# $B \to \pi\pi\ell\nu$

## Accesing theory estimates in various phase space corners

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# Motivation

- $B \to \pi\pi\ell\nu$ includes resonant $B \to \{\rho(770), f_0(980), \sigma, \ldots\}\ell\nu$
  - provides for unambigious interface between theory calculation and experimental analyses
- poses challenge to access the $B \to \pi\pi$ form factors in the entire $\pi\pi\ell\nu$ phase space
- provides laboratory to study only hadronic form factors (i.e.: no further matrix elements as in FCNC decays)

# Lorentz decomposition

[1310.6660]

- 4 $B \to \pi\pi$ form factors $F_\perp$, $F_\parallel$, $F_0$, $F_t$ in (axial) vector current

$$\langle \pi\pi | \bar{u}\gamma^\mu b | \bar{B} \rangle = i F_\perp \frac{1}{\sqrt{k^2}} \bar{q}^\mu_{(\perp)}$$

$$-\langle \pi\pi | \bar{u}\gamma^\mu \gamma_5 b | \bar{B} \rangle = F_0 \frac{2\sqrt{q^2}}{\sqrt{\lambda}} k^\mu_{(0)} + F_\parallel \frac{\bar{k}^\mu_{(\parallel)}}{\sqrt{k^2}} + F_t \frac{q^\mu}{\sqrt{q^2}}$$

- $F_i \equiv F_i(q^2, k^2, q \cdot \bar{k})$
- $\bar{q}^\mu_{(\perp)}$, $\bar{k}^\mu_{(\parallel)}$, $k^\mu_{(0)}$, $q^\mu$: orthogonal by construction
- $\lambda \equiv \lambda(M_B^2, q^2, k^2)$ Källén function

# Partial Wave Decomposition

[1310.6660]

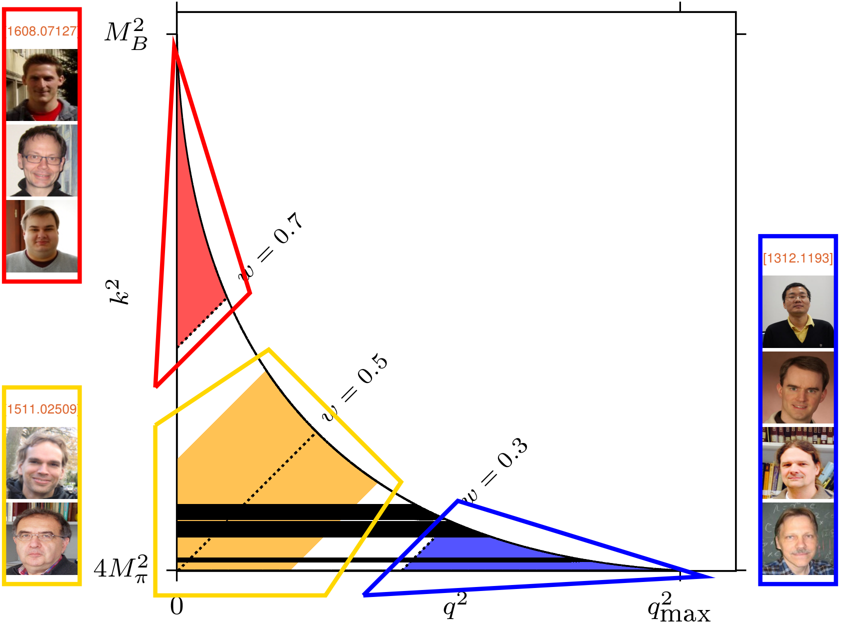
- expand in Legendre polynomials of $\cos\theta_\pi \propto q \cdot \overline{k}$ (dipion helicity angle)
- $\lambda = 0, t$ starts with S wave:

$$F_{0(t)} = F^{S}_{0(t)}(q^2, k^2) + \sqrt{3} F^{P}_{0(t)}(q^2, k^2) \cos\theta_\pi + \ldots$$

- $\lambda = \perp, \parallel$ starts with P wave:

$$F_{\perp(\parallel)} = \frac{\sqrt{3}}{\sqrt{2}} F^{P}_{\perp(\parallel)}(q^2, k^2) + \ldots$$

- result: problem reduced to to functions $F^{(\ell)}_{\lambda}(q^2, k^2)$ (Dalit-plot like)



1608.07127
1511.02509
[1312.1193]
$M_B^2$
$k^2$
$4M_\pi^2$
$w = 0.7$
$w = 0.5$
$w = 0.3$
0
$q^2$
$q^2_{\rm max}$

# Dispersion relations and $\chi$PT

[1312.1193]

– dispersion relation for fixed $q^2$

$$F(q^2,k^2) = (k^2)^n \int_{4M_\pi^2}^{\infty} \mathrm{d}s_k \, \frac{\rho(q^2,s_k)}{s_k^n(s_k - k^2 - i\varepsilon)} + n - 1 \text{ subtraction terms}\,.$$

– up to $16M_\pi^2$ (more practically: up to $4M_K^2$) the phase of the form factor relates to the $\pi\pi \to \pi\pi$ scattering phase via Watson's theorem

– Omnès representation used to solve the dispersive integral equation

– subtraction terms are fixed through use of Heavy Hadron $\chi$PT

# Light-Cone Sum Rules (LCSRs)

[1511.02509]

- sum rule with on-shell dipion state, interpolating the $B$ meson
- two $2\pi$-LCDAs are a universal, non-perturbative input
  - one LCDA normalized to $F_\pi$, the e.m. pion form factor, taken from data
  - other LCDA normalization from instanton model
- sum rule works for $k^2$ close to the threshold $4M_\pi^2$
- P wave not saturated by $B \to \rho$ transition: ample room (20 – 30%) for non-resonant P-wave contributions

# QCD-improved Factorization (QCDF)

[1608.07127]

- the energies of the pions $E_{1,2}$ are large in the $B$ rest frame: $E_{1,2} \sim M_B$
  - the soft modes in the $B$ meson factorize from the collinear modes in the pion
- the dipion mass $k^2$ is large: $k^2 \sim M_B^2$
  - the collinear modes in the first pion factorize from he anticollinear modes in the second pion
- factorization formula

$$\langle \pi_1 \pi_2 | \overline{u} \Gamma b | \overline{B} \rangle = \xi_\pi(E_2) \int_0^1 \mathrm{d}u \, \phi_\pi(u) T_\Gamma^{\mathrm{I}}(u, k^2, E_1, E_2) + \int_0^1 \mathrm{d}u \int_0^1 \mathrm{d}v \int_0^\infty \frac{\mathrm{d}\omega}{\omega} \phi_\pi(u) \, \phi_\pi(v) \, \phi_B^+(\omega) \, T_\Gamma^{\mathrm{II}}(u, v, \omega, k^2, E_1, E_2)$$

- formula confirmed by explicit calculation up to NLO in $\alpha_s$