

Semileptonic Form Factors from Lattice QCD

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Semileptonic discussion | MIAPP
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Processes

$$\ell = \mu, e$$

- CKM Determination

$$B \rightarrow \pi \ell \nu$$

$$B \rightarrow D \ell \nu$$

$$B \rightarrow D^* \ell \nu$$

(in pedagogical order)

- New Physics Search

$$B \rightarrow \pi \tau \nu$$

$$B \rightarrow D \tau \nu$$

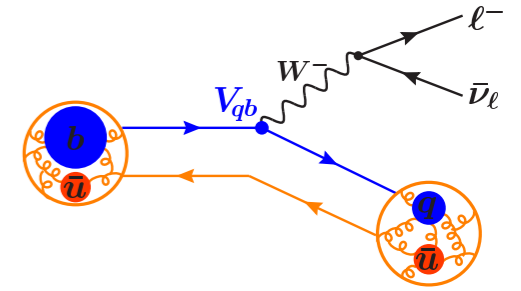
$$B \rightarrow K \nu \bar{\nu}$$

$$B \rightarrow \pi \nu \bar{\nu}$$

$$B \rightarrow K \ell^+ \ell^-$$

$$B \rightarrow \pi \ell^+ \ell^-$$

Matrix Elements and Form Factors



- Decompose amplitudes in form factors ($q = p - k = \ell + \nu$):

$$\begin{aligned} \langle \pi(k) | \bar{u} \gamma^\mu b | B(p) \rangle &= \left(p^\mu + k^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) f_+(q^2) + \frac{M_B^2 - M_\pi^2}{q^2} q^\mu f_0(q^2), \\ &= \sqrt{2M_B} [p^\mu f_{||}(q^2)/M_B + k_\perp^\mu f_\perp(q^2)] \end{aligned}$$

$$\langle \pi(k) | \bar{u} \sigma^{\mu\nu} b | B(p) \rangle = -2i \frac{p^\mu k^\nu - p^\nu k^\mu}{M_B + M_\pi} f_T(q^2),$$

$$\langle \pi(k) | \bar{u} b | B(p) \rangle = \frac{M_B^2 - M_\pi^2}{m_b - m_u} f_0(q^2),$$

PCVC: same

- The kinematic variable $q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$.

Outline

- Motivation
- Lattice QCD Calculations
- $|V_{ub}|$ and $|V_{cb}|$
- Penguin Processes
- Outlook

arXiv:1509.06235 arXiv:1510.02349 arXiv:1507.01618 arXiv:1503.07839
arXiv:1503.07237 arXiv:1403.0635

Fermilab Lattice and MILC Collaborations

Jon Bailey, Alexei Bazavov, Claude Bernard, Chris Bouchard,
Carleton DeTar, **Daping Du**, Aida El-Khadra, Elizabeth Freeland,
Elvira Gámiz, Steve Gottlieb, Urs Heller, A.S.K., Jack Laiho,
Ludmila Levkova, Yuzhi Liu, Paul Mackenzie, Yannick Meurice,
Ethan Neil, **Si-Wei Qiu**, Jim Simone, Bob Sugar,
Doug Toussaint, Ruth Van de Water, **Ran Zhou**

with special guest

Enrico Lunghi

CKM: $|V_{ub}|$ and $|V_{cb}|$

Basic Formulas for $B \rightarrow \pi l \nu$

- Relevant term in effective Hamiltonian: $\mathcal{L}_i = \bar{b} \gamma^\mu (1 - \gamma^5) u \bar{\nu}_l \gamma_\mu (1 - \gamma^5) l$
- Differential decay rate:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{ub}|^2 |\mathbf{k}|^3 |f_+(q^2)|^2 + \mathcal{O}(m_\ell^2),$$

- Steps:
 - generate numerical data at several \mathbf{k}, m_l, a ;
 - chiral continuum extrapolation;
 - extend to full kinematic range with z expansion.

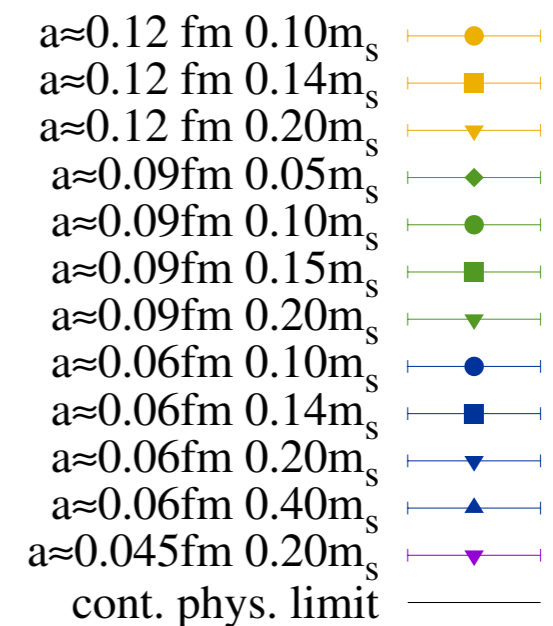
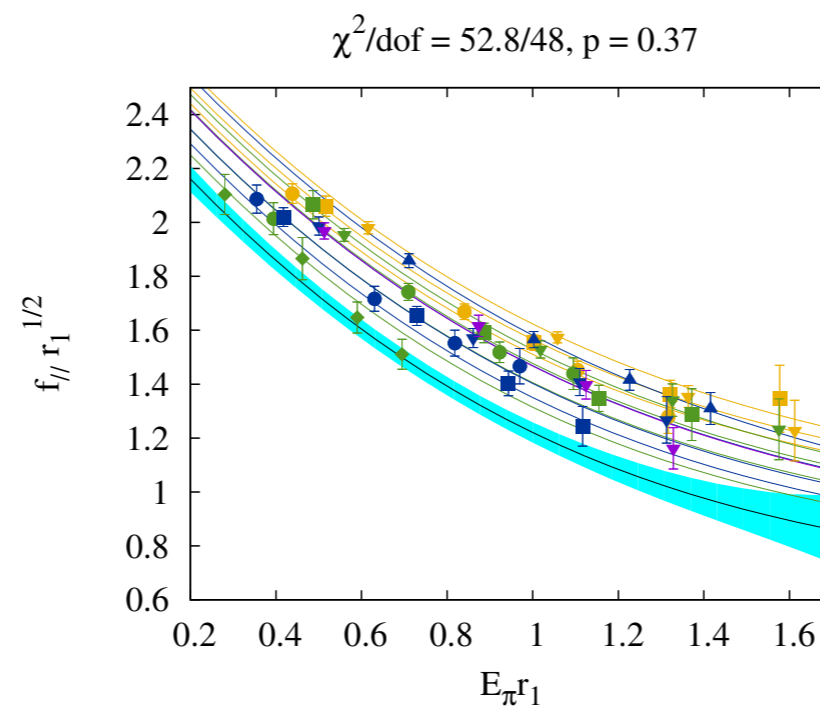
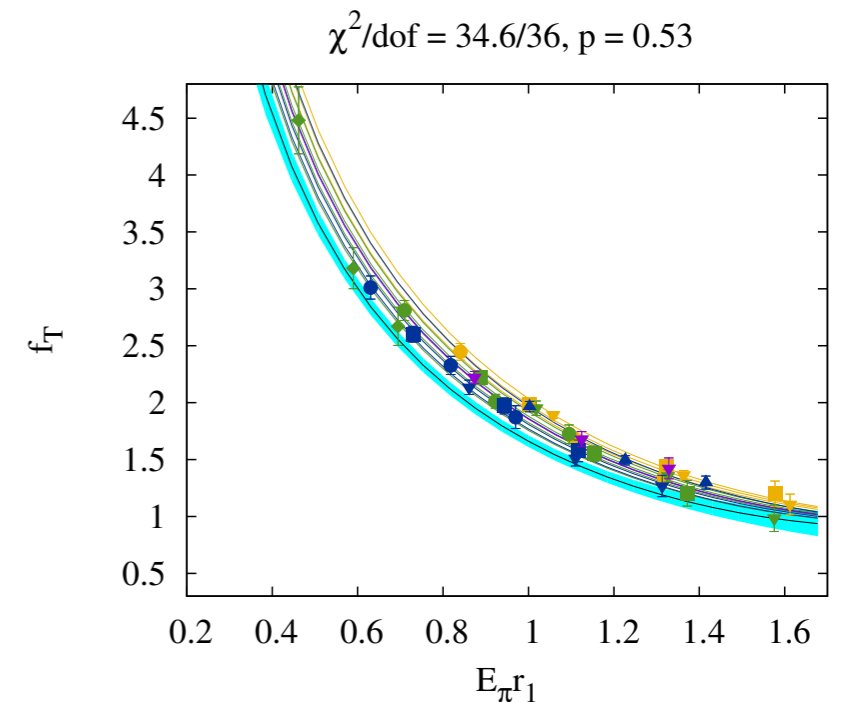
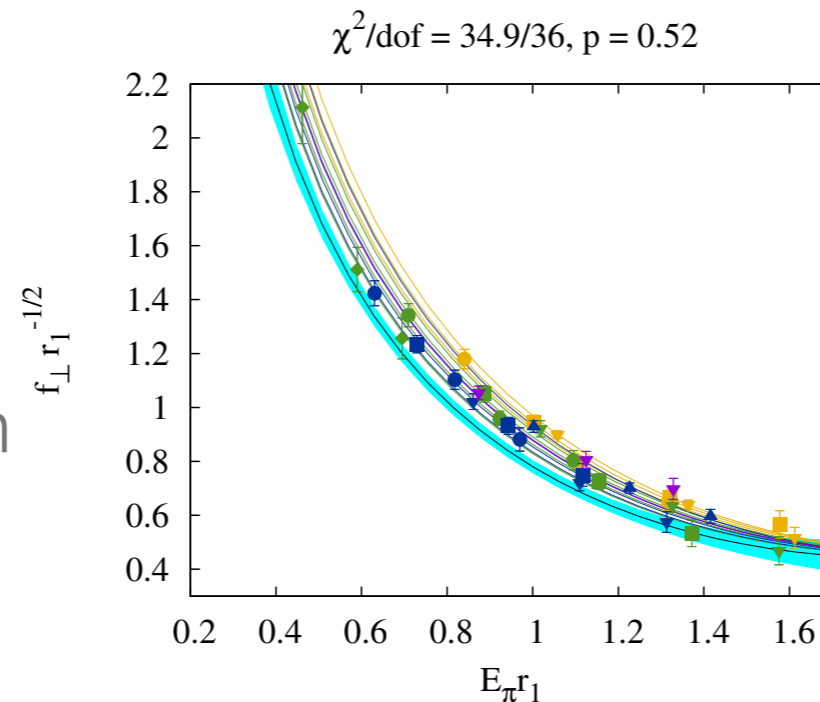
Semileptonic $B \rightarrow \pi l \nu$ for $|V_{ub}|$

arXiv:1503.07839

- Compute $f(\mathbf{k}, m_s, m_l, a)$
- Combine data with Symanzik EFT & chiral PT:

- $m_l \rightarrow \frac{1}{2}(m_u + m_d)$;
- $a \rightarrow 0$.

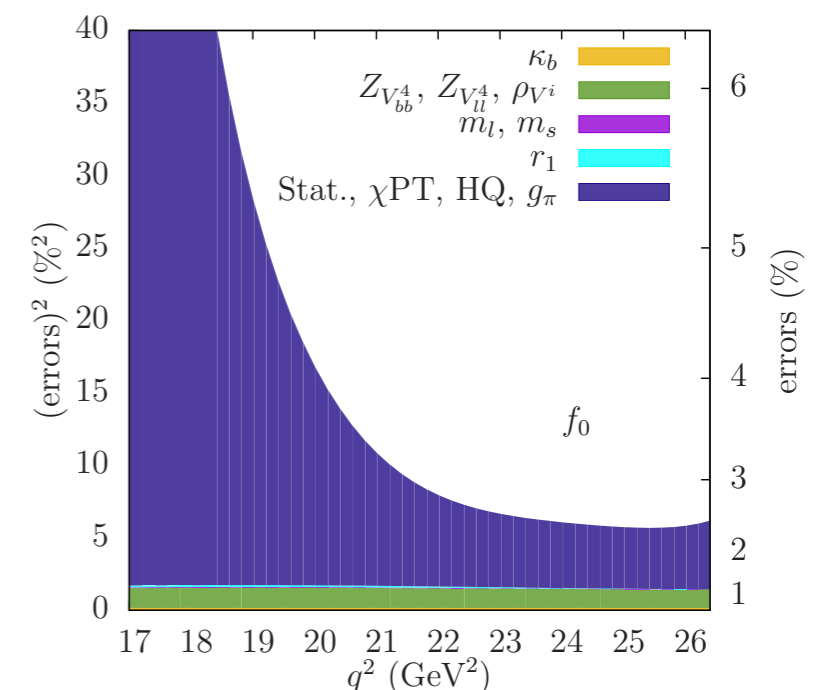
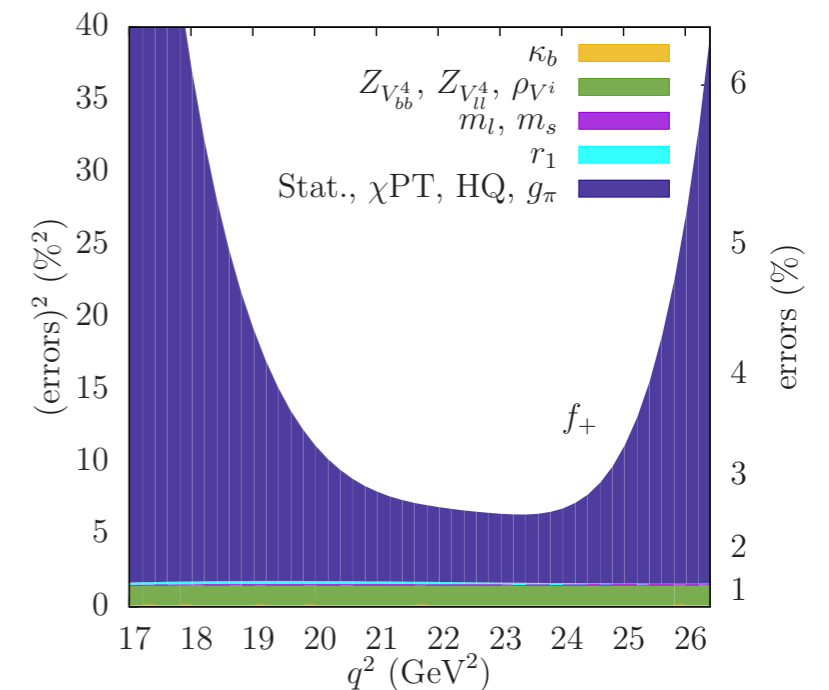
- Limited range: $|k|a \ll 1$.



Error Budgets

in region with lattice data

- The largest uncertainty, by far, comes from MC statistics, as **amplified** via the chiral-continuum extrapolation.
- Next (and independent of q^2) is matching from LGT to continuum QCD.
- Error on input parameters (m_l, m_s, κ_b) & relative scale (r_1) disappear in quadrature sum.
- Challenge: extend reach to lower q^2 , without being killed by the (amplified) statistical error.



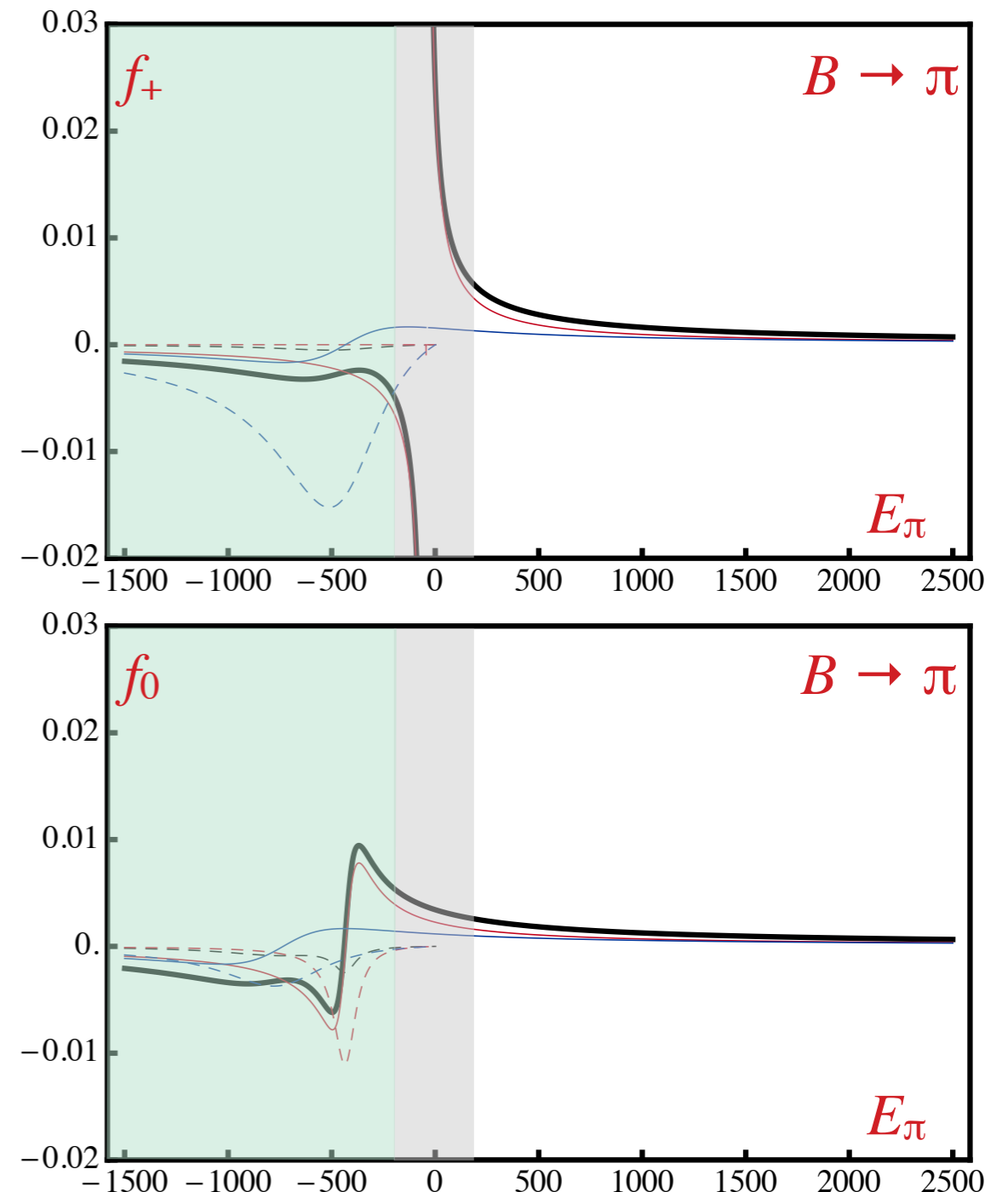
Analyticity and Unitarity



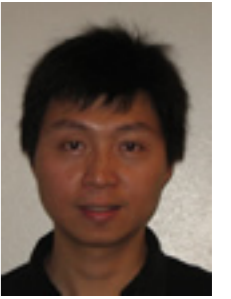
- The form factor is analytic in q^2 except a cut for $q^2 \geq (M_B + M_\pi)^2$ and (possibly) subthreshold poles $(M_B - M_\pi)^2 \leq q^2 < (M_B + M_\pi)^2$.
- With $t_+ = (M_B + M_\pi)^2$, set

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

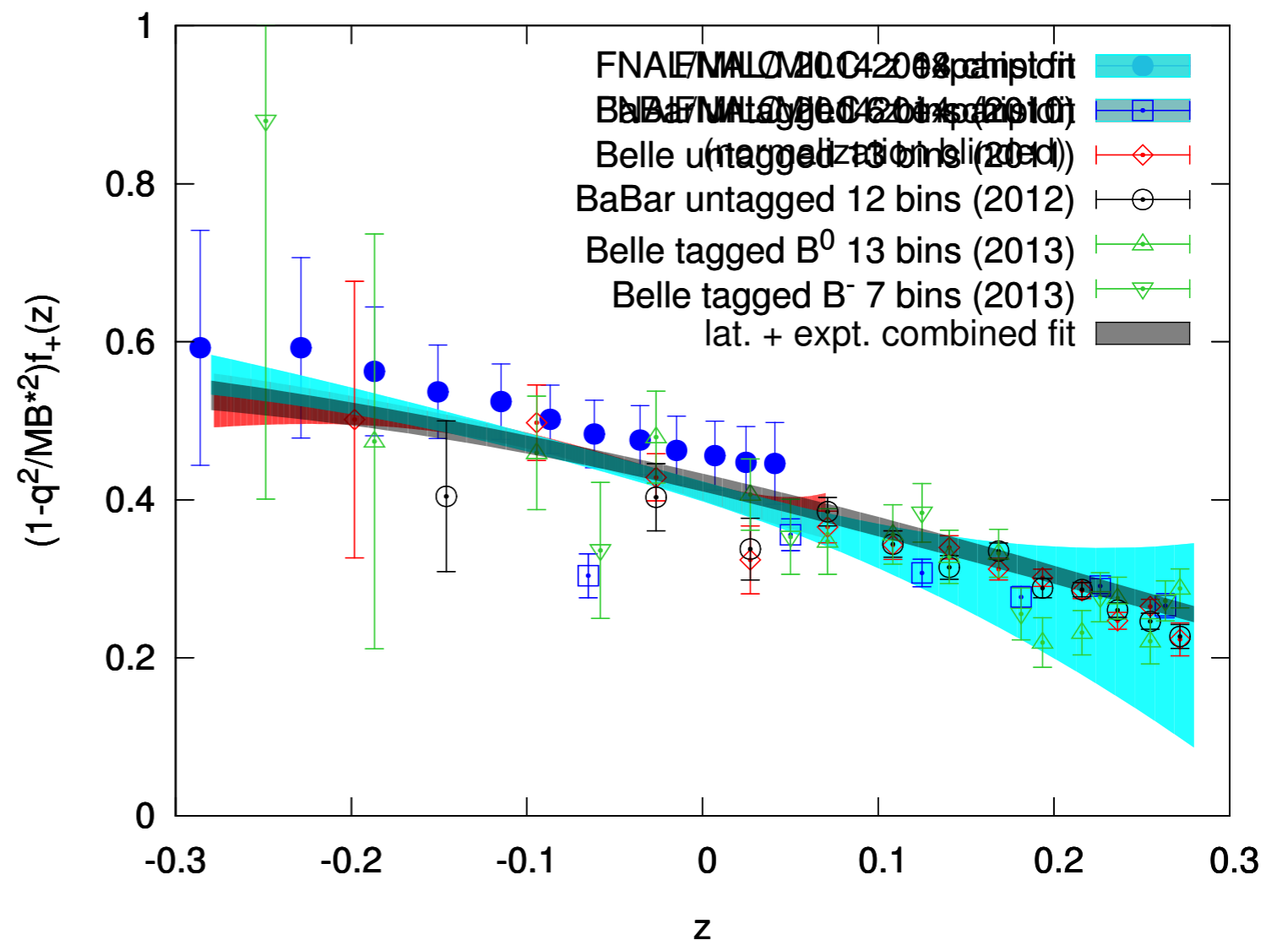
which maps cut to unit circle and semileptonic decay to real $|z| \leq 0.28$ for optimal t_0 .



Determination of $|V_{ub}|$



- Much more precise than 2008.
- **BLINDED PLOTS!!**
- z variable extends range.
- Functional fitting method.
- Relative norm'n yields $|V_{ub}|$.
- Total error on $|V_{ub}|$: 4.1%.



Reconstructing Form Factors

- For additional applications, the z expansion provides a useful summary.
- Formulas (Bourenly, Caprini, Lellouch, [arXiv:0807.2722](https://arxiv.org/abs/0807.2722)):

$$f_+(z) = \frac{1}{1 - q^2(z)/M_{B^*}^2} \sum_{n=0}^{N_z-1} b_n^+ \left[z^n - (-1)^{n-N_z} \frac{n}{N_z} z^{N_z} \right]$$

$$f_0(z) = \sum_{n=0}^{N_z} b_n^0 z^n$$

$$t_0 = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$$

- Subthreshold 1^- pole in f_+ ; first 0^+ excitation (for f_0) is unstable.

- Coefficients and correlations:

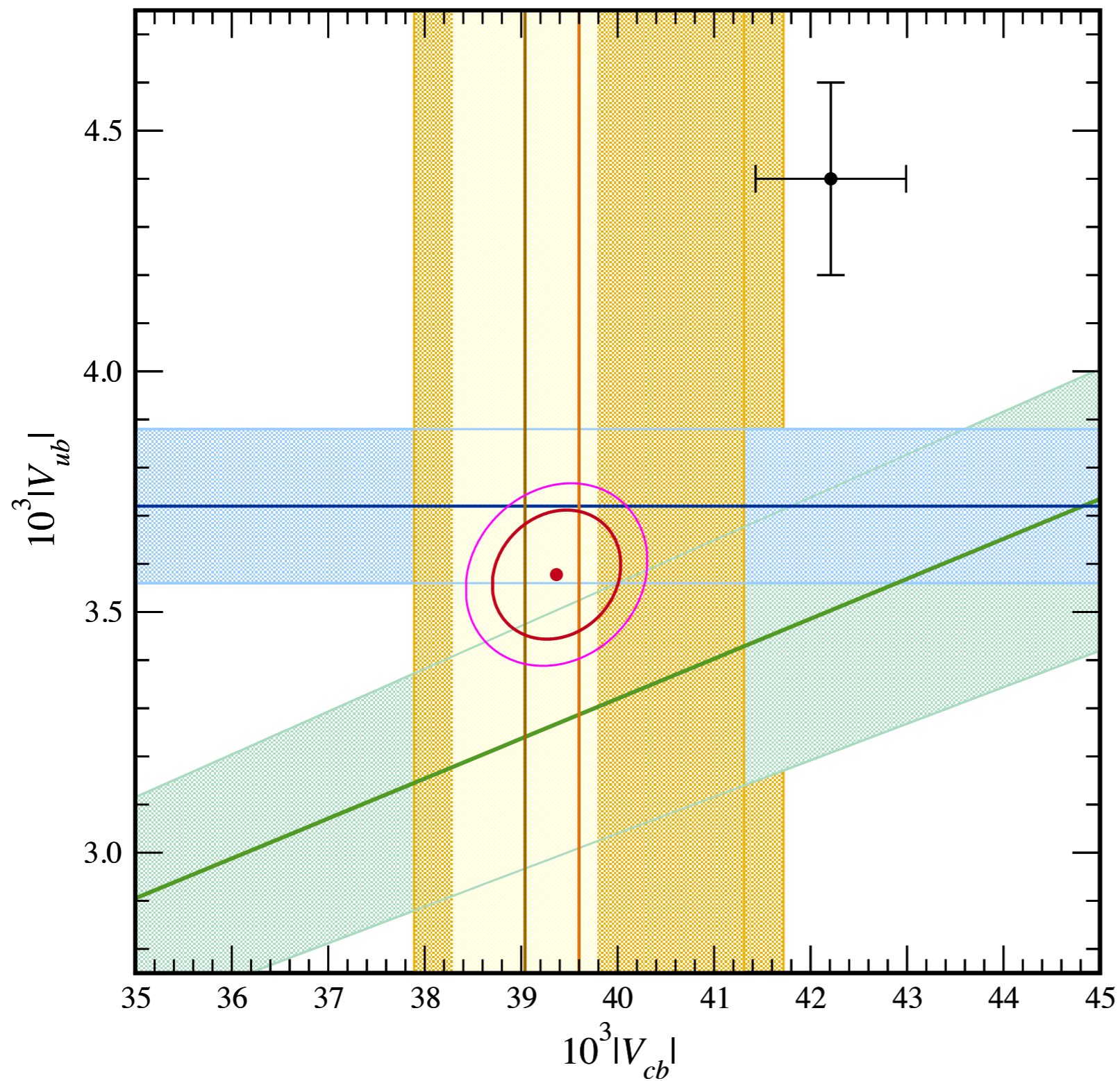
	b_0^+	b_1^+	b_2^+	b_3^+	b_0^0	b_1^0	b_2^0	b_3^0
	0.407(15)	-0.65(16)	-0.46(88)	0.4(1.3)	0.507(22)	-1.77(18)	1.27(81)	4.2(1.4)
b_0^+	1	0.451	0.161	0.102	0.331	0.346	0.292	0.216
b_1^+		1	0.757	0.665	0.430	0.817	0.854	0.699
b_2^+			1	0.988	0.482	0.847	0.951	0.795
b_3^+				1	0.484	0.833	0.913	0.714
b_0^0					1	0.447	0.359	0.189
b_1^0						1	0.827	0.500
b_2^0							1	0.838
b_3^0								1

- If you are interesting in semileptonic B decays, just take this table and the formulas from the last slide and use the resulting form factors.

Semileptonic $B \rightarrow Dlv$ for $|V_{cb}|$

arXiv:1503.07237

- Similar strategy as above:
 - compute sequence of form-factor values;
 - chiral continuum extrapolation;
 - combined z -expansion fit to obtain $|V_{cb}|$.
- Differences:
 - HQET control of cutoff effects more central [[hep-lat/0002008](#), [hep-lat/0112044](#), [hep-lat/0112045](#)];
 - use Boyd, Grinstein, Lebed form of z expansion [[hep-ph/9508211](#)].



- $|V_{ub}|/|V_{cb}|$ (latQCD + LHCb)
- $|V_{ub}|$ (latQCD + BaBar + Belle)
- $|V_{cb}|$ (latQCD + BaBar) + Belle
- $|V_{cb}|$ (latQCD + HFAG, $w = 1$)
- $p = 0.2\sigma$
- $\Delta\chi^2 = 1$
- $\Delta\chi^2 = 2$
- inclusive $|V_{xb}|$

arXiv:1503.07839

arXiv:1501.05373 RBC/UKQCD

arXiv:1503.07237

arXiv:1505.03925 HPQCD

arXiv:1403.0635

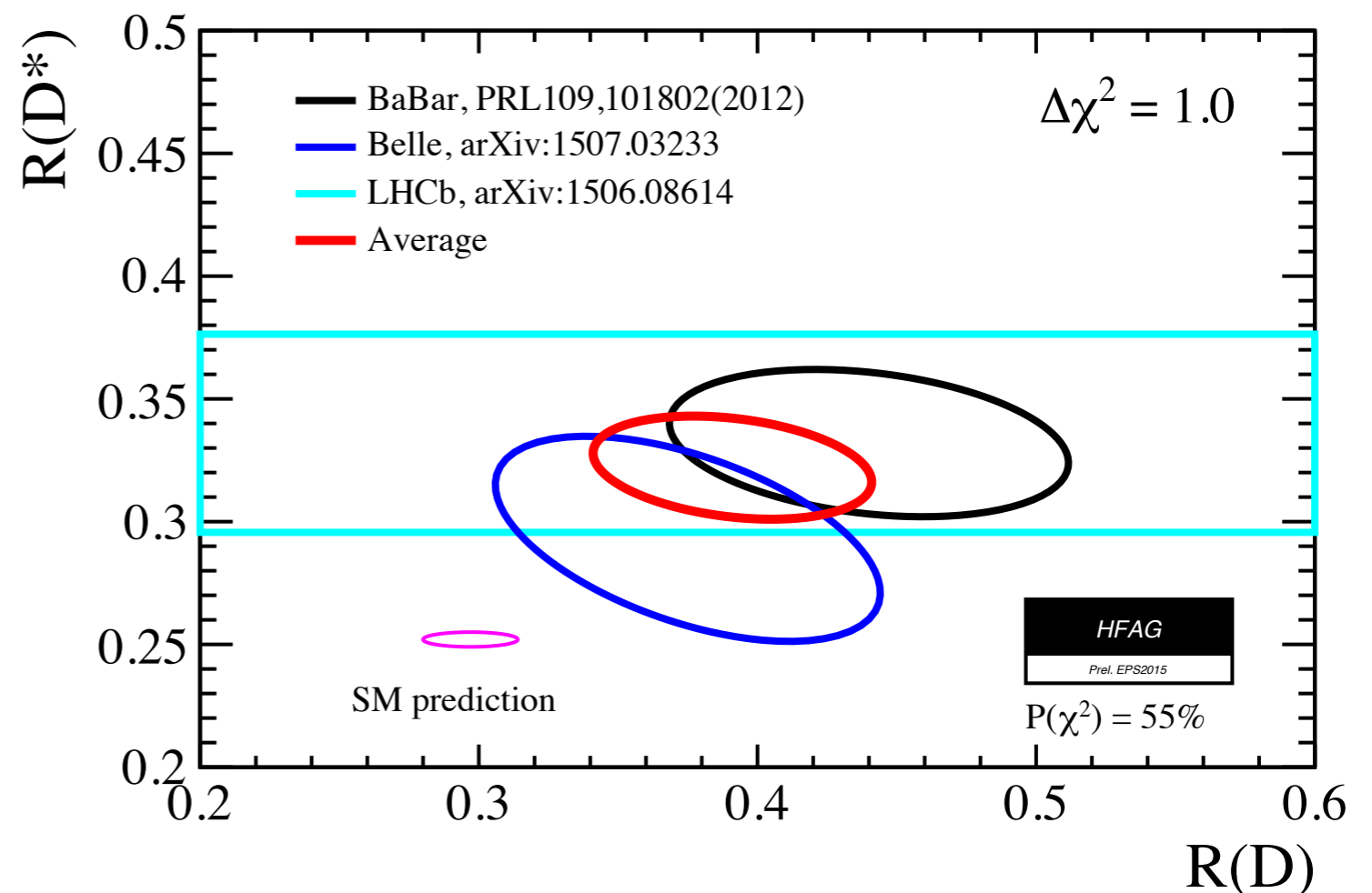
arXiv:1503.01421

Detmold, Lehner, Meinel

New Physics in $B \rightarrow D^{(*)}\tau\nu$?

BaBar, [arXiv:1205.5442](https://arxiv.org/abs/1205.5442); Belle, [arXiv:1507.03233](https://arxiv.org/abs/1507.03233); LHCb, [arXiv:1506.08614](https://arxiv.org/abs/1506.08614)

- BaBar presented evidence for an excess in both channels:
 - 2.0σ for $R(D)$; 2.7σ for $R(D^*)$; 3.4σ combined.
- With Belle & LHCb:
 - 3.9σ combined.
- Estimated form factors w/
 - HQET;
 - quenched QCD.

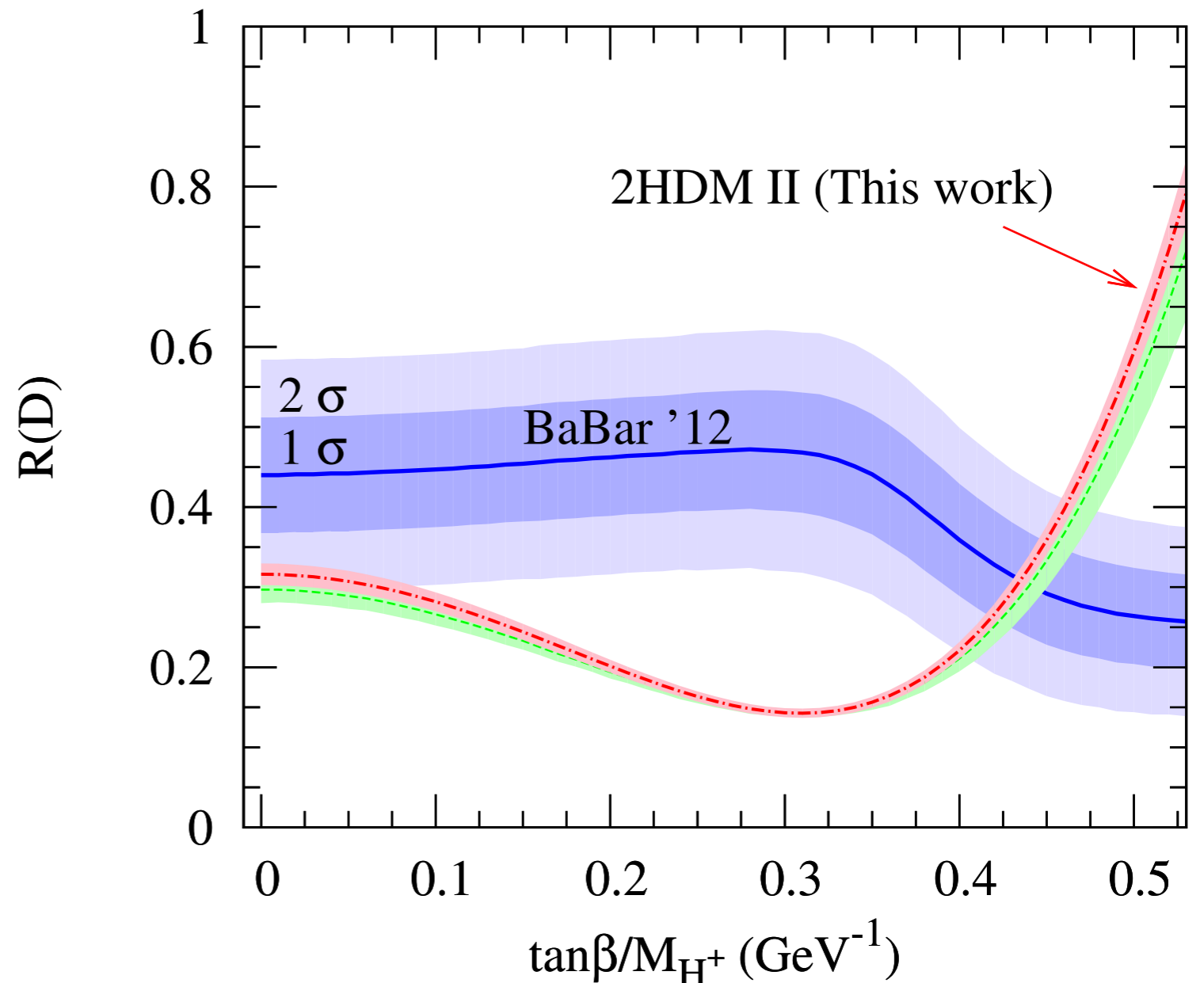


Form Factors for $B \rightarrow D^{(*)}\tau\nu$

Fermilab/MILC, [arXiv:1206.4992](https://arxiv.org/abs/1206.4992), [arXiv:1503.07237](https://arxiv.org/abs/1503.07237); HPQCD, [arXiv:1505.03925](https://arxiv.org/abs/1505.03925)

see also [arXiv:1206.4977](https://arxiv.org/abs/1206.4977).

- $R(D)$ values:
 - 0.297 ± 0.017 (est.);
 - 0.316 ± 0.014 (F/M '12);
 - 0.299 ± 0.011 (F/M '15);
 - 0.300 ± 0.008 (HPQCD).
- Lattice QCD work for $R(D^*)$ yet to come.

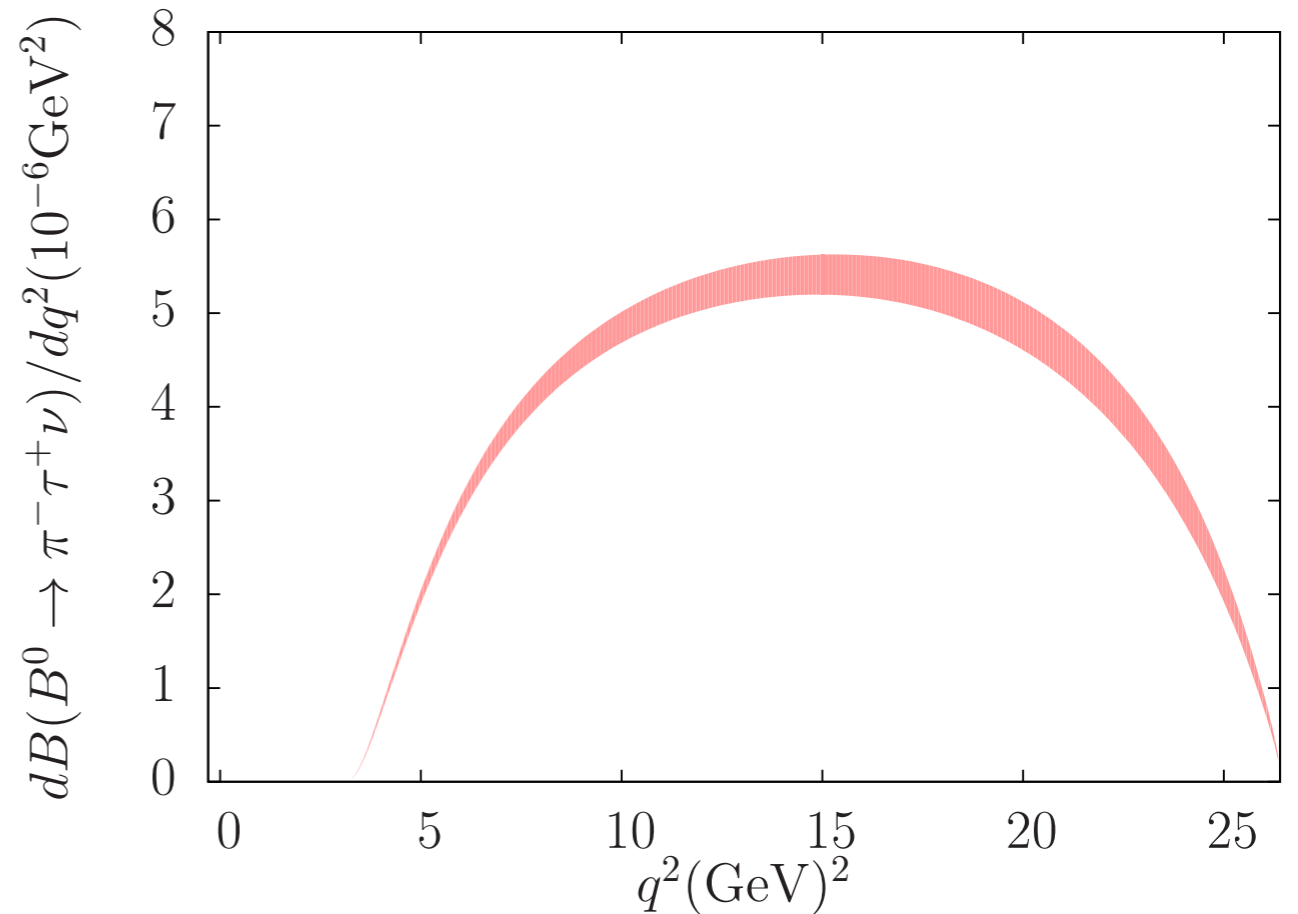


New Physics in $B \rightarrow \pi\tau\nu$?

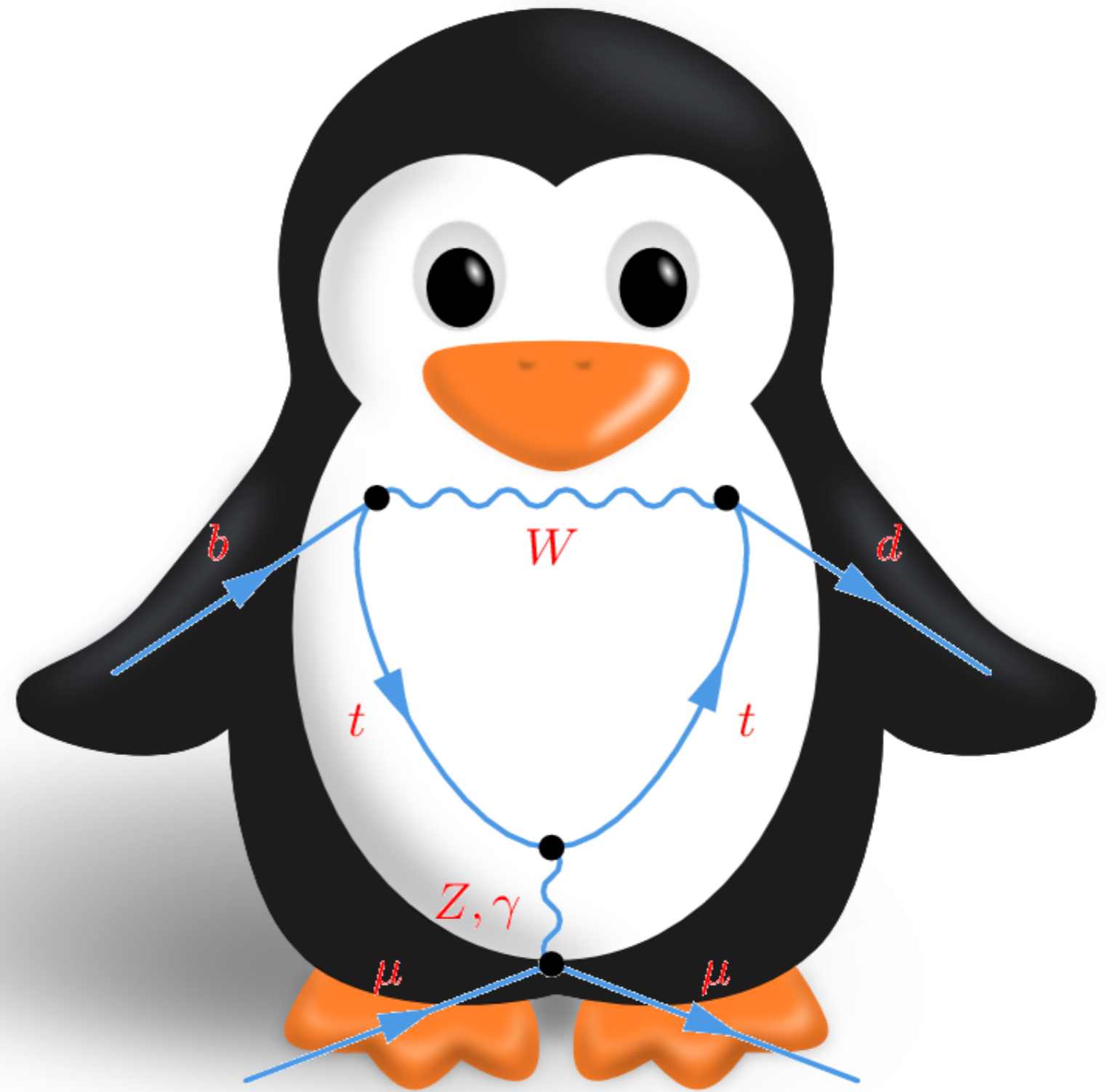
arXiv:1510.02349

- A charged Higgs mediating $b \rightarrow c$ could also mediate $b \rightarrow u$.
- SM prediction, including term $\sim m_\tau^2 |f_0|^2$.
- With the Fermilab/MILC form factors, we find

$$\begin{aligned} R(\pi) &\equiv \frac{\mathcal{B}(B \rightarrow \pi\tau\nu_\tau)}{\mathcal{B}(B \rightarrow \pi\ell\nu_\ell)} \\ &= 0.641(17) \end{aligned}$$



Penguins



graphic by Daping Du

Basic Formulas for $B^0 \rightarrow \pi^0 \nu \nu, K^0 \nu \nu$

- Relevant term in effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \frac{e^2}{16\pi^2} \frac{X_t}{\sin^2 \theta_W} \bar{q}_L \gamma_\mu b_L \sum_{\nu} \bar{\nu}_L \gamma^\mu \nu_L$$

- Differential decay rate:

$$\frac{d\mathcal{B}(B \rightarrow P \nu \bar{\nu})}{dq^2} = C_P \tau_B |V_{tb} V_{tq}^*|^2 \frac{G_F^2 \alpha^2}{32\pi^5} \frac{X_t^2}{\sin^4 \theta_W} |\mathbf{k}|^3 |f_+(q^2)|^2$$

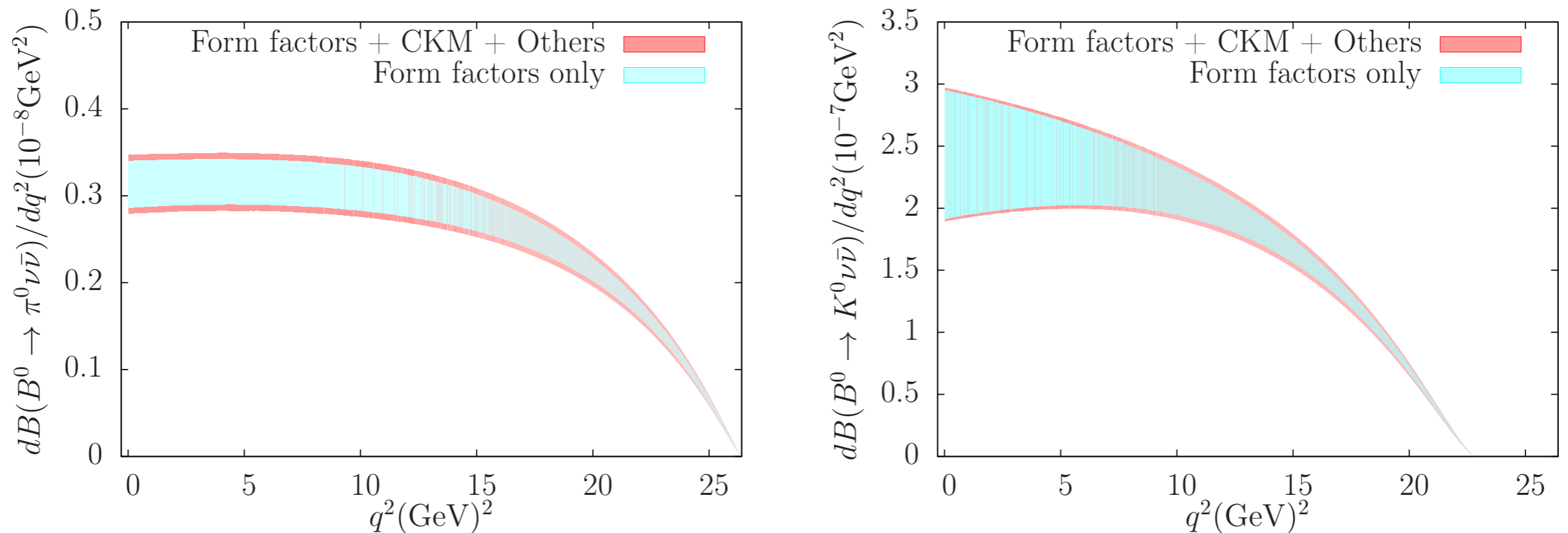
$$q = d \Rightarrow P = \pi^0, \quad C_{\pi^0} = \frac{1}{2}$$

$$q = s \Rightarrow P = K^0, \quad C_{K^0} = 1$$

- Thus, can use same form factor as above (and kaon counterpart).

SM Predictions for the Differential Rate

arXiv:1510.02349



- NB: charged counterparts receive contribution from the cascade $B \rightarrow \tau \nu$, $\tau \rightarrow \pi \nu$ (or $K \nu$), with amplitude proportional to $|V_{ub}|f_B |V_{ud}|f_\pi$ (or $|V_{us}|f_K$) [Kamenik & Smith, [arXiv:0908.1174](https://arxiv.org/abs/0908.1174)].

Basic Formulas for $B \rightarrow \pi l^+ l^-, K l^+ l^-$

cf., [arXiv:1510.02349](https://arxiv.org/abs/1510.02349), Sec. 2 & Appendix B

- One-loop effective Hamiltonian contains many operators ($q = d, s$):

$$Q_1^u = (\bar{q}_L \gamma_\mu T^a u_L) (\bar{u}_L \gamma^\mu T^a b_L)$$

$$Q_2^u = (\bar{q}_L \gamma_\mu u_L) (\bar{u}_L \gamma^\mu b_L)$$

$$Q_1 = (\bar{q}_L \gamma_\mu T^a c_L) (\bar{c}_L \gamma^\mu T^a b_L)$$

$$Q_2 = (\bar{q}_L \gamma_\mu c_L) (\bar{c}_L \gamma^\mu b_L)$$

$$Q_3 = (\bar{q}_L \gamma_\mu b_L) \sum_{q'} (\bar{q}' \gamma^\mu q')$$

$$Q_4 = (\bar{q}_L \gamma_\mu T^a b_L) \sum_{q'} (\bar{q}' \gamma^\mu T^a q')$$

$$Q_5 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum_{q'} (\bar{q}' \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q')$$

$$Q_6 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \sum_{q'} (\bar{q}' \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q')$$

$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

$$Q_8 = \frac{g}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a$$

$$Q_9 = (\bar{q}_L \gamma_\mu b_L) \sum_{\ell} (\bar{\ell} \gamma^\mu \ell)$$

$$Q_{10} = (\bar{q}_L \gamma_\mu b_L) \sum_{\ell} (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

- Matrix elements of Q_7, Q_9, Q_{10} yield form factors, including tensor f_T .

Other Matrix Elements

- Schematically,

$$\langle P\ell\ell|Q_i(y)|\bar{B}\rangle \sim (\bar{u}_\ell\gamma_\mu\nu_\ell) \int d^4x e^{iq\cdot(x-y)} \langle P|T J_{\text{em}}^\mu(x)Q_i(y)|\bar{B}\rangle$$

- Four regions:
 - when q^2 is close to M^2 of ρ , ω , ϕ : very challenging [[arXiv:1506.07760](https://arxiv.org/abs/1506.07760)];
 - when q^2 is near charmonium resonances: also very challenging;
 - low q^2 , between these two: SCET (aka QCD factorization);
 - high $q^2 \sim M_B^2$: OPE [Grinstein, Pirjol, [hep-ph/0404250](https://arxiv.org/abs/hep-ph/0404250)], leading back to the three form factors, f_+ , f_0 , & f_T .

Soft Collinear Effective Theory

[hep-ph/9905312](#), [hep-ph/0006124](#), [hep-ph/0011336](#), [hep-ph/0109045](#)

- The RHS of the equation on the last slide takes the form

$$\begin{aligned}
 C_i \langle P\ell\ell | Q_i | \bar{B} \rangle &\sim C_i [(1 + \alpha_s) f_T + (1 + \alpha_s) f_+ + \phi_B \star T \star \phi_P], & i = 1, \dots, 6, \\
 C_8 \langle P\ell\ell | Q_8 | \bar{B} \rangle &\sim C_8 [\alpha_s f_T + \alpha_s f_+ + \phi_B \star T \star \phi_P], \\
 C_7 \langle P\ell\ell | Q_7 | \bar{B} \rangle &\sim C_7 f_T \rightsquigarrow C_7^{\text{eff}} f_T, \\
 C_9 \langle P\ell\ell | Q_9 | \bar{B} \rangle &\sim C_9 f_+ \rightsquigarrow C_9^{\text{eff}} f_+, \\
 C_{10} \langle P\ell\ell | Q_{10} | \bar{B} \rangle &\sim C_{10} f_+, & \text{hep-ph/0008255, hep-ph/0106067}
 \end{aligned}$$

light-cone distribution amplitudes

- We add the nonfactorizable terms to the amplitude using the known hard-scattering kernel T and distribution amplitudes from the literature.
- SM C_7 , C_9 , C_{10} from Huber, Lunghi, Misiak, Wyler [[hep-ph/0512066](#)].

Coefficients and Correlations: $B \rightarrow Kl^+l^-$

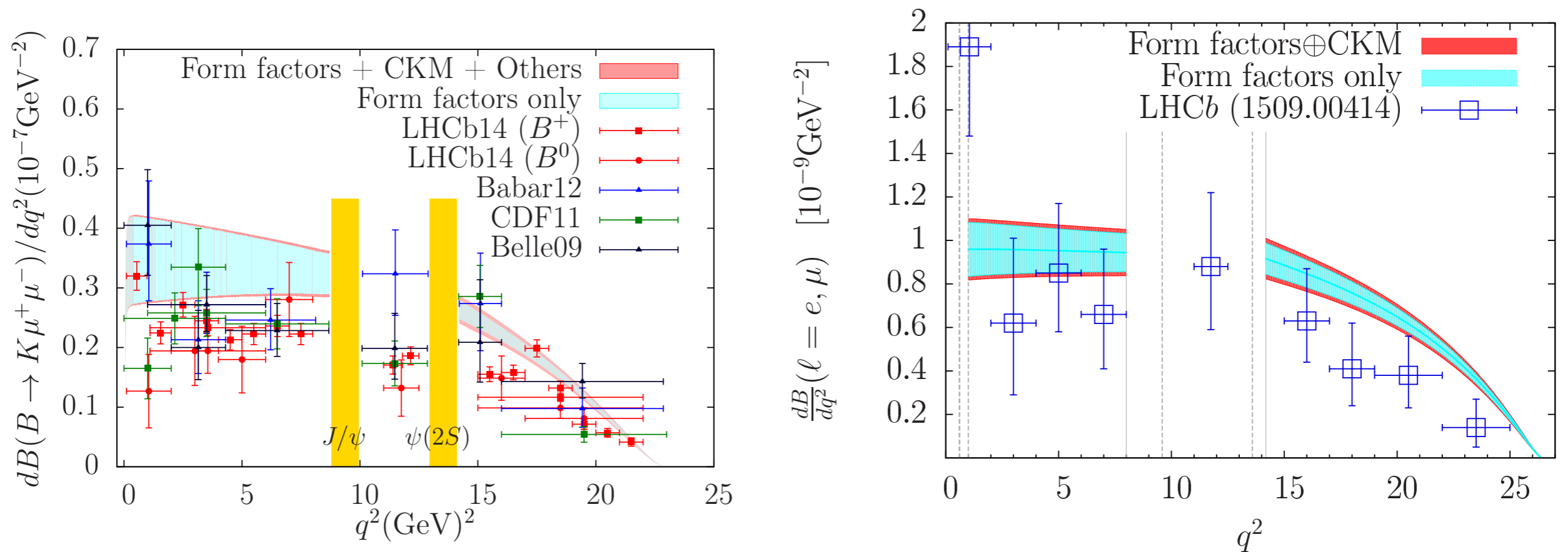
arXiv:1509.06235

	b_0^+	b_1^+	b_2^+	b_0^0	b_1^0	b_2^0	b_0^T	b_1^T	b_2^T
mean	0.466	-0.885	-0.213	0.292	0.281	0.150	0.460	-1.089	-1.114
error	0.014	0.128	0.548	0.010	0.125	0.441	0.019	0.236	0.971
b_0^+	1	0.450	0.190	0.857	0.598	0.531	0.752	0.229	0.117
b_1^+		1	0.677	0.708	0.958	0.927	0.227	0.443	0.287
b_2^+			1	0.595	0.770	0.819	-0.023	0.070	0.196
b_0^0				1	0.830	0.766	0.582	0.237	0.192
b_1^0					1	0.973	0.324	0.372	0.272
b_2^0						1	0.268	0.332	0.269
b_0^T							1	0.590	0.515
b_1^T								1	0.897
b_2^T									1

- Corresponding information for $B \rightarrow \pi l^+l^-$ in [arXiv:1503.07839](#) and [arXiv:1507.01618](#).

Kinematic Distributions

- Experimental data from LHCb [[arXiv:1403.8044](https://arxiv.org/abs/1403.8044), [arXiv:1509.00414](https://arxiv.org/abs/1509.00414)] and earlier experiments; right plot's theory **preceded** measurement:



- [arXiv:1510.02349](https://arxiv.org/abs/1510.02349) also contains predictions for the ratio, the flat terms, etc.

Other Results in [arXiv:1510.02349](https://arxiv.org/abs/1510.02349)

- Tests of heavy-quark and SU(3)-flavor symmetries.
- Comparisons with LHCb over wide bins, $q^2 \in [1 \text{ GeV}^2, 6 \text{ GeV}^2]$, and $q^2 \in [15 \text{ GeV}^2, 22 \text{ GeV}^2]$.
- SM predictions for $B \rightarrow \pi\tau^+\tau^-$, $B \rightarrow K\tau^+\tau^-$
- SM contribution to lepton-universality violation ($q^2 \in [1 \text{ GeV}^2, 6 \text{ GeV}^2]$):

$$R_{K^+}^{\mu e} \Big|_{\text{SM}} = 1.00050(43), \text{ vs. } R_{K^+}^{\mu e} \Big|_{\text{LHCb}} = 0.745_{-82}^{+97}$$

$$R_{K^+}^{\mu e} = \int_{q_{\min}^2}^{q_{\max}^2} d\mathcal{B}(B \rightarrow K\mu\mu) \left[\int_{q_{\min}^2}^{q_{\max}^2} d\mathcal{B}(B \rightarrow Kee) \right]^{-1}$$

- Neutrino modes and $B \rightarrow \pi\tau\nu$, discussed earlier.

CKM: $|V_{td}|$ and $|V_{ts}|$

- Assume that there is no new physics buried in the Wilson coefficients.
- Then the combination of our calculations with experimental measurements yield the third row of the CKM matrix.

- We find

$$|V_{td}/V_{ts}| = 0.201(20)$$

$$|V_{tb}^* V_{td}| \times 10^3 = 7.45(69)$$

$$|V_{tb}^* V_{ts}| \times 10^3 = 35.7(1.5)$$

- The uncertainty here is commensurate from the neutral B -meson mixing.
- Result for $|V_{ts}|$ **was** 1.4σ lower than that from mixing.

Oscillation Frequencies

- New results for mixing in [arXiv:1602.03560](https://arxiv.org/abs/1602.03560): 2–3 times more precise.
- Taking CKM from tree-only inputs (from CKMfitter):
- Contrast with the measured frequencies:

$$\Delta M_d^{\text{SM}} = 0.639(50)(36)(5)(13) \text{ ps}^{-1} \quad \Delta M_d^{\text{expt}} = (0.5055 \pm 0.0020) \text{ ps}^{-1}$$

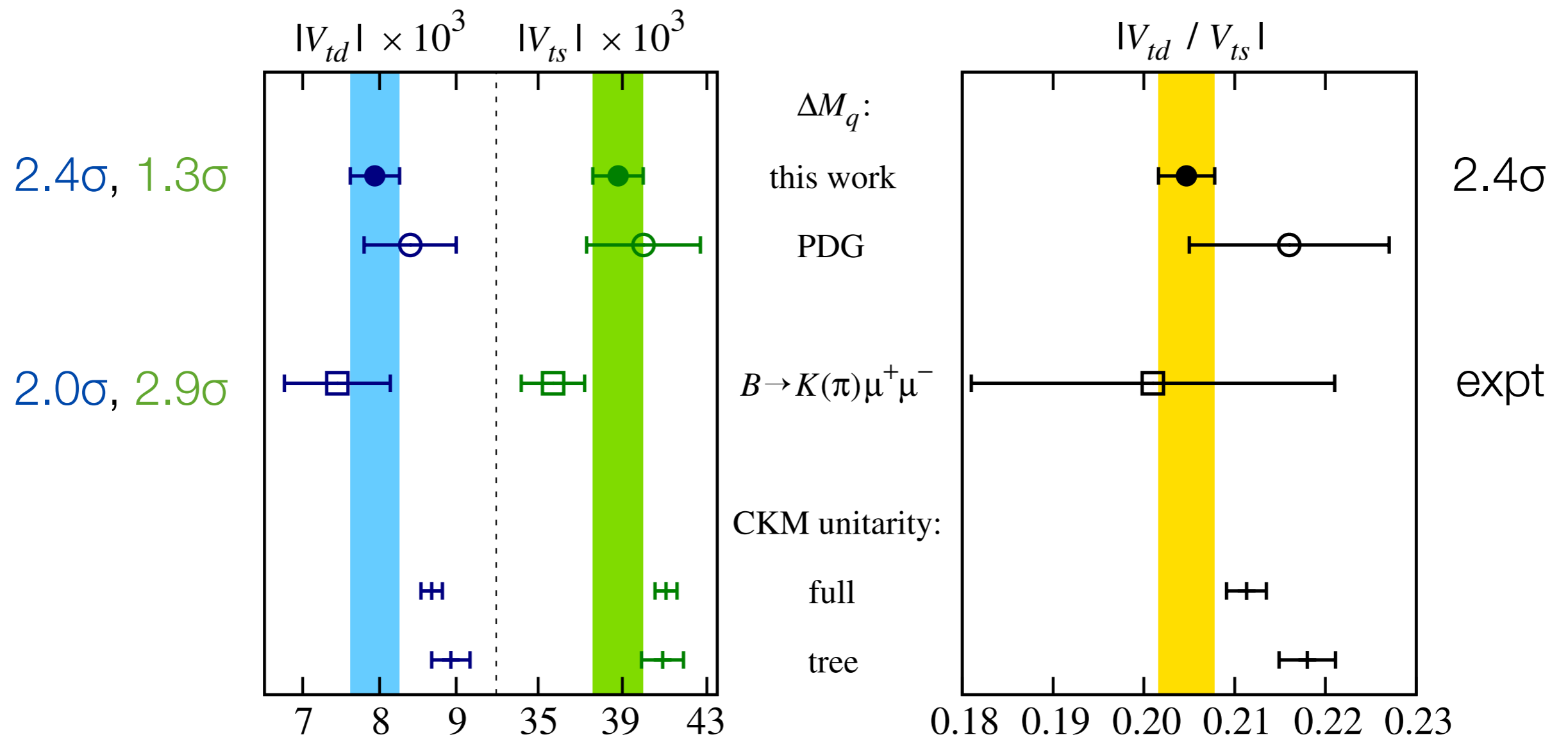
$$\Delta M_s^{\text{SM}} = 19.8(1.1)(1.0)(0.2)(0.4) \text{ ps}^{-1} \quad \Delta M_s^{\text{expt}} = (17.757 \pm 0.021) \text{ ps}^{-1}$$

$$\frac{\Delta M_d^{\text{SM}}}{\Delta M_s^{\text{SM}}} = 0.0323(9)(9)(0)(3)$$

- These amount to discrepancies of 2.1σ , 1.3σ , and 2.9σ , respectively.
- Examine these tensions with those in other FCNC processes, casting each one as a “CKM determination”.

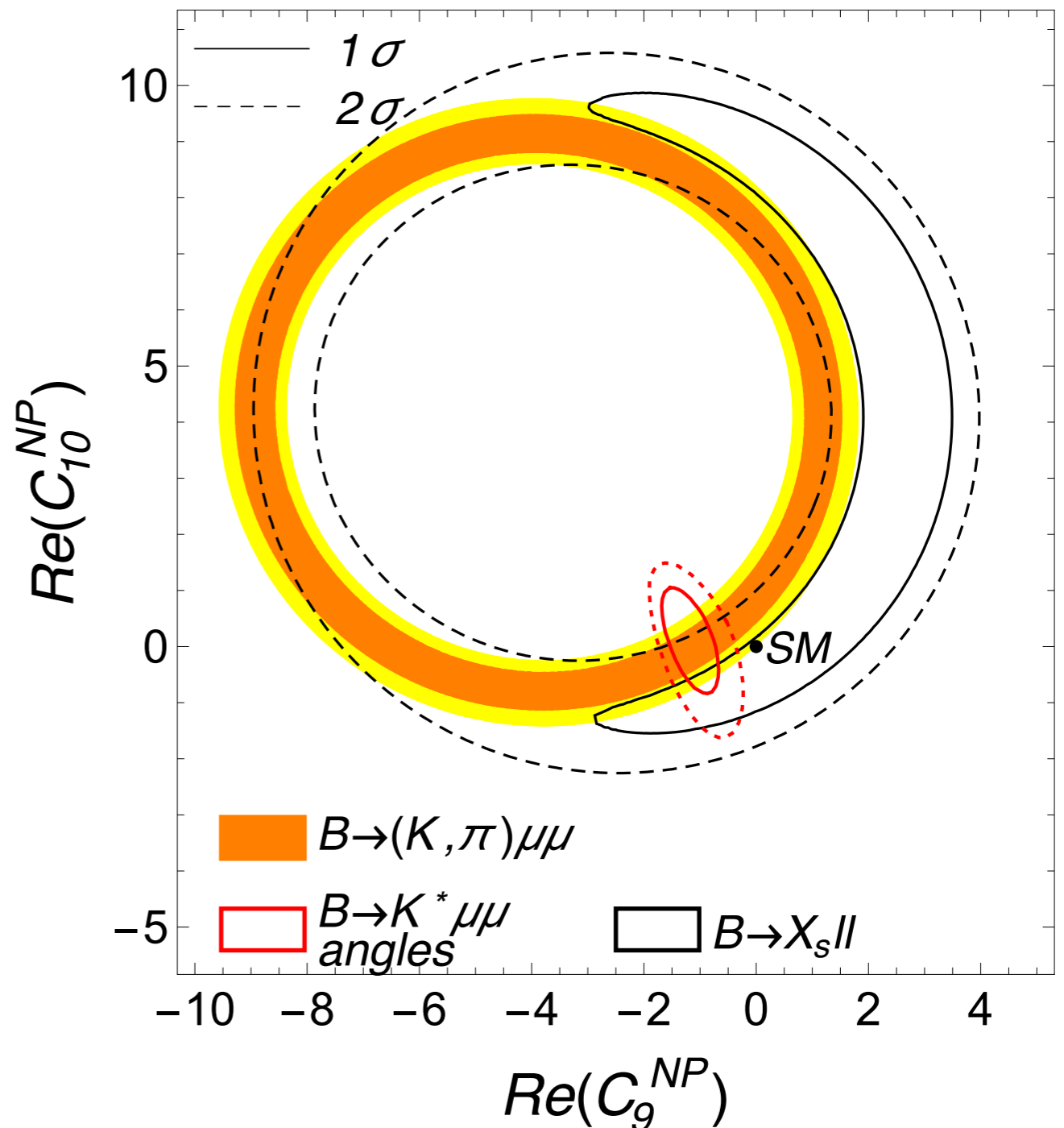
CKM Comparison

- CKM from FCNC are lower than determinations from trees and unitarity.



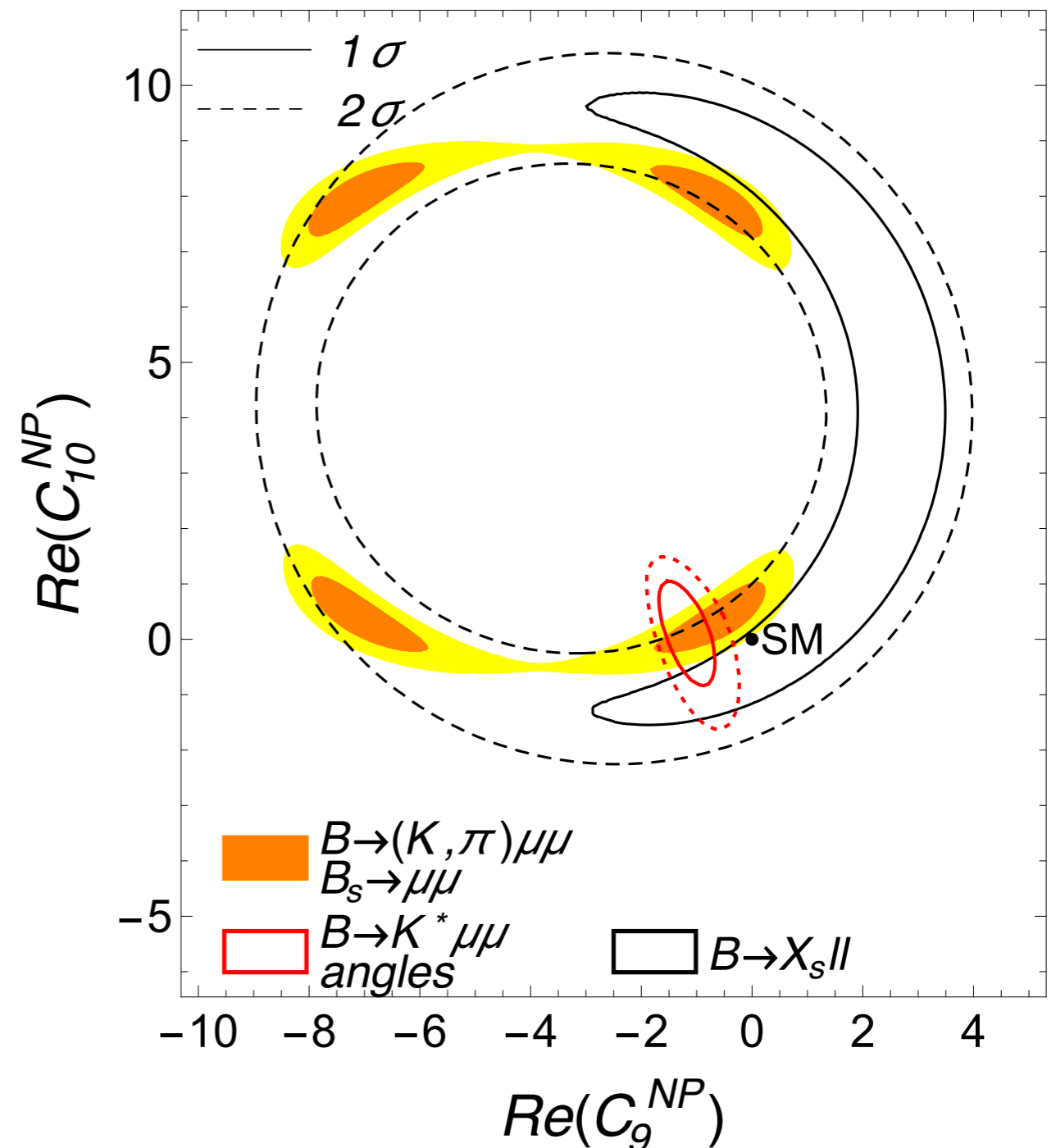
Wilson Coefficients

- Assuming no new physics is sad:
 - take the CKM matrix from a global fit;
 - determine best fit to Wilson coefficients C_9 and C_{10} .
- From the observables considered here, the SM is 2σ away from the **best fit**.
- Comparable but complementary to angular observables in $B \rightarrow K^* \mu\mu$.



Wilson Coefficients 2

- Add $B_s \rightarrow \mu\mu$, which also relies on lattice QCD $-f_{B_s}$.
- Favored region shrinks but only away from SM point.
- NB: assuming no new CPV and avoiding $b \rightarrow s\gamma$ constraints.



Outlook

- Perhaps too much information to summarize.
- The overarching take-home message:
 - we provide a convenient useful parametrization of the form factors, including correlations needed for joint fits, ratios, *etc.*;
 - the scope of application is not limited to what we've done;
 - just like collider physicists use CTEQ or MRSW parton densities, flavor physicists can use our (or other group's) form factors.
- Future work, e.g., on MILC HISQ ensembles, will improve the precision (over the coming few years).

Vielen herzlichen Dank!