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An Exact
False Vacuum Decay Rate

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with Nemevšek

Based on ArXiv: 2009.01535

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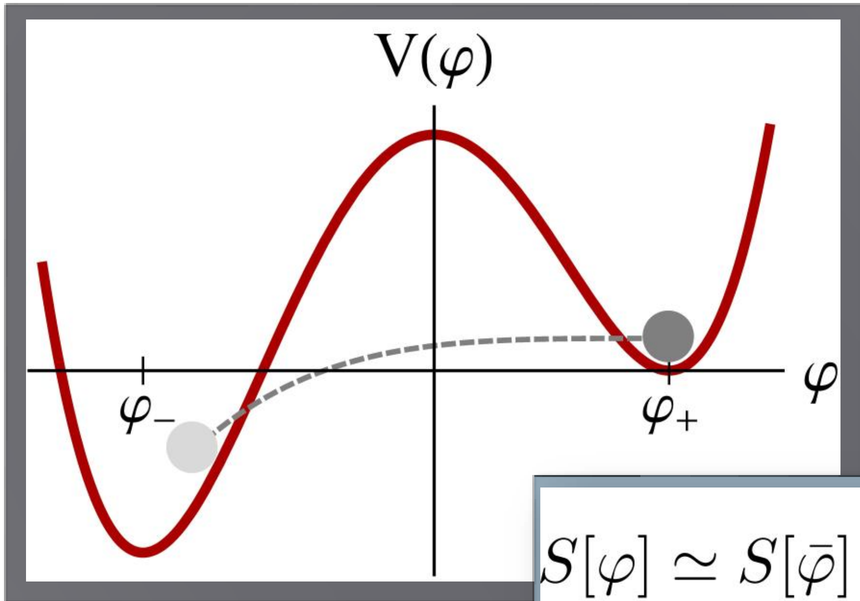
Decay Rate



a,b,c

$$\langle\varphi_+|e^{-TH}|\varphi_+\rangle \sim \int \mathcal{D}\varphi e^{-S[\varphi]}$$

$$\varphi = \bar{\varphi} + \psi$$

$$S[\varphi] \simeq S[\bar{\varphi}] + \frac{\delta S[\bar{\varphi}]}{\delta\varphi}\psi + \frac{1}{2}\psi\left(\frac{\delta^2 S[\bar{\varphi}]}{\delta\varphi^2}\right)\psi + \ldots$$

The bounce

$$\frac{\delta S[\varphi]}{\delta\varphi} = 0 \implies \bar{\varphi} \implies S[\bar{\varphi}] \equiv S_0$$

Fluctuations

$$\frac{\delta^2 S[\bar{\varphi}]}{\delta\varphi^2} \equiv \mathcal{O}$$

$$\prod_n \int \mathrm{d}c_n e^{-\frac{1}{2}\lambda_n c_n^2 2\pi} = \prod_n \sqrt{\frac{1}{\lambda_n}}$$

a. S. R. Coleman, Phys. Rev. D 15 (1977) 2929
b. C. G. Callan Jr. And S. R. Coleman, Phys. Rev. D 16 (1977)
c. A. Andreassen, D. Farhi, W. Frost and M.D. Schwartz, Phys. Rev. D 95 (2017) no.8, 085011

Decay Rate

$$\frac{\Gamma}{\mathcal{V}} = Ae^{-S_0}\left(1 + \mathcal{O}\left(\hbar\right)\right)$$

[a,b]

$$\rho = \sqrt{(\tau - \tau_0)^2 + (\mathbf{x} - \mathbf{x_0})^2}$$

The bounce action

[c,e]

$$S_0 = 2\pi^2 \int_0^\infty \rho^3 \mathrm{d}\rho \left(\frac{1}{2}\dot{\varphi}^2 + V\right)$$

Fluctuations

[d]

$$A = \left(\frac{S_0}{2\pi}\right)^2 \mathrm{Im}\sqrt{\frac{\det \mathcal{O}_{\mathrm{FV}}}{\det' \mathcal{O}}}$$

a. S. R. Coleman, V. Glaser and A. Martin, Commun.Math. Phys. 58 (1978) 211
b. K. Blum, M. Honda, R. Sato, M. Takimoto and K. Tobioka, JHEP 1705 (2017) 109
c. S. R. Coleman, Phys. Rev. D 15 (1977) 2929
d. C. G. Callan Jr. And S. R. Coleman, Phys. Rev. D 16 (1977)
e. V. Guada, M. Nemevšek and M. Pintar. Comput. Phys. Commun. 256 (2020,107480

The bounce solution

Potential

$$V = \frac{\lambda_s}{4}\left(\varphi - v_s\right)^4 + \tilde{V}_0$$

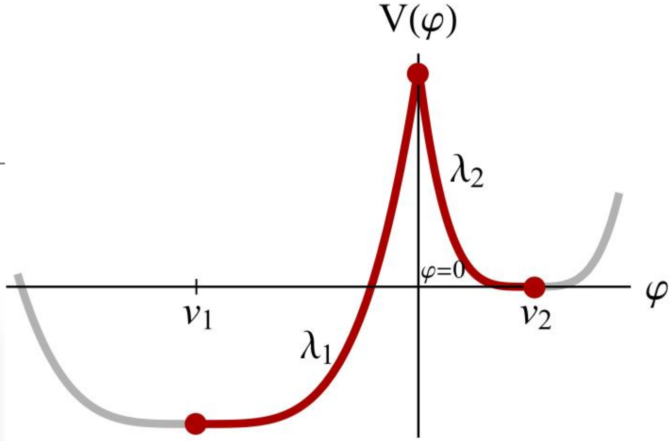
$$\frac{\delta S[\varphi]}{\delta\varphi} = 0 \implies \ddot{\varphi} + \frac{3}{\rho}\dot{\varphi} = V'$$

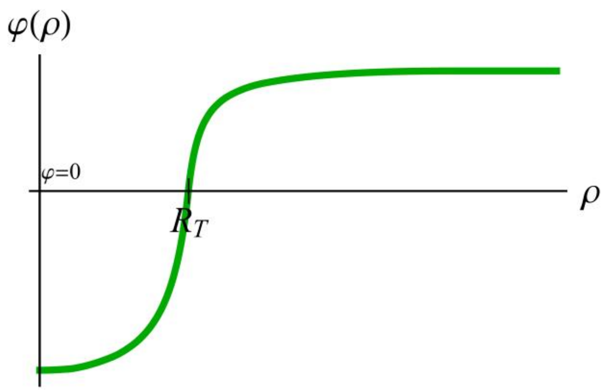
a

$$\bar{\varphi} = v_s + \sqrt{\frac{8}{\lambda_s}\frac{R_s}{R_s^2 - \rho^2}}$$

Bounce Action

$$S_0 = \left(\frac{8\pi^2}{3\lambda}\right) F\left(v_1/v_2\,, \lambda_1/\lambda_2\right)$$





a. K. Dutta, C. Hector, P. M. Vaudrevange and A. Westphal, Phys. Lett. B 708,309 (2012).

Functional determinant

$$A \propto \sqrt{\frac{\det \mathcal{O}}{\det \mathcal{O}'_{\mathrm{FV}}}}$$

4D Laplacian

$$\mathcal{O}\Psi_{\vec{n}} = \gamma_{\vec{n}}\Psi_{\vec{n}}$$

$$\Psi(\rho, \vec{\theta}) = \psi_l(\rho)\, Y_{lmm'}(\vec{\theta})$$

1D operator

$$\mathcal{O}_l \psi_{nl} = \gamma_n \psi_{nl}$$

Functional determinant

$$A \propto \sqrt{\frac{\det \mathcal{O}}{\det \mathcal{O}'_{\mathrm{FV}}}}$$

4D Laplacian

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1D operator

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Gelfand Yaglom Theorem [a]

$$\mathcal{O}_l\psi_l = -\ddot{\psi}_l - \frac{3}{\rho}\dot{\psi}_l + \frac{l\,(l+2)}{\rho^2}\psi_l + V''(\bar{\varphi})\,\psi_l = 0$$

[b]

$$\left(\frac{\det \mathcal{O}}{\det \mathcal{O}'_{\mathrm{FV}}}\right) = \prod_{l=0}^{\infty} R_l(\infty)^{(l+1)^2}$$

$$R_l \equiv \frac{\psi_l}{\psi_l^{\mathrm{FV}}}$$

a. I. Gelfand and A. Yaglom. J. Math. Phys. 1, 48 (1960)
b. H. Kleinert, EBL-schweitzer (World Scientific, 2009)

Functional determinant

$$\left(\frac{\det \mathcal{O}}{\det \mathcal{O}'_{\mathrm{FV}}}\right) = \prod_{l=0}^{\infty} R_l(\infty)^{(l+1)^2}$$

$$V''(\rho) = 3\lambda\bar{\varphi}(\rho)^2 \implies^{[a]} R_l(\infty) = \frac{l(l-1)}{(l+2)(l+3)}$$

$$V''(\rho) = \sum_s V''_s(\rho) H\left((-1)^s(\rho - R_T)\right) - \mu_V \delta(\rho - R_T)$$

$$R_l(\infty) = \frac{(l-1)\left(l^3 + c_2 l^2 + c_1 l + c_0\right)}{(l+1)(l+2)^2(l+3)}$$

[b] $l = 0$

$$\mathcal{R}_0(\infty) = -\frac{c_0}{12} < 0$$

a. A. Andtressen, W. Frost and M. D. Schwartz. Phys. Rev. D 97, 056006 (2018)
b. S. R. Coleman, Nucl. Phys. B 298 (1988)

Removing the zero modes

$l = 1$

$$\mathcal{R}_1^{\varepsilon}(\infty) = \frac{\psi_1^{\varepsilon}(\infty)}{\psi_{\mathrm{FV}1}(\infty)} \simeq \frac{\left(\mu_\varepsilon^2 + \gamma_1\right)\prod_{n=2}^{\infty}\gamma_n}{\prod_{n=1}^{\infty}\gamma_n^{\mathrm{FV}}} = \mu_\varepsilon^2 \mathcal{R}_1'(\infty)$$

$$\left(\mathcal{O}_1 + \mu_\varepsilon^2\right)\psi_1^{\varepsilon} = 0$$

$$\mathcal{R}_1'(\infty) = \lim_{\mu_\varepsilon^2 \to 0} \frac{1}{\mu_\varepsilon^2} \frac{\psi_1 + \mu_\varepsilon^2 \delta\psi_1}{\psi_1^{\mathrm{FV}}}\Bigg|_{\rho=\infty} = \frac{R_2^2}{24}\left(\frac{3\lambda_2}{8\pi^2}\right)\mathcal{S}_0\left(\frac{v_1}{v_2}\right)^6\left(\frac{\lambda_1}{\lambda_2}\right)^2$$

a. A. Andtressen, W. Frost and M. D. Schwartz. Phys. Rev. D 97, 056006 (2018)

b. S. R. Coleman, Nucl. Phys. B 298 (1988)

Finite sum

$$\left(\frac{\det \mathcal{O}}{\det \mathcal{O}'_{\mathrm{FV}}}\right) = \prod_{l=0}^{\infty} R_l(\infty)^{(l+1)^2}$$

$$\ln\left(\frac{\det \mathcal{O}_l}{\det \mathcal{O}_l^{\mathrm{FV}}}\right) = \sum_l (l+1)^2 \ln \mathcal{R}_l(\infty)$$

$$\mathcal{R}_l(\infty) = \frac{(l-1)(l^3 + c_2 l^2 + c_1 l + c_0)}{(l+1)(l+2)^2(l+3)}$$

$$\nu = l + 1$$

$$\ln\left(\frac{\nu - a}{\nu - b}\right) = \frac{b-a}{\nu} + \ldots + \mathcal{O}\left(\frac{1}{\nu^3}\right)$$

$$\Sigma_f = \sum_{\nu=1}^{\infty} \nu^2 \left(\ln \mathcal{R}_l(\infty) - \ln \mathcal{R}_l^a(\infty)\right)$$

Zeta function regularization

$$\ln \det \mathcal{O} = \sum_n \ln \gamma_n = -\frac{\mathrm{d}}{\mathrm{d}s} \sum_n \left(\frac{\mu^2}{\gamma_n}\right)^s \Bigg|_{s=0} = -\frac{\mathrm{d}}{\mathrm{d}s} \left(\mu^{2s} \zeta_{\mathcal{O}}(s)\right) \Bigg|_{s=0}$$

$$\mathrm{Re}(s) > D/2$$

Zeta function

$$\ln \left(\frac{\det \mathcal{O}}{\det \mathcal{O}_{\mathrm{FV}}}\right) = -\frac{\mathrm{d}}{\mathrm{d}s} \zeta(s)|_{s=0}$$

$$\zeta = \zeta_f + \zeta_a$$

a. S. W. Hawking, Commum. Math. Phys. 55 (1977), 133
b. K. Kirsten, Chapman & Hall/CRC Press, Boca Raton, FL, 2002, arXiv:hep-th/0005133
c. G. V. Dunne and K. Kirsten, J. Phys. A 39 (2006), 11915-11928
d. G. V. Dunne and H. Min, Phys. Rev. D 72 (2005), 125004.

Zeta function regularization

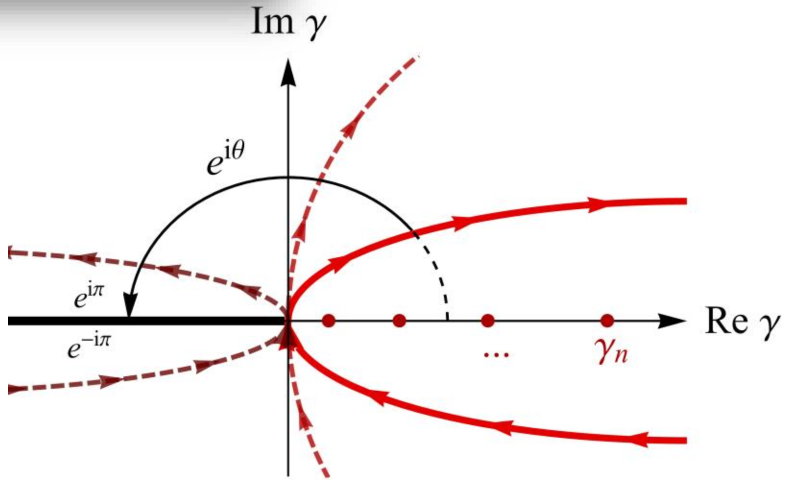
$$\zeta = \frac{\sin \pi s}{\pi} \mu^{2s} \sum_{\nu} \nu^2 \int_0^\infty \frac{\mathrm{d}\gamma}{\gamma^s} \frac{\mathrm{d}}{\mathrm{d}\gamma} \ln f_l(\gamma)$$

$\mathrm{Re}(s) > D/2$

$$\psi_l(\rho, -\gamma) = \psi_l^{FV}(\rho, -\gamma) + \int_0^\rho \mathrm{d}\rho_1 G(\rho, \rho_1) V''(\rho_1) \psi_l(\rho_1, -\gamma)$$

$$\ln f_l(\gamma) = \ln \frac{\psi_l(\infty, -\gamma)}{\psi_l^{\mathrm{FV}}(\infty, -\gamma)} \simeq \frac{1}{\nu} \ldots + \ldots + \mathcal{O}\left(\frac{1}{\nu}\right)^3$$

$$\mathcal{O}_l \psi_l(\rho, \gamma) = \gamma\, \psi_l(\rho, \gamma)$$



Zeta function renormalization

$$\zeta = \frac{\sin \pi s}{\pi} \mu^{2s} \sum_{\nu} \nu^2 \int_0^\infty \frac{d\gamma}{\gamma^s} \frac{d}{d\gamma} \ln f_l(\gamma)$$

$$\zeta = \zeta_f + \zeta_a$$

$\mathrm{Re}(s) > D/2$

$$\zeta_f = \frac{\sin \pi s}{\pi} \sum_{\nu} \nu^2 \mu^{2s} \int_0^\infty \frac{d\gamma}{\gamma^s} \frac{d}{d\gamma} \left(\ln f_l(\gamma) - \ln f_l^a(\gamma)\right)$$

$$\zeta_f'(0) = \sum_{\nu} \nu^2 \left(\ln \mathcal{R}_l(\infty) - \ln f_l^a(0)\right) = \Sigma_f$$

$$\zeta_a = \sum_{\nu=1}^{\infty} \nu^{2-2s} \left(\frac{1}{\nu} \ldots + \ldots + \mathcal{O}\left(\frac{1}{\nu}\right)^3\right)$$

$$\zeta_R(2s - 2 + 1) \qquad \zeta_R(-1) = -1/12$$

Pre-factor A

Summary

The bounce action

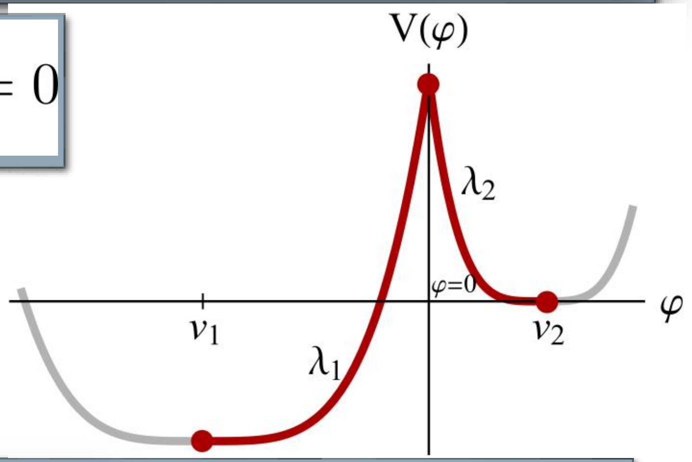
$$\frac{\Gamma}{\mathcal{V}} = \left(\frac{\mathcal{S}_0^2}{4\pi^2} e^{\frac{1}{2}\zeta'(0)} \right) e^{-\mathcal{S}_0} = v_2^4 e^{-\mathcal{S}_0 - \mathcal{S}_1}$$

Fluctuations

$$V = \frac{1}{4}\left(\lambda_2 v_2^4 - \lambda_1 v_1^4 + \lambda_1 (\varphi + v_1)^4\right) H(-\varphi) + \frac{\lambda_2}{4}(\varphi - v_2)^4 H(\varphi)$$

$$\mathcal{O}_l \psi_l = -\ddot{\psi}_l - \frac{3}{\rho}\dot{\psi}_l + \frac{l(l+2)}{\rho^2}\psi_l + V''(\bar{\varphi})\psi_l = 0$$

$$R_l(\infty) = \frac{(l-1)\left(l^3 + c_2 l^2 + c_1 l + c_0\right)}{(l+1)(l+2)^2(l+3)}$$



$$\zeta'_f(0) = \sum_{\nu} \nu^2 \left(\ln \mathcal{R}_l(\infty) - \ln f_l^a(0)\right) = \Sigma_f$$

$$\zeta = \zeta_f + \zeta_a$$

$$\zeta'_a(0) = \sum_s \frac{1}{8}\int_0^\infty d\rho\, \rho^3 V_s''^2 \left(\ln\left(\frac{\mu\rho}{2}\right) + \gamma_E + 1\right) H\left((-1)^s(\rho - R_T)\right)$$
$$- \frac{(\mu_V R_T)^2}{16} + \frac{(\mu_V R_T)^3}{24}\left(1 - \frac{3}{\mu_V^2}(V_1'' + V_2'')\big|_{R_T}\right)\left(\ln\left(\frac{\mu R_T}{2}\right) + \gamma_E + 1\right)$$

Summary

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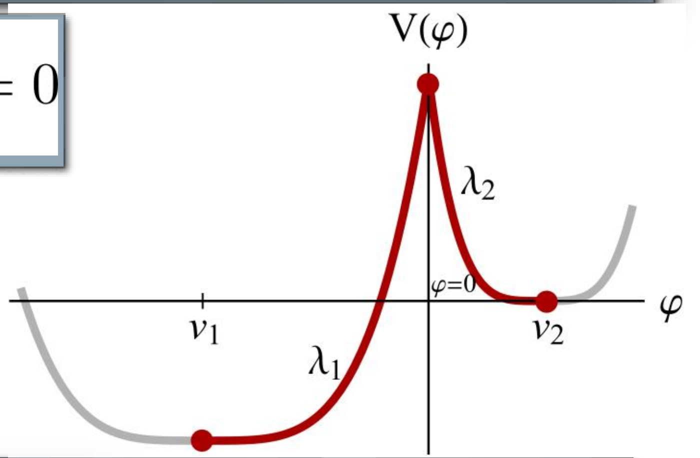

$$R_l(\infty) = \frac{(l-1)\left(l^3 + c_2 l^2 + c_1 l + c_0\right)}{(l+1)(l+2)^2(l+3)}$$

$$\zeta = \zeta_f + \zeta_a$$

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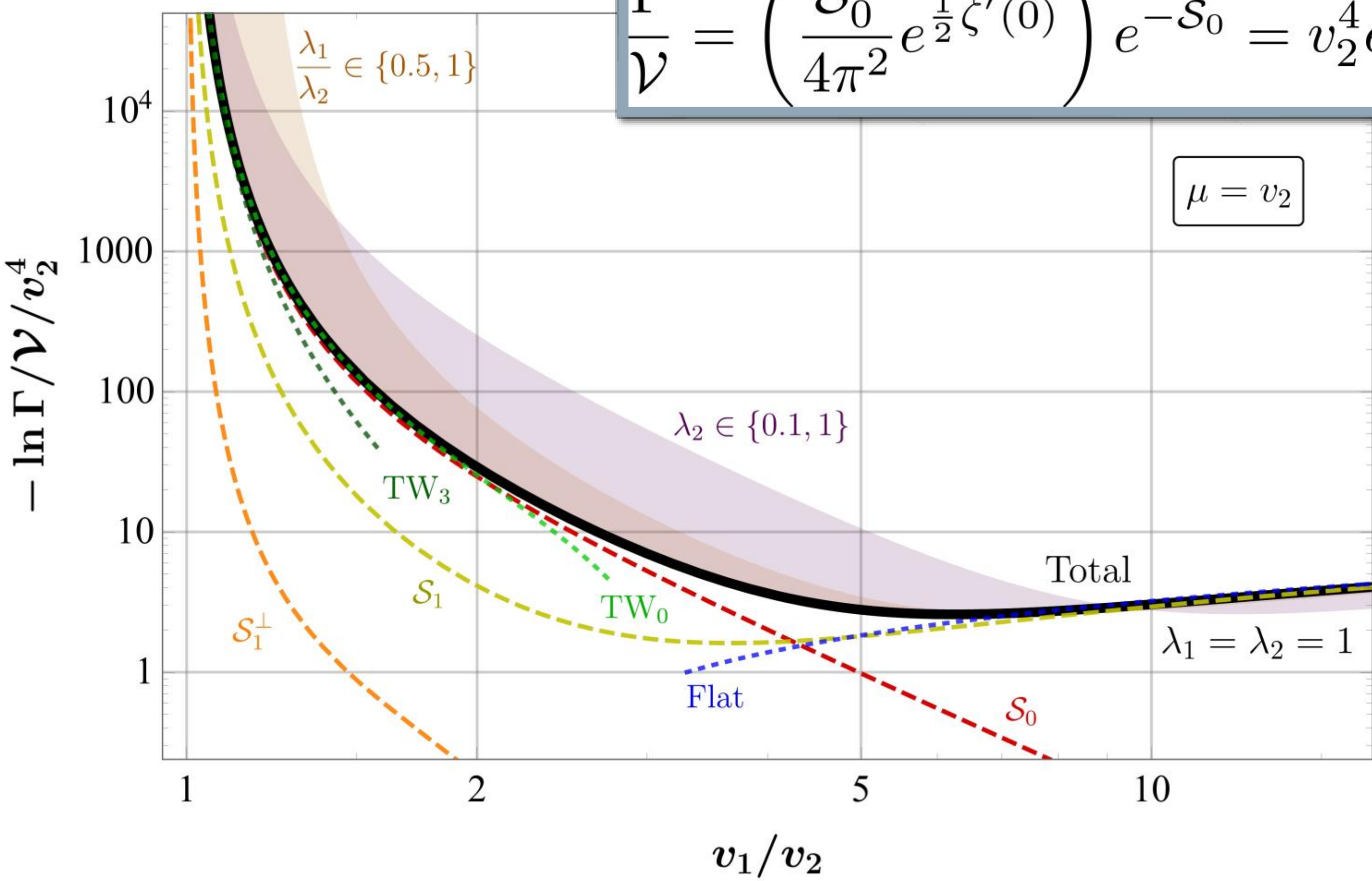
$$-\frac{(\mu_V R_T)^2}{16} + \frac{(\mu_V R_T)^3}{24}\left(1 - \frac{3}{\mu_V^2}(V_1'' + V_2'')\big|_{R_T}\right)\left(\ln\left(\frac{\mu R_T}{2}\right) + \gamma_E + 1\right)$$

Decay rate

$$\frac{\Gamma}{\mathcal{V}} = \left(\frac{\mathcal{S}_0^2}{4\pi^2} e^{\frac{1}{2}\zeta'(0)}\right) e^{-\mathcal{S}_0} = v_2^4 e^{-\mathcal{S}_0-\mathcal{S}_1}$$



$$-\ln \frac{\Gamma}{\mathcal{V}} \frac{1}{v_2^4} \simeq \begin{cases} \frac{1}{\varepsilon^3}\left(\frac{2\pi^2}{3\lambda_2} + \frac{2}{9} + \frac{\pi}{2\sqrt{3}} - \frac{1}{12}\ln\frac{2\lambda_2 v_2^2}{\mu^2}\right), & \mathrm{TW} \\ \frac{7}{12} - 2\zeta_R'(-1) + \frac{1}{3}\ln\frac{\lambda_1^2 v_1^6}{32\pi^3\mu^4 v_2^2}, & \mathrm{Flat} \end{cases}$$

Thank you!