CP Violation & Rare Decays in the Kaon System

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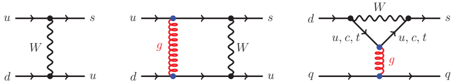
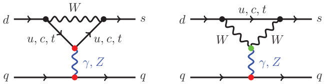
Physics contributions from different scales

Short-Distance



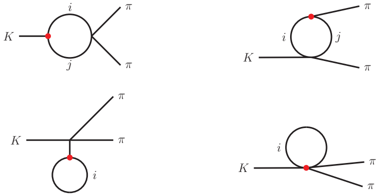
Top quark, GIM, $\mathcal{C}\not{\mathcal{P}}$

New Physics ?



Long-Distance

Chiral Dynamics



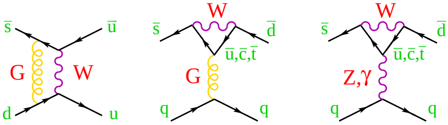
Multi-Scale Problem:

$M_\pi < M_K < m_c < m_b \ll M_W < m_t$

$\log\left(M/\mu\right)$ **OPE** , $\log\left(\nu_\chi/M_\pi\right)$ χ**PT**

Energy Scale	Fields	Effective Theory
M_W	$\boxed{\begin{array}{c} W, Z, \gamma, g \\ \tau, \mu, e, \nu_i \\ t, b, c, s, d, u \end{array}}$	Standard Model
	$\Big\Downarrow$ OPE	
$\lesssim m_c$	$\boxed{\begin{array}{c} \gamma, g \;;\; \mu, e, \nu_i \\ s, d, u \end{array}}$	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)}$, $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	$\Big\Downarrow$ $N_C \to \infty$	
M_K	$\boxed{\begin{array}{c} \gamma \;;\; \mu, e, \nu_i \\ \pi, K, \eta \end{array}}$	χPT

$\Delta S = 1$ TRANSITIONS



$$\boxed{\mathcal{L}_{\mathrm{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}}\, V_{ud} V_{us}^* \; \sum_j \; C_j(\mu)\; Q_j(\mu)}$$

$$
\begin{aligned}
Q_1 &= (\bar{s}_\alpha u_\beta)_{\mathrm{V-A}} \, (\bar{u}_\beta d_\alpha)_{\mathrm{V-A}} & Q_2 &= (\bar{s}u)_{\mathrm{V-A}} (\bar{u}d)_{\mathrm{V-A}} \\
Q_{3,5} &= (\bar{s}d)_{\mathrm{V-A}} \sum_q (\bar{q}q)_{\mathrm{V\mp A}} & Q_4 &= (\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q (\bar{q}_\beta q_\alpha)_{\mathrm{V-A}} \\
Q_{7,9} &= \tfrac{3}{2}\, (\bar{s}d)_{\mathrm{V-A}} \sum_q e_q (\bar{q}q)_{\mathrm{V\pm A}} & Q_{10} &= \tfrac{3}{2}\, (\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q e_q \, (\bar{q}_\beta q_\alpha)_{\mathrm{V-A}} \\
Q_6 &= -8 \sum_q (\bar{s}_L q_R)(\bar{q}_R d_L) & Q_8 &= -12 \sum_q e_q \, (\bar{s}_L q_R)(\bar{q}_R d_L) \\
Q_{11,12} &= (\bar{s}d)_{\mathrm{V-A}} \sum_\ell \, (\bar{\ell}\ell)_{\mathrm{V,A}} & Q_{13} &= (\bar{s}d)_{\mathrm{V-A}} \sum_\nu \, (\bar{\nu}\nu)_{\mathrm{V-A}}
\end{aligned}
$$

- $q > \mu$: $\quad C_j(\mu) = z_j(\mu) - y_j(\mu) \, (V_{td} \, V_{ts}^* / V_{ud} \, V_{us}^*)$

  NLO: $\quad O(\alpha_s^n t^n)$, $O(\alpha_s^{n+1} t^n)$ $\quad [t \equiv \log{(M/m)}]$ $\quad$ Munich / Rome, 1992-1993

  NNLO: $\quad \sim Q_{7-10,13}$ $\quad$ Buras et al, $\quad Q_{1-6}$ ongoing calculation $\quad$ M. Cerdà-Sevilla et al

- $q < \mu$: $\quad \langle \pi\pi | Q_j(\mu) | K \rangle$? $\quad$ **Physics does not depend on μ**

CHIRAL PERTURBATION THEORY (χPT)

- **Expansion in powers of p^2/Λ_χ^2 :** $\mathcal{A} = \sum_n \mathcal{A}^{(n)}$ $(\Lambda_\chi \sim 4\pi F_\pi \sim 1.2\ \mathrm{GeV})$
- **Amplitude structure fixed by chiral symmetry**

$$\mathrm{SU(3)_L} \otimes \mathrm{SU(3)_R} \to \mathrm{SU(3)_V}$$

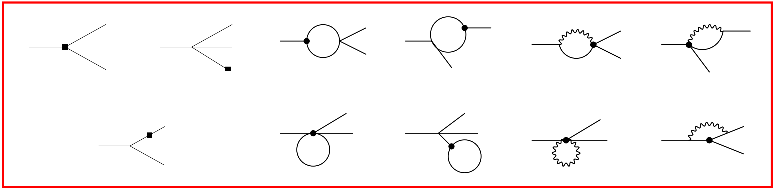
- **Short-distance dynamics encoded in Low-Energy Couplings**
- **$O(p^2)$ χPT: Goldstone interactions (π, K, η)** $\Phi \equiv \frac{1}{\sqrt{2}}\,\vec{\lambda}\,\vec{\varphi}$

$$\boxed{\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \mathrm{Tr}(\lambda L_\mu L^\mu) + G_{27}\, F^4 \left(L_{\mu 23} L^\mu_{11} + \frac{2}{3}\, L_{\mu 21} L^\mu_{13} \right)}$$

$G_R \equiv -\frac{G_F}{\sqrt{2}}\, V_{ud} V_{us}^*\, g_R$; $L_\mu = -i U^\dagger D_\mu U$; $\lambda \equiv \frac{1}{2}\,\lambda_{6-i7}$; $U \equiv \exp\left\{ i\sqrt{2}\,\Phi/F \right\}$

- **Loop corrections (χPT logarithms) unambiguously predicted**
- **LECs can be determined at $N_C \to \infty$ (matching)**
- **$O(p^2)$ LECs (G_8, G_{27}) can be phenomenologically determined**

$\mathbf{O\left[p^4, (m_u - m_d)\, p^2, e^2 p^0, e^2 p^2\right]}$ χPT



- **Nonleptonic weak Lagrangian:** $\mathcal{O}(G_F\, p^4)$

$$\mathcal{L}^{(4)}_{\rm weak} = \sum_i G_8\, N_i\, F^2\, O_i^8 + \sum_i G_{27}\, D_i\, F^2\, O_i^{27} + \text{h.c.}$$

- **Electroweak Lagrangian:** $\mathcal{O}(G_F\, e^2 p^{0,2})$

$$\mathcal{L}_{\rm EW} = e^2 F^6 G_8\, g_{ew}\, \mathrm{Tr}(\lambda U^\dagger \mathcal{Q} U) + e^2 \sum_i G_8\, Z_i\, F^4\, O_i^{EW} + \text{h.c.}$$

- $\mathcal{O}(e^2 p^{0,2})$ **Electromagnetic** $+$ $\mathcal{O}(p^4)$ **Strong:** $Z\,,\ K_i\,,\ L_i$

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	$\Downarrow$ OPE	
$\lesssim m_c$	$\gamma, g\;;\;\mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)}$, $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	$\Downarrow$ $N_C \to \infty$	
M_K	$\gamma\;;\;\mu, e, \nu_i$ π, K, η	χPT

CP Violation in $K \to \pi\pi$

$$\eta_{00} \equiv \frac{\mathcal{M}(K_L^0 \to \pi^0\pi^0)}{\mathcal{M}(K_S^0 \to \pi^0\pi^0)} \equiv \varepsilon - 2\varepsilon' \qquad , \qquad \eta_{+-} \equiv \frac{\mathcal{M}(K_L^0 \to \pi^+\pi^-)}{\mathcal{M}(K_S^0 \to \pi^+\pi^-)} \equiv \varepsilon + \varepsilon'$$

- **Indirect $\not{C}\not{P}$:** $|\varepsilon| = \frac{1}{3}\,|\eta_{00} + 2\,\eta_{+-}| = (2.228 \pm 0.011)\cdot 10^{-3}$
- **Direct $\not{C}\not{P}$:** $\mathrm{Re}(\varepsilon'/\varepsilon) = \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) = (16.6 \pm 2.3)\cdot 10^{-4}$

First evidence in 1988 by NA31

Time evolution of ε'/ε predictions: 10^{-3} units

- 1983	SD (Q_6), LO	~ 2	Gilman-Hagelin
- 1990-2000	SD, large m_t (Q_8), NLO	$\sim \mathrm{few}\cdot 10^{-1}$	Munich, Rome
	+ models of LD contributions	$\sim \mathcal{O}(1)$	Dortmund, Trieste
- 1999-2001	SD + LD (χPT) at NLO	1.7 ± 0.9	Scimemi-Pallante-Pich
- 2000-2003	models of LD contributions	$\sim \mathcal{O}(1)$	Lund, Marseille
- 2003	Isospin breaking in χPT	1.9 ± 1.0	Cirigliano-Ecker-Neufeld-Pich
- 2015	Lattice	0.14 ± 0.70	RBC-UKQCD
- 2015-2017	Dual QCD, Lattice input	0.19 ± 0.45	Munich
- 2017	χPT re-analysis	1.5 ± 0.7	Gisbert-Pich
- 2019	χPT re-analysis of IB	$1.3\,^{+0.6}_{-0.7}$	Cirigliano-Gisbert-Pich-Rodríguez

$K \to 2\pi$ Isospin Amplitudes

$$A[K^0 \to \pi^+\pi^-] \equiv A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2\, e^{i\chi_2}$$

$$A[K^0 \to \pi^0\pi^0] \equiv A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2}$$

$$A[K^+ \to \pi^+\pi^0] \equiv \frac{3}{2} A_2^+\, e^{i\chi_2^+}$$

1) $\Delta I = 1/2$ Rule: $\omega \equiv \dfrac{\mathrm{Re}\,(A_2)}{\mathrm{Re}\,(A_0)} \approx \dfrac{1}{22}$

2) Strong Final State Interactions: $\chi_0 - \chi_2 \approx \delta_0 - \delta_2 \approx 45^\circ$

$$\boxed{\varepsilon'_K = \frac{-i}{\sqrt{2}}\, e^{i(\chi_2-\chi_0)}\, \omega \left\{ \frac{\mathrm{Im}\,(A_0)}{\mathrm{Re}\,(A_0)} - \frac{\mathrm{Im}\,(A_2)}{\mathrm{Re}\,(A_2)} \right\}}$$

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$$A[K^0 \to \pi^0\pi^0] \equiv A_0 \, \mathrm{e}^{i\chi_0} - \sqrt{2} \, A_2 \, \mathrm{e}^{i\chi_2}$$

$$A[K^+ \to \pi^+\pi^0] \equiv \frac{3}{2} A_2^+ \, \mathrm{e}^{i\chi_2^+}$$

$$\boxed{\begin{aligned} A_0 \, \mathrm{e}^{i\chi_0} &= \mathcal{A}_{1/2} \\ A_2 \, \mathrm{e}^{i\chi_2} &= \mathcal{A}_{3/2} + \mathcal{A}_{5/2} \\ A_2^+ \, \mathrm{e}^{i\chi_2^+} &= \mathcal{A}_{3/2} - \frac{2}{3} \mathcal{A}_{5/2} \end{aligned}}$$

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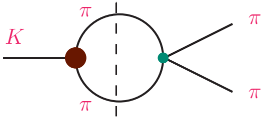
Implications of a Large Phase Shift

$$\mathcal{A}_I \;\equiv\; A_I\, \mathrm{e}^{i\delta_I} \;=\; \mathrm{Dis}\,(\mathcal{A}_I) + i\,\mathrm{Abs}\,(\mathcal{A}_I)$$

❶ **Unitarity:** $\delta_0(M_K) \;=\; (39.2 \pm 1.5)^\circ$ ➡ $A_0 \approx 1.3 \times \mathrm{Dis}\,(\mathcal{A}_0)$



$$\tan\delta_I \;=\; \frac{\mathrm{Abs}\,(\mathcal{A}_I)}{\mathrm{Dis}\,(\mathcal{A}_I)}$$

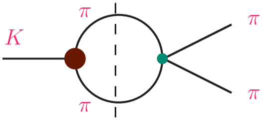
$$A_I \;=\; \mathrm{Dis}\,(\mathcal{A}_I)\;\sqrt{1+\tan^2\delta_I}$$

❷ **Analyticity:** $\Delta\,\mathrm{Dis}\,(\mathcal{A}_I)[s] \;=\; \frac{1}{\pi}\int dt\; \frac{\mathrm{Abs}\,(\mathcal{A}_I)[t]}{t-s-i\epsilon} \;+\; \mathrm{subtractions}$

$\boxed{\text{Large } \delta_0 \;\Rightarrow\; \text{Large } \mathrm{Abs}\,(\mathcal{A}_0) \;\Rightarrow\; \text{Large correction to } \mathrm{Dis}\,(\mathcal{A}_0)}$

Absorptive amplitude: on-shell intermediate $\pi\pi$ state

$$\mathcal{A}_I \;\equiv\; A_I\, \mathrm{e}^{i\delta_I} \;=\; \mathrm{Dis}\,(\mathcal{A}_I) + i\,\mathrm{Abs}\,(\mathcal{A}_I) \qquad\qquad \sigma_\pi \;\equiv\; \sqrt{1 - 4M_\pi^2/M_K^2}$$



$$\Delta_L\mathcal{A}_0/\mathcal{A}_0^{\mathrm{tree}} \;=\; (2M_K^2 - M_\pi^2)\; B_{\mathrm{loop}} + \cdots$$

$$\Delta_L\mathcal{A}_2/\mathcal{A}_2^{\mathrm{tree}} \;=\; -(M_K^2 - 2M_\pi^2)\; B_{\mathrm{loop}} + \cdots$$

$$B_{\mathrm{loop}} \;=\; \frac{1}{32\pi^2F_\pi^2}\left\{\sigma_\pi\left[\log\left(\frac{1-\sigma_\pi}{1+\sigma_\pi}\right) + i\pi\right] + \log\left(\frac{\nu_\chi^2}{M_\pi^2}\right) + 1\right\}$$

- **Finite 1-loop absorptive amplitude** (model independent)
- **Universal correction** (only depends on $\pi\pi$ quantum numbers):

  $$\mathrm{Abs}\,(\mathcal{A}_0)/\mathcal{A}_0^{\mathrm{tree}} = 0.47 \qquad , \qquad \mathrm{Abs}\,(\mathcal{A}_2)/\mathcal{A}_2^{\mathrm{tree}} = -0.21$$

- **Any (SM or NP) short-distance contribution** leads to $\Delta\mathcal{A}_I^{\mathrm{tree}} \sim g_I^{\mathrm{SD}}\mathcal{O}_I$
- **Same correction** for $\mathrm{Re}\,(g_I^{\mathrm{SD}})$ ($\mathcal{CP}$ conserving) and $\mathrm{Im}\,(g_I^{\mathrm{SD}})$ ($\cancel{\mathcal{CP}}$)

2015 Lattice Results

Isospin limit: RBC-UKQCD 1505.07863, 1502.00263

$$\sqrt{\frac{3}{2}}\,\mathrm{Re}\,A_2 = (1.50 \pm 0.04 \pm 0.14) \cdot 10^{-8}\ \mathrm{GeV} \qquad \mathrm{exp}: 1.482\,(2) \cdot 10^{-8}\ \mathrm{GeV}$$

$0.1\,\sigma$

$$\sqrt{\frac{3}{2}}\,\mathrm{Im}\,A_2 = -(6.99 \pm 0.20 \pm 0.84) \cdot 10^{-13}\ \mathrm{GeV}$$

$$\sqrt{\frac{3}{2}}\,\mathrm{Re}\,A_0 = (4.66 \pm 1.00 \pm 1.26) \cdot 10^{-7}\ \mathrm{GeV} \qquad \mathrm{exp}: 3.112\,(1) \cdot 10^{-7}\ \mathrm{GeV}$$

$1.0\,\sigma$

$$\sqrt{\frac{3}{2}}\,\mathrm{Im}\,A_0 = -(1.90 \pm 1.23 \pm 1.08) \cdot 10^{-11}\ \mathrm{GeV}$$

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = (1.38 \pm 5.15 \pm 4.59) \cdot 10^{-4} \qquad \mathrm{exp}: (16.8 \pm 1.4) \cdot 10^{-4}$$

$2.2\,\sigma$

$$\delta_0 = (23.8 \pm 4.9 \pm 1.2)^\circ \qquad \mathrm{exp}: (39.2 \pm 1.5)^\circ \qquad 2.9\,\sigma$$

$$\delta_2 = -(11.6 \pm 2.5 \pm 1.2)^\circ \qquad \mathrm{exp}: -(8.5 \pm 1.5)^\circ \qquad 1.0\,\sigma$$

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$1.0\,\sigma$

Simplified Estimate

❶ **CP violation** ➡ **Penguin operators**

❷ **Chirality** ➡ **Enhanced $(V-A)\otimes(V+A)$ operators**

$Q_6 \; = \; -8 \sum_q (\bar{s}_L q_R)\,(\bar{q}_R d_L)$, $Q_8 \; = \; -12 \sum_q e_q\,(\bar{s}_L q_R)\,(\bar{q}_R d_L)$

❸ **Large-N_C:** $\langle J\cdot J\rangle \; = \; \langle J\rangle\,\langle J\rangle\;\{1+\mathcal{O}(1/N_C)\}$

$$\mathcal{M}_{LL} \;\equiv\; \langle\pi^+\pi^-|(\bar{s}_L\gamma^\mu u_L)(\bar{u}_L\gamma_\mu d_L)|K^0\rangle \;=\; \langle\pi^+|\bar{u}_L\gamma_\mu d_L|0\rangle\,\langle\pi^-|\bar{s}_L\gamma^\mu u_L|K^0\rangle \;=\; \frac{i\sqrt{2}}{4}\,F_\pi\,(M_K^2-M_\pi^2)$$

$$\mathcal{M}_{LR}(\mu) \;\equiv\; \langle\pi^+\pi^-|(\bar{s}_L u_R)(\bar{u}_R d_L)|K^0\rangle \;=\; \langle\pi^+|\bar{u}_R d_L|0\rangle\,\langle\pi^-|\bar{s}_L u_R|K^0\rangle \;=\; \frac{i\sqrt{2}}{4}\,F_\pi\left[\frac{M_K^2}{m_d(\mu)+m_s(\mu)}\right]^2$$

At $\mu=1$ GeV, $\mathcal{M}_{LR}(\mu)/\mathcal{M}_{LL}\;\sim\;M_K^2/[m_s(\mu)+m_d(\mu)]^2\;\sim\;14$

➡ $$\boxed{\mathrm{Re}(\varepsilon'/\varepsilon)\;\approx\;2.2\cdot 10^{-3}\;\left\{B_6^{(1/2)}\,(1-\Omega_{\mathrm{eff}})-0.48\;B_8^{(3/2)}\right\}}$$

Strong Cancellation

$B_6^{(1/2)} = B_8^{(3/2)} = 1\,,\qquad \Omega_{\mathrm{eff}} = 0.12$ ➡ $\mathrm{Re}(\varepsilon'/\varepsilon)\approx 0.9\cdot 10^{-3}$

Buras et al: $B_6^{(1/2)} = 0.57\,,\quad B_8^{(3/2)} = 0.76\,,\quad \Omega_{\mathrm{eff}} = 0.15$ ➡ $\mathrm{Re}(\varepsilon'/\varepsilon)\approx 2.6\cdot 10^{-4}$

Anatomy of ε'/ε calculation

$$\boxed{\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = -\frac{\omega_+}{\sqrt{2}|\varepsilon|}\left\{\frac{\mathrm{Im}\,A_0^{(0)}}{\mathrm{Re}\,A_0^{(0)}}\,(1-\Omega_{\mathrm{eff}}) - \frac{\mathrm{Im}\,A_2^{\mathrm{emp}}}{\mathrm{Re}\,A_2^{(0)}}\right\}}$$

$$\mathcal{A}_n^{(X)} = a_n^{(X)}\left[1 + \Delta_L\,\mathcal{A}_n^{(X)} + \Delta_C\,\mathcal{A}_n^{(X)}\right]$$

Cirigliano-Gisbert-Pich-Rodríguez 2019

❶ **$O(p^4)$ χPT Loops: Large correction** (NLO in $1/N_C$) FSI

$$\Delta_L\,\mathcal{A}_{1/2}^{(8)} = 0.27 + 0.47\,i \qquad ; \qquad \Delta_L\,\mathcal{A}_{3/2}^{(g)} = -0.50 - 0.21\,i$$

❷ **$O(p^4)$ LECs fixed at $N_C\to\infty$: Small correction**

$$\Delta_C\,[\mathcal{A}_{1/2}^{(8)}]^- = 0.11 \pm 0.05 \qquad ; \qquad \Delta_C\,[\mathcal{A}_{3/2}^{(g)}]^- = -0.19 \pm 0.19$$

❸ **Isospin Breaking** $O\left[(m_u-m_d)\,p^2, e^2p^2\right]$**: Sizeable correction**

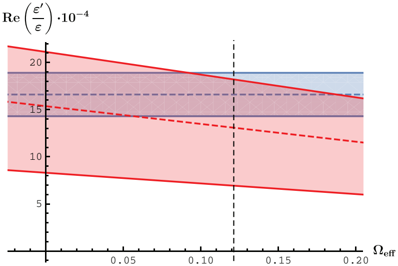
$$\Omega_{\mathrm{eff}} = 0.12 \pm 0.09$$

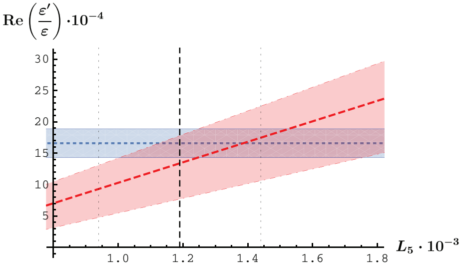
❹ **$\mathrm{Re}(g_8)$, $\mathrm{Re}(g_{27})$, $\chi_0-\chi_2$ fitted to data**

SM Prediction of ε'/ε

$$\boxed{\mathrm{Re}\,(\varepsilon'/\varepsilon)_{\mathrm{SM}} \;=\; \left(13\,{}^{+\,6}_{-\,7}\right)\cdot 10^{-4}}$$

Cirigliano, Gisbert, Pich, Rodríguez-Sánchez

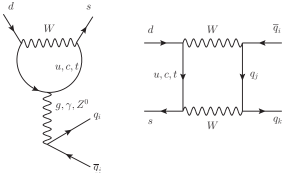




$$\mathrm{Re}\,(\varepsilon'/\varepsilon)_{\mathrm{SM}} \;=\; \left(13.1 \pm 0.4_{m_s}\,{}^{+\,2.2}_{-\,4.0}{}_{\mu}\,{}^{+\,3.0}_{-\,3.2}{}_{\nu_\chi} \pm 1.2_{\gamma_5} \pm 4.3_{L_{5,8}} \pm 1.1_{L_7} \pm 0.2_{K_i} \pm 0.3_{X_i}\right)\cdot 10^{-4}$$

Large uncertainty but no anomalies!

Rare K Decays



Cirigliano et al., 1107.6001

- Very suppressed in the SM
- Sensitive to heavy mass scales, LFV, LNV. . .
- Excellent experimental sensitivity

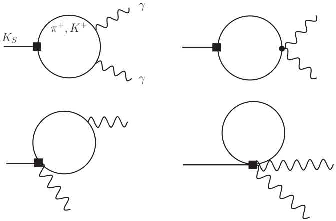
➡ **Strong constraints on New Physics**
Superb probe of flavour dynamics and $\not{CP}$

- Interesting interplay of short and long distances
- Excellent testing ground of χPT dynamics

Precise control of QCD needed when SM sensitivity is reached

$K^0 \to \gamma\gamma$

Long-distance dynamics



Finite loop: $\mathrm{Br}_{\mathrm{LO}} = 2.0 \cdot 10^{-6}$

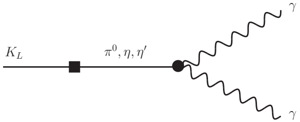
D'Ambrosio-Espriu, Goity

$$\mathrm{Br}(K_S \to \gamma\gamma) = (2.63 \pm 0.17) \cdot 10^{-6}$$

Agreement at $O(p^6)$ (FSI)

$$K_S \to \pi\pi \to \pi^+\pi^- \to \gamma\gamma$$

Kambor-Holstein, Buchalla et al



$$\mathrm{Br}(K_L \to \gamma\gamma) = (5.47 \pm 0.04) \cdot 10^{-4}$$

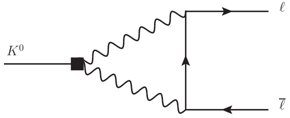
WZW Anomaly

$\mathbf{T}_{\mathrm{LO}} = \mathbf{0}$ [$\mathcal{O}(p^4)$, GMO cancel.]

$\mathcal{O}(p^6)$: SU(3) breaking, η–η' mixing

Well understood

$K^0 \to \ell^+\ell^-$



$K_S \to \ell^+\ell^-$

Long-distance dynamics

Finite 2-loop amplitude: Ecker-Pich

$$\mathrm{Br}(K_S \to e^+e^-)_{\mathrm{LO}} = 2.1 \cdot 10^{-14}$$

$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\mathrm{LO}} = 5.1 \cdot 10^{-12}$$

$$\mathrm{Br}(K_S \to e^+e^-)_{\mathrm{exp}} < 9 \cdot 10^{-9}$$

$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\mathrm{exp}} < 8 \cdot 10^{-10}$$ LHCb

(90% CL)

$K_L \to \ell^+\ell^-$

$$\mathrm{Br}(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \cdot 10^{-9}$$

$$\mathrm{Br}(K_L \to e^+e^-) = (9\,^{+6}_{-4}) \cdot 10^{-12}$$

Saturated by absorptive contrib.

Local countertem $\Longleftrightarrow$ **SD**

LD extracted from $\pi^0, \eta \to \ell^+\ell^-$

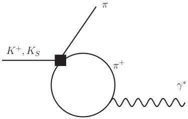
Gomez-Dumm, Pich

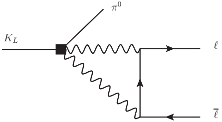
Fitted SD contrib. agrees with SM

Longitudinal Polarization: Ecker-Pich

$$|\mathcal{P}_L| = (2.6 \pm 0.4) \cdot 10^{-3}$$

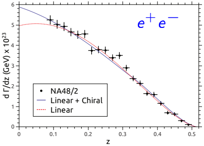
$K \to \pi \, \ell^+ \ell^-$

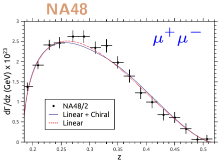




$$\mathrm{Br}(K^\pm \to \pi^\pm e^+ e^-) = 3.00\,(9) \cdot 10^{-7}$$

$$\mathrm{Br}(K^\pm \to \pi^\pm \mu^+ \mu^-) = 9.4\,(6) \cdot 10^{-8}$$





Local $\mathcal{O}(p^4)$ LECs Ecker-Pich-de Rafael

Electromagn. transition form factor

$\mathcal{O}(p^6)$ corrections D'Ambrosio et al

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 2.8 \cdot 10^{-10}$$

$$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \cdot 10^{-10}$$

(90% CL), KTeV

3 contributions: Ecker-Pich-de Rafael

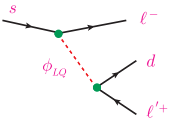
- **Direct** $\not{C}\not{P}$
- **Indirect** $\not{C}\not{P}$
- **CP conserving** (2γ)

$\not{C}\not{P}$ **dominates for e^+e^-:**

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) = 3.1\,(0.9) \cdot 10^{-11}$$

Buchalla et al

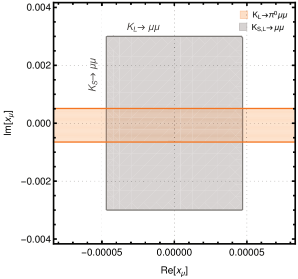
Kaon constraints on scalar leptoquarks

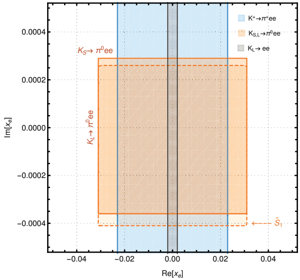


Mandal-Pich, 1908.11155

$S_1\,(\bar{3},1,1/3),\ \tilde{S}_1\,(\bar{3},1,4/3),\ R_2\,(\bar{3},2,7/6),\ \tilde{R}_2\,(\bar{3},2,1/6),\ S_3\,(\bar{3},3,1/3)$

$$\mathcal{L}_{\rm LQ} \doteq y^{ij}_{R_2}\ \bar{\ell}^i_R R_2^\dagger Q^j - y^{ij}_{\tilde{R}_2}\ \bar{d}^i_R \tilde{R}_2^T i\tau_2 L^j + y^{ij}_{\tilde{S}_1}\ \overline{d^c_R}^i \ell^j_R \tilde{S}_1 + y^{ij}_{S_3}\ \overline{Q^c}^i i\tau_2 \vec{\tau}\cdot\vec{S}_3 L^j + \text{h.c.}$$





$K_L \to e^\pm \mu^\mp$

$|a_{e\mu} + b_{e\mu}| < 1.9\cdot 10^{-5}$

$K_L \to \pi^0 e^\pm \mu^\mp$

$|a_{e\mu} - b_{e\mu}| < 2.9\cdot 10^{-4}$

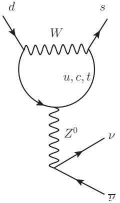
$K^+ \to \pi^+ e^\pm \mu^\mp$

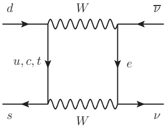
$|a_{e\mu}|, |b_{e\mu}| < 1.9\cdot 10^{-4}$

$$x_\mu = \left(\frac{1\ \text{TeV}}{M_{\rm LQ}}\right)^2 \times \begin{cases} y_{21}y^*_{22} & (R_2) \\ y_{12}y^*_{22} & (\tilde{R}_2, \tilde{S}_1, 4\times S_3) \end{cases} \quad , \quad x_e = \left(\frac{1\ \text{TeV}}{M_{\rm LQ}}\right)^2 \times \begin{cases} y_{11}y^*_{12} & (R_2) \\ y_{11}y^*_{21} & (\tilde{R}_2, \tilde{S}_1, 4\times S_3) \end{cases}$$

$$a_{e\mu} = \left(\frac{1\ \text{TeV}}{M_{\rm LQ}}\right)^2 \times y_{21}y^*_{12} \quad , \quad b_{e\mu} = \left(\frac{1\ \text{TeV}}{M_{\rm LQ}}\right)^2 \times \begin{cases} y_{22}y^*_{11} & (R_2) \\ y^*_{22}y_{11} & (\tilde{R}_2, \tilde{S}_1, 4\times S_3) \end{cases}$$

$K \to \pi\nu\bar{\nu}$





$$T \sim F\left(V_{is}^* V_{id}, \frac{m_i^2}{M_W^2}\right) \left(\bar{\nu}_L \gamma_\mu \nu_L\right) \langle\pi|\,\bar{s}_L \gamma^\mu d_L\,|K\rangle$$

Negligible long-distance contribution

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}) = (8.5 \pm 0.6) \cdot 10^{-11} \sim A^4 \left[\eta^2 + (1.4 - \rho)^2\right]$$ Buras et al

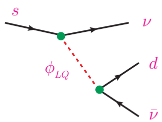
$$\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu}) = (2.9 \pm 0.3) \cdot 10^{-11} \sim A^4\,\eta^2$$ Brod et al

$\mathcal{A}(K_L \to \pi^0\nu\bar{\nu}) \neq 0$ ➡ **Direct** $\cancel{CP}$ (mixing–decay interference)

NA62 2019: $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}) < 1.85 \cdot 10^{-10}$ (90% CL)

KOTO 2015: $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu}) < 3.0 \cdot 10^{-9}$ (90% CL)

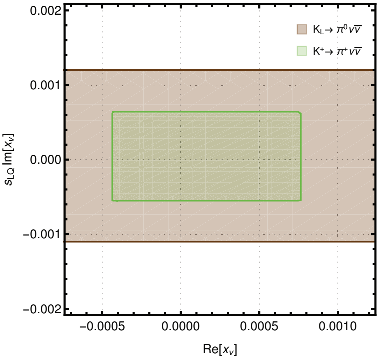
$K \to \pi\nu\bar{\nu}$ constraints on scalar leptoquarks



Mandal-Pich, 1908.11155

$S_1\,(\bar{3},1,1/3)$, $\tilde{S}_1\,(\bar{3},1,4/3)$, $R_2\,(\bar{3},2,7/6)$, $\tilde{R}_2\,(\bar{3},2,1/6)$, $S_3\,(\bar{3},3,1/3)$

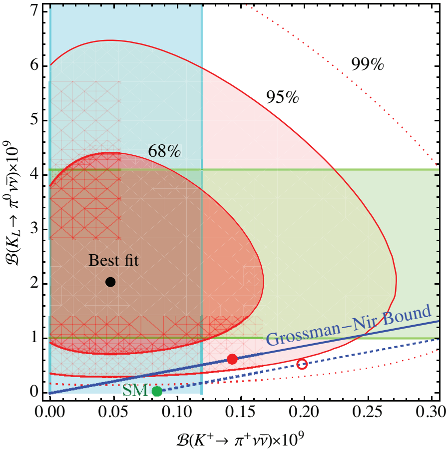
$$\mathcal{L}_{\rm LQ} \doteq -y^{ij}_{\tilde{R}_2}\;\bar{d}^i_R \tilde{R}_2^T i\tau_2 L^j + y^{ij}_{S_1}\;\overline{Q^c}^i i\tau_2 L^j S_1 + y^{ij}_{S_3}\;\overline{Q^c}^i i\tau_2 \vec{\tau}\cdot\vec{S}_3 L^j + \text{h.c.}$$



$$x_\nu = \left(\frac{1\ \text{TeV}}{M_{\rm LQ}}\right)^2 \times \left(y_{\rm LQ}\,U_{PMNS}\right)_{1\ell}\left(y_{\rm LQ}\,U_{PMNS}\right)^*_{2\ell}\qquad,\qquad s_{\rm LQ} = (-1)^{2\,I_{\rm LQ}}$$

KOTO 2019 (prel.): 4 events, exp. bkg. $= 0.05 \pm 0.02$, 1 evt suspected upstream bkg

$$\text{S.E.S.} = 6.9 \cdot 10^{-10} \quad \Rightarrow \quad \mathrm{Br}(K_L \to \pi^0 \nu \bar{\nu}) \sim 2 \cdot 10^{-9}$$



Above Grossman-Nir limit!

Kitahara et al. 1909.11111

Summary

Kaons are a wonderful laboratory to test the SM

- **Interesting interplay of short and long-distances**
- **Sensitive to heavy mass scales. New Physics?**
- **Superb probe of flavour dynamics and $\not{C}\not{P}$**
- **Excellent testing ground of χPT dynamics**

Increased sensitivities at ongoing experiments $(K \to \pi\nu\bar{\nu})$

Theoretical challenge: **precise control of QCD effects**

Successful SM prediction for ε'/ε

Cirigliano, Gisbert, Pich, Rodríguez-Sánchez

$$\boxed{\mathrm{Re}\,(\varepsilon'/\varepsilon)_{\mathrm{SM}} \;=\; \left(13\,{}^{+\,6}_{-\,7}\right)\cdot 10^{-4}}$$

$$\mathrm{Re}(\varepsilon'/\varepsilon)_{\mathrm{exp}} \;=\; \frac{1}{3}\left(1-\left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) \;=\; (16.6\pm 2.3)\cdot 10^{-4}$$

Large uncertainty but no anomalies!

Backup


18th Int. Conf. on B Physics at Frontier Machines, Beauty 2019
Ljubljana, Slovenia, 29 September – 4 October, 2019

Outlook: Needed Improvements

- **Wilson coefficients at NNLO** — Cerda et al
- **Updated value of Ω_{eff}** ✓ — Cirigliano et al
- **$g_8\, g_{\text{ew}}$ at NLO in $1/N_C$** — Rodríguez-Sánchez, A.P.
- **g_8 and higher-order LECs at NLO** — New ideas needed
- **χPT logarithms at NNLO** — Feasible
- **Improved lattice input** — Eagerly expected

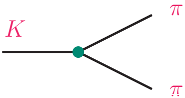
Difficult, but worth while enterprise

Best strategy: χPT (amplitudes) + Lattice (LECs)

$\mathbf{O(p^2)}$ χPT

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \langle \lambda L_\mu L^\mu \rangle \; + \; G_{27}\, F^4 \left(L_{\mu 23}\, L^\mu_{11} + \frac{2}{3}\, L_{\mu 21}\, L^\mu_{13} \right)$$

$$G_R \equiv -\frac{G_F}{\sqrt{2}}\, V_{ud} V^*_{us}\, g_R \quad ; \quad L_\mu = -iU^\dagger D_\mu U \quad ; \quad \lambda \equiv \tfrac{1}{2}\lambda_{6-i7} \quad ; \quad U \equiv \exp\left\{ i\sqrt{2}\,\Phi/F \right\}$$



$$\boxed{\begin{aligned} \mathcal{A}_{1/2} &= \sqrt{2}\, F_\pi \left(G_8 + \frac{1}{9}\, G_{27} \right) (M_K^2 - M_\pi^2) \\ \mathcal{A}_{3/2} &= \frac{10}{9}\, F_\pi\, G_{27}\, (M_K^2 - M_\pi^2) \\ \mathcal{A}_{5/2} &= 0 \quad ; \quad \delta_0 = \delta_2 = 0 \end{aligned}}$$

$$[\Gamma(K \to 2\pi) \, + \, \delta_I]_{\mathrm{Exp}} \quad \Longrightarrow \quad |g_8| \approx 5.0 \quad ; \quad |g_{27}| \approx 0.29$$

$\mathbf{O(p^2, e^2 p^0)}$ χPT

$\mathcal{Q} = \mathrm{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \langle \lambda L_\mu L^\mu \rangle + G_{27}\, F^4 \left(L_{\mu 23}\, L_{11}^\mu + \frac{2}{3}\, L_{\mu 21}\, L_{13}^\mu \right)$$

$$+ \; e^2\, F^6\, G_8\, g_{ew}\, \langle \lambda U^\dagger \mathcal{Q} U \rangle$$

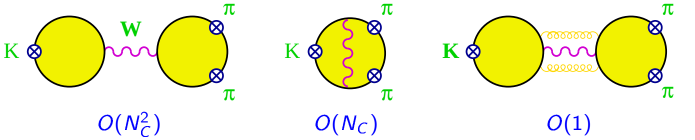
$$\mathcal{A}_{1/2} = \sqrt{2}\, F_\pi \left\{ G_8 \left[(M_K^2 - M_\pi^2) \left(1 - \frac{2}{3\sqrt{3}}\, \varepsilon^{(2)} \right) - \frac{2}{3}\, F_\pi^2\, e^2\, (g_{ew} + 2\, Z) \right] + \frac{1}{9}\, G_{27}\, (M_K^2 - M_\pi^2) \right\}$$

$$\mathcal{A}_{3/2} = \frac{2}{3}\, F_\pi \left\{ \left(\frac{5}{3}\, G_{27} + \frac{2}{\sqrt{3}}\, \varepsilon^{(2)}\, G_8 \right) (M_K^2 - M_\pi^2) - F_\pi^2\, e^2\, G_8\, (g_{ew} + 2\, Z) \right\}$$

$$\mathcal{A}_{5/2} = 0 \qquad ; \qquad \delta_0 = \delta_2 = 0$$

$\varepsilon^{(2)} = (\sqrt{3}/4)\, (m_d - m_u)/(m_s - \hat{m}) \approx 0.011$; $Z \approx (M_{\pi^\pm}^2 - M_{\pi^0}^2)/(2\, e^2 F_\pi^2) \approx 0.8$

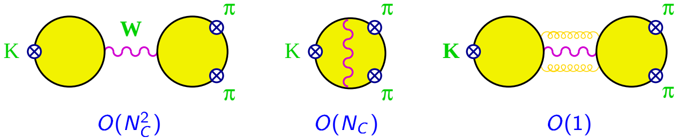
Weak Currents Factorize at Large N_C



$$A[K^0 \to \pi^0\pi^0] = 0 \quad \Longrightarrow \quad A_0 = \sqrt{2}\, A_2$$

No $\Delta I = \frac{1}{2}$ enhancement at leading order in $1/N_C$

Weak Currents Factorize at Large N_C



$$A[K^0 \to \pi^0\pi^0] = 0 \quad \longrightarrow \quad A_0 = \sqrt{2}\, A_2$$

No $\Delta I = \frac{1}{2}$ enhancement at leading order in $1/N_C$

- **Multiscale problem:** **OPE** $\quad \frac{1}{N_C} \log\left(\frac{M_W}{\mu}\right) \sim \frac{1}{3} \times 4$

Short-distance logarithms must be summed

- **Large χPT logarithms:** **FSI** $\quad \frac{1}{N_C} \log\left(\frac{\nu_\chi}{M_\pi}\right) \sim \frac{1}{3} \times 2$

Infrared logarithms must also be included $\quad [\delta_I \sim O(1/N_C)\, ,\ \delta_0 - \delta_2 \approx 45^\circ]$

Multi-Scale Problem: Summation of logarithms needed

A large $\log(M_1/M_2)$ compensates a $1/N_C$ suppression

❶ **Short-distance:** $\frac{1}{N_C}\,\log\left(M_W/\mu\right)$ Bardeen-Buras-Gerard

$$\longrightarrow \quad \begin{cases} g_8^\infty = 1.15\,{}^{+0.14}_{-0.17}{}_{\mu} \pm 0.04_{L_{5,8}} \pm 0.01_{m_s} \\ g_{27}^\infty = 0.46 \pm 0.02_{\mu} \end{cases}$$

Cirigliano et al, Pallante et al

❷ **Long-distance (χPT):** $\frac{1}{N_C}\,\log\left(\mu/m_\pi\right)$ Kambor et al, Pallante et al

$$g_8^{\rm LO} = 5.0 \quad \longrightarrow \quad g_8^{\rm NLO} = 3.6$$

$$g_{27}^{\rm LO} = 0.286 \quad \longrightarrow \quad g_{27}^{\rm NLO} = 0.288$$

Cirigliano et al

❸ **Isospin Violation:** $g_{27}^{\rm NLO} = 0.296$ Cirigliano et al

$N_C \to \infty$

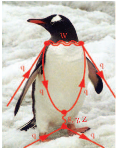
$$g_8 = \left(\frac{3}{5}\, C_2 - \frac{2}{5}\, C_1 + C_4\right) - 16\; L_5 \left(\frac{\langle \bar{q}\, q\rangle(\mu)}{F_\pi^3}\right)^2 C_6(\mu)$$

$$g_{27} = \frac{3}{5}\,(C_2 + C_1)$$

$$e^2\, g_8\, g_{ew} = -3 \left(\frac{\langle \bar{q}\, q\rangle(\mu)}{F_\pi^3}\right)^2 \left[C_8(\mu) + \frac{16}{9}\, C_6(\mu)\, e^2\, (K_9 - 2\, K_{10})\right]$$

$$\frac{\langle \bar{q}\, q\rangle(\mu)}{F_\pi^3} = \frac{M_{K^0}^2}{(m_s + m_d)(\mu)\, F_\pi} \left\{1 - \frac{8 M_{K^0}^2}{F_\pi^2}\,(2L_8 - L_5) + \frac{4 M_{\pi^0}^2}{F_\pi^2}\, L_5\right\}$$

- **Equivalent to standard calculations of B_i**
- **μ dependence only captured for $Q_{6,8}$**



Anomalous Dimension Matrix

$$\gamma_s^{(0)} = \begin{pmatrix}
-\frac{3}{N_c^2} & \frac{3}{N_c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{3}{N_c} & -\frac{3}{N_c^2} & -\frac{1}{3N_c^2} & \frac{1}{3N_c} & -\frac{1}{3N_c^2} & \frac{1}{3N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{11}{3N_c^2} & \frac{11}{3N_c} & -\frac{2}{3N_c^2} & \frac{2}{3N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{3}{N_c} - \frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} - \frac{3}{N_c^2} & -\frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{3}{N_c^2} & -\frac{3}{N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} & -\frac{n_f}{3N_c^2} & -3 + \frac{n_f}{3N_c} + \frac{3}{N_c^2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{N_c^2} & -\frac{3}{N_c} & 0 & 0 \\
0 & 0 & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & 0 & -3 + \frac{3}{N_c^2} & 0 & 0 \\
0 & 0 & \frac{1}{3N_c^2} & -\frac{1}{3N_c} & \frac{1}{3N_c^2} & -\frac{1}{3N_c} & 0 & 0 & -\frac{3}{N_c^2} & 0 \\
0 & 0 & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & 0 & 0 & 0 & -\frac{3}{N_c^2}
\end{pmatrix}$$

Only γ_{66} and γ_{88} survive the large–N_C limit

Isospin Breaking in ε'/ε

$$\boxed{\begin{aligned}\varepsilon' &\sim \omega_+ \left\{ \frac{\mathrm{Im}\, A_0^{(0)}}{\mathrm{Re}\, A_0^{(0)}} \left(1+\Delta_0+f_{5/2}\right) - \frac{\mathrm{Im}\, A_2}{\mathrm{Re}\, A_2^{(0)}} \right\} \\ &\sim \omega_+ \left\{ \frac{\mathrm{Im}\, A_0^{(0)}}{\mathrm{Re}\, A_0^{(0)}} \left(1-\Omega_{\mathrm{eff}}\right) - \frac{\mathrm{Im}\, A_2^{\mathrm{emp}}}{\mathrm{Re}\, A_2^{(0)}} \right\}\end{aligned}}$$

$$\omega \equiv \frac{\mathrm{Re}\, A_2}{\mathrm{Re}\, A_0} = \omega_+ \left(1+f_{5/2}\right) \quad ; \quad \omega_+ \equiv \frac{\mathrm{Re}\, A_2^+}{\mathrm{Re}\, A_0} \quad , \quad \Omega_{IB} = \frac{\mathrm{Re}\, A_0^{(0)}}{\mathrm{Re}\, A_2^{(0)}} \cdot \frac{\mathrm{Im}\, A_2^{\mathrm{non-emp}}}{\mathrm{Im}\, A_0^{(0)}}$$

Cirigliano, Gisbert, Pich, Rodríguez-Sánchez, 2019 (Cirigliano et al 2003)

$\times 10^{-2}$	$\alpha = 0$ LO	$\alpha = 0$ NLO	$\alpha \neq 0$ LO	$\alpha \neq 0$ NLO
Ω_{IB}	13.7	$17.1^{+8.4}_{-8.3}$	19.6 ± 4.8	26.0 ± 8.2
Δ_0	-0.002	-0.51 ± 0.12	5.6 ± 1.6	$5.7^{+1.7}_{-1.6}$
$f_{5/2}$	0	0	0	$8.2^{+2.4}_{-2.5}$
Ω_{eff}	13.7	$17.6^{+8.5}_{-8.4}$	14.0 ± 4.0	$12.1^{+9.0}_{-8.8}$

$$\boxed{\begin{aligned}\Omega_{\mathrm{eff}} &= 0.12 \pm 0.09 \\ &\equiv \Omega_{IB} - \Delta_0 - f_{5/2}\end{aligned}}$$

Phenomenological $K \to \pi\pi$ Fit

Cirigliano, Gisbert, Pich, Rodríguez-Sánchez, 2019

	LO-IC	LO-IB	NLO-IC	NLO-IB
Re g_8	4.99	5.00	3.60 ± 0.14	$3.58\,^{+0.15}_{-0.14}$
Re g_{27}	0.286	0.251	0.288 ± 0.014	$0.296\,^{+0.010}_{-0.003}$
$\chi_0 - \chi_2$	44.8°	48.0°	$(44.8 \pm 1.0)^\circ$	$(51.4 \pm 1.3)^\circ$

$$\text{IC} \equiv [m_u - m_d = \alpha = 0] \qquad ; \qquad \text{IB} \equiv [m_u - m_d \neq 0\,,\ \alpha \neq 0]$$

$\pi\pi \to \pi\pi$: $\quad \delta_0 - \delta_2 = (47.7 \pm 1.5)^\circ$ Colangelo–Gasser–Leutwyler '01

Modelling (some) non-factorizable $1/N_C$ corrections

Buras-Gérard, 1507.06326

$$B_6^{(1/2)} = 1 - \frac{3}{2}\left[\frac{F_\pi}{F_K - F_\pi}\right]\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_6^2}) = 1 - 0.66\ \ln(1 + \frac{\Lambda^2}{\tilde{m}_6^2})$$

$$B_8^{(1/2)} = 1 + \frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2}) = 1 + 0.08\ \ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2})$$

$$B_8^{(3/2)} = 1 - 2\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2}) = 1 - 0.17\ \ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2})$$

$\Rightarrow$ $B_6^{(1/2)} \leq B_8^{(3/2)} < 1$

Modelling (some) non-factorizable $1/N_C$ corrections

Buras-Gérard, 1507.06326

$$B_6^{(1/2)} = 1 - \frac{3}{2}\left[\frac{F_\pi}{F_K - F_\pi}\right]\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_6^2}) = 1 - 0.66\ \ln(1 + \frac{\Lambda^2}{\tilde{m}_6^2})$$

$$B_8^{(1/2)} = 1 + \frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2}) = 1 + 0.08\ \ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2})$$

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Not true in QCD

- FSI $(1/N_C)$ not included $\longrightarrow$ $\delta_I = 0$
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BBG Model

Bardeen-Buras-Gerard

$$\boxed{\mathcal{L}_{\rm eff} = \frac{f_\pi^2}{4}\left\{\langle D_\mu U^\dagger D^\mu U\rangle + r\,\langle m\,(U+U^\dagger)\rangle - \frac{r}{\Lambda_\chi^2}\,\langle m\,(D^2 U + D^2 U^\dagger)\rangle\right\}}$$

1. Equivalent to $\mathcal{O}(p^2)$ χPT + L_5 term $\qquad (L_i = 0\,,\ i \neq 5)$
   Most L_i are leading in N_C ➔ **$\mathcal{L}_{\rm eff}$ does not represent large-N_C QCD**

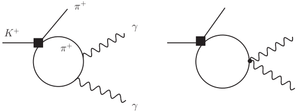
2. **Cut-off loop regularization: M $\sim$ (0.8 − 0.9) GeV**
   $f_\pi^2(M^2) = F_\pi^2 + 2\, I_2(m_\pi^2) + I_2(m_K^2)$, $\qquad I_2(m_i^2) = \frac{1}{16\pi^2}\left[M^2 - m_i^2\,\log\left(1 + \frac{M^2}{m_i^2}\right)\right]$

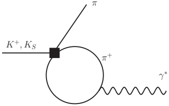
3. **Large-N_C factorization assumed to hold in the IR $(\mu = 0)$:** $\langle J \cdot J\rangle = \langle J\rangle\langle J\rangle$

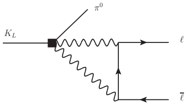
4. **M identified with SD renormalization scale μ: $C_i(\mu)$ running**
   Meson evolution ⟷ Quark evolution

5. **Vector meson loops included through Hidden U(3) Gauge Symmetry**
   Could partially account for $L_{1,2,3,9,10}$
   L_8 still missing ➔ $\langle \bar{q}q\rangle$, $Q_{6,8}$ **not quite correct even at large-N_C**

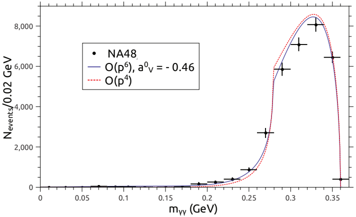
6. **$\pi\pi$ re-scattering completely missing ➔ $\delta_{0,2} = 0$, FSI absent**







$$\mathrm{Br}(K_L \to \pi^0\gamma\gamma) = (1.27 \pm 0.03)\cdot 10^{-6}$$



Finite 1-loop amplitude $[\mathcal{O}(p^4)]$**:**

$$\mathrm{Br}(K_L \to \pi^0\gamma\gamma)_{\mathrm{LO}} = 6.8 \cdot 10^{-7}$$

Ecker-Pich-de Rafael, Cappiello-D'Ambrosio, Sehgal

$\mathcal{O}(p^6)$ unitarity corrections needed

Cohen et al, Cappiello et al, D'Ambrosio-Portolés

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 2.8 \cdot 10^{-10}$$

$$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \cdot 10^{-10}$$

(90% CL), KTeV

3 contributions: Ecker-Pich-de Rafael

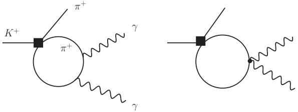
- **Direct** $\not{C}\!\!\not{P}$
- **Indirect** $\not{C}\!\!\not{P}$
- **CP conserving** (2γ)

$\not{C}\!\!\not{P}$ **dominates for** e^+e^-**:**

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) = 3.1\ (0.9)\cdot 10^{-11}$$

Buchalla et al

$K \to \pi\gamma\gamma$



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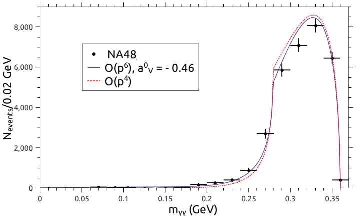

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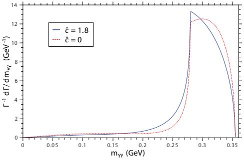
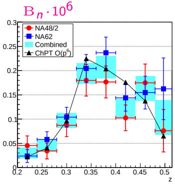
$$\mathrm{Br}(K^+ \to \pi^+\gamma\gamma) = 1.003\ (56) \cdot 10^{-6}$$

NA48/2–NA62





Local $\mathcal{O}(p^4)$ LEC:

Ecker-Pich-de Rafael

$$\hat{c} = \begin{cases} 1.72 \pm 0.21 & \mathcal{O}(p^4) \\ 1.86 \pm 0.25 & \mathcal{O}(p^6) \end{cases}$$

Small higher-order corrections

D'Ambrosio-Portolés