

# Angular analysis of $B \rightarrow K^{(*)} \mu\mu$ decays at CMS

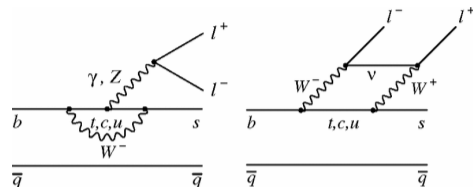
Alessio Boletti  
on behalf of CMS collaboration

Università degli Studi di Padova & INFN Padova

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# Introduction

- Flavour-changing neutral current decays  $b \rightarrow s \ell^+ \ell^-$  are doubly suppressed in the Standard Model
- Good laboratory to probe new-physics effects, through angular analysis and branching fraction measurement
- The  $B^0 \rightarrow K^{*0} \mu \mu$  decay
  - Flavour eigenstate ( $B^0 / \bar{B}^0$ ) can be identified through the  $K^{*0} \rightarrow K^- \pi^+$  decay
  - allows measuring a large set of angular parameters, sensitive to Wilson coefficients  $C_7^{(\prime)}$ ,  $C_9^{(\prime)}$ ,  $C_{10}^{(\prime)}$ ,  $C_{S,P}^{(\prime)}$
- The  $B^+ \rightarrow K^+ \mu \mu$  decay
  - allows measuring the muon forward-backward asymmetry
- Both channels are experimentally accessible, thanks to the fully-charged final states and easy-to-identify muon pair



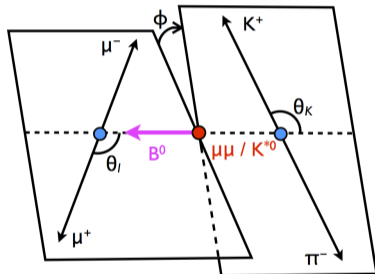


# Angular analyses in CMS

1.  $B^0 \rightarrow K^{*0} \mu\mu$  analysis with Run 1 data
2.  $B^+ \rightarrow K^+ \mu\mu$  analysis with Run 1 data
3. Prospects for the  $B^0 \rightarrow K^{*0} \mu\mu$  analysis at HL-LHC

$B^0 \rightarrow K^{*0}(892)\mu^+\mu^- \rightarrow K^+\pi^-\mu^+\mu^-$  angular analysisPhys. Lett. B 781 (2018) 517-541  
arXiv:1710.02846

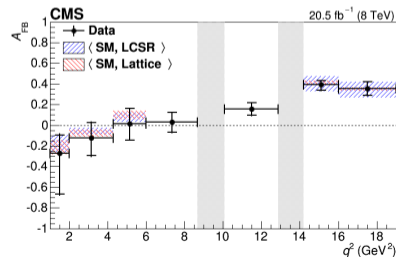
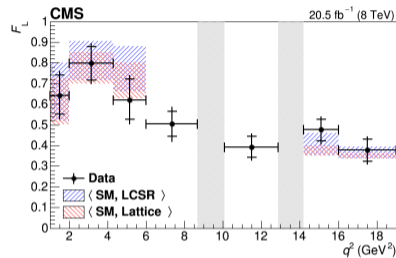
- Fully described by three angles:  $\theta_\ell, \theta_K, \phi$  and  $q^2 = M_{\mu\mu}^2$
- Robust SM calculations of several angular parameters in most of the phase space
  - forward-backward asymmetry of the muons,  $A_{FB}$
  - longitudinal polarisation fraction of the  $K^{*0}$ ,  $F_L$
  - set of clean parameters,  $P_i^{(\prime)}$
- The  $q^2$  range has been divided in 9 bins
  - 7 signal bins, in each of them the angular analysis is performed independently
  - 2 control-region bins, covering the two resonant decays
    - $B^0 \rightarrow J/\psi K^{*0}$
    - $B^0 \rightarrow \psi(2S)K^{*0}$



# Previous CMS analyses

2011 data: Phys. Lett. B 727 (2013) 77  
 2012 data: Phys. Lett. B 753 (2016) 424

- Two analyses were performed and published by CMS with 2011 and 2012 data
- The parameter space was reduced by integrating over the  $\phi$  angular variable
- $A_{FB}$  and  $F_L$  parameters and differential branching fraction were measured
- **No deviations from SM prediction**
- The analysis presented here is performed on the same dataset and uses the same selection criteria as the previous 2012 analysis



## Angular decay rate

- Final state K<sup>+</sup> π<sup>-</sup> μ<sup>+</sup> μ<sup>-</sup> has contribution from **P-wave** (K<sup>\*0</sup>), **S-wave**, and **interference**
- In total, the decay rate has 14 parameters: fold around  $\phi = 0$  and  $\theta_\ell = \pi/2$  to reduce them

$$\frac{1}{d\Gamma/dq^2} \frac{d^4\Gamma}{dq^2 d\cos\theta_l d\cos\theta_K d\phi} = \frac{9}{8\pi} \left\{ \frac{2}{3} \left[ (F_S + A_S \cos\theta_K) (1 - \cos^2\theta_l) + A_S^5 \sqrt{1 - \cos^2\theta_K} \sqrt{1 - \cos^2\theta_l} \cos\phi \right] \right. \\ \left. + (1 - F_S) \left[ 2F_L \cos^2\theta_K (1 - \cos^2\theta_l) + \frac{1}{2} (1 - F_L) (1 - \cos^2\theta_K) (1 + \cos^2\theta_l) \right] \right. \\ \left. + \frac{1}{2} P_1 (1 - F_L) (1 - \cos^2\theta_K) (1 - \cos^2\theta_l) \cos 2\phi \right. \\ \left. + 2P'_5 \cos\theta_K \sqrt{F_L (1 - F_L)} \sqrt{1 - \cos^2\theta_K} \sqrt{1 - \cos^2\theta_l} \cos\phi \right\}$$

- 6 angular parameters left:  
fit with all of them free to float not possible (low statistics, proximity of physical boundaries)

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- $F_L$ ,  $F_S$ , and  $A_S$  fixed from previous CMS measurement



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- 6 angular parameters left:  
fit with all of them free to float not possible (low statistics, proximity of physical boundaries)
- $F_L$ ,  $F_S$ , and  $A_S$  fixed from previous CMS measurement
- $P_1$  and  $P'_5$  measured,  $A_S^5$  nuisance parameter

## Fit pdf description

$$\begin{aligned}
\text{p.d.f.}(m, \cos \theta_K, \cos \theta_l, \phi) = & Y_S^C \cdot \left( S^R(m) \cdot S^a(\cos \theta_K, \cos \theta_l, \phi) \cdot \epsilon^R(\cos \theta_K, \cos \theta_l, \phi) \right. \\
& + \left. \frac{f^M}{1 - f^M} \cdot S^M(m) \cdot S^a(-\cos \theta_K, -\cos \theta_l, -\phi) \cdot \epsilon^M(\cos \theta_l, \cos \theta_K, \phi) \right) \\
& + Y_B \cdot B^m(m) \cdot B^{\cos \theta_K}(\cos \theta_K) \cdot B^{\cos \theta_l}(\cos \theta_l) \cdot B^\phi(\phi).
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Signal components for **correctly-tagged**  
and **mis-tagged** events, each composed by:

- double-Gaussian mass shape
- angular decay rate
- 3D efficiency function  
(using non-parametric Kernel-Density-Estimator)

Flavour state assignment based on  $M(K\pi)$  value

- mis-tagged event fraction 14%, measured on MC

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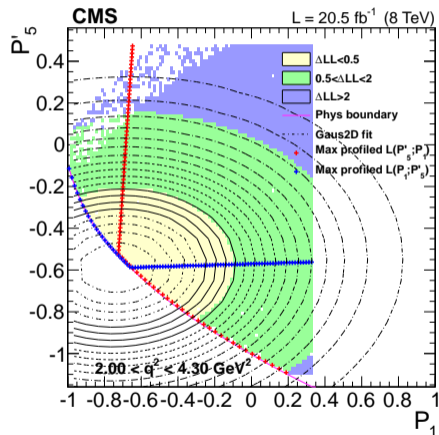
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### Background component

- exponential mass shape
- polynomial shape for each angular variable
- factorisation of angular components tested

# Fit algorithm

- Two-step fit performed for 7 (+2 control regions)  $q^2$  bins:
  - fit  $m$  side bands to determine the background shape
  - fit whole mass spectrum with 5 floating parameters ( $2$  yields,  $P_1$ ,  $P'_5$ ,  $A_s^5$ )
- Unbinned extended maximum likelihood estimator used
  - find maximum of  $\mathcal{L}$  inside the physically allowed region
- Statistical uncertainty using profiled Feldman-Cousins method
- **Blind procedure:** before fitting the signal mass region on data, the fit procedure has been fully tested and validated on simulation



# Systematic uncertainties

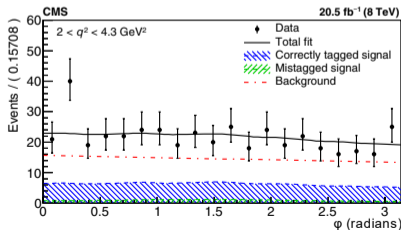
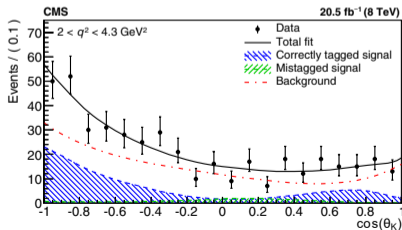
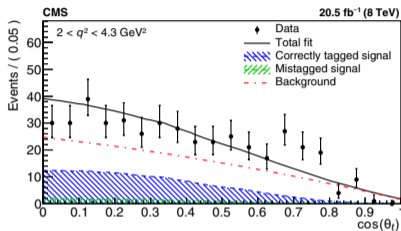
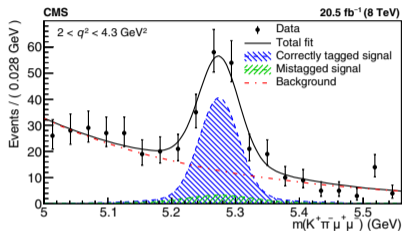
Source	$P_1 (\times 10^{-3})$	$P'_5 (\times 10^{-3})$
Simulation mismodeling	1–33	10–23
Fit bias	5–78	10–120
Finite size of simulated samples	29–73	31–110
Efficiency	17–100	5–65
$K\pi$ mistagging	8–110	6–66
Background distribution	12–70	10–51
Mass distribution	12	19
Feed-through background	4–12	3–24
$F_L, F_S, A_S$ uncertainty propagation	0–210	0–210
Angular resolution	2–68	0.1–12
Total	100–230	70–250

- **Fit bias** with cocktail signal MC + toy background from data side-bands
- **MC stat** due to limited statistics in efficiency shape evaluation
- **Efficiency**: comparing  $F_L$  on CR wrt PDG
- **$K\pi$  mistag** evaluated in  $J/\psi$  control region and propagated to all bins

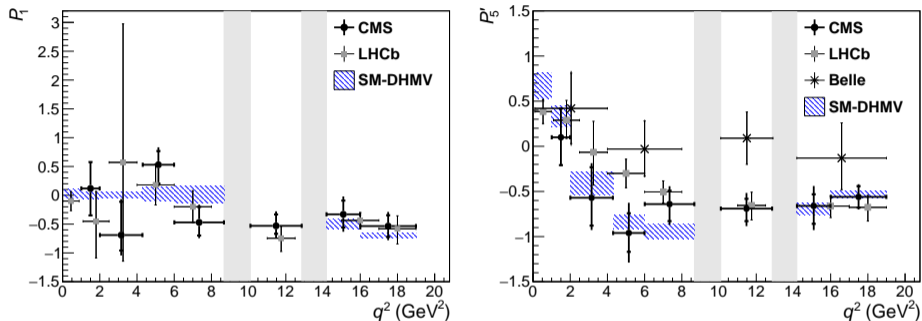
## Propagation of $F_L, F_S,$ and $A_S$ uncertainties:

- Generate pseudo experiments, with  $\times 100$  events, for each  $q^2$  bin
- Fit with  $F_L, F_S, A_S$  free to float and with  $F_L, F_S, A_S$  fixed
- Ratio of stat. uncert. on  $P_1$  and  $P'_5$  with free and fixed fit used to estimate syst uncertainties

# Results: fit projection for second bin: $2.0 < q^2 < 4.3 \text{ GeV}^2$



## Results



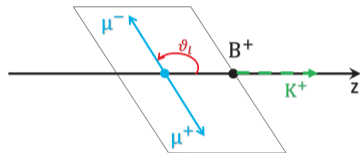
- **SM-DHMV** prediction computed using
  - soft form factors + parametrised power corrections
  - hadronic charm-loop contribution derived from calculations
- **Results compatible with SM predictions within uncertainties**
- **No significant deviations from other experimental results**



B<sup>+</sup> → K<sup>+</sup> μ<sup>+</sup> μ<sup>-</sup> angular analysis

- Fully described by the angle  $\theta_\ell$  and  $q^2 = M_{\mu\mu}^2$ ;
- Angular decay rate:

$$\frac{1}{d\Gamma/dq^2} \frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} = \frac{3}{4} (1 - F_H) (1 - \cos^2\theta_\ell) + \frac{1}{2} F_H + \mathcal{A}_{FB} \cos\theta_\ell$$



- Angular analysis to measure
  - angular parameter  $F_H$
  - forward-backward asymmetry of the muons,  $\mathcal{A}_{FB}$
- Range of  $q^2$  divided in 9 bins
  - 7 signal bins
  - 2 bins for resonant decays B<sup>+</sup> → J/ψ K<sup>+</sup> and B<sup>+</sup> → ψ(2S) K<sup>+</sup> (control channels)
  - 2 additional special bins: [1-6] GeV<sup>2</sup> (clean predictions) and [1-22] GeV<sup>2</sup> (full signal)

# Fit algorithm

$$\begin{aligned} \text{p.d.f.}(m, \cos \theta_l) = & Y_S \cdot S_i(m) \cdot S_i^a(\cos \theta_l) \cdot \epsilon_i(\cos \theta_l) \\ & + Y_B \cdot B_i^m(m) \cdot B_i^{\cos \theta_l}(\cos \theta_l) \end{aligned}$$

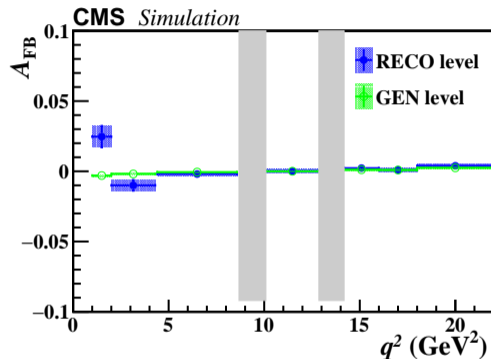
- Signal pdf component
  - double Gaussian mass shape
  - efficiency parameterised from MC with 6th-order polynomial
- Background pdf component
  - exponential mass shape
  - polynomial (3rd- or 4th-order) + Gaussian for the angular shape
- Two-step fit performed:
  - fit  $m$  side bands to determine the background shape (fixed in second step)
  - fit whole mass spectrum with 4 floating parameters (2 yields + 2 angular param)
- Unbinned extended maximum likelihood estimator used
- Statistical uncertainty using profiled Feldman-Cousins method

# Validation and systematic uncertainties

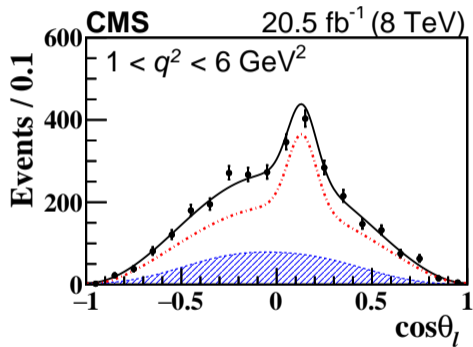
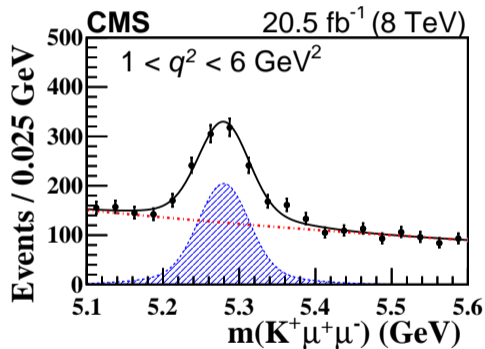
Several validation steps

- With signal MC sample (both high statistics and data-like)
- With resonant control regions

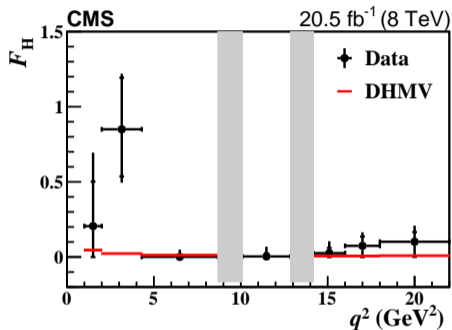
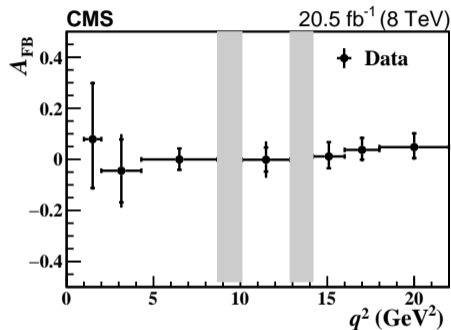
Systematic uncertainty	$A_{FB} (\times 10^{-2})$	$F_H (\times 10^{-2})$
Finite size of MC samples	0.4–1.8	0.9–5.0
Efficiency description	0.1–1.5	0.1–7.8
Simulation mismodeling	0.1–2.8	0.1–1.4
Background parametrization model	0.1–1.0	0.1–5.1
Angular resolution	0.1–1.7	0.1–3.3
Dimuon mass resolution	0.1–1.0	0.1–1.5
Fitting procedure	0.1–3.2	0.4–25
Background distribution	0.1–7.2	0.1–29
Total systematic uncertainty	1.6–7.5	4.4–39



Dominant systematic uncertainty  
from background description

Results: fit projections for special bin  $1 < q^2 < 6 \text{ GeV}^2$ 

## Results

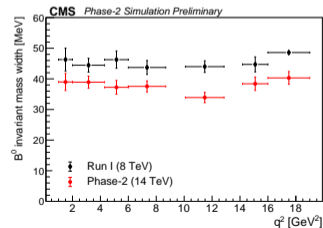
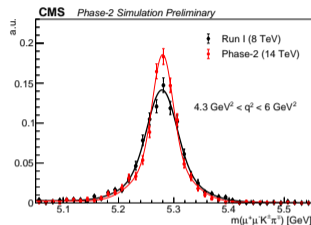


- Inner error bar is statistical uncertainty
- Full bar is total uncertainty
- Results compatible with SM predictions within uncertainties

# Prospects for $B^0 \rightarrow K^{*0} \mu\mu$ analysis at HL-LHC

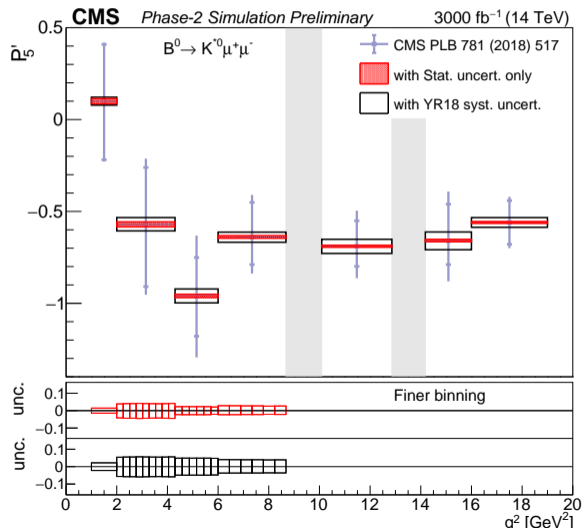
Report: CMS-PAS-FTR-18-033

- **Uncertainty on  $P_5'$**  measurement in  $B^0 \rightarrow K^{*0} \mu\mu$  angular distribution is extrapolated to **HL-LHC** scenario at  $3000 \text{ fb}^{-1}$
- Run 1 results used as baseline
- Upgraded CMS tracker detector provides **improved mass resolution**
- No changes in trigger performances and analysis strategy have been considered
- Signal yield has been obtained from MC simulations with Phase-2 detector upgrade and pileup of 200
  - Scaled to  $3000 \text{ fb}^{-1}$ :  $\sim 700\text{k}$  events in the full  $q^2$  range



# Projections on $P'_5$ uncertainty (HL-LHC)

- Run 1 statistical uncertainty scaled according to the expected yield
- Systematic uncertainties based on data control channel scaled according to statistics
- Other systematic uncertainties scaled by factor of 2
- Total uncertainty is expected to improve by 15 times wrt Run 1 result
- Large signal yield allows to split  $q^2$  range in finer bins



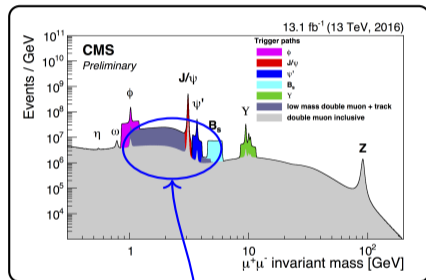
# Summary

FCNC rare decays are being extensively studied in CMS

- $B^0 \rightarrow K^{*0} \mu\mu$  angular analysis has been extended to measure  $P_1$  and  $P'_5$
- $B^+ \rightarrow K^+ \mu\mu$  angular analysis performed for the first time in CMS, to measure  $\mathcal{A}_{\text{FB}}$  and  $F_{\text{H}}$
- Prospects of  $B^0 \rightarrow K^{*0} \mu\mu$  angular analysis in HL-LHC

Currently working on Run 2 analyses

- dedicated trigger requiring two muons + 1 track with common vertex
- more decay channels to be explored
- Stay tuned



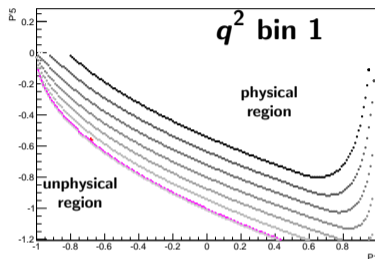


# Backup slides

## The validity range

To guarantee that the decay rate is always positively defined, the parameters have *physical boundaries*:

- interference terms  $A_S^{(5)}$  have a definition range dependent on  $F_L$ ,  $F_S$ , and  $P_1$
- to have a positive P-wave component,  $P_1$  and  $P_5'$  must satisfy:  $(P_5')^2 - 1 < P_1 < 1$ 
  - purple line in the graph
- to have a positive decay rate, an additional boundary need to be considered
  - depends on all the parameters
  - no analytic description available
  - computed numerically for each  $q^2$  bin, using the fixed values of  $F_L$ ,  $F_S$ , and  $A_S$
  - grey lines in the graphs represent the boundary for different  $A_S^5$  values within its range



**Main cause of fit convergence problems → floating parameter  $A_S^5$   
and its influence on the physical boundary**

Dataset selection for  $B^0 \rightarrow K^{*0} \mu \mu$ 

**Trig** Dedicated HLT trigger path:

Low  $p_t$  dimuon, displaced, low invariant mass

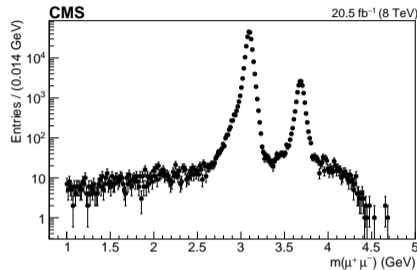
$\mu$   $p_T^\mu > 3.5$  GeV,  $p_T^{\mu\mu} > 6.9$  GeV,  
with high-quality displaced vertex

$h$   $p_T^h > 0.8$  GeV,  $|M(K\pi) - M_{K^{*0}}| < 90$  MeV,  
 $M_{KK} > 1.035$  ( $\phi$  veto), displaced from the primary vertex

$B^0$   $p_t > 8$  GeV,  $|\eta| < 2.2$ , with four-body displaced vertex requirement and global momentum alignment

- both  $B^0$  and  $\bar{B}^0$  considered
- anti radiation cut against feed-down of  $J/\psi/\psi(2S)$

**CR** :  $J/\psi$  and  $\psi(2S)$  resonances used as control regions and treated in the same way.



no PID to distinguish K from  $\pi$ ,  
flavour state assignment based on  
which hypothesis  $M(K^+\pi^-/K^-\pi^+)$   
is closer to  $M_{K^{*0}}$  (PDG)  
mistag rate 14% (MC)

# Anti-radiation cut

The signal sample is required to pass the selection:

- $m(\mu\mu) < m_{J/\psi\text{PDG}} - 3\sigma_{m(\mu\mu)}$  or
- $m_{J/\psi\text{PDG}} + 3\sigma_{m(\mu\mu)} < m(\mu\mu) < m_{\psi'\text{PDG}} - 3\sigma_{m(\mu\mu)}$  or
- $m(\mu\mu) > m_{\psi'\text{PDG}} + 3\sigma_{m(\mu\mu)}$ ;

for the control channel  $B^0 \rightarrow K^{*0}(K^+\pi^-)J/\psi(\mu^+\mu^-)$  the requirement is:

- $|m(\mu\mu) - m_{J/\psi\text{PDG}}| < 3\sigma_{m(\mu\mu)}$ .

while for the  $B^0 \rightarrow K^{*0}(K^+\pi^-)\psi'(\mu^+\mu^-)$  channel is:

- $|m(\mu\mu) - m_{\psi'\text{PDG}}| < 3\sigma_{m(\mu\mu)}$ .

To further reject feed-through from control channels →

Events are **rejected** if  $m(\mu\mu) < m_{J/\psi\text{PDG}}$ , then:

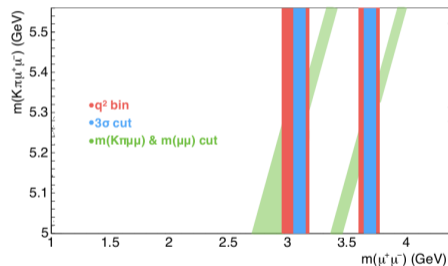
- $|(m(K\pi\mu\mu) - m_{B^0\text{PDG}}) - (m(\mu\mu) - m_{J/\psi\text{PDG}})| < 160 \text{ MeV}/c^2$ ;
- $|(m(K\pi\mu\mu) - m_{B^0\text{PDG}}) - (m(\mu\mu) - m_{\psi'\text{PDG}})| < 60 \text{ MeV}/c^2$ ;

while if  $m_{J/\psi\text{PDG}} < m(\mu\mu) < m_{\psi'\text{PDG}}$ , then:

- $|(m(K\pi\mu\mu) - m_{B^0\text{PDG}}) - (m(\mu\mu) - m_{J/\psi\text{PDG}})| < 60 \text{ MeV}/c^2$ ;
- $|(m(K\pi\mu\mu) - m_{B^0\text{PDG}}) - (m(\mu\mu) - m_{\psi'\text{PDG}})| < 60 \text{ MeV}/c^2$ ;

and if  $m(\mu\mu) > m_{\psi'\text{PDG}}$ , then:

- $|(m(K\pi\mu\mu) - m_{B^0\text{PDG}}) - (m(\mu\mu) - m_{J/\psi\text{PDG}})| < 60 \text{ MeV}/c^2$ ;
- $|(m(K\pi\mu\mu) - m_{B^0\text{PDG}}) - (m(\mu\mu) - m_{\psi'\text{PDG}})| < 30 \text{ MeV}/c^2$ .

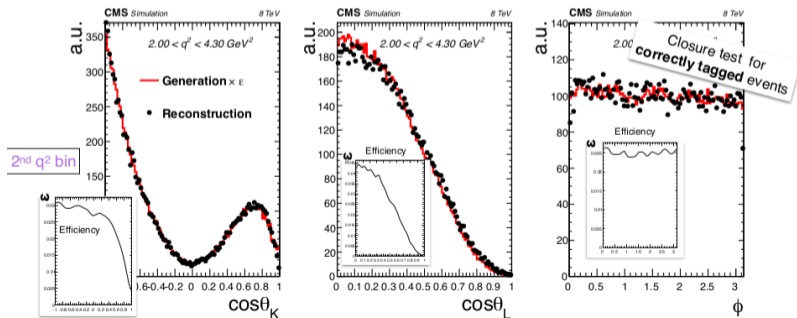


# Efficiency and closure test (right tag)

- Numerator and denominator of efficiency are separately described with nonparametric technique implemented with a kernel density estimator on unbinned distributions
- Final efficiency distributions in the p.d.f. obtained from the ratio of 3D histograms derived from the sampling of the kernel density estimators

## Closure test:

- compute efficiency with half of the MC simulation and use it to correct the other half
- same test performed both for correctly and mistagged events independently

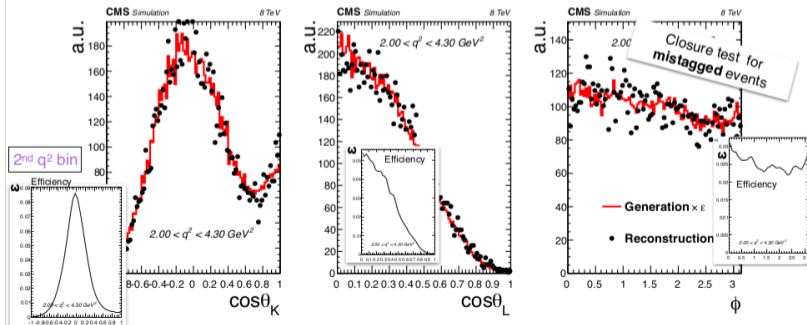


# Efficiency and closure test (wrong tag)

- Numerator and denominator of efficiency are separately described with nonparametric technique implemented with a kernel density estimator on unbinned distributions
- Final efficiency distributions in the p.d.f. obtained from the ratio of 3D histograms derived from the sampling of the kernel density estimators

## Closure test:

- compute efficiency with half of the MC simulation and use it to correct the other half
- same test performed both for correctly and mistagged events independently



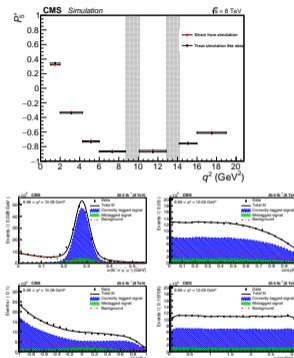
## Background considered included:

- Partially reconstructed  $B^0$  decay might pollute left  $M_{B^0}$  side bands
  - restrict left s.b. ( $5.1 < M < 5.6$  GeV, default  $5 < M < 5.6$  GeV)
  - redo fit: change in  $P_1$  and  $P'_5$  within the systematic uncertainties.
- $B^\pm \rightarrow K^\pm \mu \mu$  plus and additional random  $\pi^\mp$ :
  - distribution ends at  $M > 5.4$  GeV, further reduced by  $\cos \alpha$  cut, and BR similar to  $B^0 \rightarrow K^{*0} \mu \mu$
- $\Lambda_b \rightarrow p K J/\psi (\mu^+ \mu^-)$ 
  - look at event in the  $M_{K\pi\mu\mu} \approx M_{B^0}$  peak, reconstruct them using p, K mass hypothesis: no peak seen.
- $B^0 \rightarrow DX$ , with  $D \rightarrow hh$  and  $h$  mis-id as  $\mu$ 
  - requires two mis-id:  $P_{misld} \sim 1 \cdot 10^{-3}$ : given  $BR \sim 1 \cdot 10^{-3}$  negligible.
- $B^0 \rightarrow J/\psi (\mu \mu) K^{*0} (K\pi)$ , with one h and one  $\mu$  switched
  - $P_{misld} \mu \cdot (1 - \varepsilon_\mu) \sim 1 \cdot 10^{-4}$ ,  $Y_{B^0 \rightarrow J/\psi \mu \mu} \sim 1.6 \cdot 10^5$ : few events in bin close to  $J/\psi$
  - $J/\psi$  feed contamination in close bin included in the fit model

# Fit validation

extensive fit validation with MC: used as **systematics**

- compare fit results with MC input values (**sim mismodeling**)
- compare with data-like MC (**fit bias**)
  - signal only correct tag
  - signal correct+wrong tag
  - signal + background
- Data control channel ( $J/\psi$  and  $\psi(2S)$ ), comparing fit results with PDG ( $F_L$ ) (**efficiency**)
- compare  $P_1$  and  $P_5'$  on  $J/\psi$  and  $\psi(2S)$  w/ and w/o  $F_L$  fixed: no bias



$$\frac{\mathcal{B}(B^0 \rightarrow K^{*0} \psi(2S))}{\mathcal{B}(B^0 \rightarrow K^{*0} J/\psi)} = \frac{Y_{\psi(2S)} \epsilon_{J/\psi}}{\epsilon_{\psi(2S)} Y_{J/\psi}} \frac{\mathcal{B}(J/\psi \rightarrow \mu^+ \mu^-)}{\mathcal{B}(\psi(2S) \rightarrow \mu^+ \mu^-)} = 0.480 \pm 0.008(\text{stat}) \pm 0.055(R_{\psi}^{\mu\mu})$$

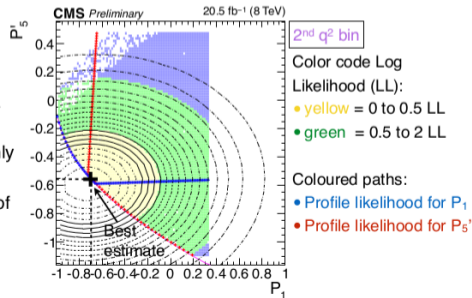
vs PDG  $0.484 \pm 0.018(\text{stat}) \pm 0.011(\text{syst}) \pm 0.012(R_{\psi}^{ee})$



# Fit procedure

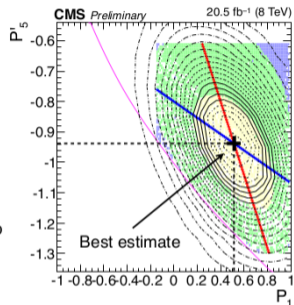
- The decay rate can become negative for certain values of the angular parameters ( $P_1$ ,  $P_5'$ ,  $A_5^s$ )
- The presence of such a physically allowed region greatly complicates the numerical maximisation process of the likelihood by MINUIT and especially the error determination by MINOS, in particular next to the boundary between physical and unphysical regions
- The **best estimate** of  $P_1$  and  $P_5'$  is computed by:
  - discretise the bi-dimensional space  $P_1$ - $P_5'$
  - maximise the likelihood as a function of  $Y_s$ ,  $Y_B$ , and  $A_5^s$  at fixed values of  $P_1$ ,  $P_5'$
  - fit the likelihood distribution with a 2D-gaussian function
  - the maximum of this function inside the physically allowed region is the best estimate

- To ensure correct coverage for the **uncertainties** of  $P_1$  and  $P_5'$ , the Feldman-Cousins method is used in a simplified form: the confidence interval's construction is performed only along two 1D paths determined by profiling the 2D-gaussian description of the likelihood inside the physically allowed region



# FC stat uncertainties determination

- To ensure correct coverage for the **uncertainties** of  $P_1$  and  $P_5'$ , the Feldman-Cousins method is used in a simplified form: the confidence interval's construction is performed only along the two 1D paths determined by profiling the 2D-gaussian description of likelihood inside the physically allowed region:
  - generate 100 pseudo-experiments for each point of the path
  - fit and rank according to the likelihood-ratio
  - confidence interval bound is found when data likelihood-ratio exceeds the 68.3% of the pseudo-experiments

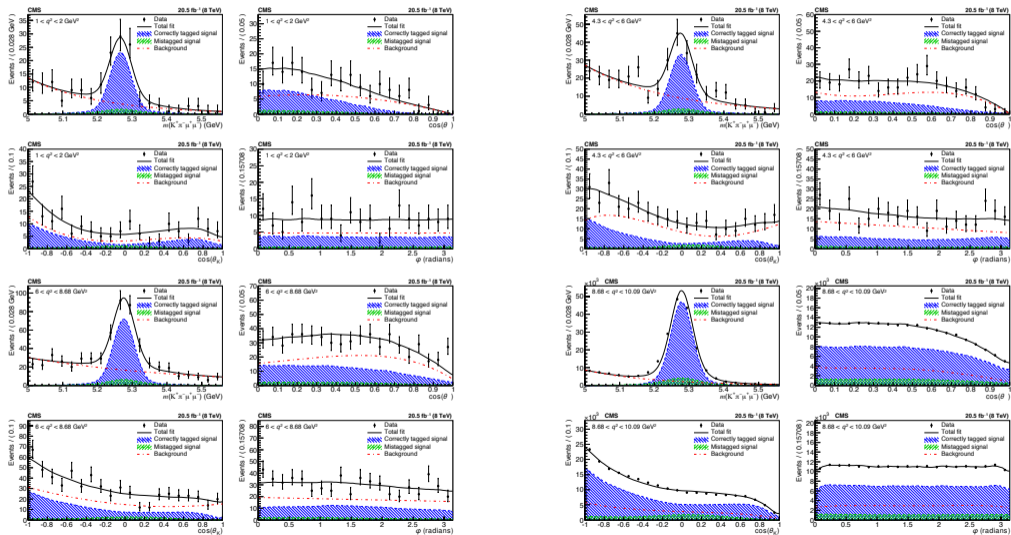


- Due to the limited number of pseudo-experiments statistical fluctuations are present
- To produce a robust result, the ranking of the data likelihood-ratio is plotted for several scan points
- The intersection is then computed using a linear fit

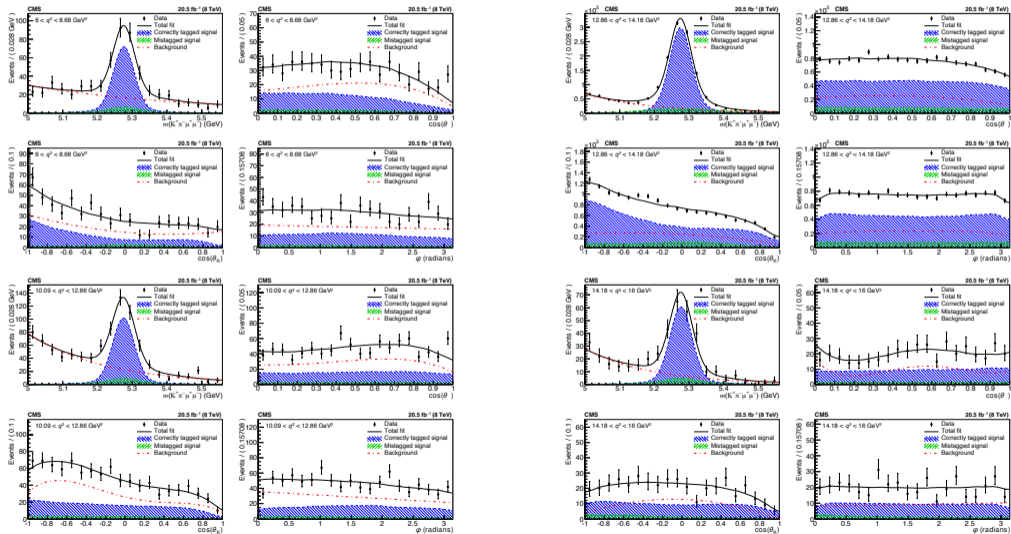
## CMS results (table)

$q^2$ (GeV <sup>2</sup> )	Signal yield	$P_1$	$P'_5$	Correlations
1.00–2.00	$80 \pm 12$	$+0.12^{+0.46}_{-0.47} \pm 0.10$	$+0.10^{+0.32}_{-0.31} \pm 0.07$	-0.0526
2.00–4.30	$145 \pm 16$	$-0.69^{+0.58}_{-0.27} \pm 0.23$	$-0.57^{+0.34}_{-0.31} \pm 0.18$	-0.0452
4.30–6.00	$119 \pm 14$	$+0.53^{+0.24}_{-0.33} \pm 0.19$	$-0.96^{+0.22}_{-0.21} \pm 0.25$	+0.4715
6.00–8.68	$247 \pm 21$	$-0.47^{+0.27}_{-0.23} \pm 0.15$	$-0.64^{+0.15}_{-0.19} \pm 0.13$	+0.0761
10.09–12.86	$354 \pm 23$	$-0.53^{+0.20}_{-0.14} \pm 0.15$	$-0.69^{+0.11}_{-0.14} \pm 0.13$	+0.6077
14.18–16.00	$213 \pm 17$	$-0.33^{+0.24}_{-0.23} \pm 0.20$	$-0.66^{+0.13}_{-0.20} \pm 0.18$	+0.4188
16.00–19.00	$239 \pm 19$	$-0.53 \pm 0.19 \pm 0.16$	$-0.56 \pm 0.12 \pm 0.07$	+0.4621

## Fit Projections



## Fit Projections



## Decay rate

$$\frac{d^4 \Gamma [B^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-]}{dq^2 d\vec{\Omega}} = \frac{9}{32\pi} \sum_i \bar{I}_i(q^2) f_i(\vec{\Omega}) \longrightarrow \text{Decay rate involving } b \text{ quark, i.e. } B^0_{\text{bar}} \text{ meson}$$

$$\frac{d^4 \bar{\Gamma} [B^0 \rightarrow K^{*0} \mu^+ \mu^-]}{dq^2 d\vec{\Omega}} = \frac{9}{32\pi} \sum_i I_i(q^2) f_i(\vec{\Omega}) \longrightarrow \text{Decay rate involving } b_{\text{bar}} \text{ quark, i.e. } B^0 \text{ meson}$$

- $\Gamma$  and  $\bar{\Gamma}$ : expression of the decay
- $f_i(\vec{\Omega})$ : combinations of spherical harmonics
- $I_i$  and  $\bar{I}_i$ :  $q^2$ -dependent angular parameters (combinations of six complex decay amplitudes)

$$\frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \frac{d^4(\Gamma + \bar{\Gamma})}{dq^2 d\vec{\Omega}} = \frac{9}{32\pi} \left[ \frac{3}{4} (1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K \right. \\ \left. + \frac{1}{4} (1 - F_L) \sin^2 \theta_K \cos 2\theta_l \right. \\ \left. - F_L \cos^2 \theta_K \cos 2\theta_l + S_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\phi \right. \\ \left. + S_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + S_5 \sin 2\theta_K \sin \theta_l \cos \phi \right. \\ \left. + \frac{4}{3} A_{\text{FB}} \sin^2 \theta_K \cos \theta_l + S_7 \sin 2\theta_K \sin \theta_l \sin \phi \right. \\ \left. + S_8 \sin 2\theta_K \sin 2\theta_l \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi \right]$$

**Assumptions / simplifications:**

- CP-averaged measurement
- Massless limit, i.e.  $q^2 \gg 4m_\mu^2$



8 independent angular parameters

# Decay rate

## Decay rate parameterisation

(JHEP 01 (2013) 048)

For example  $\mathbf{P}_5' = \frac{S_5}{\sqrt{F_L(1-F_L)}}$

$$\begin{aligned} \frac{d^4\Gamma}{dq^2 d\cos\theta_K d\cos\theta_l d\phi} = & \frac{9}{32\pi} \left[ \frac{3}{4} F_L \sin^2\theta_K + F_L \cos^2\theta_K \right. \\ & + \left( \frac{1}{4} F_L \sin^2\theta_K - F_L \cos^2\theta_K \right) \cos 2\theta_l + \frac{1}{2} P_1 F_L \sin^2\theta_K \sin^2\theta_l \cos 2\phi \\ & + \sqrt{F_L F_L} \left( \frac{1}{2} P_4' \sin 2\theta_K \sin 2\theta_l \cos \phi + P_5' \sin 2\theta_K \sin \theta_l \cos \phi \right) \\ & - \sqrt{F_L F_L} \left( P_6' \sin 2\theta_K \sin \theta_l \sin \phi - \frac{1}{2} P_8' \sin 2\theta_K \sin 2\theta_l \sin \phi \right) \\ & \left. + 2P_2 F_L \sin^2\theta_K \cos\theta_l - P_3 F_L \sin^2\theta_K \sin^2\theta_l \sin 2\phi \right] = \frac{d\Gamma^4_{\text{P-wave}}}{dq^2 d\Omega} \end{aligned}$$

Two channels can contribute to the final state  $K^+ \pi^- \mu^+ \mu^-$ :

- **P-wave** channel,  $K^+ \pi^-$  from the meson vector resonance  $K^{*0}$  decay
- **S-wave** channel,  $K^+ \pi^-$  not coming from any resonance

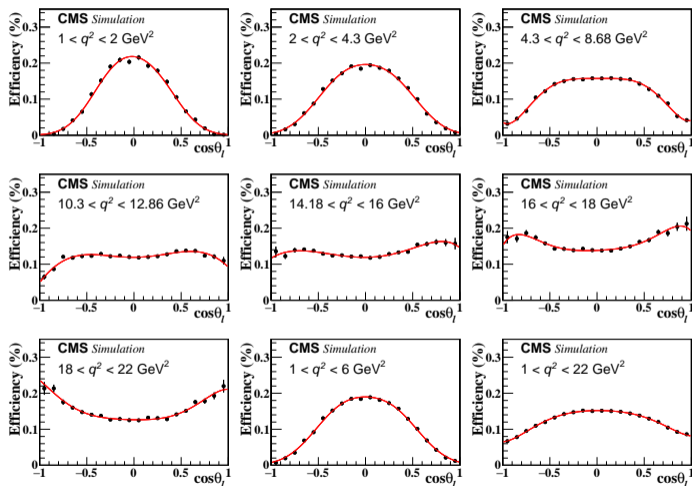
We have to parametrise both decay rates !

$$\frac{d\Gamma^4_{\text{Total}}}{dq^2 d\Omega} = (1 - F_S) \frac{d\Gamma^4_{\text{P-wave}}}{dq^2 d\Omega} + \frac{d\Gamma^4_{\text{S/SP-wave}}}{dq^2 d\Omega}$$

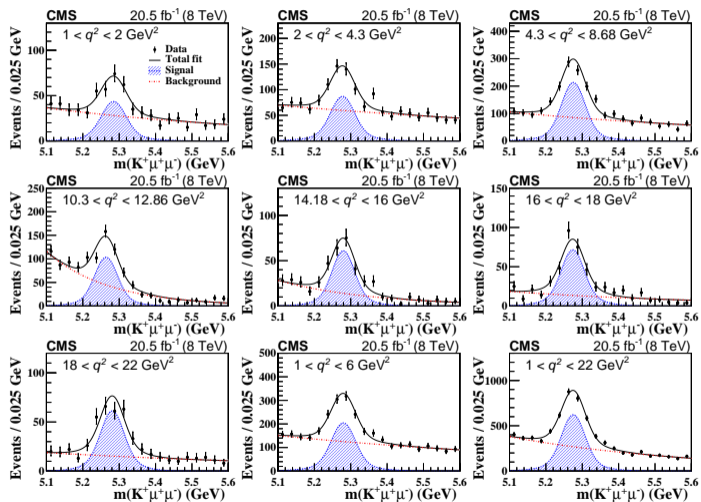
Both S-wave and S&P wave interference

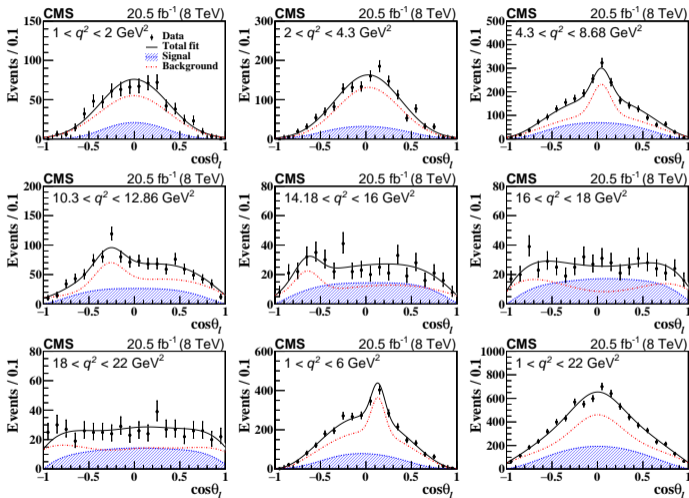
$$\begin{aligned} \frac{d\Gamma^4_{\text{S/SP-wave}}}{dq^2 d\Omega} = & \frac{3}{16\pi} \left[ F_S \sin^2\theta_\ell + A_S \sin^2\theta_\ell \cos\theta_K + A_S^4 \sin\theta_K \sin 2\theta_\ell \cos\phi \right. \\ & \left. + A_S^5 \sin\theta_K \sin\theta_\ell \cos\phi + A_S^7 \sin\theta_K \sin\theta_\ell \sin\phi + A_S^8 \sin\theta_K \sin 2\theta_\ell \sin\phi \right] \end{aligned}$$

6 independent parameters

Efficiency parameterisation -  $B^+ \rightarrow K^+ \mu \mu$ 



Fit projections -  $B^+$ -candidate mass

Fit projections -  $\cos \theta_\ell$ 

Fit results -  $B^+ \rightarrow K^+ \mu\mu$ 

$q^2$ (GeV <sup>2</sup> )	$\Upsilon_S$	$A_{FB}$	$F_H$	$F_H(\text{EOS})$	$F_H(\text{DHMV})$	$F_H(\text{FLAVIO})$
1.00–2.00	$169 \pm 22$	$0.08^{+0.22}_{-0.19} \pm 0.05$	$0.21^{+0.29}_{-0.21} \pm 0.39$	0.047	0.046	0.045
2.00–4.30	$331 \pm 32$	$-0.04^{+0.12}_{-0.12} \pm 0.07$	$0.85^{+0.34}_{-0.31} \pm 0.14$	0.024	0.023	0.022
4.30–8.68	$785 \pm 42$	$0.00^{+0.04}_{-0.04} \pm 0.02$	$0.01^{+0.02}_{-0.01} \pm 0.04$	—	0.012	0.011
10.09–12.86	$365 \pm 29$	$0.00^{+0.05}_{-0.05} \pm 0.05$	$0.01^{+0.02}_{-0.01} \pm 0.06$	—	—	—
14.18–16.00	$215 \pm 19$	$0.01^{+0.06}_{-0.05} \pm 0.02$	$0.03^{+0.03}_{-0.03} \pm 0.07$	0.007	0.007	0.006
16.00–18.00	$262 \pm 21$	$0.04^{+0.05}_{-0.04} \pm 0.03$	$0.07^{+0.06}_{-0.07} \pm 0.07$	0.007	0.007	0.006
18.00–22.00	$226 \pm 20$	$0.05^{+0.05}_{-0.04} \pm 0.02$	$0.10^{+0.06}_{-0.10} \pm 0.09$	0.008	0.009	0.008
1.00–6.00	$778 \pm 47$	$-0.14^{+0.07}_{-0.06} \pm 0.03$	$0.38^{+0.17}_{-0.21} \pm 0.09$	0.025	0.025	0.020
1.00–22.00	$2286 \pm 73$	$0.00^{+0.02}_{-0.02} \pm 0.03$	$0.01^{+0.01}_{-0.01} \pm 0.06$	—	—	—

## Experiment comparison

