

Semileptonic Penguin Decays

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MIAPP Munich 18.11.2016

Semileptonic Penguin Decays

Based on

Huber, Hurth, Lunghi arXiv:1503.0449

Inclusive $B \rightarrow X_s \ell^+ \ell^-$: Complete angular analysis and a thorough study of collinear photons

Benzke, Fickinger, Hurth, Turczyk to appear

Subleading power factorization in $B \rightarrow X_s \ell^+ \ell^-$

Hurth, Mahmoudi, Neshatpour arXiv:1603.00865

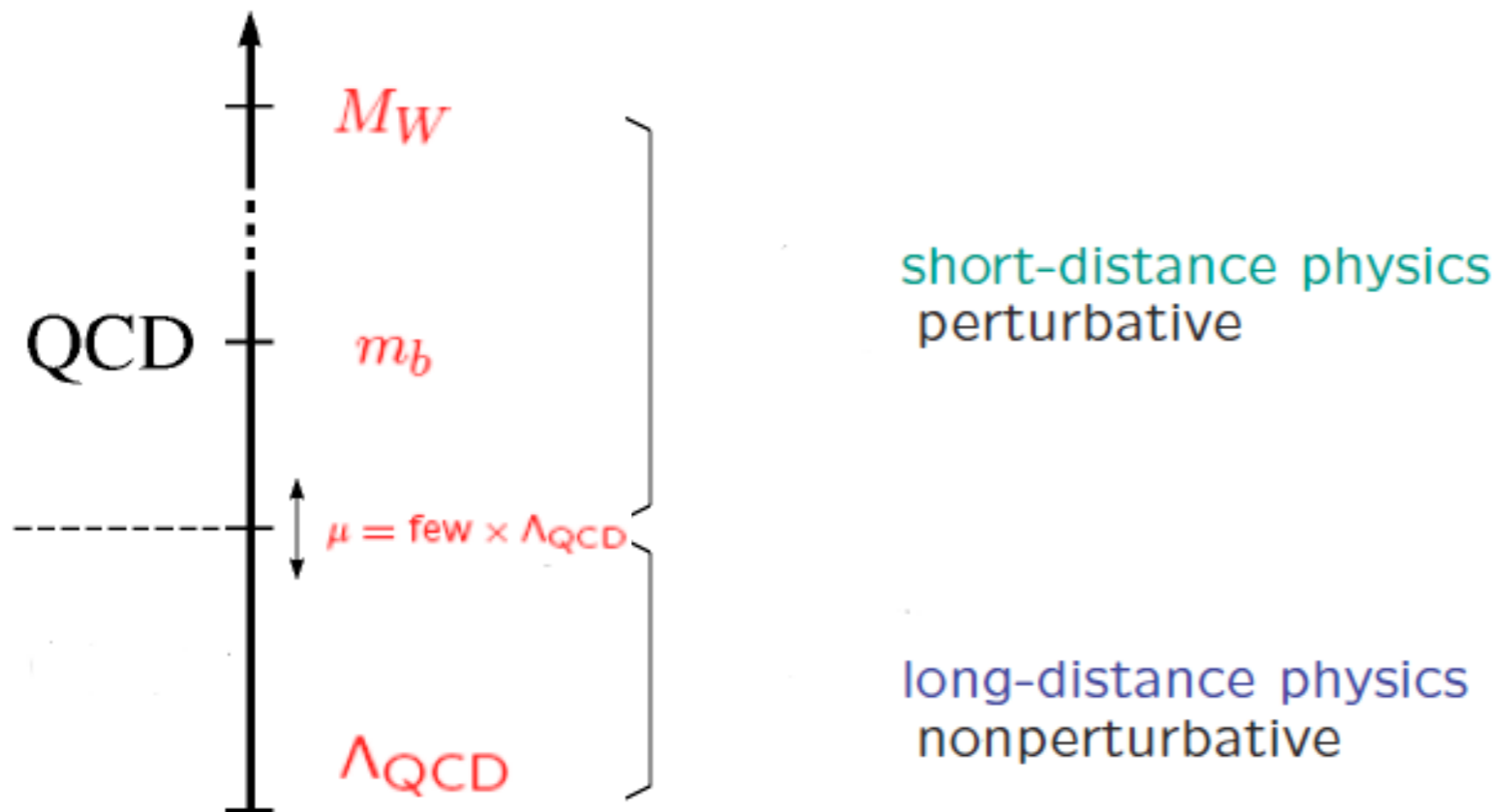
On the anomalies in the latest LHCb data

Motivation

- Radiative and semileptonic rare B decayse are highly sensitive probes for new physics
- Exclusive modes are experimentally easier (LHCb), but have larger theoretical uncertainties (issue of unknown power corrections !)
- Inclusive modes require Belle-II for full exploitation (complete angular analysis) but are theoretically very clean
- Inclusive modes allow for crosschecks of recent LHCb anomalies

Theoretical Tools

Theoretical tools for flavour precision observables



Factorization theorems: separating long- and short-distance physics

- Electroweak effective Hamiltonian: $H_{eff} = -\frac{4G_F}{\sqrt{2}} \sum C_i(\mu, M_{heavy}) \mathcal{O}_i(\mu)$
- $\mu^2 \approx M_{New}^2 \gg M_W^2$: 'new physics' effects: $C_i^{SM}(M_W) + C_i^{New}(M_W)$

How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

Inclusive modes $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$

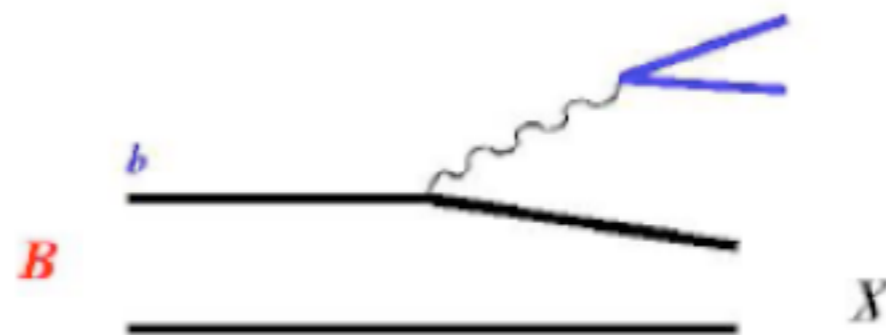
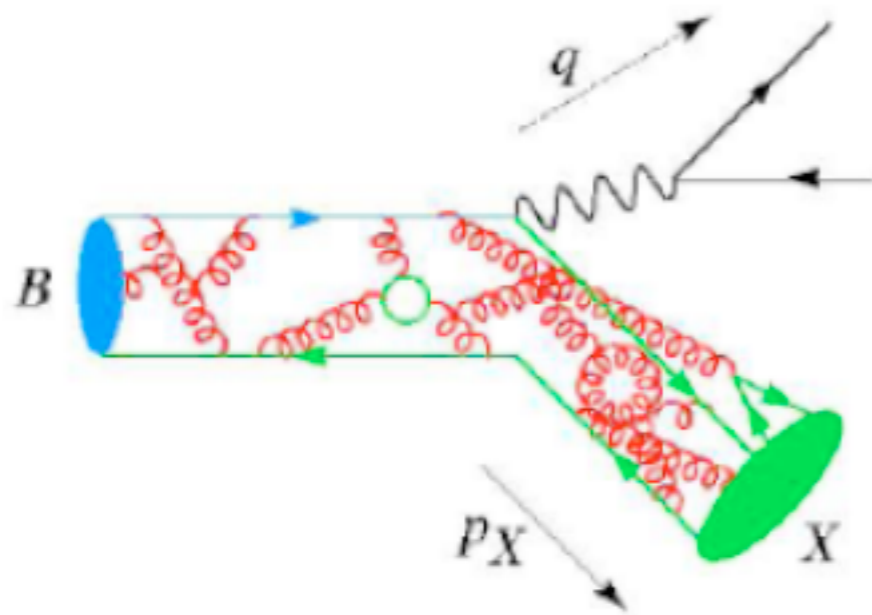
How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \rightarrow X_s \gamma) \xrightarrow{m_b \rightarrow \infty} \Gamma(b \rightarrow X_s^{\text{parton}} \gamma), \quad \Delta^{\text{nonpert.}} \sim \Lambda_{QCD}^2 / m_b^2$$

No linear term Λ_{QCD}/m_b (perturbative contributions dominant)

Chay, Georgi, Grinstein 1990



Inclusive modes $B \rightarrow X_s \gamma$ and $B \rightarrow X_s l^+ l^-$

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An old story:

- If one goes beyond the leading operator ($\mathcal{O}_7, \mathcal{O}_9$):
breakdown of local expansion

A new dedicated analysis:

naive estimate of non-local matrix elements leads to 5% uncertainty.

[Benzke, Lee, Neubert, Paz, arXiv:1003.5012](#)



Analysis in $B \rightarrow X_s l l$ in this talk; [Benzke, Fickinger, Hurth, Turczyk](#)

Exclusive modes $B \rightarrow K^{(*)} \ell \bar{\ell}$

QCD-improved factorization: BBNS 1999

$$\mathcal{T}_a^{(i)} = C_a^{(i)} \xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K^*} + O(\Lambda/m_b)$$

(Soft-collinear effective theory)

- Separation of **perturbative hard kernels** from **process-independent nonperturbative** functions like form factors
- **Relations between formfactors** in large-energy limit
- **Limitation: insufficient information on power-suppressed Λ/m_b terms**
(breakdown of factorization: 'endpoint divergences')

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"Full formfactor approach"

- we have factorizable and nonfactorizable power corrections
- using full QCD formfactors in the factorization formula takes factorizable power corrections into account automatically
- nonfactorizable contributions generated by four-quark and \mathcal{O}_8 operators

Difference between exclusive and inclusive $b \rightarrow s\gamma, \ell\ell$ modes:

Inclusive

Λ^2/m_b^2 corrections can be calculated for the leading operators in the local OPE .

Λ/m_b corrections to the subleading operators correspond to nonlocal matrix elements and can be estimated !

Exclusive

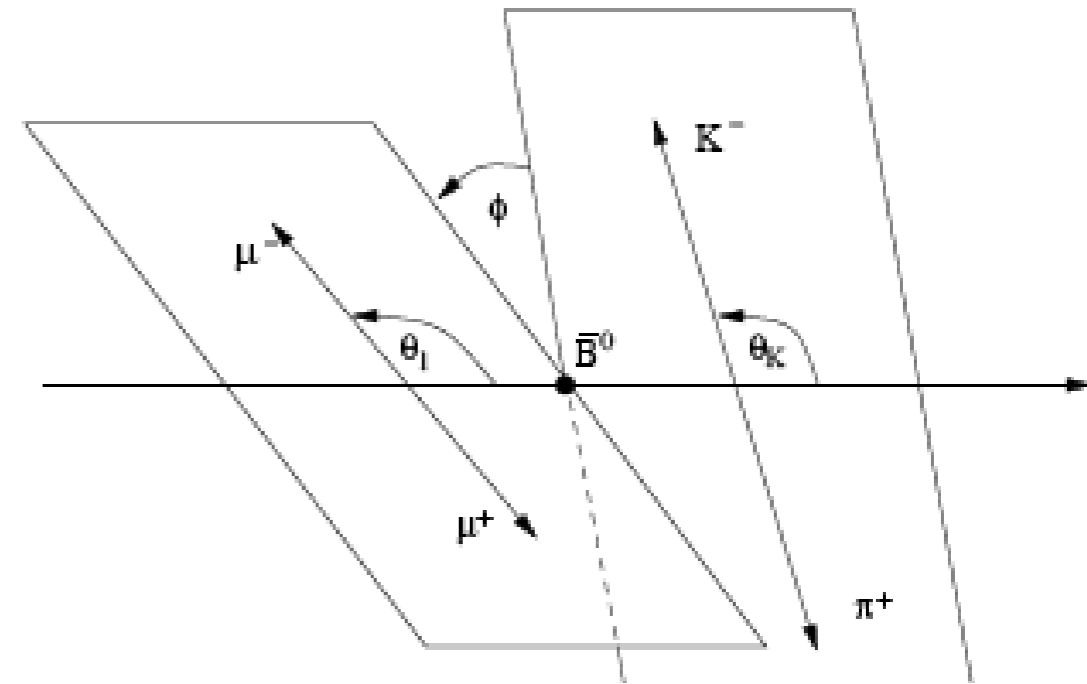
No theory of Λ/m_b corrections at all within QCD factorization formula (in the low- q^2 region); these corrections can only be "guesstimated" !

The LHCb Anomalies

Differential decay rate of $B \rightarrow K^* \ell \ell$

Assuming the \bar{K}^* to be on the mass shell, the decay $\bar{B}^0 \rightarrow \bar{K}^{*0} (\rightarrow K^- \pi^+) \ell^+ \ell^-$ described by the lepton-pair invariant mass, s , and the three angles θ_l , θ_K , ϕ .

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_l d\cos\theta_K d\phi} = \frac{9}{32\pi} J(q^2, \theta_l, \theta_K, \phi)$$



$$J(q^2, \theta_l, \theta_K, \phi) =$$

$$\begin{aligned} &= J_{1s} \sin^2 \theta_K + J_{1c} \cos^2 \theta_K + (J_{2s} \sin^2 \theta_K + J_{2c} \cos^2 \theta_K) \cos 2\theta_l + J_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\phi \\ &+ J_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_K \sin \theta_l \cos \phi + (J_{6s} \sin^2 \theta_K + J_{6c} \cos^2 \theta_K) \cos \theta_l \\ &+ J_7 \sin 2\theta_K \sin \theta_l \sin \phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin \phi + J_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi \end{aligned}$$

Large number of independent angular observables

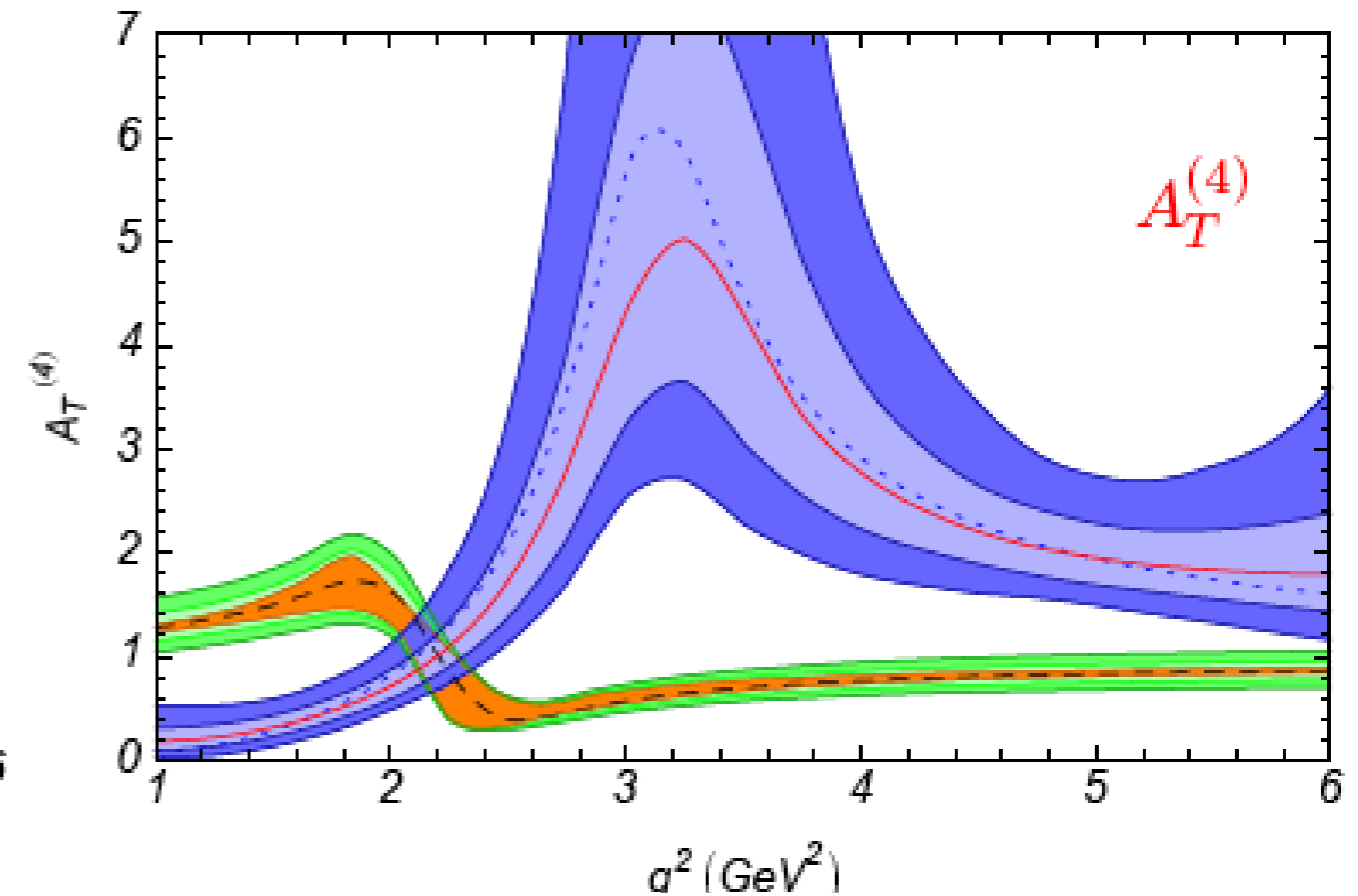
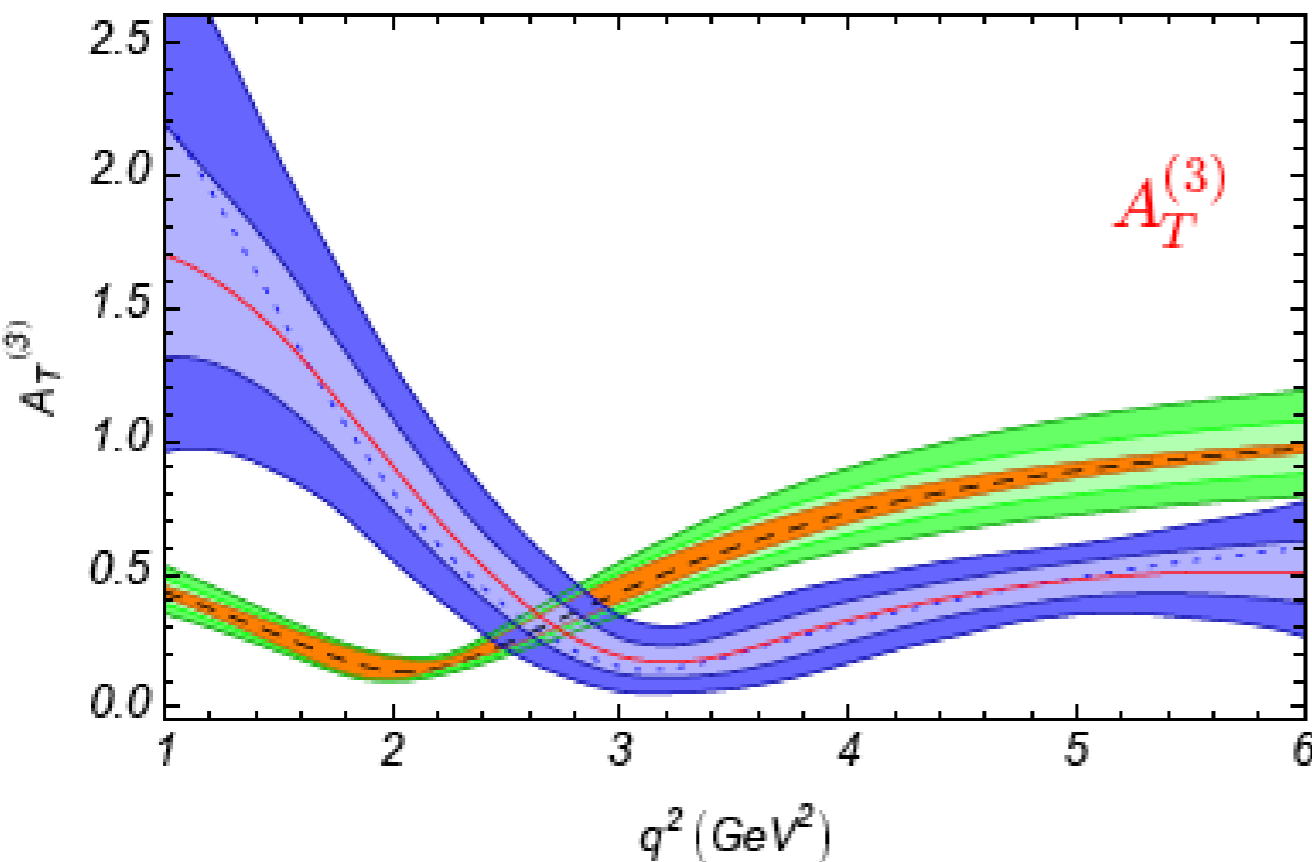
Previous predictions versus LHCb Monte Carlo (10 fb^{-1})

Egede, Hurth, Matias, Ramon, Reece, arXiv:0807.2589, arXiv:1005.0571

– unknown Λ/m_b power corrections

$$A_{\perp,\parallel,0} = A_{\perp,\parallel,0}^0 (1 + c_{\perp,\parallel,0}) \text{ vary } c_i \text{ in a range of } \pm 10\% \text{ and also of } \pm 5\%$$

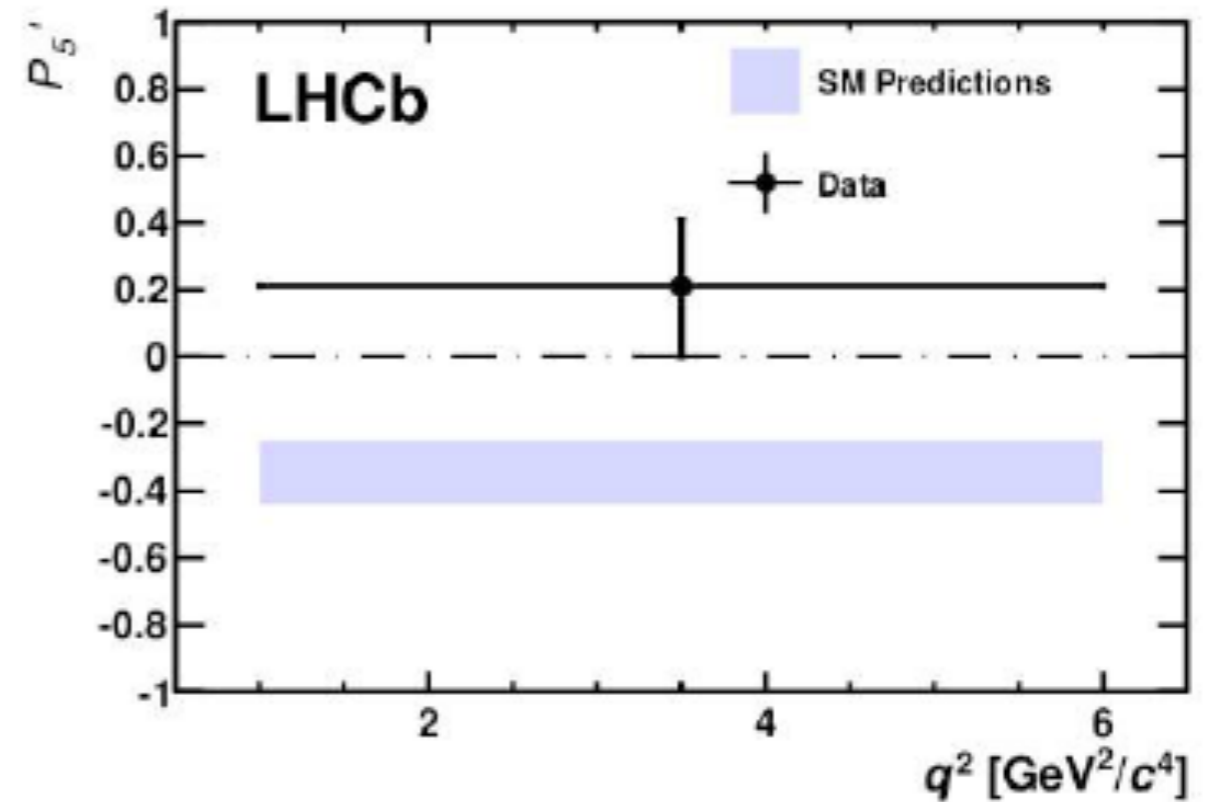
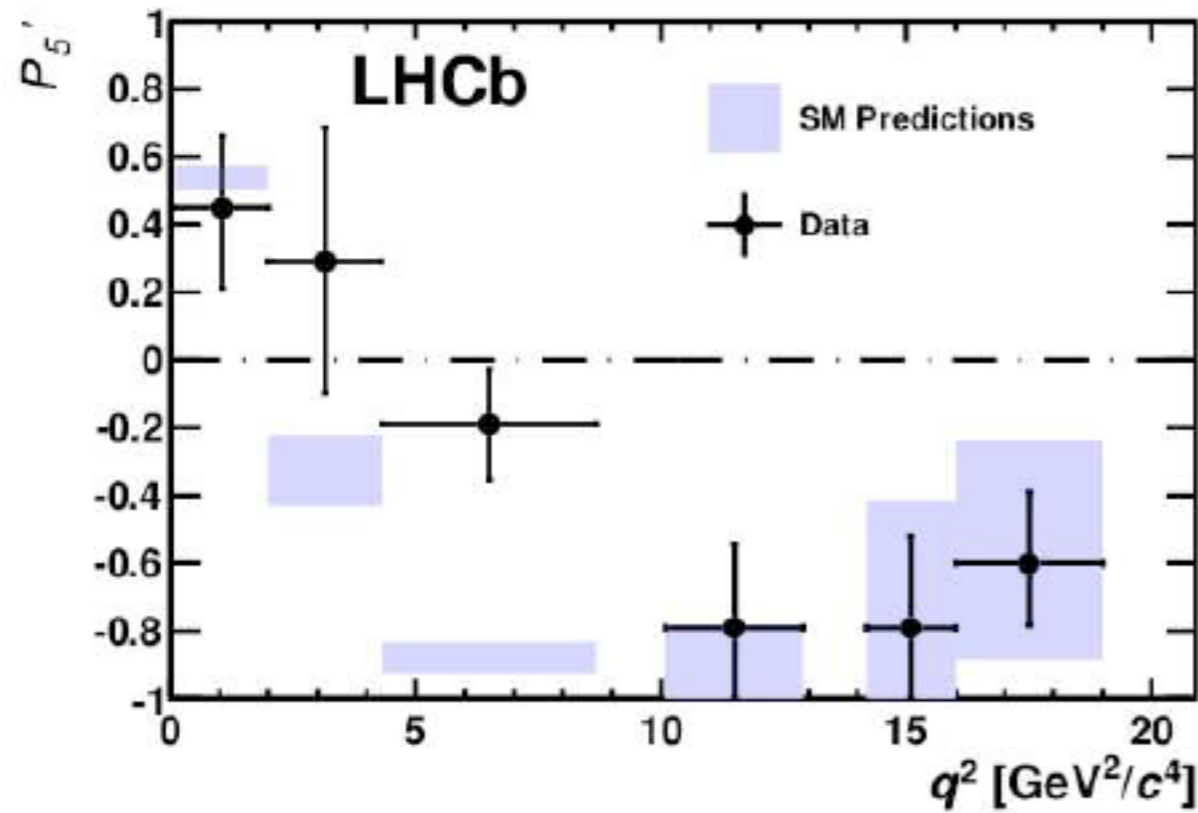
Guesstimate



The experimental errors assuming SUSY scenario (b) with large-gluino mass and positive mass insertion, is compared to the theoretical errors assuming the SM.

This was the dream in 2008

see also Altmannshofer et al., arXiv:0811.1214; Bobeth et al., arXiv:0805.2525



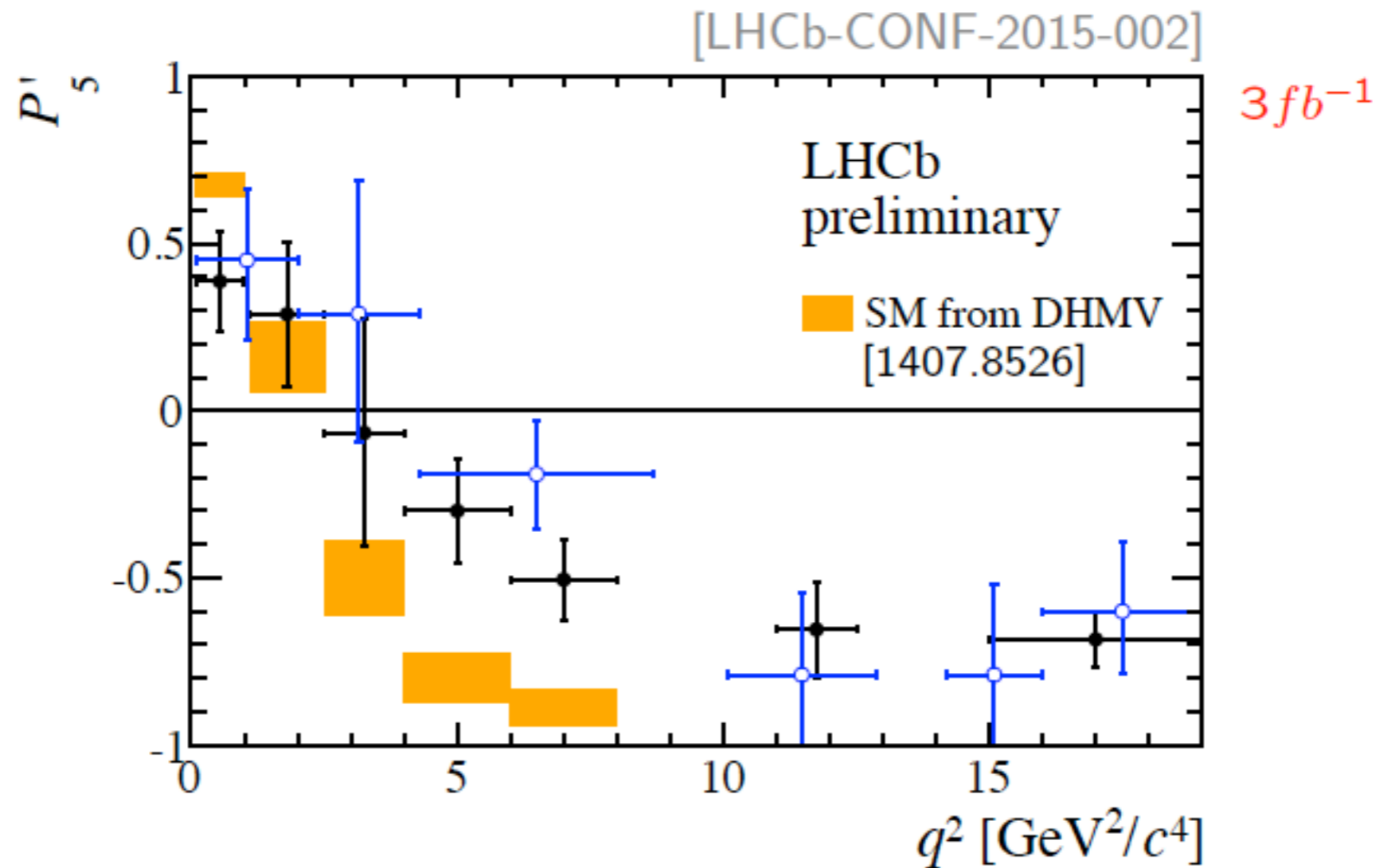
LHCb Anomaly

a statistical fluctuation, an underestimation
of Λ/m_b corrections or new physics in C_9 ?

$$C_7 \quad (B \rightarrow X_s \gamma)$$

$$C_{10} \quad (B \rightarrow \mu^+ \mu^-)$$

$$O_7^{(f)} \propto m_b \bar{s} \sigma_{\mu\nu} P_{R(L)} b F^{\mu\nu} \quad O_9^{(f)} \propto \bar{s} \gamma^\mu P_{L(R)} \bar{\ell} \gamma_\mu \ell \quad O_{10}^{(f)} \propto \bar{s} \gamma^\mu P_{L(R)} \bar{\ell} \gamma_\mu \gamma_5 \ell$$



- Tension seen in P'_5 in [PRL 111, 191801 (2013)] confirmed
- [4.0, 6.0] and [6.0, 8.0] GeV²/c⁴ show deviations of 2.9σ each
- Naive combination results in a significance of 3.7σ
- Compatible with $1fb^{-1}$ measurement

Sign for lepton non-universality ?

LHCb; arXiv:1406.6482

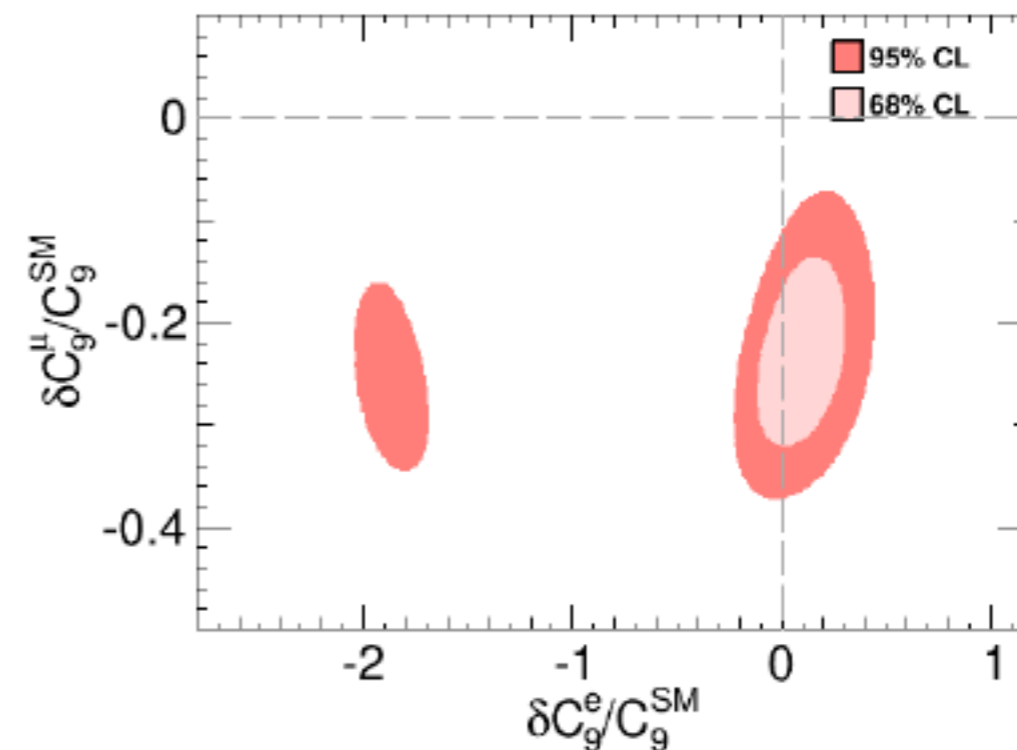
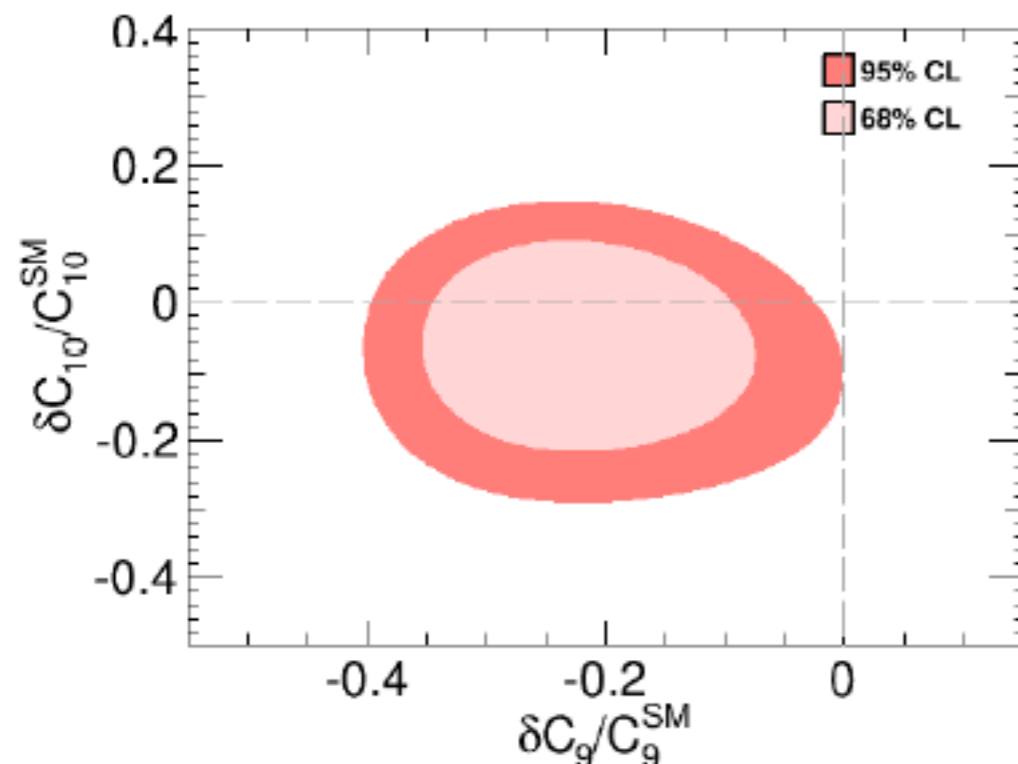
$$R_K \equiv \frac{\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \rightarrow K^+ e^+ e^-)} = 0.745_{-0.074}^{+0.090} (\text{stat}) \pm 0.036 (\text{syst})$$

2.6 σ deviation from SM

- NP tensor or scalar contribution difficult, sign for tension in C_9^μ
Alonso, Camalich, Grinstein, arXiv:1407.7044
- Theoretically rather clean, cannot explained by power corrections
- Electromagnetic corrections are taken into account

Global fits to the $b \rightarrow sll$ data

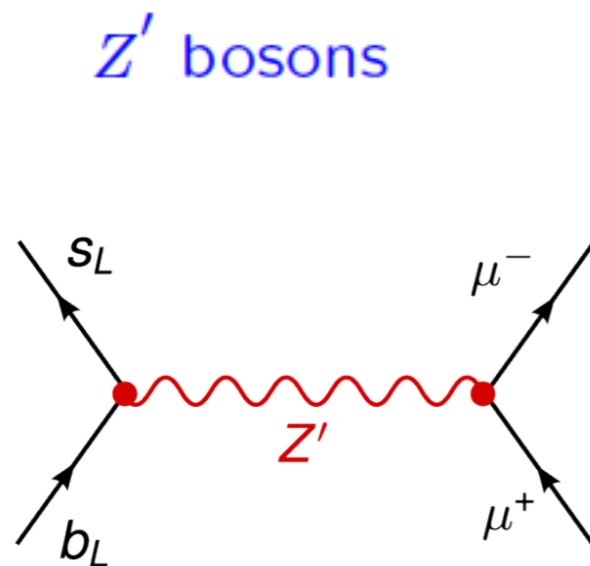
Hurth, Mahmoudi, Neshatpour, arXiv:1410.4545



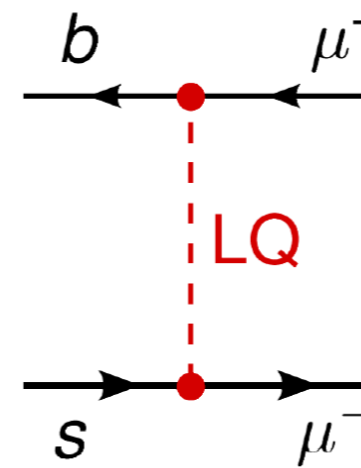
New physics explanations (1σ solutions)

Difficult to generate $\delta C_9 = -1$ at loop level (MSSM with MFV)

Various models under discussion (tree level contributions):



Leptoquarks



Altmannshofer, Straub arXiv:1308.1501

Gauld, Goertz, Haisch arXiv:1308.1959;1310.1082

Buras, De Fazio, Girschbach arXiv:1311.6729

Altmannshofer, Gori, Pospelov, Yavin arXiv:1403.1269

...

Hiller, Schmaltz arXiv:1408.1627

Sahoo, Mohanta arXiv:1501.05193

Becirevic, Fajfer, Kosnik arXiv:1503.09024

Bauer, Neubert arXiv:1511.01900 (loop)

...

Model explaining all anomalies by one leptoquark

- $$R_{D^{(*)}}^{\tau/l} = \frac{\mathcal{B}(\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}) / \mathcal{B}(\bar{B} \rightarrow D^{(*)} \tau \bar{\nu})_{SM}}{\mathcal{B}(\bar{B} \rightarrow D^{(*)} l \bar{\nu}) / \mathcal{B}(\bar{B} \rightarrow D^{(*)} l \bar{\nu})_{SM}}$$

3.9 σ deviation from $\tau - \mu/e$ universality

- $$R_K^{\mu/e} = \frac{\mathcal{B}(B \rightarrow K \mu^+ \mu^-)}{\mathcal{B}(B \rightarrow K e^+ e^-)} = 0.745_{-0.074}^{+0.090} \pm 0.036$$

2.6 σ deviation from $\mu - e$ universality

- $$(g - 2)_\mu$$

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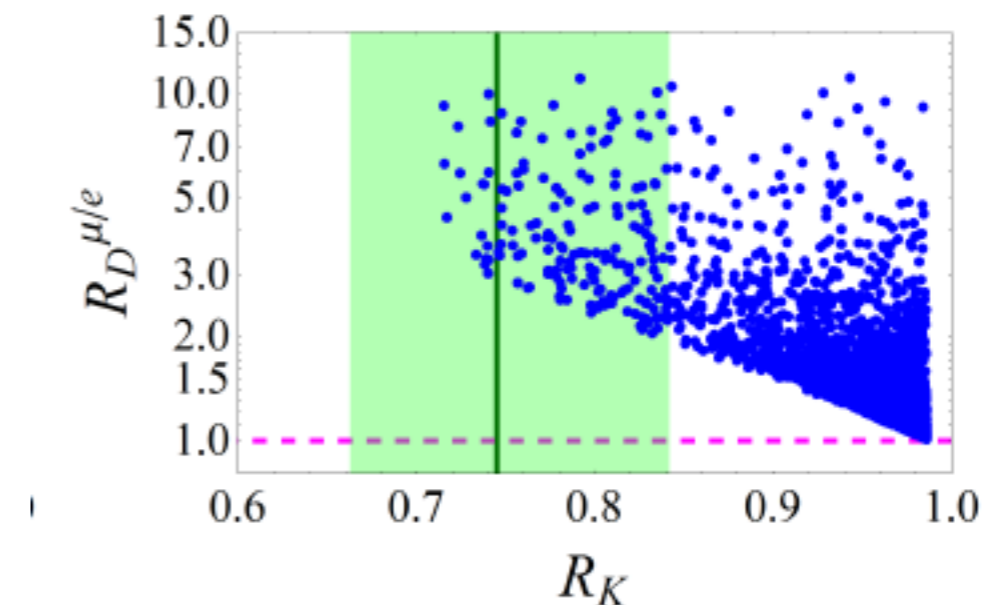
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Problem with $R_D^{\mu/e}$?

Becirevic et al. arXiv:1608.07583



Model-independent global fits to $b \rightarrow s$ data

Relevant operators: $\mathcal{O}_7, \mathcal{O}_8, \mathcal{O}'_{9\mu,e}, \mathcal{O}'_{10\mu,e}$

Scan over the values of δC_i : $C_i(\mu) = C_i^{\text{SM}} + \delta C_i$

More than 100 observables included

Experimental and theoretical correlations considered

Several groups doing global fits.

Studies based on the latest LHCb data:

Decsotes-Genon, Hofer, Matias, Virto arXiv:1510.04239

Ciuchini, Fedele, Franco, Mishima, Paul, Silvestrini, Valli arXiv:1512.07157

Hurth, Mahmoudi, Neshatpour arXiv:1603.00865

Previous studies:

Beaujean, Bobeth, Jahn arXiv:1508.01526

Altmannshofer, Straub arXiv:1503.06199, 1411.3161

Hurth, Mahmoudi, Neshatpour arXiv:1410.4545

Fit results for one operator

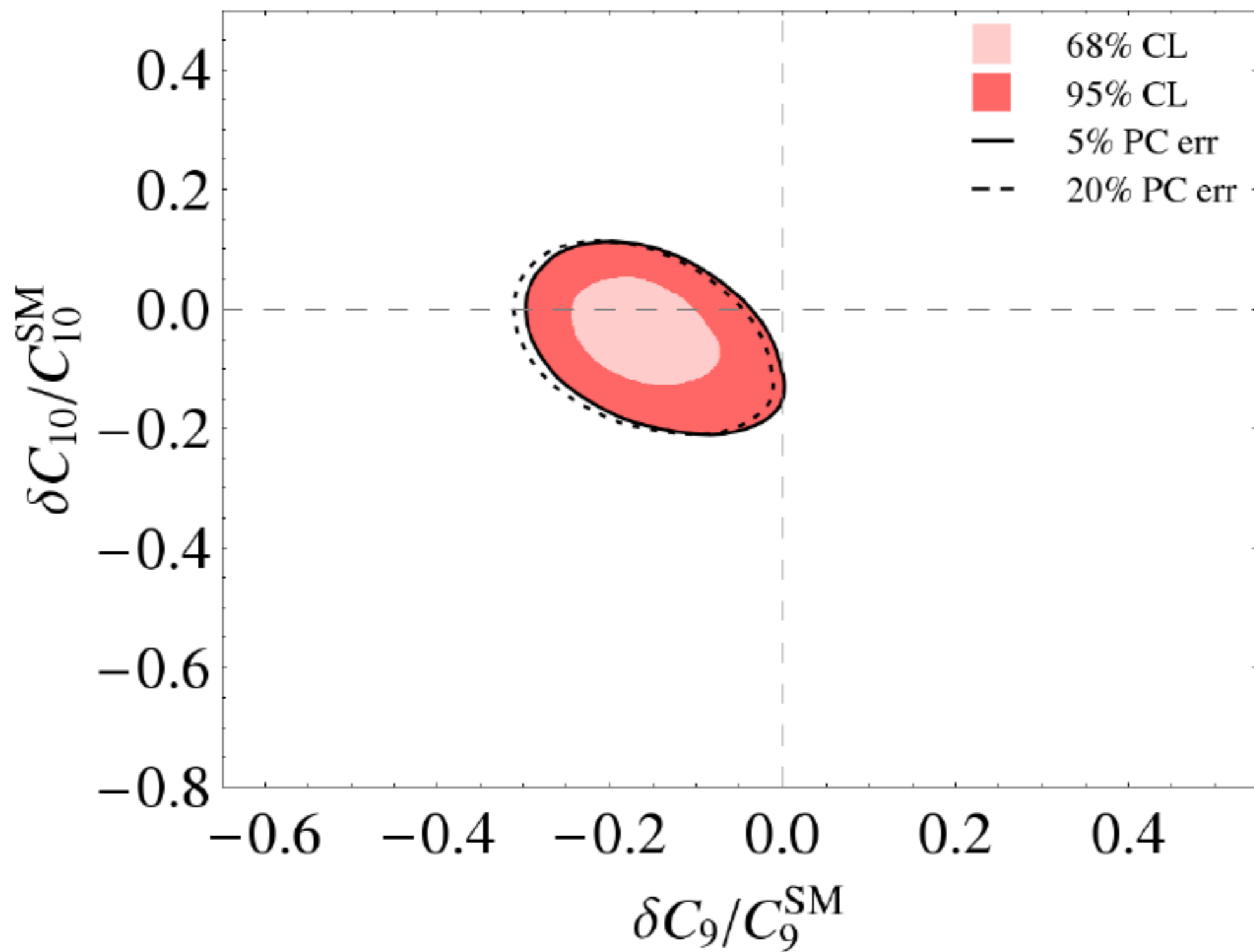
Using full QCD formfactors (Zwicky et al. arXiv:1503.0553)

Assuming 10% power correction errors (on amplitude level)

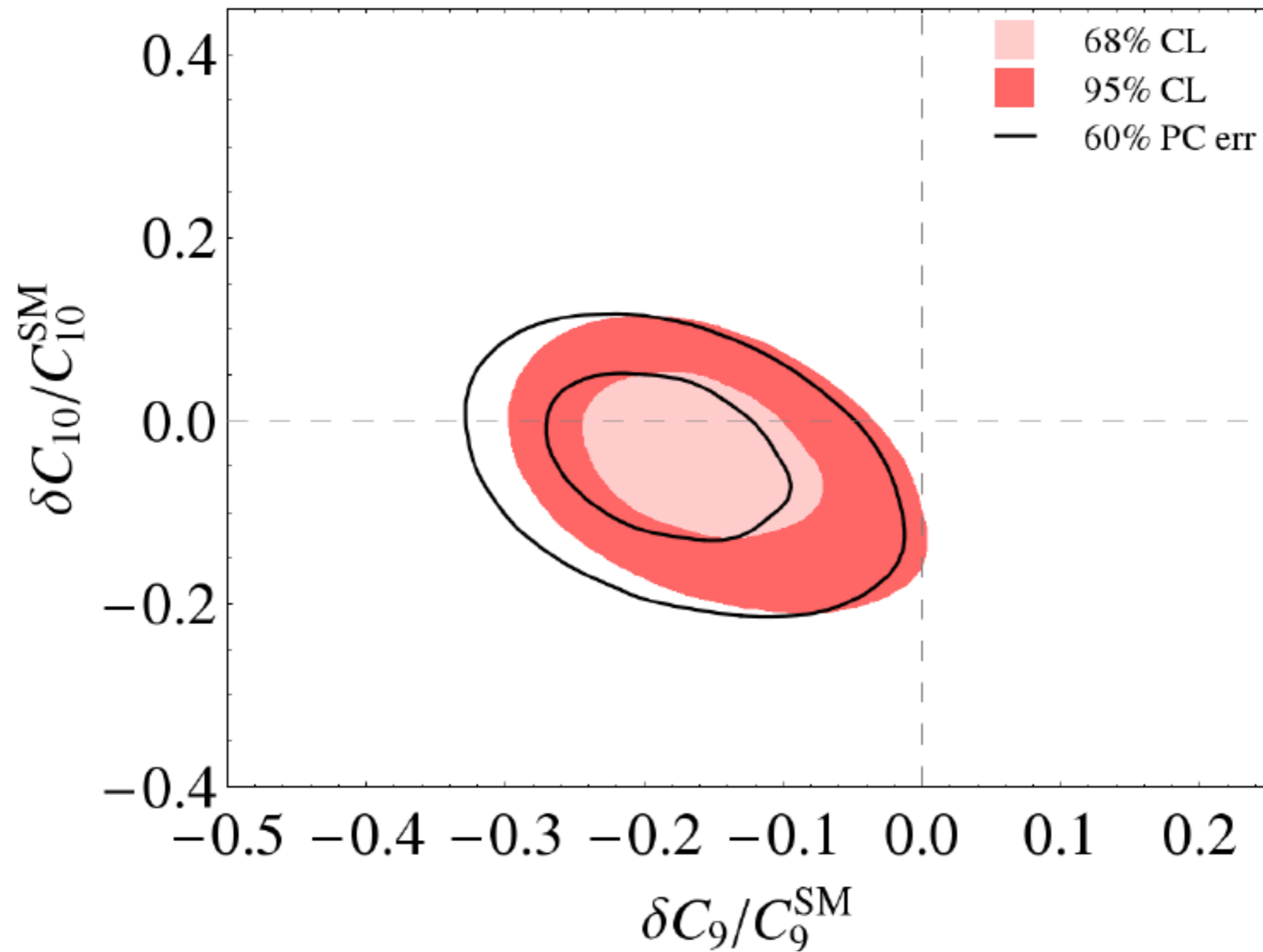
	b.f. value	χ_{\min}^2	Pull _{SM}	68% C.L.	95% C.L.
$\delta C_9/C_9^{\text{SM}}$	-0.18	123.8	3.0σ	[-0.25, -0.09]	[-0.30, -0.03]
$\delta C_9'/C_9^{\text{SM}}$	+0.03	131.9	1.0σ	[-0.05, +0.12]	[-0.11, +0.18]
$\delta C_{10}/C_{10}^{\text{SM}}$	-0.12	129.2	1.9σ	[-0.23, -0.02]	[-0.31, +0.04]
$\delta C_9^\mu/C_9^{\text{SM}}$	-0.21	115.5	4.2σ	[-0.27, -0.13]	[-0.32, -0.08]
$\delta C_9^e/C_9^{\text{SM}}$	+0.25	124.3	2.9σ	[+0.11, +0.36]	[+0.03, +0.46]

Allowing for lepton non-universality improves the fit !

Fit results for two operators

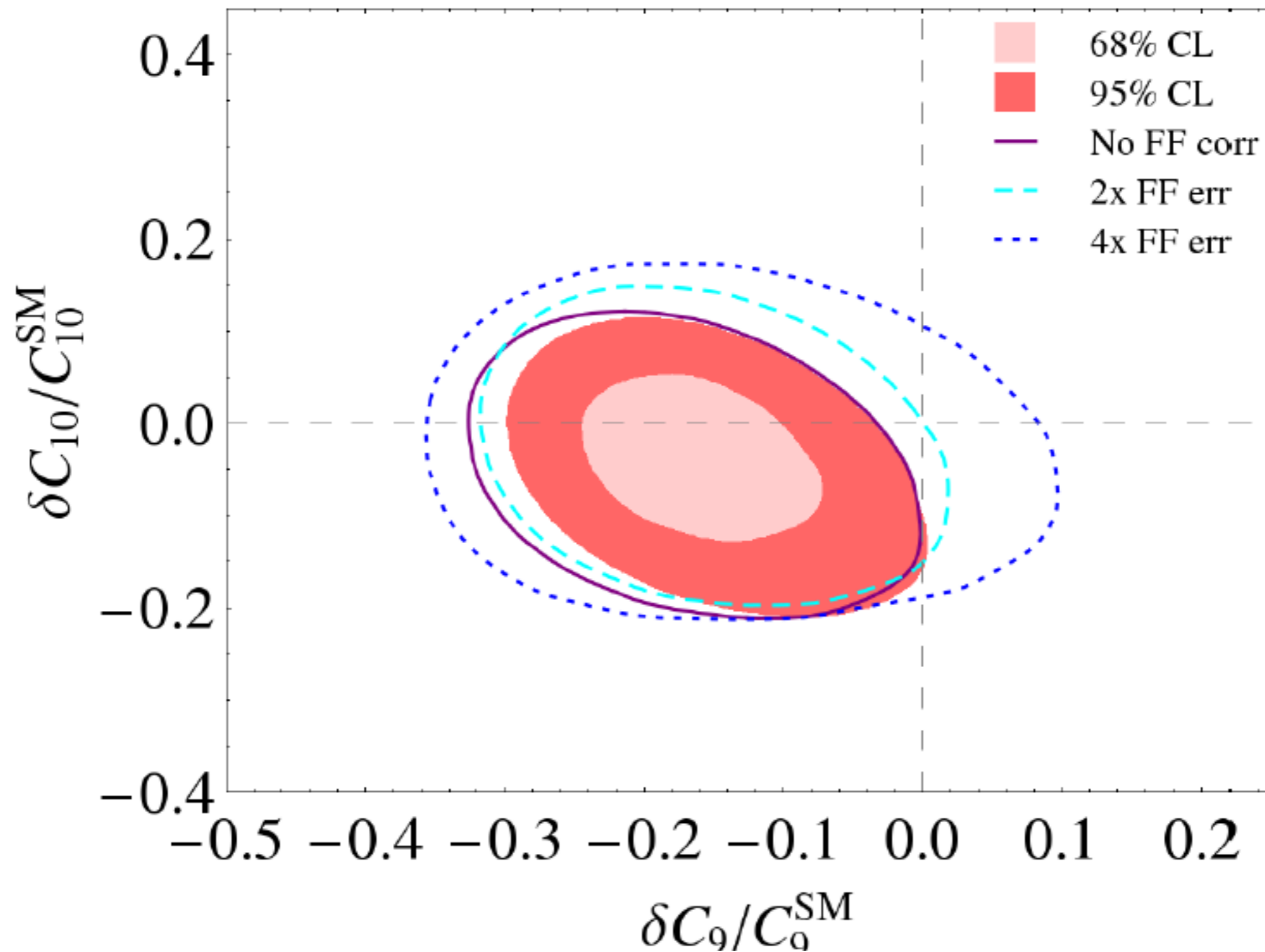
 $\{C_9, C_{10}\}$ 

Assumption of 60% power correction error (on the amplitude level)



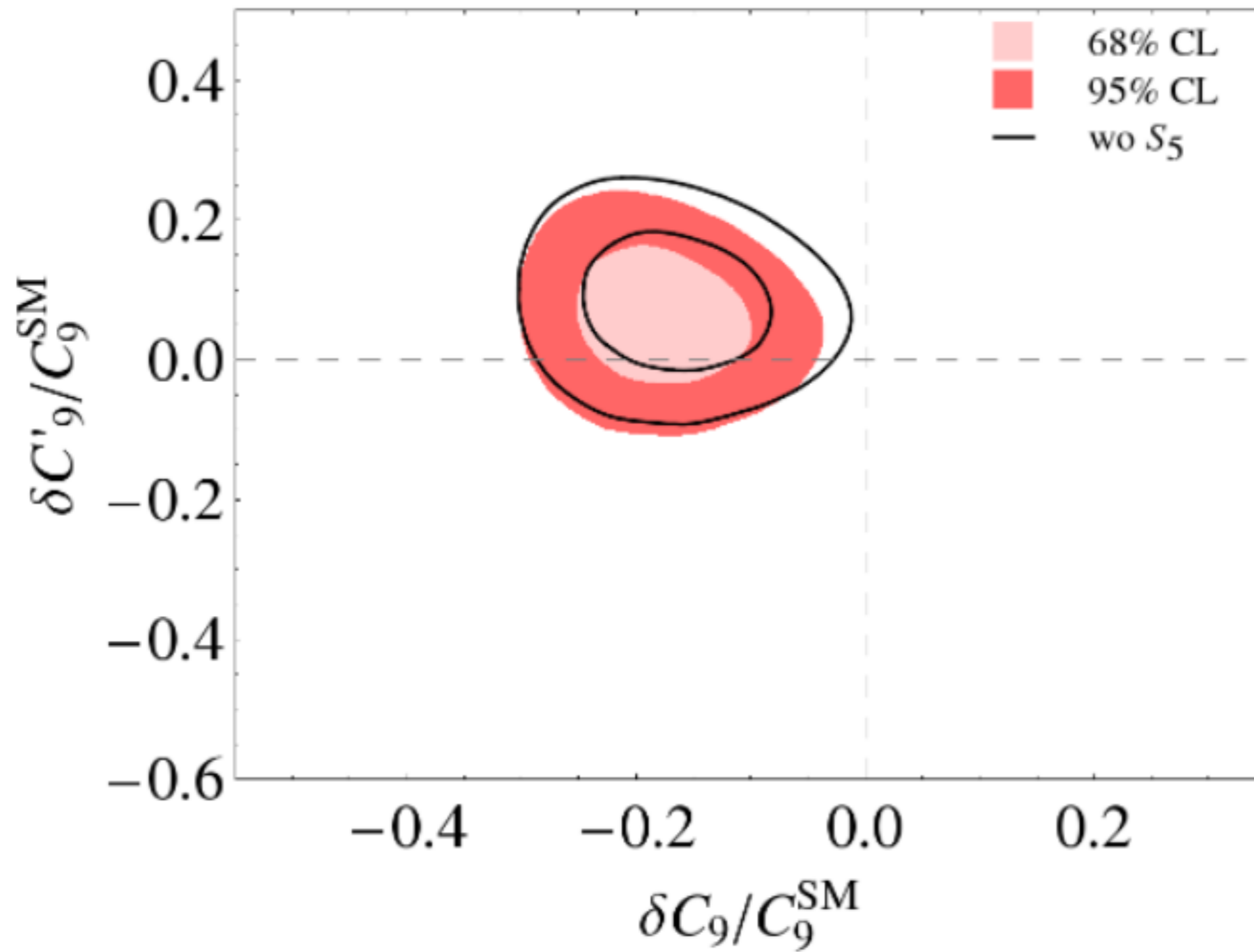
The assumption on the power correction errors have a rather mild impact on the constraints of the allowed region

Fits assuming different form factor uncertainties



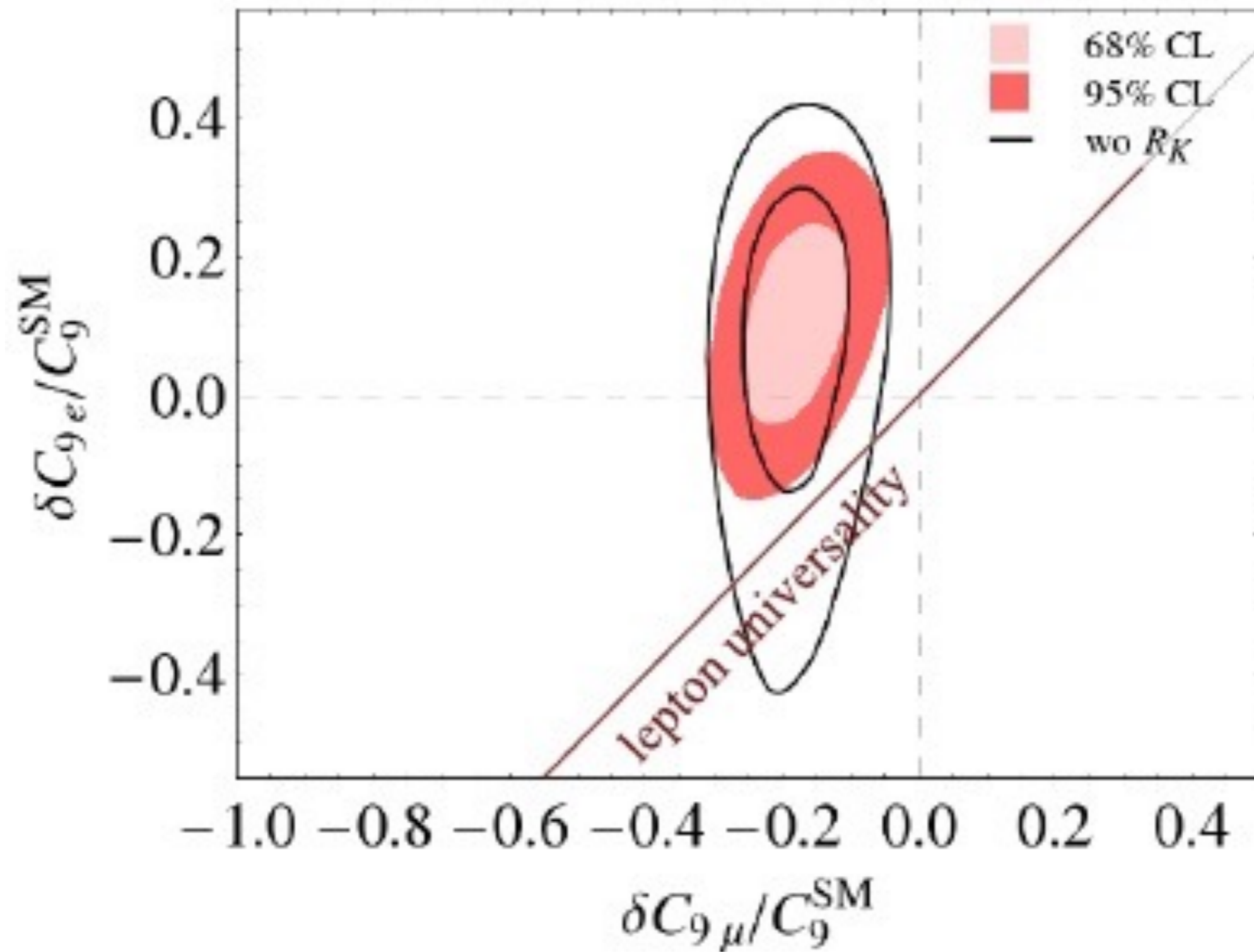
The size of the form factor errors has a crucial role in constraining the allowed region (LCSR-calculation Zwicky et al. arXiv:1503.0553)

Omitting S_5 from the fit



S_5 is not the only observable which drives $\delta C_9 / C_9^{\text{SM}}$ to negative values

Removing R_K from the fit



R_K is the main player for the best fit value for $C_9^\mu - C_9^e$
 which are in more than 2σ tension with lepton universality

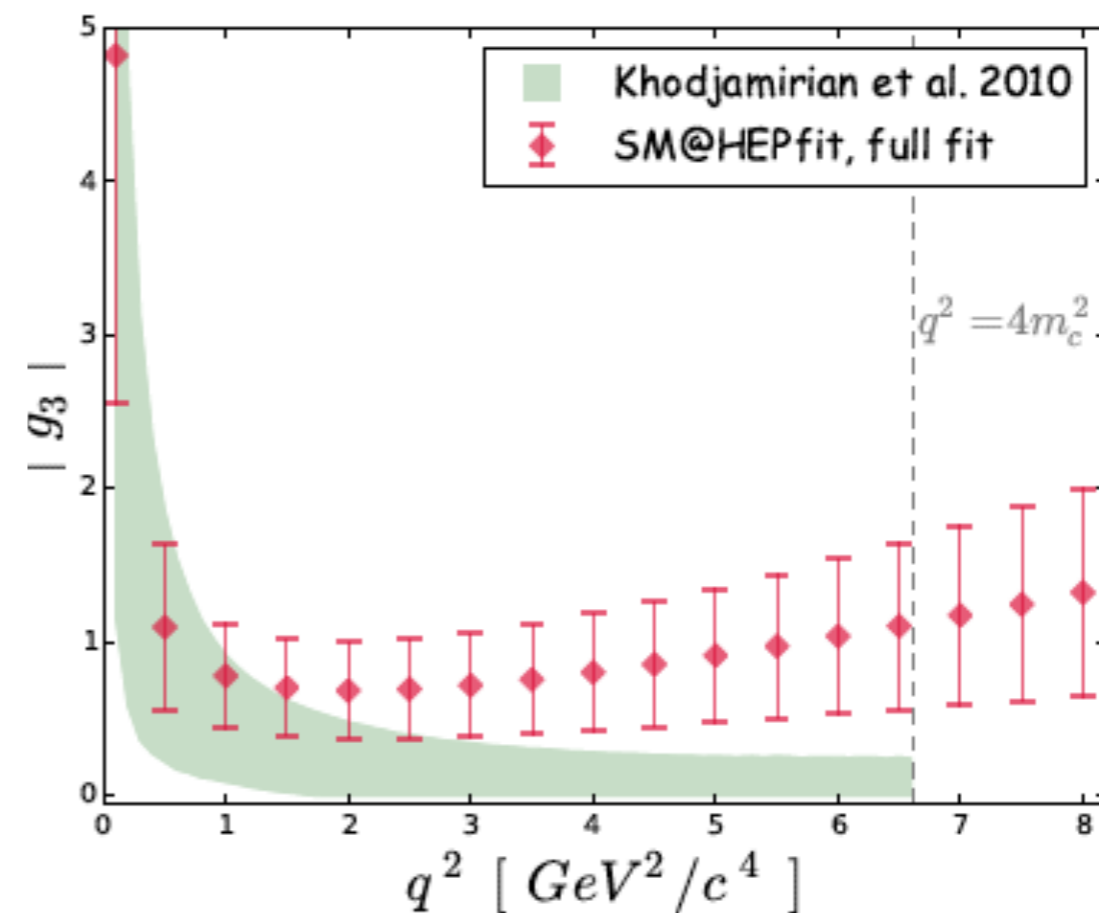
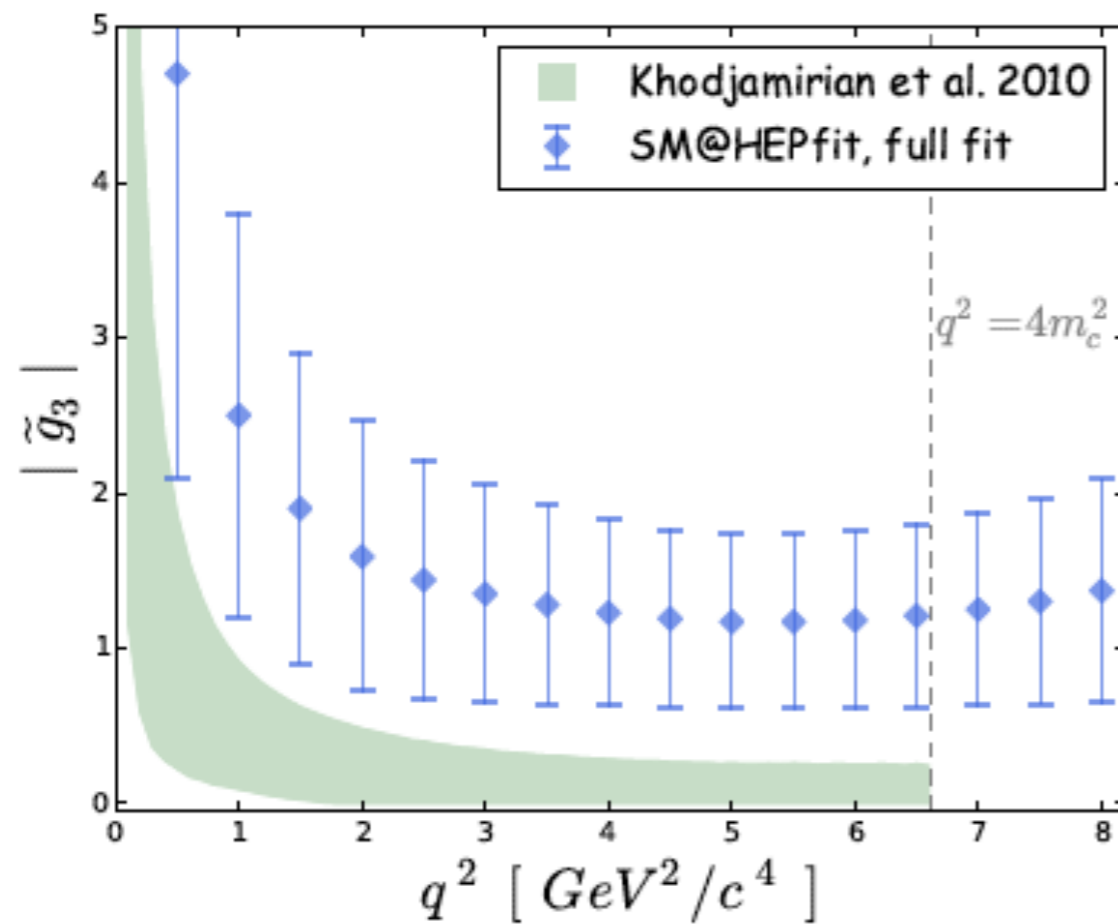
Strategy for the Future

Fit the unknown power corrections to the data

Ciuchini et al. arXiv:1512.07157

Leading SCET amplitude with general ansatz with 18 parameters
for power corrections Camalich, Jäger arXiv:1212.2263

Fit needs 20 – 50% power corrections (on the observable level)



No sign for q^2 dependence in the theory-independent fit

Significant q^2 dependence if power corrections are fixed at 1GeV
via result of LCSR calculation Khodjamirian et al. arXiv:1211.0234

Significance of the LHCb anomalies depend on the assumptions on the power corrections

Ciuchini et al. (arXiv:1512.07157): Fit produces 20-50% nonfact. power corrections on the observable level in the critical bins.

Hurth et al. (arXiv:1603.00865): Assumption of 60% nonfact. power corrections on the amplitude level lead to 17-20 % on the observable level (S_3, S_4, S_5) only.

Calculations beyond guessing numbers

Any reasonable calculation is better than a fit!

Methods offered in the analysis of $B \rightarrow K\ell^+\ell^-$ to calculate power corrections [Kjodjamirian et al. arXiv: 1211.0234](#), also [1006.4945](#)

Crosschecking errors and correlations of formfactor calculation in [Zwicky et al. arXiv: 1503.0553](#) by independent LCSR analysis

Crosscheck of LHCb anomalies with various ratios $R(e/\mu)$

Hurth, Mahmoudi, Neshatpour arXiv:1603.00865

Altmannshofer, Straub arXiv:1503.06199

R_K is theoretically rather clean compared to LHCb anomalies and its tension with the SM cannot be explained by power corrections. But both tensions might be healed by new physics in C_9^μ

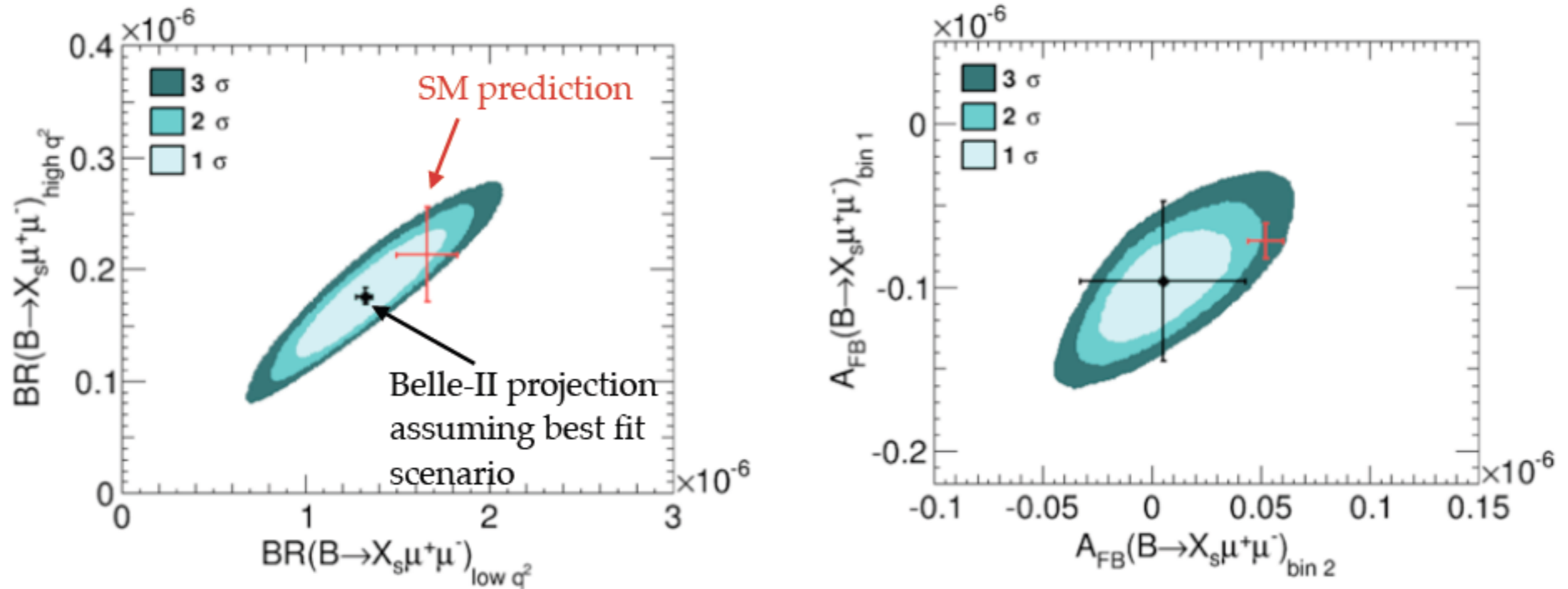
Observable	95% C.L. prediction
$\text{BR}(B \rightarrow X_s \mu^+ \mu^-) / \text{BR}(B \rightarrow X_s e^+ e^-)_{q^2 \in [1,6](\text{GeV})^2}$	[0.61, 0.93]
$\text{BR}(B \rightarrow X_s \mu^+ \mu^-) / \text{BR}(B \rightarrow X_s e^+ e^-)_{q^2 > 14.2(\text{GeV})^2}$	[0.68, 1.13]
$\text{BR}(B^0 \rightarrow K^{*0} \mu^+ \mu^-) / \text{BR}(B^0 \rightarrow K^{*0} e^+ e^-)_{q^2 \in [1,6](\text{GeV})^2}$	[0.65, 0.96]
$\langle F_L(B^0 \rightarrow K^{*0} \mu^+ \mu^-) \rangle / \langle F_L(B^0 \rightarrow K^{*0} e^+ e^-) \rangle_{q^2 \in [1,6](\text{GeV})^2}$	[0.85, 0.96]
$\langle A_{FB}(B^0 \rightarrow K^{*0} \mu^+ \mu^-) \rangle / \langle A_{FB}(B^0 \rightarrow K^{*0} e^+ e^-) \rangle_{q^2 \in [4,6](\text{GeV})^2}$	[-0.21, 0.71]
$\langle S_5(B^0 \rightarrow K^{*0} \mu^+ \mu^-) \rangle / \langle S_5(B^0 \rightarrow K^{*0} e^+ e^-) \rangle_{q^2 \in [4,6](\text{GeV})^2}$	[0.53, 0.92]
$\text{BR}(B^0 \rightarrow K^{*0} \mu^+ \mu^-) / \text{BR}(B^0 \rightarrow K^{*0} e^+ e^-)_{q^2 \in [15,19](\text{GeV})^2}$	[0.58, 0.95]
$\langle F_L(B^0 \rightarrow K^{*0} \mu^+ \mu^-) \rangle / \langle F_L(B^0 \rightarrow K^{*0} e^+ e^-) \rangle_{q^2 \in [15,19](\text{GeV})^2}$	[0.998, 0.999]
$\langle A_{FB}(B^0 \rightarrow K^{*0} \mu^+ \mu^-) \rangle / \langle A_{FB}(B^0 \rightarrow K^{*0} e^+ e^-) \rangle_{q^2 \in [15,19](\text{GeV})^2}$	[0.87, 1.01]
$\langle S_5(B^0 \rightarrow K^{*0} \mu^+ \mu^-) \rangle / \langle S_5(B^0 \rightarrow K^{*0} e^+ e^-) \rangle_{q^2 \in [15,19](\text{GeV})^2}$	[0.87, 1.01]
$\text{BR}(B^+ \rightarrow K^+ \mu^+ \mu^-) / \text{BR}(B^+ \rightarrow K^+ e^+ e^-)_{q^2 \in [1,6](\text{GeV})^2}$	[0.58, 0.95]
$\text{BR}(B^+ \rightarrow K^+ \mu^+ \mu^-) / \text{BR}(B^+ \rightarrow K^+ e^+ e^-)_{q^2 \in [15,22](\text{GeV})^2}$	[0.58, 0.95]

Table 3: Predicted ratios of observables with muons in the final state to electrons in the final state, considering the two operator fit within the $\{C_9^\mu, C_9^e\}$ set.

Crosscheck of LHCb anomalies with inclusive modes

Hurth, Mahmoudi, Neshatpour, arXiv:1410.4545

if SM deviations in R_K and P'_5 persist until Belle-II



If NP then the effect of C_9 and C'_9 are large enough to be checked at Belle-II with theoretically clean modes.

Hurth, Mahmoudi, arXiv:1312.5267 Experimental extrapolation by Kevin Flood

Inclusive modes

New physics sensitivity

Huber, Hurth, Lunghi, arXiv:1503.04849

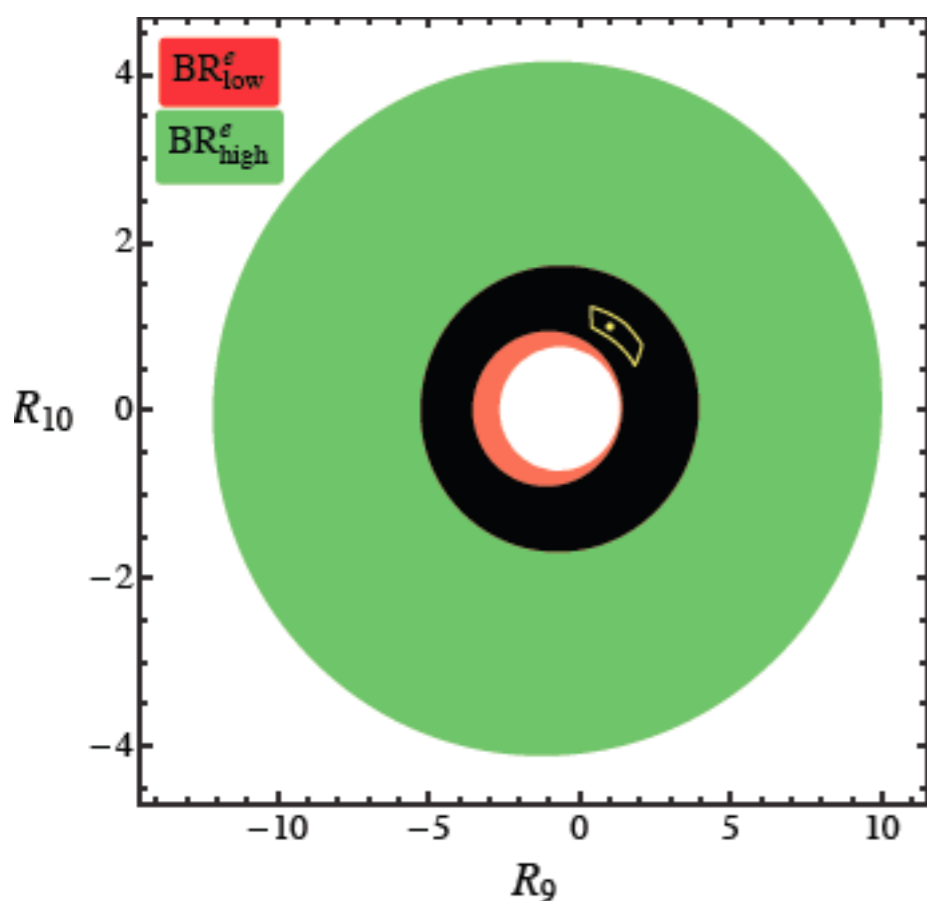
Constraints on Wilson coefficients C_9/C_9^{SM} and $C_{10}/C_{10}^{\text{SM}}$

$$R_i = \frac{C_i(\mu_0)}{C_i^{\text{SM}}(\mu_0)}$$

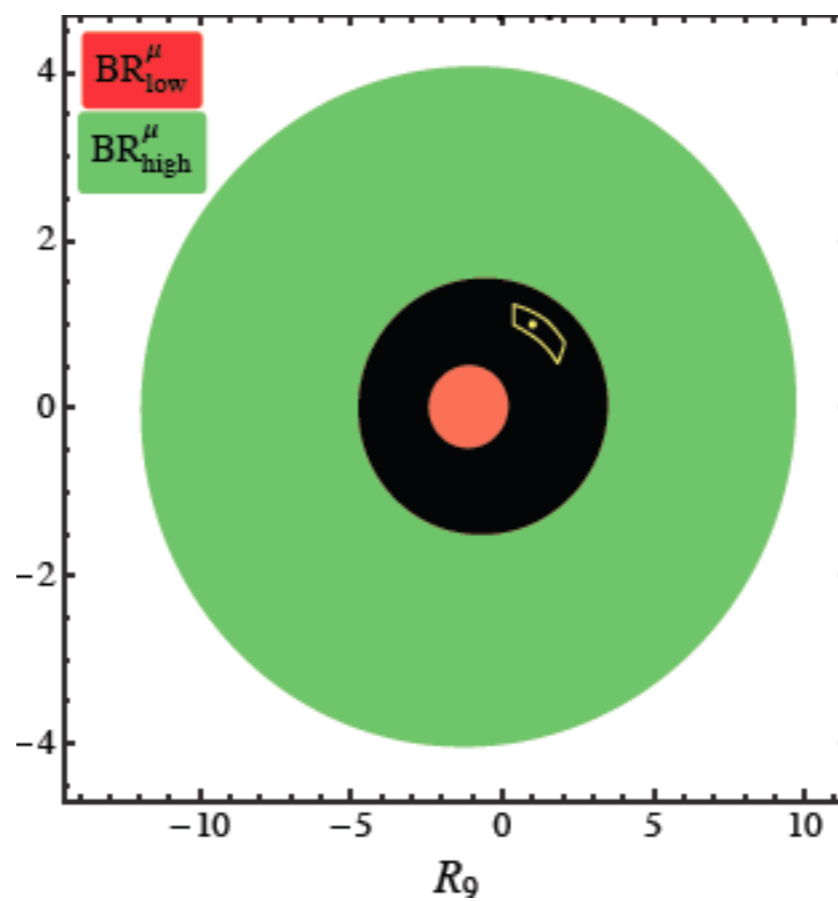
that we obtain at 95% C.L. from present experimental data
(red low q^2 , green high q^2)

that we will obtain at 95% C.L. from $50ab^{-1}$ data at Belle-II
(yellow)

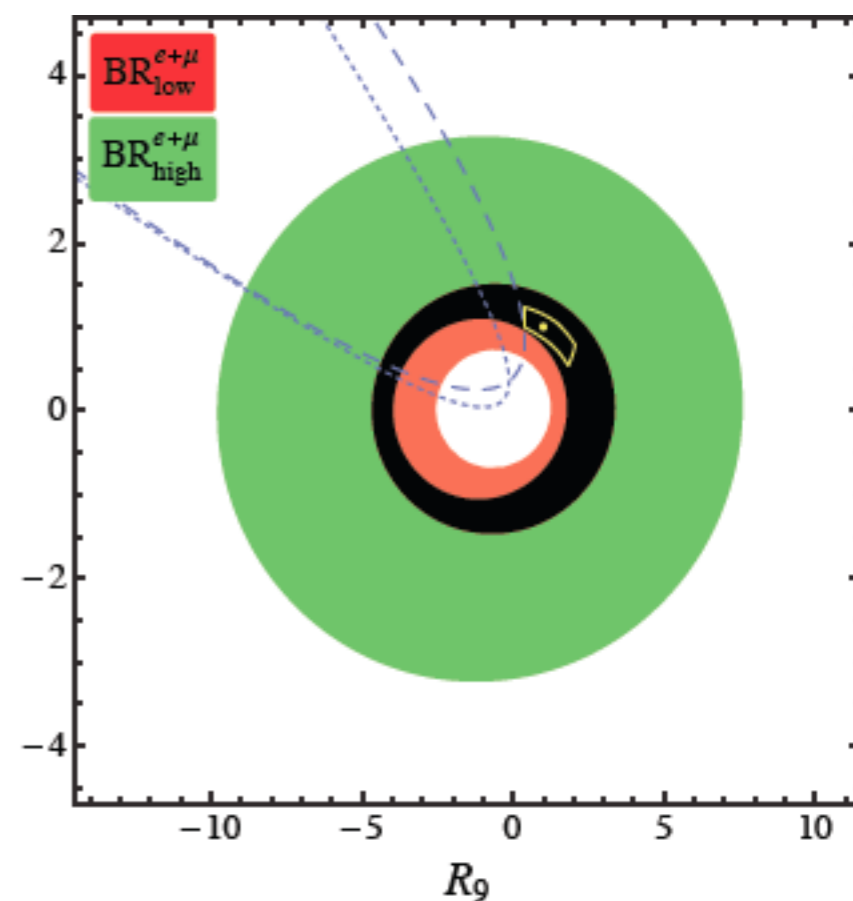
$B \rightarrow X_s e e$



$B \rightarrow X_s \mu \mu$



$B \rightarrow X_s l l$



Complete angular analysis of inclusive $B \rightarrow X_s ll$

Huber, Hurth, Lunghi, arXiv:1503.04849

- "Latest" Belle measurement of branching ratio is based on less than 30% of the total luminosity

Complete angular analysis of inclusive $B \rightarrow X_s \ell \ell$

Huber, Hurth, Lunghi, arXiv:1503.04849

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$$\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8} [(1 + z^2) H_T(q^2) + 2z H_A(q^2) + 2(1 - z^2) H_L(q^2)] \quad (z = \cos \theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \qquad \frac{dA_{\text{FB}}}{dq^2} = 3/4 H_A(q^2)$$

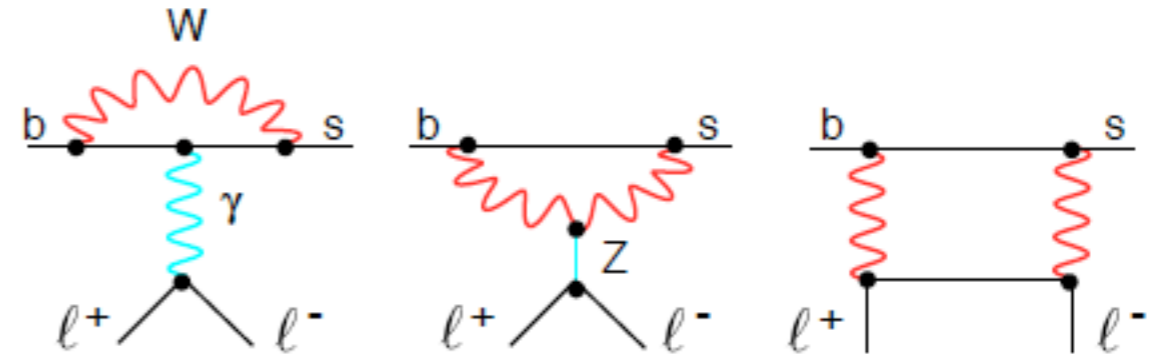
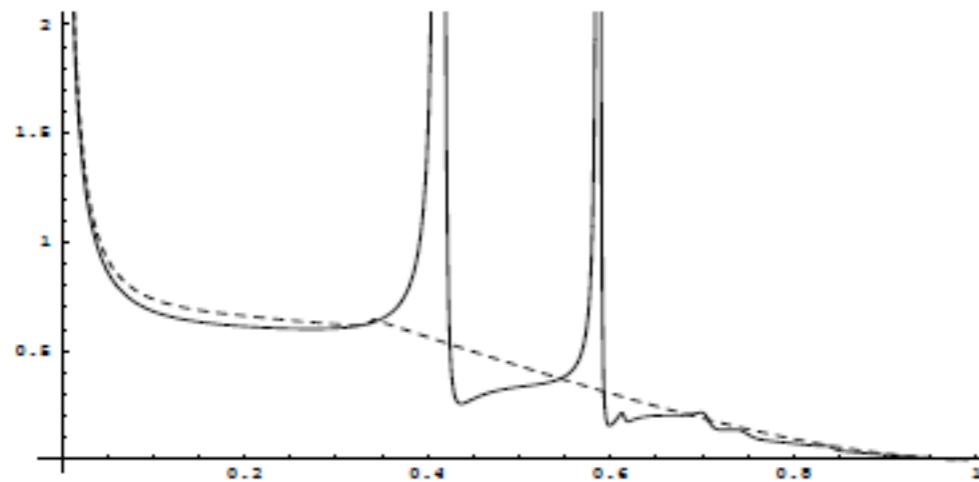
- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables
- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events

Large logs $\log(mb/m_\ell)$ different for muon and electron !

Subleading contributions in $B \rightarrow X_s l^+ l^-$

- On-shell- $c\bar{c}$ -resonances \Rightarrow cuts in dilepton mass spectrum necessary :
 $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ and $14.4\text{GeV}^2 < q^2 \Rightarrow$ perturbative contributions dominant

$$\frac{d}{ds} BR(\bar{B} \rightarrow X_s l^+ l^-) \times 10^{-5}$$



- Hadronic invariant-mass cut is imposed in order to eliminate the background like $b \rightarrow c (\rightarrow se^+ \nu) e^- \bar{\nu} = b \rightarrow se^+ e^- +$ missing energy
 - * Babar, Belle: $m_X < 1.8$ or 2.0GeV
 - * high- q^2 region not affected by this cut
 - * kinematics: X_s is jetlike and $m_X^2 \leq m_b \Lambda_{QCD} \Rightarrow$ shape function region
 - * SCET analysis: universality of jet and shape functions found:
 the 10-30% reduction of the dilepton mass spectrum can be accurately computed using the $\bar{B} \rightarrow X_s \gamma$ shape function
 5% additional uncertainty for 2.0GeV cut due to subleading shape functions

Lee, Stewart hep-ph/0511334

Lee, Ligeti, Stewart, Tackmann hep-ph/0512191

Lee, Tackmann arXiv:0812.0001 (effect of subleading shape functions)

Bell, Beneke, Huber, Li arXiv:1007.3758 (NNLO matching QCD \rightarrow SCET)

Subleading power factorization in $B \rightarrow X_s \ell^+ \ell^-$

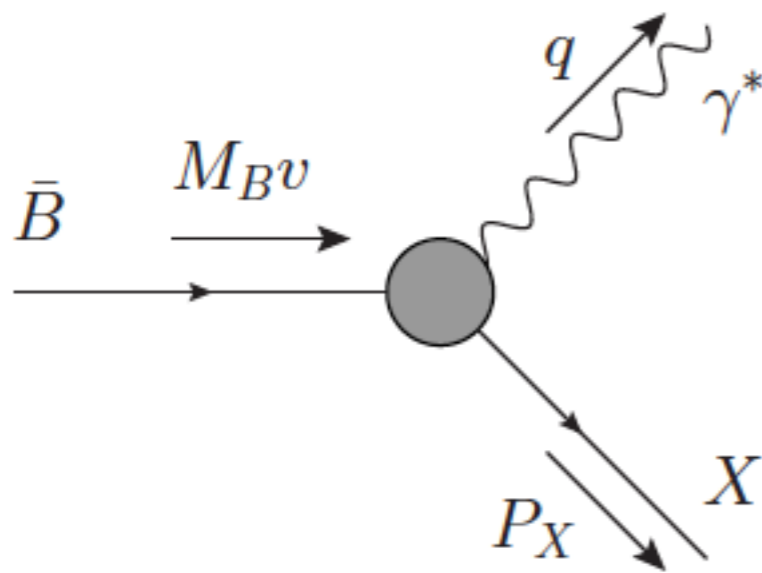
Benzke, Fickinger, Hurth, Turczyk, to appear

Hadronic cut

Additional cut in X_s necessary to reduce background
affects only low- q^2 region.

Hadronic invariant $m_X^2 < 1.8(2.0) \text{ GeV}^2$, jet-like X_s $E_X \sim \mathcal{O}(m_b)$

Multiscale problem \rightarrow SCET



$$M_B^2 \sim m_b^2 \gg m_X^2 \sim \Lambda_{\text{QCD}} m_b \gg \Lambda_{\text{QCD}}^2$$

$$m_X^2 = P_X^2 = (M_B - n \cdot q)(M_B - \bar{n} \cdot q)$$

Scaling

$$\lambda = \Lambda_{\text{QCD}}/m_b$$

Kinematics

B meson rest frame

$$q = p_B - p_X \quad 2 m_B E_X = m_B^2 + M_X^2 - q^2$$

X_s system is jet-like with $E_X \sim m_B$ and $m_X^2 \ll E_X^2$

two light-cone components $p_X^- p_X^+ = m_X^2$

$$\bar{n} p_X = p_X^- = E_X + |\vec{p}_X| \sim \mathcal{O}(m_B)$$

$$n p_X = p_X^+ = E_X - |\vec{p}_X| \sim \mathcal{O}(\Lambda_{\text{QCD}})$$

$$q^+ = n q = m_B - p_X^+, \quad q^- = \bar{n} q = m_B - p_X^-$$

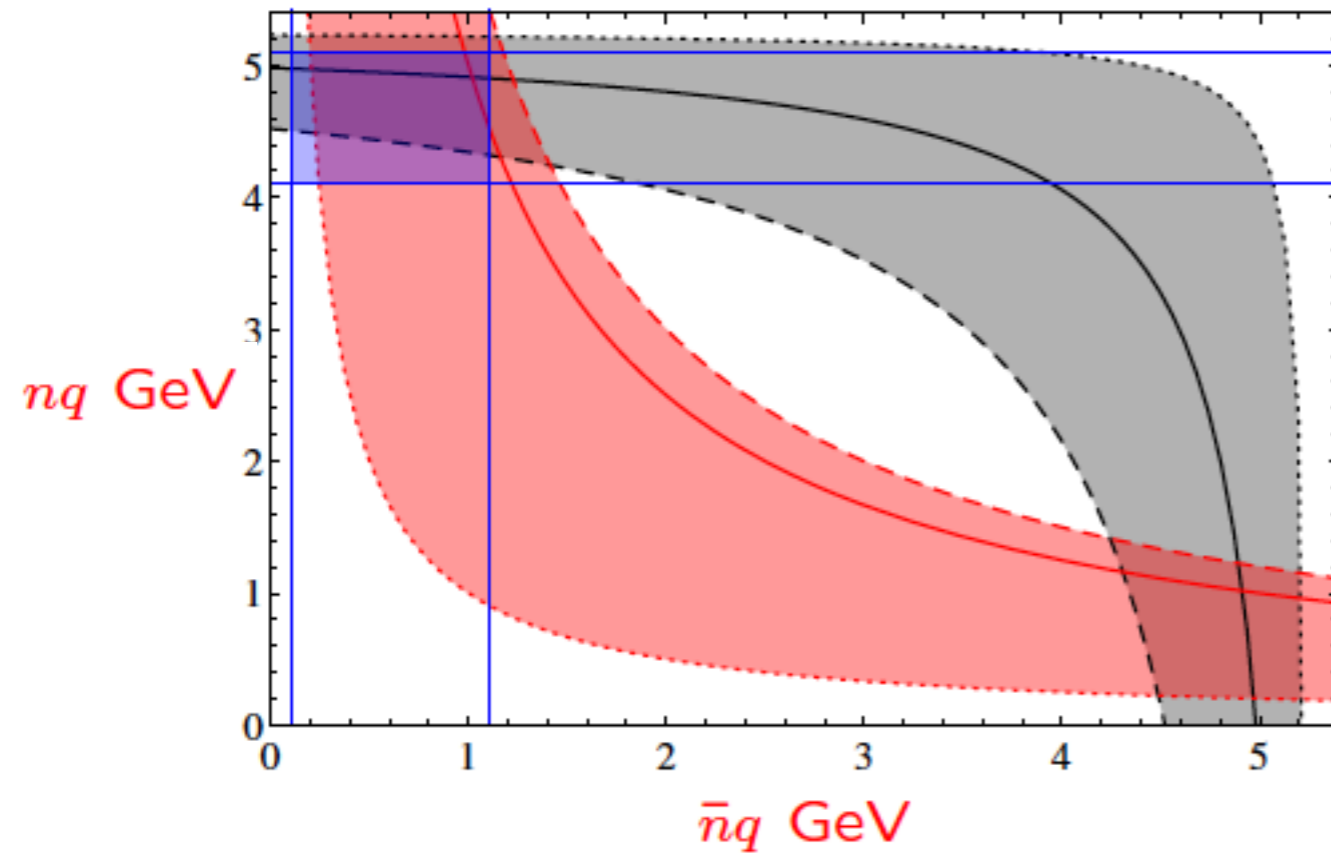
Allowed regions

low- q^2

Red: $q^2 = [1, 5, 6]$ GeV² [Dotted, Solid, Dashed]

Black: $M_x = [0.495, 1.25, 2]$ GeV [Dotted, Solid, Dashed]

Blue: anti-hard-collinear component scaling

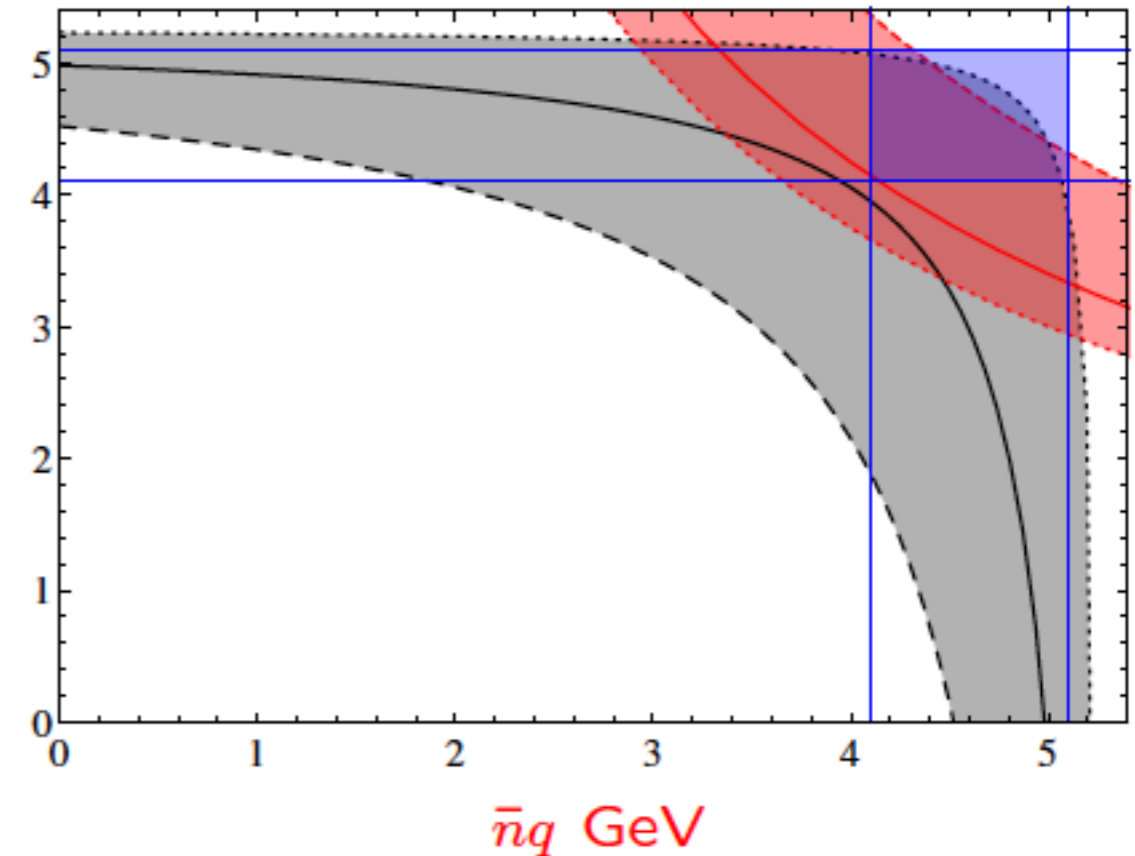


high- q^2

Red: $q^2 = [15, 17, 22]$ GeV² [Dotted, Solid, Dashed]

Black: $M_x = [0.495, 1.25, 2]$ GeV [Dotted, Solid, Dashed]

Blue: hard component scaling



Scaling

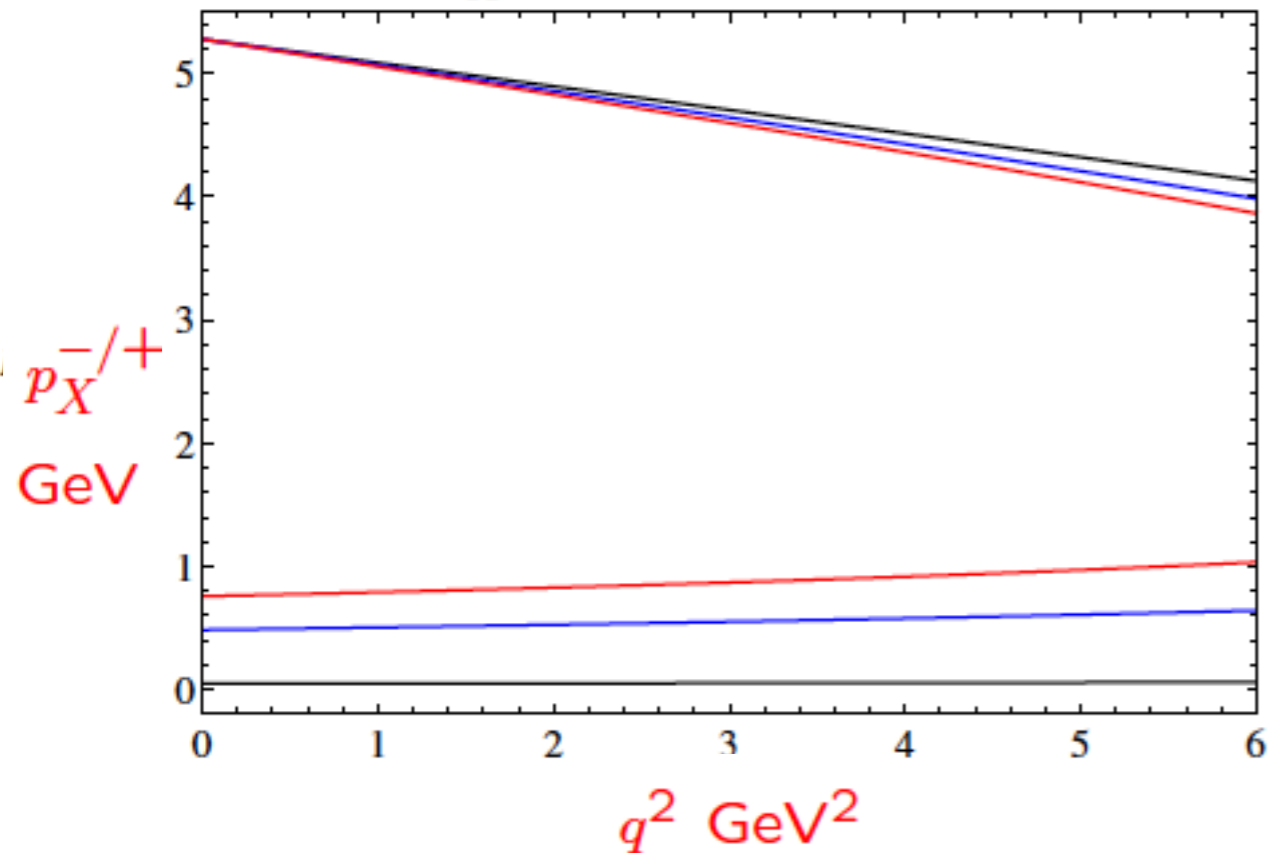
$$\lambda = \Lambda_{\text{QCD}}/m_b$$

$$m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$$

Scaling

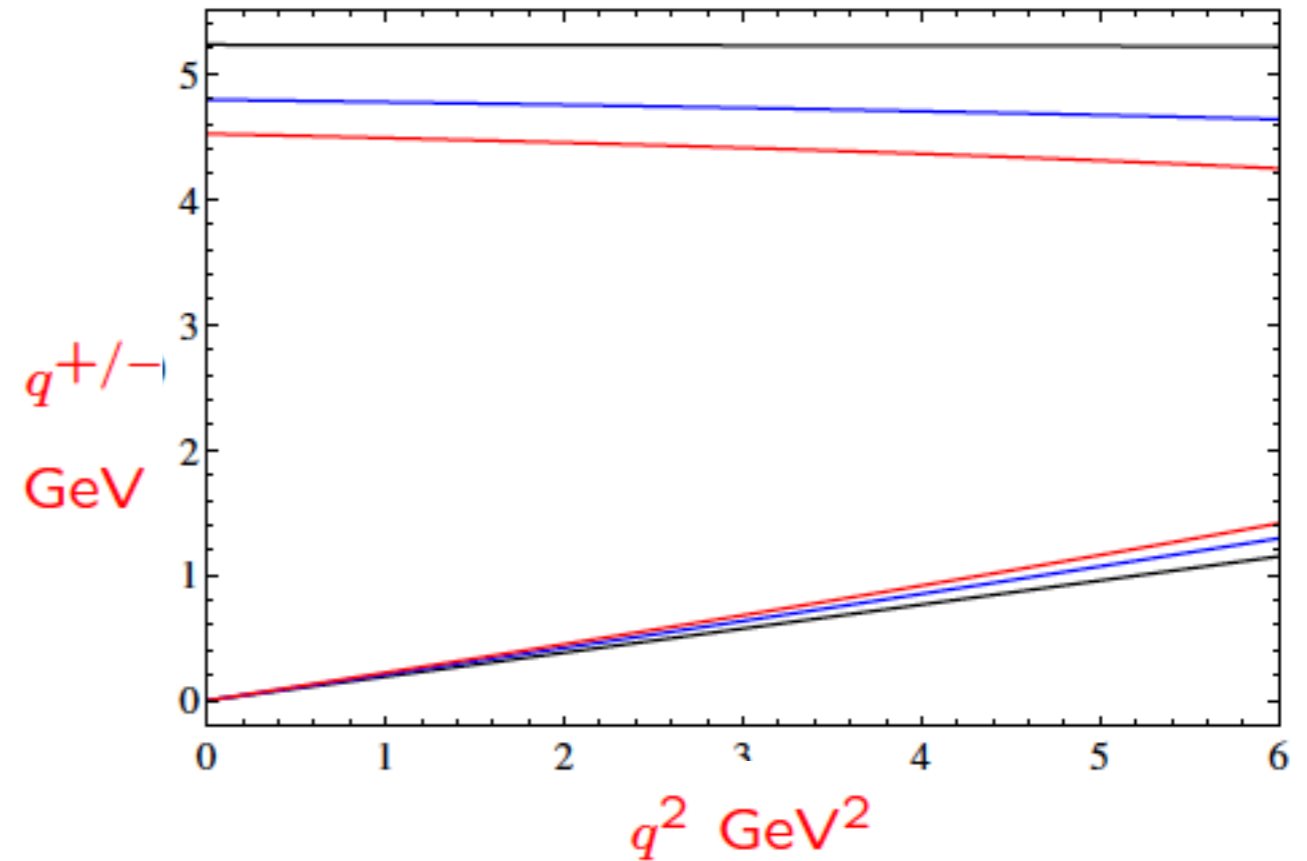
$M_X = [0.5, 1.6, 2]$ GeV [Black, Blue, Red]

Upper lines : P_X^- , lower lines : P_X^+



$M_X = [0.5, 1.6, 2]$ GeV [Black, Blue, Red]

Upper lines : q^+ , lower lines : q^-



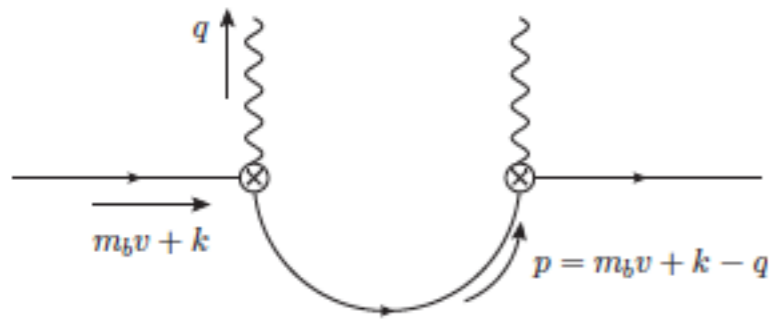
For $q^2 < 6 \text{ GeV}^2$ the scaling of np_X and $\bar{n}p_X$ implies $\bar{n}q$ is of order λ , means q anti-hard-collinear (just kinematics).

Stewart and Lee assume $\bar{n}q$ to be order 1, means q is hard.

This problematic assumption implies a different matching of SCET/QCD.

Shapefunction region

Local OPE breaks down for $m_X^2 \sim \lambda$:



$$\frac{1}{(m_b v + k - q)^2} = \frac{1}{m_b - n \cdot q} \left(1 - \frac{n \cdot k}{m_b - n \cdot q} + \dots \right) \frac{1}{m_b - \bar{n} \cdot q}$$

Resummation of leading contributions into a shape function.

(scaling of $\bar{n}q$ does not matter here; zero in case of $B \rightarrow X_s \gamma$)

Factorization theorem $d\Gamma \sim H \cdot J \otimes S$

The hard function H and the jet function J are perturbative quantities.

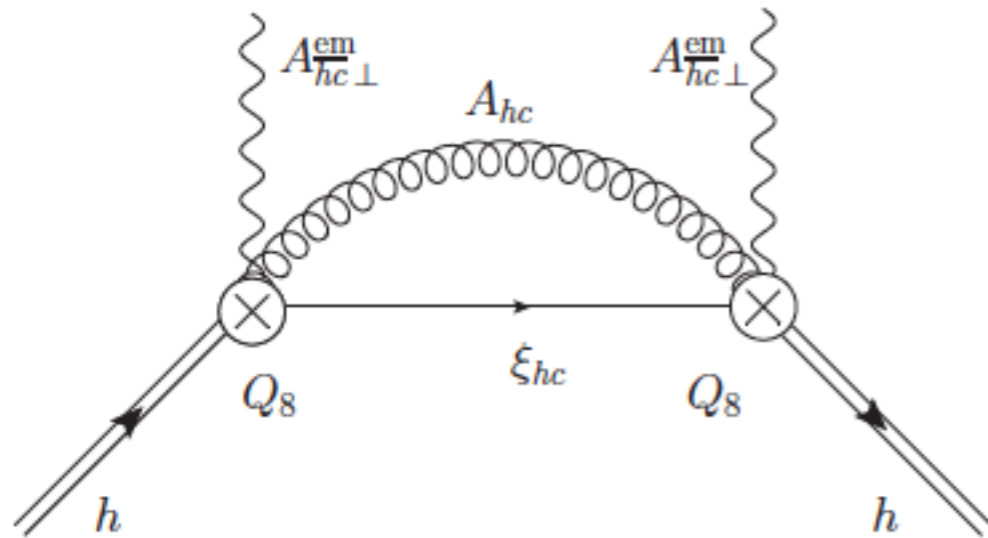
The shape function S is a non-perturbative non-local HQET matrix element.

(universality of the shape function, uncertainties due to subleading shape functions)

Calculation at subleading power

Example of **direct** photon contribution which factorizes

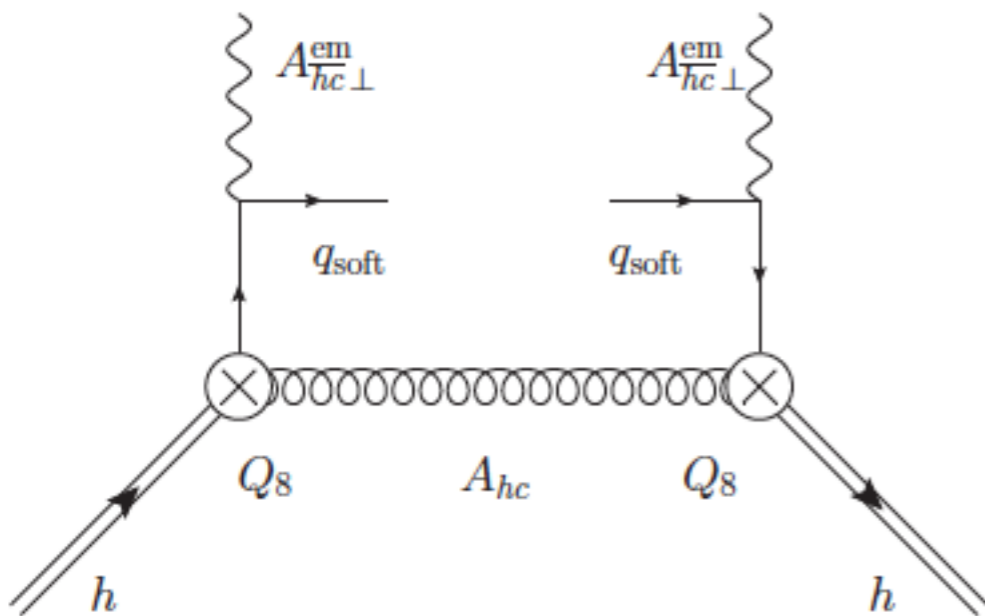
$$d\Gamma \sim H \cdot j \otimes S$$



$\rightarrow \frac{\alpha_s}{m_b}$ in low m_χ^2 region

Example of **resolved** photon contribution (double-resolved) which factorizes

$$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$$

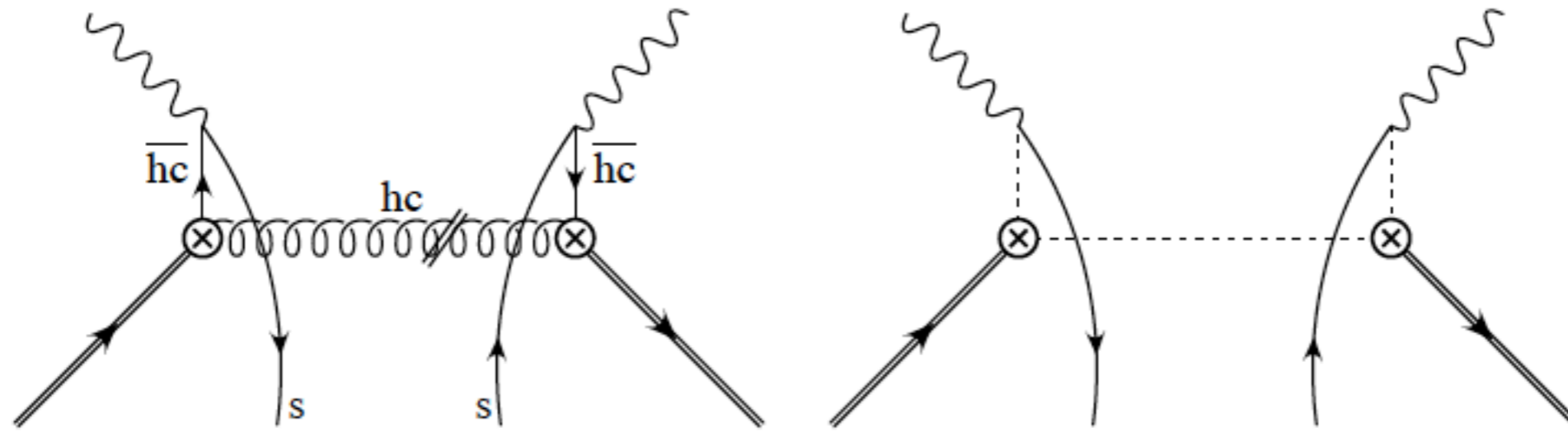


$\rightarrow \frac{\Lambda}{m_b}$

Shape function is non-local in two light-cone directions.

It survives $M_\chi \rightarrow 1$ limit (irreducible uncertainty).

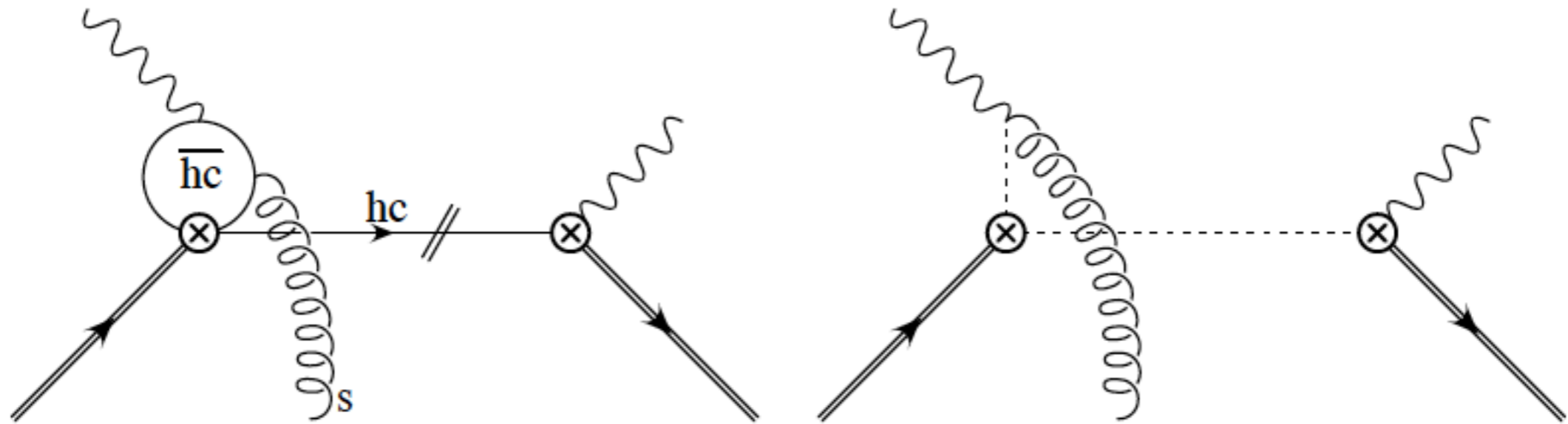
Interference of Q_8 and Q_8



$$\frac{d\Gamma^{\text{res}}}{dn \cdot q d\bar{n} \cdot q} \sim \frac{e_s^2 \alpha_s}{m_b} \int d\omega \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + \bar{n} \cdot q + i\epsilon} \int \frac{d\omega_2}{\omega_2 + \bar{n} \cdot q - i\epsilon} g_{88}(\omega, \omega_1, \omega_2)$$

$$g_{88}(\omega, \omega_1, \omega_2) = \frac{1}{M_B} \langle \bar{B} | \bar{h}(\mathbf{tn}) \dots s(\mathbf{tn} + \mathbf{u}\bar{\mathbf{n}}) \bar{s}(\mathbf{r}\bar{\mathbf{n}}) \dots h(\mathbf{0}) | \bar{B} \rangle_{\text{F.T.}}$$

Interference of Q_1 and Q_7



$$\frac{d\Gamma^{\text{res}}}{dn \cdot q d\bar{n} \cdot q} \sim \frac{1}{m_b} \int d\omega \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + i\epsilon}$$

$$\frac{1}{\omega_1} \left[\bar{n} \cdot q \left(F \left(\frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) - 1 \right) - (\bar{n} \cdot q + \omega_1) \left(F \left(\frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) - 1 \right) \right.$$

$$\left. + \bar{n} \cdot q \left(G \left(\frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) - G \left(\frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) \right) \right] g_{17}(\omega, \omega_1)$$

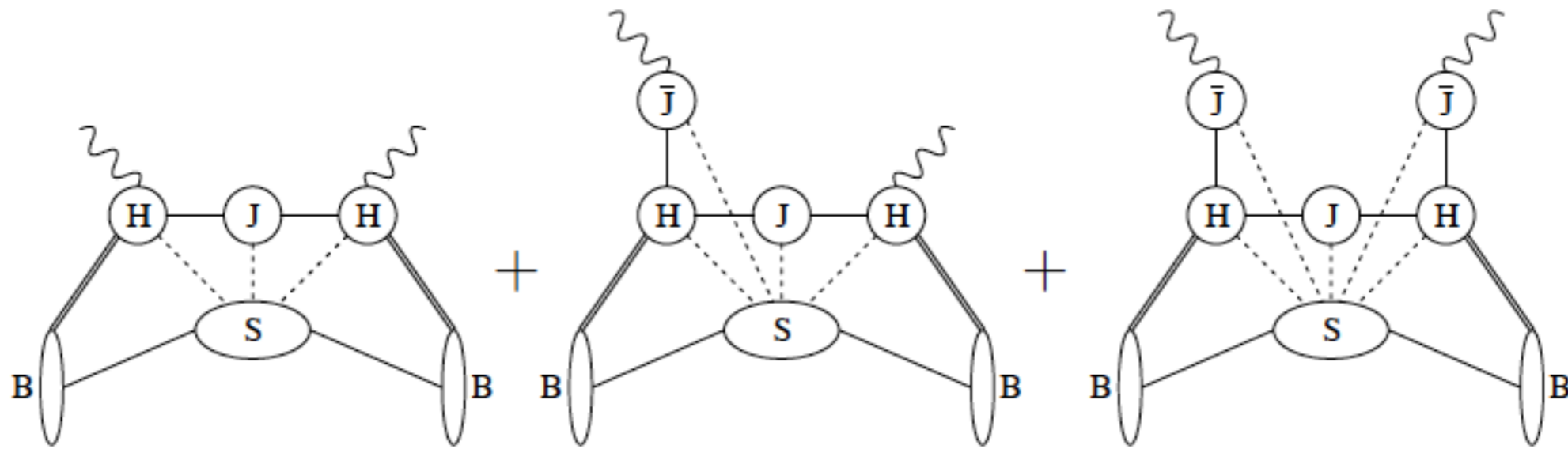
$$g_{17}(\omega, \omega_1) = \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{dt}{2\pi} e^{-i\omega t} \frac{1}{M_B} \langle \bar{B} | \bar{h}(t\mathbf{n}) \dots G_s^{\alpha\beta}(r\bar{\mathbf{n}}) \dots h(0) | \bar{B} \rangle$$

Expansion for $m_c \sim m_b$ leads to Voloshin term in the total rate ($-\lambda_2/m_c^2$), the terms stays non-local for $m_c < m_b$.

Factorization formula

In the $m_X^2 \sim \lambda$ and $q^2 \sim \lambda$ region we have the following factorization formula

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum H \cdot j_i \otimes S + \frac{1}{m_b} \sum H \cdot J \otimes s_i + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$



Numerical evaluation (work in progress)

Similar subleading shape functions as in $B \rightarrow X_s \gamma$

Use vacuum insertion approximation, PT invariance,....

Power corrections in the inclusive mode

- For q anti-hard-collinear we have identified a new type of subleading power corrections.
- In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.
- They constitute an irreducible uncertainty because they survive the $M_X \rightarrow 1$ limit.
- If q was hard then these resolved contributions would not exist

M_X cut effects in the low- q^2 region with q^2 anti-hard-collinear

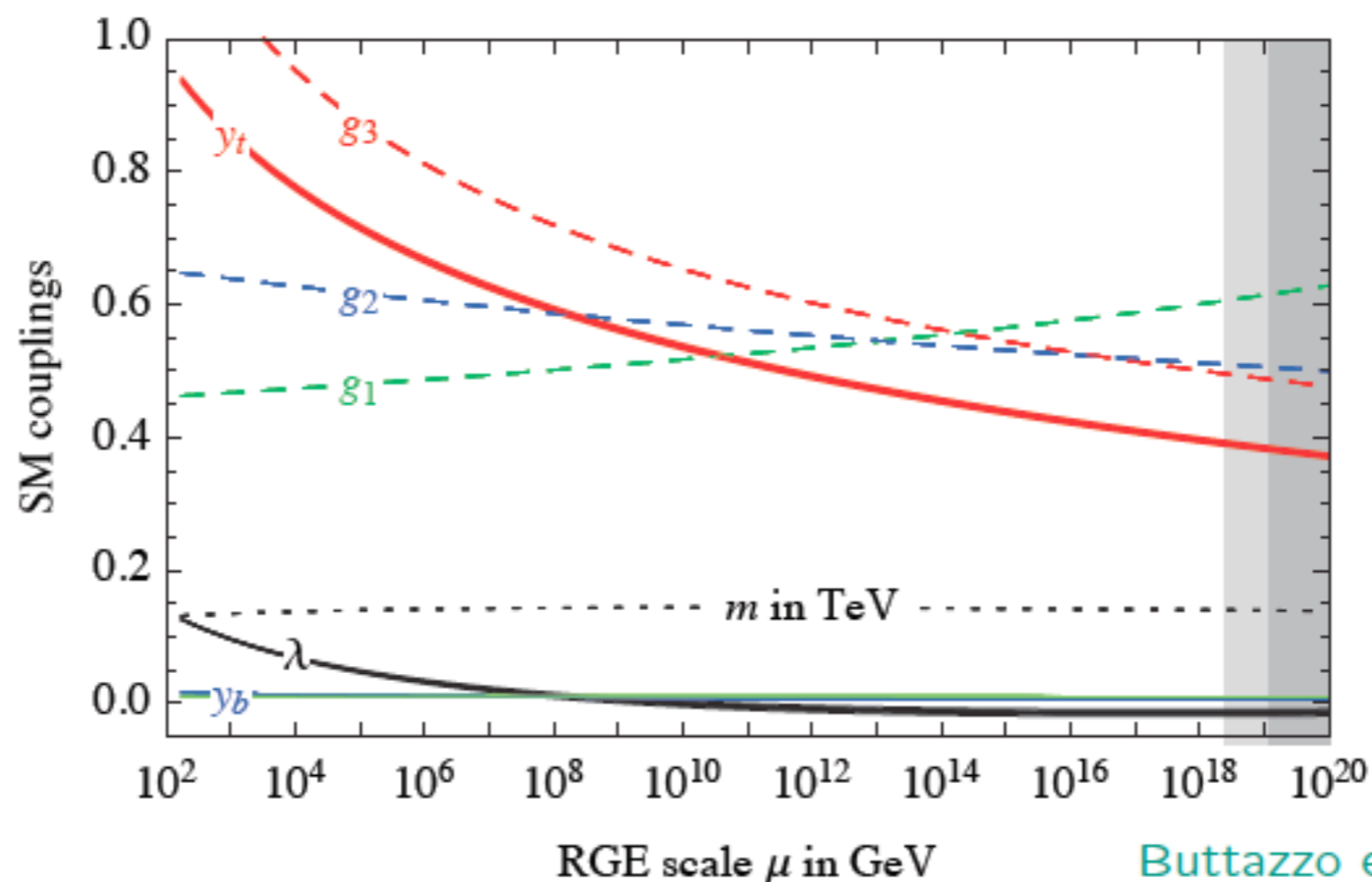
(work in progress)

Epilogue

Self-consistency of the SM

Do we need new physics beyond the SM ?

- It is possible to extend the validity of the SM up to the M_P as weakly coupled theory.



High-energy extrapolation shows that the Yukawa couplings, weak gauge couplings and the Higgs self coupling remain perturbative in the entire energy domain between the electroweak and Planck scale (no Landau poles!).

- Renormalizability implies no constraints on the free parameters of the SM Lagrangian.

Experimental evidence beyond SM

- **Dark matter** (visible matter accounts for only 4% of the Universe)
- **Neutrino masses** (Dirac or Majorana masses ?)
- **Baryon asymmetry of the Universe** (new sources of CP violation needed)

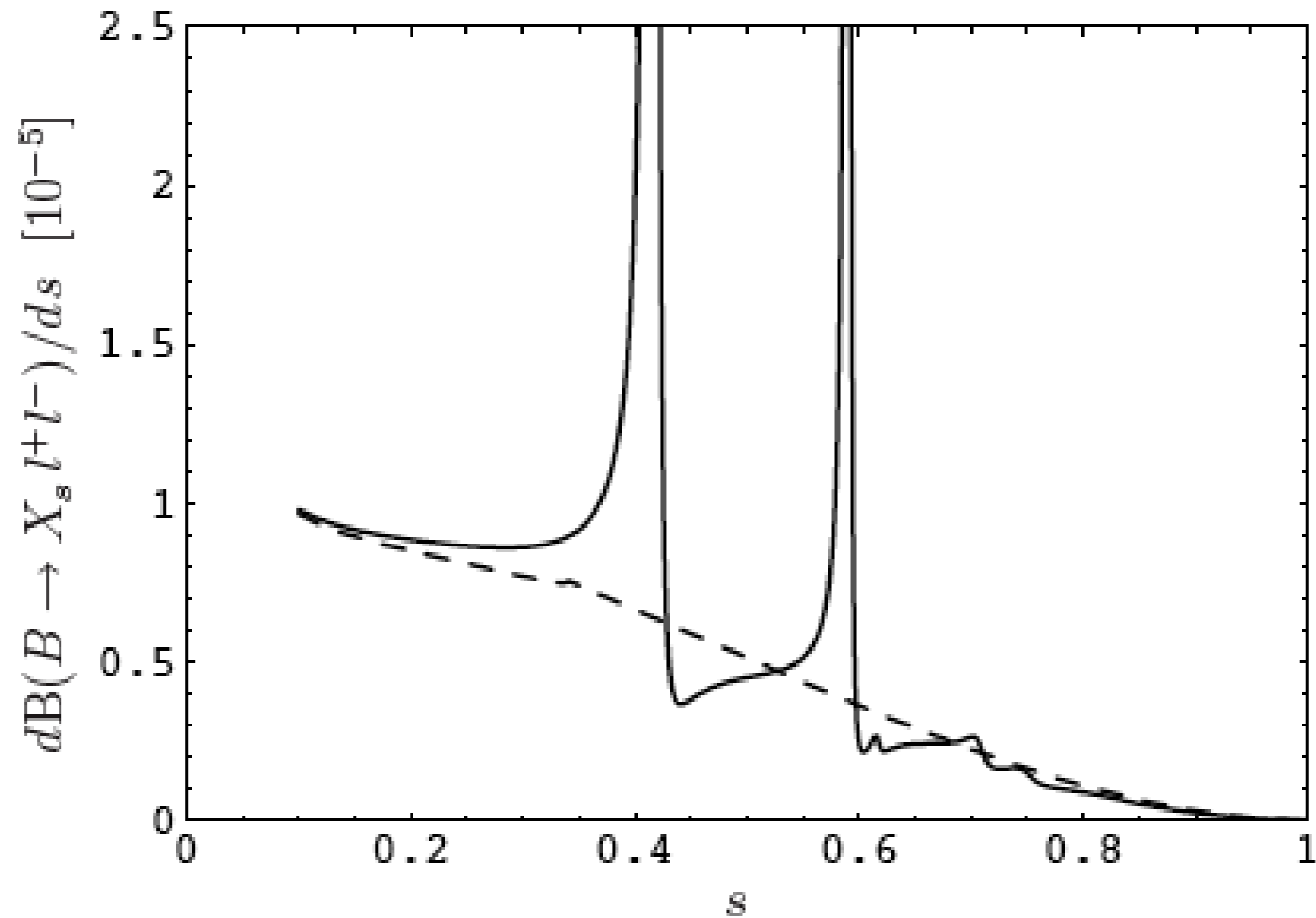
Caveat:

Answers perhaps wait at energy scales which we do not reach with present experiments.

Extra

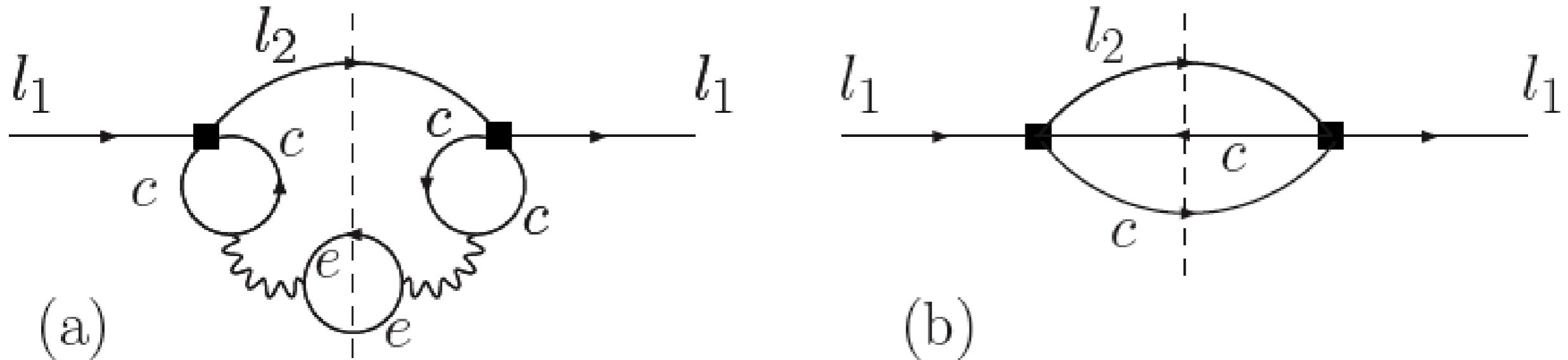
Quark-hadron duality violated in $\bar{B} \rightarrow X_s \ell^+ \ell^-$? BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances J/ψ and ψ' exceed the perturbative contributions **by two orders** of magnitude.



Quark-hadron duality violated in $\bar{B} \rightarrow X_s \ell^+ \ell^-$? BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances J/ψ and ψ' exceed the perturbative contributions **by two orders** of magnitude.



The rate $l_1 \rightarrow l_2 e^+ e^-$ (a) is connected to the integral over $|\Pi(q^2)|^2$ for which global duality is **NOT** expected to hold.

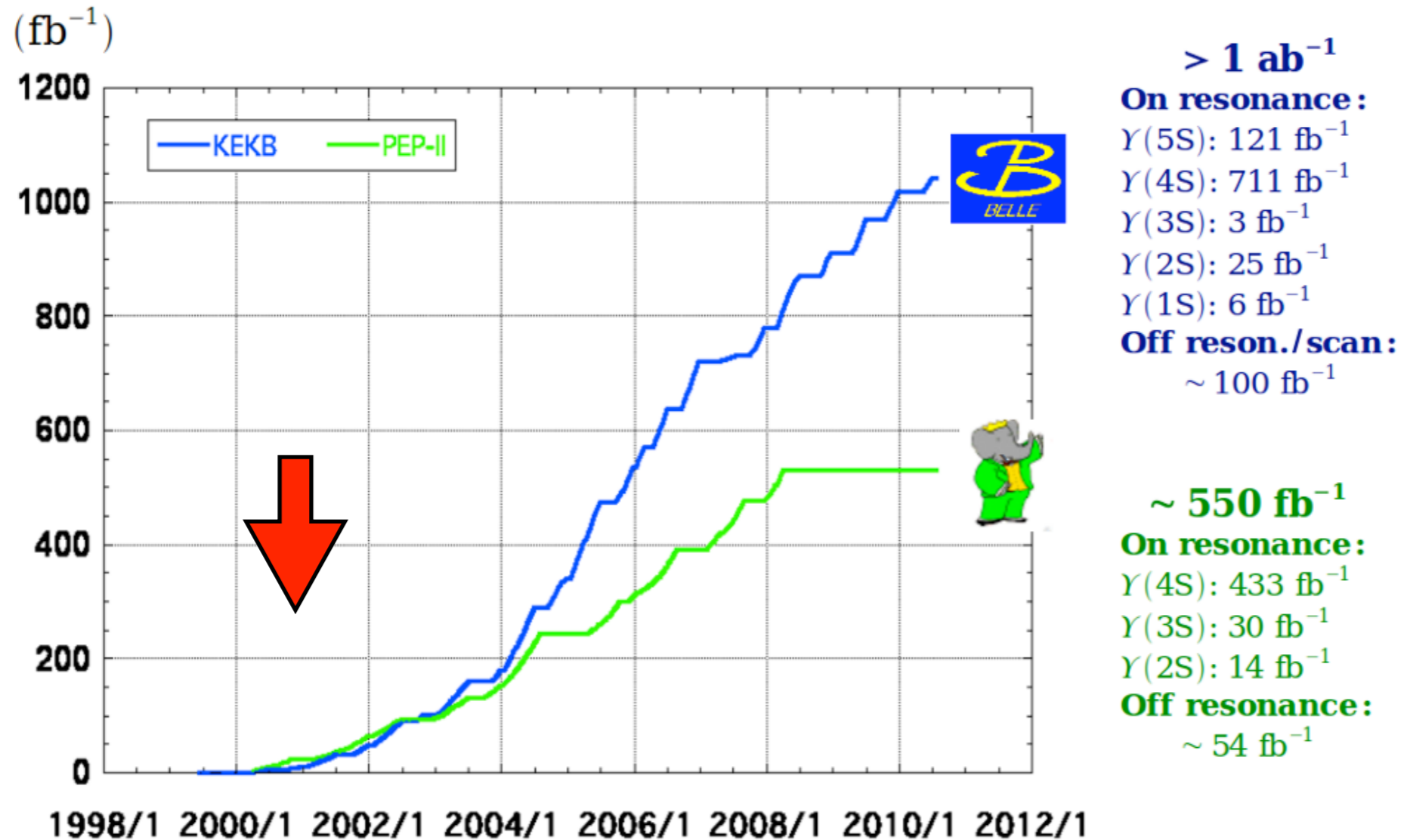
In contrast the inclusive hadronic rate $l_1 \rightarrow l_2 X$ (b) corresponds to the imaginary part of the correlator $\Pi(q^2)$.

- "Latest" Belle measurement of branching ratio is based on less than 30% of the total luminosity

Belle hep-ex/0503044 (!!!) (based $152 \times 10^6 B\bar{B}$ events)

Babar hep-ex/0404006 (!!!) (based $89 \times 10^6 B\bar{B}$ events)

Integrated luminosity of B factories



Two new analyses from the *B* factories:

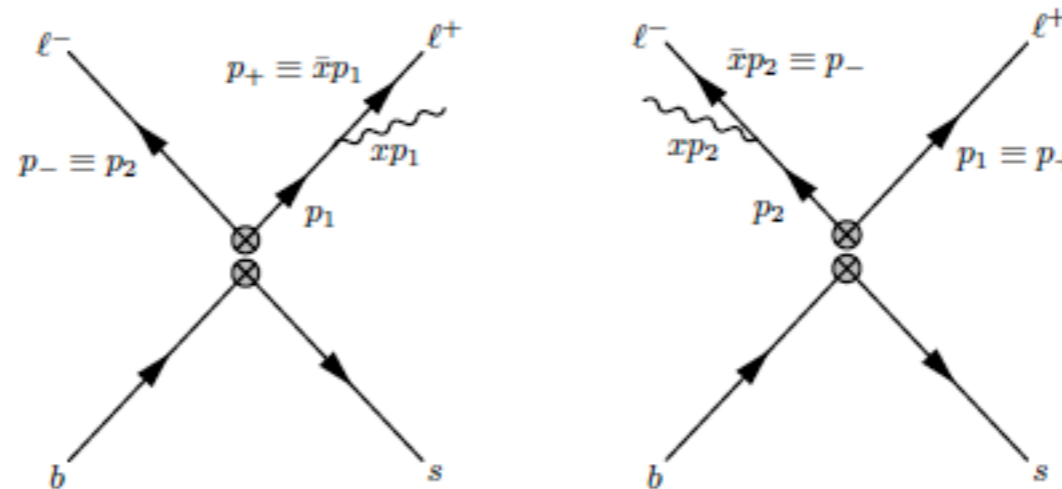
New Babar analysis on dilepton spectrum [arXiv:1312.3664](https://arxiv.org/abs/1312.3664)

New Belle analysis on AFB [arXiv:1402.7134](https://arxiv.org/abs/1402.7134)

- Collinear Photons give rise to log-enhanced QED corrections $\alpha_{\text{em}} \log(m_b^2/m_\ell^2)$
- Higher powers of z in double differential decay width
 - Definition of H_i ? Sensitivity for QED observables ?

We use Legendre polynomials for H_T and H_L and $\text{Sign}(z)$ for H_A

We can construct QED sensitive observables (vanish in absence of QED) by Legendre projectors $P_3(z)$ or $P_4(z)$: 10^{-8}



- Collinear Photons give rise to log-enhanced QED corrections $\alpha_{\text{em}} \log(m_b^2/m_\ell^2)$
- Higher powers of z in double differential decay width
 - Definition of H_i ? Sensitivity for QED observables ?
- Size of logs depend on experimental set-up

$$q^2 = (p_{e^+} + p_{e^-})^2 \quad \text{vs.} \quad q^2 = (p_{e^+} + p_{e^-} + p_{\gamma,\text{coll}})^2$$

- We assume no photons are included in the definition of q^2 (di-muon channel at Babar/Belle, di-electron at Belle)
- Babar's di-electron channel: Photons that are emitted in a cone of 35 mrad angular opening are included in q^2

Monte Carlo techniques needed to estimate this effect

$$\frac{[\mathcal{B}_{ee}^{\text{low}}]_{q=p_{e^+}+p_{e^-}+p_{\gamma,\text{coll}}}}{[\mathcal{B}_{ee}^{\text{low}}]_{q=p_{e^+}+p_{e^-}}} - 1 = 1.65\%$$

$$\frac{[\mathcal{B}_{ee}^{\text{high}}]_{q=p_{e^+}+p_{e^-}+p_{\gamma,\text{coll}}}}{[\mathcal{B}_{ee}^{\text{high}}]_{q=p_{e^+}+p_{e^-}}} - 1 = 6.8\%$$

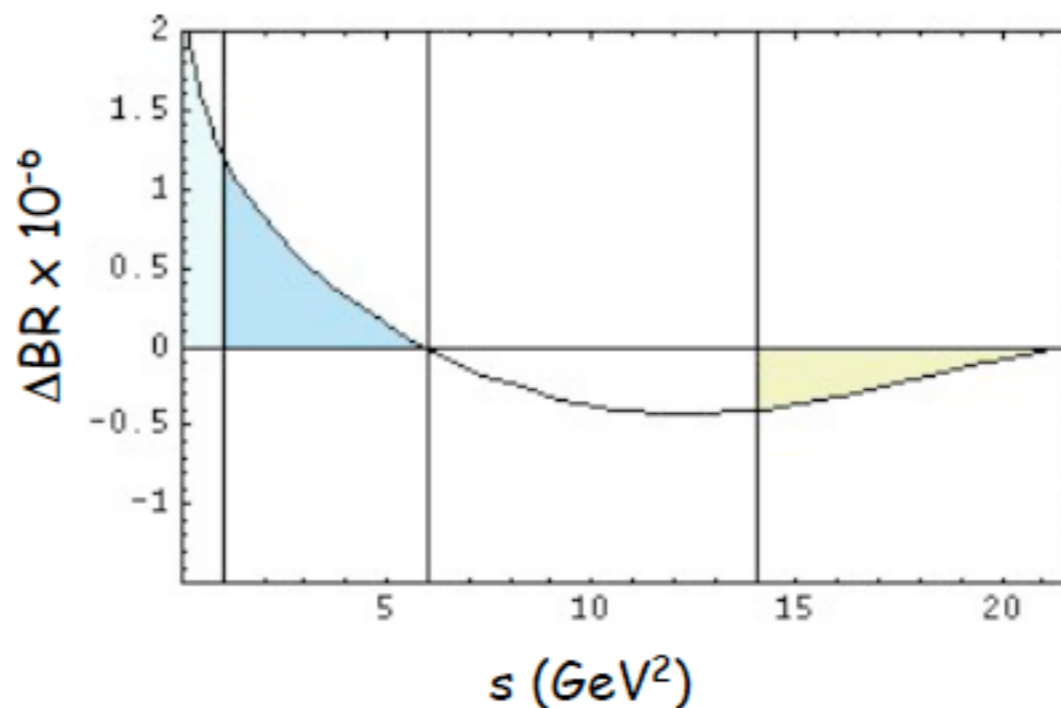
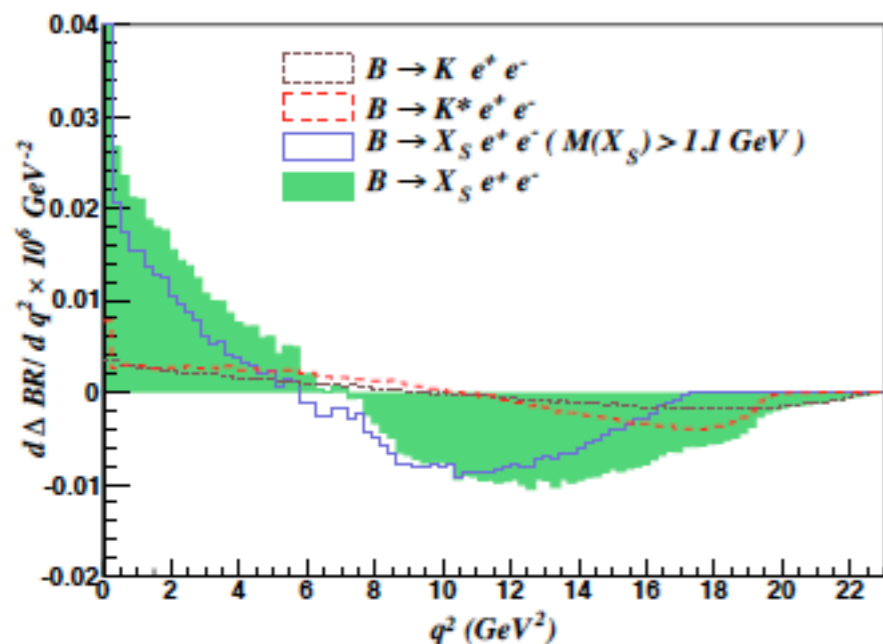
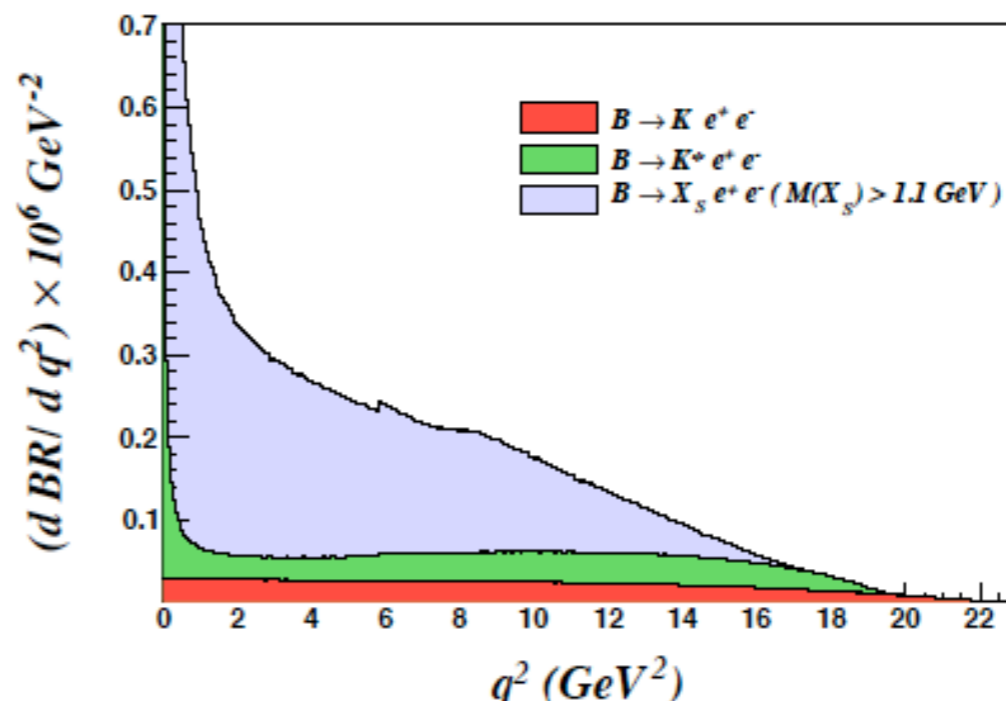
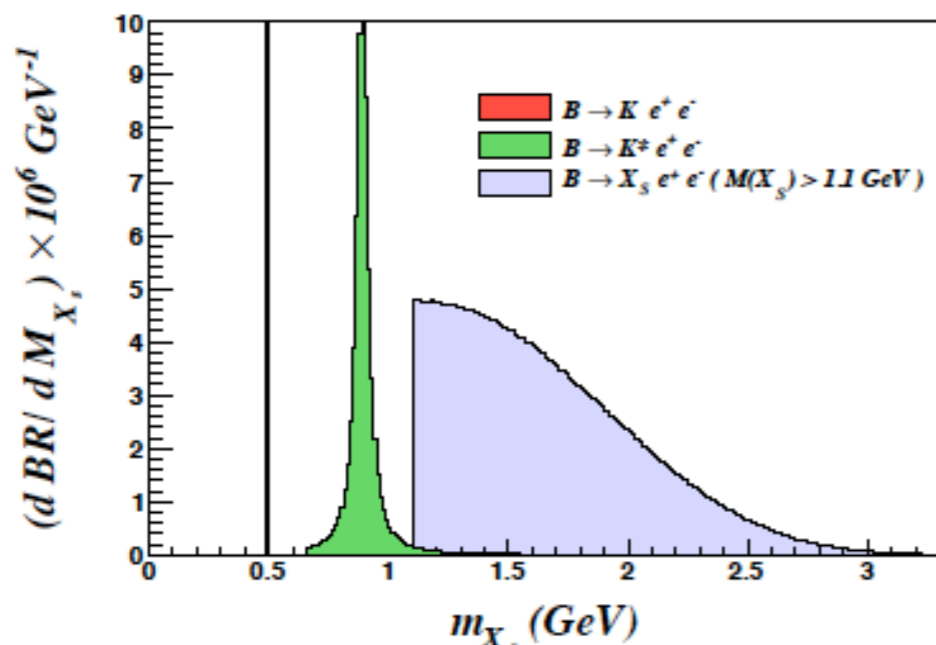
Monte Carlo analysis

Huber, Hurth, Lunghi, arXiv:1503.04849

(event generator EVTGEN, hadronization JETSET, EM radiation PHOTOS)

$$\frac{[\mathcal{B}_{ee}^{\text{low}}]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\text{coll}}}}}{[\mathcal{B}_{ee}^{\text{low}}]_{q=p_{e^+}+p_{e^-}}} - 1 = 1.65\%$$

$$\frac{[\mathcal{B}_{ee}^{\text{high}}]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\text{coll}}}}}{[\mathcal{B}_{ee}^{\text{high}}]_{q=p_{e^+}+p_{e^-}}} - 1 = 6.8\%$$



Results

Low- q^2 ($1\text{GeV}^2 < q^2 < 6\text{GeV}^2$)

$$BR(B \rightarrow X_s ee) = (1.67 \pm 0.10) 10^{-6}$$

$$BR(B \rightarrow X_s \mu\mu) = (1.62 \pm 0.09) 10^{-6}$$

Babar: $BR(B \rightarrow X_s ll) =$

$$= (1.60 (+0.41 - 0.39)_{stat} (+0.17 - 0.13)_{syst} (\pm 0.18)_{mod}) 10^{-6}$$

good agreement with SM

Results

High- q^2 , Theory: $q^2 > 14.4\text{GeV}^2$, Babar: $q^2 > 14.2\text{GeV}^2$

$$BR(B \rightarrow X_s ee) = (0.220 \pm 0.070) 10^{-6}$$

$$BR(B \rightarrow X_s \mu\mu) = (0.253 \pm 0.070) 10^{-6}$$

Babar: $BR(B \rightarrow X_s ll) =$

$$(0.57 (+0.16 - 0.15)_{stat} (+0.03 - 0.02)_{syst}) 10^{-6}$$

2σ higher than SM

Significant higher values predicted in Greub et al. due to missing power and QED corrections and different cut Greub,Pilipp,Schupbach,arXiv:0810.4077

(but perfect agreement if we use their prescriptions)

Further refinement

Normalization to semileptonic $B \rightarrow X_u l \nu$ decay rate **with the same cut** reduces the impact of $1/m_b$ corrections in the high- q^2 region significantly.

Ligeti,Tackmann arXiv:0707.1694

Theory prediction for ratio

$$R(s_0)_{ee} = (2.25 \pm 0.31) 10^{-3}$$

$$R(s_0)_{\mu\mu} = (2.62 \pm 0.30) 10^{-3}$$

Largest source of error are CKM elements (V_{ub})

Note: Additional $O(5\%)$ uncertainty due to nonlocal power corrections $O(\alpha_s \Lambda/m_b)$

Further results in units of 10^{-6}

$$H_L[1, 3.5]_{ee} = 0.64 \pm 0.03$$

$$H_L[1, 3.5]_{\mu\mu} = 0.68 \pm 0.04$$

$$H_L[3.5, 6]_{ee} = 0.50 \pm 0.03$$

$$H_L[3.5, 6]_{\mu\mu} = 0.53 \pm 0.03$$

$$H_L[1, 6]_{ee} = 1.13 \pm 0.06$$

$$H_L[1, 6]_{\mu\mu} = 1.21 \pm 0.07$$

$$H_T[1, 3.5]_{ee} = 0.29 \pm 0.02$$

$$H_T[1, 3.5]_{\mu\mu} = 0.21 \pm 0.01$$

$$H_T[3.5, 6]_{ee} = 0.24 \pm 0.02$$

$$H_T[3.5, 6]_{\mu\mu} = 0.19 \pm 0.02$$

$$H_T[1, 6]_{ee} = 0.53 \pm 0.04$$

$$H_T[1, 6]_{\mu\mu} = 0.40 \pm 0.03$$

$$H_A[1, 3.5]_{ee} = -0.103 \pm 0.005$$

$$H_A[1, 3.5]_{\mu\mu} = -0.110 \pm 0.005$$

$$H_A[3.5, 6]_{ee} = +0.073 \pm 0.012$$

$$H_A[3.5, 6]_{\mu\mu} = +0.067 \pm 0.012$$

$$H_A[1, 6]_{ee} = -0.029 \pm 0.016$$

$$H_A[1, 6]_{\mu\mu} = -0.042 \pm 0.016$$

Total error $\mathcal{O}(5 - 8\%)$. Still dominated by scale uncertainty.