IJS - Zoom

Sep 17th, 2020

Baryon and lepton number intricacies in axion models

Christopher Smith

- Outline

Jérémie Quevillon & C.S., 1903.12559, 2006.06778, 2009.xxxxx

I. The Strong CP puzzle

A. Origin of the puzzle - in a few words

In principle, there is a renormalizable CP-violating coupling in QCD:

$$\mathcal{L}_{\not{C}\not{P}} = (\theta_C - \arg\det \mathrm{Y}_u - \arg\det \mathrm{Y}_d)\frac{\alpha_S}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

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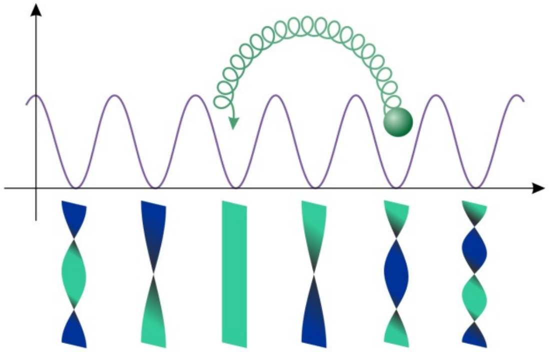
QCD has a non-trivial topology:



Explains the large η' mass ←

Violates time-reversal

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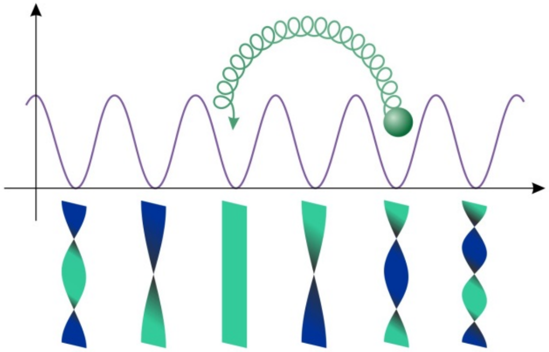

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We know they are complex.

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$\delta_{CKM} \neq 0$

from K and B physics

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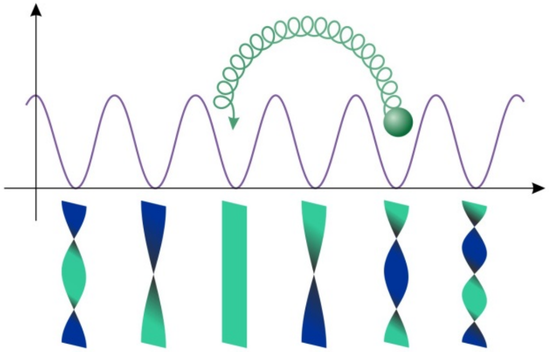

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Strong CP puzzle

Neutron EDM implies $\theta_{eff} \equiv \theta_C - \arg\det Y_u - \arg\det Y_d < 10^{-10}$!!!

B. Flavor-blind CP-violating phases in the SM

Three CPV terms are present in the SM gauge Lagrangian:

$$\mathcal{L}_{\not{CP}} = \theta_C \frac{\alpha_S}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu} + \theta_L \frac{g^2}{16\pi^2} W_{\mu\nu}\tilde{W}^{\mu\nu} + \theta_Y \frac{g'^2}{16\pi^2} B_{\mu\nu}\tilde{B}^{\mu\nu}$$

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Classically, massless quarks/lepton phases can be rotated:

$$\psi \to \exp(i\alpha_\psi)\psi \ , \ \ \psi = q_L, u_R, d_R, \ell_L, e_R$$

Quantum-mechanically, these five global $U(1)$s are anomalous.

With the appropriate rotations, all three CPV terms are eliminated:

$$\theta_C \to \theta_C - N_f(2\alpha_{q_L} + \alpha_{u_R} + \alpha_{d_R})$$
$$\theta_L \to \theta_L - N_f(3\alpha_{q_L} + \alpha_{\ell_L})$$
$$\theta_Y \to \theta_Y - N_f(1/3\alpha_{q_L} + 8/3\alpha_{u_R} + 2/3\alpha_{d_R} + \alpha_{\ell_L} + 2\alpha_{e_R})$$

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The Yukawa couplings explicitly break these $U(1)$ symmetries.

Three phases are fixed to get real quark and lepton masses.

Not enough freedom remains to get rid of all three CPV interactions:

$$\theta_C \to \theta_C - \arg\det \mathbf{Y}_u - \arg\det \mathbf{Y}_d$$

$$\theta_L \to \theta_L - N_f(3\alpha_{q_L} + \alpha_{\ell_L})$$

$$\theta_Y \to \theta_Y + N_f(3\alpha_{q_L} + \alpha_{\ell_L}) - \tfrac{8}{3}\arg\det \mathbf{Y}_u - \tfrac{2}{3}\arg\det \mathbf{Y}_d - 2\arg\det \mathbf{Y}_e$$

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Cannot be removed: Strong CP puzzle.

Removed thanks to $U(1)_{\mathcal{B}+\mathcal{L}}$ (choice for $3\alpha_{q_L} + \alpha_{\ell_L}$)

Removed by partial integration.

Not enough freedom remains to get rid of all three CPV interactions:

$$\theta_C \to \theta_C - \arg\det \mathrm{Y}_u - \arg\det \mathrm{Y}_d$$

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C. The axionic solution - Naively

$$\mathcal{L}_{axion} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{\alpha_S}{8\pi} \theta_S G_{\mu\nu} \tilde{G}^{\mu\nu} + \overline{\psi}_{L,R} i \not{D} \psi_{L,R} + y_i \overline{\psi}_L \psi_R \phi_i + V(\phi_i)$$

Step 1: Invariant under some global U(1) symmetry.

Spontaneously broken by the scalar VEVs, $\langle 0 | \phi_i | 0 \rangle = v$.

One massless goldstone boson, $\langle 0 | J^{\mu} | a^0(p) \rangle = ivp^{\mu}$.

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One massless goldstone boson, $\langle 0 | J^\mu | a^0(p) \rangle = i v p^\mu$.

Step 2: Design $\mathcal{L}_{axion}$ such that $PQ(\psi_L) \neq PQ(\psi_R)$

This makes the symmetry anomalous: $\partial_\mu J^\mu \sim G_{\mu\nu} \tilde{G}^{\mu\nu}$

The Goldstone boson shift symmetry is explicitly broken:

$$\mathcal{L}_{axion} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{\alpha_S}{8\pi} \left(\theta_S + \frac{1}{v} a^0 \mathcal{N}_C \right) G_{\mu\nu} \tilde{G}^{\mu\nu} + \partial_\mu a^0 J^\mu + \ldots$$

Removed by $a^0 \to a^0 - v\theta_S / \mathcal{N}_C$

Subleading and model-dependent

Peccei, Quinn, ’77
Weinberg ’78
Wilczek ’78

C. The axionic solution – Low-energy phenomenology

$$\mathcal{L}_{axion} \supset -\frac{\alpha_S}{8\pi}\left(\theta_S + \frac{1}{f_a}a^0\right)G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4}\left(g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a}\frac{\mathcal{N}_{em}}{\mathcal{N}_C}\right)a^0 F_{\mu\nu}\tilde{F}^{\mu\nu} + \ldots$$

Step 3: QCD non-perturbative effects are turned on.

Effect 1: Strong CP relaxes to zero

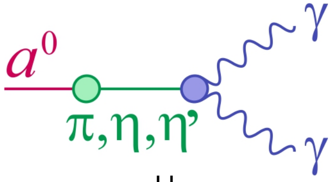
$$\frac{\alpha_S}{8\pi}\left(\theta_S + a^0 / f_a\right)G_{\mu\nu}\tilde{G}^{\mu\nu} \to V_{eff}\left(\theta_S + a^0 / f_a, \pi, \eta, \ldots\right)$$

Miminum at $\langle a^0 \rangle = -f_a \theta_S$

Effect 2: The axion acquires a mass

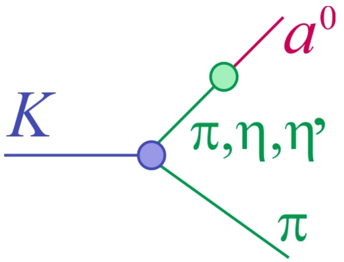
$$\frac{f_a^2 m_a^2}{f_\pi^2 m_\pi^2} = \frac{m_u m_d}{m_u + m_d} \to m_a \approx 6\mu eV \times \frac{10^{12}\,GeV}{f_a}$$

Effect 3: The axion mixes with π^0, η, η'



$$\Downarrow$$

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a}\left[\frac{\mathcal{N}_{em}}{\mathcal{N}_C} - \frac{2}{3}\frac{4m_d + m_u}{m_u + m_d}\right]$$



$\Rightarrow f_a \gtrsim 30 \times v_{EW}$

KEK, '81

C. The axionic solution – Low-energy phenomenology

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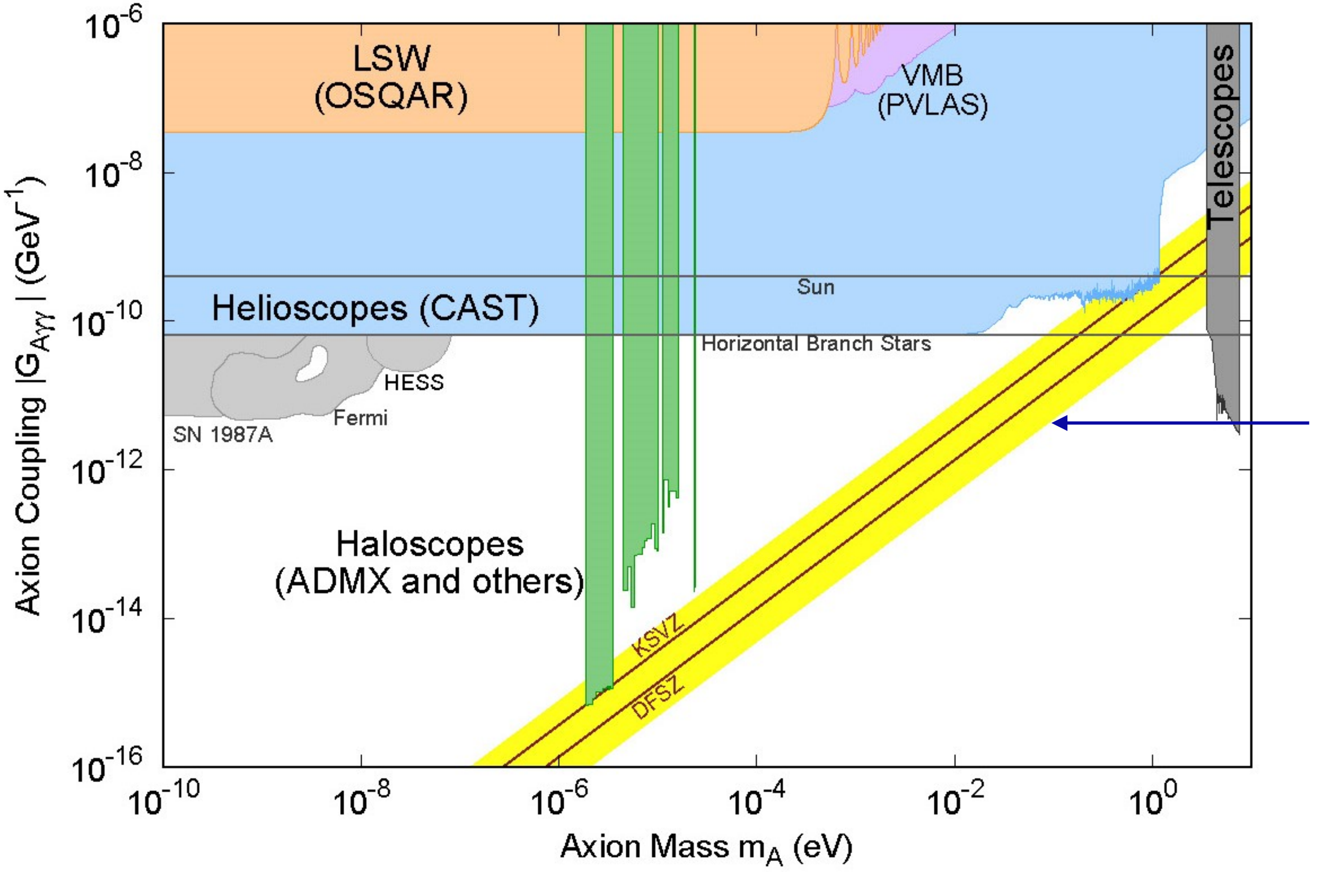
Step 4: Experimental constraints (PDG18)



Axion band:

$f_a \Rightarrow m_a, g_{a\gamma\gamma}$

III. Toy axion model

A. Toy model for a "QED axion"

Add to the usual massless QED a neutral complex scalar field:

$$\mathcal{L}_{axion} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_{L,R} i\not{D}\psi_{L,R} + (y\bar{\psi}_L\psi_R\phi + h.c.) + \partial_\mu\phi^\dagger\partial^\mu\phi - V(\phi)$$

Invariance under $U(1)_{em} \otimes U(1)_{PQ}$:

$$\phi \to \exp(-i\theta)\phi, \quad \begin{cases} \psi_L \to \exp(-i\alpha\theta)\psi_L \\ \psi_R \to \exp(-i(\alpha+1)\theta)\psi_R \end{cases}$$

The parameter α originates in the conserved fermion number.

The $U(1)_{PQ}$ symmetry is chiral hence anomalous.

Spontaneous breaking of $U(1)_{PQ}$:

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2, \quad \mu^2 < 0.$$

The associated Goldstone boson is the axion.

B. Toy model for a "QED axion": Linear representation

Linear representation for the scalar field: $\phi = \frac{1}{\sqrt{2}}(\sigma^0 + ia^0 + v)$

$$\mathcal{L}_{linear} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\not{D}\psi + m\bar{\psi}\psi\left(1+\frac{\sigma^0}{v}\right) + m\frac{a^0}{v}\bar{\psi}i\gamma_5\psi$$

$$+\frac{1}{2}\partial_\mu a^0\partial^\mu a^0 + \frac{1}{2}(\partial_\mu\sigma^0\partial^\mu\sigma^0 - m_\sigma^2(\sigma^0)^2)$$

$$-\lambda v\sigma^0((\sigma^0)^2 + (a^0)^2) - \frac{\lambda}{4}((\sigma^0)^2 + (a^0)^2)^2$$

With as usual $\begin{cases} m = yv \\ v^2 = -\mu^2/\lambda \\ m_\sigma^2 = 2\lambda v^2 \end{cases}$

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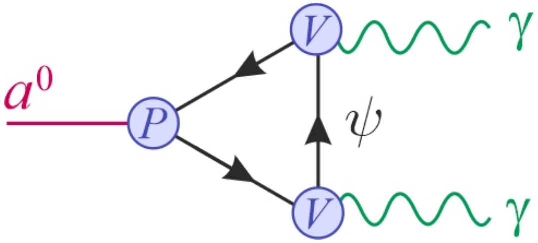
Coupling to photons:

$$\mathcal{M}(a^0 \to \gamma\gamma)_{Lin} = -i\frac{m}{v}e^2T^{\alpha\beta}_{PVV} = -i\frac{e^2}{2\pi^2 v}m^2C_0(m^2)\varepsilon^{\alpha\beta\mu\nu}q_{1\mu}q_{2\nu}$$



- Not anomalous!
- Vanishes in the $m \to 0$ limit.
- Finite as $m \to \infty$ even though $T^{\alpha\beta}_{PVV} \overset{m\to\infty}{\to} 0$.
- Effective interaction: $\mathcal{L}_{eff} = -\frac{e^2}{16\pi^2 v}a^0F_{\mu\nu}\tilde{F}^{\mu\nu}$

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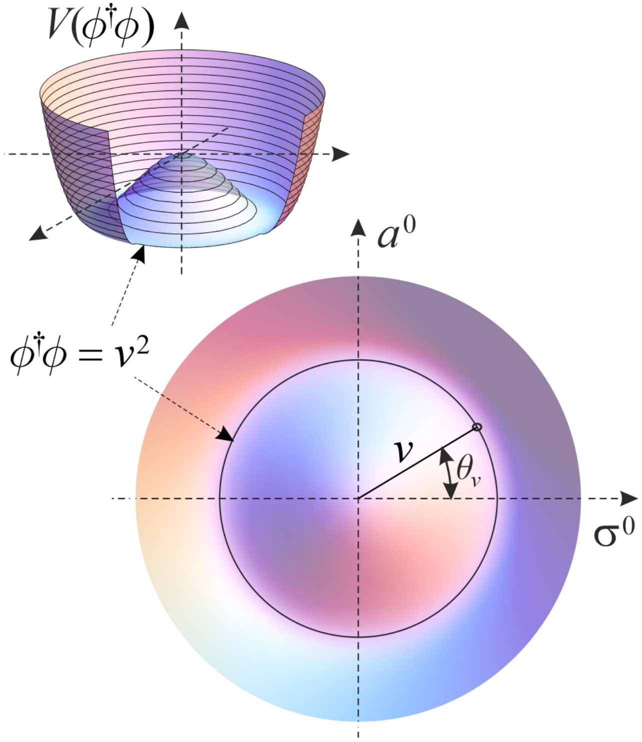
Shift symmetry: $\mathcal{L}_{linear} = \mathcal{L}_{linear} + \frac{e^2}{16\pi^2}\theta_{QED}F_{\mu\nu}\tilde{F}^{\mu\nu}$



Freedom to choose the vacuum:

$$\begin{pmatrix} \sigma^0 + v \\ a^0 \end{pmatrix} \to \begin{pmatrix} \cos\theta_v & \sin\theta_v \\ -\sin\theta_v & \cos\theta_v \end{pmatrix}\begin{pmatrix} \sigma^0 + v \\ a^0 \end{pmatrix}$$

Classically, leaves $\mathcal{L}_{linear}$ invariant.

Requires a chiral rotation since $m \to m\exp(i\theta_v)$

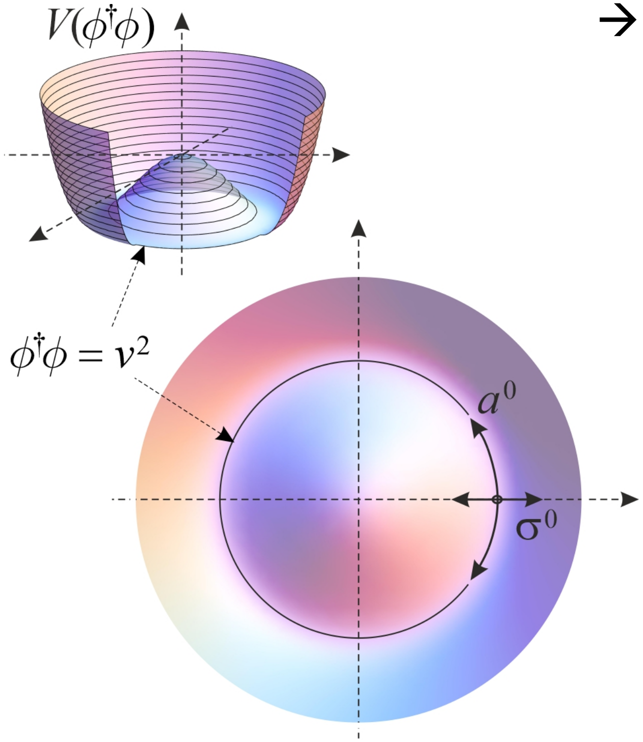
At the quantum level, $\theta_{QED} \to \theta_{QED} + \theta_v$

C. Toy model for a “QED axion”: Polar representation

Polar representation for the scalar field: $\phi = \frac{1}{\sqrt{2}}(\sigma^0 + v)\exp(-ia^0/v)$

This is a non-linear representation of $U(1)_{PQ}$:

$$\sigma^0 \to \sigma^0 \ , \ a^0 \to a^0 + v\theta_v$$



→ Spans the vacuum

→ $U(1)_{PQ}$ = true shift symmetry?

Not yet for fermions:

$$\begin{cases} \psi_L \to \exp(-i\alpha\theta_v)\psi_L \\ \psi_R \to \exp(-i(\alpha+1)\theta_v)\psi_R \end{cases}$$

C. Toy model for a “QED axion”: Polar representation

Polar representation for the scalar field: $\phi = \frac{1}{\sqrt{2}}(\sigma^0 + v)\exp(-ia^0/v)$

Reparametrization of the fermions to make them $U(1)_{PQ}$ invariant:

$$\psi_L(x) \to \exp(-i\alpha a^0(x)/v)\psi_L(x),$$
$$\psi_R(x) \to \exp(-i(\alpha+1)a^0(x)/v)\psi_R(x)$$

Consequence 1: Removes the axion from the Yukawa coupling

$$y\bar{\psi}_L\psi_R\phi \supset m\bar{\psi}_L\psi_R\exp(-ia^0/v) \to m\bar{\psi}_L\psi_R$$

Consequence 2: Non-invariance of the kinetic terms

$$\delta\mathcal{L}_{Der} = -\frac{1}{v}\partial_\mu a^0\left(\alpha\bar{\psi}_L\gamma^\mu\psi_L + (\alpha+1)\bar{\psi}_R\gamma^\mu\psi_R\right)$$

Consequence 3: Non-invariance of the fermionic measure

$$\delta\mathcal{L}_{Jac} = \frac{e^2}{16\pi^2 v}a^0\left(\underset{\substack{\uparrow \\ \psi_L}}{\alpha} - (\underset{\substack{\uparrow \\ \psi_R}}{\alpha}+1)\right)F_{\mu\nu}\tilde{F}^{\mu\nu}$$

C. Toy model for a "QED axion": Polar representation

Polar representation for the scalar field: $\phi = \frac{1}{\sqrt{2}}(\sigma^0 + v)\exp(-ia^0 / v)$

Reparametrization of the fermions to make them $U(1)_{PQ}$ invariant.

Manifest but anomalous shift symmetry:

$$\mathcal{L}_{polar} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\not{D}\psi + m\bar{\psi}\psi\left(1+\frac{\sigma^0}{v}\right)$$

$$+\frac{1}{2}\partial_\mu a^0 \partial^\mu a^0 \left(1+\frac{\sigma^0}{v}\right)^2 + \delta\mathcal{L}_{der} + \delta\mathcal{L}_{Jac}$$

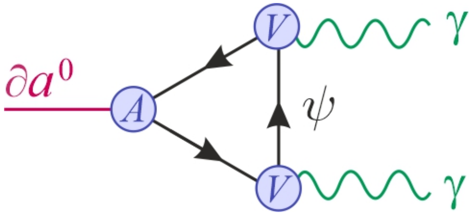
$$+\frac{1}{2}(\partial_\mu \sigma^0 \partial^\mu \sigma^0 - m_\sigma^2(\sigma^0)^2) - \lambda v(\sigma^0)^3 - \frac{\lambda}{4}(\sigma^0)^4$$

With the extra pieces: $\delta\mathcal{L}_{Der} = -\frac{1}{v}\partial_\mu a^0 \left((2\alpha+1)\bar{\psi}\gamma^\mu\psi + \bar{\psi}\gamma^\mu\gamma_5\psi\right)$

$$\delta\mathcal{L}_{Jac} = -\frac{e^2}{16\pi^2 v}a^0 F_{\mu\nu}\tilde{F}^{\mu\nu}$$

C. Toy model for a "QED axion": Polar representation

Coupling to photons:



$$\delta\mathcal{L}_{Der} = -\frac{1}{v}\partial_\mu a^0\left(\bar{\psi}\gamma^\mu\gamma_5\psi + (2\alpha+1)\bar{\psi}\gamma^\mu\psi\right)$$

$$\mathcal{M}(a^0 \to \gamma\gamma)_{Der} = -\frac{e^2}{2v}i(q_1+q_2)_\gamma\left[T_{AVV}^{\alpha\beta\gamma} + \underbrace{(2\alpha+1)T_{VVV}^{\alpha\beta\gamma}}_{=0\text{ (Furry)}}\right]$$

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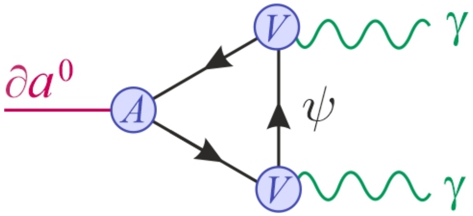

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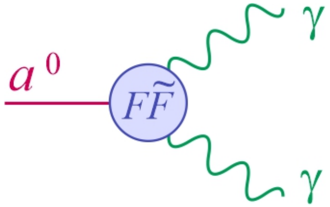

$$\delta\mathcal{L}_{Jac} = -\frac{e^2}{16\pi^2 v}a^0 F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\mathcal{M}(a^0 \to \gamma\gamma)_{Jac} = -\frac{e^2}{4\pi^2 v}\varepsilon^{\alpha\beta\mu\nu}q_{1\mu}q_{2\nu}$$

C. Toy model for a "QED axion": Polar representation

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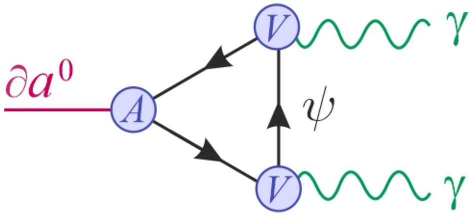

$$\delta\mathcal{L}_{Der} = -\frac{1}{v}\partial_\mu a^0\left(\bar{\psi}\gamma^\mu\gamma_5\psi + (2\alpha+1)\bar{\psi}\gamma^\mu\psi\right)$$

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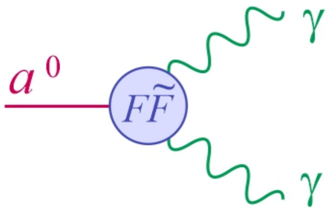
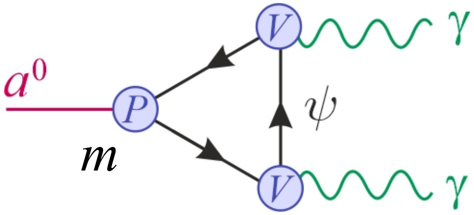


$$\delta\mathcal{L}_{Jac} = -\frac{e^2}{16\pi^2 v}a^0 F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\mathcal{M}(a^0\to\gamma\gamma)_{Jac} = -\frac{e^2}{4\pi^2 v}\varepsilon^{\alpha\beta\mu\nu}q_{1\mu}q_{2\nu}$$

=

The amplitude is indeed independent of the representation:

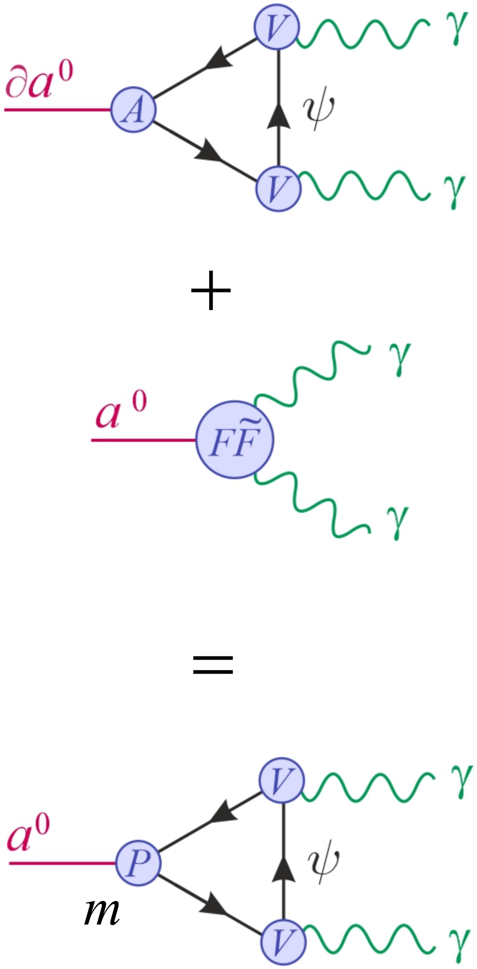


$$\mathcal{M}(a^0\to\gamma\gamma)_{Der} + \mathcal{M}(a^0\to\gamma\gamma)_{Jac} = \mathcal{M}(a^0\to\gamma\gamma)_{Lin}$$

$$i(q_1+q_2)_\gamma T^{\alpha\beta\gamma}_{AVV} - \frac{1}{2\pi^2}\varepsilon^{\alpha\beta\mu\nu}q_{1\mu}q_{2\nu} = 2imT^{\alpha\beta}_{PVV}$$

C. Toy model for a “QED axion”: Polar representation

Coupling to photons and ABJ anomaly: $\partial_\mu A^\mu - \frac{1}{8\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} = 2imP$



1- The anomaly cancels out in

$$\mathcal{M}(a^0 \to \gamma\gamma)_{Der} + \mathcal{M}(a^0 \to \gamma\gamma)_{Jac}$$

2- This cancellation ensures

$$\mathcal{M}(a^0 \to \gamma\gamma)_{Der} + \mathcal{M}(a^0 \to \gamma\gamma)_{Jac} \overset{m\to 0}{=} 0$$

3- The decay is not anomalous. Why the confusion?

Sutherland-Veltman theorem:

$\mathcal{M}(a^0 \to \gamma\gamma)_{Der} \to 0$ when $m \to \infty$

The amplitude is equal to the local term:

$$\mathcal{M}(a^0 \to \gamma\gamma)_{Lin} \overset{m\to\infty}{=} 0 + \mathcal{M}(a^0 \to \gamma\gamma)_{Jac}$$

Sutherland ‘67
Veltman ‘67

Georgi, Kaplan, Randall, ‘86
Bardeen, Peccei, Yanagida ‘87

IV. PQ and DFSZ axions

A. Prelude: The Two-Higgs Doublet Model and axions

Extra U(1) symmetry with two Higgs doublets $\Phi_{1,2} \sim \{G^{0,\pm}, h^0, H^{0,\pm}, A^0\}$:

$$U(1)_1 \otimes U(1)_2 \sim U(1)_Y \otimes U(1)_{PQ} : \; \Phi_i \to \Phi_i \exp(i\phi_i)$$

when the scalar potential is limited to:

$$\begin{aligned}\mathcal{V}_{scalar} &= m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 \\ &+ \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ &+ \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_2^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2)\end{aligned}$$

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This extra symmetry is broken spontaneously: A^0 = Goldstone boson

$$\langle \Phi_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_i \exp(i\alpha_i) \end{pmatrix} \qquad v_1^2 + v_2^2 = v^2 \ , \ v_1 / v_2 = \tan\beta = 1/x$$

Choice of relative phase between the VEVs breaks $U(1)_{PQ}$.

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Choice of relative phase between the VEVs breaks $U(1)_{PQ}$.

This extra symmetry is anomalous: $A^0 \equiv a^0$ is an axion!

VEVs phases ⇔ complex fermion masses:

$$\mathcal{L}_{fermion} = -\overline{u}_R Y_u q_L \Phi_1 - \overline{d}_R Y_d q_L \Phi_2^\dagger - \overline{e}_R Y_e \ell_L \Phi_2^\dagger + h.c.$$

Removed by an anomalous chiral rotation $\to A^0 G_{\mu\nu}^a \tilde{G}^{a,\mu\nu}$

A. Prelude: The Two-Higgs Doublet Model and axions

Question: What are the couplings of this axion to SM gauge bosons?

It is generally believed that

Georgi, Kaplan, Randall '86

$$\mathcal{L}_{PQ}^{eff} = \frac{A^0}{16\pi^2 v}\left(g_s^2 \mathcal{N}_C G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} + g^2 \mathcal{N}_L W_{\mu\nu}^i \tilde{W}^{i,\mu\nu} + g'^2 \mathcal{N}_Y B_{\mu\nu} \tilde{B}^{\mu\nu}\right)$$

$$\mathcal{N}_X = \sum_{\psi} PQ(\psi) tr(T_X^i(\psi) T_X^j(\psi))$$

Are those couplings ambiguous?

There are four entangled U(1)s: $U(1)_Y \otimes U(1)_{PQ} \otimes U(1)_{\mathcal{B}} \otimes U(1)_{\mathcal{L}}$

Are the SM fermion PQ charges well-defined?

Why no electroweak symmetry breaking effect?

Massless neutrino puzzle?

B. Axion couplings in the linear representation

Standard THDM phenomenology

Higgs basis: $$\begin{pmatrix}\Phi_h \\ \Phi_H\end{pmatrix} = \begin{pmatrix}\cos\beta & \sin\beta \\ -\sin\beta & \cos\beta\end{pmatrix}\begin{pmatrix}\Phi_2 \\ \Phi_1\end{pmatrix} \qquad (\tan\beta \equiv v_1 / v_2)$$

$$\Phi_h = \frac{1}{\sqrt{2}}\exp\left\{i\tau^j G^j / v\right\}\begin{pmatrix}0 \\ \phi_h + v\end{pmatrix}\ ,\ \Phi_H = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}H^+ \\ \phi_H + iA^0\end{pmatrix}$$

Couplings to gauge bosons: No $A^0 \to VV$ at tree level

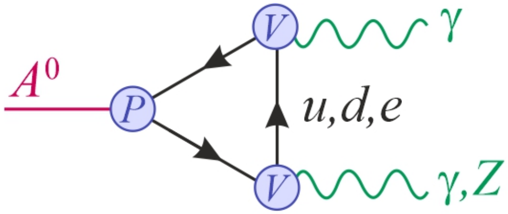
$$\mathcal{L}_{higgs} = \frac{(v+\phi_h)^2 + |\phi_H + iA^0|^2}{8}\left(2g^2 W_\mu^+ W^{-,\mu} + (g^2 + g'^2) Z_\mu Z^\mu\right)$$

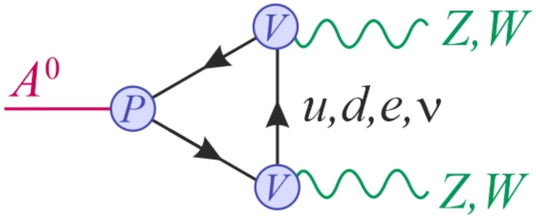
Couplings to fermions: Mass-dependent pseudoscalar couplings

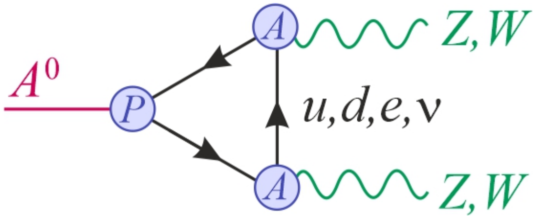
$$\mathcal{L}_{A^0 ff} = -i\sum_{f=u,d,e}\frac{m_f}{v}\chi_P^f A^0 \bar{\psi}_f \gamma_5 \psi_f\ , \qquad \chi_P^d = \chi_P^e = \frac{1}{\chi_P^u} = \tan\beta$$

B. Axion couplings in the linear representation

Couplings to gauge bosons at one loop:







$$T_{PVV}^{\alpha\beta} = -i\frac{mC_0(m^2)}{2\pi^2}\varepsilon^{\alpha\beta\rho\sigma}q_{1\rho}q_{2\sigma}$$

$$T_{PAA}^{\alpha\beta} = -i\frac{m(C_0(m^2)+2C_1(m^2))}{2\pi^2}\varepsilon^{\alpha\beta\rho\sigma}q_{1\rho}q_{2\sigma}$$

$$C_i(m^2) \equiv C_i(q_1^2, q_2^2, (q_1+q_2)^2, m^2, m'^2, m^2)$$

Amplitudes known for a long time. Gunion, Haber, Kao ‘91

Finite and non anomalous - no ambiguity of any kind!

Vanishes as $m \to 0$ since the A^0 coupling to fermion does.

B. Axion couplings in the linear representation

Couplings to gauge bosons at one loop:

$$
\begin{aligned}
\mathcal{L}^{eff}_{linear} &= \frac{A^0}{16\pi^2 v} g_s^2 \mathcal{N}_C G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \\
&+ \frac{A^0}{16\pi^2 v} e^2 \mathcal{N}_{em} F_{\mu\nu} \tilde{F}^{\mu\nu} \\
&+ \frac{A^0}{16\pi^2 v} \frac{2e^2}{c_W s_W} (\mathcal{N}_0 - s_W^2 \mathcal{N}_{em}) Z_{\mu\nu} \tilde{F}^{\mu\nu} \\
&+ \frac{A^0}{16\pi^2 v} \frac{e^2}{c_W^2 s_W^2} (\mathcal{N}_1 - 2s_W^2 \mathcal{N}_0 + s_W^4 \mathcal{N}_{em}) Z_{\mu\nu} \tilde{Z}^{\mu\nu} \\
&+ \frac{A^0}{16\pi^2 v} 2g^2 \mathcal{N}_2 W^+_{\mu\nu} \tilde{W}^{-,\mu\nu}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{N}_C &= \frac{1}{2}\left(x + \frac{1}{x} \right) \\
\mathcal{N}_{em} &= N_C \left(\frac{4}{9} x + \frac{1}{9x} \right) + \frac{1}{x} \\
\mathcal{N}_0 &= \frac{1}{4}\left(N_C \left(\frac{2}{3} x + \frac{1}{3x} \right) + \frac{1}{x} \right) \\
\mathcal{N}_1 &= \frac{1}{12}\left(N_C \left(x + \frac{1}{x} \right) + \frac{1}{x} \right) \\
\mathcal{N}_2 &= \frac{1}{12}\left(N_C \left(x + \frac{1}{x} \right) + \frac{3}{2x} \right)
\end{aligned}
$$

Non-decoupling when $m_{u,d,e} \to \infty$.

$(x \equiv \cot\beta \equiv v_2 / v_1)$

Breaks EW symmetry $\mathcal{N}_0 \neq \mathcal{N}_1 \neq \mathcal{N}_2$.

C. Axion couplings in the polar representation

Polar representation for the two doublets:

Exponentiate the GB of $U(1)_1 \otimes U(1)_2$: $\Phi_i = \frac{1}{\sqrt{2}} \exp\{i\eta_i / v_i\} \begin{pmatrix} \sqrt{2} H_i^+ \\ v_i + H_i^0 \end{pmatrix}$

Identify the GB:

$$\begin{array}{l} U(1)_Y \rightarrow \\ U(1)_{PQ} \rightarrow \end{array} \begin{pmatrix} G^0 \\ a^0 \end{pmatrix} = \frac{1}{v} \begin{pmatrix} v_2 & v_1 \\ -v_1 & v_2 \end{pmatrix} \begin{pmatrix} \eta_2 \\ \eta_1 \end{pmatrix}$$

Hence: $\Phi_1 = \frac{1}{\sqrt{2}} \exp i \left\{ 1 \frac{G^0}{v} + x \frac{a^0}{v} \right\} \begin{pmatrix} \sqrt{2} H_1^+ \\ v_1 + H_1^0 \end{pmatrix}$

$$\Phi_2 = \frac{1}{\sqrt{2}} \exp i \left\{ \underset{\overset{\uparrow}{Y}}{1} \frac{G^0}{v} - \underset{\overset{\uparrow}{PQ}}{\frac{1}{x}} \frac{a^0}{v} \right\} \begin{pmatrix} \sqrt{2} H_2^+ \\ v_2 + H_2^0 \end{pmatrix}$$

$(x \equiv \cot\beta \equiv v_2 / v_1)$

$\Rightarrow$ PQ not orthogonal to Y !

Defined in the broken phase!

C. Axion couplings in the polar representation

PQ charge assignment for the fermions:

$$\mathcal{L}_{fermion} = -\bar{u}_R Y_u q_L \Phi_1 - \bar{d}_R Y_d q_L \Phi_2^\dagger - \bar{e}_R Y_e \ell_L \Phi_2^\dagger + h.c.$$

$$PQ(\Phi_1) = x \Rightarrow \begin{cases} PQ(q_L) = \alpha \\ PQ(u_R) = \alpha + x \end{cases} \qquad PQ(\Phi_2) = -\frac{1}{x} \Rightarrow \begin{cases} PQ(d_R) = \alpha + \frac{1}{x} \\ PQ(\ell_L) = \beta \\ PQ(e_R) = \beta + \frac{1}{x} \end{cases}$$

Free parameters related to $U(1)_\mathcal{B}$ and $U(1)_\mathcal{L}$.

In the linear representation: - $\mathcal{B}$ and $\mathcal{L}$ are clearly unbroken.
- These parameters do not appear

Are we free to fix them? Are there constraints on them?

C. Axion couplings in the polar representation

Fermion reparametrization: $\psi \to \exp\left\{ i\frac{PQ(\psi)}{v} a^0 \right\}\psi$

Consequence 1: Non-invariance of the kinetic terms

$$\delta\mathcal{L}_{Der} = -\frac{1}{v}\partial_\mu a^0 \sum_{q_L, u_R, d_R, \ell_L, e_R} PQ(\psi)\bar{\psi}\gamma^\mu\psi \longleftarrow$$

Only couplings to SM fermions

Consequence 2: Non-invariance of the fermionic measure

$$\delta\mathcal{L}_{Jac} = \frac{a^0}{16\pi^2 v} g_s^2 \mathcal{N}_C G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + \frac{a^0}{16\pi^2 v} g^2 \mathcal{N}_L W^i_{\mu\nu}\tilde{W}^{i,\mu\nu} + \frac{a^0}{16\pi^2 v} g'^2 \mathcal{N}_Y B_{\mu\nu}\tilde{B}^{\mu\nu}$$

$$\mathcal{N}_C = \frac{1}{2}\left(x + \frac{1}{x} \right)$$

$$\mathcal{N}_L = -\frac{1}{2}(3\alpha + \beta)$$

$$\mathcal{N}_Y = \frac{1}{2}(3\alpha + \beta) + \frac{4}{3}x + \frac{1}{3x} + \frac{1}{x}$$

Both manifestly $SU(2)_L \otimes U(1)_L$ symmetric and both ambiguous!

C. Axion couplings in the polar representation

Fermion reparametrization: $\psi \to \exp\left\{ i \frac{PQ(\psi)}{v} a^0 \right\} \psi$

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$\longleftarrow$ Only couplings to SM fermions

Remember: $\partial_\mu J^\mu_{\mathcal{B}} = \partial_\mu J^\mu_{\mathcal{L}} = -\frac{1}{16\pi^2}\left(\frac{1}{2} g^2 W^i_{\mu\nu}\tilde{W}^{i,\mu\nu} - \frac{1}{2} g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu} \right)$

Remember that $a^0 \to W^+W^-$ occurs in the linear representation

$$+\frac{a^0}{16\pi^2 v} g^2 \mathcal{N}_L W^i_{\mu\nu}\tilde{W}^{i,\mu\nu} \qquad \mathcal{N}_L = -\frac{1}{2}(3\alpha + \beta)$$

$$+\frac{a^0}{16\pi^2 v} g'^2 \mathcal{N}_Y B_{\mu\nu}\tilde{B}^{\mu\nu} \qquad \mathcal{N}_Y = \frac{1}{2}(3\alpha + \beta) + \frac{4}{3}x + \frac{1}{3x} + \frac{1}{x}$$

Both manifestly $SU(2)_L \otimes U(1)_L$ symmetric and both ambiguous!

C. Axion couplings in the polar representation

Fermion reparametrization: $\psi \to \exp\{ i$

	u	d	e	ν
χ_V	$2\alpha + x$	$2\alpha + \frac{1}{x}$	$2\beta + \frac{1}{x}$	β
χ_A	x	$\frac{1}{x}$	$\frac{1}{x}$	$-\beta$

Consequence 1: Non-invariance of the

$$\delta\mathcal{L}_{Der} = -\frac{1}{2v}\partial_\mu a^0 \sum_{u,d,e,\nu} \chi_V^f \bar{\psi}_f \gamma^\mu \psi_f + \chi_A^f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$$

Remember: $\partial_\mu J_{\mathcal{B}}^\mu = \partial_\mu J_{\mathcal{L}}^\mu = -\frac{1}{16\pi^2}\left(\frac{1}{2} g^2 W_{\mu\nu}^i \tilde{W}^{i,\mu\nu} - \frac{1}{2} g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right)$

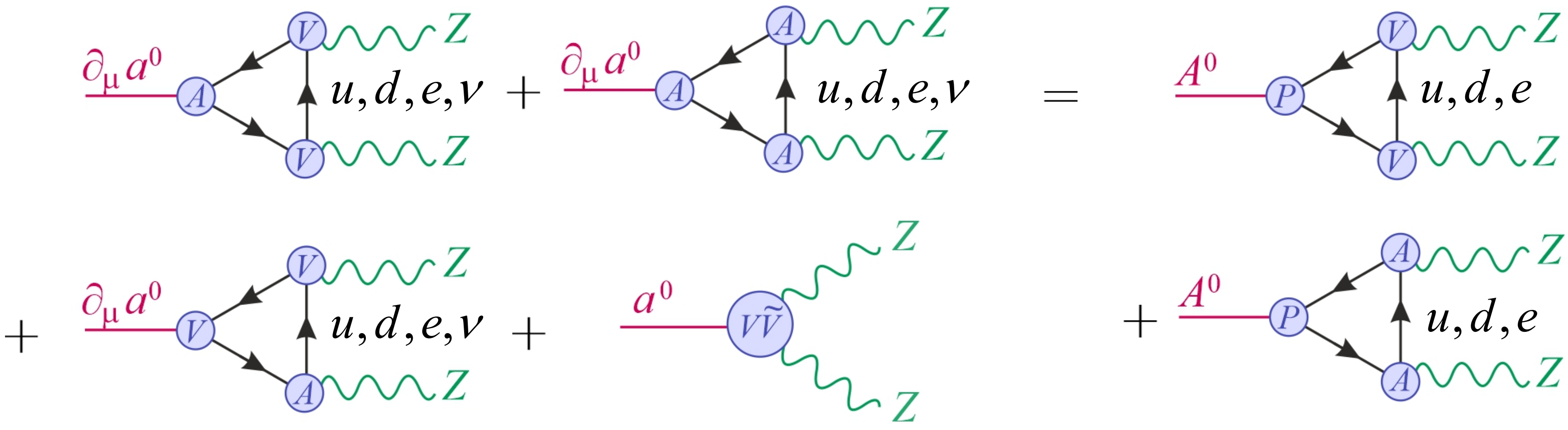
Remember that $a^0 \to W^+W^-$ occurs in the linear representation

$$+\frac{a^0}{16\pi^2 v} g^2 \mathcal{N}_L W_{\mu\nu}^i \tilde{W}^{i,\mu\nu} \qquad \mathcal{N}_L = -\frac{1}{2}(3\alpha + \beta)$$

$$+\frac{a^0}{16\pi^2 v} g'^2 \mathcal{N}_Y B_{\mu\nu}\tilde{B}^{\mu\nu} \qquad \mathcal{N}_Y = \frac{1}{2}(3\alpha + \beta) + \frac{4}{3}x + \frac{1}{3x} + \frac{1}{x}$$

Both manifestly $SU(2)_L \otimes U(1)_Y$ symmetric and both ambiguous!

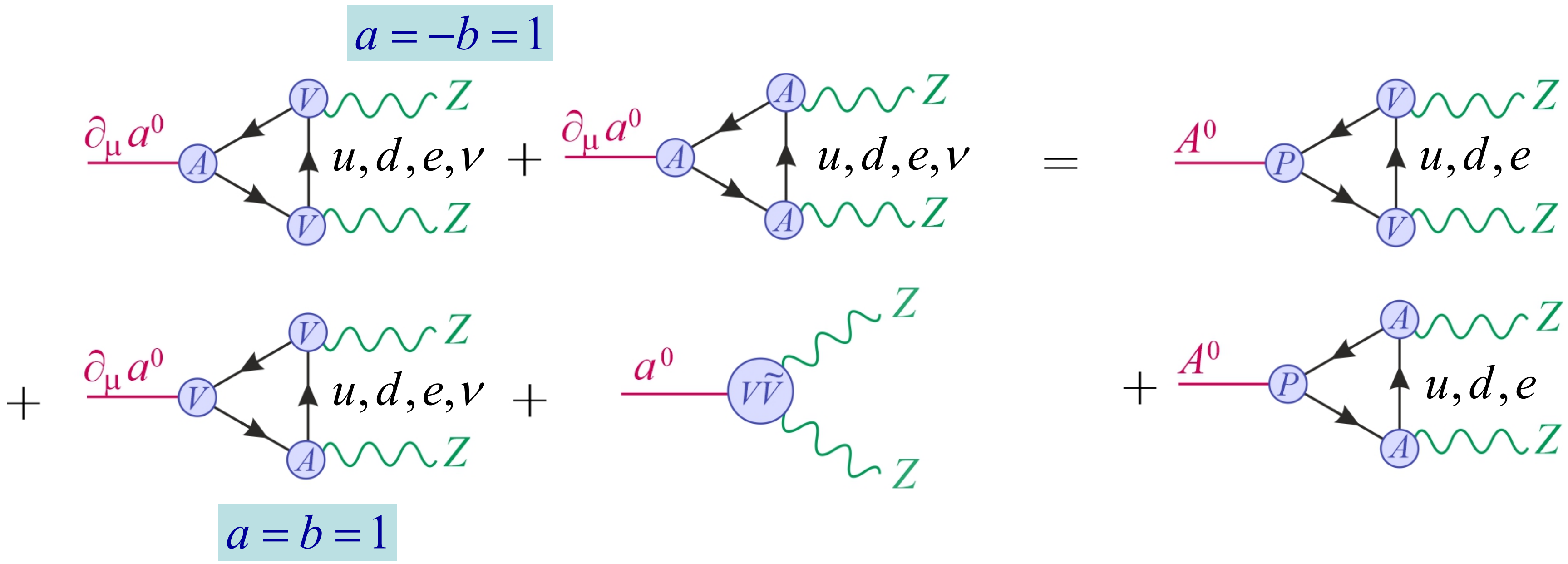
D. Equivalence of the linear and polar representations



Polar representation

Linear representation

D. Equivalence of the linear and polar representations



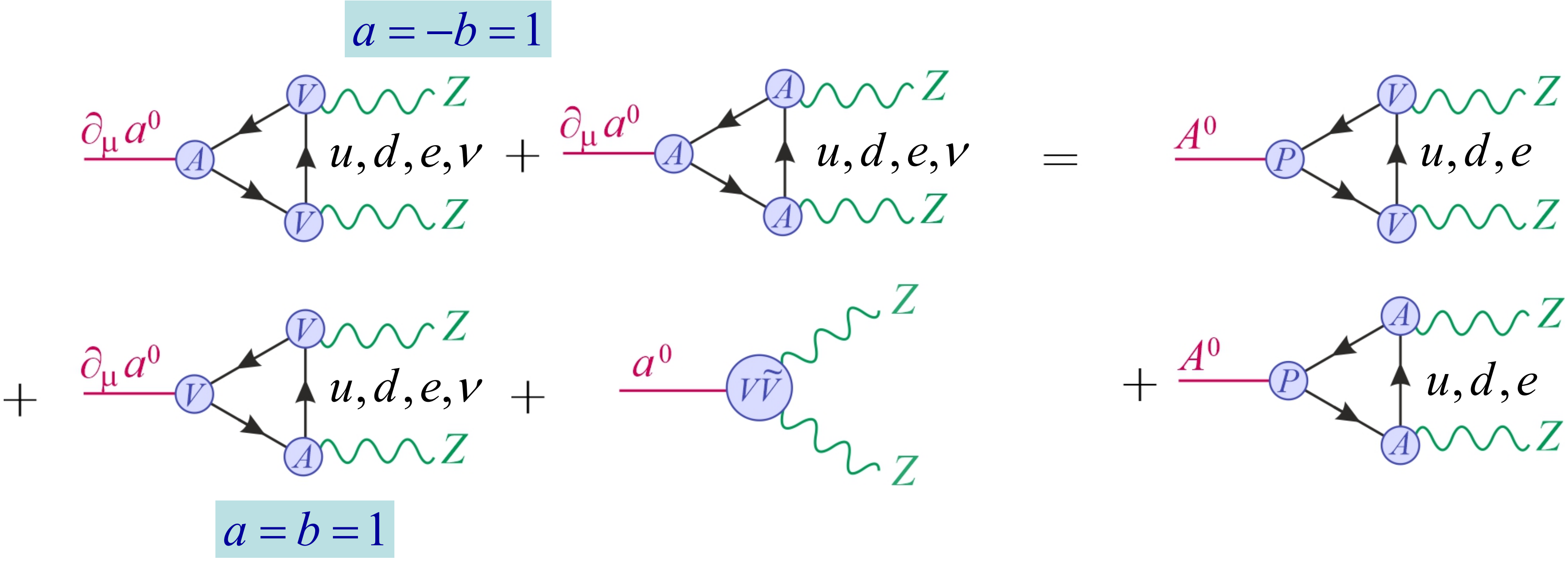
Massive AVV triangle, with the anomaly anywhere:

$$i(q_1+q_2)_\alpha T^{\alpha\beta\gamma}_{AVV} = \frac{1}{4\pi^2}(a-b)\varepsilon^{\beta\gamma\mu\nu}q_{1\mu}q_{2\nu} + 2imT^{\beta\gamma}_{PVV}$$

$$-i(q_1)_\beta T^{\alpha\beta\gamma}_{AVV} = \frac{1}{4\pi^2}(1+b)\varepsilon^{\gamma\alpha\mu\nu}q_{1\mu}q_{2\nu}$$

$$-i(q_2)_\gamma T^{\alpha\beta\gamma}_{AVV} = \frac{1}{4\pi^2}(1-a)\varepsilon^{\alpha\beta\mu\nu}q_{1\mu}q_{2\nu}$$

D. Equivalence of the linear and polar representations



Sutherland-Veltman is violated by VAV triangles:

$$i(q_1+q_2)_\alpha T^{\alpha\beta\gamma}_{AVV}\,|_{m\to\infty} = \frac{1}{4\pi^2}(a-b-2)\varepsilon^{\beta\gamma\mu\nu}q_{1\mu}q_{2\nu}$$

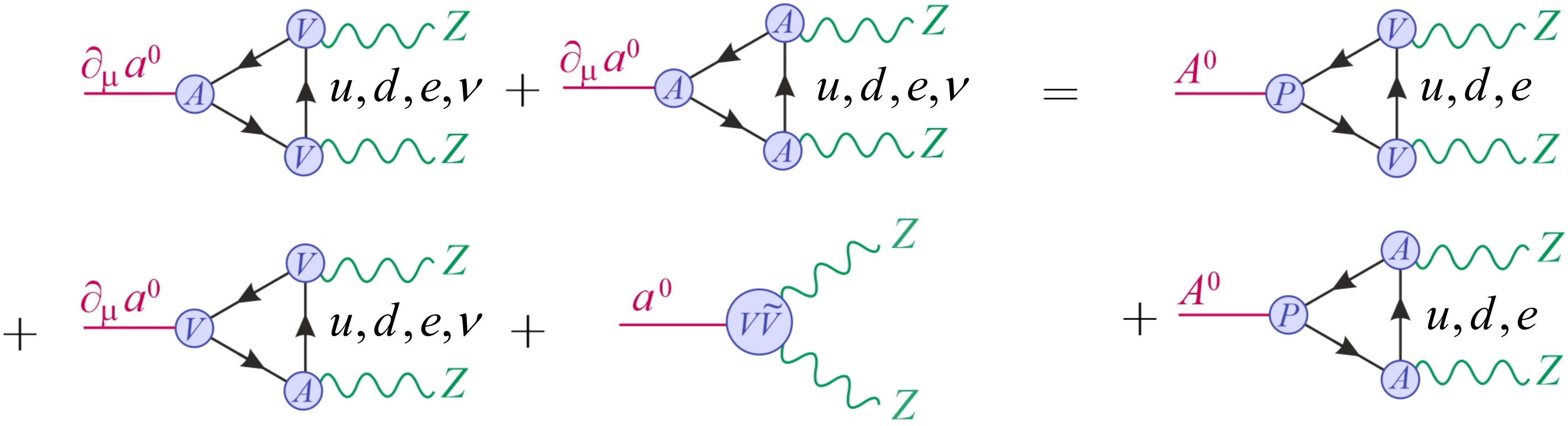
$$-i(q_1)_\beta T^{\alpha\beta\gamma}_{AVV}\,|_{m\to\infty} = \frac{1}{4\pi^2}(1+b)\varepsilon^{\gamma\alpha\mu\nu}q_{1\mu}q_{2\nu}$$

$$-i(q_2)_\gamma T^{\alpha\beta\gamma}_{AVV}\,|_{m\to\infty} = \frac{1}{4\pi^2}(1-a)\varepsilon^{\alpha\beta\mu\nu}q_{1\mu}q_{2\nu}$$

All three vanish only for $a=-b=1$

D. Equivalence of the linear and polar representations

$a=-b=1$



Massive AAA triangle, with the anomaly anywhere:

$$i(q_1+q_2)_\alpha T^{\alpha\beta\gamma}_{AAA} = \frac{1}{4\pi^2}(a-b)\varepsilon^{\beta\gamma\mu\nu}q_{1\mu}q_{2\nu} + 2imT^{\beta\gamma}_{PAA}$$

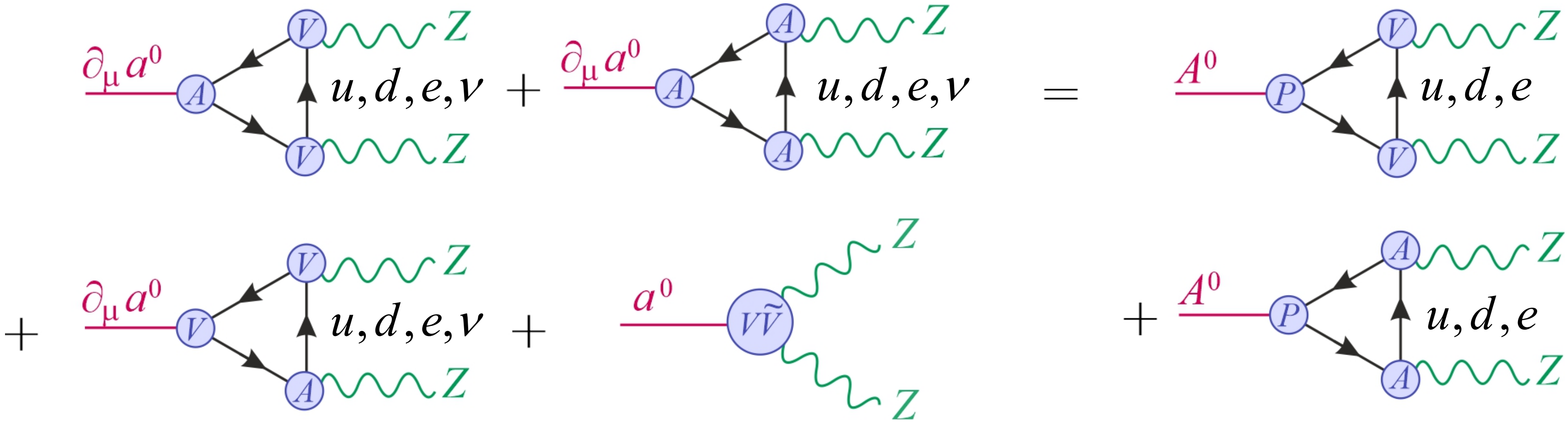
$$-i(q_1)_\beta T^{\alpha\beta\gamma}_{AAA} = \frac{1}{4\pi^2}(1+b)\varepsilon^{\gamma\alpha\mu\nu}q_{1\mu}q_{2\nu} + 2imT^{\alpha\gamma}_{PAA}$$

$$-i(q_2)_\gamma T^{\alpha\beta\gamma}_{AAA} = \frac{1}{4\pi^2}(1-a)\varepsilon^{\alpha\beta\mu\nu}q_{1\mu}q_{2\nu} + 2imT^{\alpha\beta}_{PAA}$$

Bose symmetric

D. Equivalence of the linear and polar representations

$a = -b = 1$



Sutherland-Veltman only for the Bose-symmetric anomaly:

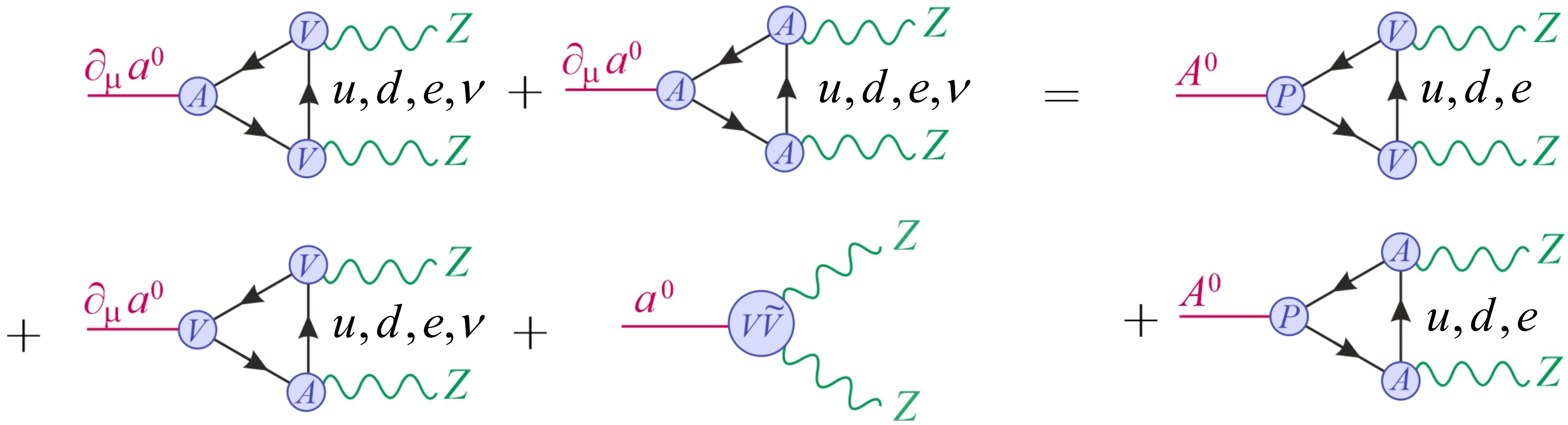
$$i(q_1+q_2)_\alpha T^{\alpha\beta\gamma}_{AAA}\,|_{m\to\infty} = \frac{1}{4\pi^2}\left(a-b-\frac{2}{3}\right)\varepsilon^{\beta\gamma\mu\nu}q_{1\mu}q_{2\nu}$$

$$-i(q_1)_\beta T^{\alpha\beta\gamma}_{AAA}\,|_{m\to\infty} = \frac{1}{4\pi^2}\left(1+b-\frac{2}{3}\right)\varepsilon^{\gamma\alpha\mu\nu}q_{1\mu}q_{2\nu}$$

$$-i(q_2)_\gamma T^{\alpha\beta\gamma}_{AAA}\,|_{m\to\infty} = \frac{1}{4\pi^2}\left(1-a-\frac{2}{3}\right)\varepsilon^{\alpha\beta\mu\nu}q_{1\mu}q_{2\nu}$$

All three vanish only for $a = -b = 1/3$

D. Equivalence of the linear and polar representations



The anomalies in the AVV, VAV, AAA triangles kill the local terms

The ambiguous parameters α and β disappear!

Ensures the vanishing as $m \to 0$ (hence ν disappears)

Only the mass-dependent parts of the AVV and AAA triangles survive

$SU(2)_L \otimes U(1)_Y$ is broken via the chiral fermion masses.

The AAA and VAV triangles violate Sutherland Theorem:

When $m \to \infty$, effective interactions $\neq$ local anomalous terms

F. Extension to DFSZ axions

Dine, Fischler, Srednicki '81
Zhitnitskii '80

Tuned by the EW scale, the PQ axion couplings are too large.

DFSZ axion from a THDM extended with a complex scalar singlet:

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \qquad \Rightarrow \; v_S^2 = -\mu^2 / \lambda$$

$$V(\phi, \Phi_1, \Phi_2) = \phi^\dagger \phi (a \Phi_1^\dagger \Phi_1 + b \Phi_2^\dagger \Phi_2) + [\lambda_{12}^2 \phi^2 \Phi_1^\dagger \Phi_2 + h.c.]$$

In the polar representation, GB cancel out except for

$$V(\eta_S, \eta_1, \eta_2) = \frac{1}{4} \lambda_{12}^2 v_S^2 v_1 v_2 \cos\left(\frac{\eta_2}{v_2} - \frac{\eta_1}{v_1} + 2\frac{\eta_S}{v_S} \right)$$

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$$V(\phi)=\mu^2\phi^\dagger\phi+\lambda(\phi^\dagger\phi)^2 \qquad \Rightarrow \; v_S^2=-\mu^2/\lambda$$

$$V(\phi,\Phi_1,\Phi_2)=\phi^\dagger\phi(a\Phi_1^\dagger\Phi_1+b\Phi_2^\dagger\Phi_2)+[\lambda_{12}^2\phi^2\Phi_1^\dagger\Phi_2+h.c.]$$

The axion is dominantly in the singlet:

$$\left(\pi^0\sim\frac{\eta_2}{v_2}-\frac{\eta_1}{v_1}+2\frac{\eta_S}{v_S}\right)\times\left(G^0\sim\frac{v_1\eta_1+v_2\eta_2}{v}\right)=\left(a^0\sim\eta_S+\frac{v_2\eta_1-v_1\eta_2}{v_S}\sin 2\beta\right)$$

The PQ charges of the scalars are thus ($\varphi_{polar}=\exp(i\eta_\varphi/v_\varphi)\varphi_{scalar}$)

$$PQ(\Phi_1,\Phi_2,\phi)=(2\cos^2(\beta),\,-2\sin^2(\beta),\,1)$$

All the previous results identical, but for $v\rightarrow v_S/\sin 2\beta$.

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The PQ charges of the scalars are thus ($\varphi_{polar}=\exp(i\eta_\varphi/v_\varphi)\varphi_{scalar}$)

$$PQ(\Phi_1,\Phi_2,\phi)=(\ x,\ -1/x,\ 1/2(x+1/x))$$

All the previous results identical, but for $v\to v_S/\sin 2\beta$.

Turning on $\mathcal{B}$ and $\mathcal{L}$ violation

A. The $\mathcal{B}$ and $\mathcal{L}$ ambiguities in the PQ charges

Fermionic PQ charges are ambiguous.

$$PQ(\Phi_1) = x \qquad PQ(q_L) = \alpha \qquad PQ(\ell_L) = \beta$$

$$PQ(\Phi_2) = -\frac{1}{x} \qquad PQ(u_R) = \alpha + x \qquad PQ(e_R) = \beta + \frac{1}{x}$$

$$PQ(d_R) = \alpha + \frac{1}{x}$$

Originates in the $\mathcal{B}$ and $\mathcal{L}$ invariance of the Yukawa couplings.

The incorrect way to use these parameters:

Set them to some value, $\alpha = \beta = 0$ say, which forbids for example:

$$\mathcal{L}^{eff}_{Majo} = \frac{1}{M}(\overline{\ell}^C_L \Phi_1^T)(\Phi_1 \ell_L) \ \rightarrow PQ(\mathcal{L}^{eff}_{Majo}) = 2(\beta + x)$$

Yet, in the linear representation, adding this operator is harmless!

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Fermionic PQ charges are ambiguous.

$$PQ(\Phi_1) = x \qquad PQ(q_L) = \alpha \qquad PQ(\ell_L) = \beta$$

$$PQ(\Phi_2) = -\frac{1}{x} \qquad PQ(u_R) = \alpha + x \qquad PQ(e_R) = \beta + \frac{1}{x}$$

$$PQ(d_R) = \alpha + \frac{1}{x}$$

Originates in the $\mathcal{B}$ and $\mathcal{L}$ invariance of the Yukawa couplings.

The correct way to use these parameters:

Keep them free to accommodate possible $\mathcal{B}$ and/or $\mathcal{L}$ violations.

If two models differ by their values: equivalent phenomenology!

What happens with too much $\mathcal{B}$ and/or $\mathcal{L}$ violations?

B. Lepton number violation: Introducing neutrino masses

Adding a seesaw of Type I: $\mathcal{L}^{\nu}_{Type\,I} = \overline{\nu}_R \Phi_i \ell_L + \overline{\nu}^C_R M \nu_R$

In the presence of M, the $U(1)_1 \otimes U(1)_2$ symmetry exists only if

$$i = 1 : PQ(\nu_R) = 0 = \beta + x \rightarrow \beta = -x$$

$$i = 2 : PQ(\nu_R) = 0 = \beta - 1/x \rightarrow \beta = 1/x$$

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Merging the seesaw with DFSZ: $\mathcal{L}^{\nu}_{\nu DFSZ} = \overline{\nu}_R \Phi_i \ell_L + \overline{\nu}^C_R \lambda \nu_R \phi$

This unifies two mechanisms: ϕ = Majoron = axion.

Clarke, Volkas '16

$i=1$	ϕ	Φ_1	Φ_2	q_L	u_R	d_R	ℓ_L	e_R	ν_R
$U(1)_1$	$+1/2$	1	0	α	$\alpha+1$	α	$-5/4$	$-5/4$	$-1/4$
$U(1)_2$	$-1/2$	0	1	$1/3-\alpha$	$1/3-\alpha$	$-\alpha-2/3$	$+1/4$	$-3/4$	$+1/4$

The PQ symmetry is defined only after it is broken as

$$PQ = xU_1 - U_2 / x$$

$\mathcal{L}$ as we know it no longer exists; it is not in $U(1)_1 \otimes U(1)_2$.

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This unifies two mechanisms: ϕ = Majoron = axion.

$\mathcal{L}$ as we know it no longer exists; it is not in $U(1)_1 \otimes U(1)_2$.

ν_R are no longer neutral:

$$i = 1: PQ(\nu_R) = -PQ(\phi)/2 = \beta + x \Rightarrow \beta = -(5x + 1/x)/4$$

$$i = 2: PQ(\nu_R) = -PQ(\phi)/2 = \beta - 1/x \Rightarrow \beta = -(x - 3/x)/4$$

B. Lepton number violation: Introducing neutrino masses

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$$i = 2 : PQ(\nu_R) = -PQ(\phi)/2 = \beta - 1/x \Rightarrow \beta = -(x - 3/x)/4$$

Adding a seesaw of Type II:

$$\mathcal{L}^{\nu}_{Type\,II} = -\overline{\ell}_L^C Y_\Delta \ell_L \Delta - \lambda_{\nu 1} \phi^2 \Phi_1^\dagger \Phi_2 - \lambda_{\nu 2} \phi \Phi_1^T \Delta^\dagger \Phi_2 - \lambda_{\nu 3} \Phi_1^T \Delta^\dagger \Phi_2$$

Bertolini, Santamaria ’91
Bertolini, L. Di Luzio, H. Kolešová and M. Malinský ’15

$$\lambda_{\nu 2} = 0 : PQ(\Phi_1, \Phi_2, \phi, \Delta) = (x,\ -1/x,\ 1/2\,(x + 1/x),\ x - 1/x) + \mathcal{O}(v_\Delta / v)$$

$$\lambda_{\nu 3} = 0 : PQ(\Phi_1, \Phi_2, \phi, \Delta) = (x,\ -1/x,\ 1/2\,(x + 1/x),\ 3/2\,x - 1/2x) + \mathcal{O}(v_\Delta / v)$$

$$G^0 \sim v_1 \eta_1 + v_2 \eta_2 + 2 v_\Delta \eta_\Delta$$

B. Lepton number violation: Introducing neutrino masses

Adding a seesaw of Type I: $\mathcal{L}^{\nu}_{Type\,I} = \overline{\nu}_R \Phi_i \ell_L + \overline{\nu}^C_R M \nu_R$

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Adding a seesaw of Type II:

$$\mathcal{L}^{\nu}_{Type\,II} = -\overline{\ell}^C_L Y_\Delta \ell_L \Delta - \lambda_{\nu 1} \phi^2 \Phi_1^\dagger \Phi_2 - \lambda_{\nu 2} \phi \Phi_1^T \Delta^\dagger \Phi_2 - \lambda_{\nu 3} \Phi_1^T \Delta^\dagger \Phi_2$$

$$\lambda_{\nu 2} = 0 : \beta = (3x + 1/x)/4$$

$$\lambda_{\nu 3} = 0 : \beta = (-x + 1/x)/2$$

In all those cases: - the PQ charges differ only in the value of β,

- the low-energy axion phenomenology is identical.

C. Baryon number violation

Explicit $\mathcal{B}$-violating couplings are tightly restricted by $U(1)_1 \otimes U(1)_2$:

$$\mathcal{H}_{eff} = \frac{1}{\Lambda^2}\left(\underbrace{\ell_L q_L^3}_{3\alpha+\beta} + \underbrace{e_R u_R^2 d_R}_{3\alpha+\beta+2\tilde{x}} + \underbrace{e_R u_R q_L^2 + \ell_L q_L d_R u_R}_{3\alpha+\beta+\tilde{x}}\right)$$

$$\tilde{x} \equiv x + 1/x$$

$$\mathcal{H}_{eff} = \frac{1}{\Lambda^5}\left(\underbrace{e_R \ell_L^2 u_R^3}_{3\alpha+3\beta+2x+\tilde{x}} + \underbrace{\ell_L^3 q_L u_R^2}_{3\alpha+3\beta+2x} + \underbrace{d_R^4 u_R^2}_{6\alpha+2/x+2\tilde{x}} + \underbrace{d_R^3 u_R q_L^2}_{6\alpha+2/x+\tilde{x}} + \underbrace{d_R^2 q_R^4}_{6\alpha+2/x}\right)$$

If β is fixed by ν masses, at most two $\mathcal{B}$-violating operators.

Or, the other operators are higher-dimensional:

They require some ϕ insertions, since $PQ(\phi) = \tilde{x}/2$

C. Baryon number violation

Explicit $\mathcal{B}$-violating couplings are tightly restricted by $U(1)_1 \otimes U(1)_2$:

At most two leading operators allowed simultaneously.

Others are very suppressed when $v_S \ll \Lambda_{\Delta\mathcal{B}}$.

Fixes one combination of α and β to some value.

Dynamical $\mathcal{B}$-violation typically occur in GUT settings:

$$\begin{aligned}\delta\mathcal{L}_{Jac} &= \frac{a^0}{16\pi^2 v} g_s^2 \mathcal{N}_C G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \\ &+ \frac{a^0}{16\pi^2 v} g^2 \mathcal{N}_L W^i_{\mu\nu} \tilde{W}^{i,\mu\nu} \\ &+ \frac{a^0}{16\pi^2 v} g'^2 \mathcal{N}_Y B_{\mu\nu} \tilde{B}^{\mu\nu}\end{aligned}$$

$$\begin{aligned}\mathcal{N}_C &= \frac{1}{2}\left(x + \frac{1}{x}\right) \\ \mathcal{N}_L &= -\frac{1}{2}(3\alpha + \beta) \\ \mathcal{N}_Y &= \frac{1}{2}(3\alpha + \beta) + \frac{4}{3}x + \frac{1}{3x} + \frac{1}{x}\end{aligned}$$

$$\mathcal{N}_C = \mathcal{N}_L = 5/3\mathcal{N}_Y \Rightarrow 3\alpha + \beta = -x - 1/x$$

C. Baryon number violation

Explicit $\mathcal{B}$-violating couplings are tightly restricted by $U(1)_1 \otimes U(1)_2$:

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Dynamical $\mathcal{B}$-violation typically occur in GUT settings:

$$\mathcal{N}_C = \mathcal{N}_L = 5/3\mathcal{N}_Y \Rightarrow 3\alpha + \beta = -x - 1/x$$

Dynamical $\mathcal{B}$-violation always occur via electroweak instantons:

$$\mathcal{L}_{eff} \sim (\ell_L q_L^3)^3 \Rightarrow 3\alpha + \beta = 0$$

At this level, it is difficult to reconcile all the constraints!

C. What happens if too much $\mathcal{B}$ and/or $\mathcal{L}$ violation is present?

The $U(1)_1 \otimes U(1)_2$ symmetry cannot be exact.

The axion becomes massive,

All vacua are no longer equivalent,

The $\mathcal{B}$ and/or $\mathcal{L}$ violating effects move the axion to the true vacuum

Strong CP is solved only if that drive is weaker than that of QCD

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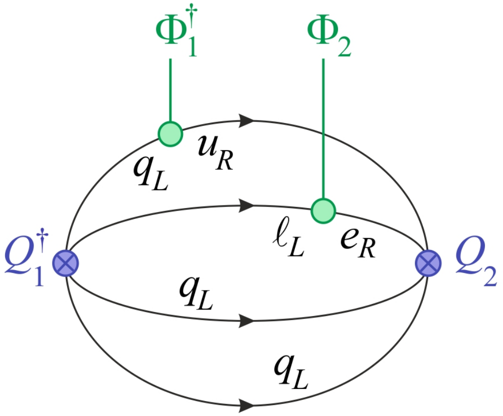
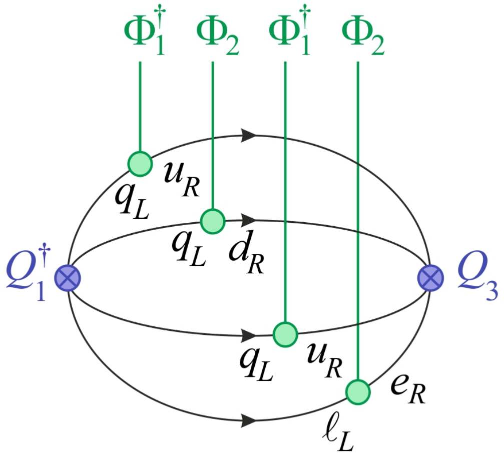
Example 1: Imagine several Weinberg operators are present



$$\mathcal{L}_{eff} = \lambda_{eff}\Phi_1^\dagger\Phi_2 \to m_a^2 \sim \Lambda^2$$



$$\mathcal{L}_{eff} = \lambda_{eff}(\Phi_1^\dagger\Phi_2)^2 \to m_a^2 \sim v^2$$

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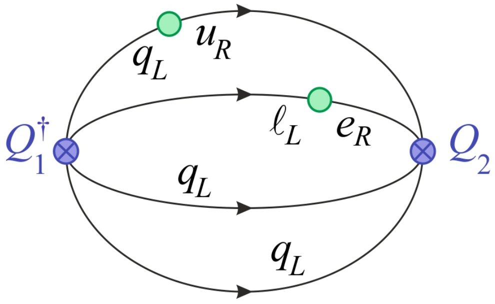
Example 1: Imagine several Weinberg operators are present

Without PQ, $\psi \to \exp\left\{iPQ(\psi)a^0/v\right\}\psi$ fails to remove the axion.



$$\sim \langle \Omega | Q_1^\dagger Q_2 | \Omega \rangle \exp(i\lambda a^0/v)$$

$$\sim \left|\langle 0 | Q_1^\dagger Q_2 | 0 \rangle\right| \cos(\lambda(a^0+\omega)/v+\theta_{12})$$

To be compared to $\left|\langle 0 | aG\tilde{G} | 0 \rangle\right| \sim m_\pi^2 f_\pi^2 \cos((a^0+\omega)/v+\theta_{QCD})$

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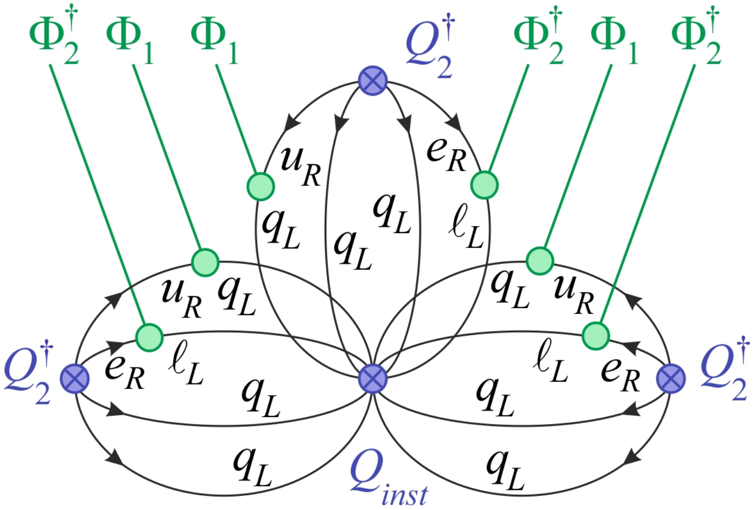
Example 2: Weinberg operators and EW instantons



$$\mathcal{L}_{eff} = \lambda_{eff}(\Phi_1^\dagger \Phi_2)^3 \rightarrow m_a^2 \sim v^4 / \Lambda^2$$

Much smaller than QCD...

Except at high temperature!

Conclusion

Three main messages:

The PQ symmetry is entangled with $\mathcal{B}$ and $\mathcal{L}$

Some ambiguities in the fermion PQ charges are present.

Low-energy axion couplings are independent of these ambiguities

Freedom to include $\mathcal{B}$ and $\mathcal{L}$ in the PQ current

Beware of the anomalous vector couplings $\partial_\mu a^0 \overline{\psi}_f \gamma^\mu \psi_f$.

These ambiguities are crucial to accommodate explicit $\mathcal{B}$ and/or $\mathcal{L}$ violation

Adding the seesaws to the PQ, DFSZ, SU(5),... axion is possible!

Permits to relate seemingly different models:

Equivalence of DFSZ, DFSZ + Seesaw type I, II, νDFSZ

In practice: - Be careful ruling out a coupling based on the PQ charges!

- Instead, start by identifying the global symmetries
- Define the PQ symmetry in the broken phase, via its GB