

# $CP$ violation and mixing in charm decays at LHCb

Maxime Schubiger  
On behalf of the LHCb collaboration

Beauty 2019 - Ljubljana, Slovenia  
3 October 2019



# Introduction

- In the Standard Model (SM), charge-parity violation (CPV) in the quark sector comes only from the complex phase in the CKM matrix
  - Order of magnitudes too small to explain our matter dominated universe
- Look for other sources in New Physics (NP) processes that enhance CPV

- CPV in the decay
  - Difference of decay rate between two  $CP$  conjugated states

$$|A(i \rightarrow f)|^2 \neq |A(\bar{i} \rightarrow \bar{f})|^2$$

- CPV in mixing
  - Difference of transition rate between two flavour eigenstates

$$|A(i \rightarrow \bar{i})|^2 \neq |A(\bar{i} \rightarrow i)|^2$$

- CPV in the interference between mixing and decay
  - Interference between the decay with and without mixing

$$|A(i \rightarrow f)|^2 \neq |A(i \rightarrow \bar{i} \rightarrow f)|^2$$

# $CP$ violation in charm

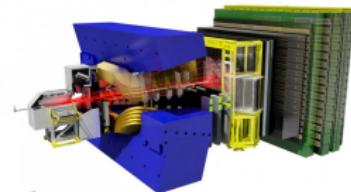
Why look for CPV in charm ?

- Prediction of CPV in charm from the SM are small
  - Lots of room for NP enhancement
- Only way to probe for CPV in up-type hadrons
  - Complementary to other searches in  $B$  or  $K$



Why look for CPV in charm at LHCb ?

- Largest sample of charm decays
  - Large  $c\bar{c}$  cross-section:
$$\sigma(pp \rightarrow c\bar{c}X) = (2369 \pm 3 \pm 152 \pm 118) \mu\text{b},$$
at 13 TeV and for  $p_T < 8 \text{ GeV}/c$ ,  $2.0 < y < 4.5$  [JHEP 03 (2016) 159]
    - Large charm yields ( $\mathcal{O}(100 \text{ M})$ )  $D^0 \rightarrow K^- \pi^+$  tagged decays)
- Good momentum resolution (0.5 – 1%)
- Good tracking efficiency (over 95%) [Int. J. Mod. Phys A30 (2015) 1530022]
- Excellent vertex resolution (IP resolution  $(15 + 29/p_T) \mu\text{m}$ )

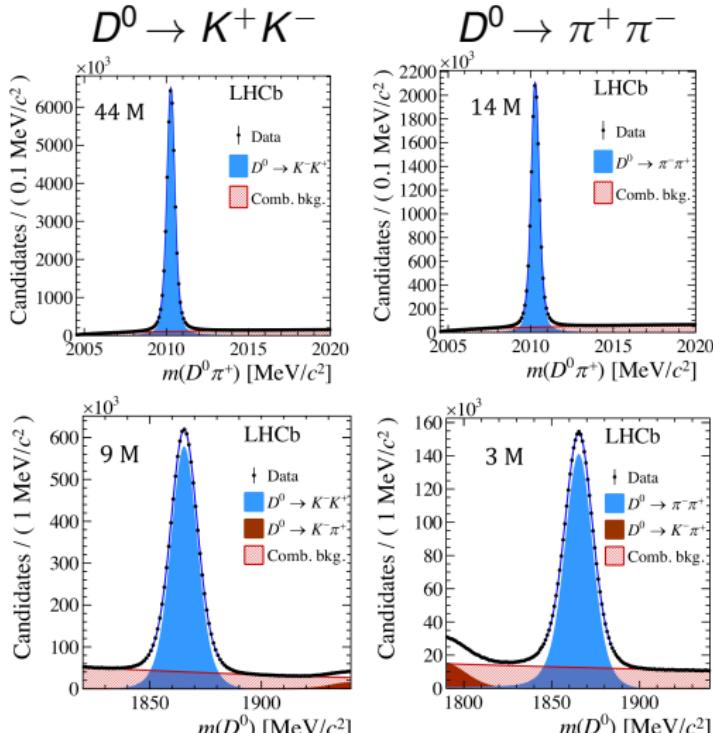


# **Observation of $CP$ violation in charm decays**

[Phys. Rev. Lett. 122 (2019) 211803]

- Dataset :  $5.9 \text{ fb}^{-1}$ , Run 2
- Comparison between 2 Cabibbo-suppressed decays :

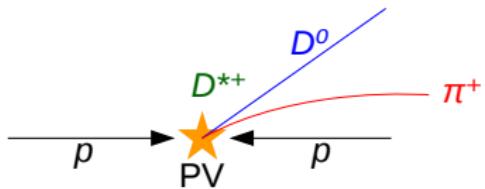
Prompt



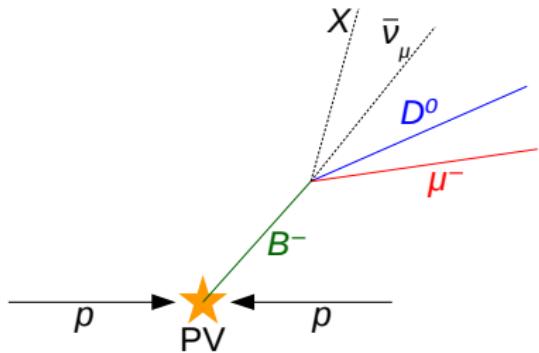
Semileptonic

2 independent tagging methods are used :

Prompt  
 $D^{*+} \rightarrow D^0 \pi^+$



Semileptonic  
 $B^- \rightarrow D^0 \mu^- \bar{\nu}_\mu X$



The experimental observable is not directly  $A_{CP}$ , but  $A_{\text{raw}}$  :

$$A_{\text{raw}} \approx A_{CP} + A_P + A_D + A_{\text{tag}}$$

- The production asymmetry  $A_P$  : Potential asymmetry between the production of  $D^{*+}$  ( $B^+$ ) and  $D^{*-}$  ( $B^-$ )
- The detection asymmetry  $A_D$  : Mesons and anti-mesons have different behaviours in matter (= 0 in symmetric final states such as  $K^+K^-$  and  $\pi^+\pi^-$ )
- The tagging asymmetry  $A_{\text{tag}}$  : The tagging particle also has different behaviour in matter according to its charge
- The  $CP$  asymmetry  $A_{CP}$  : The interesting physical quantity

$$A_{CP} = \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow \bar{f})}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow \bar{f})}$$

- Experimental asymmetries are difficult to measure
  - They can be made independent of the  $D^0$  decay by equalising the  $K^+K^-$  and  $\pi^+\pi^-$  kinematics
- They can be subtracted

$$\begin{aligned}\Delta A_{CP} &= A_{\text{raw}}(D^0 \rightarrow K^+K^-) - A_{\text{raw}}(D^0 \rightarrow \pi^+\pi^-) \\ &= \color{blue}{A_{CP}(D^0 \rightarrow K^+K^-)} + \color{green}{A_P(D^{*+})} + \color{red}{A_{\text{tag}}(\pi^+)} \\ &\quad - \color{blue}{A_{CP}(D^0 \rightarrow \pi^+\pi^-)} - \color{green}{A_P(D^{*+})} - \color{red}{A_{\text{tag}}(\pi^+)} \\ &= \color{blue}{A_{CP}(D^0 \rightarrow K^+K^-)} - \color{blue}{A_{CP}(D^0 \rightarrow \pi^+\pi^-)}\end{aligned}$$

Similarly for the semileptonic sample

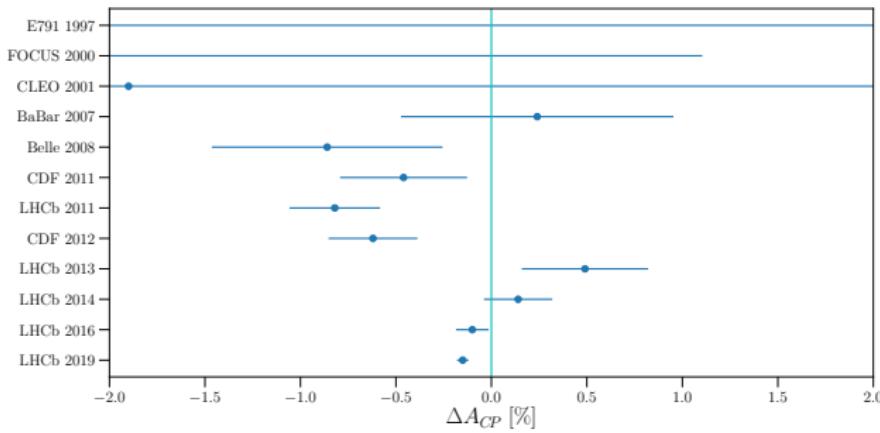
## Results of this analysis

- $\Delta A_{CP}^{prompt} = [-18.2 \pm 3.2 \pm 0.9] \times 10^{-4}$
- $\Delta A_{CP}^{SL} = [-9 \pm 8 \pm 5] \times 10^{-4}$

Combination of the two, plus previous Run 1 analyses :

- $\Delta A_{CP} = [-15.4 \pm 2.9] \times 10^{-4}$

⇒ First observation of  $CP$  violation in charm at  $5.3\sigma$  !



# Search for time-dependent $CP$ violation in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays

Preliminary results

[LHCb-CONF-2019-001]

[LHCb-PAPER-2019-032]

## Prompt analysis [LHCb-CONF-2019-001]

- Dataset :  $1.9 \text{ fb}^{-1}$ , 2015-2016
- Production mode :  $D^{*+} \rightarrow D^0 \pi^+$
- Yields : 17 M  $D^0 \rightarrow K^+ K^-$ , 5 M  $D^0 \rightarrow \pi^+ \pi^-$

## Semileptonic analysis [LHCb-PAPER-2019-032]

- Dataset :  $5.4 \text{ fb}^{-1}$ , 2016-2018
- Production mode :  $B^- \rightarrow D^0 \mu X^-$
- Yields : 9 M  $D^0 \rightarrow K^+ K^-$ , 3 M  $D^0 \rightarrow \pi^+ \pi^-$

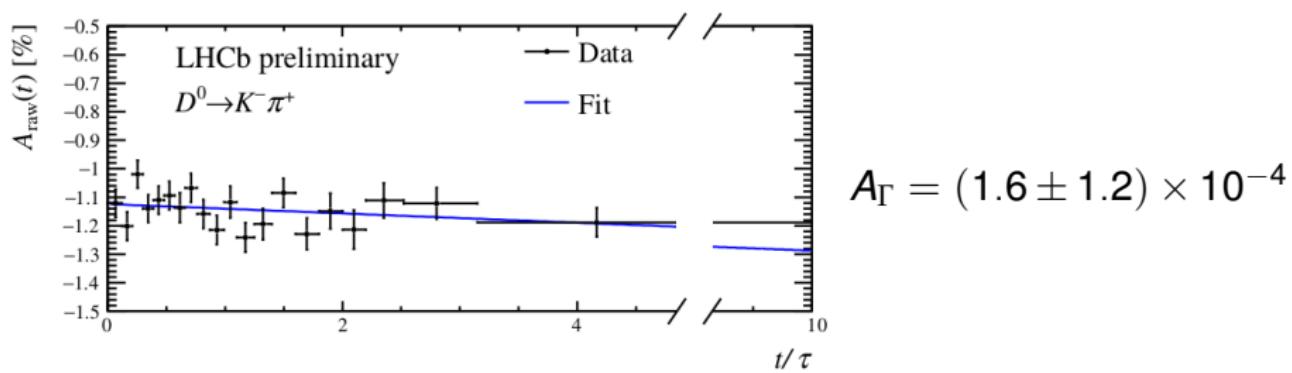
## Formalism

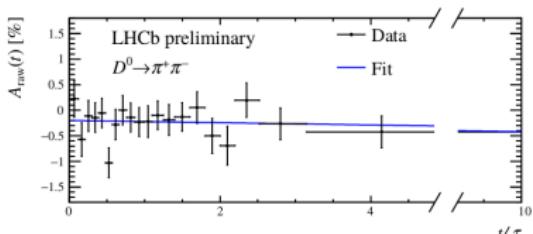
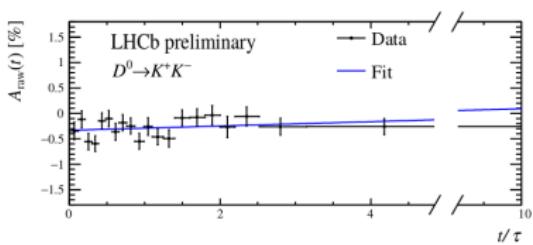
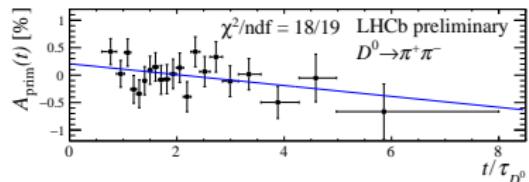
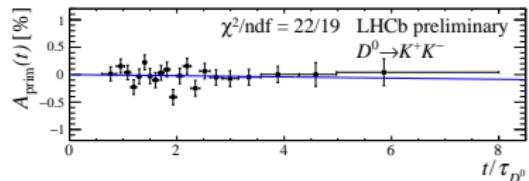
$$A_{CP} = \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow \bar{f})}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow \bar{f})} \approx A_{CP}^{\text{dir}} - A_\Gamma \frac{t}{\tau}$$

$$A_{\text{raw}}(D^0 \rightarrow f; t) \approx A_{CP}(D^0 \rightarrow f; t) + A_P(B(D^*)) + A_D(\mu(\pi))$$

Control channel :  $D^0 \rightarrow K^- \pi^+$

$A_\Gamma$  expected to be well below experimental sensitivity





Prompt results :

$$A_\Gamma(D^0 \rightarrow K^+K^-) = (1.3 \pm 3.5 \pm 0.7) \times 10^{-4}$$

$$A_\Gamma(D^0 \rightarrow \pi^+\pi^-) = (11.3 \pm 6.9 \pm 0.8) \times 10^{-4}$$

Semileptonic results :

$$A_\Gamma(D^0 \rightarrow K^+K^-) = (-4.3 \pm 3.6 \pm 0.5) \times 10^{-4}$$

$$A_\Gamma(D^0 \rightarrow \pi^+\pi^-) = (2.2 \pm 7.0 \pm 0.8) \times 10^{-4}$$

Prompt combination :

$$A_\Gamma(K^+K^- + \pi^+\pi^-) = (0.9 \pm 2.1 \pm 0.7) \times 10^{-4}$$

Semileptonic combination

$$A_\Gamma(K^+K^- + \pi^+\pi^-) = (-2.9 \pm 2.0 \pm 0.6) \times 10^{-4}$$

SM prediction [[A. Cerri et al., arxiv:1812.07638](#)]

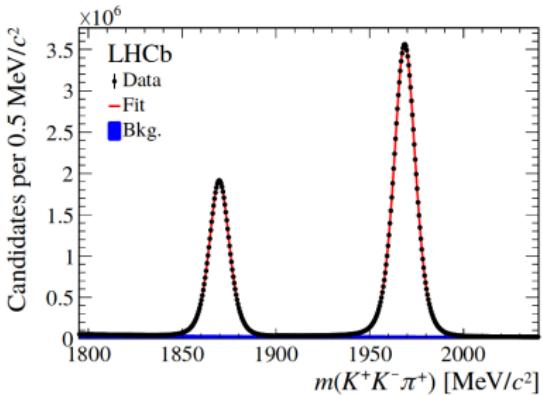
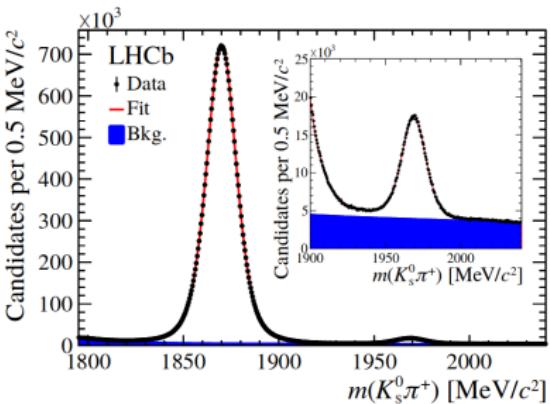
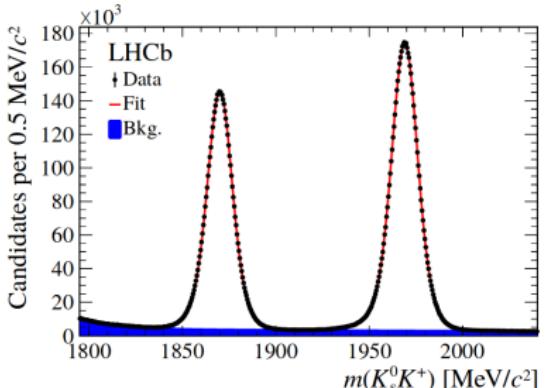
$$A_\Gamma \approx 3 \times 10^{-5}$$

# **Search for $CP$ violation in $D_s^+ \rightarrow K_s^0 \pi^+$ , $D^+ \rightarrow K_s^0 K^+$ and $D^+ \rightarrow \phi \pi^+$ decays**

[Phys. Rev. Lett. 122 (2019) 191803]

# Search for CPV in $D_s^+$ and $D^+$ decays [PRL 122 (2019) 191803]

- Dataset :  $3.8 \text{ fb}^{-1}$ , 2015-2017
- 3 decay modes
  - $D_s^+ \rightarrow K_s^0 \pi^+ : 600 \text{ k}$
  - $D^+ \rightarrow K_s^0 K^+ : 5.1 \text{ M}$
  - $D^+ \rightarrow \phi \pi^+ : 53.3 \text{ M}$
- 3 CF control samples
  - $D^+ \rightarrow K_s^0 \pi^+ : 30.5 \text{ M}$
  - $D_s^+ \rightarrow K_s^0 K^+ : 6.5 \text{ M}$
  - $D_s^+ \rightarrow \phi \pi^+ : 107 \text{ M}$



- $CP$  asymmetry measured from raw asymmetry

$$A_{\text{raw}}(D_{(s)}^+ \rightarrow f^+) \approx A_{CP}(D_{(s)}^+ \rightarrow f^+) + A_P(D_{(s)}^+) + A_D(f^+)$$

- Assume no CPV in CF control samples

$$A_{CP}(D_s^+ \rightarrow K_s^0 \pi^+) \approx A_{\text{raw}}(D_s^+ \rightarrow K_s^0 \pi^+) - A_{\text{raw}}(D_s^+ \rightarrow \phi \pi^+) - A_D(\bar{K}^0)$$

$$\begin{aligned} A_{CP}(D^+ \rightarrow K_s^0 K^+) &\approx A_{\text{raw}}(D^+ \rightarrow K_s^0 K^+) - A_{\text{raw}}(D^+ \rightarrow K_s^0 \pi^+) \\ &\quad - A_{\text{raw}}(D_s^+ \rightarrow K_s^0 K^+) + A_{\text{raw}}(D_s^+ \rightarrow \phi \pi^+) - A_D(\bar{K}^0) \end{aligned}$$

$$A_{CP}(D^+ \rightarrow \phi \pi^+) \approx A_{\text{raw}}(D^+ \rightarrow \phi \pi^+) - A_{\text{raw}}(D^+ \rightarrow K_s^0 \pi^+)$$

- Results

$$A_{CP}(D_s^+ \rightarrow K_s^0 \pi^+) = (1.6 \pm 1.7 \pm 0.5) \times 10^{-3}$$

$$A_{CP}(D^+ \rightarrow K_s^0 K^+) = (-0.04 \pm 0.61 \pm 0.45) \times 10^{-3}$$

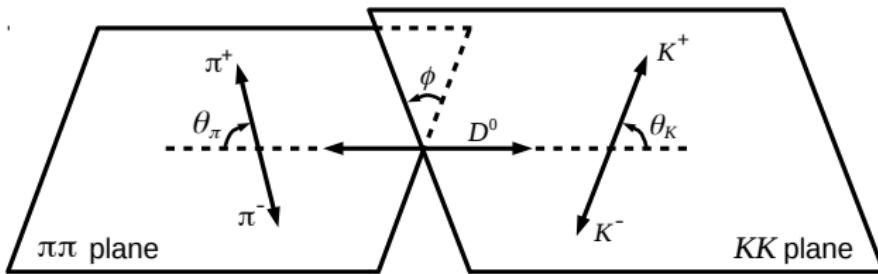
$$A_{CP}(D^+ \rightarrow \phi \pi^+) = (0.03 \pm 0.40 \pm 0.29) \times 10^{-3}$$

→ Compatible with  $CP$  conservation

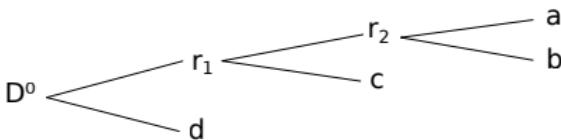
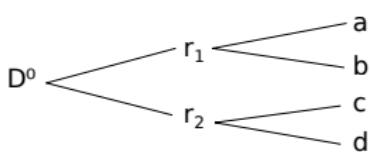
# **Search for $CP$ violation through an amplitude analysis of $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ decays**

[JHEP 02 (2019) 126]

- Dataset :  $3.0 \text{ fb}^{-1}$ , Run 1
- Production mode :  $B^- \rightarrow D^0 \mu^- \bar{\nu}_\mu X$
- Yield : 160 k signal candidates
- Large number of interfering amplitudes could enhance CPV
- 4-body spinless decay  $\rightarrow$  5D phase space
  - $m(K^+ K^-)$ ,  $m(\pi^+ \pi^-)$ ,  $\cos(\theta_K)$ ,  $\cos(\theta_\pi)$ ,  $\phi_{KK,\pi\pi}$



- Use the isobar model to describe the signal PDF



- The signal PDF

$$a(\mathbf{x}; \mathbf{c}) = \frac{\epsilon_s(\mathbf{x}) S(\mathbf{x}; \mathbf{c}) \mathcal{R}_4(\mathbf{x})}{\int \epsilon_s(\mathbf{x}) S(\mathbf{x}; \mathbf{c}) \mathcal{R}_4(\mathbf{x}) d^5 \mathbf{x}} \quad \text{with} \quad S(\mathbf{x}; \mathbf{c}) = \left| \sum_k c_k A_k(\mathbf{x}) \right|^2$$

- The background PDF

$b(\mathbf{x})$  is taken from the  $D^0$  mass sidebands

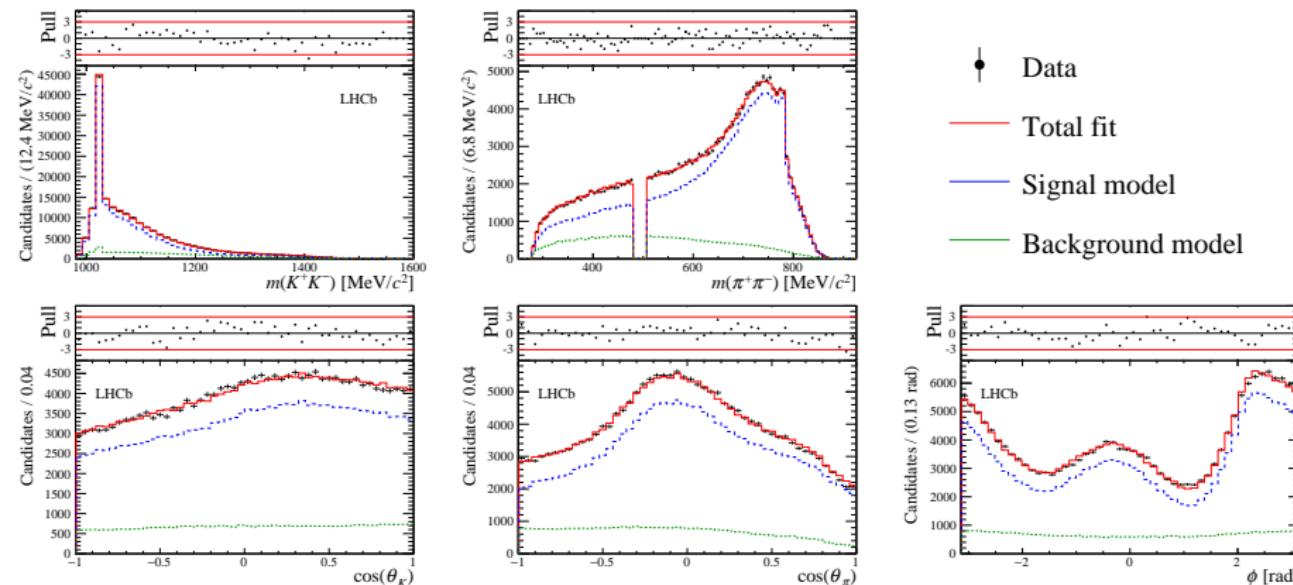
## Model Building Method

- ① Create a list of all possible amplitudes
- ② Start with following minimal model :
  - $D^0 \rightarrow \phi(1020)^0 [K^+, K^-] (\rho - \omega)^0 [\pi^+, \pi^-]$  in S,P & D waves
  - $D^0 \rightarrow K^*(892)^0 [K^+, \pi^-] \bar{K}^*(892)^0 [K^-, \pi^+]$  in S,P & D waves
- ③ Fit model + 1 new amplitude from the list
- ④ Add to the model the amplitude that produces the largest decrease in  $-2 \ln(\mathcal{L})$
- ⑤ Iterate steps 3 & 4

## Stopping criteria

- Goodness of fit :  $\chi^2$
- Sum of fit fractions : interference

26 amplitudes have been selected to describe the signal



- Simultaneous fit of  $D^0$  and  $\bar{D}^0$  decays
- CPV parametrisation :

$$\overline{|c_k|} = \frac{|c_k|_{D^0} + |c_k|_{\bar{D}^0}}{2}$$

$$\overline{\arg(c_k)} = \frac{\arg(c_k)_{D^0} + \arg(c_k)_{\bar{D}^0}}{2}$$

$$A_{|c_k|} = \frac{|c_k|_{D^0} - |c_k|_{\bar{D}^0}}{|c_k|_{D^0} + |c_k|_{\bar{D}^0}}$$

$$\Delta \arg(c_k) = \frac{\arg(c_k)_{D^0} - \arg(c_k)_{\bar{D}^0}}{2}$$

- Fit fraction asymmetry:

$$A_{\mathcal{F}_k} = \frac{\mathcal{F}_k^{D^0} - \mathcal{F}_k^{\bar{D}^0}}{\mathcal{F}_k^{D^0} + \mathcal{F}_k^{\bar{D}^0}}$$

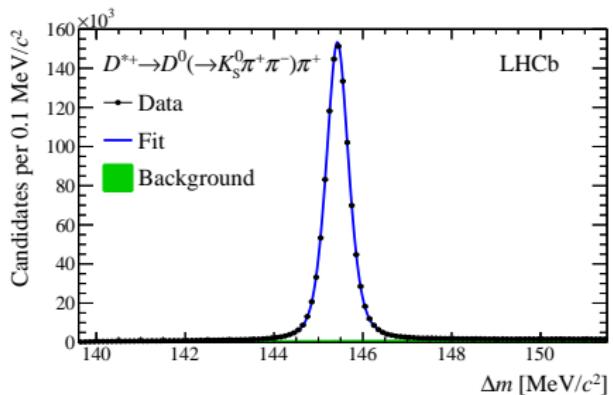
- Consistent with  $CP$  conservation with a sensitivity ranging from 1% to 15%

# **Measurement of the mass difference between neutral charm-meson eigenstates in $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ decays**

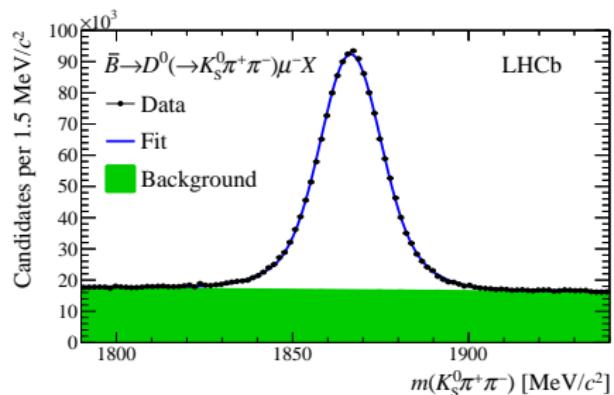
[Phys. Rev. Lett. 122 (2019) 231802]

- Dataset :  $3.0 \text{ fb}^{-1}$ , Run 1
- Production modes :
  - $D^{*+} \rightarrow D^0 \pi^+$  : 1.3 M signal candidates
  - $B \rightarrow D^0 \mu^- X$  : 1 M signal candidates

Prompt



Semileptonic



- Mass eigenstates

$$|D_{1,2}\rangle \equiv p|D^0\rangle \pm q|\bar{D}^0\rangle$$

- Mixing parameters

$$x \equiv \frac{m_1 - m_2}{\Gamma}, \quad y \equiv \frac{\Gamma_1 - \Gamma_2}{2\Gamma}, \quad \text{with } \Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$$

- CPV parametrisation

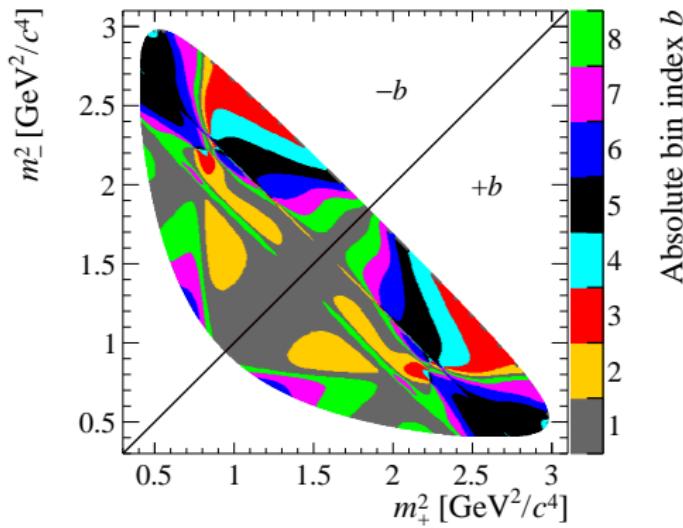
$$x_{CP} \equiv -\text{Im}(z_{CP}), \quad y_{CP} \equiv -\text{Re}(z_{CP})$$

$$\Delta x \equiv -\text{Im}(\Delta z), \quad \Delta y \equiv -\text{Re}(\Delta z)$$

$$\text{with } z_{CP} \pm \Delta z \equiv -(q/p)^{\pm 1}(y + ix)$$

The bin-flip method [A. Di Canto et al., PRD 99 (2019) 012007]

- Description of the phase space
- Bins of nearly constant strong-phase difference between  $D^0$  &  $\bar{D}^0$
- Simultaneous least-square fit of yields in all phase space and decay time bins



Results of  $CP$  parameters

$$x_{CP} = [2.7 \pm 1.6 \pm 0.4] \times 10^{-3}$$

$$\Delta x = [-0.53 \pm 0.70 \pm 0.22] \times 10^{-3}$$

$$y_{CP} = [7.4 \pm 3.6 \pm 1.1] \times 10^{-3}$$

$$\Delta y = [0.6 \pm 1.6 \pm 0.3] \times 10^{-3}$$

## Derived mixing parameters

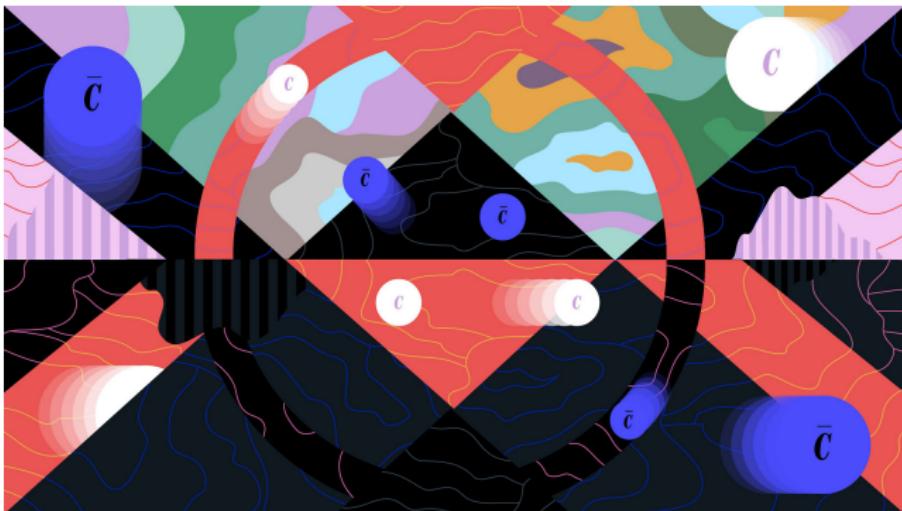
Parameter	Value	95.5% CL interval
$x [10^{-2}]$	$0.27^{+0.17}_{-0.15}$	$[-0.05, 0.60]$
$y [10^{-2}]$	$0.74 \pm 0.37$	$[0.00, 1.50]$
$ q/p $	$1.05^{+0.22}_{-0.17}$	$[0.55, 2.15]$
$\phi$	$-0.09^{+0.11}_{-0.16}$	$[-0.73, 0.29]$

Combination with world average on  $x$ 

$$x = (3.9^{+1.1}_{-1.2}) \times 10^{-3} \quad \rightarrow \quad \text{first evidence for mass difference !}$$

# Conclusion

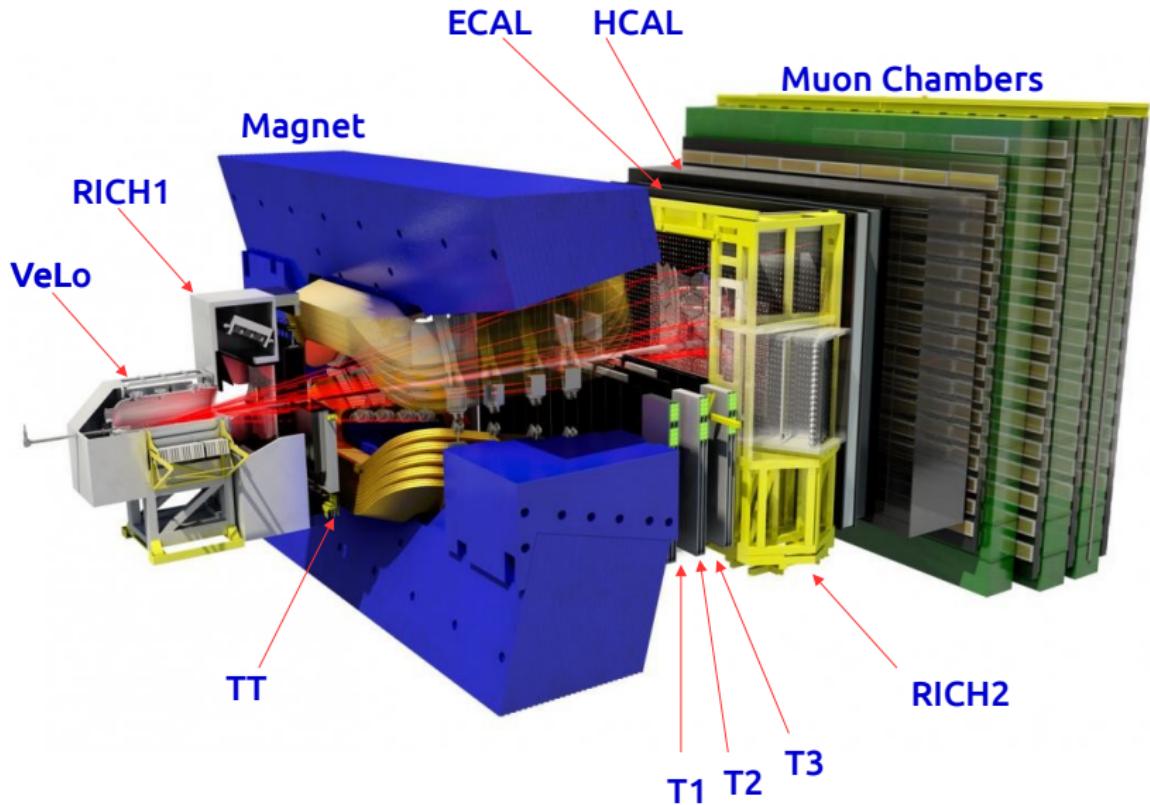
- Highlight of recent analyses from LHCb
- CPV has been observed for the first time in charm decays
- Many other analyses ongoing to complete the picture
- Working hard on Run 2 analyses and towards the upgrade for even better results



Artwork by Sandbox Studio, Chicago with Ana Kova

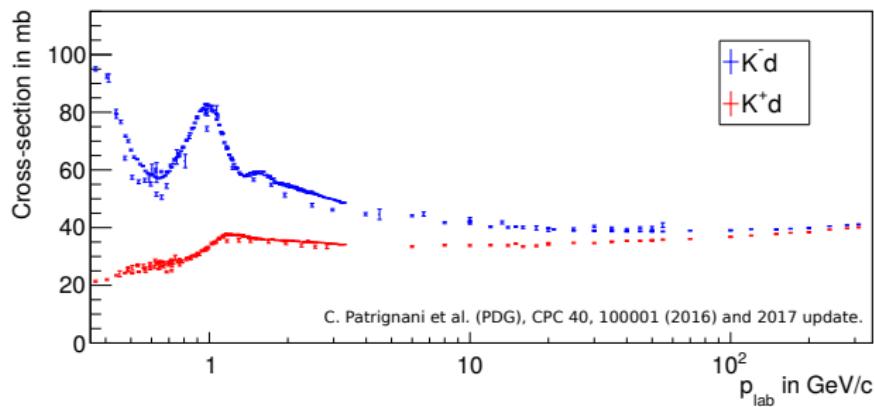
# **BACKUP**

# The LHCb detector



# Detection asymmetry

- Detection asymmetry reduced by flipping magnet polarity regularly
- Residual detection asymmetry due to intrinsic different cross-section between particles of opposite charge when interacting with the detector's material



# Signal PDF $D^0 \rightarrow K^+K^-\pi^+\pi^-$

$$a(\mathbf{x}; \mathbf{c}) = \frac{\epsilon_s(\mathbf{x}) S(\mathbf{x}; \mathbf{c}) \mathcal{R}_4(\mathbf{x})}{\int \epsilon_s(\mathbf{x}) S(\mathbf{x}; \mathbf{c}) \mathcal{R}_4(\mathbf{x}) d^5 \mathbf{x}} \quad \text{with} \quad S(\mathbf{x}; \mathbf{c}) = \left| \sum_k c_k A_k(\mathbf{x}) \right|^2$$

- The normalisation integral is done by summing over a MC sample
- The efficiencies are taken care of by using a fully-simulated MC sample that has been reconstructed like the data sample
- Breit-Wigner, Flatté and K-matrix formalisms for the lineshapes
- Covariant formalism used for the spin factors

# $D^0 \rightarrow K^+K^-\pi^+\pi^-$ signal model

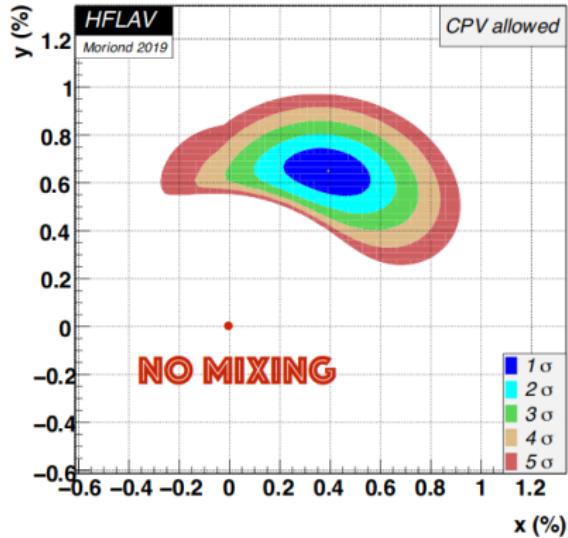
Amplitude	$ c_k $	$\arg(c_k)$ [rad]	Fit fraction [%]	$\sigma$
$D^0 \rightarrow \phi(1020)^0(\rho - \omega)^0$	1	0	$23.82 \pm 0.38 \pm 0.50$	$> 40$
$D^0 \rightarrow K_1(1400)^+K^-$	$0.614 \pm 0.011 \pm 0.031$	$1.05 \pm 0.02 \pm 0.05$	$19.08 \pm 0.60 \pm 1.46$	$> 40$
$D^0 \rightarrow [K^-\pi^+]_{L=0}[K^+\pi^-]_{L=0}$	$0.282 \pm 0.004 \pm 0.008$	$-0.60 \pm 0.02 \pm 0.10$	$18.46 \pm 0.35 \pm 0.94$	$> 40$
$D^0 \rightarrow K_1(1270)^+K^-$	$0.452 \pm 0.011 \pm 0.017$	$2.02 \pm 0.03 \pm 0.05$	$18.05 \pm 0.52 \pm 0.98$	$> 40$
$D^0 \rightarrow K^*(892)^0\bar{K}^*(892)^0$	$0.259 \pm 0.004 \pm 0.018$	$-0.27 \pm 0.02 \pm 0.03$	$9.18 \pm 0.21 \pm 0.28$	$> 40$
$D^0 \rightarrow K^*(1680)^0[K^-\pi^+]_{L=0}$	$2.359 \pm 0.036 \pm 0.624$	$0.44 \pm 0.02 \pm 0.03$	$6.61 \pm 0.15 \pm 0.37$	$> 40$
$D^0 \rightarrow [K^*(892)^0\bar{K}^*(892)^0]_{L=1}$	$0.249 \pm 0.005 \pm 0.017$	$1.22 \pm 0.02 \pm 0.03$	$4.90 \pm 0.16 \pm 0.18$	$> 40$
$D^0 \rightarrow K_1(1270)^-K^+$	$0.220 \pm 0.006 \pm 0.011$	$2.09 \pm 0.03 \pm 0.07$	$4.29 \pm 0.18 \pm 0.41$	$> 40$
$D^0 \rightarrow [K^+K^-]_{L=0}[\pi^+\pi^-]_{L=0}$	$0.120 \pm 0.003 \pm 0.018$	$-2.49 \pm 0.03 \pm 0.16$	$3.14 \pm 0.17 \pm 0.72$	37
$D^0 \rightarrow K_1(1400)^-K^+$	$0.236 \pm 0.008 \pm 0.018$	$0.04 \pm 0.04 \pm 0.09$	$2.82 \pm 0.19 \pm 0.39$	33
$D^0 \rightarrow K^*(1680)^0\bar{K}^*(892)^0$	$0.823 \pm 0.023 \pm 0.218$	$2.99 \pm 0.03 \pm 0.05$	$2.75 \pm 0.15 \pm 0.19$	37
$D^0 \rightarrow [\bar{K}^*(1680)^0K^*(892)^0]_{L=1}$	$1.009 \pm 0.022 \pm 0.276$	$-2.76 \pm 0.02 \pm 0.03$	$2.70 \pm 0.11 \pm 0.09$	$> 40$
$D^0 \rightarrow \bar{K}^*(1680)^0[K^+\pi^-]_{L=0}$	$1.379 \pm 0.029 \pm 0.373$	$1.06 \pm 0.02 \pm 0.03$	$2.41 \pm 0.09 \pm 0.27$	$> 40$
$D^0 \rightarrow [\phi(1020)^0(\rho - \omega)^0]_{L=2}$	$1.311 \pm 0.031 \pm 0.018$	$0.54 \pm 0.02 \pm 0.02$	$2.29 \pm 0.08 \pm 0.08$	$> 40$
$D^0 \rightarrow [K^*(892)^0\bar{K}^*(892)^0]_{L=2}$	$0.652 \pm 0.018 \pm 0.043$	$2.85 \pm 0.03 \pm 0.04$	$1.85 \pm 0.09 \pm 0.10$	$> 40$
$D^0 \rightarrow \phi(1020)^0[\pi^+\pi^-]_{L=0}$	$0.049 \pm 0.001 \pm 0.004$	$-1.71 \pm 0.04 \pm 0.37$	$1.49 \pm 0.09 \pm 0.33$	30
$D^0 \rightarrow [K^*(1680)^0\bar{K}^*(892)^0]_{L=1}$	$0.747 \pm 0.021 \pm 0.203$	$0.14 \pm 0.03 \pm 0.04$	$1.48 \pm 0.08 \pm 0.10$	$> 40$
$D^0 \rightarrow [\phi(1020)^0(\rho(1450)^0)]_{L=1}$	$0.762 \pm 0.035 \pm 0.068$	$1.17 \pm 0.04 \pm 0.04$	$0.98 \pm 0.09 \pm 0.05$	24
$D^0 \rightarrow a_0(980)^0f_2(1270)^0$	$1.524 \pm 0.058 \pm 0.189$	$0.21 \pm 0.04 \pm 0.19$	$0.70 \pm 0.05 \pm 0.08$	27
$D^0 \rightarrow a_1(1260)^+\pi^-$	$0.189 \pm 0.011 \pm 0.042$	$-2.84 \pm 0.07 \pm 0.38$	$0.46 \pm 0.05 \pm 0.22$	17
$D^0 \rightarrow a_1(1260)^-\pi^+$	$0.188 \pm 0.014 \pm 0.031$	$0.18 \pm 0.06 \pm 0.43$	$0.45 \pm 0.06 \pm 0.16$	14
$D^0 \rightarrow [\phi(1020)^0(\rho - \omega)^0]_{L=1}$	$0.160 \pm 0.011 \pm 0.005$	$0.28 \pm 0.07 \pm 0.03$	$0.43 \pm 0.05 \pm 0.03$	18
$D^0 \rightarrow [K^*(1680)^0\bar{K}^*(892)^0]_{L=2}$	$1.218 \pm 0.089 \pm 0.354$	$-2.44 \pm 0.08 \pm 0.15$	$0.33 \pm 0.05 \pm 0.06$	14
$D^0 \rightarrow [K^+K^-]_{L=0}(\rho - \omega)^0$	$0.195 \pm 0.015 \pm 0.035$	$2.95 \pm 0.08 \pm 0.29$	$0.27 \pm 0.04 \pm 0.05$	15
$D^0 \rightarrow \phi(1020)^0f_2(1270)^0$	$1.388 \pm 0.095 \pm 0.257$	$1.71 \pm 0.06 \pm 0.37$	$0.18 \pm 0.02 \pm 0.07$	14
$D^0 \rightarrow K^*(892)^0\bar{K}_2(1430)^0$	$1.530 \pm 0.086 \pm 0.131$	$2.01 \pm 0.07 \pm 0.09$	$0.18 \pm 0.02 \pm 0.02$	20
Sum of fit fractions			$129.32 \pm 1.09 \pm 2.38$	
$\chi^2/\text{ndf}$			$9242/8121 = 1.14$	

# Mass difference measurement details

$$\chi^2 \equiv \sum_{\text{pr, sl}} \sum_{l, d} \sum_{+, -} \sum_{b, j} \frac{(N_{-bj}^\pm - N_{+bj}^\pm R_{+bj}^\pm)^2}{(\sigma_{-bj}^\pm)^2 + (\sigma_{+bj}^\pm R_{+bj}^\pm)^2}$$
$$+ \sum_{b, b'} (X_b^{\text{CLEO}} - X_b) (V_{\text{CLEO}}^{-1})_{bb'} (X_{b'}^{\text{CLEO}} - X_{b'}) .$$
$$R_{bj}^\pm \approx \frac{r_b + \frac{1}{4} r_b \langle t^2 \rangle_j \text{Re}(z_{CP}^2 - \Delta z^2) + \frac{1}{4} \langle t^2 \rangle_j |z_{CP} \pm \Delta z|^2 + \sqrt{r_b} \langle t \rangle_j \text{Re}[X_b^*(z_{CP} \pm \Delta z)]}{1 + \frac{1}{4} \langle t^2 \rangle_j \text{Re}(z_{CP}^2 - \Delta z^2) + r_b \frac{1}{4} \langle t^2 \rangle_j |z_{CP} \pm \Delta z|^2 + \sqrt{r_b} \langle t \rangle_j \text{Re}[X_b(z_{CP} \pm \Delta z)]}.$$

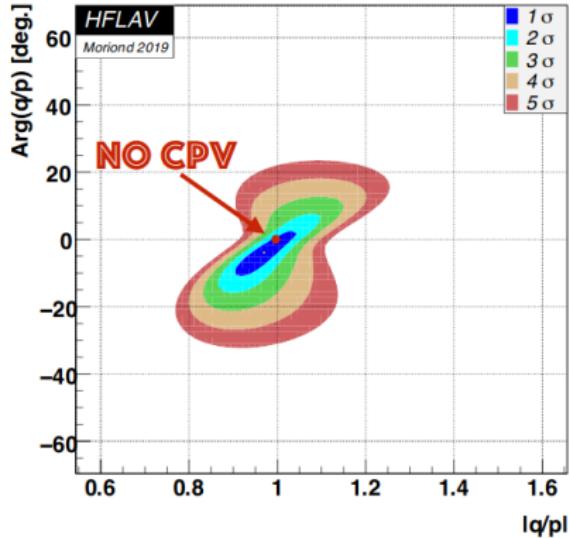
$N$  are the yields and  $\sigma$  are the yields uncertainties.  $X_b$  are the strong phase difference in each phase space bin and  $r_b$  are the yields ratios in these bins.

# World average



$$x = (0.39^{+0.11}_{-0.12}) \%$$

$$y = (0.651^{+0.063}_{-0.069}) \%$$



$$|q/p| = (0.969^{+0.050}_{-0.045})$$

$$\phi = (-3.9^{+4.5}_{-4.6})^\circ$$