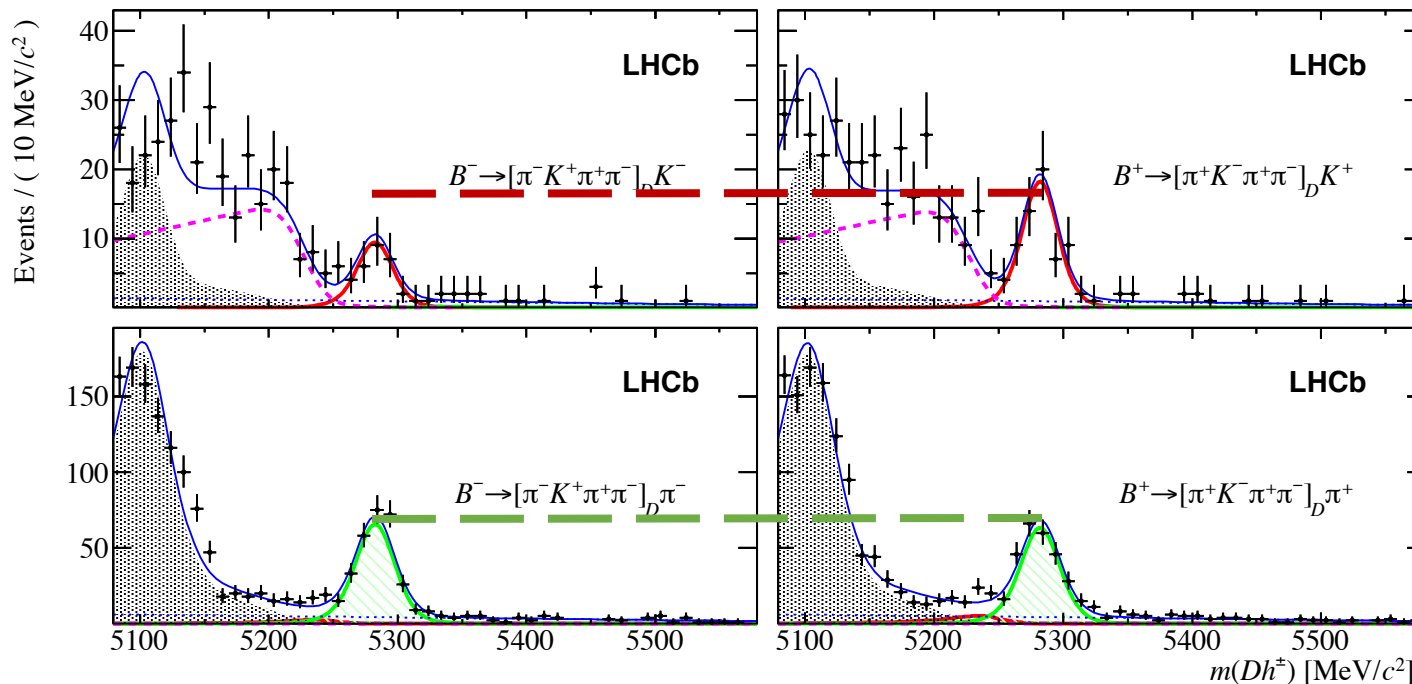
$$A_{\text{ADS}(K)}^{\pi K} = -0.403 \pm 0.056 \pm 0.011$$

$$A_{\text{ADS}(\pi)}^{\pi K} = 0.100 \pm 0.031 \pm 0.009$$

4 body ADS suppressed signals

- Visible CP asymmetry
 - Close to 3 sigma evidence

$$\begin{array}{ll}
 B^\pm \rightarrow [\pi^\pm K^\mp \pi^+ \pi^-]_D K^\pm & 159 \pm 17 \\
 B^\pm \rightarrow [\pi^\pm K^\mp \pi^+ \pi^-]_D \pi^\pm & 539 \pm 26
 \end{array}$$



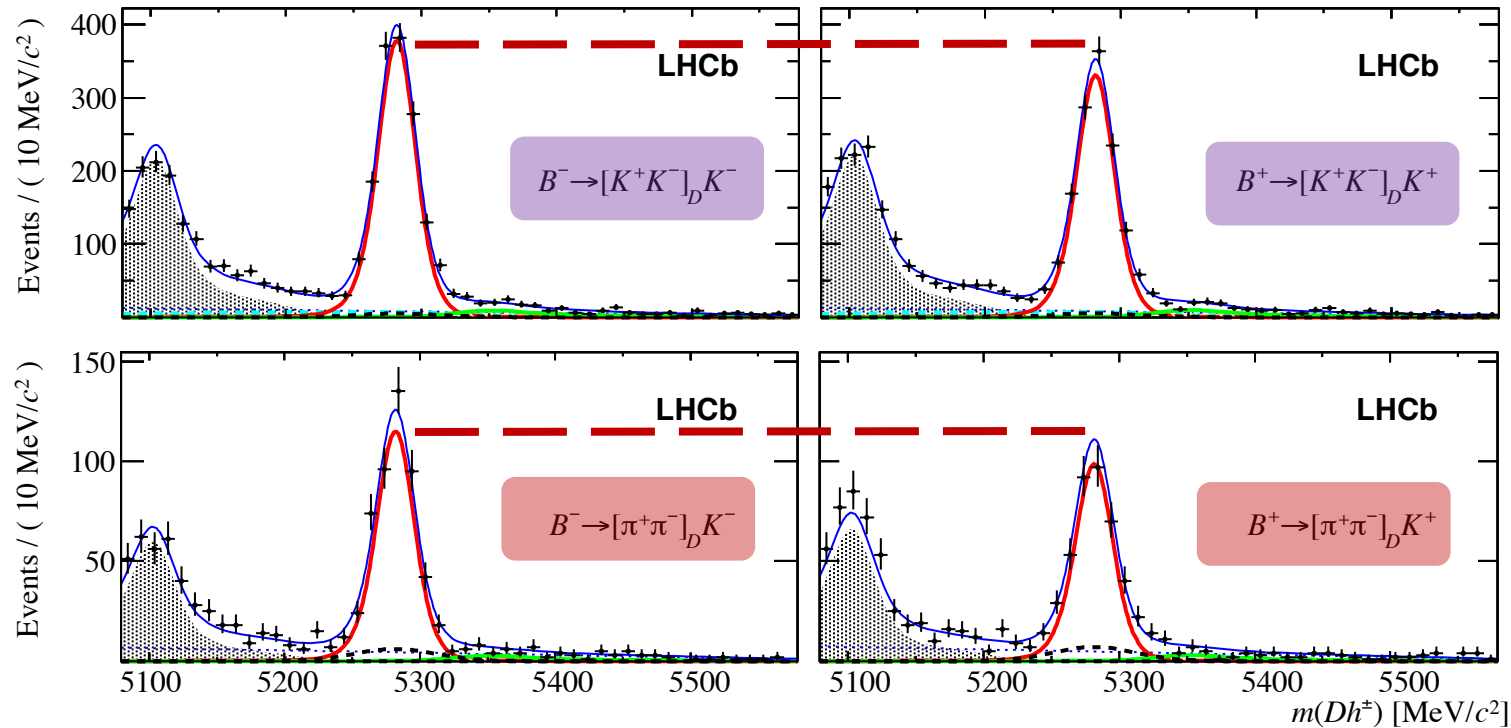
$$A_{\text{ADS}(K)}^{\pi K \pi \pi} = -0.313 \pm 0.102 \pm 0.038 \quad A_{\text{ADS}(\pi)}^{\pi K \pi \pi} = 0.023 \pm 0.048 \pm 0.005$$

2 body GLW modes

- Small asymmetries seen
 - About 5 sigma combined!

$$B^\pm \rightarrow [K^+K^-]_D K^\pm \quad 3816 \pm 92$$

$$B^\pm \rightarrow [\pi^+\pi^-]_D K^\pm \quad 1162 \pm 48$$



$$A_K^{KK} = 0.087 \pm 0.020 \pm 0.008$$

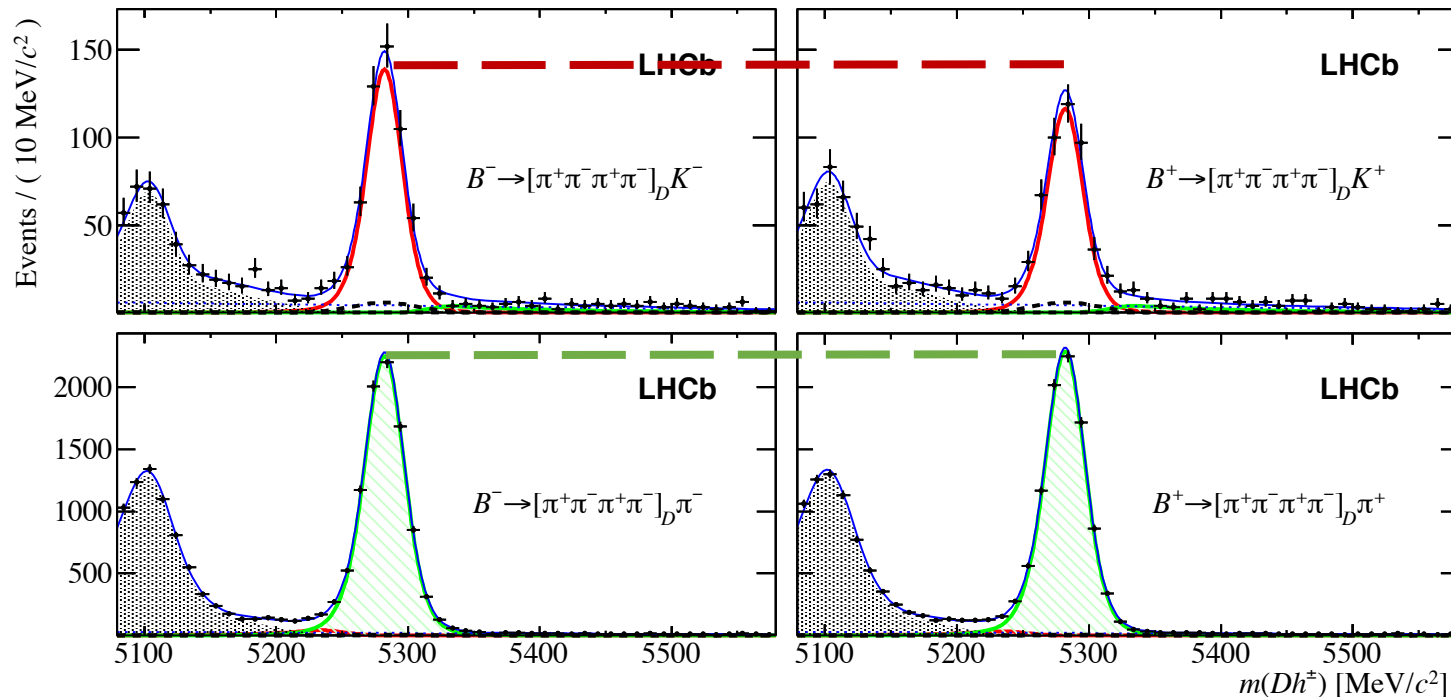
$$A_K^{\pi\pi} = 0.128 \pm 0.037 \pm 0.012$$

4 body GLW mode

- First use of this mode!
- Decay $\sim 75\%$ CP even

$$B^\pm \rightarrow [\pi^+\pi^-\pi^+\pi^-]_D K^\pm \quad 1497 \pm 60$$

$$B^\pm \rightarrow [\pi^+\pi^-\pi^+\pi^-]_D \pi^\pm \quad 19,360 \pm 150$$



$$A_K^{\pi\pi\pi\pi} = 0.100 \pm 0.034 \pm 0.018$$

$$A_\pi^{\pi\pi\pi\pi} = -0.0041 \pm 0.0079 \pm 0.0024$$

- Several new or updated inputs into the combination

- Two body GLW/ADS

$$B^{\pm} \rightarrow DK^{\pm}, D \rightarrow h^{+}h'^{-}$$

- Four body GLW/ADS

$$B^{\pm} \rightarrow DK^{\pm}, D \rightarrow h^{+}\pi^{-}\pi^{+}\pi^{-}$$

- GGSZ

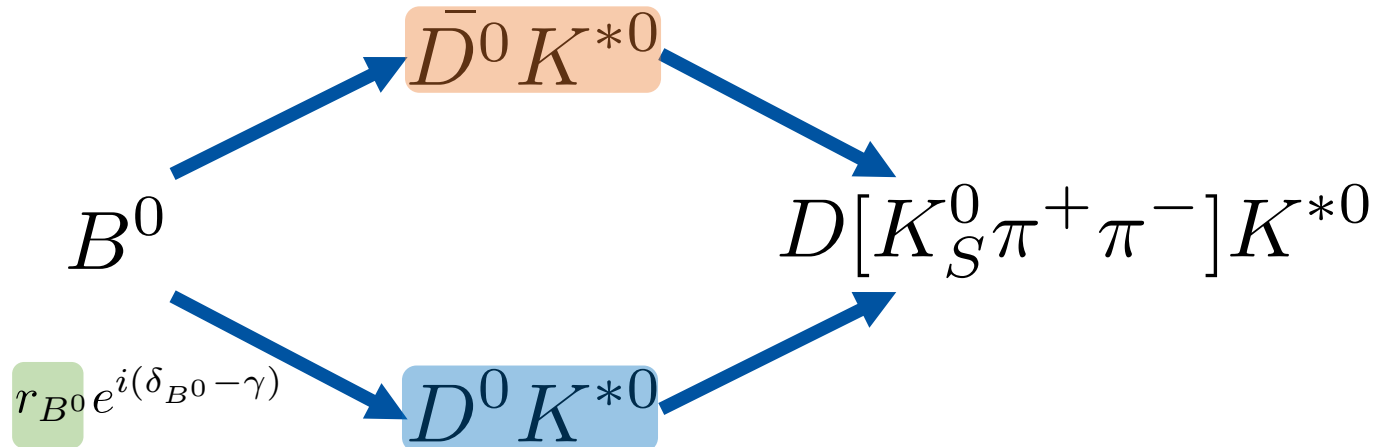
$$B^0 \rightarrow DK^{*0}, D \rightarrow K_S^0\pi^{+}\pi^{-}$$

- GLW-Dalitz

$$B^0 \rightarrow DK^{+}\pi^{-}, D \rightarrow h^{+}h^{-}$$

GGSZ analysis

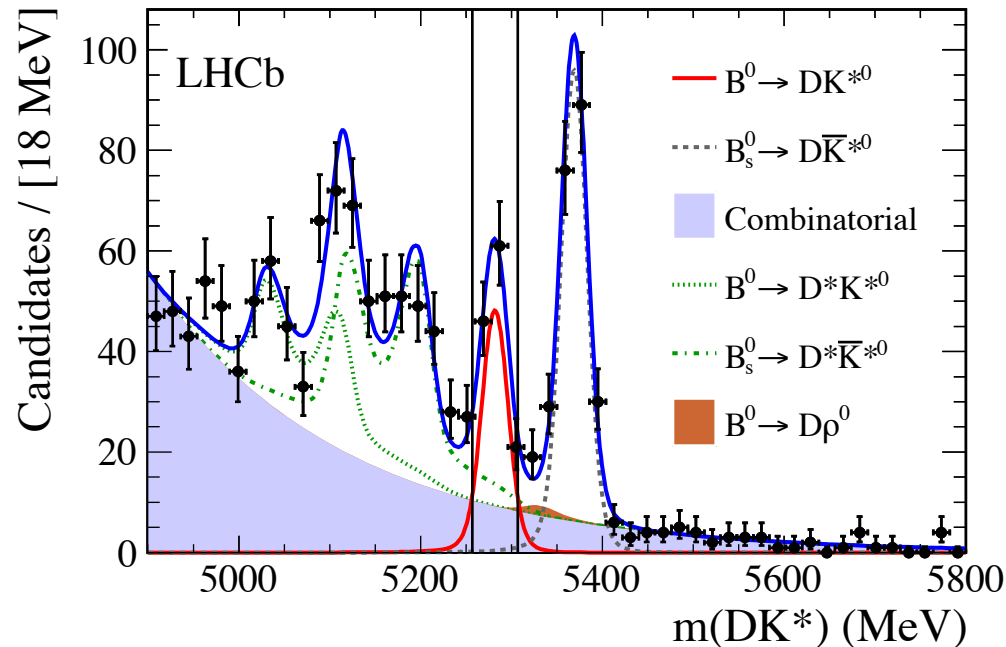
- Can also consider the neutral B channels
 - Decay chain $B^0 \rightarrow DK^{*0}, D \rightarrow K_S^0 \pi^+ \pi^-$
 - Yields lower than $B^\pm \rightarrow DK^\pm$ but $r_{B^0} \approx 3r_B$
 - Model dependent uses model from BaBar (focus of this talk)
 - Model independent takes input from CLEO
 - Totally consistent results with comparable uncertainties



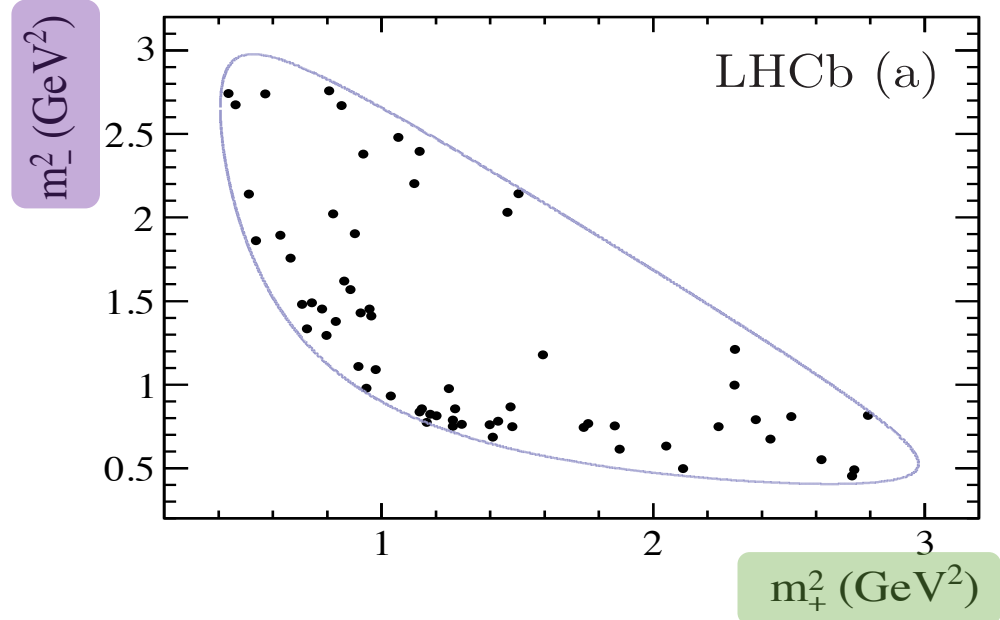
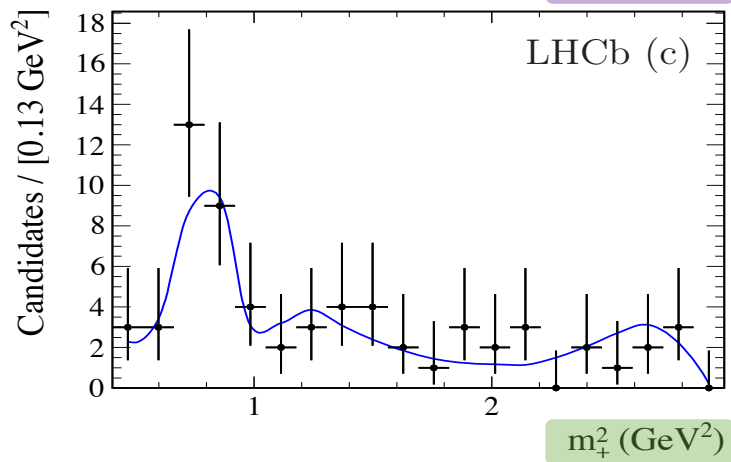
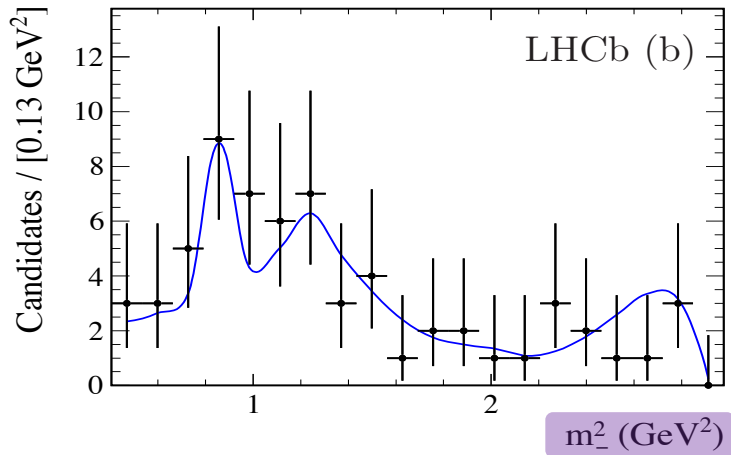
GGSZ analysis

- Analysis strategy (model dependent)
 - First perform a fit to the B mass
 - Candidates within the black lines used in the CP fit
 - Fit the D Dalitz plot with the BaBar model
 - Note we couldn't use the Belle model

89 ± 11 events



GGSZ analysis



$$m_+^2 \equiv m^2(K_S^0 \pi^+)$$

$$m_-^2 \equiv m^2(K_S^0 \pi^-)$$

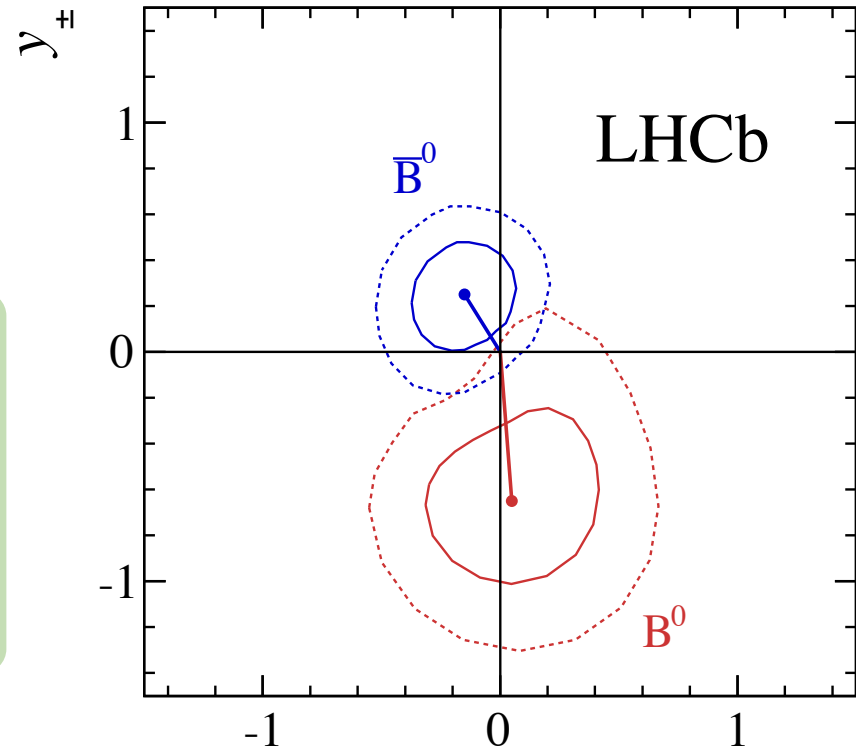
B^0 plots

GGSZ analysis

- Model dependent results
 - Extract cartesian parameters from CP fit
 - No CPV at 2 sigma
 - 3rd uncertainty from DP model
 - Stat uncertainties dominate

$$\begin{aligned}x_- &= -0.15 \pm 0.14 \pm 0.03 \pm 0.01, \\y_- &= 0.25 \pm 0.15 \pm 0.06 \pm 0.01, \\x_+ &= 0.05 \pm 0.24 \pm 0.04 \pm 0.01, \\y_+ &= -0.65^{+0.24}_{-0.23} \pm 0.08 \pm 0.01,\end{aligned}$$

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma) \quad y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$



- Several new or updated inputs into the combination

- Two body GLW/ADS

$$B^{\pm} \rightarrow DK^{\pm}, D \rightarrow h^{+}h'^{-}$$

- Four body GLW/ADS

$$B^{\pm} \rightarrow DK^{\pm}, D \rightarrow h^{+}\pi^{-}\pi^{+}\pi^{-}$$

- GGSZ

$$B^0 \rightarrow DK^{*0}, D \rightarrow K_S^0\pi^{+}\pi^{-}$$

- GLW-Dalitz

$$B^0 \rightarrow DK^{+}\pi^{-}, D \rightarrow h^{+}h^{-}$$

GLW-Dalitz analysis

- Measure γ using $B^0 \rightarrow DK^+\pi^-$ decays

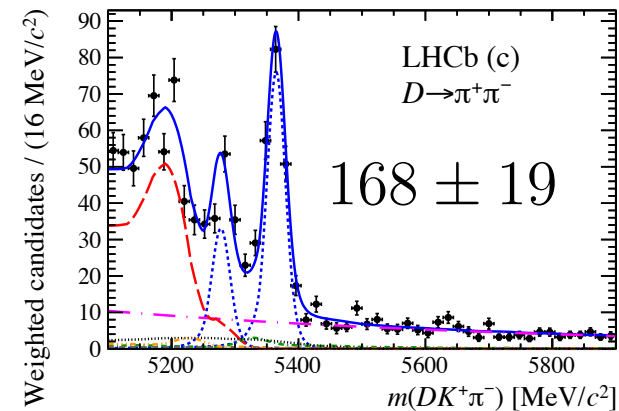
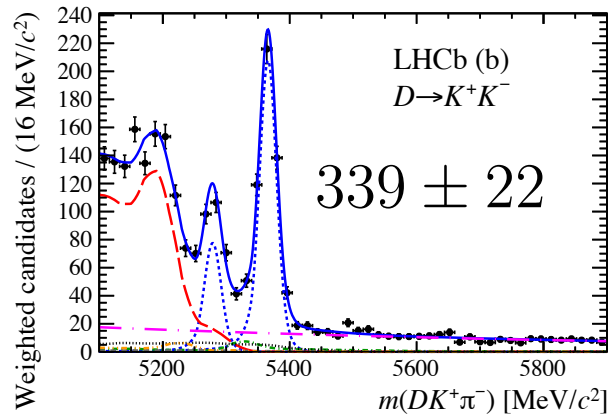
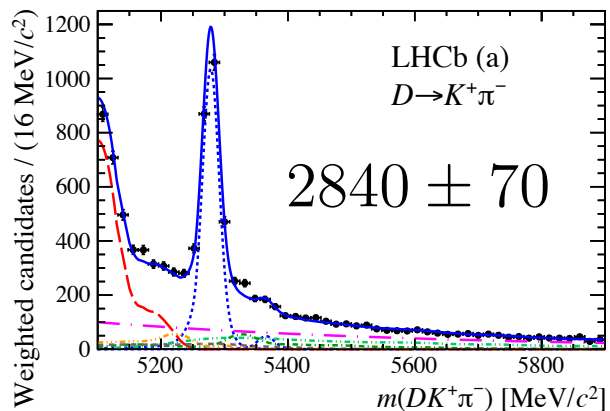
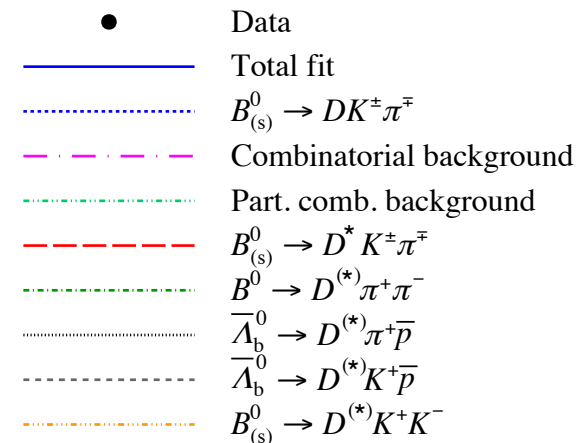
- Use CP even modes $D \rightarrow K^+K^-, \pi^+\pi^-$

- Bin in output of the neural network

- Simultaneous DP fit

- Purity of the GLW modes hurt by

$B_s^0 \rightarrow D^{(*)}K^-\pi^+$ contributions

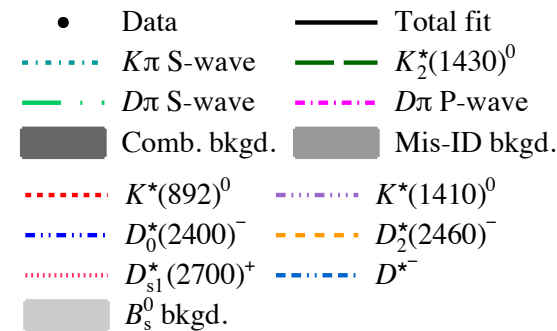
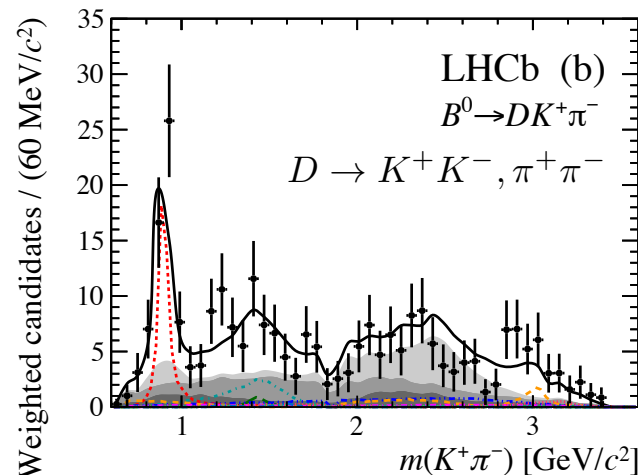
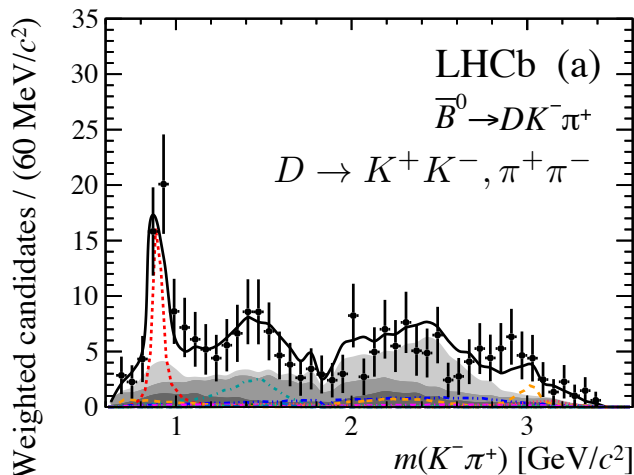


GLW-Dalitz analysis

- Amplitude fit based on previous results (see backups)
 - First time this method has been used
 - Use to $D \rightarrow K^+ \pi^-$ help guide the fit – no CPV allowed
 - Include CPV for $K^*(892)$ amplitude

$$\mathcal{A}(m^2(D\pi^-), m^2(K^+\pi^-)) = \sum_{j=1}^N c_j F_j(m^2(D\pi^-), m^2(K^+\pi^-))$$

$c_j \rightarrow \begin{cases} c_j & \text{for a } D\pi^- \text{ resonance,} \\ c_j [1 + x_{\pm,j} + iy_{\pm,j}] & \text{for a } K^+\pi^- \text{ resonance} \end{cases}$



GLW-Dalitz analysis

- Cartesian parameters reported

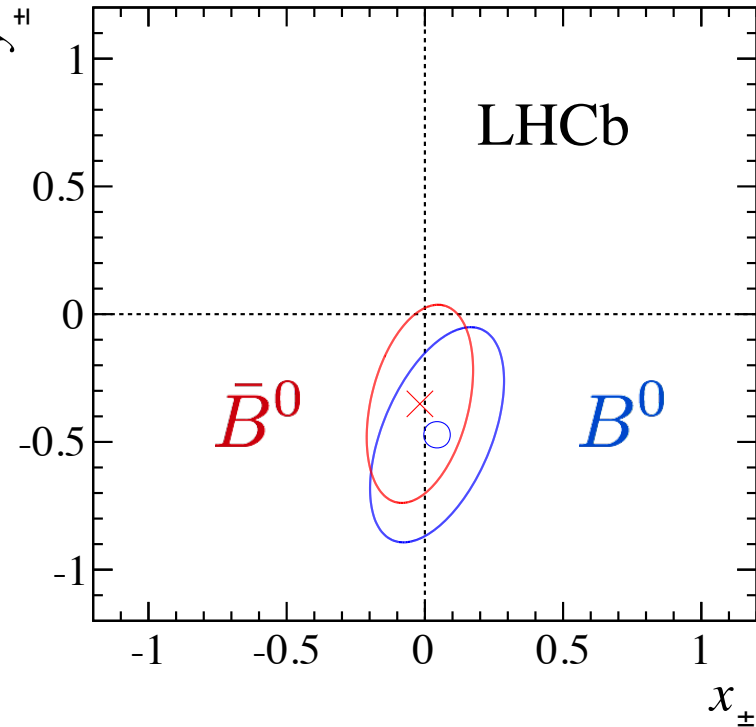
- Consistent with no CPV
- Statistics the biggest factor

$$x_+ = 0.04 \pm 0.16 \pm 0.11$$

$$x_- = -0.02 \pm 0.13 \pm 0.14$$

$$y_+ = -0.47 \pm 0.28 \pm 0.22$$

$$y_- = -0.35 \pm 0.26 \pm 0.41$$



- Report quasi-two-body values

- Help combine with $B^0 \rightarrow DK^{*0}$ results
- Coherence factor, relative magnitudes and strong phases

$$\kappa = 0.958^{+0.005}_{-0.010} {}^{+0.002}_{-0.045},$$

$$\bar{R}_B = 1.02^{+0.03}_{-0.01} \pm 0.06,$$

$$\Delta\bar{\delta}_B = 0.02^{+0.03}_{-0.02} \pm 0.11$$

- Several new or updated inputs into the combination

- Two body GLW/ADS

$$B^{\pm} \rightarrow DK^{\pm}, D \rightarrow h^+ h'^-$$

- Four body GLW/ADS

Update of LHCb γ combination

$$B^{\pm} \rightarrow DK^{\pm}, D \rightarrow h^+ \pi^- \pi^+ \pi^-$$

- GGSZ of $B \rightarrow DK$ like decays

$$B^0 \rightarrow DK^{*0}, D \rightarrow K_S^0 \pi^+ \pi^-$$

- GLW-Dalitz

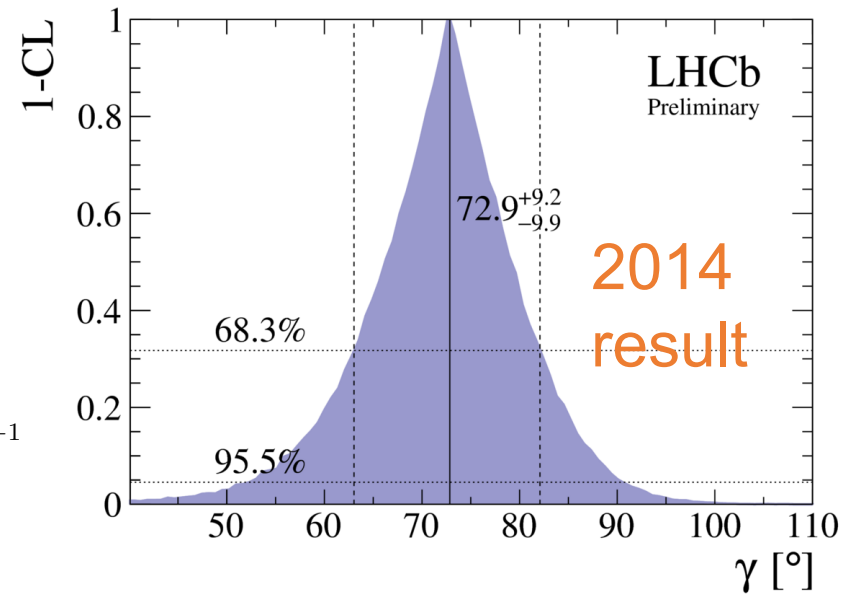
$$B^0 \rightarrow DK^+ \pi^-, D \rightarrow h^+ h^-$$

DK Combination

- Best knowledge from combining all LHCb analyses

- Previous result from 2014
- Only $B \rightarrow DK$ like decays

- $B^+ \rightarrow DK^+$, $D \rightarrow h^+h^-$, GLW/ADS, 3 fb^{-1}
- $B^+ \rightarrow DK^+$, $D \rightarrow h^+\pi^-\pi^+\pi^-$, quasi-GLW/ADS, 3 fb^{-1}
- $B^+ \rightarrow DK^+$, $D \rightarrow h^+h^-\pi^0$, quasi-GLW/ADS, 3 fb^{-1}
- $B^+ \rightarrow DK^+$, $D \rightarrow K_s^0 h^+h^-$, model-independent GGSZ, 3 fb^{-1}
- $B^+ \rightarrow DK^+$, $D \rightarrow K_s^0 K^+\pi^-$, GLS, 3 fb^{-1}
- $B^0 \rightarrow DK^+\pi^-$, $D \rightarrow h^+h^-$, GLW-Dalitz, 3 fb^{-1}
- $B^0 \rightarrow DK^{*0}$, $D \rightarrow K^+\pi^-$, ADS, 3 fb^{-1}
- $B^0 \rightarrow DK^{*0}$, $D \rightarrow K_s^0 \pi^+\pi^-$, model-dependent GGSZ, 3 fb^{-1}
- $B^+ \rightarrow DK^+\pi^+\pi^-$, $D \rightarrow h^+h^-$, GLW/ADS, 3 fb^{-1}
- $B_s^0 \rightarrow D_s^\mp K^\pm$, time-dependent, 1 fb^{-1}



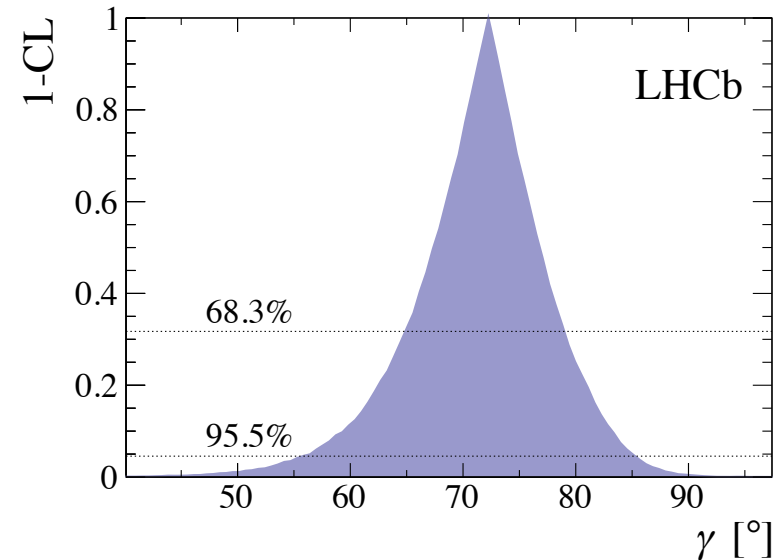
New or updated since
the above result

Work in progress: 3 fb^{-1}

DK Combination

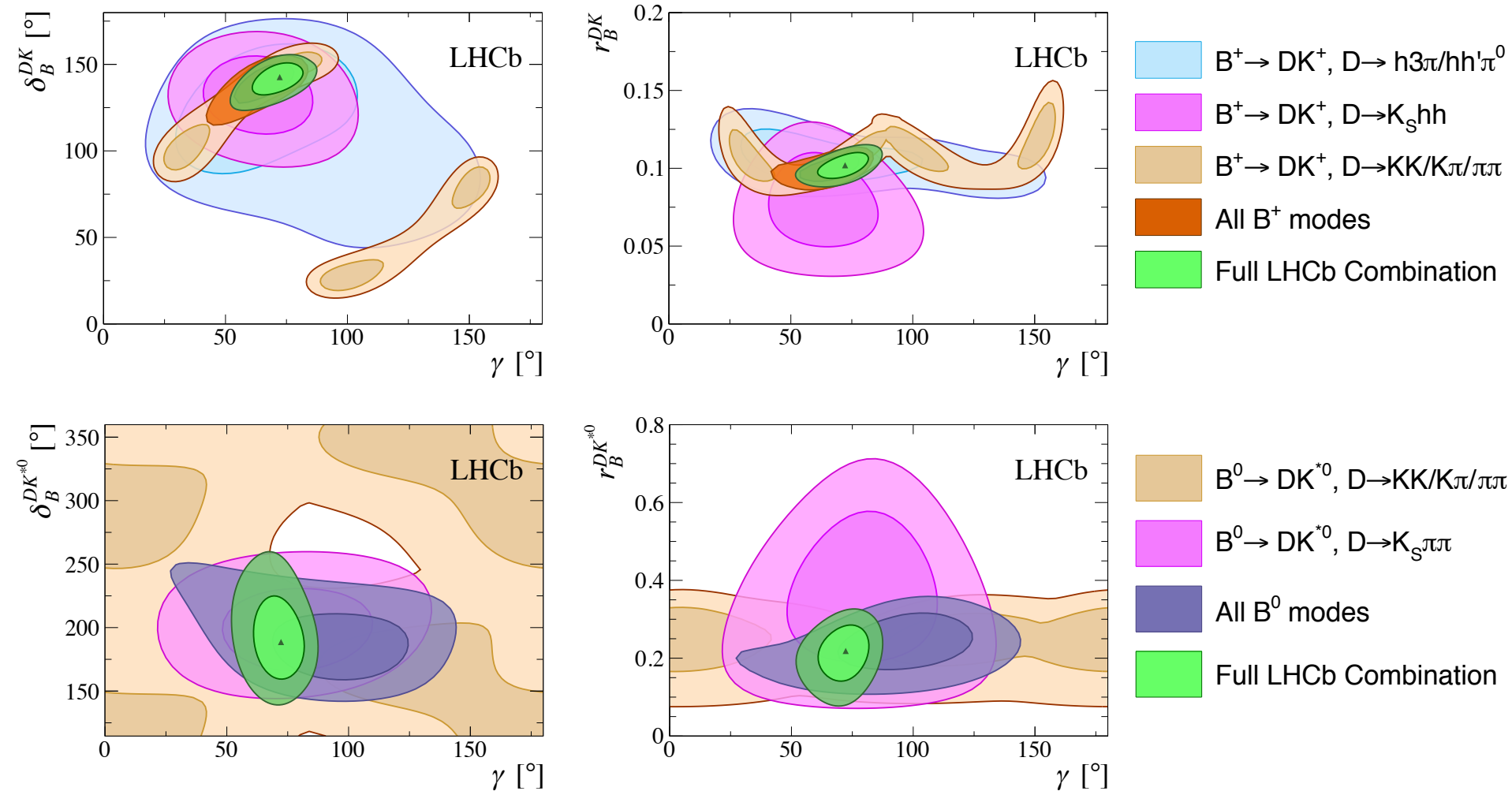
- New DK combination
 - Frequentist results here
 - Bayesian results agree well
 - Coverage is good

$$\gamma = (72.2^{+6.8}_{-7.3})^\circ$$



Observable	Central value	68.3% Interval	95.5% Interval	99.7% Interval
γ ($^\circ$)	72.2	[64.9, 79.0]	[55.9, 85.2]	[43.7, 90.9]
r_B^{DK}	0.1019	[0.0963, 0.1075]	[0.0907, 0.1128]	[0.0849, 0.1182]
δ_B^{DK} ($^\circ$)	142.6	[136.0, 148.3]	[127.8, 153.6]	[116.2, 158.7]
$r_B^{DK^{*0}}$	0.218	[0.171, 0.263]	[0.118, 0.305]	[0.000, 0.348]
$\delta_B^{DK^{*0}}$ ($^\circ$)	189	[169, 212]	[148, 241]	[123, 283]

DK Combination



DK Combination

- Improvement on 2014 result
 - Around 2 or 3 degrees more precise

$$\gamma \in [64.9, 79.0]^\circ \text{ at } 68.3\% \text{ CL}$$

$$\gamma \in [55.9, 85.2]^\circ \text{ at } 95.5\% \text{ CL}$$

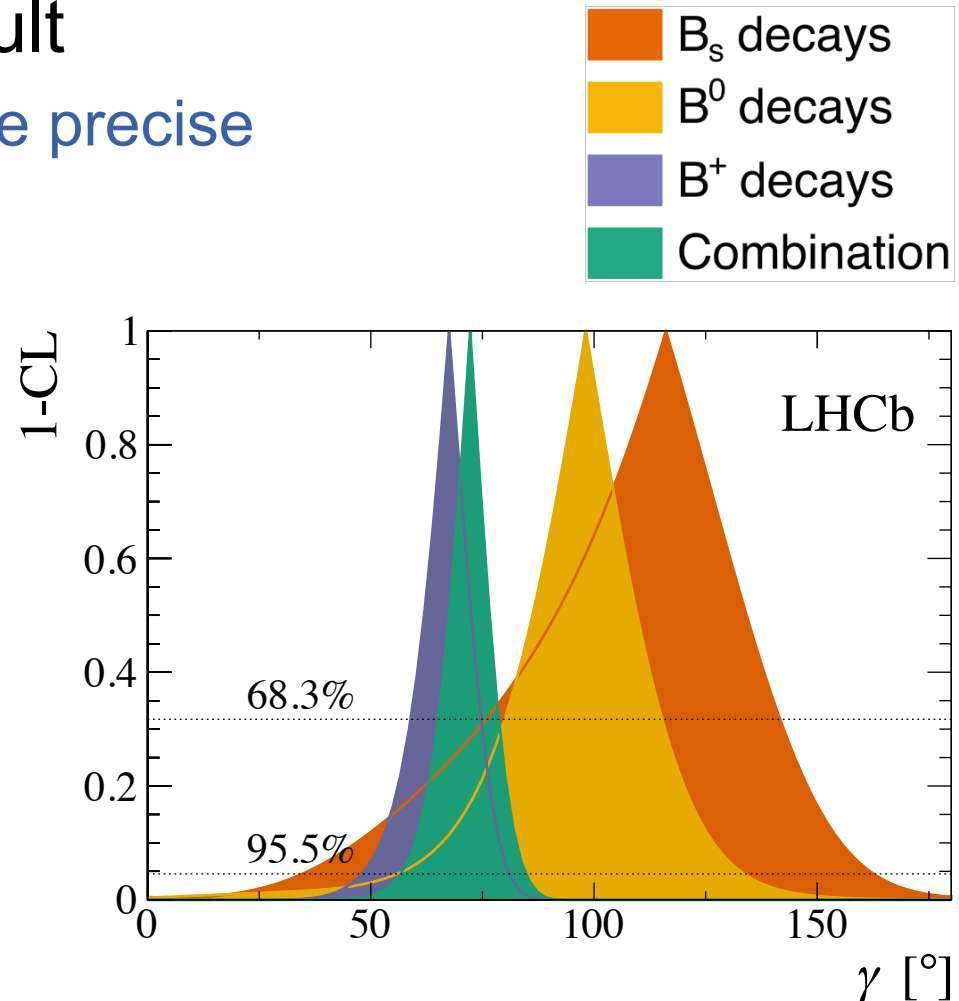
$$\gamma = (72.2^{+6.8}_{-7.3})^\circ$$

- Still more from Run I?

$$B_s^0 \rightarrow D_s^\mp K^\pm$$

$$B^\pm \rightarrow D^* K^\pm$$

$$B^\pm \rightarrow DK^{*\pm}$$



- Several new or updated inputs into the combination

- Two body GLW/ADS

$$B^\pm \rightarrow DK^\pm, D \rightarrow h^+ h'^-$$

- Four body GLW/ADS

Update of LHCb γ combination

$$B^\pm \rightarrow DK^\pm, D \rightarrow h^+ \pi^- \pi^+ \pi^-$$

- GGSZ

of $B \rightarrow Dh$ like decays

$$B^0 \rightarrow DK^{*0}, D \rightarrow K_S^0 \pi^+ \pi^-$$

- GLW-Dalitz

$$B^0 \rightarrow DK^+ \pi^-, D \rightarrow h^+ h^-$$

Dh Combination

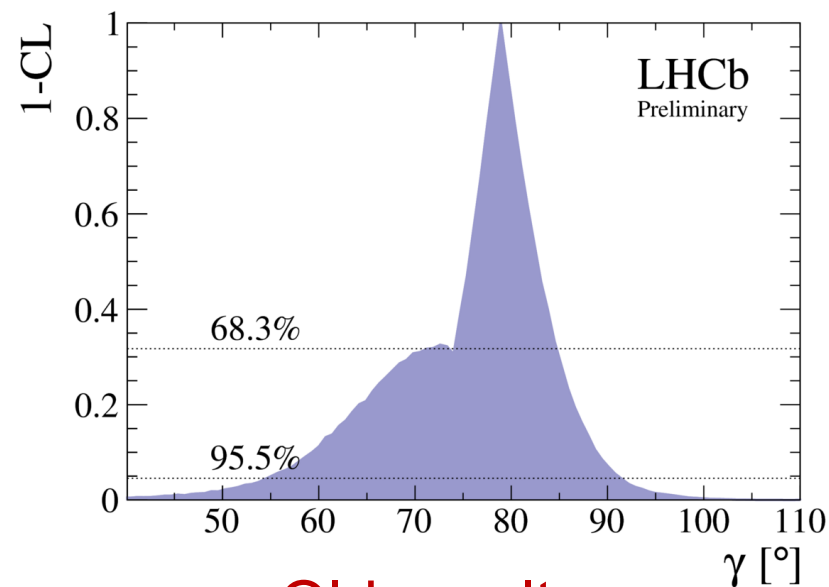
- Try to squeeze sensitivity from $B \rightarrow D\pi$ like decays
 - Previous result from 2014
 - Several additional inputs w.r.t. the DK only combination

$$B^+ \rightarrow D\pi, \quad D \rightarrow h^+ h^-$$

$$B^+ \rightarrow D\pi, \quad D \rightarrow h^+ \pi^- \pi^+ \pi^-$$

$$B^+ \rightarrow D\pi, \quad D \rightarrow h^+ h^- \pi^0$$

$$B^+ \rightarrow D\pi\pi\pi, \quad D \rightarrow h^+ h^-$$



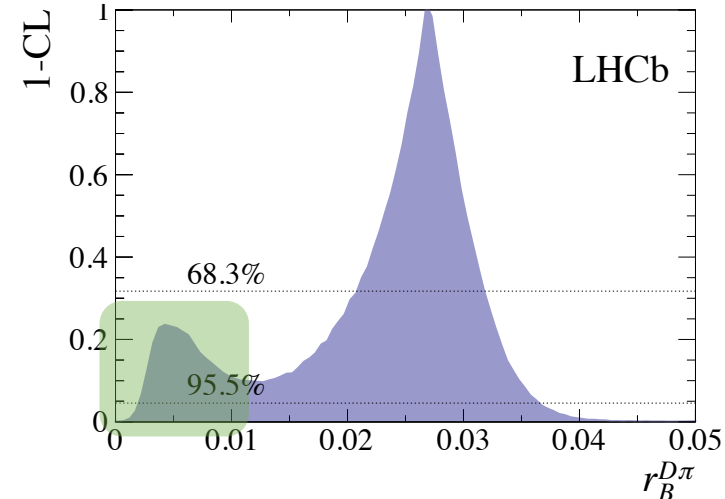
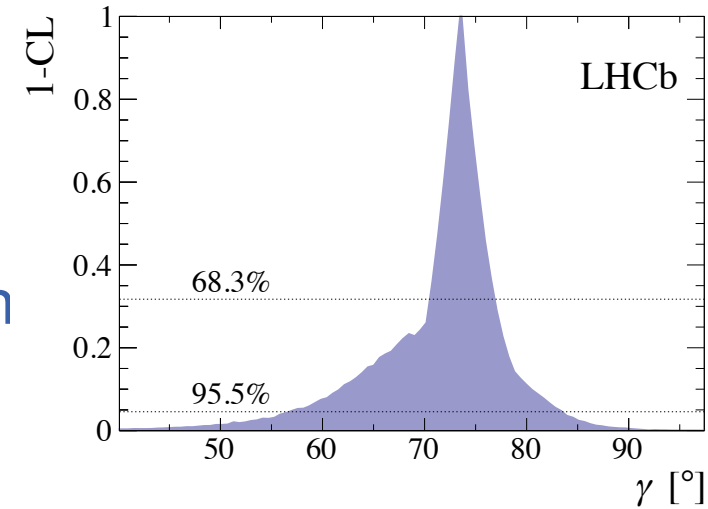
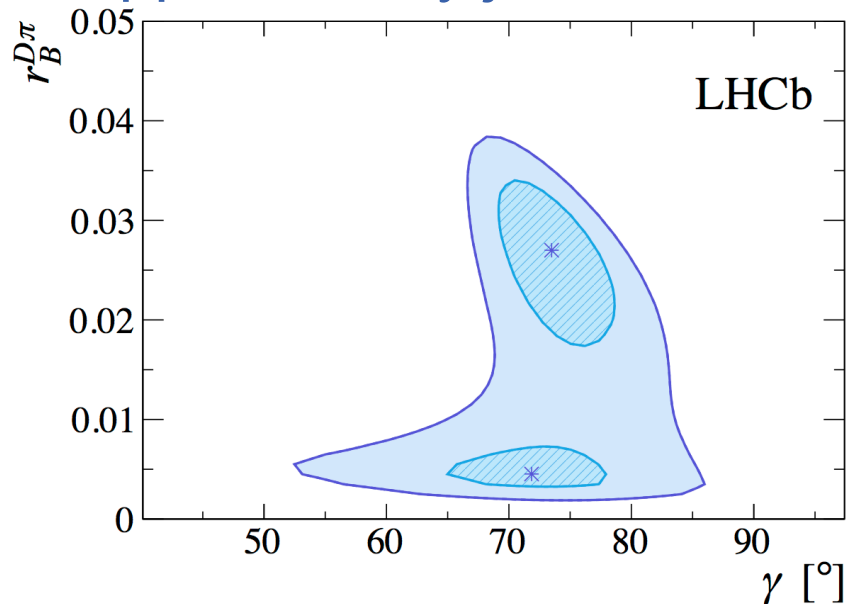
Old result

- Observables: 71 \rightarrow 89 and parameters 32 \rightarrow 38

Dh Combination

- New Dh combination

- Frequentist results here
- Two solutions observed in $r_B^{D\pi}$,
low solution agrees with expectation
suppressed by just over 1 sigma



Dh Combination

Observable	Central value	68.3% Interval	95.5% Interval	99.7% Interval
γ ($^\circ$)	73.5	[70.5, 76.8]	[56.7, 83.4]	[40.1, 90.8]
r_B^{DK}	0.1017	[0.0970, 0.1064]	[0.0914, 0.1110]	[0.0844, 0.1163]
δ_B^{DK} ($^\circ$)	141.6	[136.6, 146.3]	[127.2, 151.1]	[114.6, 155.7]
$r_B^{DK^*0}$	0.220	[0.173, 0.264]	[0.121, 0.307]	[0.000, 0.355]
$\delta_B^{DK^*0}$ ($^\circ$)	188	[168, 211]	[148, 239]	[120, 280]
$r_B^{D\pi}$	0.027	[0.0207, 0.0318]	[0.0020, 0.0365]	[0.0008, 0.0425]
$\delta_B^{D\pi}$ ($^\circ$)	348.3	[343.2, 352.9]	[220.5, 356.4]	[192.9, 359.8]

Bayesian combination favours low $r_B^{D\pi}$ solution instead,
but everything is consistent at the 2 sigma level

Dh Combination

Observable	Central value	68.3% Interval	95.5% Interval	99.7% Interval
γ ($^\circ$)	73.5	[70.5, 76.8]	[56.7, 83.4]	[40.1, 90.8]
r_B^{DK}	0.1017	[0.0970, 0.1064]	[0.0914, 0.1110]	[0.0844, 0.1163]
δ_B^{DK} ($^\circ$)	141.6	[136.6, 146.3]	[127.2, 151.1]	[114.6, 155.7]
r_B^{DK*0}	0.220	[0.173, 0.264]	[0.121, 0.307]	[0.000, 0.355]
δ_B^{DK*0} ($^\circ$)	100	[100, 111]	[100, 120]	[100, 130]

Such differences are not uncommon in the presence of a highly non-Gaussian likelihood function

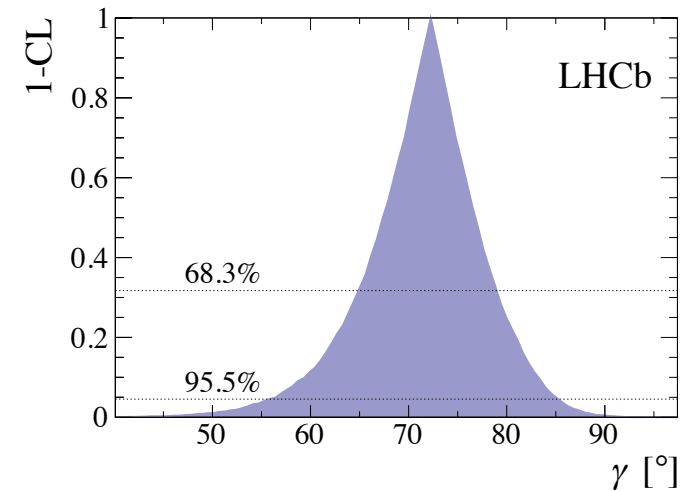
Note that both frameworks find the same χ^2 minima

Summary

- New inputs to the combinations
 - New decay modes and methods used for the first time
 - Both DK only and Dh combinations performed
 - Reached the expected Run I sensitivity

$$\gamma = (72.2^{+6.8}_{-7.3})^\circ$$

- Time for a rest?
 - Plenty still to do!
 - New decay modes and updates from Run I
 - Should more than double the data sample with Run II



Outlook

- What comes next?

- More channels for both B and D decays from Run I and II

$$B^+ \rightarrow D^{*0} K^+$$

$$B^+ \rightarrow D^0 K^{*+}$$

$$B^0 \rightarrow D^{\mp} \pi^{\pm}$$

$$B_s^0 \rightarrow D_s^{\mp} K^{\pm}$$

$$B_s^0 \rightarrow D_s^{*\mp} K^{\pm}$$

$$D \rightarrow K K \pi \pi$$

$$D \rightarrow K_S^0 \pi \pi \pi^0$$

All are being investigated, some in preparation for CKM 2016

Outlook

- Looking deeper into the crystal ball

Sample	\mathcal{L} (fb ⁻¹)	Units of Run I
Run I	3	1
Run II	5	3
Upgrade	≈ 50	≈ 60
Future upgrade	≈ 300	≈ 360

- Beauty cross section up by more than a factor 2 and small increases in trigger and selection efficiencies
- Hadronic trigger efficiency should roughly double

Outlook

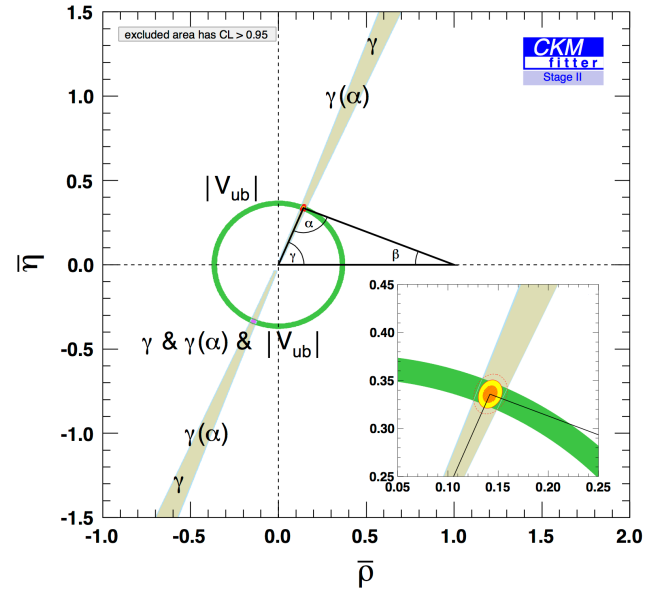
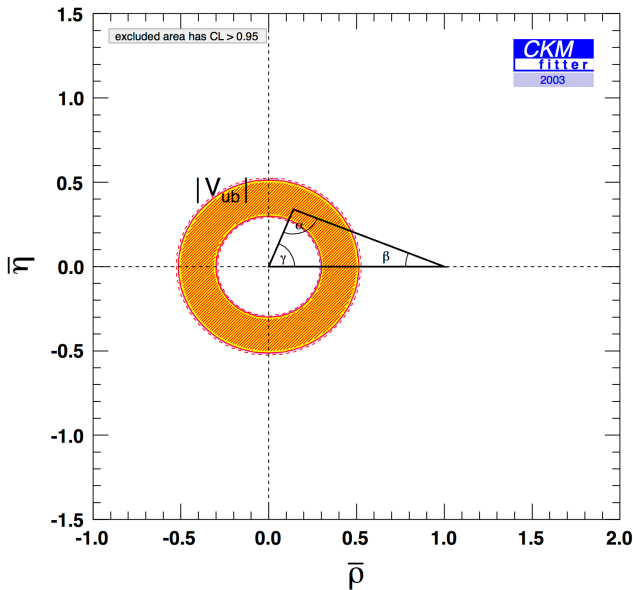
- Target sub-degree precision
 - Indirect measurements give γ to $(+1.0)^\circ$
 $(-3.7)^\circ$
 - Lattice improvements will decrease the uncertainties
 - Expect statistical uncertainties to scale with data samples
 - Systematic uncertainties should also decrease

Sample	$\sigma_{\text{stat}} (^\circ)$
Run I	8
Run II	4
Upgrade	≈ 1
Future upgrade	< 0.5

- Anticipate similar precision to Belle II in upgrade era

Conclusion

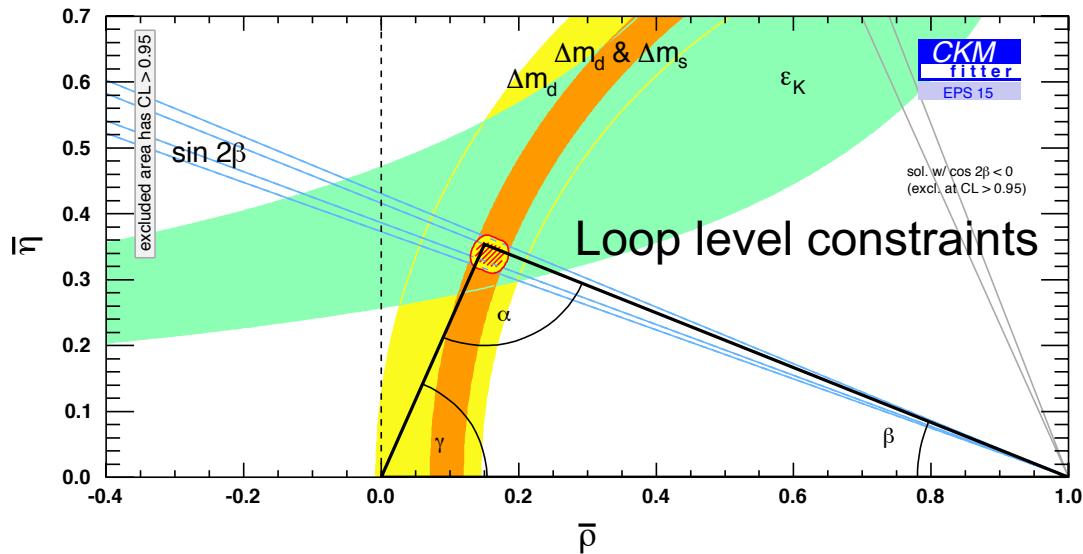
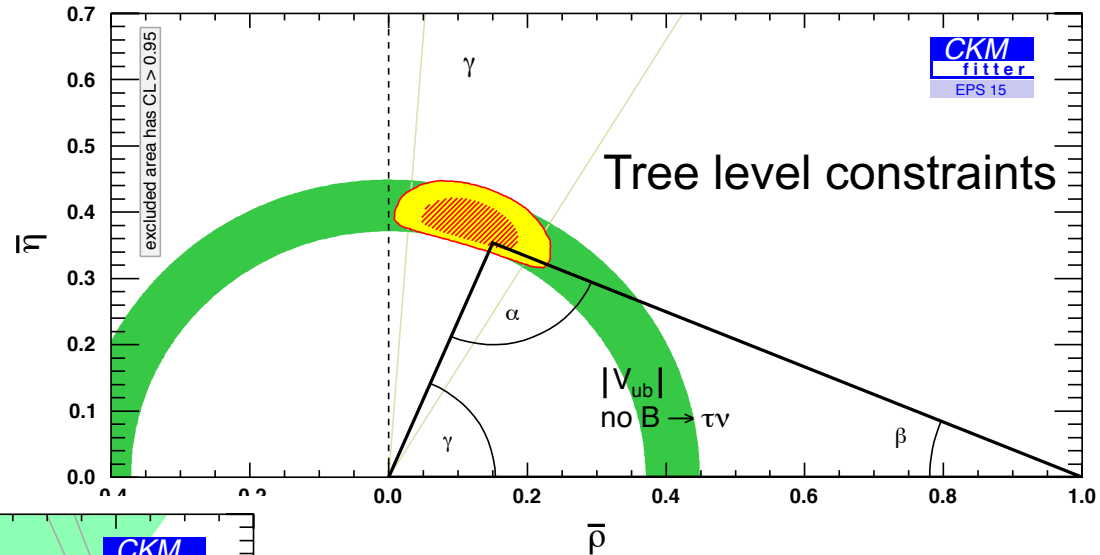
- LHCb has made a big impact measuring γ
 - Hope to halve our uncertainties in Run II
 - Things should get exciting in the upgrade era (2021 ->)
 - We look forward to healthy competition with Belle II
 - Many areas where we can complement each other



Backups

CKM picture

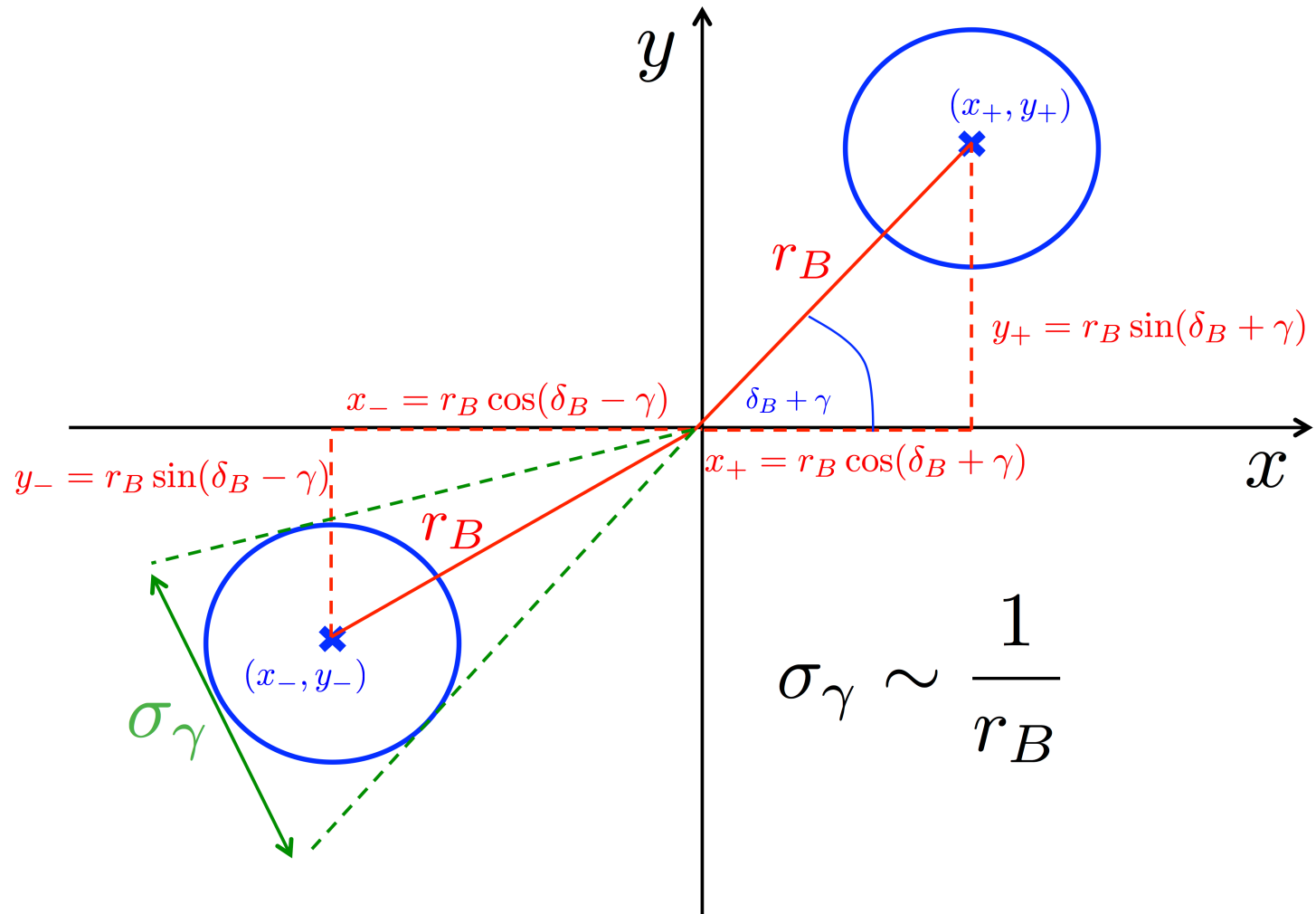
Room for improvement!



Gammacombo

- Framework available for anyone to use
 - See HEP Forge for more details
 - <http://gammacombo.hepforge.org/web/HTML/index.html>
- Used for the combinations in this talk
 - Frequentist treatment

Reading x and y

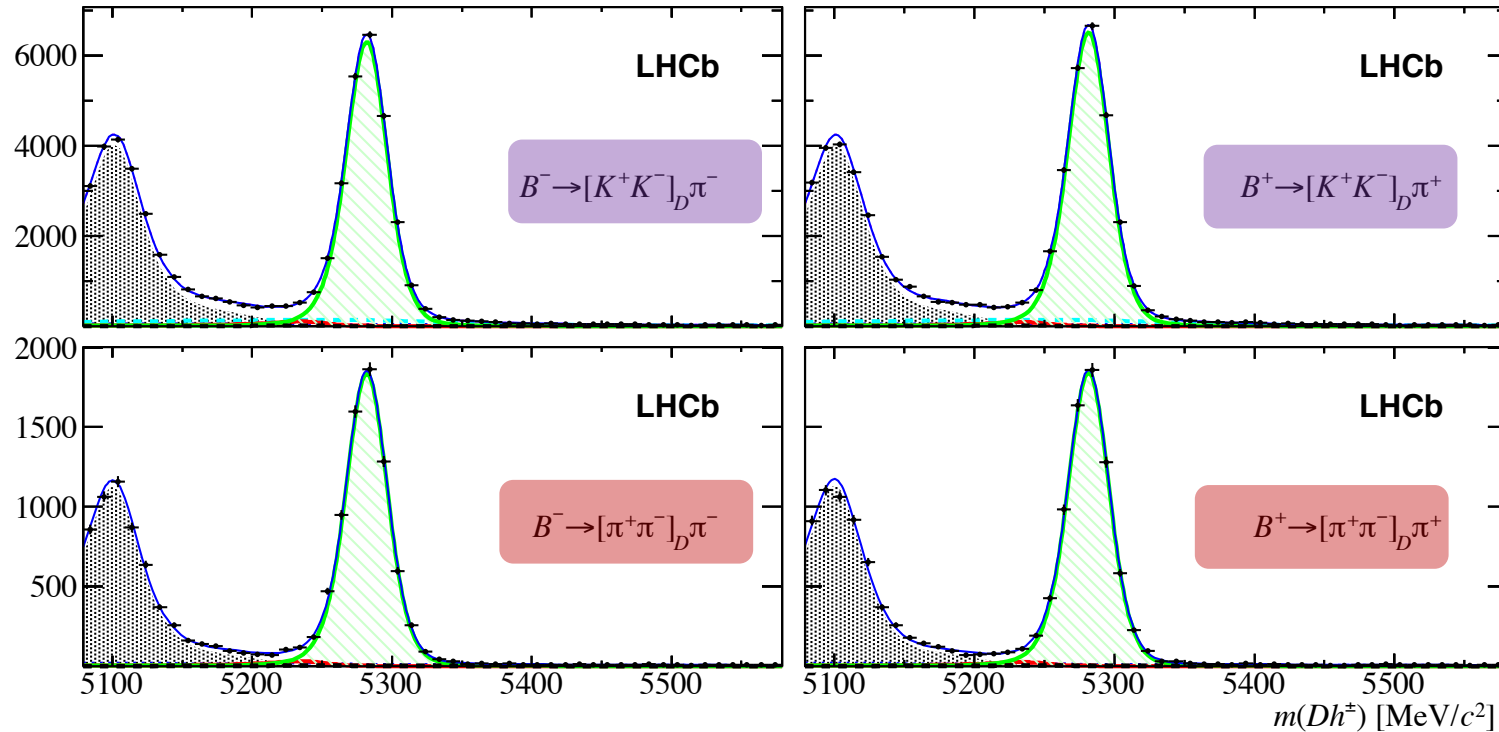


2 body GLW modes

- Not much to see...

$$B^\pm \rightarrow [K^+K^-]_D \pi^\pm \quad 50,140 \pm 270$$

$$B^\pm \rightarrow [\pi^+\pi^-]_D \pi^\pm \quad 14,680 \pm 130$$



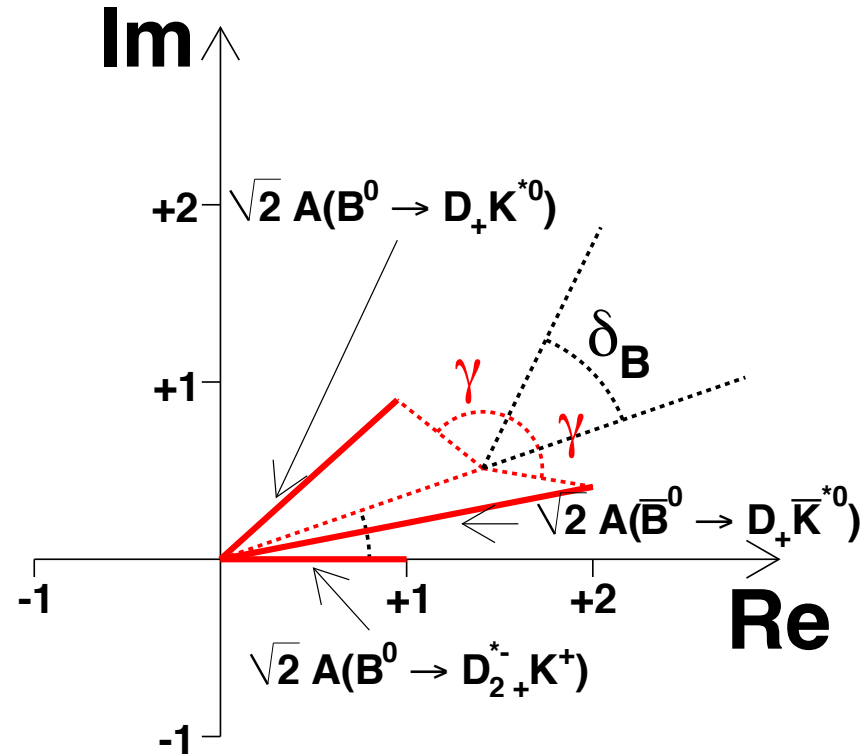
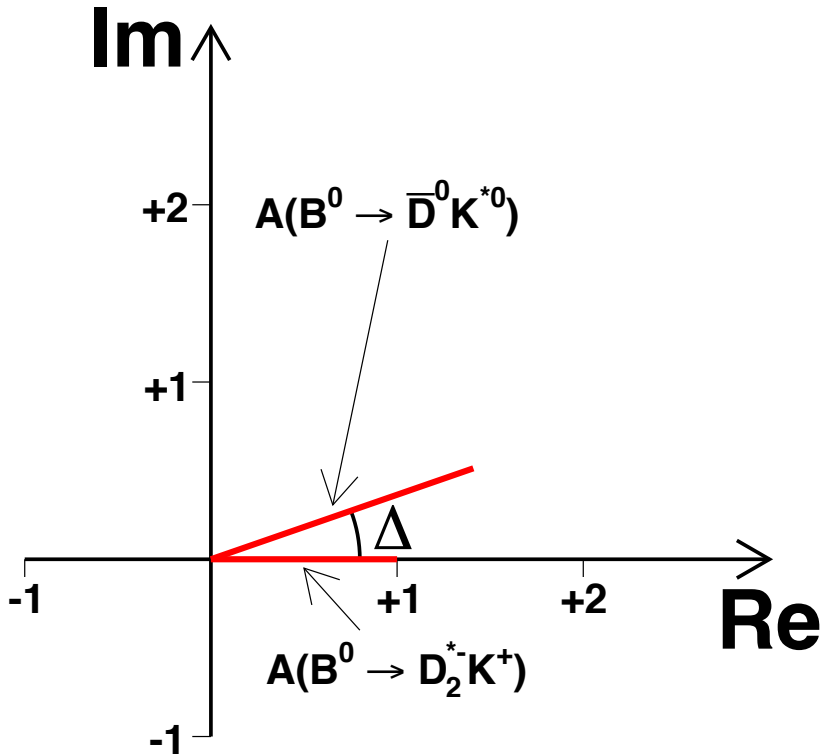
$$A_\pi^{KK} = -0.0145 \pm 0.0050 \pm 0.0017 \quad A_\pi^{\pi\pi} = 0.0043 \pm 0.0086 \pm 0.0031$$

GLW-Dalitz

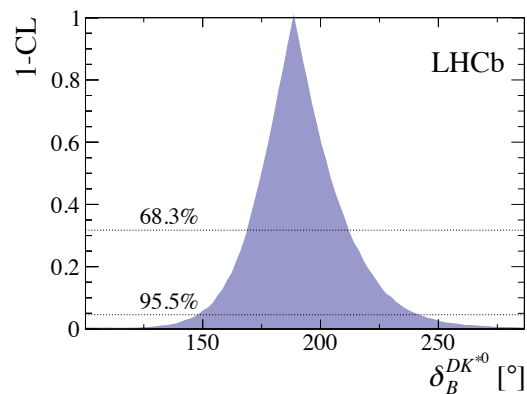
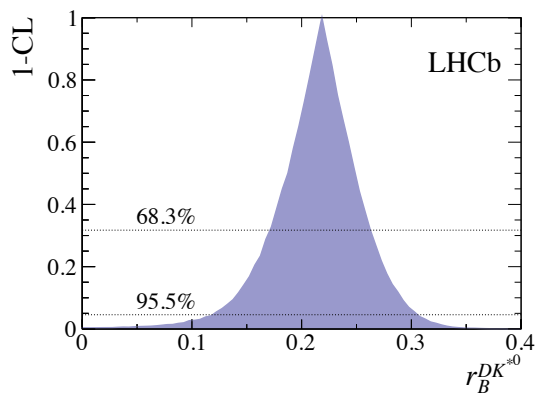
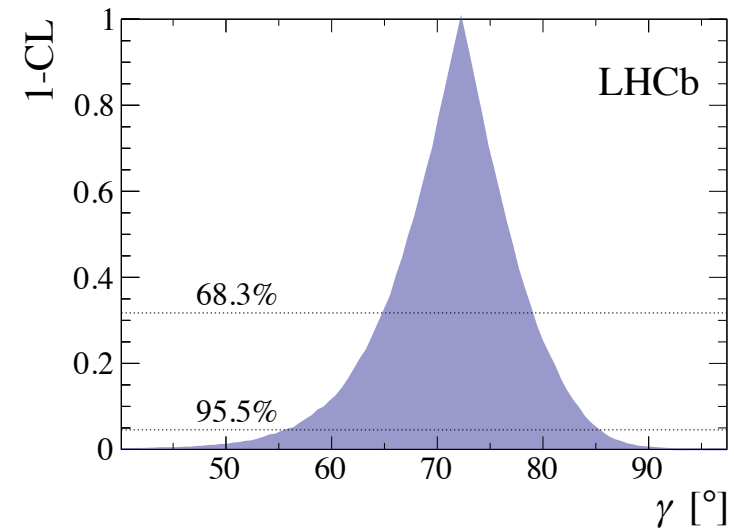
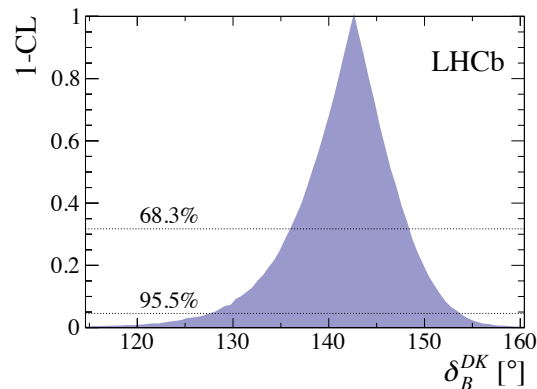
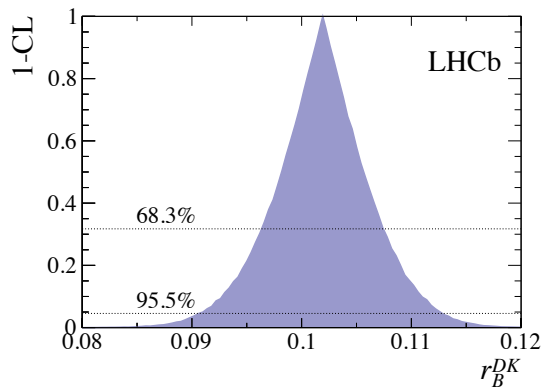
Table 8: Results for the complex coefficients c_j from the fit to data. Uncertainties are statistical only. All reported quantities are free to vary in the fit, except that the $D_2^*(2460)^-$ component is fixed as a reference amplitude, and the magnitude of the $D_{s1}^*(2700)^+$ component is constrained. The $K^+\pi^-$ S-wave is the coherent sum of the $K_0^*(1430)^0$ and the nonresonant $K\pi$ S-wave component [45].

Resonance	Real part	Imaginary part
$K^*(892)^0$	-0.07 ± 0.10	-1.19 ± 0.04
$K^*(1410)^0$	0.16 ± 0.04	0.21 ± 0.06
$K_0^*(1430)^0$	0.40 ± 0.08	0.67 ± 0.06
Nonresonant $K\pi$ S-wave	0.37 ± 0.07	0.69 ± 0.07
$K_2^*(1430)^0$	-0.01 ± 0.06	-0.48 ± 0.04
$D_0^*(2400)^-$	-1.10 ± 0.05	-0.18 ± 0.07
$D_2^*(2460)^-$	1.00	0.00
Nonresonant $D\pi$ S-wave	-0.44 ± 0.06	0.02 ± 0.07
Nonresonant $D\pi$ P-wave	-0.61 ± 0.05	-0.08 ± 0.06
$D_{s1}^*(2700)^+$	0.57 ± 0.05	-0.09 ± 0.19

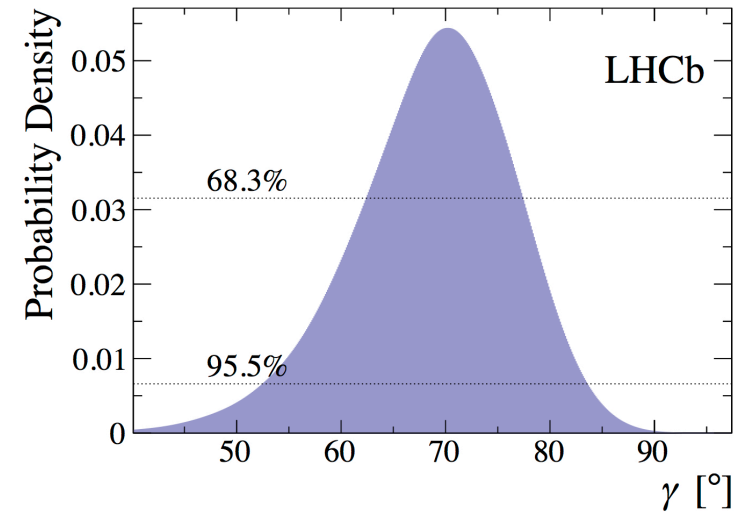
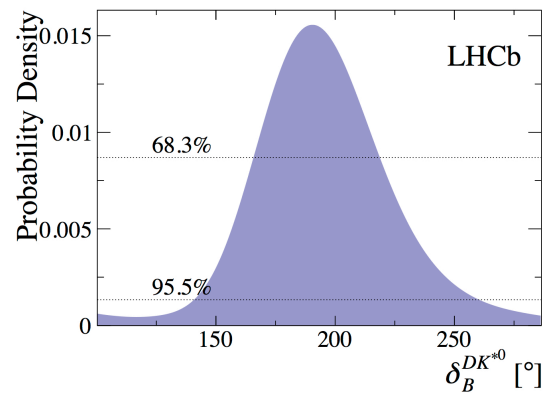
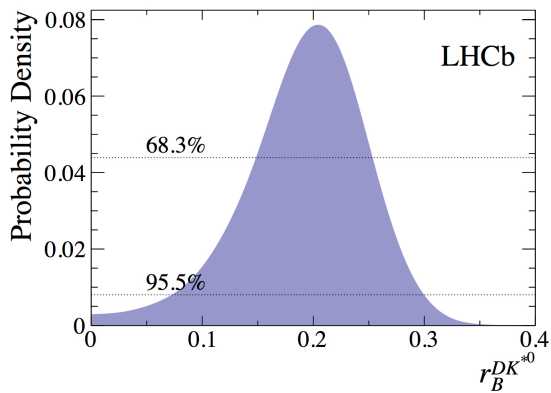
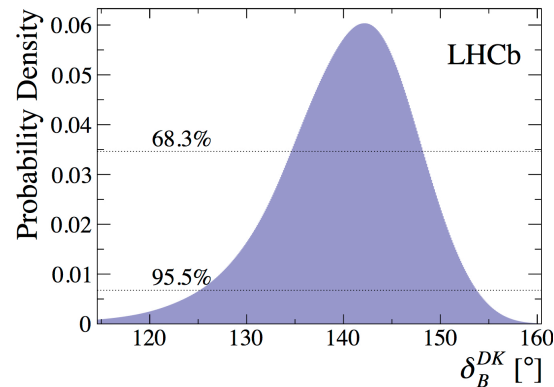
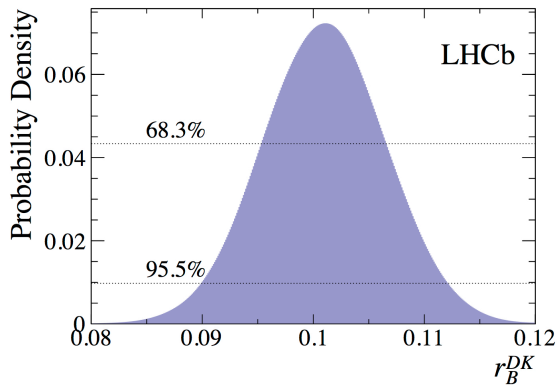
GLW-Dalitz



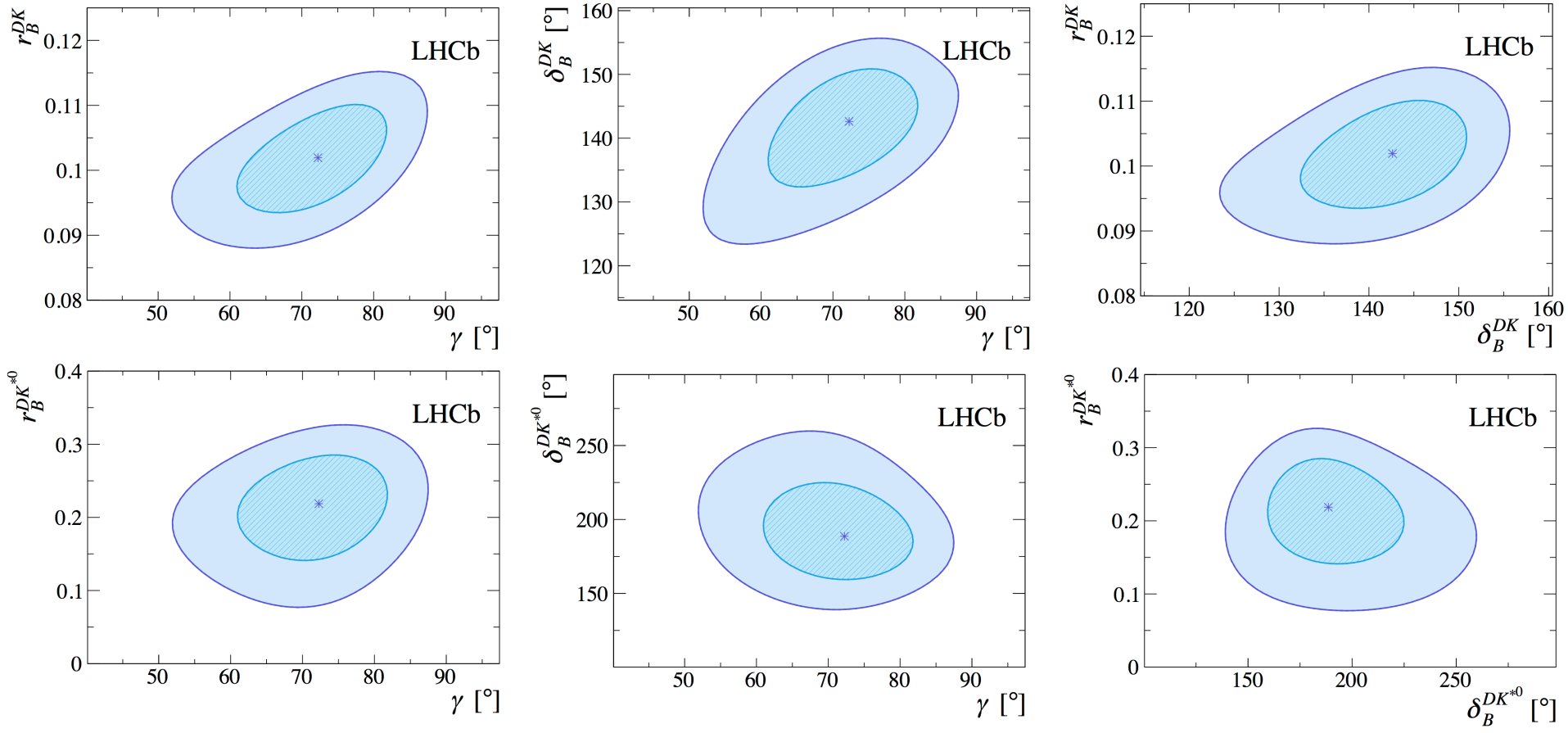
DK combination (frequentist)



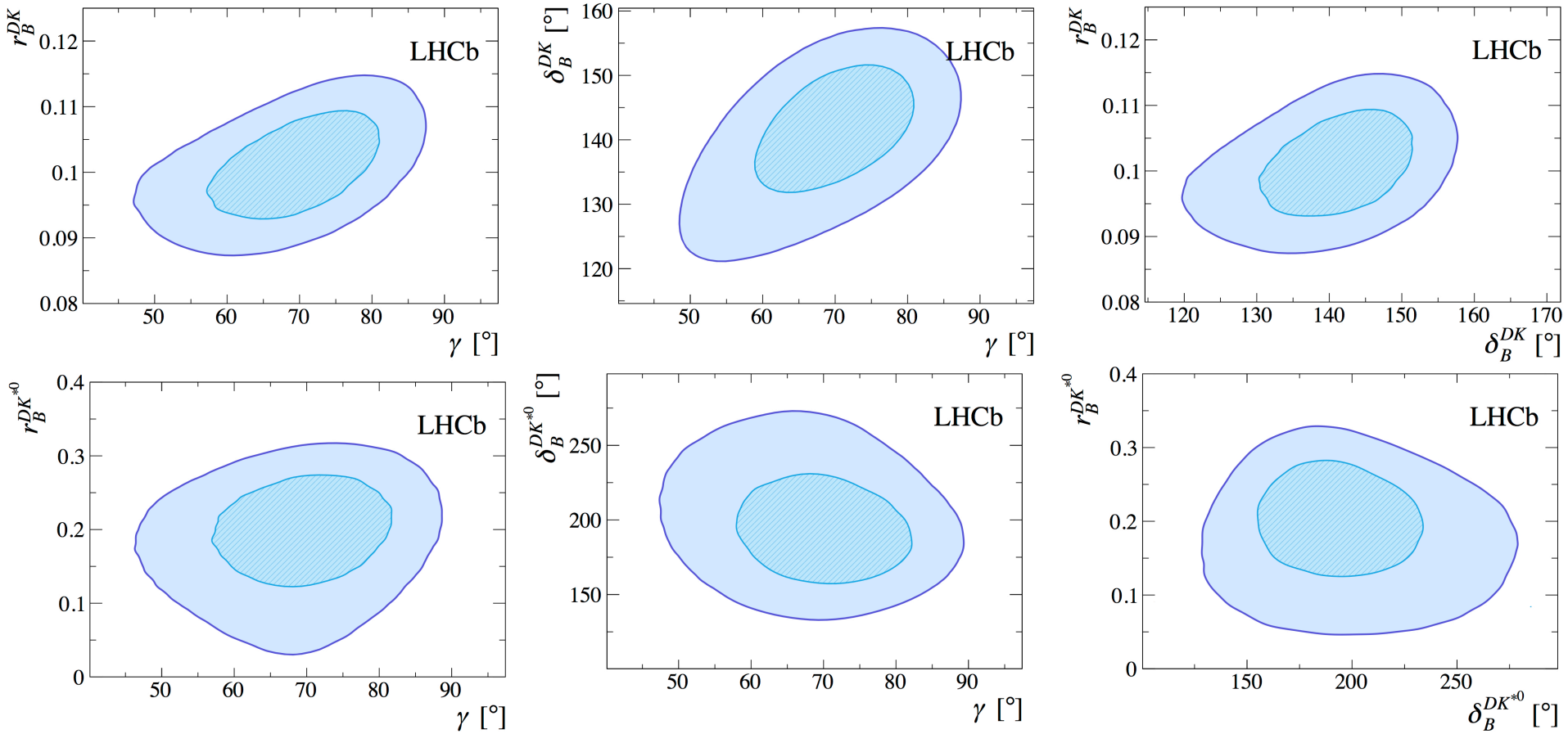
DK combination (Bayesian)



DK combination 2D (frequentist)



DK combination 2D (Bayesian)



DK results summary

Table 3: Confidence intervals and central values for the parameters of interest in the frequentist DK combination.

frequentist

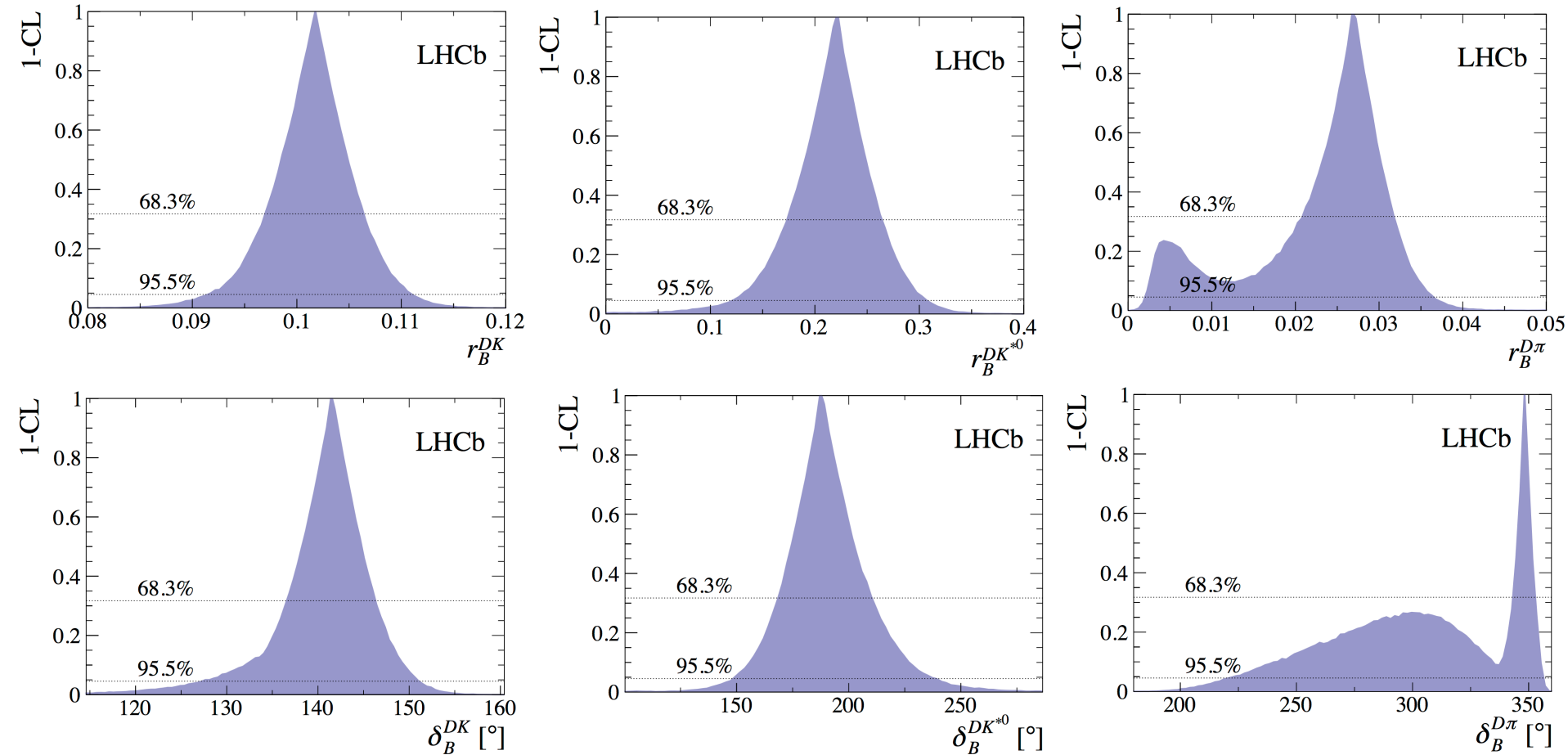
Observable	Central value	68.3% Interval	95.5% Interval	99.7% Interval
$\gamma(^{\circ})$	72.2	[64.9, 79.0]	[55.9, 85.2]	[43.7, 90.9]
r_B^{DK}	0.1019	[0.0963, 0.1075]	[0.0907, 0.1128]	[0.0849, 0.1182]
$\delta_B^{DK}(^{\circ})$	142.6	[136.0, 148.3]	[127.8, 153.6]	[116.2, 158.7]
$r_B^{DK^{*0}}$	0.218	[0.171, 0.263]	[0.118, 0.305]	[0.000, 0.348]
$\delta_B^{DK^{*0}}(^{\circ})$	189	[169, 212]	[148, 241]	[123, 283]

Table 6: Credible intervals and most probable values for the hadronic parameters extracted from the DK Bayesian combination.

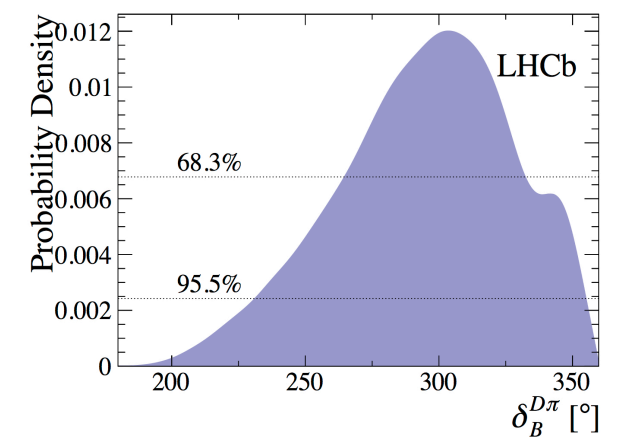
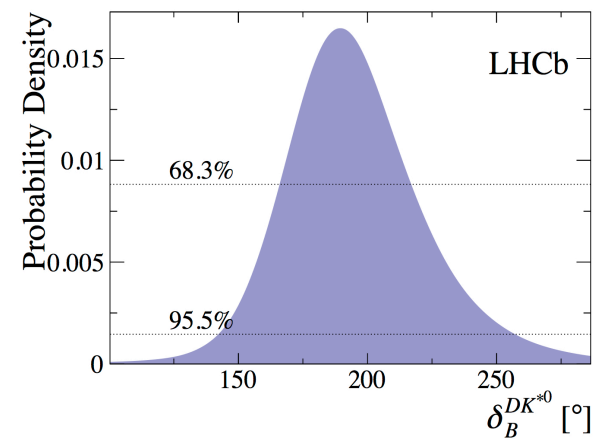
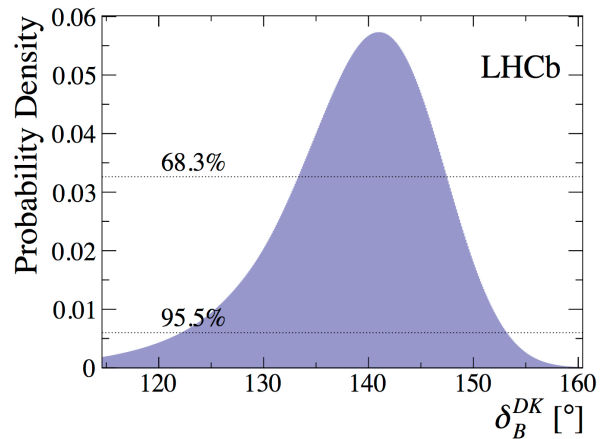
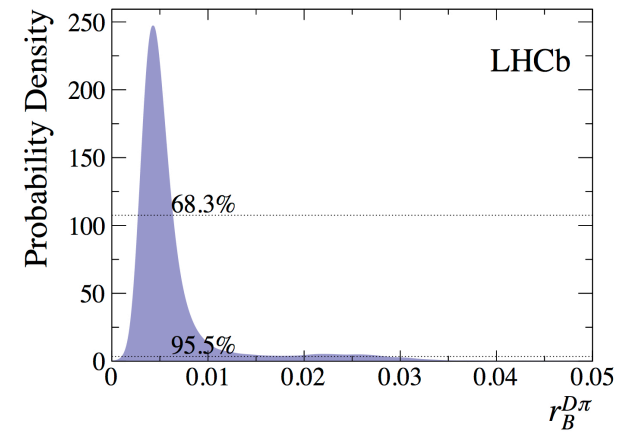
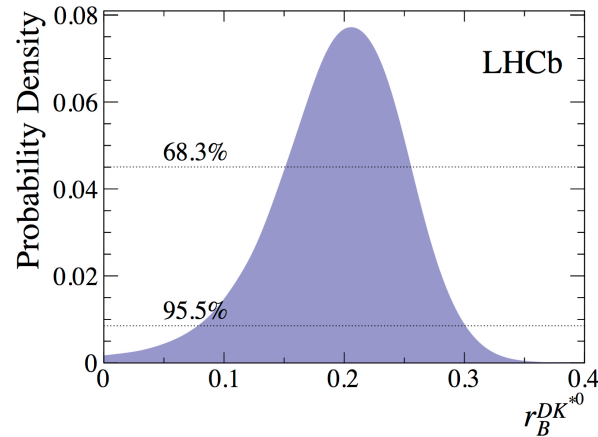
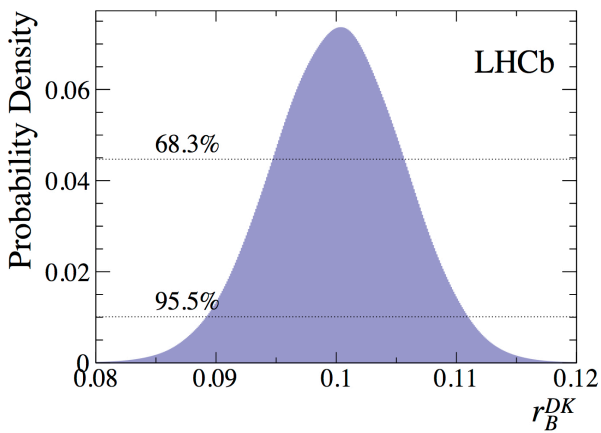
Bayesian

Observable	Central value	68.3% Interval	95.5% Interval	99.7% Interval
$\gamma(^{\circ})$	70.3	[62.4, 77.4]	[52.6, 83.5]	[42.1, 88.4]
r_B^{DK}	0.1012	[0.0954, 0.1064]	[0.0900, 0.1120]	[0.0846, 0.1171]
$\delta_B^{DK}(^{\circ})$	142.2	[134.7, 148.1]	[125.3, 153.7]	[113.2, 157.9]
$r_B^{DK^{*0}}$	0.204	[0.149, 0.253]	[0.073, 0.299]	[0.000, 0.322]
$\delta_B^{DK^{*0}}(^{\circ})$	190.3	[165.8, 218.4]	[139.5, 263.4]	[117.8, 292.4]

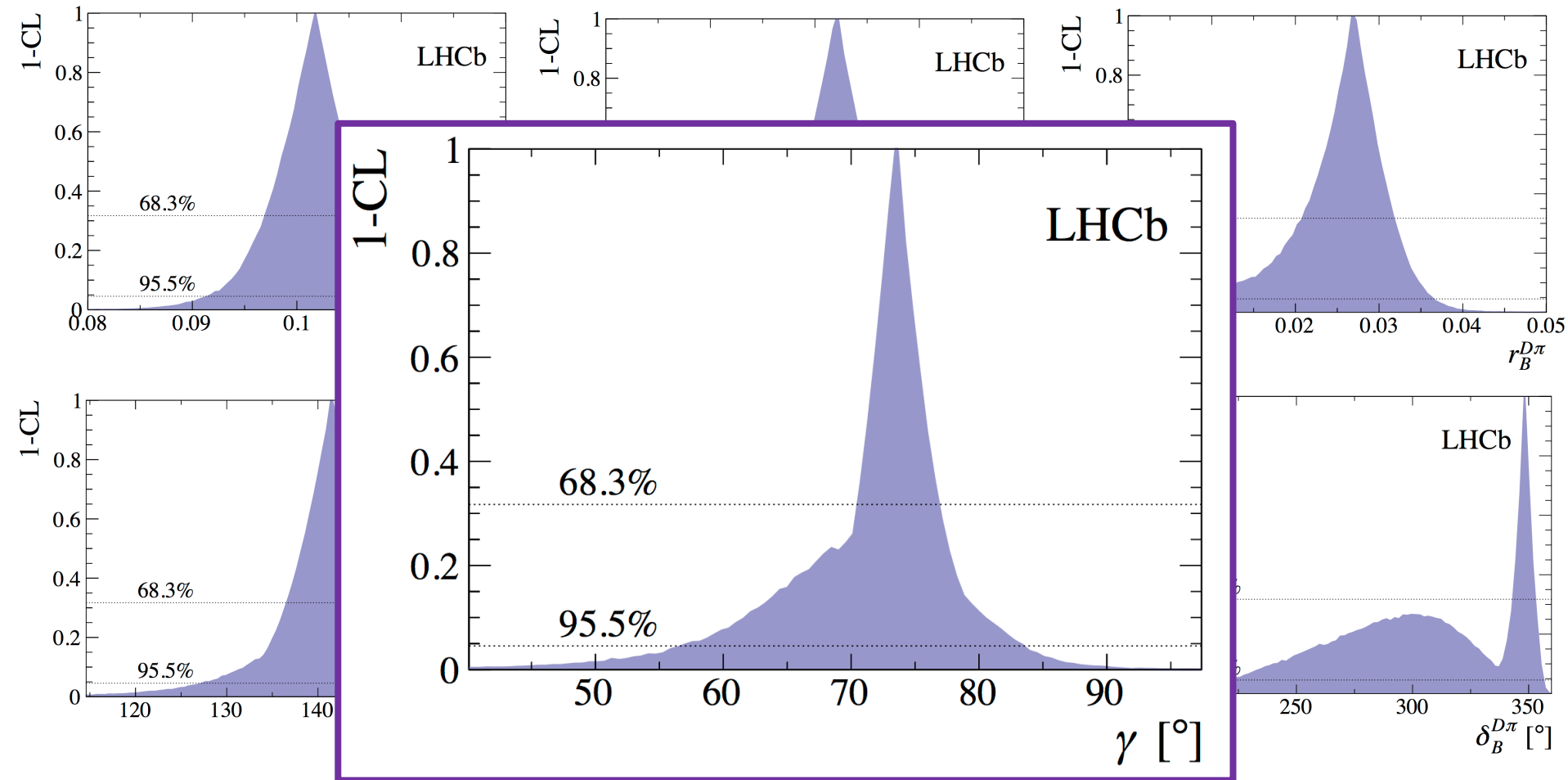
Dh combination (frequentist)



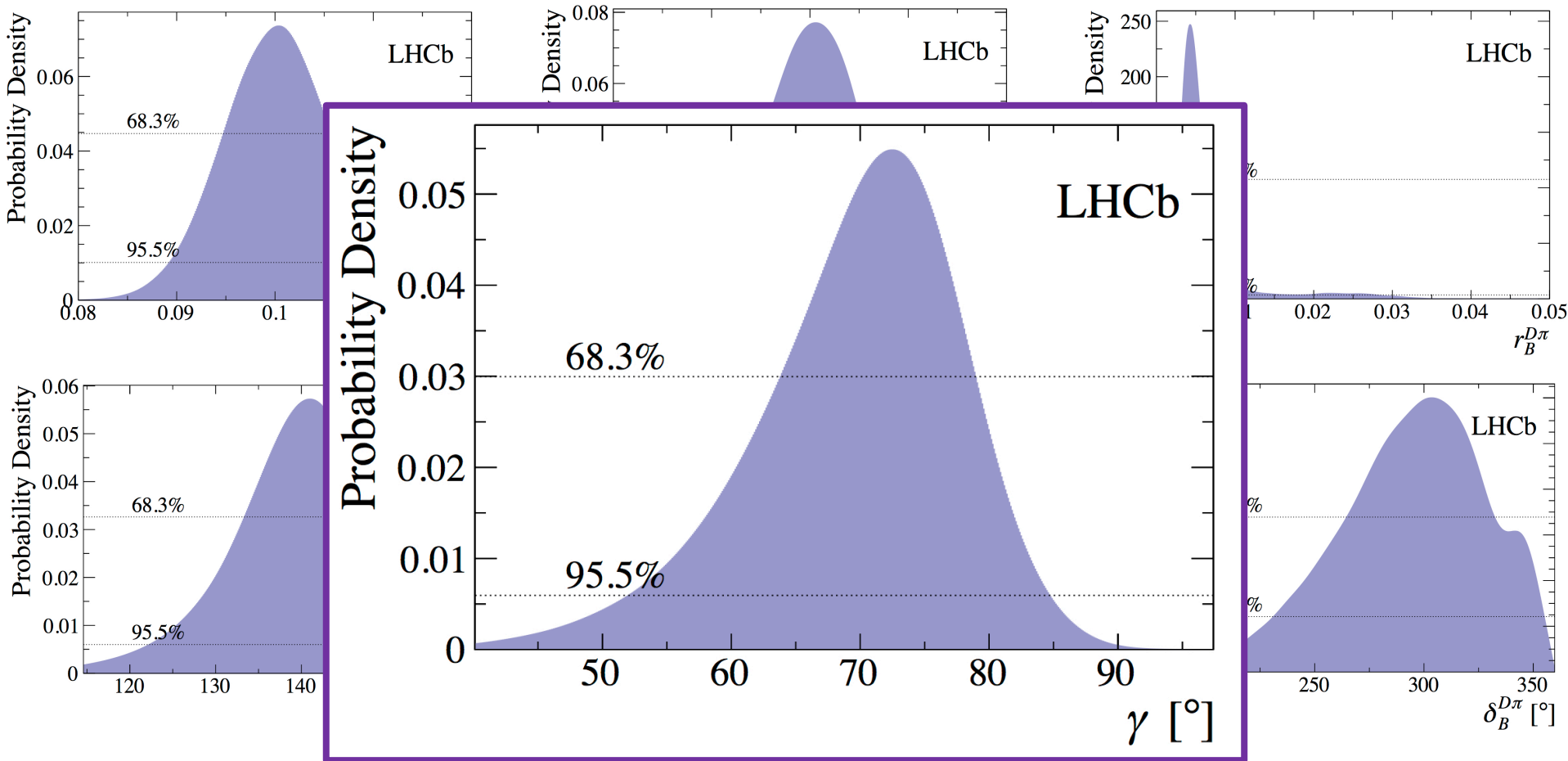
Dh combination (Bayesian)



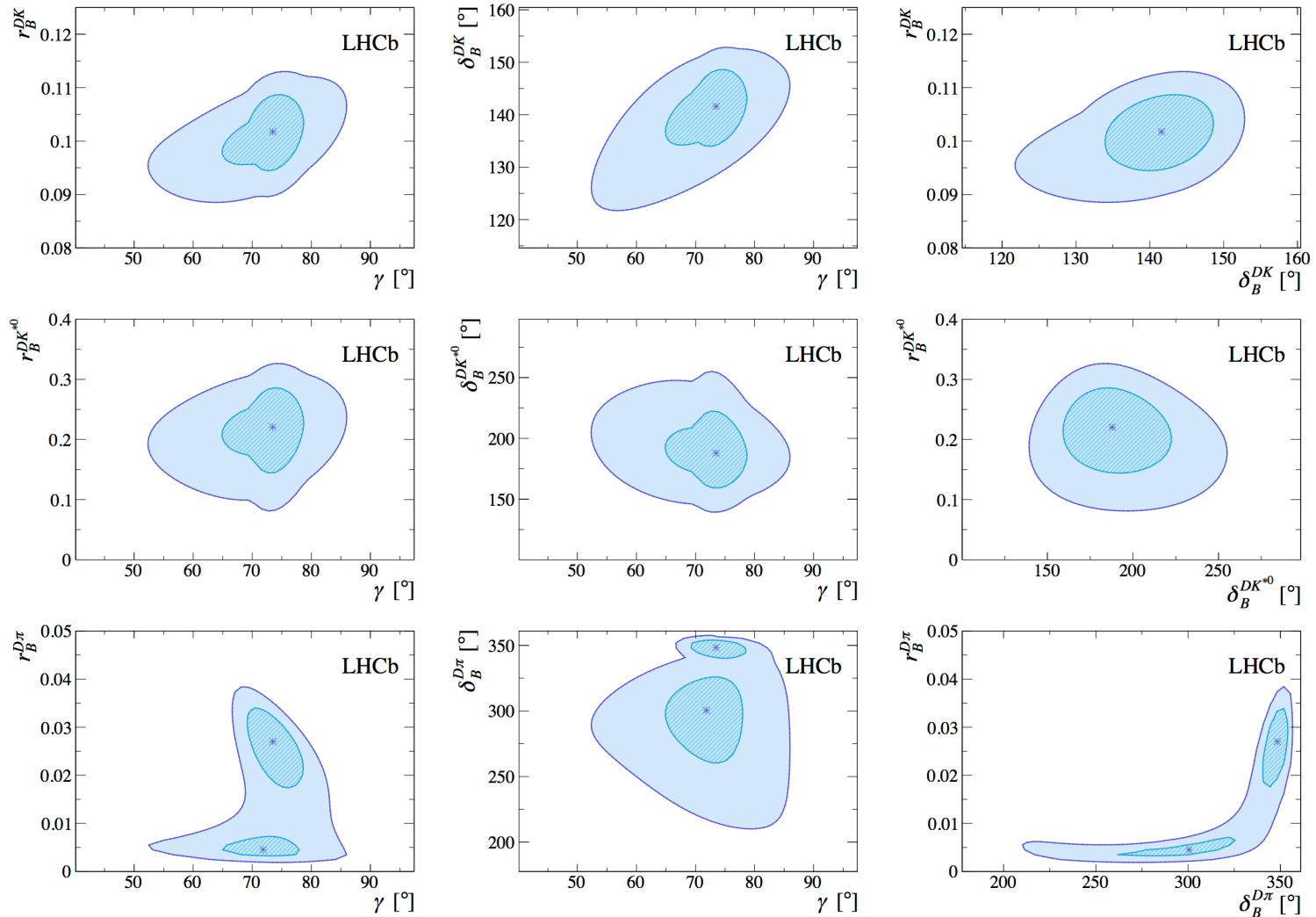
Dh combination (frequentist)



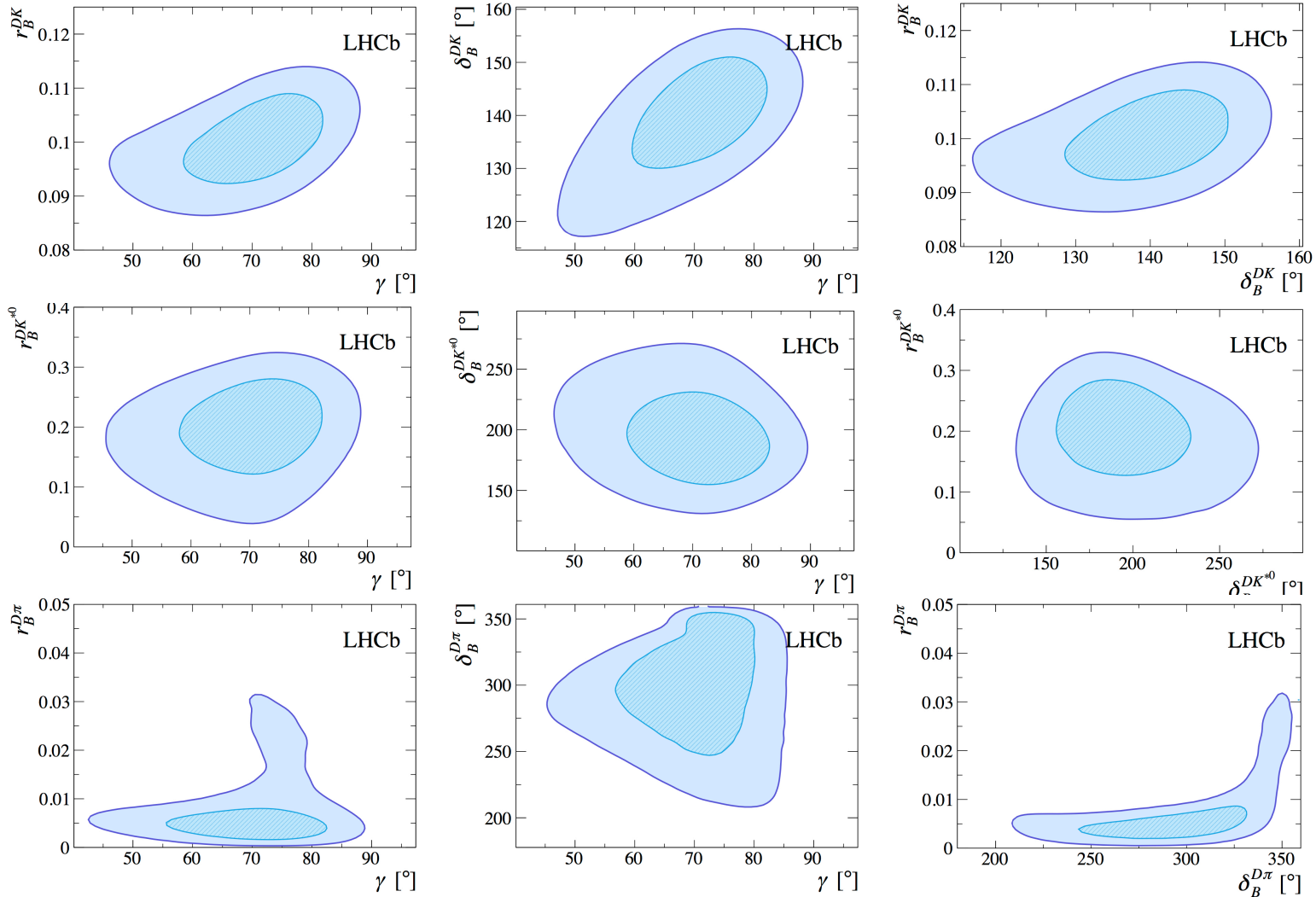
Dh combination (Bayesian)



Dh combination 2D (frequentist)



Dh combination 2D (Bayesian)



Dh results summary

Table 4: Confidence intervals and central values for the parameters of interest in the frequentist Dh combination.

frequentist

Observable	Central value	68.3% Interval	95.5% Interval	99.7% Interval
$\gamma(^{\circ})$	73.5	[70.5, 76.8]	[56.7, 83.4]	[40.1, 90.8]
r_B^{DK}	0.1017	[0.0970, 0.1064]	[0.0914, 0.1110]	[0.0844, 0.1163]
$\delta_B^{DK}(^{\circ})$	141.6	[136.6, 146.3]	[127.2, 151.1]	[114.6, 155.7]
$r_B^{DK^{*0}}$	0.220	[0.173, 0.264]	[0.121, 0.307]	[0.000, 0.355]
$\delta_B^{DK^{*0}}(^{\circ})$	188	[168, 211]	[148, 239]	[120, 280]
$r_B^{D\pi}$	0.027	[0.0207, 0.0318]	[0.0020, 0.0365]	[0.0008, 0.0425]
$\delta_B^{D\pi}(^{\circ})$	348.3	[343.2, 352.9]	[220.5, 356.4]	[192.9, 359.8]

Table 7: Credible intervals and most probable values for the hadronic parameters extracted from the Dh Bayesian combination.

Bayesian

Observable	Central value	68.3% Interval	95.5% Interval	99.7% Interval
$\gamma(^{\circ})$	72.4	[63.9, 79.0]	[52.1, 84.6]	[40.1, 89.5]
r_B^{DK}	0.1003	[0.0948, 0.1057]	[0.0893, 0.1109]	[0.0838, 0.1159]
$\delta_B^{DK}(^{\circ})$	141.0	[133.3, 147.5]	[122.1, 153.1]	[108.6, 157.5]
$r_B^{DK^{*0}}$	0.2072	[0.1514, 0.2555]	[0.0788, 0.3007]	[0.0031, 0.3291]
$\delta_B^{DK^{*0}}(^{\circ})$	189.8	[166.3, 216.5]	[143.9, 255.2]	[120.2, 286.0]
$r_B^{D\pi}$	0.0043	[0.0027, 0.0063]	[0.0011, 0.0281]	[0.0008, 0.0329]
$\delta_B^{D\pi}(^{\circ})$	303.7	[264.7, 332.7]	[231.5, 355.2]	[202.7, 359.0]

Systematic limitations

Limiting factors in the high-statistics era

Where will we become limited, as things stand:

- Most¹ $B \rightarrow DK$ modes rely on CLEO strong phase measurements at the $\psi(3770)$
- Allows for model independence; crucial in the high-statistics era
- Current systematic due to CLEO inputs $\sim 2^\circ$
- Some D modes not analysed by CLEO; some would benefit from D -phasespace-binned analysis

Available now:

- Quadruplication of the CLEO dataset at BES III (\rightarrow systematic $\sim 1^\circ$)
 - ▶ Measurement in $D \rightarrow K\pi$ ([Int.J.Mod.Phys.Conf.Ser. 31 1460305](#))
 - ▶ Preliminary results in $D \rightarrow K_S^0 \pi\pi$
- Supplement (but not match) with strong phase measurements in charm mixing

To avoid systematic limitation in the upgrade era:

- Full spectrum of strong phase measurements with full $15\text{-}20 \text{ fb}^{-1}$ at BES III

¹not, e.g., $B_s^0 \rightarrow D_s^+ K$