BSM in the sky

Tutorial - Answers

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Problem 1 - Survival of the baryon abundance

- **Answer 1 :** The relic abundance of proton and antiproton is

$$\frac{n_{p\bar{p}}}{s}\Big|_{\infty} = 2\frac{n_p}{s}\Big|_{\infty} = 2\frac{H_{\rm FO}}{\sigma_{p\bar{p}}\, v_{\rm rel}\, s_{\rm FO}} = \frac{1.51}{\sqrt{g_{\rm SM}^{\rm FO}}}\frac{x_{\rm FO}}{M_{\rm pl}\, M_{\rm DM}\, \sigma_{p\bar{p}}\, v_{\rm rel}} \tag{1}$$

where the freeze-out temperature, $x_{\rm FO} \equiv M_{\rm DM}/T_{\rm FO}$, in the instantaneous freeze-out approximation, is solution of

$$n_p\, \sigma_{p\bar{p}}\, v_{\rm rel} \simeq H, \tag{2}$$

namely,

$$x_{\rm FO} \simeq {\rm Log}\left[0.192\frac{\rm g_p}{\sqrt{\rm g_{SM}^{FO}}}\, {\rm M_{pl}\, M_{DM}}\, \sigma_{\rm p\bar{p}}\, {\rm v_{rel}\, x_{FO}}\right] \quad \rightarrow \quad {\rm x_{FO}} \simeq 49, \tag{3}$$

where we have plugged $g_p = 4$, $g_{\rm SM}^{\rm FO} = 10.75$ and $c_1 = 1$. Therefore, the surviving baryon abundance $n_B \equiv n_{p\bar{p}}$ is

$$\frac{n_B}{s}\Big|_{\infty} \simeq 1.8 \times 10^{-19}. \tag{4}$$

- **Question 2 :** The entropy to photon number density today reads

$$\frac{s_0}{n_\gamma} = \frac{\frac{2\pi^2}{45}\, 2(1+\frac{7}{8}\left(\frac{4}{11}\right) N_{\rm eff})T_\gamma^3}{\frac{1.2}{\pi^2}\, 2\, T_\gamma^3} \simeq 7.1. \tag{5}$$

Hence, we find the baryon-to-entropy ratio

$$\frac{n_B}{s}\Big|_{\infty}^{\rm BBN} \simeq \frac{1}{7.1}\frac{n_\gamma}{s}\Big|_{\infty}^{\rm BBN} \simeq 8.7 \times 10^{-11} \gg 1.8 \times 10^{-19}. \tag{6}$$

We conclude that it must pre-exist an excess of baryons over antibaryons, namely a baryon asymmetry $\Delta n_B \equiv n_B - n_{\bar{B}}$, which survives the annihilation

$$\frac{\Delta n_B}{s}\Big|_{\rm early\ U} \simeq \frac{n_B}{s}\Big|_{\infty}^{\rm BBN} \simeq 8.7 \times 10^{-11}. \tag{7}$$

Problem 2 - Upper bound on the mass of thermal DM

- **Answer 1 :**

$$\sigma_{\rm ine}^{(L)} \leq \frac{\pi(2L+1)}{p_i^2}. \tag{8}$$

In the early universe, the momentum of the particle i can be written as

$$p_i^2 = E_i^2 - M_{\rm DM}^2 = (\gamma_i^2 - 1)M_{\rm DM}^2 = \frac{M_{\rm DM}^2 v_i^2}{(1-v_i^2)} \approx M_{\rm DM}^2 v_i^2, \tag{9}$$

Then the relative velocity $v_{\rm rel}$ is easily related to the individual velocity v_i in the center of mass

$$v_{\rm rel}^2 = (\vec{v}_1^2 - \vec{v}_2^2)^2 = \vec{v}_1^1 + \vec{v}_2^2 - 2\vec{v}_1 \cdot \vec{v}_2 = 4v_i^2, \tag{10}$$

hence leading to

$$\sigma_{\rm ine}^{(L)} v_{\rm rel} \leq \frac{4\pi(2L+1)}{M_{\rm DM}^2 v_{\rm rel}}. \tag{11}$$

- **Answer 2 :** From [1], we read the s-wave annihilation cross-section at freeze-out for ~ 100 TeV DM

$$\langle \sigma v_{\rm rel} \rangle_{\rm FO} = \begin{cases} 2.4 \times 10^{-26}\ {\rm cm^3/s} & {\rm Majorana} \\ 4.8 \times 10^{-26}\ {\rm cm^3/s} & {\rm Dirac} \end{cases} \tag{12}$$

The upper bound on the DM mass comes from

$$\langle \sigma v_{\rm rel} \rangle_{\rm FO} \leq \langle \sigma v_{\rm rel} \rangle_{\rm uni}^{\rm max} \quad \rightarrow \quad M_{\rm DM}^2 \leq \frac{4\pi(2L+1)}{\langle \sigma v_{\rm rel} \rangle_{\rm FO}} \left\langle \frac{1}{v_{\rm rel}} \right\rangle. \tag{13}$$

From using $\langle 1/v_{\rm rel} \rangle = \sqrt{x_{\rm FO}/\pi}$ [2], we get

$$M_{\rm DM} \lesssim \begin{cases} \lesssim 140\sqrt{2L+1}\ {\rm TeV} & {\rm Majorana} \\ \lesssim 100\sqrt{2L+1}\ {\rm TeV} & {\rm Dirac} \end{cases} \tag{14}$$

- **Answer 3 :** However, at such a large DM mass we expect non-perturbative (Sommerfeld) effects to change the s-wave scaling of $\langle \sigma v_{\rm rel} \rangle_{\rm FO} \sim v_{\rm rel}^0$ to $\langle \sigma v_{\rm rel} \rangle_{\rm FO} \sim v_{\rm rel}^{-1}$. Hence, the values in eq.(12) given by [1] are modified. We can recompute them following the method of [3] that we recall. For a given velocity dependence of the annihilation cross-section $\langle \sigma v_{\rm rel} \rangle_{\rm FO} = \sigma_0\, x^{-n}$, the freeze-out occurs at $x_{\rm FO} \equiv M_{\rm DM}/T_{\rm FO}$, solution of

$$x_{\rm FO} = {\rm Log}\left[0.038({\rm n}+1)\frac{\rm g_D}{\sqrt{\rm g_{SM}}}\ {\rm M_{pl}\ M_{DM}}\ \sigma_0\right] - \left({\rm n}+\frac{1}{2}\right){\rm Log}\left[{\rm x_{FO}}\right]. \tag{15}$$

where $g_{\rm D} = 2$ for Majorana and $g_{\rm D} = 4$ for Dirac fermion DM. Then, the DM relic abundance is

$$\Omega_{\rm DM} h^2 = \frac{g_{\rm D}}{2} \frac{s_0}{3M_{\rm pl}^2 H_0^2}\, M_{\rm DM}\, \frac{3.79(n+1)x_{\rm FO}^{n+1}}{\sqrt{g_{\rm SM}}\, M_{\rm pl}\, M_{\rm DM}\, \sigma_0} \tag{16}$$

with $H_0 = 100$ km/s/Mpc and $s_0 = 2913\ {\rm cm}^3$.[1] The 'Dirac-to-Majorana' factor $g_{\rm D}/2$ is needed for counting DM and anti-DM when DM is a Dirac fermion. Therefore, we deduce the required annihilation cross-section at freeze-out to get the correct DM abundance for $M_{\rm DM} = 100$ TeV

$$\langle \sigma v_{\rm rel} \rangle_{\rm FO} = \begin{cases} 2.4 \times 10^{-26}\ {\rm cm^3/s}, & {\rm Majorana\ and\ n = 0} \\ 1.1 \times 10^{-26}\ {\rm cm^3/s}, & {\rm Majorana\ and\ n = -1/2} \\ 4.9 \times 10^{-26}\ {\rm cm^3/s}, & {\rm Dirac\ and\ n = 0} \\ 2.4 \times 10^{-26}\ {\rm cm^3/s}, & {\rm Dirac\ and\ n = -1/2} \end{cases} \tag{17}$$

[1] $s_0 = 2913\ {\rm cm}^3$ is the SM value with $N_{\rm eff} \simeq 3.045$ [4, 5].

Hence, we deduce the upper-bound on the DM mass from unitarity, for perturbative $n = 0$ or Sommerfeld-enhanced $n = -1/2$, both of them being s-wave annihilation $L = 0$,

$$M_{\rm DM} \lesssim \begin{cases} 138 \text{ TeV}, & \text{Majorana and n = 0} \\ 197 \text{ TeV}, & \text{Majorana and n = -1/2} \\ 96 \text{ TeV}, & \text{Dirac and n = 0} \\ 137 \text{ TeV}, & \text{Dirac and n = -1/2} \end{cases} \tag{18}$$

We did not include the effect of delayed annihilation and bound states formation which are sub-dominant [6]. Note that the similarity of $M_{\rm DM}^{\rm max}$ for Majorana and $n = 0$ with $M_{\rm DM}^{\rm max}$ for Dirac and $n = -1/2$ comes from the almost exact cancellation between the Dirac-to-Majorana factor and the velocity dependence of the non-perturbative (Sommerfeld-enhanced) cross-section $\langle \sigma v_{\rm rel} \rangle \propto 1/v_{\rm rel}$.

- **Answer 4 :** *Show that a cross-section larger than few times the geometrical cross-section would violate unitarity.* Consider the scattering between two extended objects (balls) of diameter $2R_{\rm DM}$. The highest partial wave contributing to the collision is

$$L_{\rm max} = M_{\rm DM}\, v_{\rm rel}\, 2R_{\rm DM}. \tag{19}$$

Indeed, for $L > L_{\rm max}$, the impact parameter $L/(M_{\rm DM} v_{\rm rel})$ is too large and the objects miss each other. Then

$$(\sigma v_{\rm rel})_{\rm max}^{\rm uni} = \frac{4\pi}{M_{\rm DM}^2 v_{\rm rel}} \sum_{L=0}^{L_{\rm max}} (2L+1) \tag{20}$$

$$= 16\pi R_{\rm DM} v_{\rm rel} \tag{21}$$

$$\simeq \pi R_{\rm DM}^2 \tag{22}$$

where we have used $\sum_{L=0}^{L_{\rm max}} L = L_{\rm max}(L_{\rm max}+1)$. We conclude that the cross-section can not be larger than the geometrical cross-section without violating unitarity.

Problem 3 - Relaxing the unitarity bound by injecting entropy

- **Answer 1 :** The temperature $T_{\rm dom}$ at which the heavy relic dominates the energy density of the universe must satisfy $\rho_{\rm rad} = \rho_V$, so

$$\frac{\pi^2}{30} g_{\rm SM} T_{\rm dom}^4 = m_V Y_V \frac{2\pi^2}{45} g_{\rm SM} T_{\rm dom}^3 \quad \rightarrow \quad T_{\rm dom} = \frac{4}{3}\, Y_V\, m_V. \tag{23}$$

- **Answer 2 :** Just after the decay of the cold relic when $H = \Gamma_V$, the universe is dominated by SM radiation. From Friedman's equation, we deduce

$$\frac{\pi^2}{90 M_{\rm pl}^2} g_{\rm SM} \left(T_{\rm dec}^{\rm after}\right)^4 = \Gamma_V^2 \quad \rightarrow \quad T_{\rm dec}^{\rm after} = \left(\frac{90}{\pi^2 g_{\rm SM}}\right)^{1/4} \sqrt{\Gamma_V M_{\rm pl}}. \tag{24}$$

- **Answer 3 :** If we assume that the decay occurs instantaneously when $H = \Gamma_V$, then we can neglect the universe expansion and the energy density is conserved through the decay

$$\rho_{\rm dec}^{\rm before} = \rho_{\rm dec}^{\rm after} \quad \rightarrow \quad m_V\, Y_V\, \frac{2\pi^2}{45} g_{\rm SM} \left(T_{\rm dec}^{\rm before}\right)^3 = \frac{\pi^2}{30} g_{\rm SM} \left(T_{\rm dec}^{\rm after}\right)^4, \tag{25}$$

$$\rightarrow \quad \frac{4}{3}\, m_V\, Y_V = \frac{\left(T_{\rm dec}^{\rm after}\right)^4}{\left(T_{\rm dec}^{\rm before}\right)^3}, \tag{26}$$

$$\rightarrow \quad \left(T_{\rm dec}^{\rm before}\right)^3 = \frac{\left(T_{\rm dec}^{\rm after}\right)^4}{T_{\rm dom}}. \tag{27}$$

- **Answer 4 :** We deduce the dilution factor

$$D \equiv \frac{S^{\rm after}}{S^{\rm before}} = \left(\frac{T_{\rm dec}^{\rm after}}{T_{\rm dec}^{\rm before}}\right)^3 = \frac{T_{\rm dom}}{T_{\rm dec}^{\rm after}} = \frac{4}{3} Y_V \left(\frac{\pi^2 g_{\rm SM}}{90}\right)^{1/4} \frac{m_V}{\sqrt{\Gamma_V m_{\rm pl}}}. \tag{28}$$

Answer 5 : We compute

$$\left.\frac{n_{\rm DM}}{s}\right|_0 = \left.\frac{n_{\rm DM}\ a^3}{s\ a^3}\right|_{\rm after} = \frac{1}{D}\left.\frac{n_{\rm DM}\ a^3}{s\ a^3}\right|_{\rm before} = \frac{1}{D}\left.\frac{n_{\rm DM}}{s}\right|_{\rm FO} \tag{29}$$

where we have used conservation of the number of DM particles through the decay. Hence, injection of entropy during the decay dilutes the DM relic abundance as

$$\left.\frac{n_{\rm DM}}{s}\right|_0^{\rm dil} = \frac{1}{D}\left.\frac{n_{\rm DM}}{s}\right|_0^{\rm Std}, \tag{30}$$

implying that the required cross-section at freeze-out is decreased by a factor D

$$\left.\langle\sigma v\rangle\right|_{\rm FO}^{\rm dil} \ / \ \left.\langle\sigma v\rangle\right|_{\rm FO}^{\rm Std} = 1/D. \tag{31}$$

- **Answer 6 :** Hence the upper bound on the DM mass from unitarity is increased by a factor $\sqrt{D}$

$$M_{\rm DM} \lesssim 140\ \sqrt{D}\ {\rm TeV} \tag{32}$$

where we have assumed perturbative Majorana DM or Sommerfeld-enhanced Dirac DM, both s-wave, c.f. eq. (18). Strongly diluting the DM reopens the parameter space, hence leading to dub this kind of scenario 'Homeopathic Dark Matter' [7].

- **Answer 7 :** Plugging $m_V = 100$ PeV, $\Gamma_V = (0.03\ {\rm s})^{-1}$, $Y = 10^{-2}$ and $g_{\rm SM} = 106.75$, we get the maximal dilution factor compatible with BBN

$$D \simeq 3.4 \times 10^8\ \left(\frac{Y}{10^{-2}}\right)\left(\frac{m_V}{100\ {\rm PeV}}\right)\left(\frac{\tau_V}{0.03\ {\rm s}}\right)^{1/2}, \tag{33}$$

implying the maximal upper bound on the DM mass

$$M_{\rm DM}^{\rm max} \simeq 2.6\ {\rm EeV}\left(\frac{\rm m_V}{100\ {\rm PeV}}\right)^{1/2}\left(\frac{\tau_V}{0.03\ {\rm s}}\right)^{1/4}. \tag{34}$$

Hence BBN provides a limit on the dilution (as opposed to non-allopathic medecine [8]).

Problem 4 - Gamma-ray at Earth from DM decay

- **Answer 1 :**
  - Scalar DM: $\frac{\lambda}{\Lambda}\bar{f}\gamma^\mu(1+r\gamma_5)f\partial_\mu\mathbf{S}$, $\frac{\lambda}{\Lambda}\mathbf{S}F_{\mu\nu}F^{\mu\nu}$, $\frac{\lambda}{\Lambda}\mathbf{S}F^{\mu\nu}\tilde{F}^{\sigma\lambda}$,
  - Fermion DM: $\frac{\lambda}{\Lambda}\bar{\psi}\not{D}LH$.

  For more examples, you can have a look at [9].

- **Answer 2 :**

$$\Gamma = \frac{1}{8\pi}\frac{M_{\rm DM}^3}{\Lambda^2}. \tag{35}$$

- **Answer 3 :**

$$\tau \simeq 10^{26}\ {\rm s}\left(\frac{\Lambda}{10^{16}\ {\rm GeV}}\right)^2\left(\frac{3\ {\rm keV}}{\rm M_{DM}}\right)^3. \tag{36}$$

Problem 5 - Gamma-ray at Earth from DM annihilation

- **Answer 1 :** The J-factor averaged over the disk $[0,\ 1^\circ]$ centred on the GC, assuming the NFW profile is

$$\bar{J}_{\rm NFW}^{\theta<1^\circ} \simeq 1116. \tag{37}$$

- **Answer 2 :** Now assuming a core of size 0.5 kpc, we get

$$\bar{J}_{\rm NFW+core}^{\theta<1^\circ} \simeq 83 \simeq \frac{1}{13}\ \bar{J}_{\rm NFW}^{\theta<1^\circ}. \tag{38}$$

The impact on the annihilation cross-section is

$$\frac{\langle\sigma v\rangle_{\rm NFW+core}}{\langle\sigma v\rangle_{\rm NFW}} = \frac{\bar{J}_{\rm NFW}}{\bar{J}_{\rm NFW+core}}. \tag{39}$$

The error done by neglecting the existence of a core decreases with the angle θ.

	$\bar{J}_{\rm NFW}$	$\bar{J}_{\rm NFW+core}$	$\langle\sigma v\rangle_{\rm NFW+core} \,/\, \langle\sigma v\rangle_{\rm NFW}$
$\theta < 0.5^\circ$	2273	87	26
$\theta < 1^\circ$	1116	83	13
$\theta < 2^\circ$	541	75	7

- **Answer 3 :**

$$\frac{\langle\sigma v\rangle_{\rm NFW+core}}{\langle\sigma v\rangle_{\rm NFW}}\Big|_{0.3^\circ<\theta<1^\circ} \simeq 10. \tag{40}$$

The HESS constraints in [10] using a NFW profile may underestimate the upper bound (meaning that their bounds may be too aggressive[2]) on $\langle\sigma v\rangle$ by one order of magnitude if the MW center has a core of size 0.5 kpc. For a more detailed analysis of the impact of a core on the HESS constraints, you can check [12].

- **Answer 4 :**

$$\int_{\rm disk} ds\, d\Omega\, \rho\,[r(s)] = \rho_0^2 \int_{r_\odot \cos\theta - \sqrt{r_0^2 - r_\odot^2 \sin^2\theta}}^{r_\odot \cos\theta + \sqrt{r_0^2 - r_\odot^2 \sin^2\theta}} ds\, d\Omega \tag{41}$$

$$= 2\pi\rho_0^2 \int_0^{\theta_{\rm max}} d\theta\, 2\sin\theta\, \sqrt{r_0^2 - r_\odot^2 \sin^2\theta} \tag{42}$$

$$= 2\pi\rho_0^2 \left[\frac{\left(r_0^2 - r_\odot^2 \sin^2\theta\right)^{3/2}}{-\frac{3}{2} r_\odot^2} \right]_0^{\theta_{\rm max}} \tag{43}$$

$$= \frac{4\pi}{3} \rho_0^2 \frac{r_0^3}{r_\odot^2} \tag{44}$$

Finally

$$J_{\rm disk} = \int_{\rm disk} \frac{ds}{r_\odot}\, d\Omega\, \frac{\rho\,[r(s)]}{\rho_\odot} = \frac{4\pi}{3} \left(\frac{\rho_0}{\rho_\odot}\right)^2 \left(\frac{r_0}{r_\odot}\right)^3. \tag{45}$$

[2] However, the approach of using NFW without a core (or even Einasto), other than strengthening the constraints, can be justified by the still on-going cusp VS core debate and the possibility for the existence of small scale substructure, DM sub-halos [11], which can potentially increase $\bar{J}$ (the opposit effect as the existence of a core).

Problem 6 - Analogue of Sommerferld enhancement in classical gravity

- **Answer :** Conservation of angular momentum reads

$$m\,v\,b_{\rm max} = m\,v(R)\,R \quad \rightarrow \quad v(R) = \frac{b_{\rm max}}{R}v, \tag{46}$$

which, with conservation of energy gives

$$\frac{1}{2}m\,v^2 = \frac{1}{2}m\,v(R)^2 - \frac{G\,M\,m}{R}, \tag{47}$$

$$v^2 = v^2\frac{b_{\rm max}^2}{R^2} - 2\frac{G\,M}{R}, \tag{48}$$

$$1 = \frac{b_{\rm max}^2}{R^2} - \left(\frac{v_{\rm esc}}{v}\right)^2. \tag{49}$$

Hence, we get

$$\sigma = \pi^2\,b_{\rm max} = \sigma_0\left(1 + \frac{v_{\rm esc}^2}{v^2}\right). \tag{50}$$

The exercise comes from [13].

Problem 7 - γ-ray constraints on $U(1)_D$ model

- **Answer 1 :** Satisfying the correct DM abundance fixes

$$\frac{\pi\alpha_D^2}{M_{\rm DM}^2} \simeq 2.4\times10^{-26}\ {\rm cm^3/s} \quad \rightarrow \quad \alpha \simeq 0.0256. \tag{51}$$

- **Answer 2 :** By looking at fig 6 from [14], we can see that the HESS upper bounds on $\langle\sigma v_{\rm rel}\rangle_{\rm MW}$ are well above (at least 8×10^{-26} cm^3/s for the more stringent bounds which assumes DM annihilation into $\tau^-\tau^+$) the annihilation cross-section in the MW which, being velocity-independent, is the same as the freeze-out cross-section $\langle\sigma v_{\rm rel}\rangle_{\rm FO} \simeq 2.4\times10^{-26}$ cm^3/s. Assuming the validity of the perturbative cross-section $\langle\sigma v_{\rm rel}\rangle = \frac{\pi\alpha_D^2}{M_{\rm DM}^2}$, HESS is unable to constrain the DM model.

- **Answer 3 :** We compute the Sommerfeld enhancement factor in the Coulomb regime of DM ($\alpha_D \simeq 0.0256$) in the MW ($v_{\rm rel} \simeq 220$ km/s $\simeq 0.7\times10^{-3}$)

$$\langle\sigma v\rangle|_{\rm MW}/\langle\sigma v\rangle|_{\rm FO} = 2\pi\frac{\alpha_D}{v_{\rm rel}}\frac{1}{1-e^{-2\pi\frac{\alpha_D}{v_{\rm rel}}}} \simeq 219, \tag{52}$$

and in Dwarfs ($v_{\rm rel} \simeq 10$ km/s $\simeq 3\times10^{-5}$)

$$\langle\sigma v\rangle|_{\rm DW}/\langle\sigma v\rangle|_{\rm FO} \simeq 4818. \tag{53}$$

The Sommerfeld enhancement acts as a boost factor for the ID.

- **Answer 4 :** By looking at table 1 in [14], we read that the HESS upper bound on annihilation cross-section in the MW assuming decay into 2 pairs of $\tau^-\tau^+$ is 8×10^{-26} cm^3/s. Since they assume Majorana particles, the relevant upper bound for the model we study is twice larger: 1.6×10^{-25} cm^3/s. This has to be compared with the annihilation cross-section in the MW multiplied by two times (because two pairs of $\tau^-\tau^+$) the branching ratio of dark photon decay to $\tau^-\tau^+$

$$\frac{\pi\alpha_D^2}{M_{\rm DM}^2}\, S_H\, 2\, BR\left(V \to \tau^-\tau^+\right) \lesssim 1.6 \times 10^{-25}\ {\rm cm}^3/{\rm s}. \tag{54}$$

where S_H is the Sommerfeld enhancement factor in the Hulthen approximation

$$S_H = \frac{2\pi\alpha_D}{v_{\rm rel}} \frac{\sinh(\pi M_{\rm DM} v_{\rm rel}/m_*)}{\cosh(\pi M_{\rm DM} v_{\rm rel}/m_*) - \cosh(\pi\sqrt{M_{\rm DM}^2 v_{\rm rel}^2/m_*^2 - 4M_{\rm DM}\alpha_D/m_*})}, \tag{55}$$

with $m_* = 1.68\ m_V$. We neglect the Sommerfeld enhancement during freeze-out such that the correct DM abundance is still satisfied for $\alpha \simeq 0.0256$. Assuming the branching ratio $BR\left(V \to \tau^-\tau^+\right) \sim 15$ % and the DM mass $M_{\rm DM} = 1$ TeV, we solve for the value of the dark photon mass m_V saturating eq. (54)

$$m_V \lesssim 22\ {\rm TeV}. \tag{56}$$

A more precise calculation taking into account other decay channel leads to [7]

$$m_V \lesssim 100\ {\rm TeV}. \tag{57}$$

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