Measurement of the τ g-2, status & perspectives

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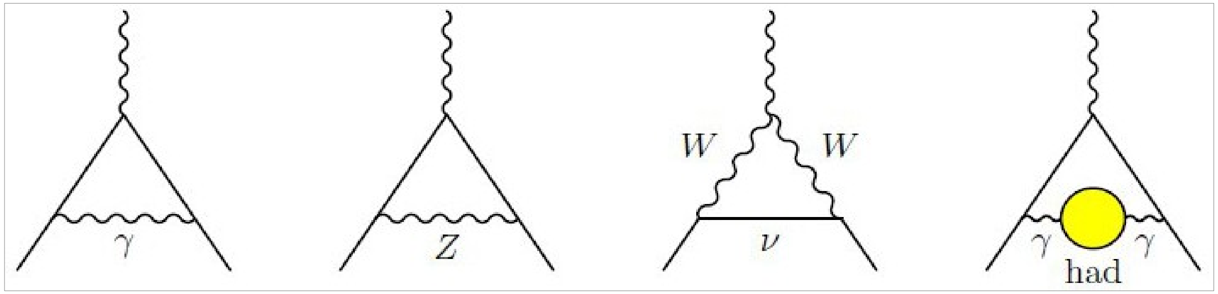
g-2

- $a_l = (g_l - 2)/2$ is the QFT correction to the Dirac equation



- Actual value depends on the mass of the involved leptons
- SM predictions for the τ:

	mu	τ
aEW/aH	1/45	1/7
aEW/δaH	3	10

a_τ^{SM} =	117324 (2)	x 10^{-8}	QED
+	47.4 (0.5)	x 10^{-8}	EW
+	337.5 (3.7)	x 10^{-8}	HLO
+	7.6 (0.2)	x 10^{-8}	HHO (vac)
+	5 (3)	x 10^{-8}	HHO (lbl)

Eidelman et al., JHEP03(2016)140

- Precision test of the SM and opportunity for NP searches
- g-2 has been measured with high precision for e and μ

$a_e = (1159.65218076 \pm 0.00000027) \times 10^{-6}$ (PDG 2014)$_e$

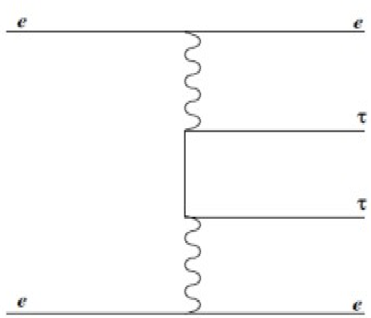
$a_\mu = (1165.9209 \pm 0.0006) \times 10^{-6}$ (PDG 2014)$_\mu$

τ g-2 status

- What about the τ?
- Too short lifetime, have to infer on g-2 from cross-section/decays
- Current limit on PDG by DELPHi using $e^-e^+ \rightarrow e^-e^+\tau^-\tau^+$ (assumes any deviation from tree diagram due to g-2)

a_τ=0.018 ± 0.017 **$-0.052 < a_\tau < 0.013$ at 95% CL (PDG 2014)**

J. Abdallah et al. [DELPHI Collaboration], EPJ C35 159



$e^-e^+ \rightarrow e^-e^+\tau^-\tau^+$ proposed by:

F. Cornet and J.I. Illana, Phys. Rev. D 53 (1996) 118

- To be compared to SM prediction **a_τ=1177.21(5) x 10^{-6}**
- Model independent limits for NP:

$-0.007 < a_\tau^{NP} < 0.005$ at 95% CL

Gonzalez-Sprinberg, et al. Nucl. Phys. B 582 (2000) 3

τ g-2 @ B-factories

- Two different proposals in “recent” years:
- Measure g-2 in production via spin correlations of decays products:

J. Bernabeu et al., JHEP01(2009)062

- Measure g-2 in decays via leptonic radiative decays:

M. Passera et al., JHEP03(2016)140

- Caution: if the τ is off mass-shell what one measures is not g-2 but $F_2(q^2)$
- Most general $ff\gamma$ vertex:

$$\Gamma^\mu(q^2) = -ieQ_f\left\{\gamma^\mu F_1(q^2) + \frac{\sigma^{\mu\nu}q_\nu}{2m_f}\left[iF_2(q^2) + F_3(q^2)\gamma_5\right] + \left(\gamma^\mu - \frac{2q^\mu m_f}{q^2}\right)\gamma_5\, F_4(q^2)\right\}$$

- $F_2(q^2) \longrightarrow \mathbf{a}_\tau,\ q^2 \longrightarrow 0$
- For $e^- e^+ \to \tau^- \tau^+$ @ 1 loop $F_2(s) = \left(\frac{\alpha}{2\pi}\right)\frac{2m_\tau^2}{s}\frac{1}{\beta}\left(\log\frac{1+\beta}{1-\beta} - i\,\pi\right), \quad \text{for} \quad q^2 = s > 4m_\tau^2$

$$F_2(M_\Upsilon^2) = (2.65 - 2.45\,i) \times 10^{-4}$$ @ Y(4S)

τ g-2 from asymmetries

- Spin dependent cross-section for $e^- e^+ \to \tau^- \tau^+$:

$$\frac{d\sigma(\vec{s}_+, \vec{s}_-)}{d\Omega_{\tau^-}} = \frac{d\sigma^0(\vec{s}_+, \vec{s}_-)}{d\Omega_{\tau^-}} + \frac{d\sigma^S(\vec{s}_+, \vec{s}_-)}{d\Omega_{\tau^-}} + \frac{d\sigma^{SS}(\vec{s}_+, \vec{s}_-)}{d\Omega_{\tau^-}}$$

- We consider 2 body (hν) final states; integrating out $\cos\theta_{\tau-}$ one gets

$$d^4\sigma^{SS} = \frac{2\pi\alpha^2\beta}{3\,s}\left[(s_+^x s_-^x)\,\mathcal{XX} + (s_+^y s_-^y)\,\mathcal{YY} + (s_+^z s_-^z)\,\mathcal{ZZ}\right] \times \frac{d\Omega_{h^+}}{4\pi}\frac{d\Omega_{h^-}}{4\pi}\,Br_+\,Br_-$$

$$\mathcal{XX} = (2-\beta^2)\,|F_1|^2 + 4\mathrm{Re}\,\{F_2\}$$
$$\mathcal{YY} = -|F_1|^2\,\beta^2$$
$$\mathcal{ZZ} = (1+\beta^2)\,|F_1|^2 + 2\mathrm{Re}\,\{F_2\}$$

- Integrating out, e.g., $\theta_\pm^*$ & performing asymmetric int. on $\varphi_\pm$ one gets A_{TT}:

$$d^2\sigma_{TT} = \frac{\pi\alpha^2\beta}{96\,s}\,[-(\alpha_-\alpha_+)]\,(\mathcal{XX})\,(\cos\phi_-\ \cos\phi_+)\,d\phi_+\,d\phi_-\,Br_+\,Br_-$$

$$A_{TT} \equiv -\frac{\alpha_-\alpha_+}{\sigma}\left(\int_{-\pi/2}^{\pi/2} d\phi_- - \int_{\pi/2}^{3\pi/2} d\phi_-\right)\left(\int_{-\pi/2}^{\pi/2} d\phi_+ - \int_{\pi/2}^{3\pi/2} d\phi_+\right) d^2\sigma_{TT}$$
$$= -\frac{\pi\alpha^2\beta}{6\,s}\,\frac{\alpha_-\alpha_+}{\sigma}\left[(2-\beta^2)\,|F_1|^2 + 4\mathrm{Re}\,\{F_2\}\right] Br_+\,Br_-$$

- Using similar techniques one can get A_{NN} (asym. $\varphi_\pm$) and A_{LL} (asym. $\theta_\pm^*$)

$$A_{NN} = \frac{\pi\alpha^2\beta}{6\,s}\,\frac{(\alpha_-\alpha_+)}{\sigma}\,\beta^2\,|F_1|^2\,Br_+\,Br_- \qquad A_{LL} = -\frac{\pi\alpha^2\beta}{6\,s}\,\frac{(\alpha_-\alpha_+)}{\sigma}\left[(1+\beta^2)\,|F_1|^2 + 2\mathrm{Re}\,\{F_2\}\right] Br_+\,Br_-$$

τ g-2 from asymmetries (2)

- With a similar approach, one can define $A_{LT} \sim Re\{F_2\}$ (asym. int. on $\cos\theta_{\tau-}$) and $A_N \sim Im\{F_2\}$ (from linear x-sec term, asym. int. on $\cos\theta_{\tau-}$)
- Expected sensitivities for (excl.) $\tau \rightarrow h\nu$, $h=\pi, \rho$, 100% eff. (?), (no syst.):

		EXPERIMENT (ab = attobarn = $10^{-18}b$)		
		Babar+Belle	Super B/Flavour Factory (1 yr. running)	Super B/Flavour Factory (5 yrs. running)
	Correlations	$2ab^{-1}$	$15ab^{-1}$	$75ab^{-1}$
$Re\{F_2\}$	$TT--LT$	7.6×10^{-5}	2.8×10^{-5}	1.2×10^{-5}
	$LL--LT$	5.2×10^{-5}	1.9×10^{-5}	8.5×10^{-6}
	$NN--LT$	5.1×10^{-5}	1.8×10^{-5}	8.3×10^{-6}
	Global	2.9×10^{-5}	1.1×10^{-5}	4.7×10^{-6}
$Im\{F_2\}$ (from ref. [10])	Normal single-τ Asymm.	2.1×10^{-5}	7.8×10^{-6}	3.5×10^{-6}

τ g-2 from asymmetries: issues

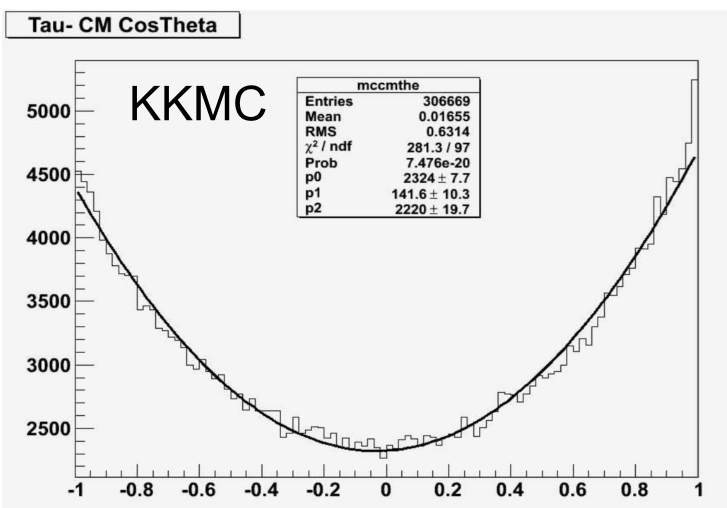
- Main assumption to extract F_2 from asymmetries: <u>being on top of the resonance boxes and interference can be neglected</u>

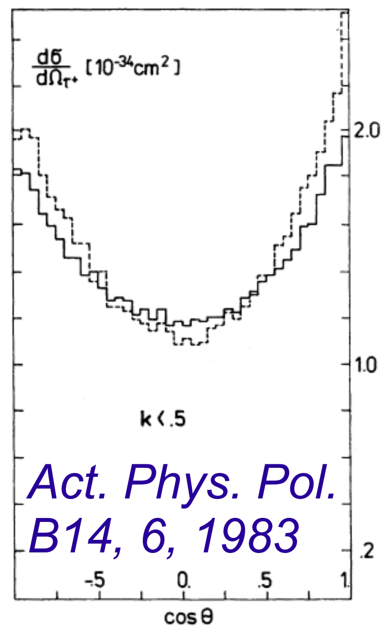
$$\frac{d\sigma}{dcos\theta_{\tau^-}} = \frac{\pi\,\alpha^2}{2s}\beta\,\left[(2-\beta^2\sin^2\theta_{\tau^-})\,|F_1(s)|^2 + 4\,\mathrm{Re}\,\{F_2(s)\}\right] \times Br(\tau^- \to h^-\nu_\tau)\,Br(\tau^+ \to h^+\bar{\nu}_\tau)$$

Enhancement to res. amplitude

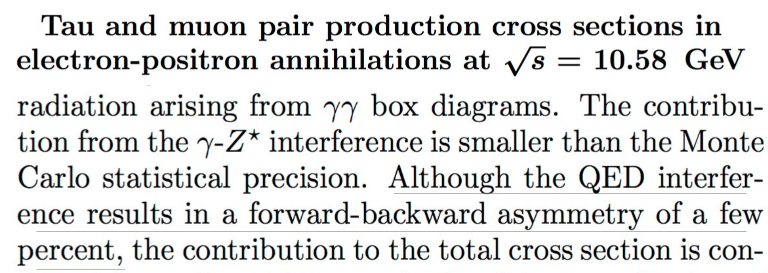
$$H(M_\Upsilon^2) = \frac{4\pi\alpha Q_b^2}{M_\Upsilon^2}\frac{|F_\Upsilon(M_\Upsilon^2)|^2}{i\Gamma_\Upsilon M_\Upsilon} = -i\,\frac{3}{\alpha}Br\left(\Upsilon \to e^+e^-\right)$$

- From simulation it can be seen that this is not quite true





Act. Phys. Pol. B14, 6, 1983



Tau and muon pair production cross sections in electron-positron annihilations at $\sqrt{s} = 10.58$ GeV

radiation arising from $\gamma\gamma$ box diagrams. The contribution from the γ-$Z^\star$ interference is smaller than the Monte Carlo statistical precision. Although the QED interference results in a forward-backward asymmetry of a few percent, the contribution to the total cross section is con-

Banerjee et al., Phys.Rev.D77:054012,2008

- Can we somehow account for box diagrams in order to use this method?

g-2 from radiative τ decays

- Dates back to an idea by Laursen et al.

Radiation zeros and a test for the g value of the τ lepton

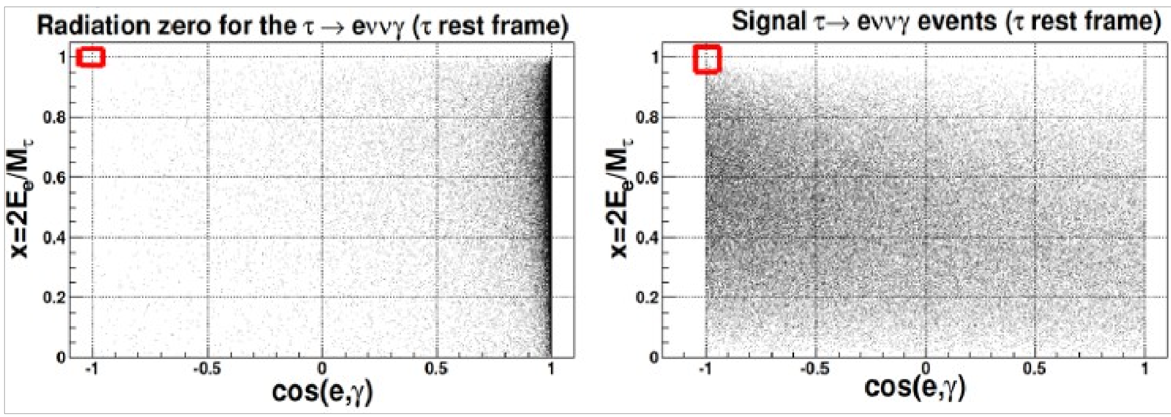
Laursen, Samuel, Sen, PRD29 (1984) 2652

M. L. Laursen,* Mark A. Samuel, and Achin Sen

Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078

(Received 21 November 1983)

- Take advantage of "radiation zero"



Study of the tau g-2 via its radiative leptonic decays

Massimo Passera
INFN Padova

TAU 2012
Nagoya
September 18 2012

M. Passera et al. arxiv1301.5302

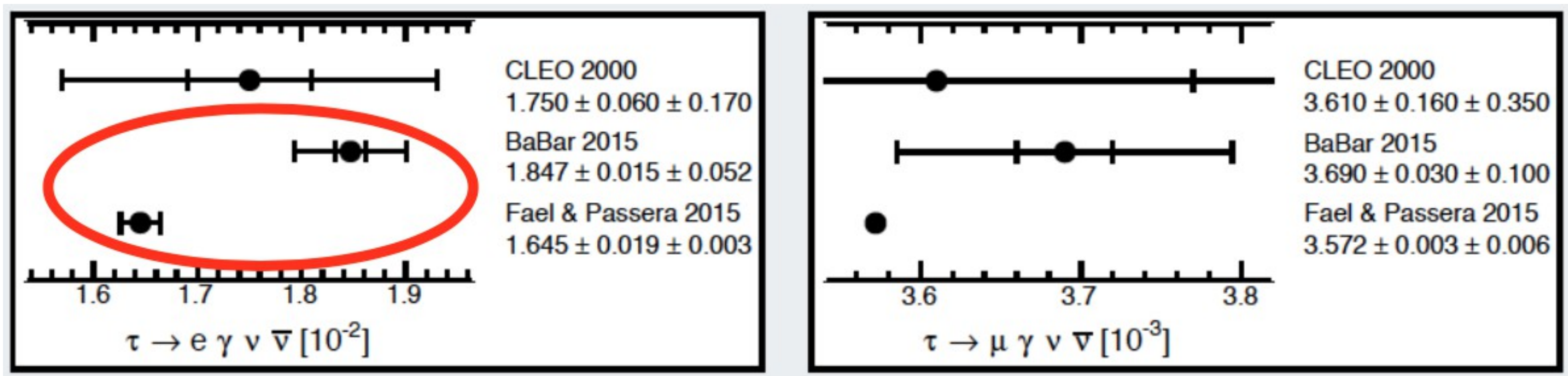
- Only sensitive to large values of g-2 ($N_{evt} \sim \mathbf{a_\tau}^2$)
- Recently M. Passera et al. suggested to use an effective lagrangian approach and full NLO QED corrections and fit on full phase-space

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + c_a \frac{e}{4\Lambda}\, \mathcal{O}_a - c_d \frac{i}{2\Lambda}\, \mathcal{O}_d,$$

$$\mathcal{O}_a = \bar{\tau}\sigma_{\mu\nu}\tau\, F^{\mu\nu}, \quad \mathcal{O}_d = \bar{\tau}\sigma_{\mu\nu}\gamma_5\tau\, F^{\mu\nu}.$$

g-2 from radiative τ decays (2)

- Radiative τ decays recently measured by BaBar, only other measurement on PDG (and only existing for electron channel) by CLEO



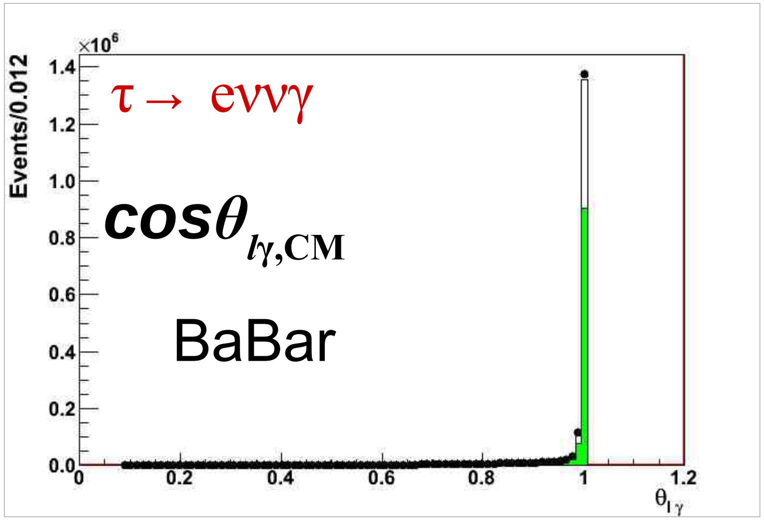
- CLEO 2000: T. Bergfeld et al., PRL 84 (2000) 830
- *BABAR* 2015: PRD 91, 051103 (2015)
- Fael & Passera 2015: NLO calculation, JHEP 07 (2015) 153, arXiv:1602.00457 [hep-ph]
- 3.5σ discrepancy between *BABAR* 2015 and NLO calculation, to be investigated

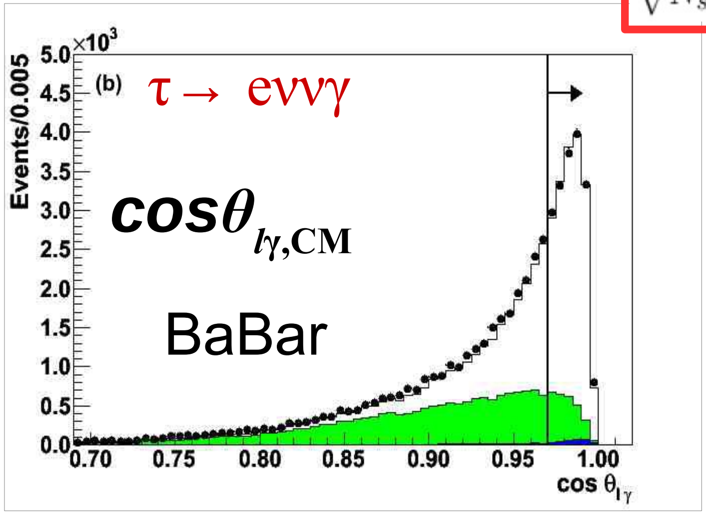
From M. Passera'a talk @ Belle2 Italian Meeting

- Experimentally $\tau \to \mu\nu\nu\gamma$ relatively easy to measure $\tau \to e\nu\nu\gamma$ more complicated due to bremsstrahlung photons with similar signature as signal (@ BaBar hard-cut $m_{l\gamma} > 0.14$ GeV/c^2)

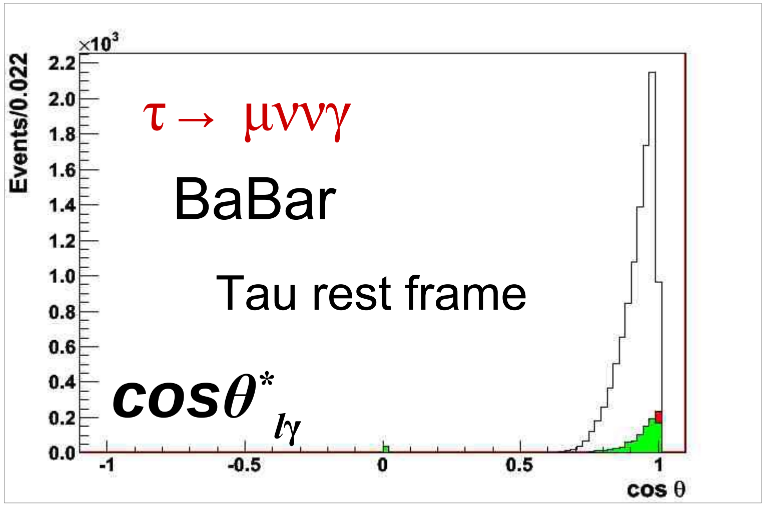
g-2 from radiative τ decays (3)

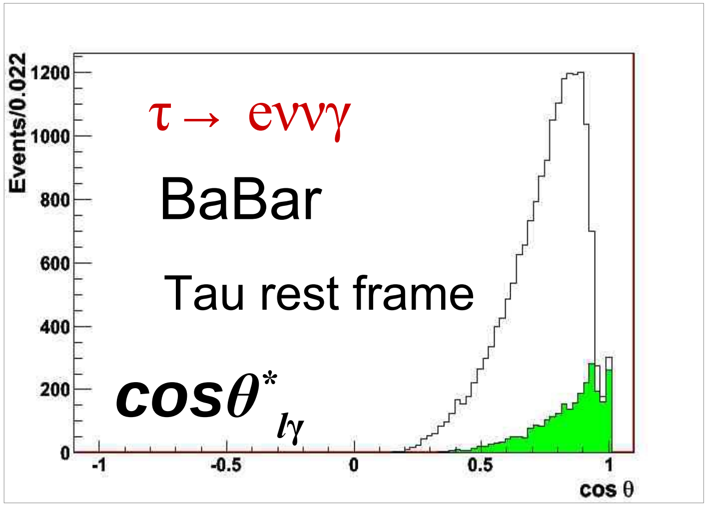
- To reduce (poorly known) bkg, selection is optimized using $\frac{N_{sig}}{\sqrt{N_{sig} + N_{bkg}(1 + \alpha^2 N_{bkg}) + \beta^2}}$





- Due to selection requirements most sensitive PS region is lost





PRD 91, 051103 (2015)

Summary

- **a_τ** is very poorly known experimentally
- Use of spin correlations is attractive but effect of boxes has to be accounted → not usable yet. Possible?
- Radiative decays might be a viable way → detailed studies have to be performed and better detector performance is required
- Belle2 should finally measure **a_τ**!
- Some work from both theory and experiment is required

Backups

Asymmetries ref. frame

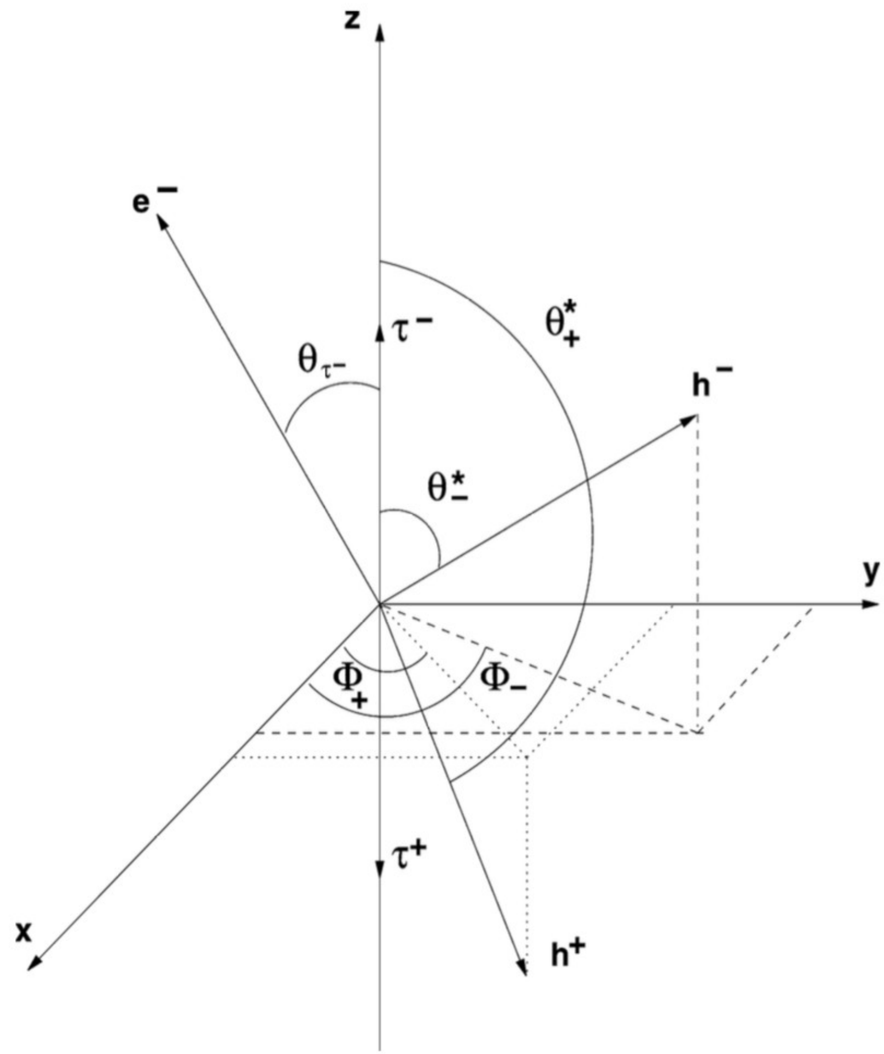


Fig. 2. Coordinate system for $h^{\pm}$ production from the $\tau^{\pm}$.

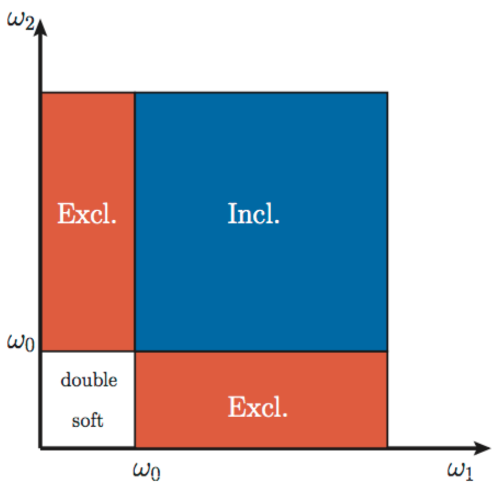
Double real soft photon

Inclusive and exclusive branching ratios at NLO

τ

The branching ratio of radiative μ and τ leptonic decays for a minimum photon energy ω_0:

$$\mathcal{B}(\omega_0) \propto \int d\Phi_4 \left(d\Gamma_{\rm LO} + d\Gamma_{\rm virt}\right) + \int d\Phi_5 \, d\Gamma_{\gamma\gamma}$$



- $\mathcal{B}^{\rm Exc}(\omega_0)$: only one γ of energy $\omega > \omega_0$, additional second soft photon $\omega' < \omega_0$.

  $\mathcal{B}^{\rm Exc}(\omega_0) = \blacksquare$ (red)

- $\mathcal{B}^{\rm Inc}(\omega_0)$: at least one γ of energy $\omega > \omega_0$.

  $\mathcal{B}^{\rm Inc}(\omega_0) = \blacksquare + \blacksquare$ (red + blue)

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Double real soft photon

B.R. of radiative τ leptonic decays ($\omega_0 = 10$ MeV)		
	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\mathcal{B}_{\rm LO}$	1.834×10^{-2}	3.663×10^{-3}
$\mathcal{B}^{\rm Inc}_{\rm NLO}$	$-1.06\,(1)_n(10)_N \times 10^{-3}$	$-5.8\,(1)_n(2)_N \times 10^{-5}$
$\mathcal{B}^{\rm Exc}_{\rm NLO}$	$-1.89\,(1)_n(19)_N \times 10^{-3}$	$-9.1\,(1)_n(3)_N \times 10^{-5}$
$\mathcal{B}^{\rm Inc}$	$1.728\,(10)_{\rm th}(3)_\tau \times 10^{-2}$	$3.605\,(2)_{\rm th}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\rm Exc}$	$1.645\,(19)_{\rm th}(3)_\tau \times 10^{-2}$	$3.572\,(3)_{\rm th}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\dagger}_{\rm EXP}$	$1.847\,(15)_{\rm st}(52)_{\rm sy} \times 10^{-2}$	$3.69\,(3)_{\rm st}(10)_{\rm sy} \times 10^{-3}$

(n): numerical errors

(N): uncomputed NNLO corr.

$\sim (\alpha/\pi) \ln r \ln(\omega_0/M) \times \mathcal{B}^{\rm Exc/Inc}_{\rm NLO}$

(th): combined $(n) \oplus (N)$

(τ): experimental error of τ lifetime: $\tau_\tau = 2.903(5) \times 10^{-13}$ s

† BaBar, PRD 91 (2015) 051103;
B. Oberhof, arXiv:1502.01810.

Fael, Mercolli and MP, 1506.03416 (JHEP 2015)
Fael and MP, 1602.00457

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SM contributions to g-2

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+	5 (3)	x 10^{-8}	HHO (lbl)	

QED: Up to 3 loops
M.Passera,Phys.Rev.D75(2007)013002
2 ln2 (m /me)(/ 4) from
un-calculated 4-loop term

EW: Up to 2 loops
A. Czarnecki et al., PRL 76 (1996) 3267
T.V. Kukhto et al., Nucl. Phys. B 371 (1992) 567

HLO: Dispersion integral
S.Eidelman et al., Mod.Phys.Lett.A22(2007)159

HHO (vac): **B.Krause, Phys.Lett.B390 (1997) 392**

HHO (lbl): **S.Eidelman et al., Mod.Phys.Lett.A22(2007)159**

Spare

- .....