

Quantum-correlated $D\bar{D}$ inputs to CKM angle γ from BESIII



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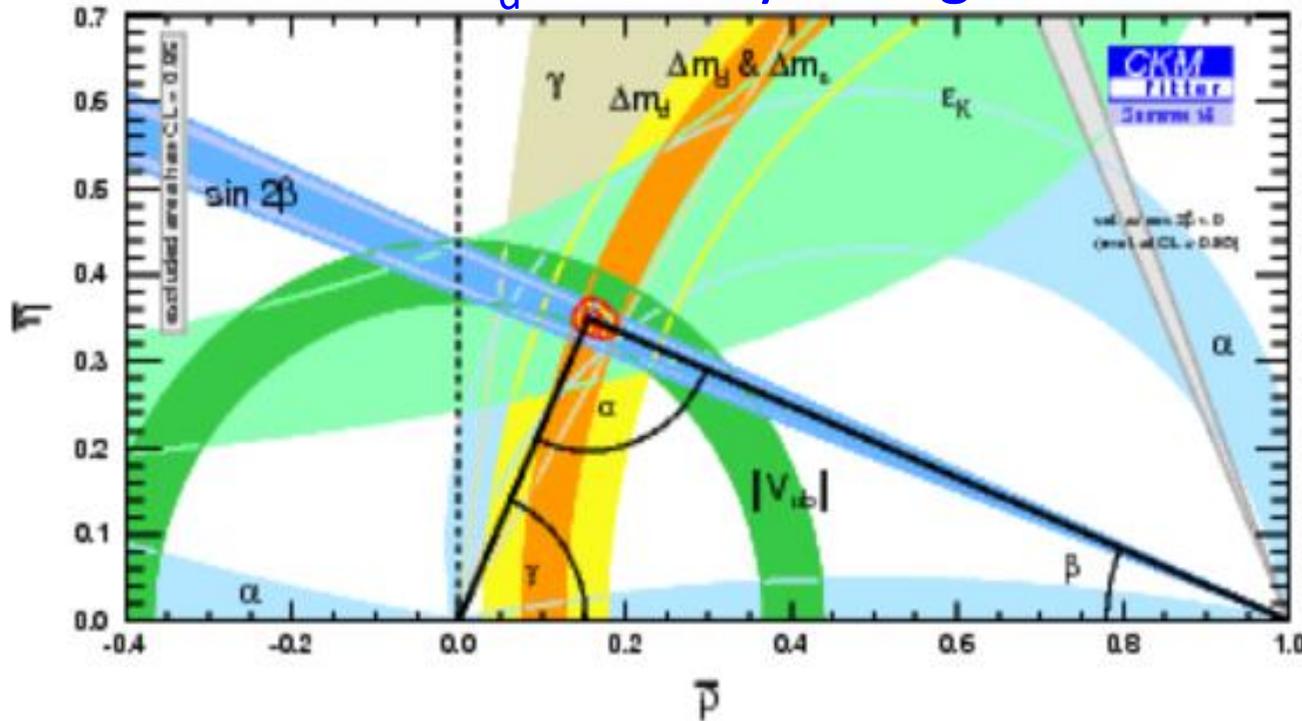
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Introduction

- γ/ϕ_3 is the only CKM angle that can be measured in tree-level processes, in which the contribution of non-SM effects is expected to be small [JHEP 01(2014)051].
- Measurement of γ provides a benchmark of the SM with negligible theoretical uncertainty.

B_d Unitarity Triangle



Phases of CKM elements: *Moriond 2018*

$$\beta = \varphi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \quad \beta = (22.0 \pm 0.7)^\circ$$

$$\alpha = \varphi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \quad \alpha = (84.9^{+5.1}_{-4.5})^\circ$$

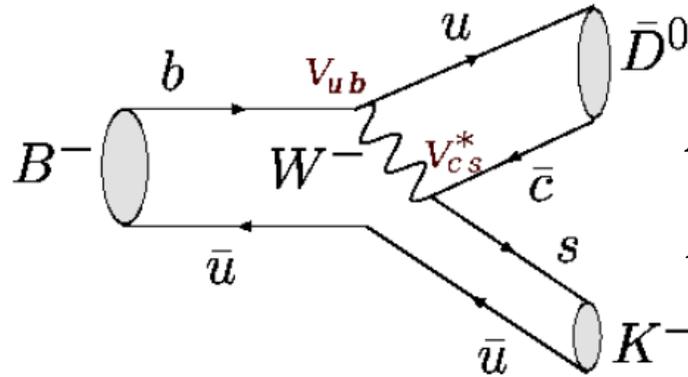
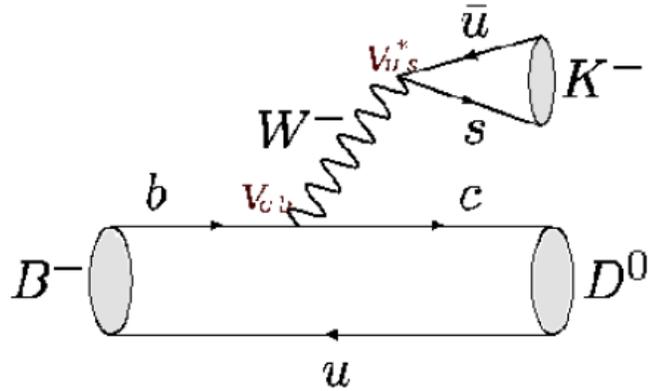
$$\gamma = \varphi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \quad \gamma = (73.5^{+4.2}_{-5.1})^\circ$$

~ an order of magnitude worse than that on β .

$$\gamma \text{ from CKM fitter: } \gamma = (65.8^{+1.0}_{-1.7})^\circ$$

- The current world-average of γ deviates the indirect determination from CKM fitter by $\sim 1.5\sigma$.
- Clearly, an improved knowledge of the measurement of γ is important to further test the SM and probe for new physics.

□ γ/ϕ_3 can be measured by studying the interference between $B^- \rightarrow D^0 K^-$ and $B^- \rightarrow \bar{D}^0 K^-$



$$A(B^- \rightarrow D^0 K^-) = A_B A_D$$

$$A(B^- \rightarrow \bar{D}^0 K^-) = A_B r_B e^{i(\delta_B - \gamma)} A_{\bar{D}}$$

where r_B is ratio of suppressed to favored amplitudes, δ_B is the strong-phase difference between the favoured and suppressed amplitudes.

□ Generally, three methods were proposed to measure γ/ϕ_3 :

- ✓ GLW ^[1]: via $D^0 \rightarrow$ CP eigenstate, $K^+ K^-$, $\pi^+ \pi^-$, $K_S^0 \pi^0$ etc.
- ✓ ADS ^[2]: via $D^0 \rightarrow$ CF and DCS, such as $K^+ \pi^-$, $K^+ \pi^- \pi^0$, $K^+ \pi^- \pi^- \pi^+$ etc.
- ✓ GGSZ ^[3]: via with $D^0 \rightarrow$ Multi-body self-conjugate decays, $K_S^0 \pi^+ \pi^-$ etc.

Measurements from strong phases are key inputs.

For GGSZ:
$$d\Gamma(B^\pm \rightarrow DK^\pm) = |\mathcal{A}_D|^2 + r_B^2 |\mathcal{A}_{\bar{D}}|^2 + 2r_B |\mathcal{A}_D| |\mathcal{A}_{\bar{D}}| \times [\cos \Delta \delta_D \cos(\delta_B \pm \phi_3) + \sin \Delta \delta_D \sin(\delta_B \pm \phi_3)]$$

[1] M. Gronau, D. London, Phys. Lett. B 253, 483 (1991); M. Gronau, D. Wyler, Phys. Lett. B 265, 172 (1991).

[2] D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. 78, 3257 (1997).

[3] A. Giri, Y. Grossman, A. Soffer and J. Zupan, Phys. Rev. D 68, 054018 (2003).

The Quantum Correlated $D\bar{D}$ meson pairs

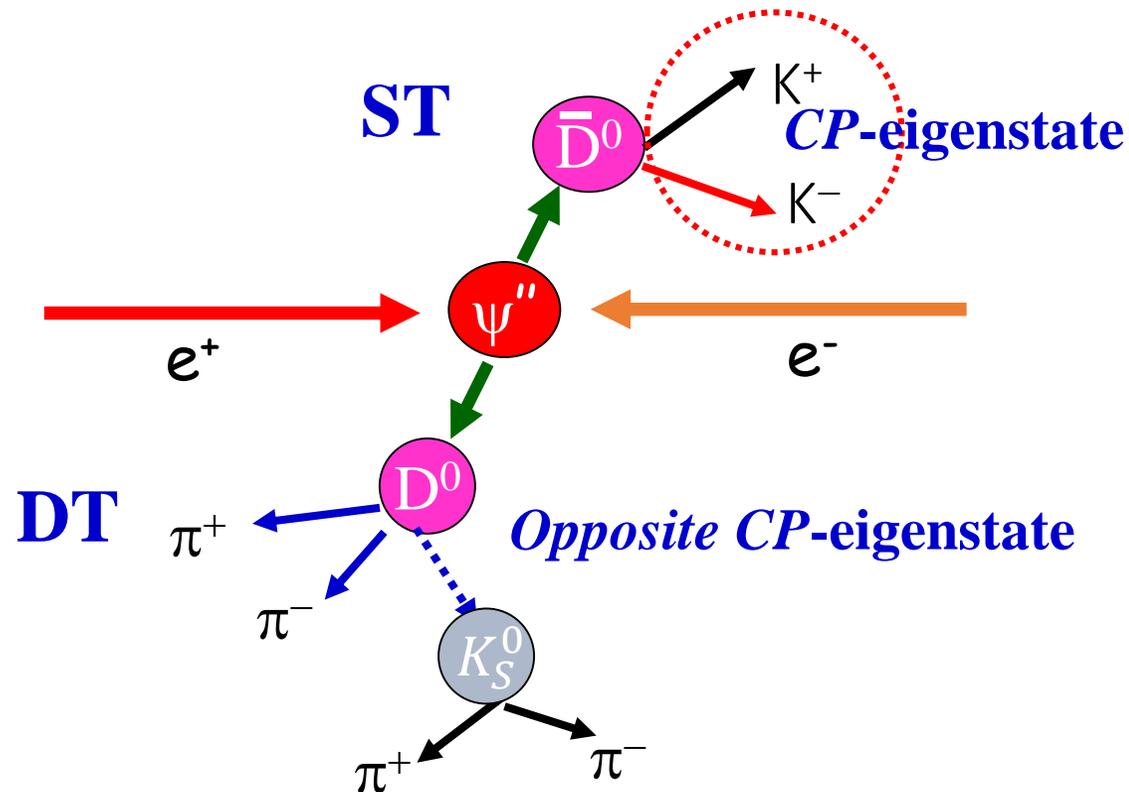
□ $\psi(3770)$ is a spin -1 state and therefore the amplitude of $\psi(3770) \rightarrow D^0\bar{D}^0$:

$$(|D^0\rangle|\bar{D}^0\rangle - |\bar{D}^0\rangle|D^0\rangle)/\sqrt{2} \quad [\text{anti-symmetric wave function}]$$

The amplitude for two D mesons to decay to states F and G is [D. Atwood and A. Soni, PRD68, 033003 (2003)]:

$$\Gamma(F|G) = \Gamma_0 [A_F^2\bar{A}_G^2 + \bar{A}_F^2A_G^2 - 2R_FR_GA_F\bar{A}_FA_G\bar{A}_G\cos[\delta_D^F - \delta_D^G]]$$

The coherence factors R_F , the strong-phase difference (or the related parameters) δ_D^F , can be extracted based on the study of the quantum correlated DD meson pairs.



- ✓ Single tag (ST) samples: decay products of only one D meson are reconstructed
- ✓ Double tag (DT) samples: decay products of both D mesons are reconstructed
- ✓ Some typical reconstructed D decay modes

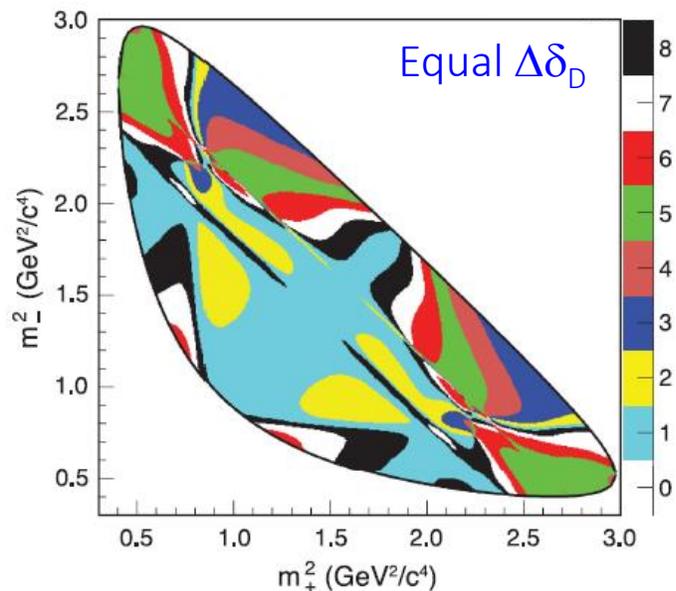
Tag group	Flavor
	$K^+\pi^-, K^+\pi^-\pi^0, K^+\pi^-\pi^-\pi^+, K^+e^-\bar{\nu}_e$
CP-even	$K^+K^-, \pi^+\pi^-, K_S^0\pi^0\pi^0, K_L^0\pi^0, \pi^+\pi^-\pi^0$
CP-odd	$K_S^0\pi^0, K_S^0\eta, K_S^0\omega, K_S^0\eta', K_L^0\pi^0\pi^0$
Mixed-CP	$K_S^0\pi^+\pi^-$

The DT mode K^+K^- vs. $K_S^0\pi^+\pi^-$ is selected as an example.

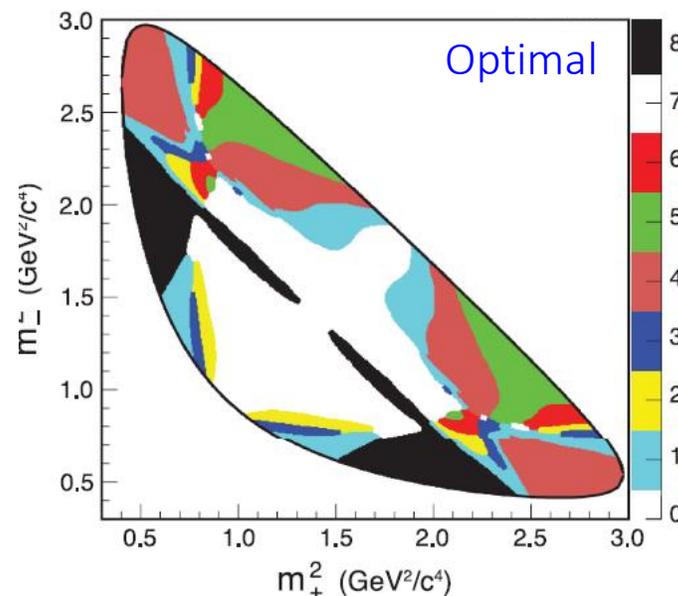
✓ Using this method, CLEO had performed lots of important and excellent measurements related to strong phases.

Strong-phase parameters in $D \rightarrow K_S^0 \pi^+ \pi^-$

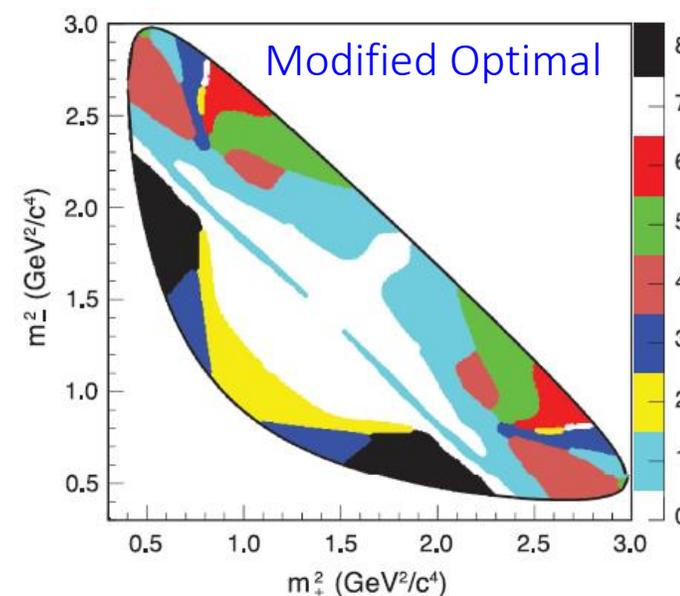
Three typical binning schemes [J. Libby et al. (CLEO Collaboration), Phys. Rev. D 82,112006 (2010)]



DD-mixing¹, β measurements²
[minimum variation in $\Delta\delta_D$]



γ measurements^{3,4}
[Optimized sensitivity]



γ in Low yields
[Optimization including backgrounds]

- ✓ “BaBar K-matrix” $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ model as in Ref. [Phys. Rev. D 78, 034023 (2008)].
- ✓ It should be noted that although the choice of binning is model-dependent, however, a poor choice of model results only in a loss of precision, instead of bias in measuring γ/ϕ_3 .

[1] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 122, 231802 (2019); JHEP 04(2016) 033.

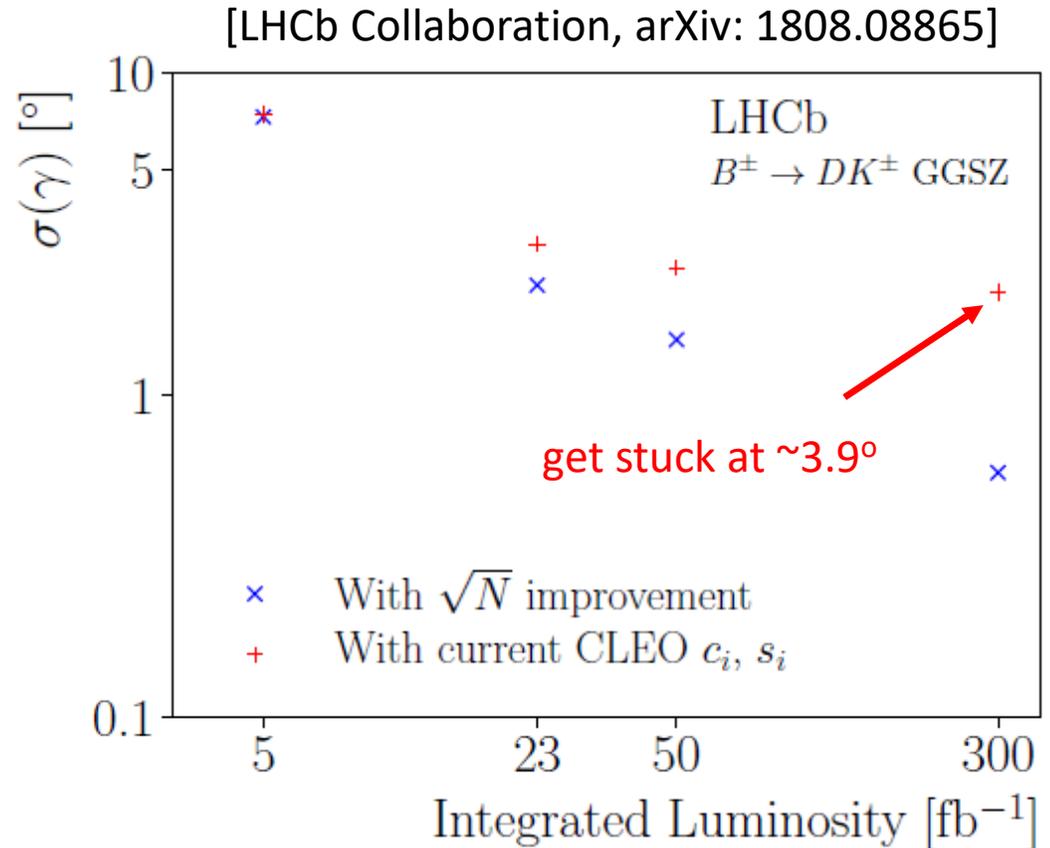
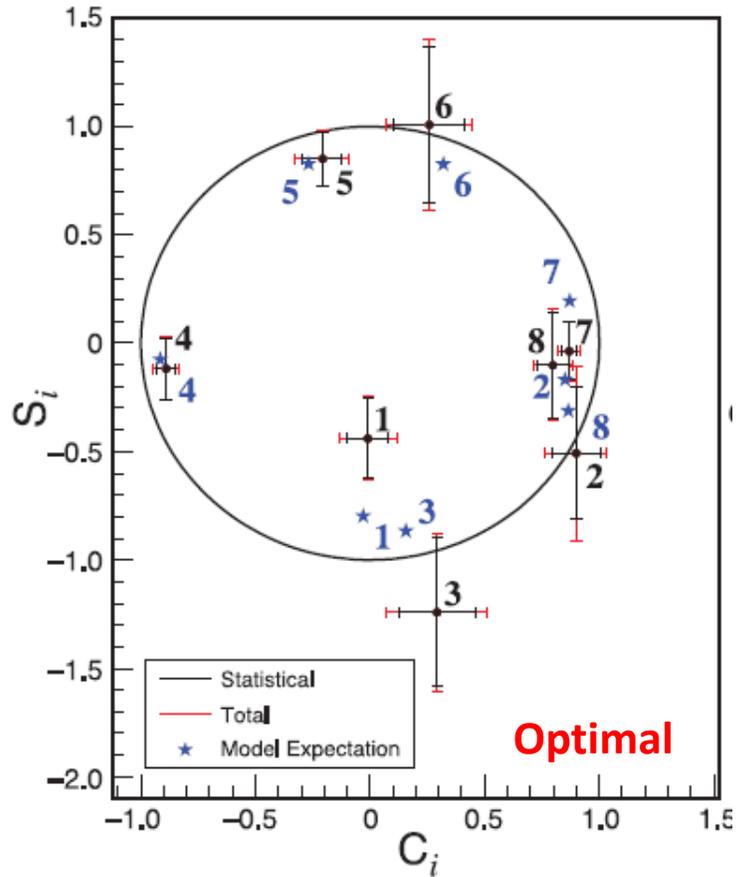
[2] V. Vorobyev et al. (Belle Collaboration), Phys. Rev. D 94, 052004 (2016).

[3] R. Aaij et al. (LHCb Collaboration), Phys. Lett. B 718, 43 (2012); JHEP 10 (2014) 097; JHEP 06 (2016) 131; JHEP 08 (2018) 176.

[4] H. Aihara et al. (Belle Collaboration), Phys. Rev. D 85, 112014 (2012).

Strong-phase parameters in $D \rightarrow K_{S/L}^0 \pi^+ \pi^-$

- ✓ Results of c_i and s_i in optimal binning from CLEO experiments.

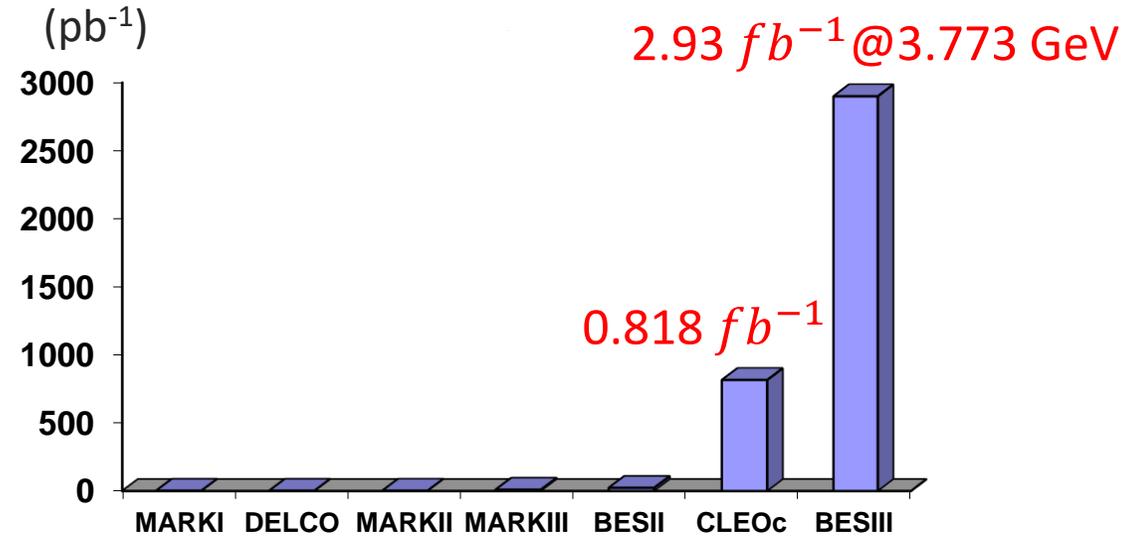
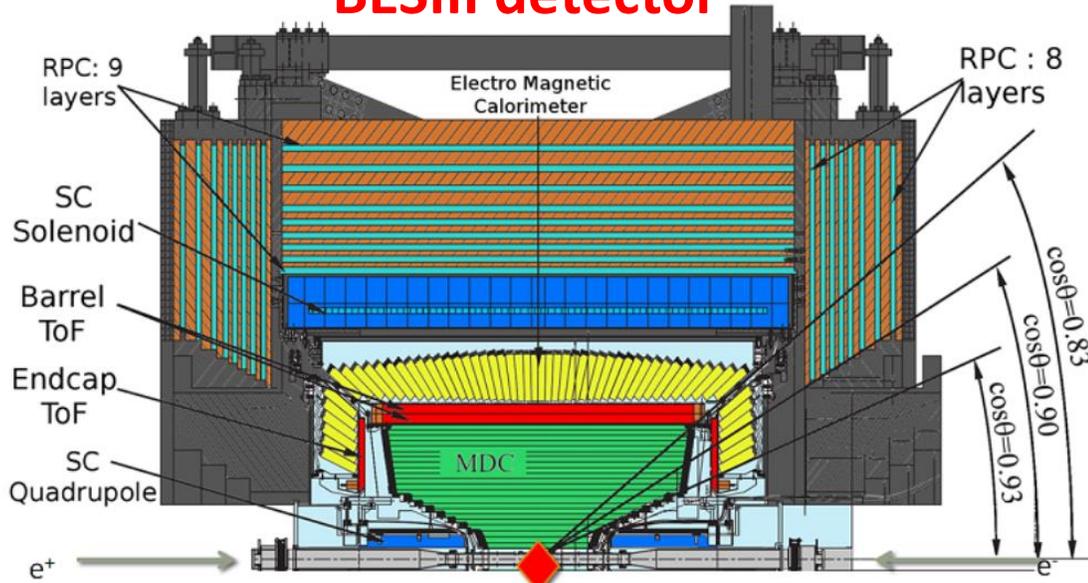


- ✓ The systematic uncertainty in measurement of γ due to input strong-phase parameters is 3.9° for optimal binning. The overall sensitivity is limited to $\sim 3.9^\circ$ for model-independent GGSZ approach.
- ✓ *Therefore, improved measurements in c_i & s_i from BESIII are essential for degree-level precision of measuring γ via model-independent GGSZ approach.*

$\psi(3770) \rightarrow D^0 \bar{D}^0$ samples at BESIII

- Uncertainty of strong-phase inputs from CLEO-c contribute $\sim 2^\circ$ to γ , and will be comparable with the experimental statistical uncertainty at LHCb-Run2.
- BESIII is only machine running at τ -charm energy region. Related quantum-correlated studies are key to constrain the γ measurement at LHCb upgrades 1(2) and Belle II.

BESIII detector



MDC: $\sigma_p/p = 0.5\%$ (1GeV/c) **EMC:** $\frac{\sigma_E}{\sqrt{E}} = 2.5$ (5.0)% (1GeV):
 $\sigma_{dE/dx} = 6\%$ for barrel (endcap)

TOF: $\sigma_{TOF} = 68$ ps for barrel, 110 ps for endcap.

- ✓ Good performance of BESIII detector: high tracking & PID efficiencies, high purity samples
- ✓ Largest $\psi(3770)$ data sample

Strong-phase parameters in $D \rightarrow K_{S/L}^0 \pi^+ \pi^-$ at BESIII

□ $\psi(3770)$ is a spin -1 state and therefore the amplitude of $\psi(3770) \rightarrow D^0 \bar{D}^0$:

$$(|D^0\rangle|\bar{D}^0\rangle - |\bar{D}^0\rangle|D^0\rangle)/\sqrt{2}. \quad \text{[anti-symmetric wave function]}$$

✓ For CP-tagged $K_S^0 \pi^+ \pi^-$, its amplitude is expressed by:

$$f_{CP\pm} = \frac{1}{\sqrt{2}} [f_D(m_+^2, m_-^2) \pm f_D(m_-^2, m_+^2)]$$

The expected yields in Dalitz Plot (DP) bins:

$$\rightarrow M_i^\pm = h_{CP\pm} (K_i \pm 2c_i \sqrt{K_i K_{-i}} + K_{-i})$$

✓ Similarly, for $K_S^0 \pi^+ \pi^-$ vs. $K_S^0 \pi^+ \pi^-$, the expected yields in DP bins is:

$$\rightarrow M_{ij} = h_{\text{corr}} [K_i K_{-j} + K_{-i} K_j - 2\sqrt{K_i K_{-j} K_{-i} K_j} (c_i c_j + s_i s_j)]$$

c_i & s_i are obtained by studying DT events:
 $K_S^0 \pi^+ \pi^-$ vs. CP-tag &
 $K_S^0 \pi^+ \pi^-$ vs. $K_S^0 \pi^+ \pi^-$ tags

Here $h_{cp\pm}$ and h_{corr} are the normalization factors related to yields of single tags and the number of neutral D meson pairs.

From above equations, the precision on s_i is limited by yield of $K_S^0 \pi^+ \pi^-$ vs. $K_S^0 \pi^+ \pi^-$.

Measurements of strong-phase parameters $c_i^{(\prime)}$ & $s_i^{(\prime)}$ at BESIII

- To improve the precision of measuring s_i , the $K_S^0\pi^+\pi^-$ vs. $K_L^0\pi^+\pi^-$ events are added, which is dependent on $(c_i, s_i, c_i'$ and $s_i')$. Due to similarities between the decays, weak model assumptions^[1,2,3] can provide a constraint on the differences between c_i and c_i' , s_i and s_i' .

$$K_S^0\pi^+\pi^- \text{ vs. } K_L^0\pi^+\pi^- \quad M'_{ij} = h'_{\text{corr}} \left[K_i K'_{-j} + K_{-i} K'_j + 2\sqrt{K_i K'_{-j} K_{-i} K'_j} (c_i c'_j + s_i s'_j) \right]$$

$$CP \text{ tag vs. } K_L^0\pi^+\pi^- \quad M_i^{\pm} = h'_{CP\pm} (K'_i \mp 2c'_i \sqrt{K'_i K'_{-i}} + K'_{-i})$$

- The c_i' and s_i' parameters are useful for Belle-II experiment if they use the decay mode $B \rightarrow DK$, with $D \rightarrow K_L^0\pi^+\pi^-$ to measure γ .
- The strong-phase parameters are obtained by minimizing the log-likelihood function constructed by using the observed and expected yields.

[1] R. A. Briere et al. (CLEO Collaboration), Phys. Rev. D 80, 032002 (2009).

[2] J. Libby et al. (CLEO Collaboration), Phys. Rev. D 82, 112006 (2010).

[3] I.I. Bigi and H. Yamamoto, Phys. Lett. B 349, 363 (1995).

Beam-constrained mass distributions

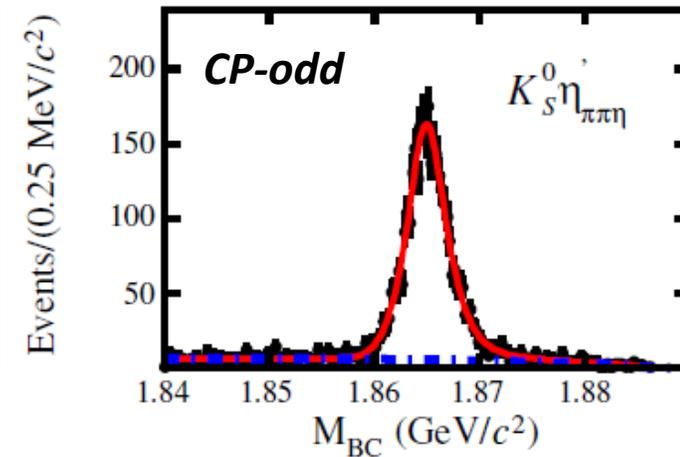
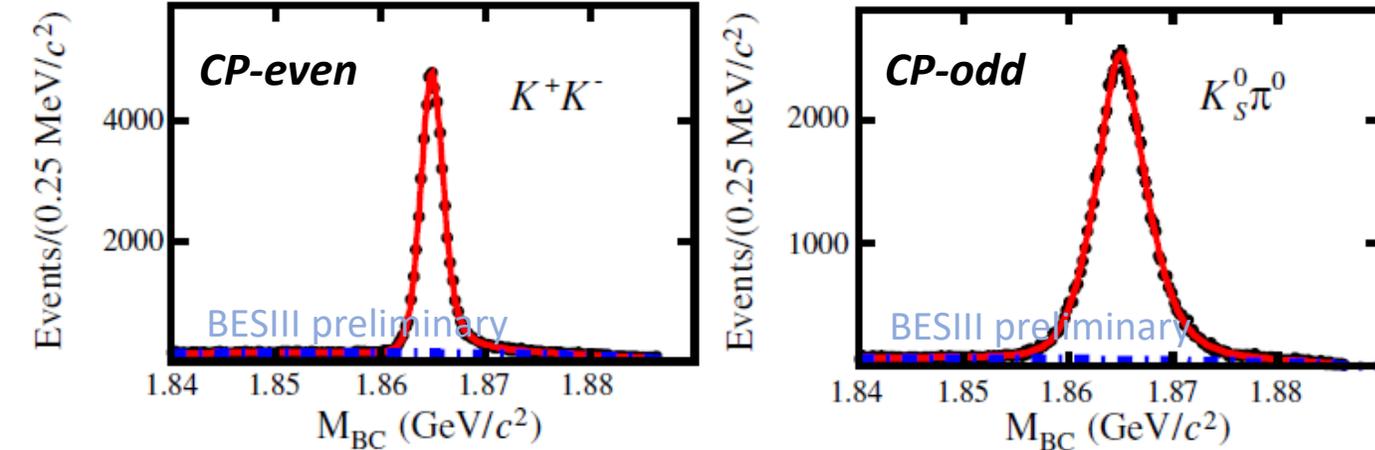
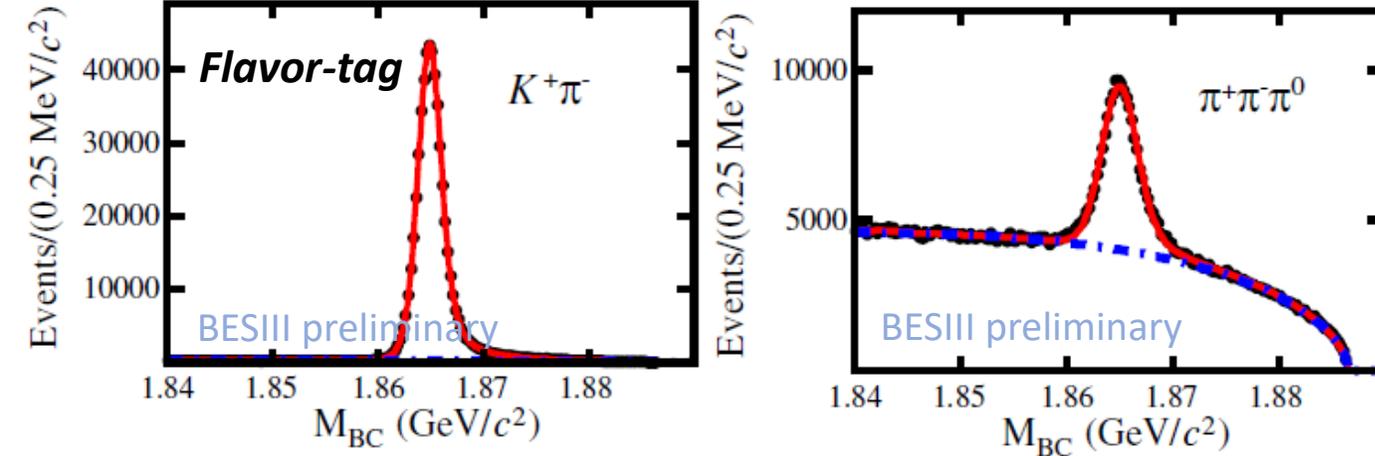
The M_{BC} distributions for ST D mesons

✓ A list of tag decay modes used in this analysis

Tag group	Flavor
Flavor	$K^+\pi^-, K^+\pi^-\pi^0, K^+\pi^-\pi^-\pi^+, K^+e^-\bar{\nu}_e$
CP-even	$K^+K^-, \pi^+\pi^-, K_S^0\pi^0\pi^0, K_L^0\pi^0, \pi^+\pi^-\pi^0$
CP-odd	$K_S^0\pi^0, K_S^0\eta, K_S^0\omega, K_S^0\eta', K_L^0\pi^0\pi^0$
Mixed-CP	$K_S^0\pi^+\pi^-$

✓ Beam-constrained mass (M_{BC})

$$M_{BC} = \sqrt{(\sqrt{s}/2)^2 - |\vec{p}_{D_{tag}}|^2}$$

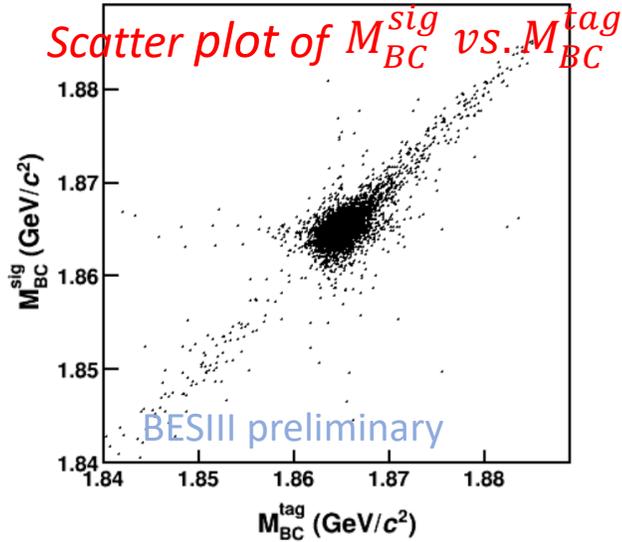


✓ $D \rightarrow \pi^+\pi^-\pi^0$ is not fully CP-even and the corrections for the decay is always applied.

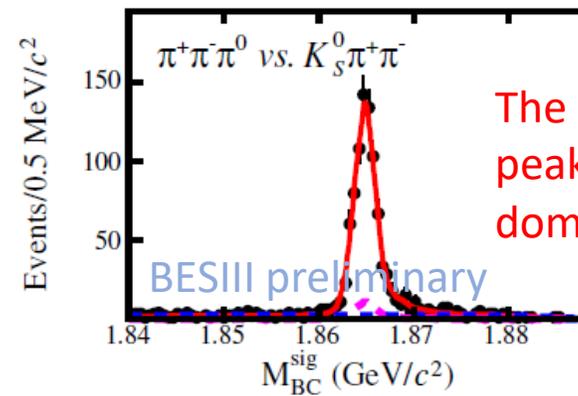
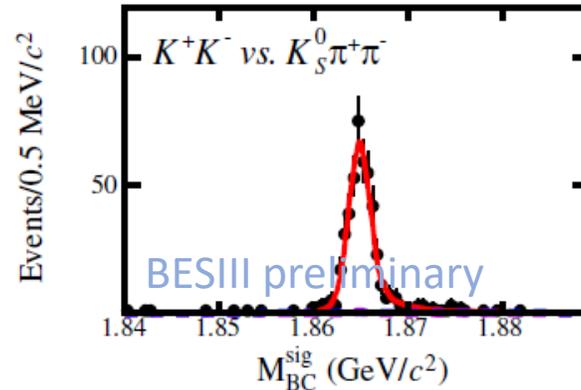
✓ The fractional CP-even content of $D \rightarrow \pi^+\pi^-\pi^0$: $F_+^{\pi^+\pi^-\pi^0} = 0.973 \pm 0.017$ [PLB747, 9 (2015)].

DT samples of $D \rightarrow K_S \pi^+ \pi^-$ and $D \rightarrow K_L \pi^+ \pi^-$

- ✓ The fully reconstructed DT $K_S \pi^+ \pi^-$ events are obtained by searching for the $K_S \pi^+ \pi^-$ signals in the recoiling system of fully-reconstructed ST events.

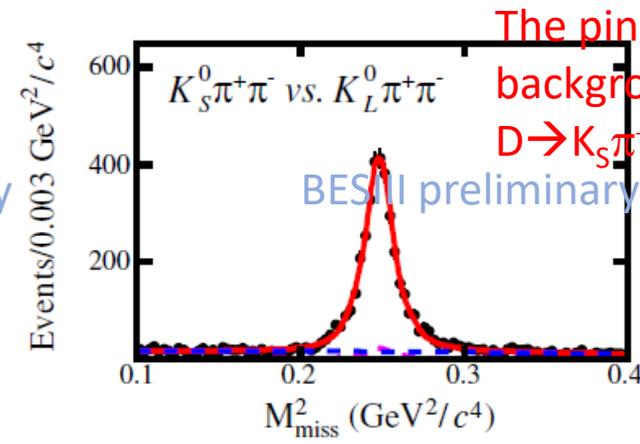
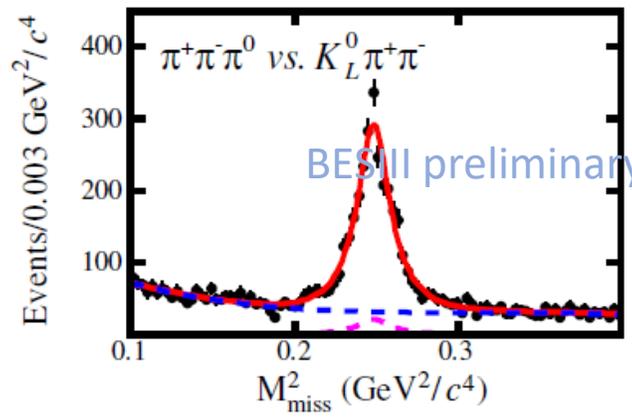
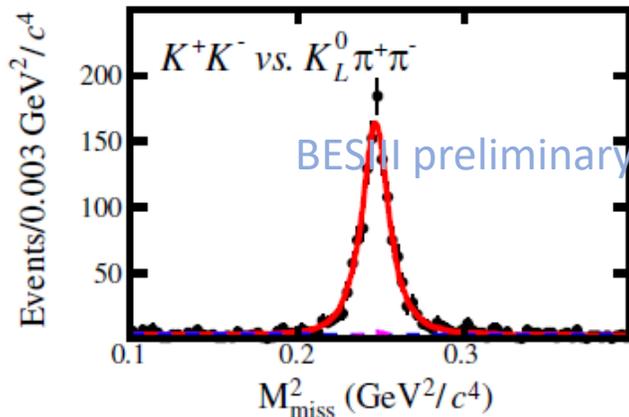


projections of the two-dimensional fits on M_{BC}^{sig}



The pink curves show peaking backgrounds dominated by $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$.

- ✓ DT events containing K_L^0 particles are identified via kinematic variable missing-mass-square (M_{miss}^2).



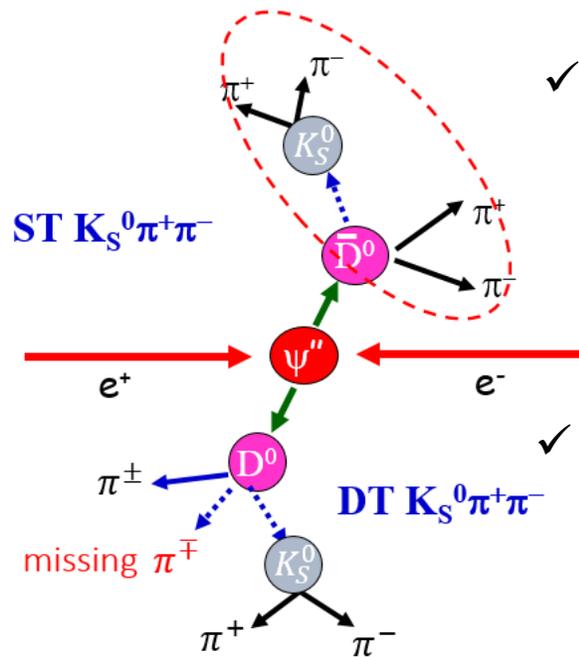
The pink curves show peaking backgrounds dominated by $D \rightarrow K_S \pi^+ \pi^-$ with $K_S^0 \rightarrow \pi^0 \pi^0$.

DT samples of $K_S^0\pi^+\pi^-$ vs. $K_S^0\pi^+\pi^-$

- ✓ To increase the sensitivity of measuring s_i , the partially-reconstructed $K_S^0\pi^+\pi^-$ vs. $K_S^0\pi^+\pi^-$ events are introduced into analysis to increase the yield of $K_S^0\pi^+\pi^-$ vs. $K_S^0\pi^+\pi^-$ events.
- ✓ Two partially reconstructed samples are selected at BESIII.

1) Missing a charged pion originating from D^0 , denoted as $K_S^0\pi^+\pi_{miss}^-$ vs. $K_S^0\pi^+\pi^-$

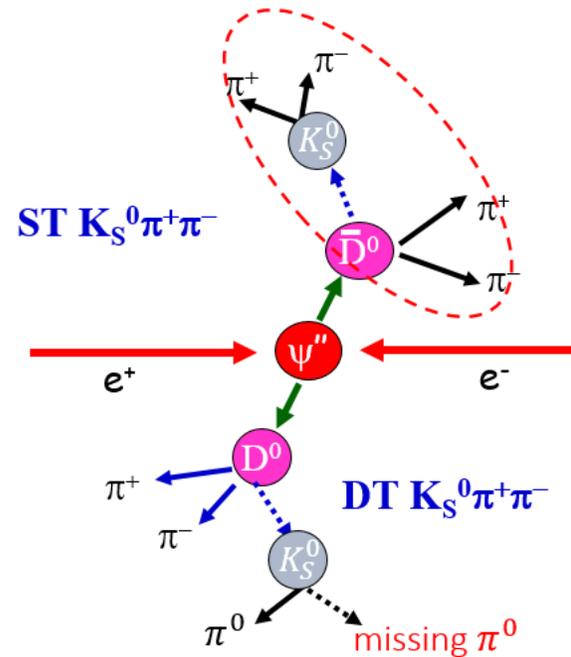
2) Missing a neutral pion originating from $K_S^0 \rightarrow \pi^0\pi^0$, denoted as $K_S^0(\pi^0\pi_{miss}^0)\pi^+\pi^-$ vs. $K_S^0\pi^+\pi^-$



✓ Searching for a charged pion plus a K_S^0 in the system recoiling against ST $K_S^0\pi^+\pi^-$ events.

✓ The missing-mass-square distribution is calculated to identify the desired signal.

$K_S^0\pi^+\pi_{miss}^-$ vs. $K_S^0\pi^+\pi^-$



✓ identifying $\pi^+\pi^-$ from D^0 decays and a π^0 unused in ST reconstruction, in the system recoiling against ST $K_S^0\pi^+\pi^-$ events.

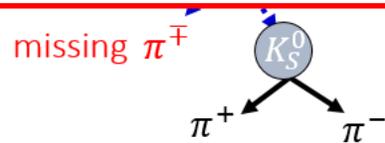
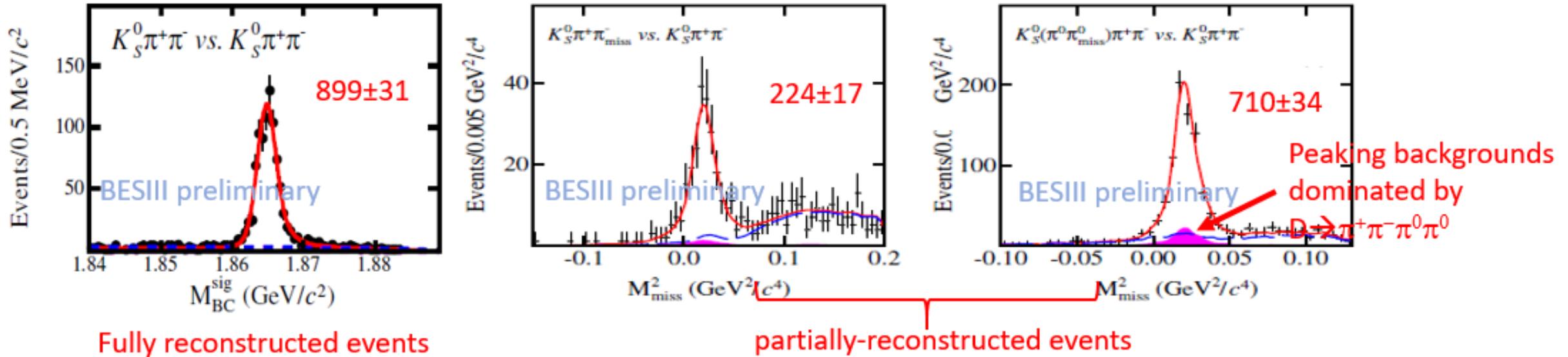
✓ the peaking $\pi^+\pi^-\pi^0\pi^0$ background are suppressed by requiring missing-mass-square of $\pi^+\pi^-$ combination be within in K_S^0 mass region.

$K_S^0(\pi^0\pi_{miss}^0)\pi^+\pi^-$ vs. $K_S^0\pi^+\pi^-$

Distributions of DT $K_S^0\pi^+\pi^-$ vs. $K_S^0\pi^+\pi^-$ events

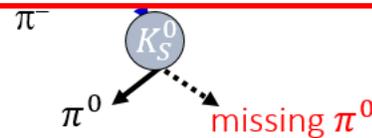
- ✓ To increase the sensitivity of measuring s_i , the partially-reconstructed $K_S^0\pi^+\pi^-$ vs. $K_S^0\pi^+\pi^-$ events are introduced into analysis to increase the yield of $K_S^0\pi^+\pi^-$ vs. $K_S^0\pi^+\pi^-$ events.
- ✓ Two partially reconstructed samples are selected at BESIII.

The yield of DT $K_S^0\pi^+\pi^-$ vs. $K_S^0\pi^+\pi^-$ is doubled by adding the partially-reconstructed samples.



$K_S^0\pi^+\pi^-_{miss}$ vs. $K_S^0\pi^+\pi^-$

to identify the desired signal.



$K_S^0(\pi^0\pi^0_{miss})\pi^+\pi^-$ vs. $K_S^0\pi^+\pi^-$

missing-mass-square of $\pi^+\pi^-$ combination be within in K_S^0 mass region.

Comparisons of DT events between BESIII and CLEO

Mode	BESIII [signal yields]		CLEO [raw yields]	
	$N_{K_S^0 \pi^+ \pi^-}^{DT}$	$N_{K_L^0 \pi^+ \pi^-}^{DT}$	$N_{K_S^0 \pi^+ \pi^-}^{DT}$	$N_{K_L^0 \pi^+ \pi^-}^{DT}$
Flavor tags				
$K^+ \pi^-$	4740 ± 71	9511 ± 115	1444	2857
$K^+ \pi^- \pi^0$	8899 ± 95	19225 ± 176	2759	5133
$K^+ \pi^- \pi^- \pi^+$	5695 ± 78	11906 ± 132	2240	4100
$K^+ e^- \bar{\nu}_e$	4123 ± 75		1191	
CP-even tags				
$K^+ K^-$	443 ± 22	1289 ± 41	124	357
$\pi^+ \pi^-$	184 ± 14	531 ± 28	61	184
$K_S^0 \pi^0 \pi^0$	198 ± 16	612 ± 35	56	— <i>new modes</i>
$\pi^+ \pi^- \pi^0$	790 ± 31	2571 ± 74	—	—
$K_L^0 \pi^0$	913 ± 41		237	
CP-odd tags				
$K_S^0 \pi^0$	643 ± 26	861 ± 46	189	288
$K_S^0 \eta \gamma \gamma$	89 ± 10	105 ± 15	39	43
$K_S^0 \eta \pi^+ \pi^- \pi^0$	23 ± 5	40 ± 9	—	—
$K_S^0 \omega$	245 ± 17	321 ± 25	83	—
$K_S^0 \eta' \pi^+ \pi^- \eta$	24 ± 6	38 ± 8	—	—
$K_S^0 \eta' \gamma \pi^+ \pi^-$	81 ± 10	120 ± 14	—	—
$K_L^0 \pi^0 \pi^0$	620 ± 32		—	—
Mixed CP tags				
$K_S^0 \pi^+ \pi^-$	899 ± 31	3438 ± 72	473	1201
$K_S^0 \pi^+ \pi^-_{\text{miss}}$	224 ± 17			
$K_S^0 (\pi^0 \pi^0_{\text{miss}}) \pi^+ \pi^-$	710 ± 34		—	—

✓ More tag decay modes are used in BESIII analysis.

✓ DT events detected at BESIII and comparisons with CLEO [J. Libby et al. (CLEO Collaboration), Phys. Rev. D 82,112006 (2010)]:

CP – eigenstate vs. $K_S^0 \pi^+ \pi^-$: 5.3×CLEO

CP – eigenstate vs. $K_L^0 \pi^+ \pi^-$: 9.2×CLEO

$K_S^0 \pi^+ \pi^-$ vs. $K_S^0 \pi^+ \pi^-$: 3.9×CLEO

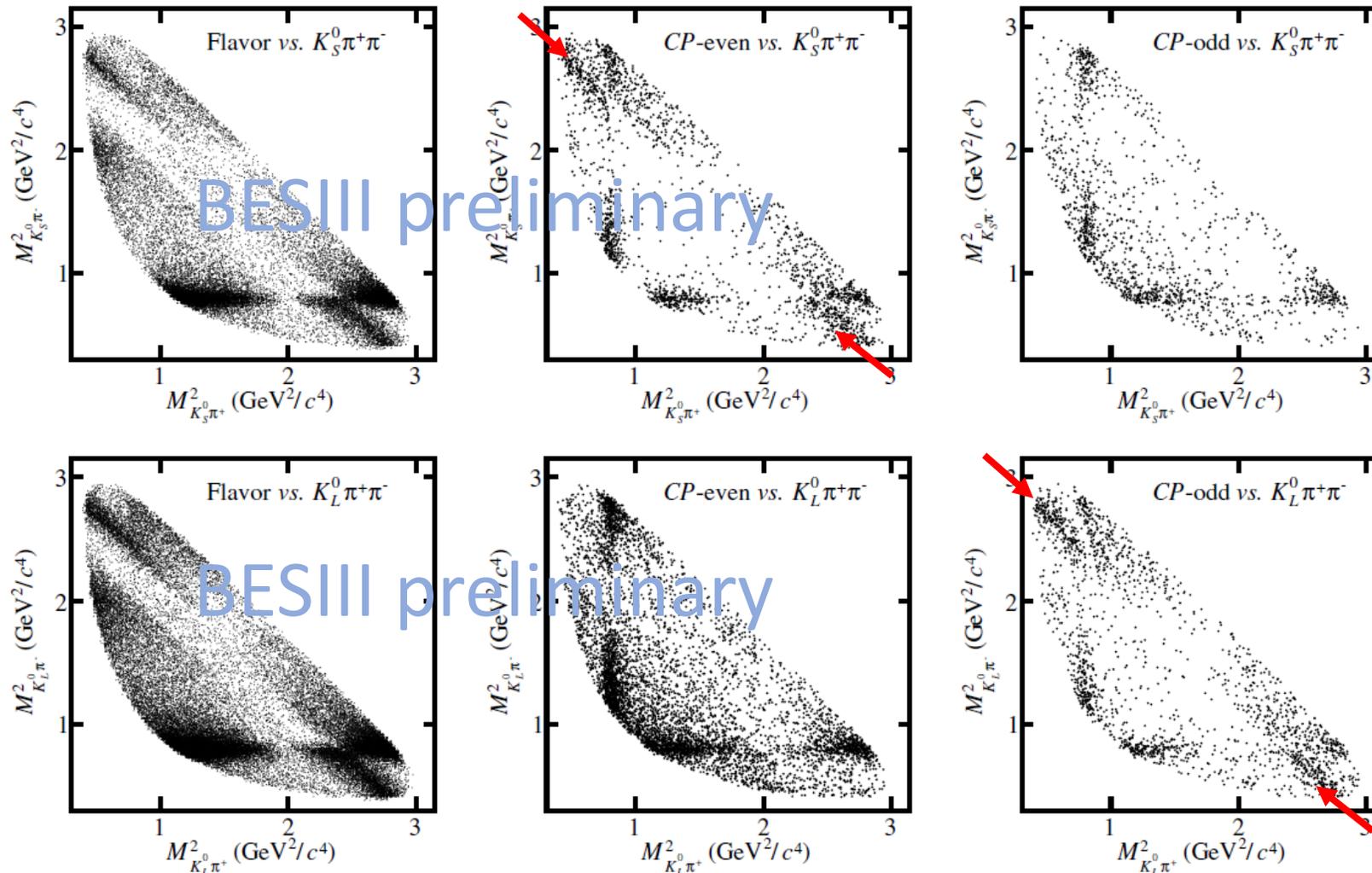
$K_L^0 \pi^+ \pi^-$ vs. $K_S^0 \pi^+ \pi^-$: 2.9×CLEO

✓ “— —” stands for unused mode in CLEO analysis.

Dalitz plots observed in data

The effect of quantum correlation is immediately seen in Dalitz plots.

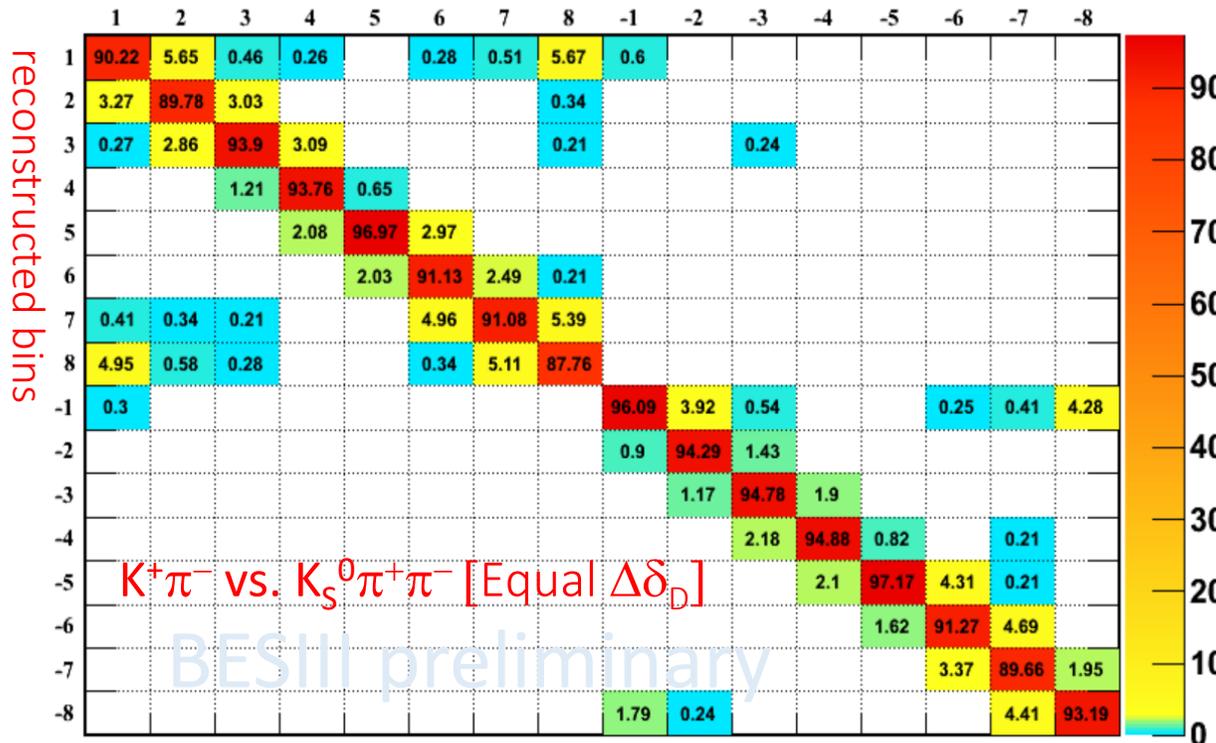
- ✓ The CP-odd component $K_S^0 \rho(770)^0$ is visible in CP-even tagged $K_S^0 \pi^+ \pi^-$ decays, but is absent in CP-odd tagged $K_S^0 \pi^+ \pi^-$ decays.



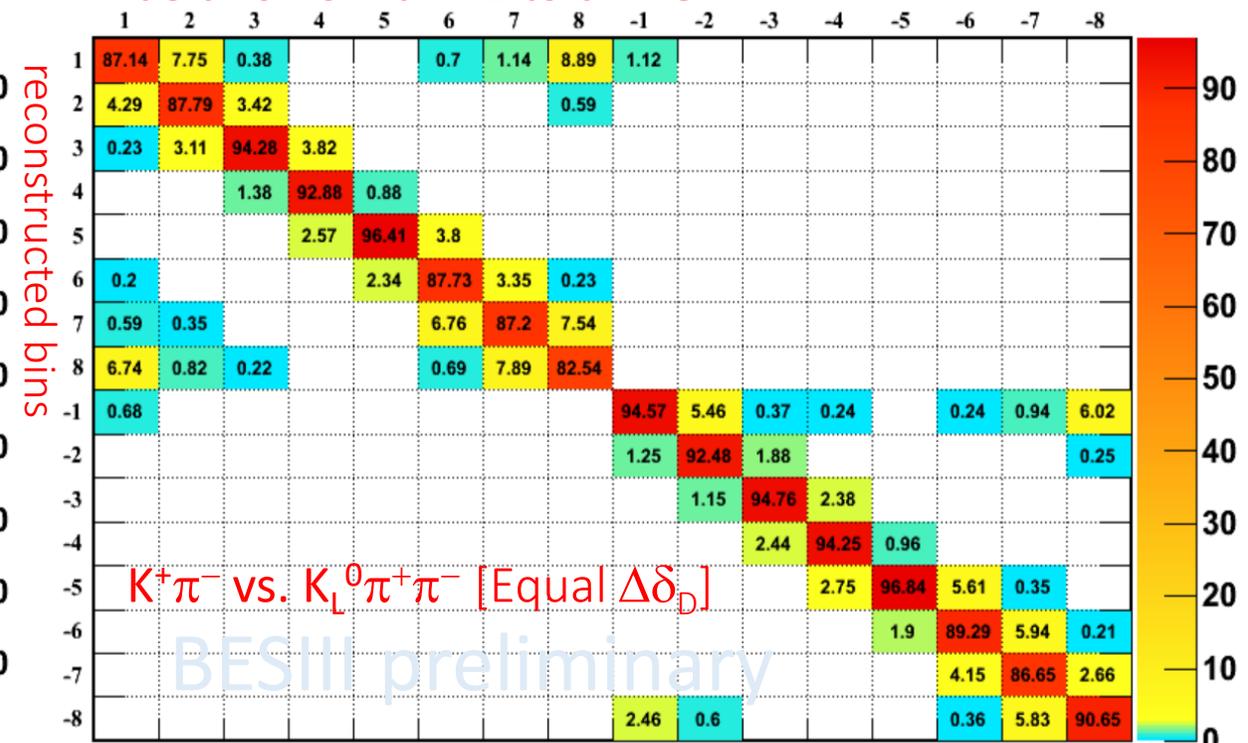
Bin migration effects

- ❑ The finite detector resolution can cause some of the selected events to migrate between Dalitz plot bins after reconstruction.
- ❑ Due to the irregular shape of binning, the bin migrations in different bins could differ greatly. This effects can be evaluated by studying the efficiency matrix obtained from the simulated data.

True bins from bin +1 to bin -8



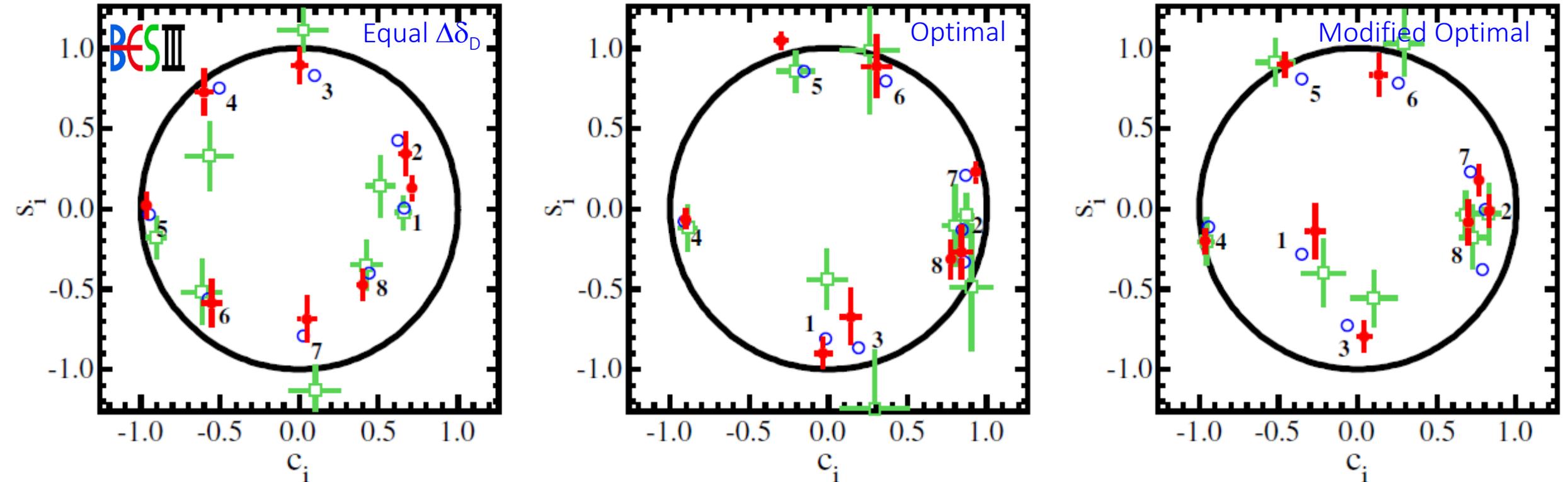
True bins from bin +1 to bin -8



In above tables, the blank places stand for the bin migrations where are less than 0.2%.

- ❑ Due to the difference of resolutions ($0.0068 \text{ GeV}^2/c^4$ for $D \rightarrow K_S^0\pi^+\pi^-$ and $0.0105 \text{ GeV}^2/c^4$ for $D \rightarrow K_L^0\pi^+\pi^-$), the effects of bin migrations across bins ranges from (3-12)% for $K_S\pi\pi$ and (3-18)% for $K_L\pi\pi$, respectively.
- ❑ Neglecting bin migration leads to $\sim 0.7(0.3) \times \sigma_{\text{stat}}$ deviation in the measurement of $\text{ci}(\text{si})$.

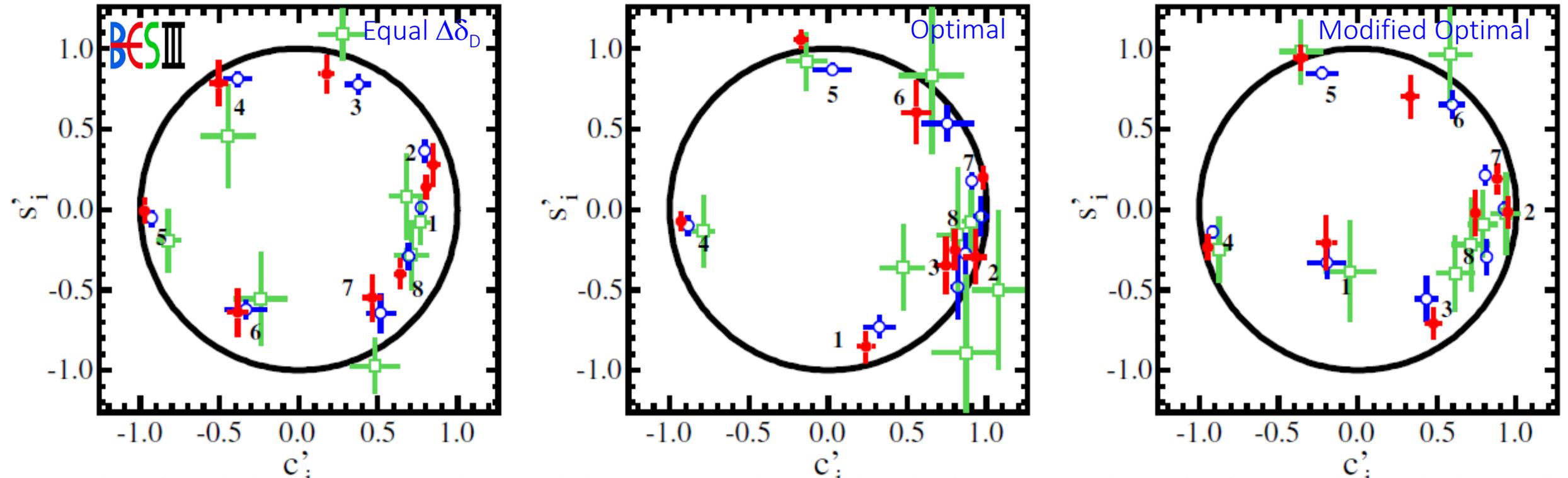
The preliminary strong-phase parameters



The c_i and s_i measured in this work (red dots with error bars), the expected results from Ref. [I. Adachi *et al.* (BaBar and Belle Collaborations), Phys. Rev. D 98, 110212 (2018)] (blue open circles) and the CLEO results (green open squares with error bars) [J. Libby *et al.* (CLEO Collaboration), Phys. Rev. D 82, 112006 (2010)].

- ✓ The strong-phase parameters are limited by statistical errors.
- ✓ *There is no single dominant systematic uncertainty in measurement of c_i & s_i .*
- ✓ on average a factor of ~ 2.5 (2.0) more precise for c_i (s_i) than CLEO measurements.
- ✓ Using BESIII results, the associated uncertainty on γ/ϕ_3 is expected to be approximately a factor of three smaller than that from CLEO analysis, if using an analysis of $B^- \rightarrow DK^-$, $D \rightarrow K_S^0 \pi^+ \pi^-$.

The preliminary strong-phase parameters



The c'_i and s'_i measured in this work (red dots with error bars), the expected results from Ref. [I. Adachi *et al.* (BaBar and Belle Collaborations), Phys. Rev. D 98, 110212 (2018)] (blue open circles) and the CLEO results (green open squares with error bars) [J. Libby *et al.* (CLEO Collaboration), Phys. Rev. D 82, 112006 (2010)].

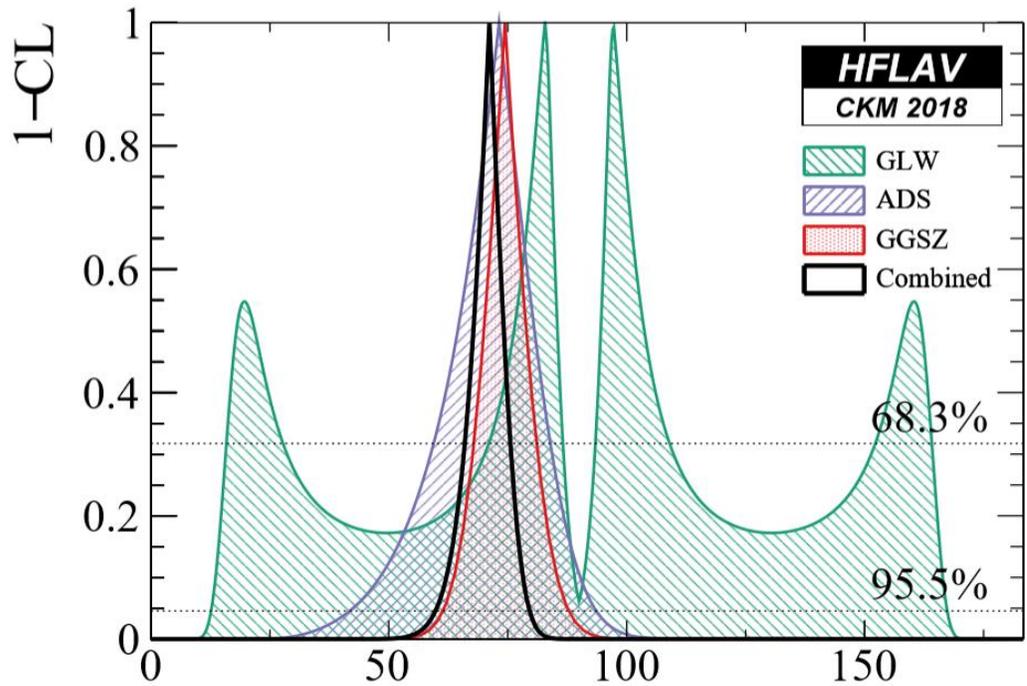
- ✓ The strong-phase parameters are limited by statistical errors.
- ✓ *There is no single dominant systematic uncertainty in measurement of c'_i & s'_i .*
- ✓ on average a factor of ~ 2.8 (2.2) more precise for c'_i (s'_i) than CLEO measurements.
- ✓ The improved precision on c'_i and s'_i are important for Belle-II experiment in γ measurement, if using an analysis of $B^- \rightarrow DK^-$, $D \rightarrow K_L^0 \pi^+ \pi^-$.

Summary

- BESIII is a unique experiment which is the only machine running at τ -charm factory.
- The largest $\psi(3770)$ data can be used to improve the measurements of strong-phase parameters, which provide the key inputs in ranges of γ measurements, $D^0\bar{D}^0$ mixing and CPV studies.
- Studies in $D \rightarrow K_{S/L} \pi^+ \pi^-$ show excellent preliminary results. These results will have important impacts over a wide range of studies in flavour physics.
- A range of quantum-correlated studies are undergoing at BESIII.
- More 10 fb^{-1} $\psi(3770)$ data has been proposed to collect in the next few years.

THANKS

Status and prospects of γ measurements



$$\gamma_{\text{CKM2018}}^{\text{HFLAV}} = (71.1_{-5.3}^{+4.6})^\circ$$

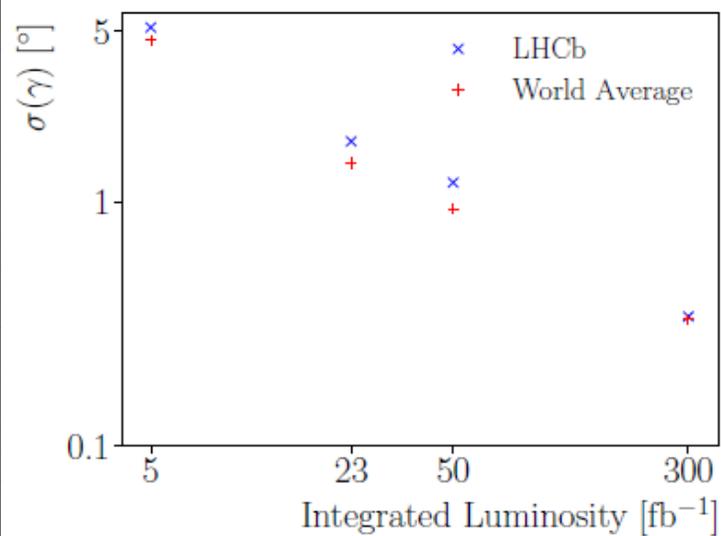
- ✓ dominated by LHCb measurements
- The latest LHCb γ combination gives

$$\gamma = (74.0_{-5.8}^{+5.0})^\circ$$

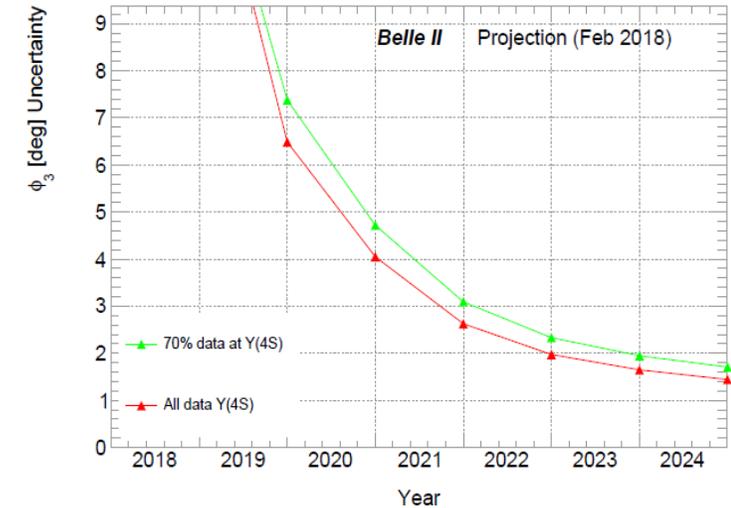
- ✓ The degree-level precision on γ can be expected in near future.



[arXiv: 1808.08865]



[arXiv: 1808.10567]



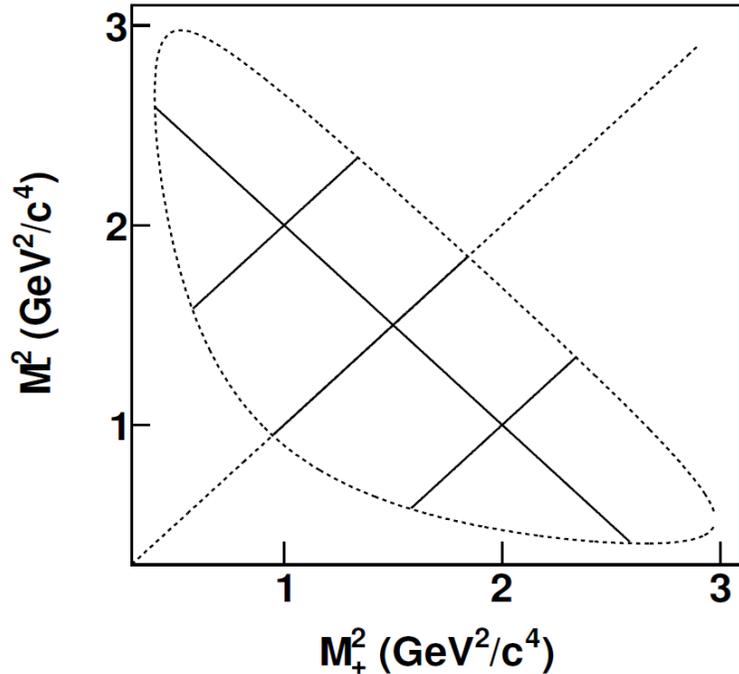
	Collected/ Expected luminosity	Year attained	γ/ϕ_3 sensitivity
LHCb Run-1 [7, 8 TeV]	3 fb ⁻¹	2012	5.5°
LHCb Run-2 [13 TeV]	5 fb ⁻¹	2018	2.8°
LHCb phase-1 upgrade [14 TeV]	50 fb ⁻¹	2030	0.71°
LHCb phase-2 upgrade [14 TeV]	300 fb ⁻¹	2035 (?)	0.28°
Belle-II Run	50 ab ⁻¹	2025	1.5°

■ In above table, sensitivity from LHCb is obtained by scaling Run-I statistical error.

✓ GGSZ approach [A. Giri, Y. Grossman, A. Soffer and J. Zupan, Phys. Rev. D 68, 054018] :

$B^- \rightarrow D^0 K^-$ with $D^0 \rightarrow$ Multi-body self-conjugate decays, $K_s^0 \pi^+ \pi^-$ etc.

Amplitude: $f_{B^-}(m_+^2, m_-^2) \propto f_D(m_+^2, m_-^2) + r_B e^{i(\delta_B - \gamma)} f_{\bar{D}}(m_+^2, m_-^2),$



- Dividing the DP into symmetrically bins
- Produced events in DP bins:

$$K_i = A_D \int_{D_i} |f_D(m_+^2, m_-^2)|^2 dm_-^2 dm_+^2 \equiv A_D F_i$$

In model-dependent GGSZ approach, the (square) binning scheme is further developed by Bondar and Poluektov. [A. Bondar, A. Poluektov, Eur. Phys. J. C 47, 347(2006); 55, 51 (2008)].

The strong-phase interference between D^0 and \bar{D}^0 can be parameterized as:

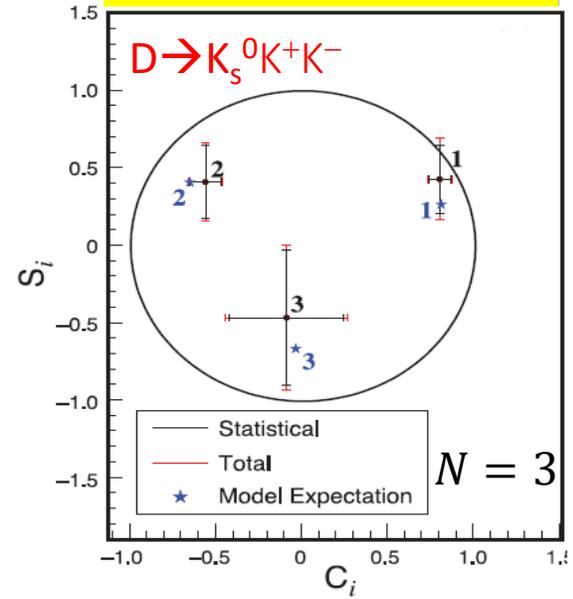
$$c_i = \frac{1}{\sqrt{F_i F_{-i}}} \int_i |f_D(m_+^2, m_-^2)| |f_{\bar{D}}(m_-^2, m_+^2)| \cos[\Delta\delta_D(m_+^2, m_-^2)] dm_+^2 dm_-^2$$

$$s_i = \frac{1}{\sqrt{F_i F_{-i}}} \int_i |f_D(m_+^2, m_-^2)| |f_{\bar{D}}(m_-^2, m_+^2)| \sin[\Delta\delta_D(m_+^2, m_-^2)] dm_+^2 dm_-^2 \quad 23$$

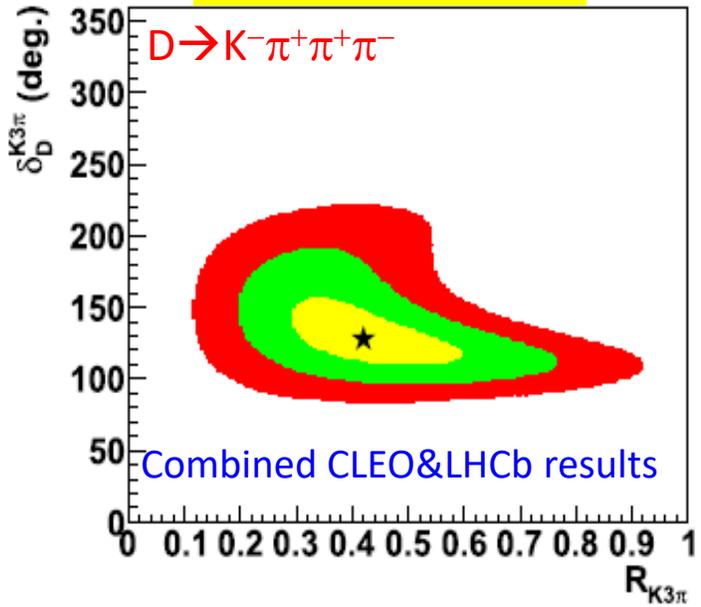
- In model-independent GGSZ approach, strong-phase parameters (c_i & s_i) measured from quantum-correlated DD decays are key input parameters.

Other related strong-phase measurements at CLEO

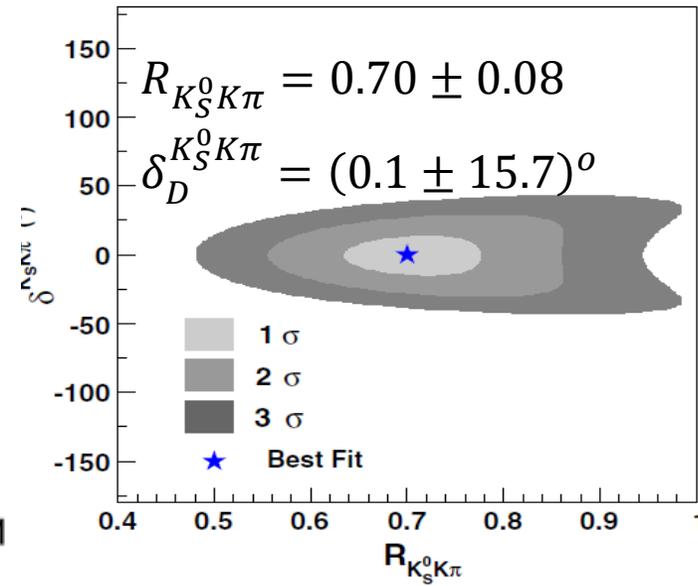
[PRD82, 112006(2010)]



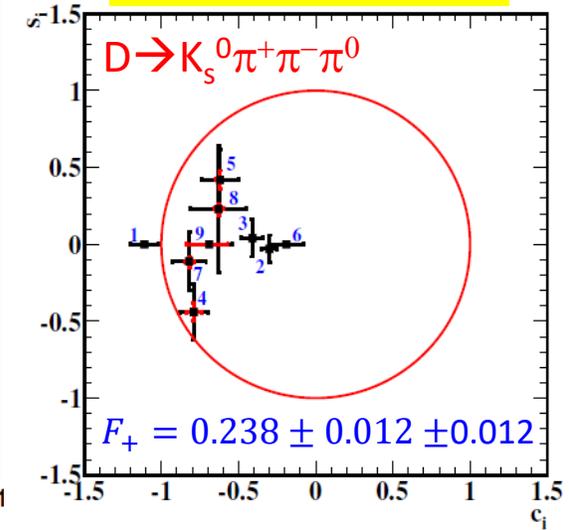
[PLB757, 520(2016)]



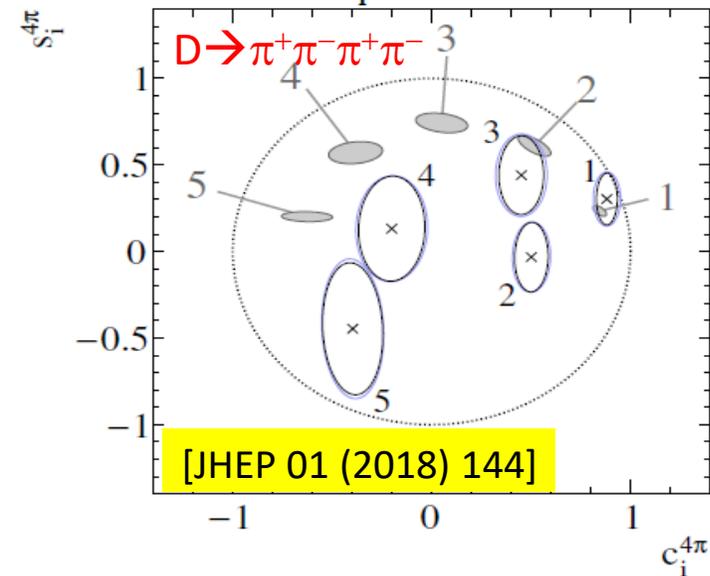
[PRD85, 092016(2012)]



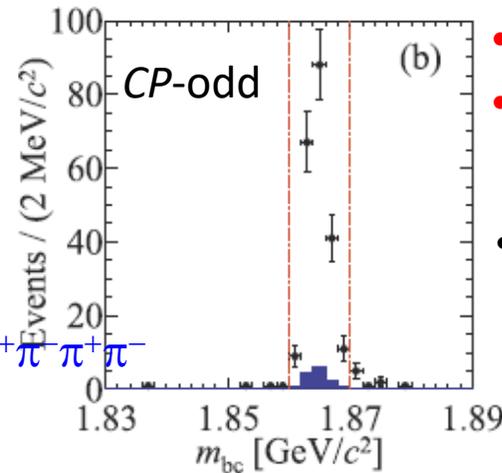
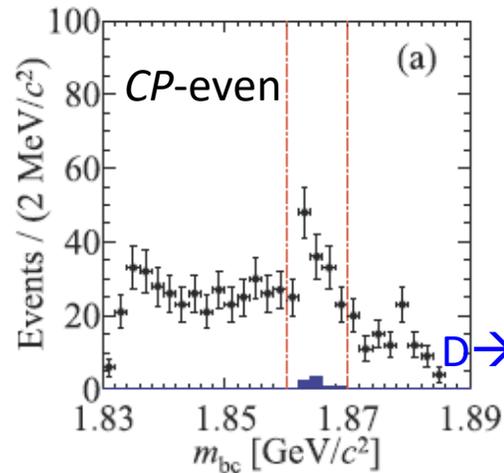
[JHEP 01 (2018) 082]



Equal $\Delta\delta_p^{4\pi}$ binning



✓ CP-even fractions: $F_+ = \frac{N_{CP+}}{N_{CP+} + N_{CP-}}$



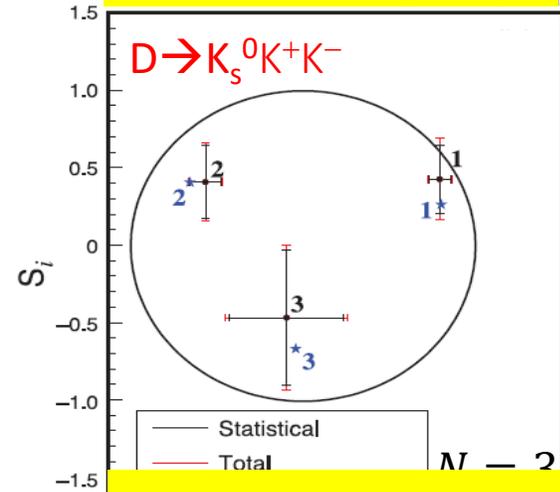
- $D \rightarrow \pi^+ \pi^- \pi^0, K^+ K^- \pi^0$
- $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

[PLB740, 1(2015); PLB 747, 9 (2015)]

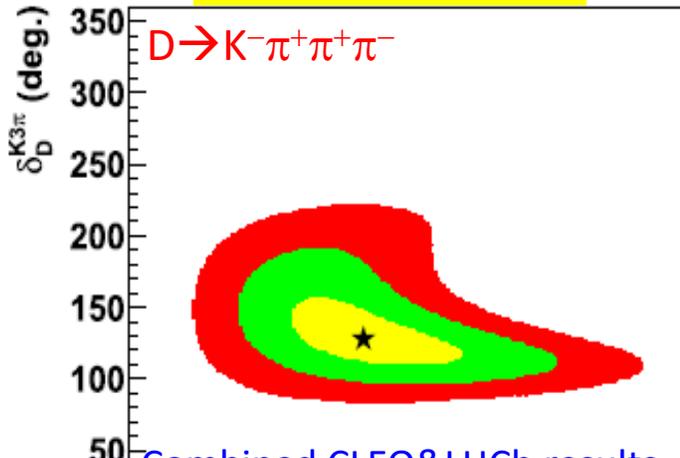
- $\delta_D^{K\pi}$ was also measured by using quantum-correlated data at CLEO, but mixing inputs dominate. [PRD78,012001 & PRL100,221801]

Other related strong-phase measurements at CLEO

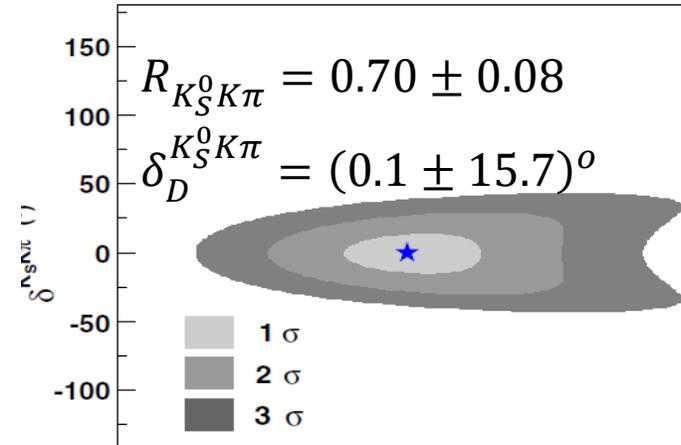
[PRD82, 112006(2010)]



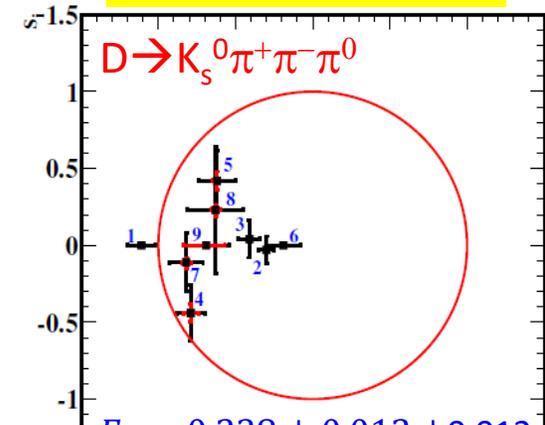
[PLB757, 520(2016)]



[PRD85, 092016(2012)]

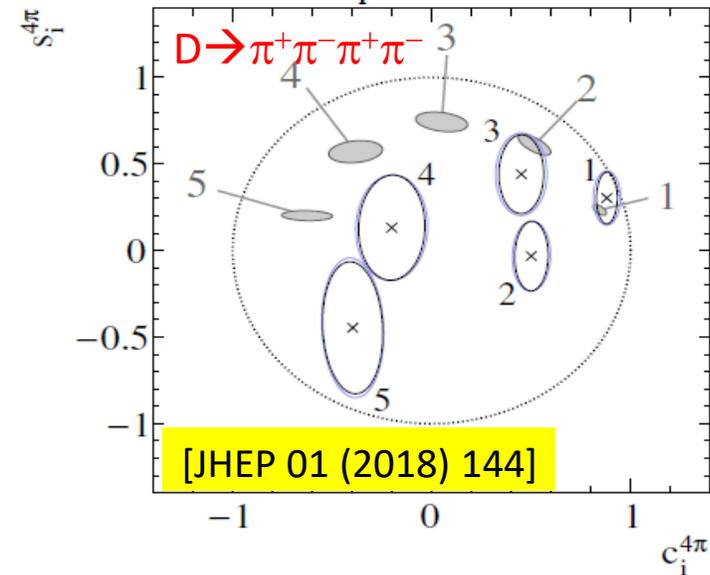


[JHEP 01 (2018) 082]



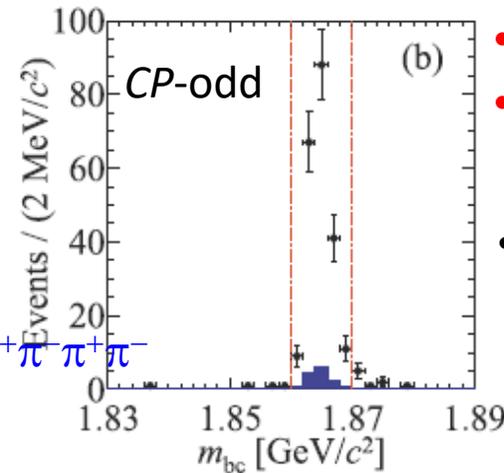
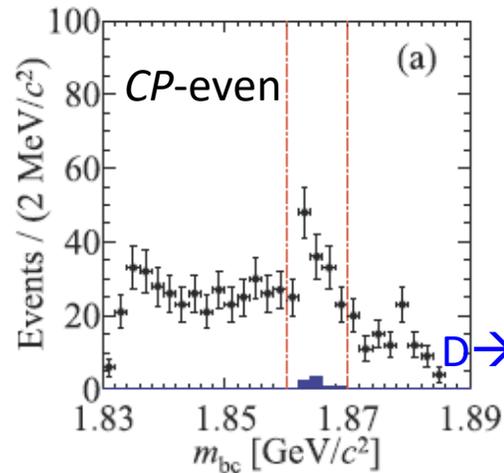
BESIII intends to expand the programme performed at CLEO.

Equal $\Delta\delta_p^{4\pi}$ binning



[JHEP 01 (2018) 144]

✓ CP-even fractions: $F_+ = \frac{N_{CP+}}{N_{CP+} + N_{CP-}}$



- $D \rightarrow \pi^+ \pi^- \pi^0, K^+ K^- \pi^0$
- $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

[PLB740, 1(2015); PLB 747, 9 (2015)]

- $\delta_D^{K\pi}$ was also measured by using quantum-correlated data at CLEO, but mixing inputs dominate.

[PRD78,012001 & PRL100,221801]

□ Taking the bin migrations into account,

- ✓ For CP-tagged $K_S^0\pi^+\pi^-$, the expected yield in each of the Dalitz-plot bins is expressed by:

$$N_i^{\text{exp}\pm} = h_{CP\pm} \sum_j^{N_{\text{bins}}} \epsilon_{ij}^{\text{CP}} (K_j \pm 2c_j \sqrt{K_j K_{-j}} + K_{-j}) \longleftrightarrow M_i^{\pm} = h_{CP\pm} (K_i \pm 2c_i \sqrt{K_i K_{-i}} + K_{-i})$$

No migration effects

- ✓ For $K_S^0\pi^+\pi^-$.vs. $K_S^0\pi^+\pi^-$, the expected yield in each of the Dalitz-plot bins is expressed by:

$$N_n^{\text{exp}} = h_{\text{corr}} \sum_{m=1}^{N'_{\text{bins}}} \epsilon_{nm}^{K_S^0\pi^+\pi^-} [K_{im} K_{-jm} + K_{-im} K_{jm} - 2\sqrt{K_{im} K_{-jm} K_{-im} K_{jm}} (c_{im} c_{jm} + s_{im} s_{jm})]$$

where $h_{CP\pm}$ and h_{corr} are the normalization factors, ϵ_{ij} is the efficiency matrix determined from the simulated data. K_i and K_j are the produced flavour-tagged yields of $D \rightarrow K_S^0\pi^+\pi^-$ in each of the Dalitz-plot bins.

The amplitudes of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ and $D^0 \rightarrow K_L^0 \pi^+ \pi^-$ can be separated into a CF decay ($D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$) and a DCS decay ($D^0 \rightarrow K^0 \pi^+ \pi^-$). These amplitudes can be expressed as:

$$A(D^0 \rightarrow K_S^0 \pi^+ \pi^-) = [A(D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-) + A(D^0 \rightarrow K^0 \pi^+ \pi^-)] / \sqrt{2}$$

and

$$A(D^0 \rightarrow K_L^0 \pi^+ \pi^-) = [A(D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-) - A(D^0 \rightarrow K^0 \pi^+ \pi^-)] / \sqrt{2}$$

If considering the largest four resonances [K^{*-} (CF), K^{*+} (DCS), ρ^0 and f_0 (CF+DCS)], these amplitudes can be expressed as

$$A(K_S^0 \pi^+ \pi^-) = \frac{1}{\sqrt{2}} \left[\sum_i a_i K^{*-} \pi^+ + \sum_i b_i K^{*+} \pi^- + \sum_j (a_j + b_j) K^0 \rho^0 + \sum_k (a_k + b_k) K^0 f_0 \right]$$

and

$$A(K_L^0 \pi^+ \pi^-) = \frac{1}{\sqrt{2}} \left[\sum_i a_i K^{*-} \pi^+ - \sum_i b_i K^{*+} \pi^- + \sum_j (a_j - b_j) K^0 \rho^0 + \sum_k (a_k - b_k) K^0 f_0 \right]$$

in which we have assumed the coefficients a_i and b_i to CF decays and DCS decays, respectively. From the above equation, we can see that there is no change in amplitudes for $K^{*-} \pi^+$ components. For $K^{*+} \pi^-$, the sign of its amplitudes in $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ and $D^0 \rightarrow K_L^0 \pi^+ \pi^-$ is opposite. For CP-eigenstate $K^0 \rho^0$ and $K^0 f_0$, the K_L^0 coefficients will be smaller than K_S^0 cases by

$$\frac{a_j - b_j}{a_j + b_j} \approx (1 - 2r e^{i\delta}),$$

where $r = \tan^2 \theta_C$, where θ_C is the cabibbo angle and δ can take any value from 0 to 2π .