# SEMILEPTONIC B DECAYS 

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## IMPORTANCE OF $\left|V_{x b}\right|$

$V_{c b}$ plays an important role in the determination of UT

$$
\begin{aligned}
& \quad \varepsilon_{K} \approx x\left|V_{c b}\right|^{4}+\ldots \\
& \text { and in the prediction of }
\end{aligned}
$$ FCNC: $\propto\left|V_{t b} V_{t s}\right|^{2} \simeq\left|V_{c b}\right|^{2}\left[1+O\left(\lambda^{2}\right)\right]$ where it often dominates the theoretical uncertainty.

$V_{u b} / V_{c b}$ constrains directly the UT


Since several years, exclusive decays prefer smaller $\left|V_{u b}\right|$ and $\left|V_{c b}\right|$

## SEMITAUONIC ANOMALY

$$
R\left(D^{(*)}\right)=\frac{\mathcal{B}\left(B \rightarrow D^{(*)} \tau \nu\right)}{\mathcal{B}\left(B \rightarrow D^{(*)} \mu \nu\right)}
$$



## INCLUSIVE SEMILEPTONIC B DECAYS

OPE allows us to write inclusive observables as double series in $\Lambda / m_{b}$ and $\alpha_{s}$

$$
\begin{gathered}
M_{i}=M_{i}^{(0)}+\frac{\alpha_{s}}{\pi} M_{i}^{(1)}+\left(\frac{\alpha_{s}}{\pi}\right)^{2} M_{i}^{(2)}+\left(M_{i}^{(\pi, 0)}+\frac{\alpha_{s}}{\pi} M_{i}^{(\pi, 1)}\right) \frac{\mu_{\pi}^{2}}{m_{b}^{2}} \\
\quad+\left(M_{i}^{(G, 0)}+\frac{\alpha_{s}}{\pi} M_{i}^{(G, 1)}\right) \frac{\mu_{G}^{2}}{m_{b}^{2}}+M_{i}^{(D, 0)} \frac{\rho_{D}^{3}}{m_{b}^{3}}+M_{i}^{(L S, 0)} \frac{\rho_{L S}^{3}}{m_{b}^{3}}+\ldots \\
\left.\left.\mu_{\pi}^{2}(\mu)=\left.\frac{1}{2 M_{B}}\langle B| \vec{b}(\vec{D})^{2} b_{\psi}\right|_{\psi}\right\rangle_{\mu} \quad \mu_{G}^{2}(\mu)=\left.\frac{1}{2 M_{B}}\langle B| \bar{b} \frac{i}{v} \frac{\sigma_{\mu v} G^{\mu v}}{\psi}\right|_{\psi}\right\rangle_{\mu}
\end{gathered}
$$

OPE valid for inclusive enough measurements, away from perturbative singularities
Current fits includes 6 non-pert parameters
$m_{b, c} \quad \mu_{\pi, G}^{2} \quad \rho_{D, L S}^{3}$
and all known corrections up to $O\left(\Lambda^{3} / m_{b}{ }^{3}\right)$

## EXTRACTION OF THE OPE PARAMETERS

$\mathrm{E}_{1}$ spectrum

hadronic mass spectrum


Global shape parameters (first moments of the distributions) tell us about $m_{b}, m_{c}$ and the B structure, total rate about $\left|V_{c b}\right|$

OPE parameters describe universal properties of the B meson and of the quarks $\rightarrow$ useful in many applications (rare decays, $V_{u b}, \ldots$ )

## THE SEMILEPTONIC FIT

Alberti, Healey, Nandi, PG 1411.6560

- kinetic scheme calculation based on 1107.3100; hep-ph/0401063
- includes all $\mathrm{O}\left(\alpha_{s}^{2}\right)$ and $\mathrm{O}\left(\alpha_{s} / \mathrm{mb}_{\mathrm{b}}{ }^{2}\right)$ corrections
- reassessment of theoretical errors, realistic correlations following Schwanda, PG, 1307.4551
- external constraints: precise heavy quark mass determinations, mild constraints on $\mu^{2}{ }_{G}$ from hyperfine splitting and $\varrho^{3}{ }_{L S}$ from sum rules

Previous global fits: Buchmuller, Flaecher hep-ph/0507253, Bauer et al, hep-ph/0408002 (1S scheme)

## CHARM MASS DETERMINATIONS



Remarkable improvement in recent years.
$m_{c}$ can be used as precise input to fix $m_{b}$ instead of radiative moments

## FIT RESULTS

| $m_{b}^{k i n}$ | $\bar{m}_{c}(3 \mathrm{GeV})$ | $\mu_{\pi}^{2}$ | $\rho_{D}^{3}$ | $\mu_{G}^{2}$ | $\rho_{L S}^{3}$ | $\mathrm{BR}_{c \ell \nu}$ | $10^{3}\left\|V_{c b}\right\|$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4.553 | 0.987 | 0.465 | 0.170 | 0.332 | -0.150 | 10.65 | 42.21 |
| 0.020 | 0.013 | 0.068 | 0.038 | 0.062 | 0.096 | 0.16 | 0.78 |

## WITHOUT MASS

 CONSTRAINTS$$
m_{b}^{k i n}(1 \mathrm{GeV})-0.85 \bar{m}_{c}(3 \mathrm{GeV})=3.714 \pm 0.018 \mathrm{GeV}
$$

- results depend little on assumption for correlations and choice of inputs, $1.8 \%$ determination of $\mathrm{V}_{\mathrm{cb}}$
- 20-30\% determination of the OPE parameters




## RESULTS: BOTTOM MASS



The fit gives $\boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{\operatorname { k i n }}}(\mathbf{1 G e V})=\mathbf{4 . 5 5 3 ( 2 0 )} \mathbf{G e V}$ scheme translation error $m_{b}^{k i n}(1 \mathrm{GeV})=m_{b}\left(m_{b}\right)+0.37(3) \mathrm{GeV}$ $\bar{m}_{b}\left(\bar{m}_{b}\right)=4.183(37) \mathrm{GeV}$

## CHARM MASS DEPENDENCE



FIT PERFORMED WITH ETM CHARM MASS: $m_{c}(3 \mathrm{GeV})=1.056(16) \mathrm{GeV}$ $V_{c b}$ only slightly smaller

## HIGHER ORDER EFFECTS

- Reliability of the method depends on our ability to control higher order effects. Quark-hadron duality violation would manifest as inconsistency in the fit.
- Purely perturbative corrections complete at NNLO, small residual error (kin scheme)Melnikov,Biswas,Gzarnecki,Pak,PG
- Mixed corrections perturbative corrections to power suppressed coefficients completed at $O\left(\alpha_{s} / m_{b}{ }^{2}\right)$
Becher, Boos, Lunghi, Alberti, Ewerth, Nandi, PG, Mannel,Pivovarov, Rosenthal


## HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters and powers of $1 / m_{c}$ starting $1 / \mathrm{m}^{5}$. At $1 / \mathrm{m}_{\mathrm{b}}{ }^{4}$

$$
\begin{aligned}
2 M_{B} m_{1} & =\left\langle\left((\vec{p})^{2}\right)^{2}\right\rangle \\
2 M_{B} m_{2} & =g^{2}\left\langle\vec{E}^{2}\right\rangle \\
2 M_{B} m_{3} & =g^{2}\left\langle\vec{B}^{2}\right\rangle \\
2 M_{B} m_{4} & =g\langle\vec{p} \cdot \operatorname{rot} \vec{B}\rangle
\end{aligned}
$$

$$
\begin{aligned}
& 2 M_{B} m_{5}=g^{2}\langle\vec{S} \cdot(\vec{E} \times \vec{E})\rangle \\
& 2 M_{B} m_{6}=g^{2}\langle\vec{S} \cdot(\vec{B} \times \vec{B})\rangle \\
& 2 M_{B} m_{7}=g\langle(\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B})\rangle \\
& 2 M_{B} m_{8}=g\left\langle(\vec{S} \cdot \vec{B})(\vec{p})^{2}\right\rangle \\
& 2 M_{B} m_{9}=g\langle\Delta(\vec{\sigma} \cdot \vec{B})\rangle
\end{aligned}
$$

Mannel,Turczyk,Uraltsev 1009.4622
can be estimated by Lowest Lying State Saturation approx by truncating

$$
\langle B| O_{1} O_{2}|B\rangle=\sum_{\substack{n \\ \text { see also Heinonen,Mannel 1407.4384 }}}\langle B| O_{1}|n\rangle\langle n| O_{2}|B\rangle
$$

and relating higher dimensional to lower dimensional matrix elements, e.g.

$$
\rho_{D}^{3}=\epsilon \mu_{\pi}^{2} \quad \rho_{L S}^{3}=-\epsilon \mu_{G}^{2} \quad \epsilon \sim 0.4 \mathrm{GeV}
$$

excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases (Mannel, Uraltsev, PG, 2012) In LLSA good convergence of the HQE.
We used LLSA as loose constraint ( $60 \%$ gaussian uncertainty, dimensional estimate for vanishing matrix elements) in the fit including higher powers

## SENSITIVITY TO HIGHER POWER CORRECTIONS

PG,Healey,Turczyk 1606.06174

$$
\left|V_{c b}\right|=42.11(74) \times 10^{-3}(0.25 \% \text { reduction })
$$

$\left|V_{c b}\right|=42.00(64) \times 10^{-3}$ if one uses $m_{c}(2 \mathrm{GeV})$ and includes PDG average for $m_{b}$ 1.5\% uncertainty



DEPENDENCE ON LLSA UNCERTAINTY


WE RESCALE ALL LLSA UNCERTAINTIES BY A FACTOR $\xi$

## EXCITATION ENERGY DEPENDENCE



## PROSPECTS

- Theoretical uncertainties already dominant
- $O\left(\alpha_{s} / m_{b}{ }^{3}\right)$ calculation under way
- $O\left(1 / m_{Q^{4,5}}\right)$ effects need further investigation but small effect on $\mathrm{V}_{\mathrm{cb}}$
- NNNLO corrections to total width feasible, needed for 1\% uncertainty?
- Electroweak (QED) corrections
- New observables in view of Belle-II: FB asymmetry proposed by S.Turczyk could be measured already by Babar and Belle now
- Lattice QCD information on local matrix elements is the next frontier, e.g.

$$
M_{H_{Q}}=m_{Q}+\bar{\Lambda}+\frac{\mu_{\pi}^{2}-a_{H} \mu_{G}^{2}}{2 m_{Q}}+\ldots
$$

## CUTS IN $B \rightarrow X_{u} l v$

Experiments often use kinematic cuts to avoid the $\mathrm{b} \rightarrow \mathrm{clv}$ background:

$$
\mathrm{m}_{\mathrm{X}}<\mathrm{M}_{\mathrm{D}} \quad \mathrm{E}_{\mathrm{l}}>\left(\mathrm{M}_{\mathrm{B}}^{2}-\mathrm{M}_{\mathrm{D}}^{2}\right) / 2 \mathrm{M}_{\mathrm{B}} \quad \mathrm{q}^{2}>\left(\mathrm{M}_{\mathrm{B}}-\mathrm{M}_{\mathrm{D}}\right)^{2} \ldots
$$

The cuts destroy convergence of the OPE that works so well in $b \rightarrow c$. OPE expected to work only away from pert singularities

Rate becomes sensitive to local b-quark wave function properties like Fermi motion. Dominant nonpert contributions can be resummed into a SHAPE FUNCTION $f(\mathrm{k}+$ ).
Equivalently the SF is seen to emerge from soft gluon resummation


## HOW TO ACCESS THE SF?

$$
\frac{d^{3} \Gamma}{d p_{+} d p_{-} d E_{\ell}}=\frac{G_{F}^{2}\left|V_{u b}\right|^{2}}{192 \pi^{3}} \int d k C\left(E_{\ell}, p_{+}, p_{-}, k\right) F(k)+O\left(\frac{\Lambda}{m_{b}}\right)
$$

$\underset{\text { e.g. at } \mathrm{q}^{2}=0}{\text { OPE constrints }} \int_{-\infty}^{\bar{\Lambda}} k^{2} F(k) d k=\frac{\mu_{\pi}^{2}}{3}+O\left(\frac{\Lambda^{3}}{m_{b}}\right)$ etc.

| Predictions based on <br> resummed pQCD <br> DGE, ADFR | OPE constraints + <br> parameterization <br> without/with resummation <br> GGOU, BLNP |
| :---: | :---: |
| Fit semileptonic (and radiative) data |  |
| SIMBA, NNVub |  |

## $V_{u b} \mid$ DETERMINATIONS

## Inclusive: 5\% total error

| HFAG 2014 | Average IV |
| :---: | :---: |
| DGE | $4.52(16)(16)$ |
| BLNP | $4.45(16)(22)$ |
| GGOU | $4.51(16)(15)$ |

UT fit (without direct $\mathrm{V}_{\text {ub }}$ ):

$$
V_{u b}=3.66(12) 10^{-3}
$$

Recent experimental results are theoretically cleanest ( $2 \%$ ) but based on background modelling.
 Signal simulation also relies on theoretical models...

## NEW preliminary Babar endpoint analysis

High sensitivity of the BR on the shape of the signal
in the endpoint region. GGOU: $\left|V_{u b}\right|=4.03_{-0.22}^{+0.20} \times 10^{-3}$


solid squares and triangles $-X_{c}$ with $m c$ constraint fit and $X_{c}+X_{s} \nu$ fit of $S F$ parameters (BLNP and GGOU)
solid and open - translation "kinetic" to "shape-function" with $\mu=2.0 \mathrm{GeV}$
and $\mu=1.5 \mathrm{GeV}$ (BLNP), respectively
results based on $0.8-2.6 \mathrm{GeV} / \mathrm{c}$ momentum range
HFAG 2014 average based on tagged and untagged measurements
Consistent with and more precise than our previous result:
BaBar, Phys.Rev. D73(2006)012006 ( $\mathrm{p}_{\mathrm{e}}>2 \mathrm{GeV} / \mathrm{c}$ )
Y.SKOVPEN, EPS-PH 2015

NB Belle multivariate analysis uses GGOU+DN for the inclusive part

## FUNCTIONAL FORMS



About 100 forms considered in GGOU, large variety, double max discarded. Small uncertainty

$$
(1-2 \%) \text { on } V_{u b}
$$



A more systematic method by Ligeti et al. arXiv:0807.1926 Plot shows 9 SFs that satisfy all the first three moments

Only 2 parameters FF, is that good enough?

## The NNVub Project

K.Healey, C. Mondino, PG, 1604.07598


- Use Artificial Neural Networks to parameterize shape functions without bias and extract $\mathrm{V}_{\mathrm{ub}}$ from theoretical constraints and data, together with HQE parameters in a model independent way (without assumptions on functional form). Similar to NNPDF. Applies to $b \rightarrow u l v, b \rightarrow s \gamma, b \rightarrow s l+l-$
- Belle-II will be able to measure some kinematic distributions, thus constraining directly the shape functions. NNVub will provide a flexible tool to analyse data.

Selection of NN replicas trained on the first three moments only. They are not sufficient. We know photon spectrum in bsgamma: single peak dominance, not too steep

Beware: sampling can be biased by implementation!



Comparison with 2007 paper, same inputs


NNVub GGOU(2007)

| Experimental cuts (in GeV or $\mathrm{GeV}^{2}$ ) | $\left\|V_{u b}\right\| \times 10^{3}$ | $\left\|V_{u b}\right\| \times 10^{3}[15]$ |
| :---: | :---: | :---: |
| $M_{X}<1.55, E_{\ell}>1.0$ Babar [44] | $\left.4.30(20){ }^{26}{ }_{27}\right)$ | 4.29(20) ${ }_{22}^{21}$ ) |
| $M_{X}<1.7, E_{\ell}>1.0$ Babar [44] | $4.05(23)\binom{19}{20}$ | $4.09(23)\binom{18}{19}$ |
| $M_{X} \leq 1.7, q^{2}>8, E_{\ell}>1.0$ Babar[44] | $4.23(23){ }^{(28)}$ | $4.32(23)\left(\begin{array}{l}\text { (30) }\end{array}\right.$ |
| $E_{\ell}>2.0$ Babar [41] | $4.47(26)\binom{27}{27}$ | $4.50(26)\binom{18}{25}$ |
| $E_{\ell}>1.0$ Belle [45] | $4.58(27)\binom{10}{11}$ | $4.60(27)\binom{10}{11}$ |

Inputs for constraints from sl fit by Alberti et al, 2014 with full uncertainties and correlations

## PROSPECTS

- Learning @ Belle-II from kinematic distributions, e.g. MX spectrum
- OPE parameters checked/ improved in $\mathrm{b} \rightarrow$ ulv (moments): global NN+OPE fit
- alternative approach SIMBA Tackmann, Ligeti, Stewart
- include all relevant information with correlations
- check signal dependence at endpoint
- full phase space

implementation of $\alpha_{s}{ }^{2}$ and $\alpha_{\mathrm{s}} / \mathrm{mb}^{2}{ }^{2}$ corrections
- model/exclude high $\mathrm{q}^{2}$ tail

At Belle-II we can expect to bring inclusive $\mathrm{V}_{\mathrm{ub}}$ at almost the same level as $\mathrm{V}_{\mathrm{cb}}$

## ExCLusive $B \rightarrow D^{*} \ell v$

At zero recoil, where rate vanishes, the ff is

$$
\mathcal{F}(1)=\eta_{A}\left[1+O\left(\frac{1}{m_{c}^{2}}\right)+\ldots\right]
$$

Thanks to measurement of slopes and shape parameters, exp error only ~1.3\% extrapolation to zero recoil with CLN parameterization

The ff $F(I)$ cannot be experimentally determined. Lattice $Q C D$ is the best hope to compute it. Only one unquenched Lattice calculation:

$$
\mathrm{F}(\mathrm{I})=0.906(\mathrm{l} 3) \mathrm{V}_{\mathrm{cb}} \mid=39.25(49)_{\exp }(53)_{\mathrm{lat}}(19)_{\text {QED }} 10^{-3}
$$

## I.9\% error (adding in quadrature)

~2.9 $\sigma$ or $\sim 8 \%$ from inclusive determination
NB Heavy Quark Sum Rules estimate $\mathrm{F}(\mathbf{1})=0.86(2) \quad$ PG, Mannel, Uraltsev 2012

## NEW RESULTS FOR $B \rightarrow D l v$ FFs

FNAL/MILC 1503.07237

$w$

| Source | $f_{+}(\%)$ |
| :--- | :---: |
| Statistics+matching $+\chi$ PT cont. extrap. | 1.2 |
| $\quad$ (Statistics) | $(0.7)$ |
| $\quad$ (Matching) | $(0.7)$ |
| $\quad(\chi \mathrm{PT} /$ cont. extrap.) | $(0.6)$ |
| Heavy-quark discretization | 0.4 |
| Lattice scale $r_{1}$ | 0.2 |
| Total error | 1.2 |

## HPQCD 1505.03925




## NEW BELLE SPECTRUM 1510.03657



## FORM FACTORS

$$
\left\langle D\left(p^{\prime}\right)\right| V^{\mu}|\bar{B}(p)\rangle=f_{+}\left(q^{2}\right)\left(p+p^{\prime}\right)^{\mu}+f_{-}\left(q^{2}\right)\left(p-p^{\prime}\right)^{\mu} \quad q^{2}=\left(p-p^{\prime}\right)^{2}
$$

$$
\begin{gathered}
\frac{d \Gamma}{d q^{2}}\left(B \rightarrow D l \nu_{l}\right)=\frac{\eta_{e w}^{2} G_{F}^{2}\left|V_{c b}\right|^{2} m_{B} \lambda^{1 / 2}}{192 \pi^{3}}\left(1-\frac{m_{l}^{2}}{q^{2}}\right)^{2}\left[c_{+}^{l} f_{+}\left(q^{2}\right)^{2}+c_{0}^{l} f_{0}\left(q^{2}\right)^{2}\right] \\
r=m_{D} / m_{B}, \lambda=\left(q^{2}-m_{B}^{2}-m_{D}^{2}\right)^{2}-4 m_{B}^{2} m_{D}^{2} \\
\eta_{e w}=1+\alpha / \pi \ln M_{Z} / m_{b} \approx 1.0066 \\
c_{+}^{l}=\frac{\lambda}{m_{B}^{4}}\left(1+\frac{m_{l}^{2}}{2 q^{2}}\right), \quad c_{0}^{l}=\left(1-r^{2}\right)^{2} \frac{3 m_{l}^{2}}{2 q^{2}} \\
f_{0}\left(q^{2}\right)=f_{+}\left(q^{2}\right)+\frac{q^{2}}{m_{B}^{2}-m_{D}^{2}} f_{-}\left(q^{2}\right) \quad f_{+}(0)=f_{0}(0)
\end{gathered}
$$

## UNITARITY CONSTRAINTS

CROSSING + ANALITYCITY


CUT FOR
$q^{2} \geq\left(m_{B}+m_{D}\right)^{2}$


USING QUARK-HADRON DUALITY. DISPERSION RELATIONS $\rightarrow$ GLOBAL QHD

## UNITARITY CONSTRAINTS

$$
\begin{gathered}
\left(-g^{\mu \nu}+\frac{q^{\mu} q^{\nu}}{q^{2}}\right) \Pi^{T}\left(q^{2}\right)+\frac{q^{\mu} q^{\nu}}{q^{2}} \Pi^{L}\left(q^{2}\right) \equiv i \int d^{4} x e^{i q x}\langle 0| T J^{\mu}(x) J^{\dagger \nu}(0)|0\rangle \\
\chi^{L}\left(q^{2}\right)=\frac{\partial \Pi^{L}}{\partial q^{2}}, \quad \chi^{T}\left(q^{2}\right)=\frac{1}{2} \frac{\partial^{2} \Pi^{T}}{\partial\left(q^{2}\right)^{2}}
\end{gathered}
$$

SATISFY UNSUBTRACTED DISP REL, PERT CALCULATION FOR $q^{2}=0$ Boyd, Grinstein, Lebed 1995

$$
\begin{aligned}
\chi^{T}(0) & =\left[5.883+0.552_{\alpha_{s}}+0.050_{\alpha_{s}^{2}}\right] 10^{-4} \mathrm{GeV}^{-2}=6.486(48) 10^{-4} \mathrm{GeV}^{-2} \\
\chi^{L}(0) & =\left[5.456+0.782_{\alpha_{s}}-0.034_{\alpha_{s}^{2}}\right] 10^{-3}=6.204(81) 10^{-3}
\end{aligned}
$$

USING UP-TO-DATE QUARK MASSES AND BLOOP CALCULATION Grigo et al 2012

$$
\tilde{\chi}^{T}(0)=\chi^{T}(0)-\sum_{n=1,2} \frac{f_{n}^{2}\left(B_{c}^{*}\right)}{M_{n}^{4}\left(B_{c}^{*}\right)} \quad \begin{gathered}
\text { BOUND state } \\
\text { contributions }
\end{gathered}
$$

| Type | Mass (GeV) | Decay constants (GeV) |
| :---: | :---: | :---: |
| $1^{-}$ | $6.329(3)$ | $0.422(13)$ |
| $1^{-}$ | $6.920(20)$ | $0.300(30)$ |
| $1^{-}$ | 7.020 |  |
| $1^{-}$ | 7.280 |  |
| $0^{+}$ | 6.716 |  |
| $0^{+}$ | 7.121 |  |

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## UNITARITY CONSTRAINTS

$$
\begin{gathered}
z=\frac{\sqrt{1+w}-\sqrt{2}}{\sqrt{1+w}+\sqrt{2}} \quad w=\frac{m_{B}^{2}+m_{D}^{2}-q^{2}}{2 m_{B} m_{D}}
\end{gathered} \quad 0<z<0.0646
$$

$$
\sum_{n=0}^{\infty}\left(a_{n}^{+, 0}\right)^{2}<1
$$

WEAK UNITARITY CONSTRAINTS

BGL PARAMETERIZATION: TRUNCATE EXPANSION AT $n=\mathcal{N}$ PROBLEMS AT THRESHOLD AND WITH LARGE $q^{2}$ SCALING

BCL PARAMETERIZATION BOURELLY CAPRINI LELLOUCH 2008

$$
\begin{equation*}
f_{+}(z)=\frac{1}{1-q^{2} / M_{+}^{2}} \sum_{n=0}^{N} a_{n}^{+}\left[z^{n}-(-1)^{n-N-1} \frac{n}{N+1} z^{N+1}\right] \tag{BCL}
\end{equation*}
$$

## STRONG UNITARITY CONSTRAINTS

If one knows something about the other channels the constraints become tighter In the heavy quark limit all $B^{(*)} \rightarrow D^{(*)}$ form factors either vanish or are prop to the IsgurWise function

$$
\sum_{i=1}^{H} \sum_{n=0}^{\infty} b_{i n}^{2} \leq 1 \quad \sum_{n} b_{i n} z^{n}=c_{i}(z) f_{+}(z)
$$

CAPRINI
LELLOUCH

$$
\begin{gathered}
\left.f_{+}(z) \simeq f_{+}(0)\left[1-8 \rho_{1}^{2} z+\left(51 \rho_{1}^{2}-10\right) z^{2}-\left(252 \rho_{1}^{2}-84\right) z^{3}\right)\right] \\
\frac{f_{0}(z)}{f_{+}(z)} \simeq\left(\frac{2 \sqrt{r}}{1+r}\right)^{2} \frac{1+w}{2} 1.0036\left[1-0.0068 w_{1}+0.0017 w_{1}^{2}-0.0013 w_{1}^{3}\right] \\
w_{1}=w-1
\end{gathered}
$$

neubert
CLN 1998

CLN exploit NLO HQET relations between form factors to reduce to only 2 parameters... but $1 / \mathrm{m}^{2}$ corrections can be sizable For ex at zero recoil

$$
\begin{array}{r}
\frac{F_{D^{*}}(z=0)}{f_{+}(z=0)}=0.948 \neq 0.860(14) \\
\text { NLO HQET LATTICE (FNAL) }
\end{array} \quad \frac{f_{+}(0)}{f_{0}(0)}=0.775 \neq 0.753(3)
$$

CLN parameterization has intrinsic uncertainties that can no longer be neglected.
We use HQET expressions only in derivation of unitarity bounds and have checked that results are unaffected

## Global fit to $B \rightarrow D l v$



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## RESULTS

| exp data | lattice data | N,par | $10^{3} \times\left\|V_{c b}\right\|$ | $\chi^{2} /$ dof | $R(D)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| all | all | 2,BGL | $40.62(98)$ | $22.1 / 26$ | $0.302(3)$ |
| all | all | 3, BGL | $40.47(97)$ | $18.2 / 24$ | $0.299(3)$ |
| all | all | 4,BGL | $\mathbf{4 0 . 4 9 ( 9 7 )}$ | $19.0 / 22$ | $\mathbf{0 . 2 9 9 ( 3 )}$ |
| Belle | all | 3,BGL | $40.92(1.12)$ | $11.6 / 14$ | $0.300(3)$ |
| BaBar | all | 3, BGL | $40.11(1.55)$ | $12.6 / 14$ | $0.301(4)$ |
| all | FNAL | 3,BGL | $40.17(1.05)$ | $10.4 / 18$ | $0.293(4)$ |
| all | HPQCD | 3, BGL | $40.51+1.82$ | $101 / 18$ | $0.299(7)$ |
| all | all | CLN | $40.85(95)$ | $77.1 / 29$ | $0.305(3)$ |
| all | $f_{+}$only | CLN | $40.33(99)$ | $20.0 / 23$ | $0.305(3)$ |
| all | all | 2,BCL | $40.49(98)$ | $18.2 / 26$ | $0.299(3)$ |
| all | all | $3, B C L$ | $40.48(96)$ | $18.2 / 24$ | $0.299(3)$ |
| all | all | 4,BCL | $40.48(97)$ | $17.9 / 22$ | $0.299(3)$ |

## Global fit to $B \rightarrow D / v$

- $\left|V_{c b}\right|=40.49(0.97) 10^{-3} \quad(\mathrm{BGL}, \mathrm{N}=4)$ compatible with both inclusive and $B \rightarrow D^{*}$
- $\boldsymbol{R}(D)=0.299(3) 2 \sigma$ from HFAG average
- Constrained fit with strong unitarity bounds
- weak bounds leads to very similar results with slightly larger errors
- BGL and BCL parameterizations give almost identical results
- assumes no correlation between FNAL and HPQCD, 3\% syst error on Babar data, correct treatment of last bin, no finite size bin effect.
- Non-zero recoil lattice results are crucial: only zero recoil leads to $\left|V_{c b}\right|=39.6(2.0) 10^{-3}$ (BGL)
- Belle only fit has higher $\mathrm{V}_{\mathrm{cb}}$
- Possible improvements from more precise data (Belle-II, reanalysis of Babar data) and lattice calculations


## WEAK vs STRONG BOUNDS




Figure 2: Form factor $f_{+}(z)$ in the $N=4$ BGL fit to lattice data for $f_{+, 0}(z)$ with weak (brown band) and strong (gray band) unitarity constraints. The $N=2$ band (independent of unitarity constraints) is shown in dashed lines for comparison. FNAL/MILC synthetic data are shown in red, HPQCD in blue. On the right, enlarged detail of the tail.

$\mathrm{B} \rightarrow \mathrm{D}^{*}$ analyses based on CLN: errors underestimated.
However the spectrum is measured precisely and extrapolation to zero-recoil is a small effect. New Belle analysis under way...

## PROSPECTS FOR EXCLUSIVE $V_{c b}$

- Need for more lattice calculations and extension of $\mathbf{B} \rightarrow \mathbf{D}^{*}$ ff to non-zero recoil. Matching at $1 / \mathrm{m}_{\mathrm{Q}}{ }^{3}$ for lattice discretization effects under study by FNAL/MILC. Simulations at physical pion mass and $m_{b} a \leq 1$ ?
- Heavy quark sum rules (with BPS arguments) favor smaller $F(1)=0.86(2)$ leading to agreement with inclusive. Difficult to improve, how good is the BPS limit?
- QED/EW corrections: SD log OK, SD remainder tiny if $\mathrm{G}_{\mu}$ employed, soft/ collinear radiation subtracted out by Photos, intermediate photons (IR finite) are structure dependent: lattice calculations? exp cuts? relevance of Coulomb enhancement for $\mathrm{B}^{\circ}$ decay rate?
- New channels (Bc, Bs, $\Lambda_{b}$ ) at Belle-II and LHCb, can also be combined for unitarity bounds, better understanding of $\mathrm{D}^{* *}$


## RECENT LATTICE $B \rightarrow \pi$

RBC/UKQCD 1501.05373


FNAL/MILC 1503.07839


FNAL ${ }^{\sigma^{2}} 3.72(16) 10^{-3}$ only $4.3 \%$ error
$2.2 \sigma$ from inclusive RBC/UKQCD 3.61(32) $10^{-3}$
$1.9 \sigma$ from inclusive
LCSR 3.32(26) $10^{-3}$
$2.9 \sigma$ from inclusive
LHCb depends
on $\mathrm{V}_{\mathrm{cb}}$ employed but low

## RECENT LATTICE RESULTS

1503.07839



| exp data | lattice data | N,par | $10^{3} \times\left\|V_{c b}\right\|$ | $\chi^{2} /$ dof | $R(D)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| all | all | 2,BGL | 40.62 (98) | 22.1/26 | $0.302(3)$ |
| all | all | $3 . \mathrm{BGL}$ | 40.47 (97) | 18.2/24 | 0.299(3) |
| all | all | 4,BGI | 40.49(97) | 19.0/22 | $0.299(3)$ |
| Belle | all | 3,BGI | 40.92(1.12) | 11.6/14 | $0.300(3)$ |
| BaBar | all | 3,BCL | 40.11(1.55) | 12.6/14 | 0301 (4) |
| all | FNAL | 3,BGL | 40.17(1.05) | 10.4/18 | $0.293(4)$ |
| all | HPQCD | 3,BGL | $40.51 \pm 1.82$ | 10.1/18 | $0.299(7)$ |
| all | all | CLN | 40.85(95) | $77.1 / 29$ | $0.305(3)$ |
| all | $f_{+}$only | CLN | 40.33 (99) | 20.0/23 | $0.305(3)$ |
| all | all | 2,BCL | 40.49(98) | 18.2/26 | $0.299(3)$ |
| all | all | 3,BCL | 40.48(96) | 18.2/24 | $0.299(3)$ |
| all | all | 4,BCL | 40.48(97) | 17.9/22 | $0.299(3)$ |

Prospects: further improvements in LQCD, much more data @ BelleII, $\mathrm{B}_{\mathrm{s}} \rightarrow \mathrm{Klv}$ and other channels @Belle-II and LHCb

## VISUAL SUMMARY



## NEW PHYSICS?

The difference in $\mathrm{V}_{\mathrm{cb}}$ incl vs excl $\mathrm{D}^{*}$ with FNAL/MILC form factor is large: $3 \sigma$ or about $8 \%$. The perturbative corrections to inclusive $\mathrm{V}_{\mathrm{cb}}$ total $5 \%$, the power corrections about $4 \%$.

Right Handed currents now excluded since

$$
\begin{aligned}
& \left|V_{c b}\right|_{i n c l} \simeq\left|V_{c b}\right|\left(1+\frac{1}{2}|\delta|^{2}\right) \\
& \left|V_{c b}\right|_{B \rightarrow D^{*}} \simeq\left|V_{c b}\right|(1-\delta) \\
& \left|V_{c b}\right|_{B \rightarrow D} \simeq\left|V_{c b}\right|(1+\delta)
\end{aligned}
$$

Chen,Nam,Crivellin,Buras,Gemmler, Isidori,Mannel,...
$\delta=\epsilon_{R} \frac{\tilde{V}_{c b}}{V_{c b}} \approx 0.08$

Most general SU(2) invariant dim 6 NP (without RH light neutrino) can explain results, but it is incompatible with $\mathrm{Z} \rightarrow \mathrm{b} \overline{\mathrm{b}}$ data

Crivellin, Pokorski 1407.1320
(though this may need update after new $\mathrm{B} \rightarrow$ Dlv result...)

## RH CURRENTS DON'T HELP Vub EITHER

- Can ease $\left|\mathrm{V}_{\mathrm{ub}}\right|$ tension by allowing small righthanded contribution to Standard-Model weak current [Crivellin, PRD81 (2010) 031301]
- RH currents disfavored by $\wedge_{\mathrm{b}}$ decays (taking $\left|\mathrm{V}_{\mathrm{cb}}\right|$ from $B \rightarrow D^{*} l v+$ HFAG to obtain $\left|\mathrm{V}_{\mathrm{ub}}\right|$ )

[based on Bernlochner et al., PRD 90, 094003 (2014)]
R. van de Water

Also here $\mathrm{SU}(2) \mathrm{xU}(1)$ invariant NP cannot explain discrepancies 1407.1320

## SUMMARY

- Improvements of OPE approach to s.l. decays continue. No sign of inconsistency in this approach so far, competitive $\boldsymbol{m}_{\boldsymbol{b}}-\boldsymbol{m}_{\boldsymbol{c}}$ determination.
- Exclusive/incl. tension in $V_{c b}$ remains ( $2.9 \sigma, 8 \%$ ) only in the $\mathrm{D}^{*}$ channel. The $\mathbf{D}$ channel is becoming competitive and is compatible with both. The remaining tension calls for new lattice analyses and new data (ongoing Belle analysis, Belle-II)
- New fit allows for precise SM determination $R(D)=0.299$ (3)
- Exclusive/incl tension in $V_{u b}$ seems receding because of new FNAL/ MILC and HPQCD results and of preliminary Babar results. Significant progress will come with Belle-II and further LHCb data ( $\mathrm{B} \rightarrow$ tv etc).
- NNVub framework permits implementation of Belle-II experimental data and OPE constraints, reducing the SFs uncertainty. Comparison with data will validate inclusive approach to $\mathrm{V}_{\mathrm{ub}}$ in a much more stringent way.
- New physics explanations quite constrained for both $V_{u b}$ and $V_{c b .}$.

