# SEMILEPTONIC B DECAYS, $|V_{xb}|$ and $R(D,D^*)$

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new analyses of B-factories data, new calculations of FFs by several lattice collaborations and for light-cone sum rules, rising to the challenges of a precision measurement

### PDG AVERAGES



### NEW PHYSICS?



Differential distributions constrain NP, SMEFT interpretation incompatible with LEP data. For a recent detailed analysis see Jung & Straub 1801.01112 the green band on the left is actually larger than it should

# The importance of $|V_{xb}|$

The most important CKM unitarity test is the Unitarity Triangle (UT)  $V_{cb}$  plays an important role in UT  $\varepsilon_K \approx x |V_{cb}|^4 + ...$ and in the prediction of FCNC:  $\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2 \left[1 + O(\lambda^2)\right]$ where it often dominates the

where it often dominates the theoretical uncertainty. V<sub>ub</sub>/V<sub>cb</sub> constrains directly the UT



### VIOLATION OF LFU WITH TAUS

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_{\tau}\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_{\ell}\right)}$$



### EXCLUSIVE DECAYS



There are I(2) and 3(4) FFs for D and D<sup>\*</sup> for light (heavy) leptons, for instance  $\langle D|ar{c}\gamma^\mu b|B
angle\propto f_{+,0}(q^2)$ 

Information on FFs comes from LQCD (at high q<sup>2</sup>), LCSR (at low q<sup>2</sup>), exp...

#### MODEL INDEPENDENT FF PARAMETRIZATION



USING QUARK-HADRON DUALITY + DISPERSION RELATIONS

#### UNITARITY CONSTRAINTS

n=0

$$z = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}} \qquad w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}} \qquad 0 < z < 0.056$$



assuming saturation by single hadron channel

#### TRUNCATED AT ORDER N

#### STRONG UNITARITY CONSTRAINTS

Information on other channels with same quantum numbers makes the bounds tighter. HQS implies that all  $B^{(*)} \rightarrow D^{(*)}$  ff either vanish or are prop to the Isgur-Wise function: any ff F<sub>j</sub> can be expressed as

$$F_j(z) = \left(\frac{F_j}{F_i}\right)_{\text{HQET}} F_i(z)$$

which leads to (hyper)ellipsoids in the ai space for S, P, V, A currents

**Caprini Lellouch Neubert (CLN, 1998)** exploit NLO HQET relations between form factors + QCD sum rules to **reduce parameters** for ffs "up to 2% uncertainty", **never included in exp analysis**. The *practical version of CLN* is

$$h_{A1}(z) = h_{A1}(1) \left[ 1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3 \right]$$
  

$$R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2,$$
  

$$R_2(w) = R_2(1) + 0.11(w - 1) - 0.06(w - 1)^2,$$

only 2+2 parameters! but uncertainty? bias?

## HQS breaking in FF relations

**HQET:** 
$$F_i(w) = \xi(w) \left[ 1 + c^i_{\alpha_s} \frac{\alpha_s}{\pi} + c^i_b \epsilon_b + c^i_c \epsilon_c + \dots \right] \quad \epsilon_{b,c} = \overline{\Lambda}/2m_{b,c}$$

c<sub>b,c</sub> can be computed using subleading IW functions from QCD sumrules Neubert, Ligeti, Nir 1992-93, Bernlochner et al 1703.05330

**RATIOS** 
$$\frac{F_j(w)}{V_1(w)} = A_j \left[ 1 + B_j w_1 + C_j w_1^2 + D_j w_1^3 + \dots \right] \qquad w_1 = w - 1$$
  
Roughly  $\epsilon_c \sim 0.25, \qquad \epsilon_c^2 \sim 0.06$  but coefficients??

In a few cases we can compare these ratios with recent lattice results: there are 5-13% differences, always > NLO correction. For ex.:

$$\frac{A_1(1)}{V_1(1)}\Big|_{\text{LQCD}} = 0.857(15), \qquad \frac{A_1(1)}{V_1(1)}\Big|_{\text{HQET@NLO}} = 0.966(28)$$

Looking at NLO HQET corrections, NNLO can be sizeable, naturally O(10-20)%

#### LATTICE + EXP FIT for $B \rightarrow D/V$ Bigi, PG 1606.08030



# $|V_{cb}|$ from $B \rightarrow D^* / v$ new HFLAV (2019)

LQCD provided only light lepton FF at zero recoil, w=1, where rate vanishes. Experimental results must therefore be **extrapolated to zero-recoil** 

**Exp error only ~1.1%!!!**  $\mathcal{F}(1)\eta_{ew}|V_{cb}| = 35.27(38) \times 10^{-3}$ 

HFLAV extrapolate with CLN parametr. (no error), but  $\chi^2$ /dof of=42.3/23!

Two unquenched lattice calculations

 $\mathcal{F}(I) = 0.906(I3)$ Bailey et al 1403.0635 (FNAL/MILC)

Using their average 0.904(12):

F(I) =0.895(26)

Harrison et al 1711.11013 (HPQCD)

 $|V_{cb}| = 38.76(69) \ 10^{-3}$ 

~ 3.4 $\sigma$  or ~ 8% from inclusive determination 42.00(65) 10<sup>-3</sup>

PG,Healey,Turczyk 2016

Heavy quark sum rules F(1) < 0.925 and estimate of inelastic contribution F(1)≈0.86 Mannel, Uraltsev, PG, 2012

## 2017 tagged Belle analysis (preliminary)

1702.01521

w and angular deconvoluted distributions (independent of parameterization). All previous analyses are CLN based.



# Updating Strong Unitarity Bounds

Fit to new Belle's data + total branching ratio (world average) in 1707.09509 with UPDATED strong unit. bounds (including uncertainties & LQCD inputs)

for reference CLN fit  $|V_{cb}|=0.0392(12)$ 

BGL Fit:	Data + lattice	Data + lattice + LCSR	Data + lattice	Data + lattice + LCSR
unitarity	weak	weak	$\operatorname{strong}$	strong
$\chi^2/dof$	28.2/33	32.0/36	29.6/33	33.1/36
$ V_{cb} $	0.0424(18)	0.0413(14)	0.0415(13)	$0.0406 \begin{pmatrix} +12\\ -13 \end{pmatrix}$
A				

LCSR: Light Cone Sum Rule results from Faller, Khodjamirian, Klein, Mannel, 0809.0222

Using strong unitarity bounds brings BGL closer to CLN and reduce uncertainties but a 3.5-5% difference persists

### 2018 UNTAGGED BELLE ANALYSIS



- Full Belle dataset, most precise study to date; provides data in a way that can be reanalysed with different assumptions.
- CLN and a somewhat simplified BGL analysis lead to very similar results, suggesting low |V<sub>cb</sub>|=38.4(0.9) 10<sup>-3</sup>.
- We used  $BGL^{(222)}$  to fit the data, taking into account D'Agostini effect and got  $|V_{cb}|=39.1(+1.5-1.3)$  10<sup>-3</sup> 1905.08209





#### CONSISTENCY OF DATASETS (poor theorist approach)



## A GLOBAL FITTO 2017 & 2018 DATA

Jung, Schacht, PG 1905.08209



• No parametrization dependence (CLN and BGL give the same result)

- About I sigma higher than HFLAV, comparable uncertainty, p-value ~24% but not well-defined. I.9 $\sigma$  from inclusive
- We truncate the BGL series when additional terms do not change the fit (no overfitting!). Fit stable. Strong constraints irrelevant.

#### COMPARISON WITH HQS, DECAYS WITH TAU

$$R_1(w) = \frac{V_4(w)}{A_1(w)}$$

$$R_2(w) = \frac{w - r}{w - 1} \left( 1 - \frac{1 - r}{w - r} \frac{A_5(w)}{A_1(w)} \right)$$

Comparison of *R*<sub>1,2</sub> from BGL fit to 2017+2018 data vs HQET+QCD sum rule predictions (with parametric + 15% th uncertainty) 2017 data only have problems...



- Decays with tau require pseudoscalar FF undetermined from fit, no lattice calculation yet.
- We use kinematic constraint at q<sup>2</sup>=0 and HQET with conservative uncert.

 $\begin{aligned} R(D^*) &= 0.254^{+0.007}_{-0.006} \,, \\ P_{\tau} &= -0.476^{+0.037}_{-0.034} \,, \\ F_L^{D^*} &= 0.476^{+0.015}_{-0.014} \,, \end{aligned}$ 

# PRELIMINARY JLQCD RESULTS 1811.00794



# BLINDED FNAL-MILC RESULTS 1901.00216

Unquenched calculation of all B→D\* form factors at small recoil Discretization errors are still missing





Blinding is introduced as a global normalisation factor close to 1, which multiplies all form factors in the same way.





### THE IMPORTANCE OF THE SLOPE



Here we use **new improved LCSR** results by Gubernari, Kokulu, van Dyk, 1811.00983 that lead to a minor change wrt 0809.0222

Blinding affects only marginally the slope of the ff F, which is the key to  $V_{cb}$ .

Plot suggests large slope, dF/dw~-1.4.

Assuming -1.40(7) we see that the fit can still accomodate a high  $V_{cb}$ 

Constraints	103 V <sub>cb</sub>	$\chi^2$
slope	40.8(0.8)	84.5/73
slope+LCSR	40.8(0.8)	88.0/76

#### EXPLOITING LCSRS Bordone, Jung, Van Dyk 1908.

#### Theory determination of $b \rightarrow c$ Form Factors

SM: BGL fit to data + FF normalization  $\rightarrow |V_{cb}|$ NP: can affect the  $q^2$ -dependence, introduces additional FFs To determine general NP, FF shapes needed from theory

In [MJ/Straub'18,Bordone/MJ/vDyk'19], we use all available theory input:

Unitarity bounds (using results from [BGL,Bigi/Gambino(/Schacht)'16'17])

 $A_1^{B \to D^*}$ 

• LQCD for  $f_{+,0}(q^2)$   $(B \rightarrow D)$ ,  $h_{A_1}(q^2_{\max})$   $(B \rightarrow D^*)$ [HPQCD'15,'17,Fermilab/MILC'14,'15]

• LCSR for all FFs [Gubernari/Kokulu/vDyk'18]



Including both  $B \rightarrow D, D^*$  they get V<sub>cb</sub>=40.3(0.8)10<sup>-3</sup>

M.Jung

 $\dot{10}$ 

## BABAR REANALYSIS 1903.10002

Reanalysis of tagged B<sup>0</sup> and B<sup>+</sup> data, unbinned 4 dimensional fit with BGL only About 6000 events No data provided yet



BGL form factors compared with CLN HFLAV

V<sub>cb</sub>=0.03836(90)

See next talk



#### INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in  $\Lambda/m_b$  and  $\alpha_s$ 

$$\begin{split} M_{i} = & M_{i}^{(0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(1)} + \left(\frac{\alpha_{s}}{\pi}\right)^{2} M_{i}^{(2)} + \left(M_{i}^{(\pi,0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(\pi,1)}\right) \frac{\mu_{\pi}^{2}}{m_{b}^{2}} \\ & + \left(M_{i}^{(G,0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(G,1)}\right) \frac{\mu_{G}^{2}}{m_{b}^{2}} + M_{i}^{(D,0)} \frac{\rho_{D}^{3}}{m_{b}^{3}} + M_{i}^{(LS,0)} \frac{\rho_{LS}^{3}}{m_{b}^{3}} + \dots \end{split}$$

Global shape parameters (first moments of the distributions, <u>with various lower</u> <u>cuts on EI</u>) tell us about m<sub>b</sub>, m<sub>c</sub> and the B structure, total rate about |V<sub>cb</sub>|

OPE parameters describe universal properties of the B meson and of the quarks: they are useful in many applications (rare decays, V<sub>ub</sub>,...)

Reliability of the method depends on our control of higher order effects. Quarkhadron duality violation would manifest as inconsistency in the fit.

**kinetic scheme** fit includes all corrections  $O(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3)$ , m<sub>c</sub> constraint from sum rules/lattice

# FIT RESULTS



Alberti, Healey, Nandi, PG, 1411.6560

WITHOUT MASS CONSTRAINTS

$$m_b^{kin}(1 \text{GeV}) - 0.85 \,\overline{m}_c(3 \text{GeV}) = 3.714 \pm 0.018 \,\text{GeV}$$

- results depend little on assumption for correlations and choice of inputs, 1.8% determination of V<sub>cb</sub>
- 20-30% determination of the OPE parameters
- b mass determination in agreement with recent lattice and sum rules results



# HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters starting  $1/m^4$ : 9 at dim 7, 18 at dim 8 In principle relevant: HQE contains  $O(1/m_h^n 1/m_c^k)$  Mannel, Turc

Mannel,Turczyk,Uraltsev 1009.4622

Lowest Lying State Saturation Approx (LLSA) truncating

 $\langle B|O_1O_2|B\rangle = \sum \langle B|O_1|n\rangle \langle n|O_2|B\rangle$ 

see also Heinonen, Mannel 1407.4384

and relating higher dimensional to lower dimensional matrix elements, e.g.

$$\rho_D^3 = \epsilon \,\mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon \,\mu_G^2 \quad \epsilon \sim 0.4 \text{GeV}$$

excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases 1206.2296

We use LLSA as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers. The rest of the fit is unchanged, with slightly smaller theoretical errors

$$|V_{cb}| = 42.00(64) \times 10^{-3}$$

# PROSPECTS for INCLUSIVE $V_{cb}$

- Theoretical uncertainties already dominant
- $O(\alpha_s/m_b^3)$  calculation completed for width (Mannel, Pivovarov) in progress for the moments (Nandi, PG)
- $O(1/m_Q^{4,5})$  effects need further investigation but small effect on  $V_{cb}$
- $O(\alpha_{s^3})$  corrections to total width feasible, needed for 1% uncertainty?
- Electroweak (QED) corrections require attention
- New observables in view of Belle-II: FB asymmetry proposed by S.Turczyk could be measured already by Babar and Belle now, q<sup>2</sup> moments...
- Lattice QCD information on local matrix elements is the next frontier (e.g. heavy meson masses and Hashimoto's proposal)

#### MESON MASSES FROM ETMC Melis, Simula, PG 1704.06105



- on the lattice one can compute mesons for arbitrary quark masses
- see also Kronfeld & Simone hep-ph/0006345, 1802.04248 We used both pseudoscalar and vector mesons
- Direct 2+1+1 simulation, a=0.62-0.89 fm, m $_{\pi}$ =210-450 MeV, heavy masses from m<sub>c</sub> to  $3m_c$ , ETM ratio method with extrapolation to static point.
- Kinetic scheme with cutoff at IGeV, good sensitivity up to 1/m<sup>3</sup> corrections
- Results consistent with s.l. fits



 $|V_{ub}| = (3.70 \pm 0.10 \,(\text{exp}) \pm 0.12 \,(\text{theo})) \times 10^{-3} \,(\text{data} + \text{LQCD}),$  $|V_{ub}| = (3.67 \pm 0.09 \,(\text{exp}) \pm 0.12 \,(\text{theo})) \times 10^{-3} \,(\text{data} + \text{LQCD} + \text{LCSR}),$ 

- Theory good: two LQCD and LCSR agree well
- bad chi2/dof, situation would improve (and V<sub>ub</sub> would increase) by considering discrepant results with care

# INCLUSIVE Vub

HFLAV 1909.12524

BLNP	DGE	GGOU	ADFR	BLL		
Input parameters						
SF	$\overline{MS}$	kinetic	$\overline{MS}$	1S		
[572, 573]	Ref. [574]	see Sec. 6.2.2	Ref. [575]	Ref. [556]		
$4.582 \pm 0.026$	$4.188 \pm 0.043$	$4.554 \pm 0.018$	$4.188 \pm 0.043$	$4.704 \pm 0.029$		
$0.145 \begin{array}{c} +0.091 \\ -0.097 \end{array}$		$0.414 \pm 0.078$	-			
$ V_{ub} $ values $[10^{-3}]$						
$4.22 \pm 0.49^{+0.29}_{-0.34}$	$3.86 \pm 0.45^{+0.25}_{-0.27}$	$4.23 \pm 0.49^{+0.22}_{-0.31}$	$3.42 \pm 0.40^{+0.17}_{-0.17}$	-		
$4.51 \pm 0.47^{+0.27}_{-0.29}$	$4.43 \pm 0.47^{+0.19}_{-0.21}$	$4.52 \pm 0.48^{+0.25}_{-0.28}$	$3.93 \pm 0.41^{+0.18}_{-0.17}$	$4.68 \pm 0.49^{+0.30}_{-0.30}$		
$4.93 \pm 0.46^{+0.26}_{-0.29}$	$4.82 \pm 0.45^{+0.23}_{-0.23}$	$4.95 \pm 0.46^{+0.16}_{-0.21}$	$4.48 \pm 0.42^{+0.20}_{-0.20}$	-		
$4.41 \pm 0.12^{+0.27}_{-0.27}$	$3.85 \pm 0.11^{+0.08}_{-0.07}$	$3.96 \pm 0.10^{+0.17}_{-0.17}$	-	-		
$4.71 \pm 0.32^{+0.33}_{-0.38}$	$4.35 \pm 0.29^{+0.28}_{-0.30}$	-	$3.81 \pm 0.19^{+0.19}_{-0.18}$			
$4.50 \pm 0.27^{+0.20}_{-0.22}$	$4.62 \pm 0.28^{+0.13}_{-0.13}$	$4.62 \pm 0.28^{+0.09}_{-0.10}$	$4.50 \pm 0.30^{+0.20}_{-0.20}$	-		
$4.24 \pm 0.19^{+0.25}_{-0.25}$	$4.47 \pm 0.20^{+0.19}_{-0.24}$	$4.30 \pm 0.20^{+0.20}_{-0.21}$	$3.83 \pm 0.18^{+0.20}_{-0.19}$	-		
$4.03 \pm 0.22^{+0.22}_{-0.22}$	$4.22 \pm 0.23^{+0.21}_{-0.27}$	$4.10 \pm 0.23^{+0.16}_{-0.17}$	$3.75 \pm 0.21^{+0.18}_{-0.18}$	1991 - 1991 - 1995 - 1995 - 1995 - 1995 - 1995 - 1995 - 1995 - 1995 - 1995 - 1995 - 1995 - 1995 - 1995 - 1995 -		
$4.32 \pm 0.23^{+0.26}_{-0.28}$	$4.24 \pm 0.22^{+0.18}_{-0.21}$	$4.33 \pm 0.23^{+0.24}_{-0.27}$	$3.75 \pm 0.20^{+0.17}_{-0.17}$	$4.50 \pm 0.24^{+0.29}_{-0.29}$		
$4.09 \pm 0.25^{+0.25}_{-0.25}$	$4.17 \pm 0.25^{+0.28}_{-0.37}$	$4.25 \pm 0.26^{+0.26}_{-0.27}$	$3.57 \pm 0.22^{+0.19}_{-0.18}$	-		
$4.33 \pm 0.24^{+0.19}_{-0.21}$	$4.45 \pm 0.24^{+0.12}_{-0.13}$	$4.44 \pm 0.24^{+0.09}_{-0.10}$	$4.33 \pm 0.24^{+0.19}_{-0.19}$	-		
$4.34 \pm 0.27^{+0.20}_{-0.21}$	$4.43 \pm 0.27^{+0.13}_{-0.13}$	$4.43 \pm 0.27^{+0.09}_{-0.11}$	$4.28 \pm 0.27^{+0.19}_{-0.19}$	-		
-			-	$5.01 \pm 0.39^{+0.32}_{-0.32}$		
$4.44_{-0.14-0.22}^{+0.13+0.21}$	$3.99 \pm 0.10^{+0.09}_{-0.10}$	$4.32 \pm 0.12^{+0.12}_{-0.13}$	$3.99 \pm 0.13^{+0.18}_{-0.12}$	$4.62 \pm 0.20^{+0.29}_{-0.29}$		
	$\begin{array}{r} \text{BLNP} \\ & \\ & \\ \text{SF} \\ [572, 573] \\ 4.582 \pm 0.026 \\ 0.145 \stackrel{+0.091}{_{-0.097}} \\ 4.22 \pm 0.49 \stackrel{+0.29}{_{-0.34}} \\ 4.51 \pm 0.47 \stackrel{+0.27}{_{-0.29}} \\ 4.93 \pm 0.46 \stackrel{+0.26}{_{-0.29}} \\ 4.93 \pm 0.46 \stackrel{+0.26}{_{-0.29}} \\ 4.41 \pm 0.12 \stackrel{+0.27}{_{-0.27}} \\ 4.71 \pm 0.32 \stackrel{+0.33}{_{-0.38}} \\ 4.50 \pm 0.27 \stackrel{+0.20}{_{-0.22}} \\ 4.24 \pm 0.19 \stackrel{+0.25}{_{-0.25}} \\ 4.03 \pm 0.22 \stackrel{+0.22}{_{-0.22}} \\ 4.32 \pm 0.23 \stackrel{+0.26}{_{-0.25}} \\ 4.09 \pm 0.25 \stackrel{+0.25}{_{-0.25}} \\ 4.34 \pm 0.27 \stackrel{+0.20}{_{-0.21}} \\ 4.44 \stackrel{+0.13+0.21}{_{-0.14-0.22}} \\ \end{array}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $		

Importance of the model used to simulate the signal

# THE NNVUB PROJECT

K.Healey, C. Mondino, PG, 1604.07598



- Use Artificial Neural Networks to parameterize shape functions without bias and extract V<sub>ub</sub> from theoretical constraints and data, together with HQE parameters in a model independent way (without assumptions on functional form). Similar to NNPDF. Applies to *b→ulv*, *b→sy*, *b→sl+l-*
- Belle-II will be able to measure some kinematic distributions, thus constraining directly the shape functions. NNVub will provide a flexible tool to analyse data.

# PROSPECTS @ BELLE-II

- Learning @ Belle-II from kinematic distributions, e.g. M<sub>X</sub> spectrum
- OPE parameters checked/ improved in b→ulv (moments): global NN+OPE fit
- alternative approach SIMBA Bernlochner, Tackmann, Ligeti, Stewart
- include all relevant information with correlations
- check signal dependence at endpoint
- full phase space implementation of  $\alpha_{s^2}$  and  $\alpha_{s}/m_{b^2}$  corrections
- model/exclude high q<sup>2</sup> tail

At Belle-II we can hope to bring inclusive  $V_{ub}$  at almost the same level as  $V_{cb}$ 



# CONCLUSIONS

- Revisiting the exclusive b→c decays has been useful: uncertainties were underestimated. Several lattice coll. are computing all necessary FFs, in parallel with Belle-II improved measurements (also for the inclusive moments).
- Inclusive/Exclusive tensions remain, but weaker.
   Hopefully, they will disappear.
- Experiments should provide deconvoluted spectra or alternative but equivalent information. Theoretical prejudice is transient by definition and should never be hardwired into precision measurements.
- Lattice calculations may have an impact on inclusive analyses: unexplored land.
- Something is moving also for V<sub>ub</sub> and more will come by the enhanced possibilities at Belle-II







#### SOMETIMES UNITARITY IS NOT ENOUGH...



The BGL fit with 2017 data tends to prefer **unphysical solutions** for  $V_4$  with a positive w-slope. Imposing LCSR at  $w_{max}$  cures the problem, but one could equally well ask for a negative slope at w=1 or use HQET ratios with appropriate uncertainty. They all tend to lower  $V_{cb}$  slightly.

The size of NLO corrections varies strongly. Some ff are protected by Luke's theorem (no 1/m corrections at zero recoil), others are linked by kinematic relations at max recoil to those protected

NNLO corrections can be sizeable and are naturally O(10-20)%

$$\frac{F_j(w)}{V_1(w)} = A_j \left[ 1 + B_j w_1 + C_j w_1^2 + D_j w_1^3 + \dots \right]$$

$F_{j}$	$A_j$	$B_{j}$	$C_{j}$	$D_j$
$S_1$	1.0208	-0.0436	0.0201	-0.0105
$S_2$	1.0208	-0.0749	-0.0846	0.0418
$S_3$	1.0208	0.0710	-0.1903	0.0947
$P_1$	1.2089	-0.2164	0.0026	-0.0007
$P_2$	0.8938	-0.0949	0.0034	-0.0009
$P_3$	1.0544	-0.2490	0.0030	-0.0008
$V_1$	1	0	0	0
$V_2$	1.0894	-0.2251	0.0000	0.0000
$V_3$	1.1777	-0.2651	0.0000	0.0000
$V_4$	1.2351	-0.1492	-0.0012	0.0003
$V_5$	1.0399	-0.0440	-0.0014	0.0004
$V_6$	1.5808	-0.1835	-0.0009	0.0003
$V_7$	1.3856	-0.1821	-0.0011	0.0003
$A_1$	0.9656	-0.0704	-0.0580	0.0276
$A_2$	0.9656	-0.0280	-0.0074	0.0023
$A_3$	0.9656	-0.0629	-0.0969	0.0470
$A_4$	0.9656	-0.0009	-0.1475	0.0723
$A_5$	0.9656	0.3488	-0.2944	0.1456
$A_6$	0.9656	-0.2548	0.0978	-0.0504
$A_7$	0.9656	-0.0528	-0.0942	0.0455

#### SENSITIVITY TO HIGHER POWER CORRECTIONS

