Z_b tetraquark channel with lattice QCD

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Contents

Quantum chromodynamics (QCD)

- fundamental quantum field theory of quarks (q) and gluons (g)
- the Lagrangian:

$$\mathscr{L} = \sum_q \overline{\psi}_{q,a} \left(i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} A^C_\mu - m_q \delta_{ab} \right) \psi_{q,b} - \frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu}$$

ψ — quark field

A_μ — gluon gauge field

- ab-initio predictive methods for QCD:
  - quantum chromodynamics on the lattice (lattice QCD)
  - perturbative expansions in the coupling

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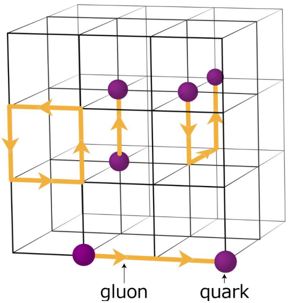
Lattice QCD

- The theory is discretised onto a four-dimensional spacetime lattice
  - quark fields - lattice sites
  - gauge fields - links between sites
- lattice spacing a
- quantities extracted from Euclidean (imaginary-time) correlation functions
- Monte-Carlo methods



Hadrons

mesons - baryon number
$B = 0$

$q\bar{q}$ states

baryons - baryon number
$B = 1$

qqq states

exotic mesons:

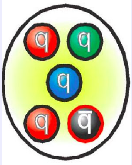
$qq\overline{qq}$ - tetraquarks

states made of bound gluons - glueballs

$q\bar{q}$-pairs with an excited gluon - hybrids

exotic baryons:

$q\bar{q}qqq$ - pentaquarks



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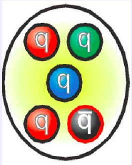
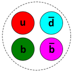
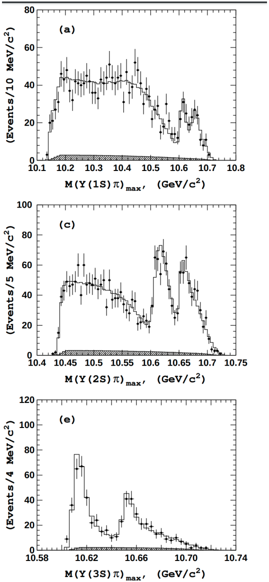


Experimental discovery of the Z_b tetraquarks

- the Belle collaboration first observed two charged bottomonium-like resonances in $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-, h_b(mP)\pi^+\pi^-,$ $(n = 1, 2, 3;\ m = 1, 2)$

name	valence quark content	m [MeV]	$I^G(J^{PC})$	discovery year	reference
$Z_b(10610)$		10607.2	$1^+(1^{+-})$	2011	[A. Bondar *et al.* (Belle), Phys. Rev. Lett. 108, 122001 (2012).] and
$Z_b(10650)$		10652.2	$1^+(1^{+-})$	2011	[A. Garmash *et al.* (Belle), Phys. Rev. D91, 072003 (2015).] for both

- $Z_b(10610)$ and $Z_b(10650)$ decay also to $B\overline{B}^*$ and $B^*\overline{B}^*$ respectively
  - these are the dominant decay channels



Experimental discovery of the Z_b tetraquarks

- properties incompatible with a $q\overline{q}$ structure
- masses few MeV above the thresholds for the open beauty channel $B\overline{B}^*$ and $B^*\overline{B}^*$
- this suggests a "molecular" nature of these states, which might explain most of their observed properties

Lattice QCD - scattering analysis

- Lüscher scattering formalism:
  - eigen-energies of a correlation function
    $\Rightarrow$ two hadron scattering amplitudes
    $\Rightarrow$ mass and decay width
- rigorously treating Z_b tetraquark with lattice QCD:
  - scattering matrix for at least 7 coupled channels
  - a severe challenge

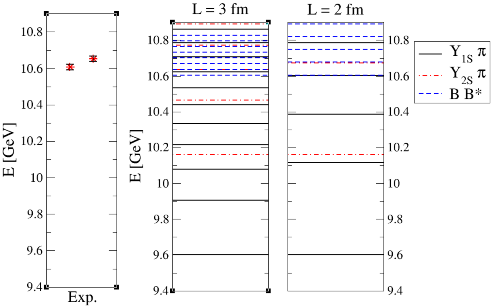
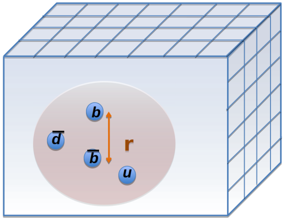


Figure: non-interacting energies $E(L) = \sum_{i=1,2} \sqrt{m_i^2 + p_i^2} + \Delta E$, $\vec{p}_i = \frac{2\pi}{L}\vec{n}_i$

Lattice QCD - static quark limit → Born-Oppenheimer approximation

- b and $\overline{b}$ infinitely heavy ($m_b \to \infty$) and static
  - on a distance r
  - their spins – conserved quantities
- compute the potential $V(r)$ of the static quarks in the presence of the light quarks, $\Rightarrow$ potential of B and $\overline{B}$



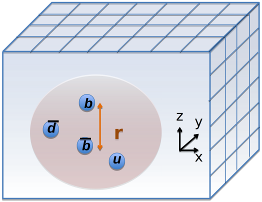
Born-Oppenheimer approximation:

- solve the Schrödinger equation with the computed potential $V(r)$

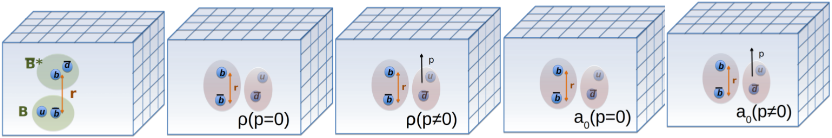
Symmetries of the static limit

- $j_{z,\mathrm{light}}$ = z-component of the angular momentum of the light quarks
- $C \circ P$ = combined parity and charge conjugation
- $\epsilon = P_x$ = x-parity = reflection along an axis orthogonal to the separation axis e. g. x-axis
- my advisor has considered the case $S_{\mathrm{heavy}} = 1,\ j_{z,\mathrm{light}} = 0,\ CP = -1,\ \epsilon = -1$

  [S. Prelovšek, H. Bahtiyar and J. Petković (2019), [arXiv:1912.02656].]



My work on Z_b



shematic presentation of operators for a specific set of quantum numbers

$$(S_{\text{heavy}} = 0,\ j_{z,\text{light}} = 0,\ CP = +1,\ \varepsilon = +1):$$

1. $O_1 = O^{B\bar{B}^*} \propto (P_-\gamma_3)_{AB}(P_-\gamma_5)_{CD}\ (\bar{b}_C q_B)\ (\bar{q}_A b_D)$
2. $O_2 = O^{B\bar{B}^{*\prime}} \propto (P_-\gamma_3)_{AB}(P_-\gamma_5)_{CD}\ (\bar{b}_C \nabla^2 q_B)\ (\nabla^2 \bar{q}_A b_D)$
3. $O_3 = O^{\eta_b \rho(0)} \propto [\bar{b} P_- \gamma_5 b]\ [\bar{q}\gamma_3 q]_{\vec{p}=\vec{0}}$
4. $O_4 = O^{\eta_b \rho(1)} \propto [\bar{b} P_- \gamma_5 b]\ \left([\bar{q}\gamma_3 q]_{\vec{p}=\hat{e}_z} + [\bar{q}\gamma_3 q]_{\vec{p}=-\hat{e}_z}\right)$
5. $O_5 = O^{\eta_b \rho(0)} \propto [\bar{b} P_- \gamma_5 b]\ [\bar{q}\gamma_4\gamma_3 q]_{\vec{p}=\vec{0}}$
6. $O_6 = O^{\eta_b \rho(1)} \propto [\bar{b} P_- \gamma_5 b]\ \left([\bar{q}\gamma_4\gamma_3 q]_{\vec{p}=\hat{e}_z} + [\bar{q}\gamma_4\gamma_3 q]_{\vec{p}=-\hat{e}_z}\right)$
7. $O_7 = O^{h_b a_0(0)} \propto [\bar{b} P_- \gamma_5 b]\ [\bar{q}\mathbb{I} q]_{\vec{p}=\vec{0}}$
8. $O_8 = O^{h_b a_0(1)} \propto [\bar{b} P_- \gamma_5 b]\ \left([\bar{q}\mathbb{I} q]_{\vec{p}=\hat{e}_z} + [\bar{q}\mathbb{I} q]_{\vec{p}=-\hat{e}_z}\right)$

Generalized eigenvalue problem (GEVP)

- two-point correlation functions

$$C_{ij}(t) = \langle 0|O_i(t)O_j^\dagger(0)|0\rangle = \sum_n \langle 0|O_i|n\rangle\langle n|O_j^\dagger|0\rangle e^{-E_n t}$$

- overlaps:

$$Z_i^{(n)} = \langle 0|\hat{O}_i|n\rangle, \qquad Z_j^{(n)*} = \langle n|\hat{O}_j^\dagger|0\rangle$$

- eigen-energies E_n extracted using the GEVP approach

$$\sum_{j=1}^{N} C_{ij}(t)v_j^{(n)}(t,t_0) = \sum_{j=1}^{N} \lambda^{(n)}(t,t_0)C_{ij}(t_0)v_j^{(n)}(t,t_0),$$

where $n = 1,\ldots,N$ and $t > t_0$

$$\lambda^{(n)}(t,t_0) = e^{-E_n(t-t_0)} = Ae^{-E_n t}$$

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Preliminary results

- effective energies:

$$E_{\text{eff}}^{(n)}(t,t_0) = -\frac{\partial \log\left(\lambda^{(n)}(t,t_0)\right)}{\partial t}, \qquad E_n = \lim_{t\to\infty} E_{\text{eff}}^{(n)}(t,t_0)$$

example for $r=1,\ t_0=2$ and these 5 interpolating operators:

1. $O_1 = O^{B\bar{B}^*} \propto (P_-\gamma_3)_{AB}(P_-\gamma_5)_{CD}\ \left(\bar{b}_C q_B\right)\ \left(\bar{q}_A b_D\right)$
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5. $O_5 = O^{\eta_b\rho(2)} \propto \left[\bar{b}P_-\gamma_5 b\right]\left(\left[\bar{q}\gamma_3 q\right]_{\vec{p}=2\hat{e}_z} + \left[\bar{q}\gamma_3 q\right]_{\vec{p}=-2\hat{e}_z}\right)$

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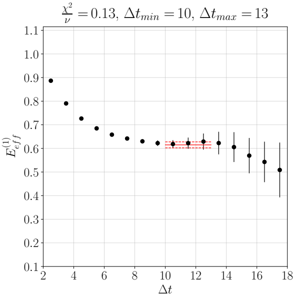
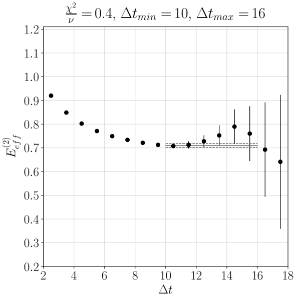
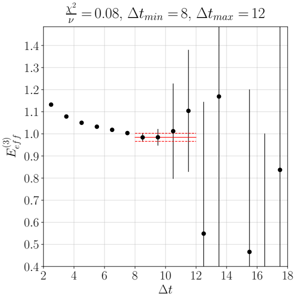
example for $S_{\text{heavy}} = 0,\ j_{z,\text{light}} = 0,\ CP = +1,\ \epsilon = +1,\ r = 1,\ t_0 = 2$:



$n = 1$



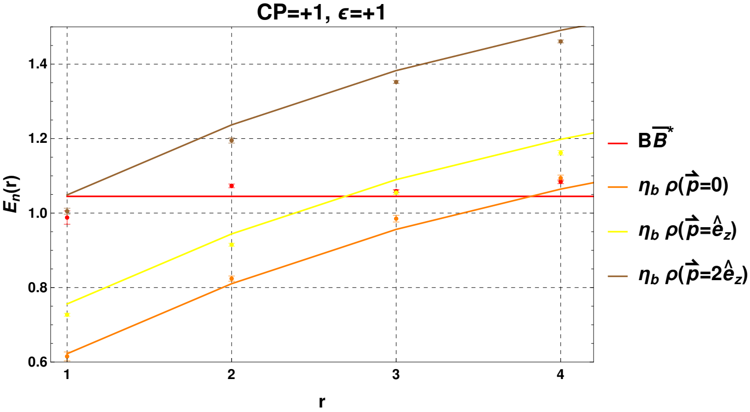
$n = 2$

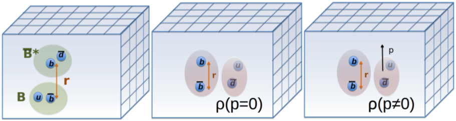


$n = 3$

Preliminary results

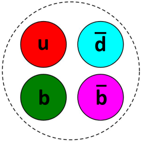
comparing eigen energies with non-interacting energies





Conclusion

- there are many phenomenological studies on spectroscopy of exotic hadrons with heavy quarks
- but only one group (except of my supervisor) made studies based on the fundamental theory – lattice QCD – considering the Z_b tetraquark
- this tetraquark is due to experimental discovery of great interest



- looks like no attraction in my channel

Thank you for your attention.