

Pedro Schwaller

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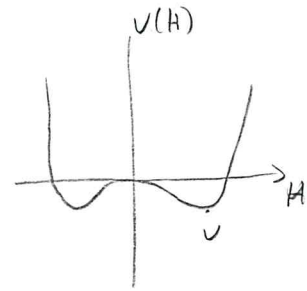
Phase transitions in the early universe

The Higgs potential of the SM is given

$$\text{by } V(H) = -\mu^2 |H|^2 + \lambda |H|^4$$

At tree level the minimum $\langle H \rangle = \left(\begin{smallmatrix} 0 \\ v/\sqrt{2} \end{smallmatrix} \right)$ is given by

$$v^2 = \frac{\mu^2}{\lambda} \quad (\text{if } \mu^2 > 0).$$



In QFT the potential receives radiative corrections. To incorporate them one introduces the effective potential $V_{\text{eff}}(\phi_{cl})$, where ϕ_{cl} is the classical background field. The minimum is then found by imposing

$$\frac{\partial V_{\text{eff}}}{\partial \phi_{cl}} = 0.$$

Methods for computing the one loop corrected eff. potential can be found e.g. in Peskin.

For the SM, the renormalized 1-loop result is

$$V(\phi_{cl}) = V_0(\phi_{cl}) + \frac{1}{64\pi^2} \sum_{\substack{i=\omega, Z \\ u, t, \chi}} u_i m_i^4(\phi_{cl}) \left[\log \frac{m_i^2(\phi_{cl})}{\mu^2} - C_i \right]$$

↑ #dof
↑ $\frac{5}{6}$ gauge bosons

↑ $\frac{3}{2}$ else

Literature:

- M. Quiros : Finite T field theory and PT's (hep-ph/9901372)
- Laine, Vuorinen : 1701.01554
- Books by Kapusta, Le Bellac

This is the vacuum case. The early universe is a messy place however, a hot plasma with all SM particles in thermal equilibrium. This induces additional, finite temperature contributions to V_{eff} .

Intuitive picture: A particle moving through the plasma is constantly hit and scattered. This makes it more difficult to accelerate it \leftrightarrow the particle gets an additional thermal mass.

The contributions to V_{eff} separate into a $T=0$ and a finite temperature part. One finds:

$$\Delta V^{(1, \text{loop})}(\varphi_{cl}, T) = \frac{T^4}{2\pi^2} \left[\sum_{i=W, Z} n_i \mathcal{J}_B\left(\frac{m_i^2(\varphi_{cl})}{T^2}\right) + n_f \mathcal{J}_F\left(\frac{m_f^2(\varphi_{cl})}{T^2}\right) \right]$$

$$\left. \begin{aligned} &\hookrightarrow -\frac{\pi^4}{45} + \frac{\pi^2}{12} \frac{m^2}{T^2} + \dots \\ &\left. \begin{aligned} &\frac{7\pi^4}{360} - \frac{\pi^2}{24} \frac{m^2}{T^2} + \dots \end{aligned} \right\}$$

Finally the total eff-potential at finite T can be written as

$$\bigcirc \quad V(\varphi_{cl}, T) = D(T^2 - T_0^2)\varphi_{cl}^2 - ET\varphi_{cl}^3 + \frac{\lambda(T)}{4}\varphi_{cl}^4$$

$$D = \frac{1}{8v^2} (2m_W^2 + m_Z^2 + 2m_t^2)$$

$$E = \frac{1}{4\pi v^3} (2m_W^3 + m_Z^3) \quad [\text{no fermions contribution}]$$

$$T_0^2 = \frac{m_h^2 - 8Bv^2}{4D}$$

$$B = \frac{3}{64\pi^2 v^4} (2m_W^4 + m_Z^4 - 4m_t^4)$$

The important point here is that at $T > T_0$, the global minimum of $V_{\text{eff}}(T)$ is at $\phi_{cl} = 0$, i.e. the electroweak symmetry is restored.

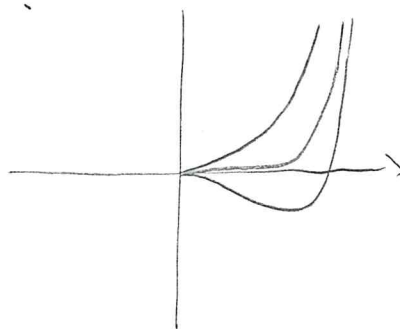
In cosmology, the universe after the big bang (and inflation) reheats to high temperatures ($T \gg T_0$) so that EW symmetry is unbroken initially. The universe undergoes a transition from $\phi_{cl} = 0$ to $\phi_{cl} \sim v$ around $T_0 \sim 10^2$ GeV.
[also QCD PT near GeV scale]

Consider $V(\phi, T) = D(T^2 - T_0^2) \phi^2 + \frac{\lambda(T)}{4} \phi^4$.

stationary points: $\frac{dV}{d\phi} = 0$

$$\phi_1(T) = 0$$

$$\phi_2(T) = \sqrt{\frac{2D(T_0^2 - T^2)}{\lambda(T)}} \quad (T < T_0)$$



At $T > T_0$, $\phi = 0$ is the only solution.

At $T = T_0$, both solutions are at $\phi = 0$. For $T < T_0$, ϕ_2 is the global minimum and ϕ_1 becomes a local maximum. There is no barrier between the minima, the field can adiabatically follow the vacuum state.
→ 2nd order PT.

For more complex potentials, a barrier might exist. E.g.

$$V(\phi, T) = D(T^2 - T_0^2) \phi^2 - \epsilon T \phi^3 + \frac{\lambda(T)}{4} \phi^4$$

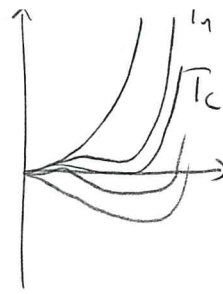
$T > T_1$: $\phi=0$ is min.

At T_1 : Second min. appears

At T_c : Both minima have equal value

$T < T_c$: $\phi=0$ becomes meta-stable

$T < T_c$: barrier goes away, $\phi=0$ is max.



The PT can start at $T \leq T_c$. If the tunneling probability is small, $T < T_c$. Also models with tree level barriers exist $\Rightarrow T_c = 0$.

Thermal tunneling

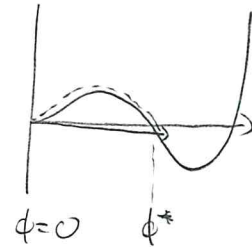
Bubble formation

Tunneling rate: $\frac{\Gamma}{V} \sim A(T) e^{-S_3/T}$

$$S_3 = \int d^3x \left(\frac{1}{2} (\nabla\phi)^2 + U(\phi, T) \right)$$

with ϕ being the bounce solution

path from $\phi=0$ to ϕ^* that minimises action.



Explain bubble expansion

\hookrightarrow energy gain vs. surface tension

Compete with expansion of universe $\rightarrow \frac{S_3}{T} \sim 140$

Motivations

1. First order PT = deviation from th. eqo \Rightarrow Baryogenesis
2. GWs in range of planned expts.