Gravitational waves from first order phase transitions

Marek Lewicki

Kings College London & University of Warsaw

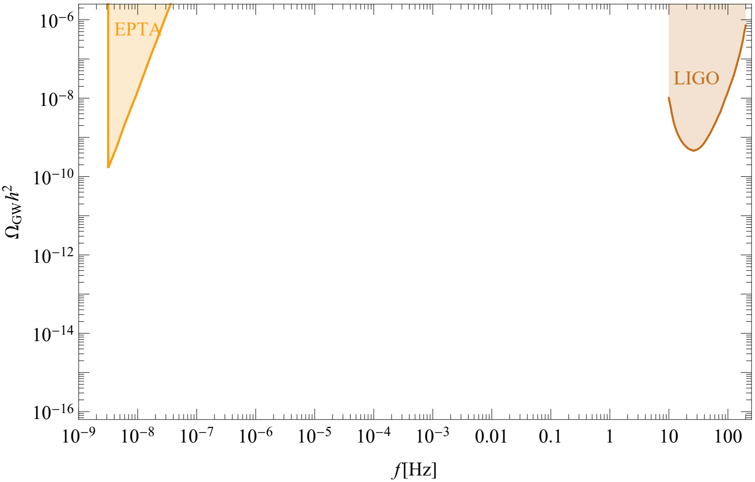
2 VII 2020

Based on:
J. Ellis, ML, J. M. No arXiv:1809.08242, 2003.07360
J. Ellis, ML, J. M. No, V. Vaskonen arXiv:1903.09642
ML, V. Vaskonen arXiv: 1912.00997
J. Ellis, M. Fairbairn, ML, J. M. No, V. Vaskonen, A Wickens 1907.04315, 2005.05278

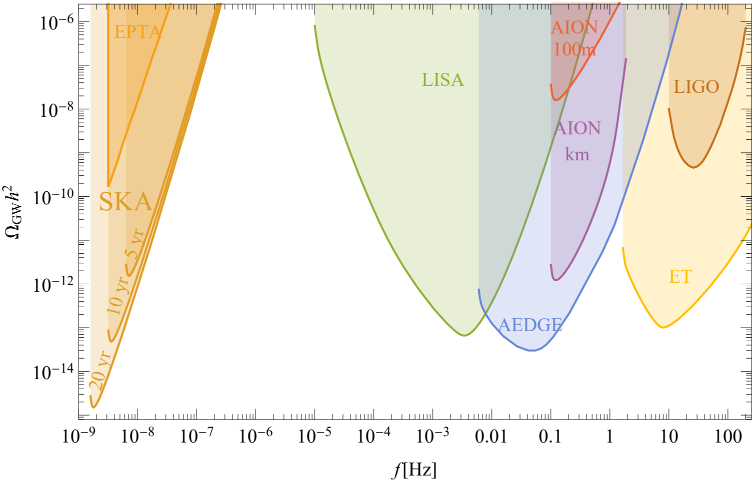
Plan

- Experimental prospects
- Introduction to first order phase transitions
- Energy stored in the bubble walls
- Lifetime of the sound wave source
- Polarised GW signals
- Conclusions

Experimental outlook



Experimental outlook



Phase Transitions

Bubble nucleation

Bubble: static field configuration passing the barrier (excited through thermal fluctuations)

- decay rate

$$\Gamma(T) \approx T^4 \exp\left(-\frac{S_3(T)}{T}\right),$$

- $\mathcal{O}(3)$ symmetric action

$$S_3(T) = 4\pi \int dr r^2 \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi, T)\right].$$

- EOM → bubble profile

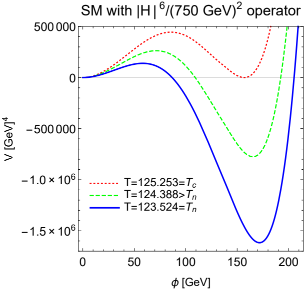
$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{\partial V(\phi,T)}{\partial \phi} = 0,$$

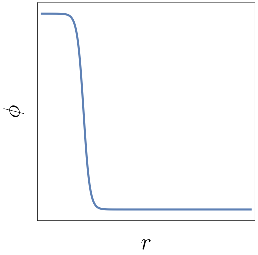
$$\phi(r \to \infty) = 0 \quad \text{and} \quad \dot{\phi}(r=0) = 0.$$

- nucleation temperature

$$N(T_n) = \int_{t_c}^{t_n} dt \frac{\Gamma(t)}{H(t)^3} = \int_{T_n}^{T_c} \frac{dT}{T}\frac{\Gamma(T)}{H(T)^4} = 1$$

Linde '81 '83





Gravitational waves from a PT

- Strength of the transition

$$\alpha \approx \left.\frac{\Delta V}{\rho_R}\right|_{T=T_*}, \quad \Delta V = V_f - V_t$$

- Characteristic scale

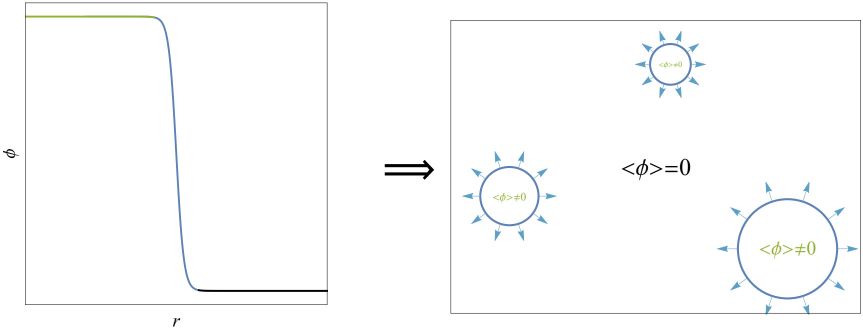
$$HR_* = (8\pi)^{\frac{1}{3}} \left(\frac{\beta}{H}\right)^{-1}$$

- Signals are produced by three main mechanisms:
  - collisions of bubble walls: $\Omega_{\rm col} \propto \left(\kappa_{\rm col} \frac{\alpha}{\alpha+1}\right)^2 (HR_*)^2$

    Kamionkowski '93, Huber '08, Hindmarsh '18,
  - sound waves: $\Omega_{\rm sw} \propto \left(\kappa_{\rm sw} \frac{\alpha}{\alpha+1}\right)^2 (HR_*)(H\tau_{sw})$

    Hindmarsh '13 '15 '17, Ellis '18 '19 '20
  - turbulence $\Omega_{\rm turb} \propto \left(\kappa_{\rm sw} \frac{\alpha}{\alpha+1}\right)^{\frac{3}{2}} (HR_*)(1 - H\tau_{sw})$

    Caprini '09, Ellis '19 '20
- Sound wave period lasts $H\tau_{\rm sw} \equiv \min\left[1, \frac{HR_*}{U_f}\right]$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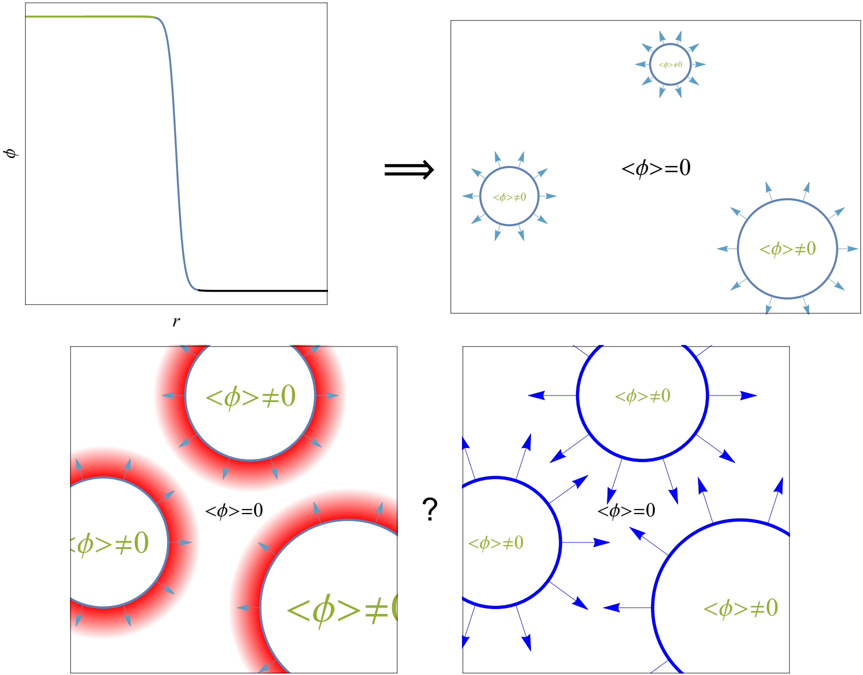

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- Energy of the bubble

$$\mathcal{E} = 4\pi R^2 \sigma \gamma - \frac{4\pi}{3} R^3 p\,, \qquad \gamma = \frac{1}{\sqrt{1-\dot{R}^2}}$$

- Vacuum pressure on the wall
  Coleman '73

$$p_0 = \Delta V$$

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- Leading order plasma contribution
  Bodeker '09 Caprini '09

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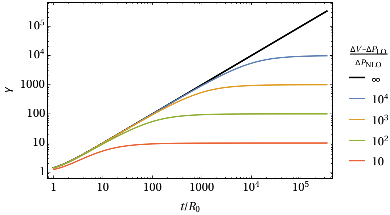
- Next-To-Leading order plasma contribution
  Bodeker '17

$$p = \Delta V - \Delta P_{\rm LO} - \gamma \Delta P_{\rm NLO} \approx \Delta V - \frac{\Delta m^2 T^2}{24} - \gamma\, g^2 \Delta m_V T^3 \,.$$



- terminal velocity γ factor and the value in absence of friction

$$\gamma_{\rm eq} \equiv \frac{\Delta V - \Delta P_{\rm LO}}{\Delta P_{\rm NLO}}\,, \qquad \gamma_* \equiv \frac{2}{3}\frac{R_*}{R_0}\,,$$

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- Finally the efficiency factors read

$$\kappa_{\rm col} = \frac{E_{\rm wall}}{E_V} = \begin{cases} \frac{\gamma_{\rm eq}}{\gamma_*}\left[1 - \frac{\Delta P_{\rm LO}}{\Delta V}\left(\frac{\gamma_{\rm eq}}{\gamma_*}\right)^2\right]\,, & \gamma_* > \gamma_{\rm eq} \\ 1 - \frac{\Delta P_{\rm LO}}{\Delta V}\,, & \gamma_* \leq \gamma_{\rm eq}\,, \end{cases}$$

$$\kappa_{\rm sw} = \frac{\alpha_{\rm eff}}{\alpha}\frac{\alpha_{\rm eff}}{0.73 + 0.083\sqrt{\alpha_{\rm eff}} + \alpha_{\rm eff}}\,, \quad \text{with} \quad \alpha_{\rm eff} = \alpha(1 - \kappa_{\rm col})\,.$$

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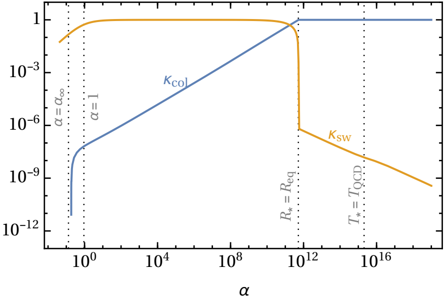
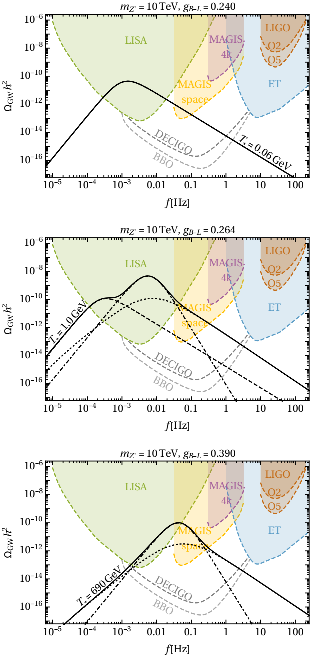
- $U(1)_{B-L}$ Example: $V(\varphi, T) \simeq \frac{3g_{\rm B-L}^4\varphi^4}{4\pi^2}\left[\log\left(\frac{\varphi^2}{v_\varphi^2}\right) - \frac{1}{2}\right] + g_{\rm B-L}^2 T^2\varphi^2$





$m_{Z'} = 10\,\text{TeV}$, $g_{B-L} = 0.240$
$m_{Z'} = 100\,\text{TeV}$, $g_{B-L} = 0.25$
$m_{Z'} = 10\,\text{TeV}$, $g_{B-L} = 0.264$
$m_{Z'} = 100\,\text{TeV}$, $g_{B-L} = 0.29$
$m_{Z'} = 10\,\text{TeV}$, $g_{B-L} = 0.390$
$m_{Z'} = 100\,\text{TeV}$, $g_{B-L} = 0.34$
$\Omega_{\rm GW} h^2$
f[Hz]
LISA
MAGIS space
MAGIS 4k
LIGO
O2
O5
ET
DECIGO
BBO
$T_* = 0.06\,\text{GeV}$
$T_* = 0.14\,\text{GeV}$
$T_* = 1.0\,\text{GeV}$
$T_* = 96\,\text{GeV}$
$T_* = 690\,\text{GeV}$
$T_* = 1.7\,\text{TeV}$

Plasma related GW sources

- Sound wave spectrum reduction and earlier onset of turbulence

$$\Omega_{\mathrm{sw}} \propto H\tau_{\mathrm{sw}} = \frac{HR_*}{U_f}\,, \quad \Omega_{\mathrm{turb}} \propto 1 - H\tau_{\mathrm{sw}} = 1 - \frac{HR_*}{U_f}$$

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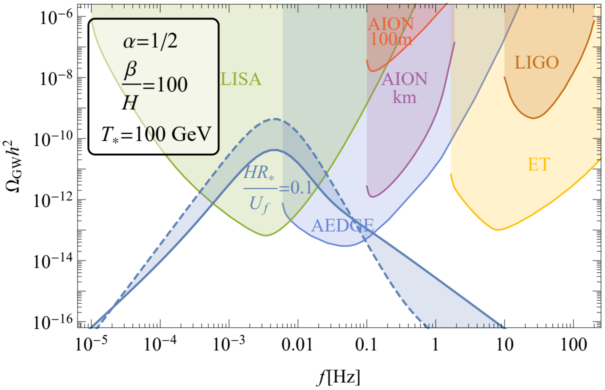


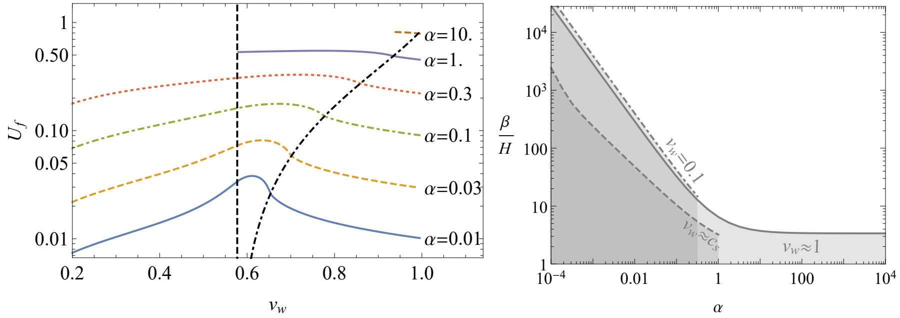
Plasma related GW sources

- Bubble size and transition time scale

$$H\,\tau_{\rm sh} \sim \frac{H\,R_*}{U_f} \approx \frac{(8\pi)^{\frac{1}{3}}\,{\rm Max}(v_w, c_s)}{U_f}\left(\frac{\beta}{H}\right)^{-1},$$

- Root-mean-square four-velocity of the plasma

$$U_f \approx \sqrt{\frac{3}{4}\frac{\kappa_{\rm sw}\alpha}{1+\alpha}} \xrightarrow{v_w\approx 1} \frac{\sqrt{3}\alpha}{2(1+\alpha)\sqrt{0.73+0.083\sqrt{\alpha}+\alpha}}.$$



Purely thermal transition: no $T = 0$ potential barrier

- Simple polynomial potential

$$V(\phi) = m^2\phi^2 - a\,\phi^3 + \lambda\phi^4$$

- Has a semi-analytical solution

$$\frac{S_3}{T} = \frac{a}{T\lambda^{3/2}} \frac{8\pi\sqrt{\delta}\left(\beta_1\delta + \beta_2\delta^2 + \beta_3\delta^3\right)}{81(2-\delta)^2}, \quad \text{where} \quad \delta \equiv \frac{8\lambda m^2}{a^2}$$

Adams '93

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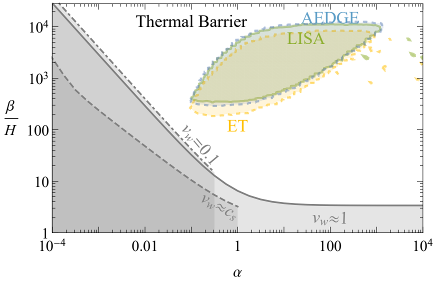
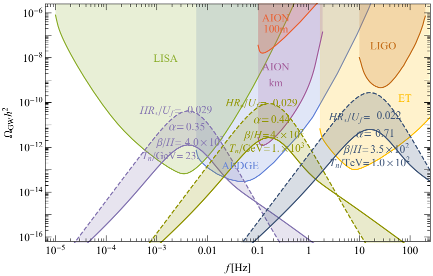
Adams '93

- Useful for high temperature expansion

$$V(\phi, T) = \frac{g_{m^2}}{24}\left(T^2 - T_0^2\right)\phi^2 - \frac{g_m}{12\pi}T\phi^3 + \lambda\phi^4\,, \quad T_0^2 > 0$$

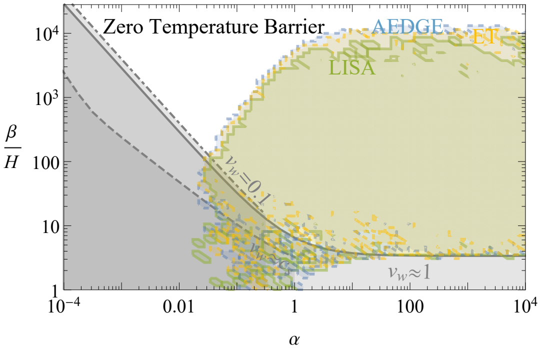




Tree-level potential barrier at $T = 0$

- High temperature expansion with a dominant tree level barrier

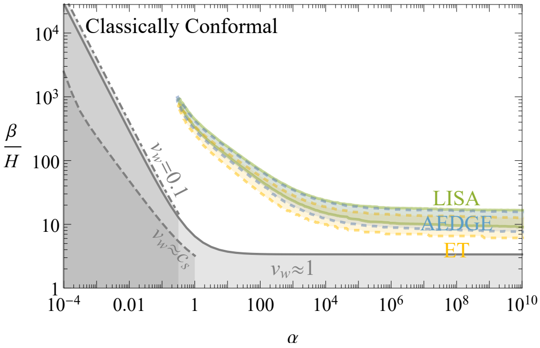
$$V(\phi, T) = \frac{g_{m^2}}{24}\left(T^2 - T_0^2\right)\phi^2 - A\phi^3 + \lambda\phi^4\,, \quad T_0^2 < 0$$



Classically scale-invariant CW-like potential

- High temperature expansion with a dominant tree level barrier

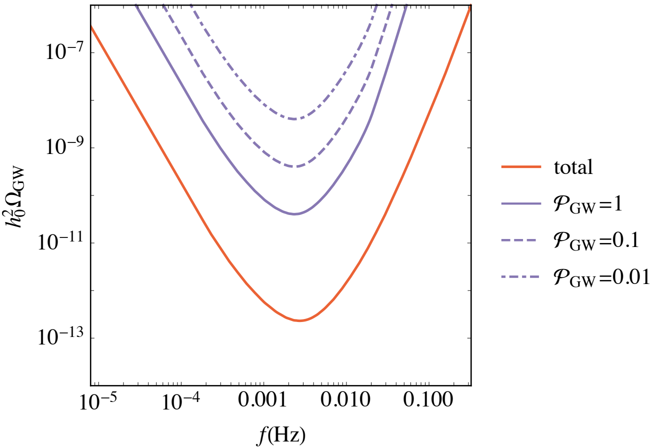
$$V(\phi, T) = g^2 T^2 \phi^2 + \frac{3g^4}{4\pi^2}\phi^4 \left(\log\left(\frac{\phi^2}{v^2}\right) - \frac{1}{2} - \frac{g^2 T^2}{2v^2}\right)$$



Polarisation of the GW signal

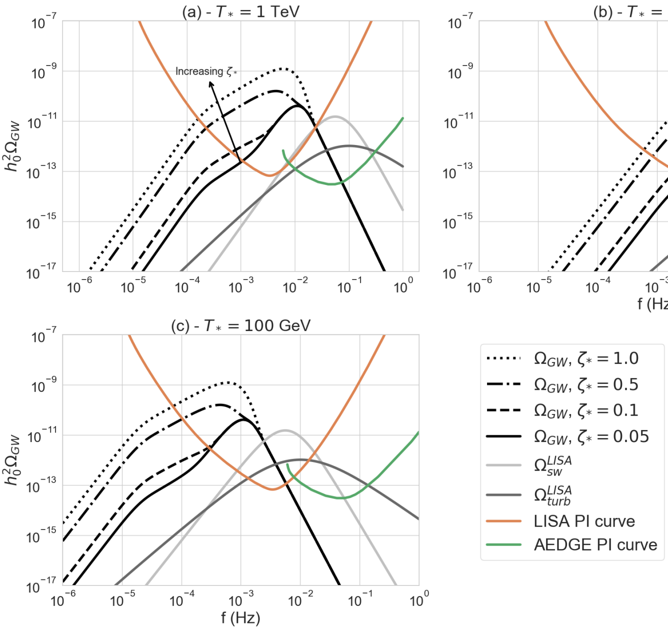
- Polarisation fraction

$$\mathcal{P}_{\rm GW}(k) = \frac{\Omega^{+}_{\rm GW}(k) - \Omega^{-}_{\rm GW}(k)}{\Omega_{\rm GW}(k)}$$

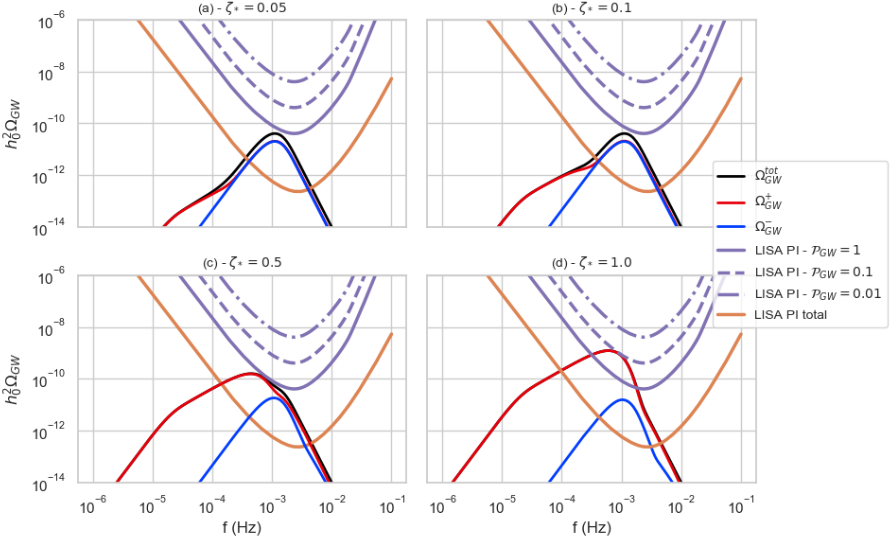


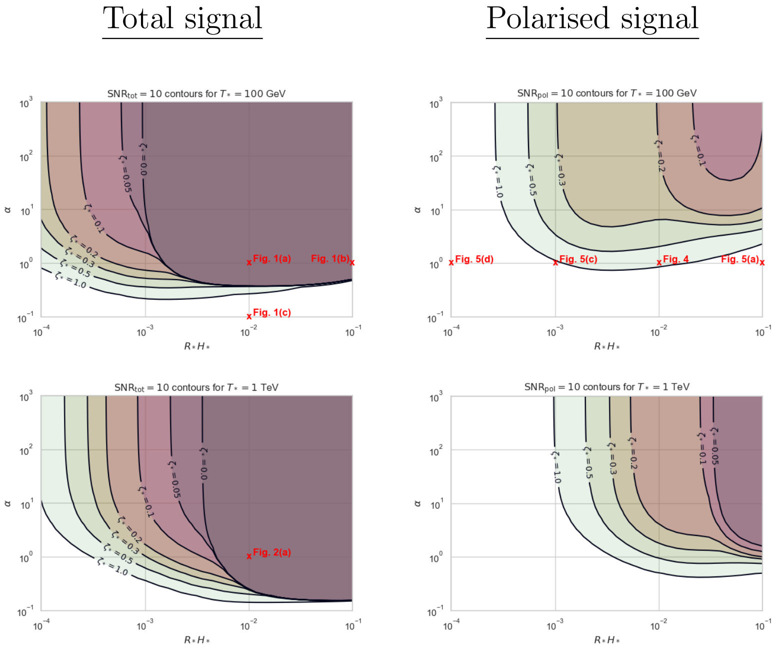
Domcke '20, Ellis '20

Different modelling of plasma sources



- Signal for $\alpha = 1$ and $\beta/H = 100$



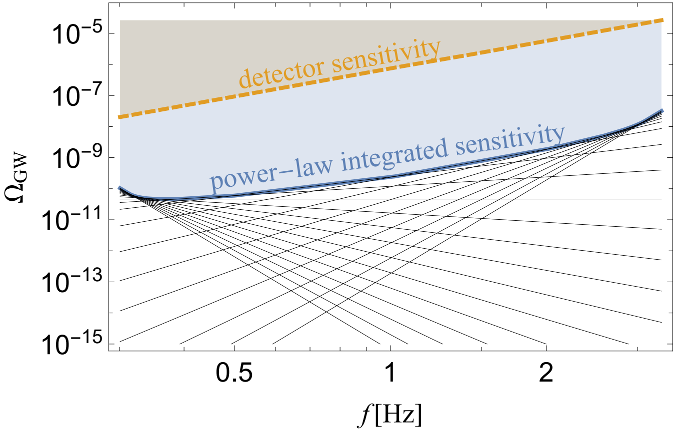

Total signal
Polarised signal
SNR$_{tot}$ = 10 contours for T_* = 100 GeV
SNR$_{pol}$ = 10 contours for T_* = 100 GeV
SNR$_{tot}$ = 10 contours for T_* = 1 TeV
SNR$_{pol}$ = 10 contours for T_* = 1 TeV
α
$R_* H_*$
$\zeta_* = 0.0$
$\zeta_* = 0.05$
$\zeta_* = 0.1$
$\zeta_* = 0.2$
$\zeta_* = 0.3$
$\zeta_* = 0.5$
$\zeta_* = 1.0$
Fig. 1(a)
Fig. 1(b)
Fig. 1(c)
Fig. 2(a)
Fig. 5(d)
Fig. 5(c)
Fig. 4
Fig. 5(a)

Conclusions

- Observable bubble collision GW signal requires very significant supercooling $\alpha > 10^{10}$.
  - → Observing a bubble collision signal would indicate a scale invariant potential for the field undergoing the transition.
- Sound wave period generically last less than a Hubble time.
  - → This leads to a much weaker sound wave sourced GW signal and potentially a significant increase in the signal sourced by turbulence.
  - → length of the sound-wave period can also carry information on the potential.
- Observing polarisation would require a very strong signal.
- PTs an also have leave other complimentary signals including production of primordial magnetic fields.

Power-law integrated sensitivity

$$\Omega_{GW}^{noise} = \frac{2\pi}{3}\frac{f^3 S_h}{H_0^2}, \qquad \text{SNR} = \sqrt{\mathcal{T}\int df \left(\frac{\Omega_{GW}^{signal}}{\Omega_{GW}^{noise}}\right)^2}$$



Thrane, Romano ’13

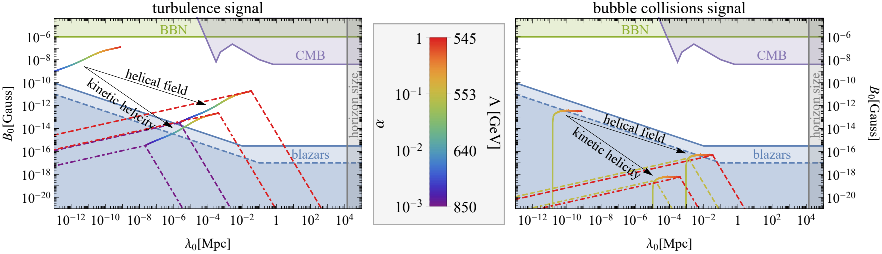
Primordial Magnetic Fields

- Energy density and correlation length of produced Magnetic Field

  Durrer '13 Brandenburg '17 Vachaspati '19

$$\rho_{B,*} = 0.1\kappa_{\text{col/sw}} \frac{\alpha}{1+\alpha}\rho_* \qquad \lambda_* = R_*$$

- In the $SM + H^6$ model



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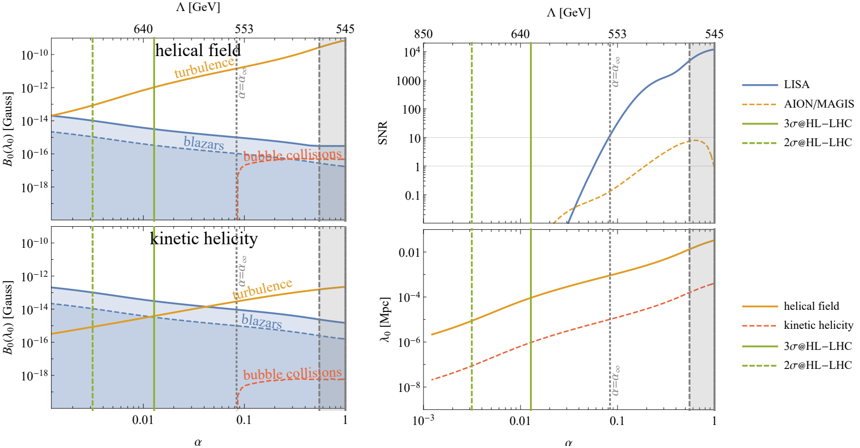
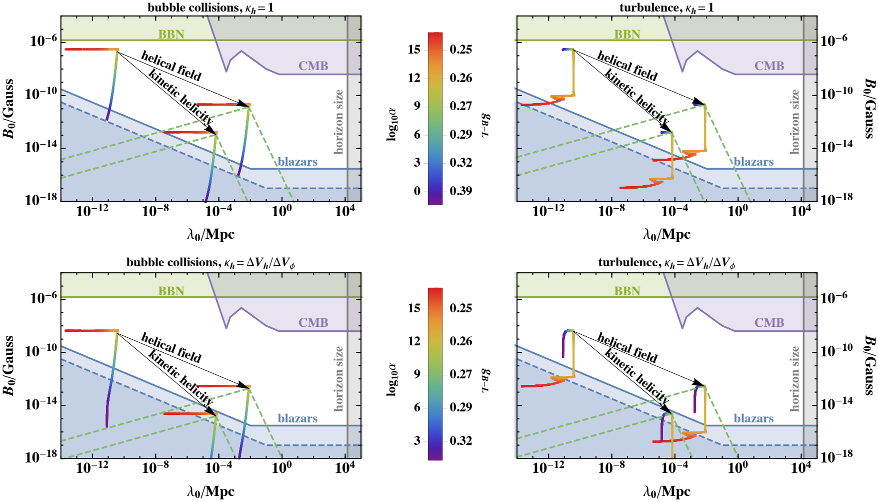


Primordial Magnetic Fields

- In the $U(1)_{\mathrm{B-L}}$ model with $m_{Z'} = 4\mathrm{TeV}$



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