Seminar@Ljubljana, April 1 2021

THE HEBREW UNIVERSITY OF JERUSALEM

Precise Calculation of the Decay Rate of False Vacuum with Multi-Field Bounce

Yutaro Shoji

(Racah Institute of Physics, Hebrew University of Jerusalem)

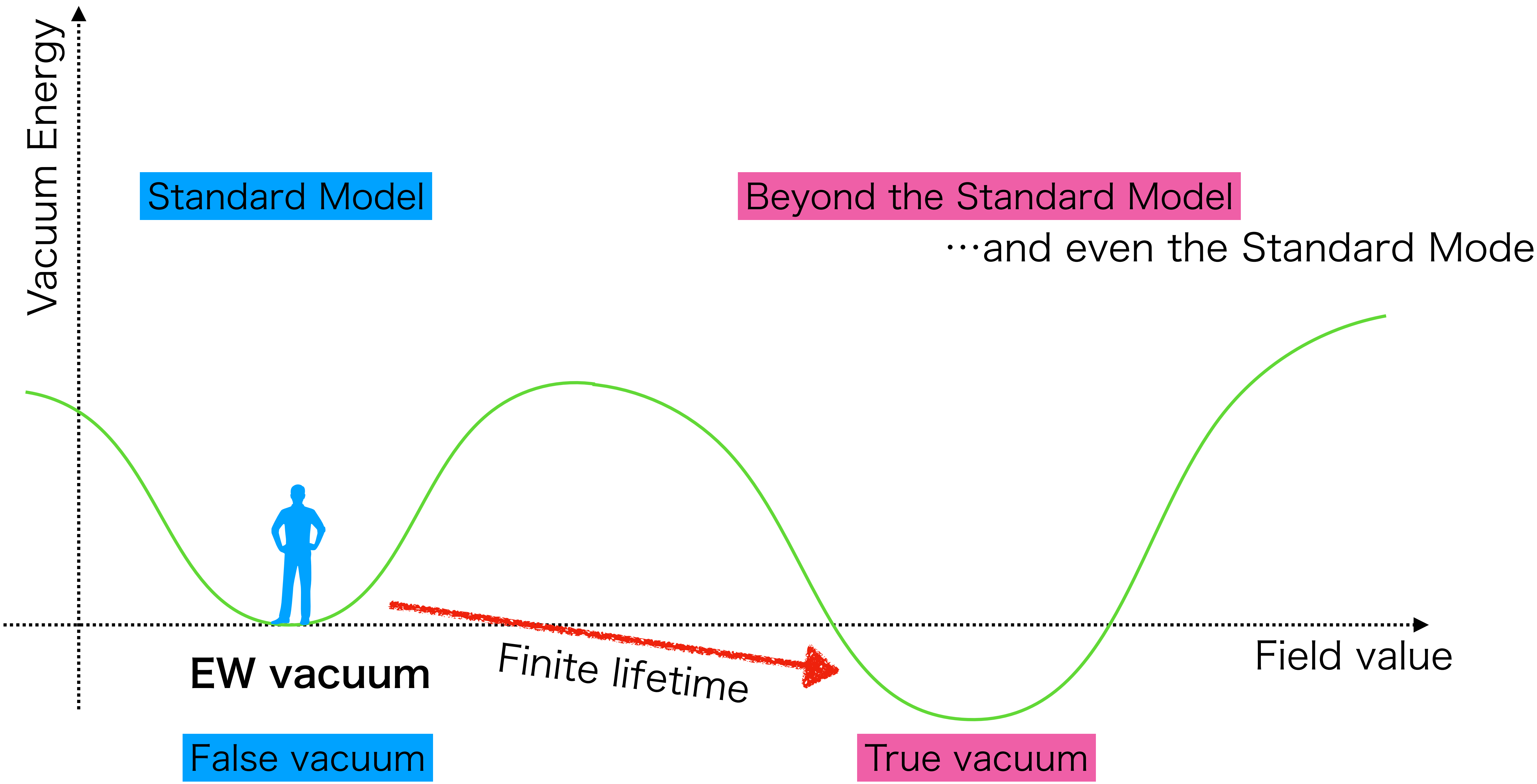
Phys. Lett. B771 (2017) 281; M. Endo, T. Moroi, M. M. Nojiri, YS

JHEP11(2017)074; M. Endo, T. Moroi, M. M. Nojiri, YS

JHEP 11 (2020) 006; S. Chigusa, T. Moroi, YS

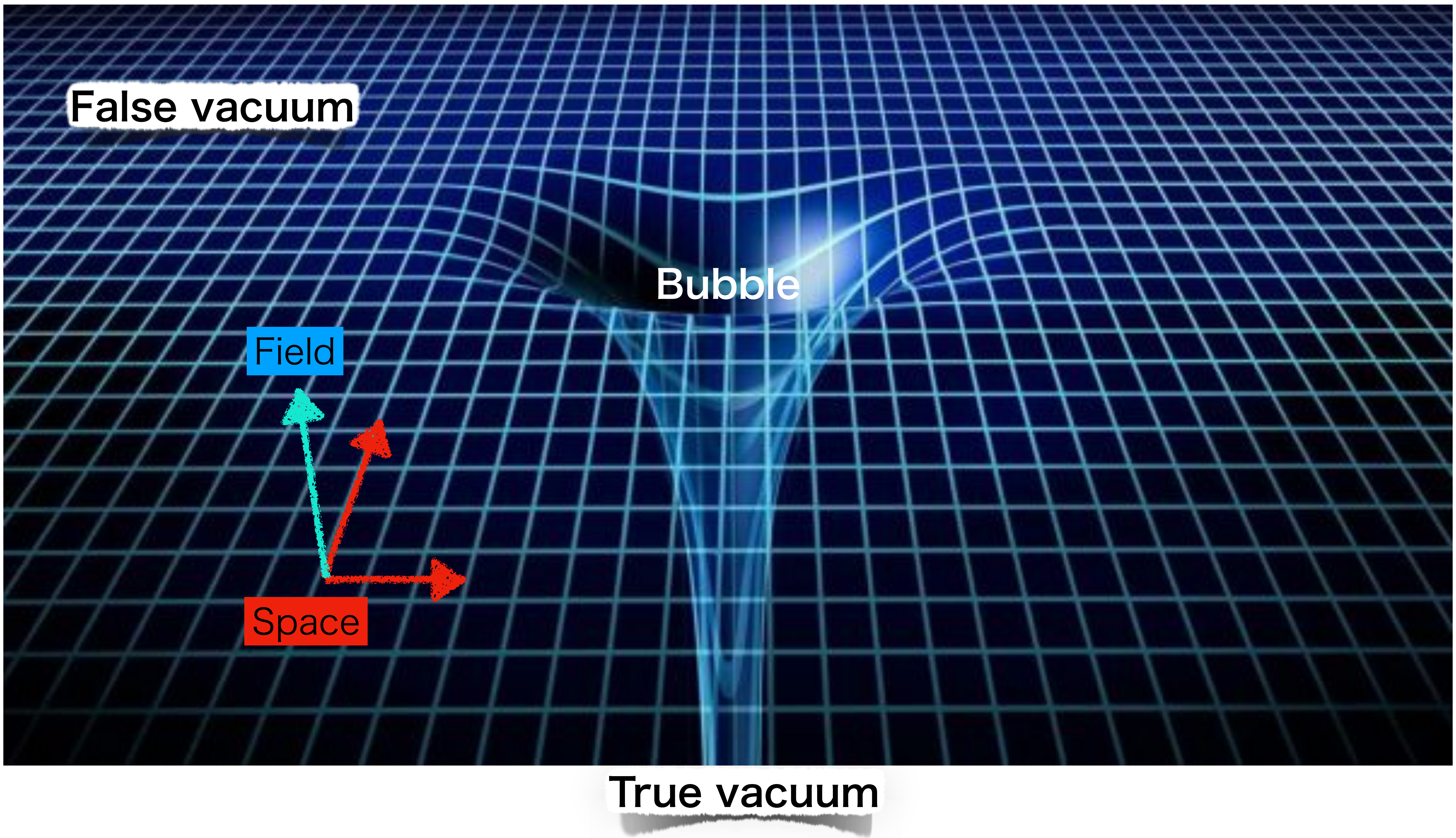
Introduction

Vacuum decay

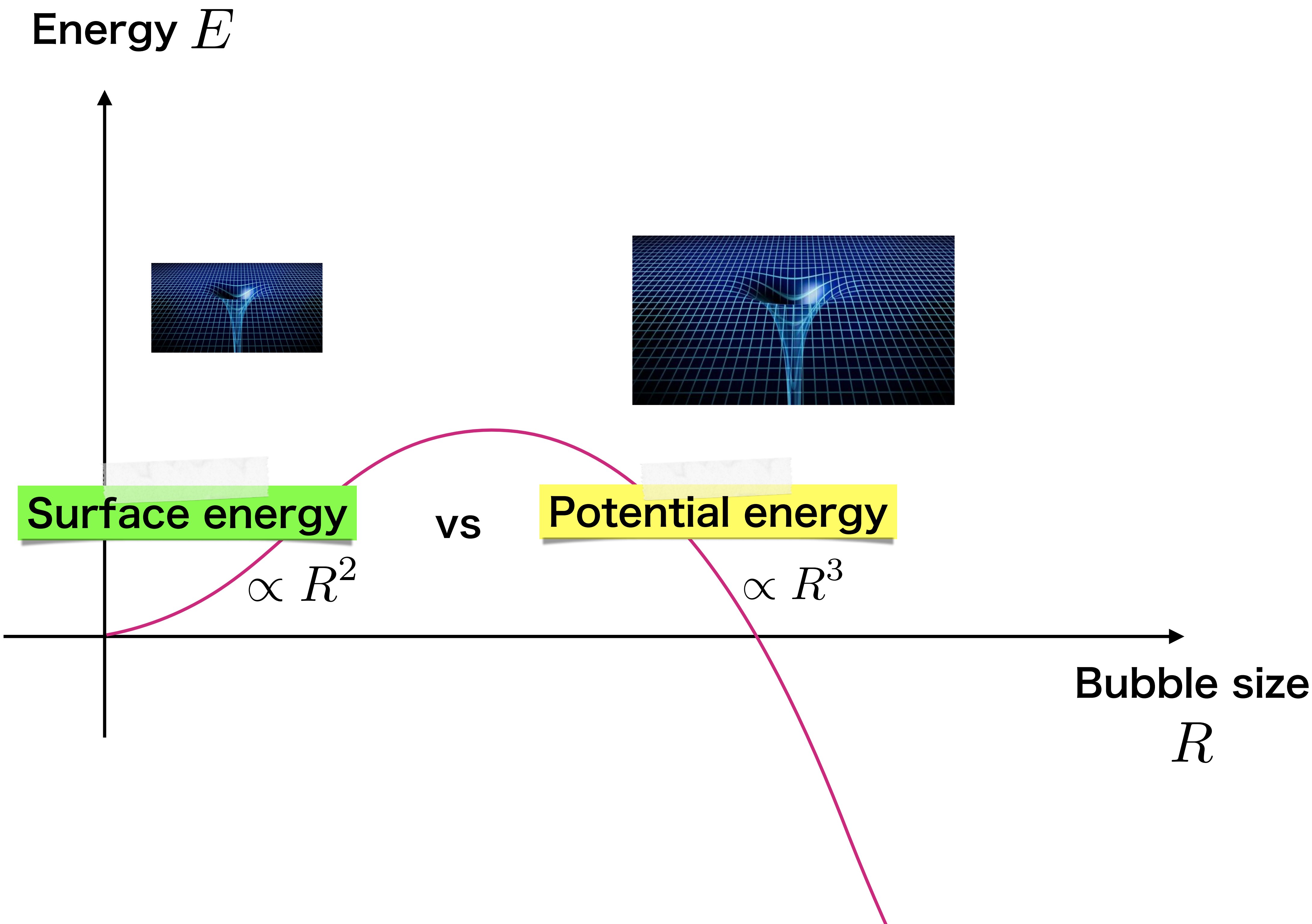


Vacuum decay

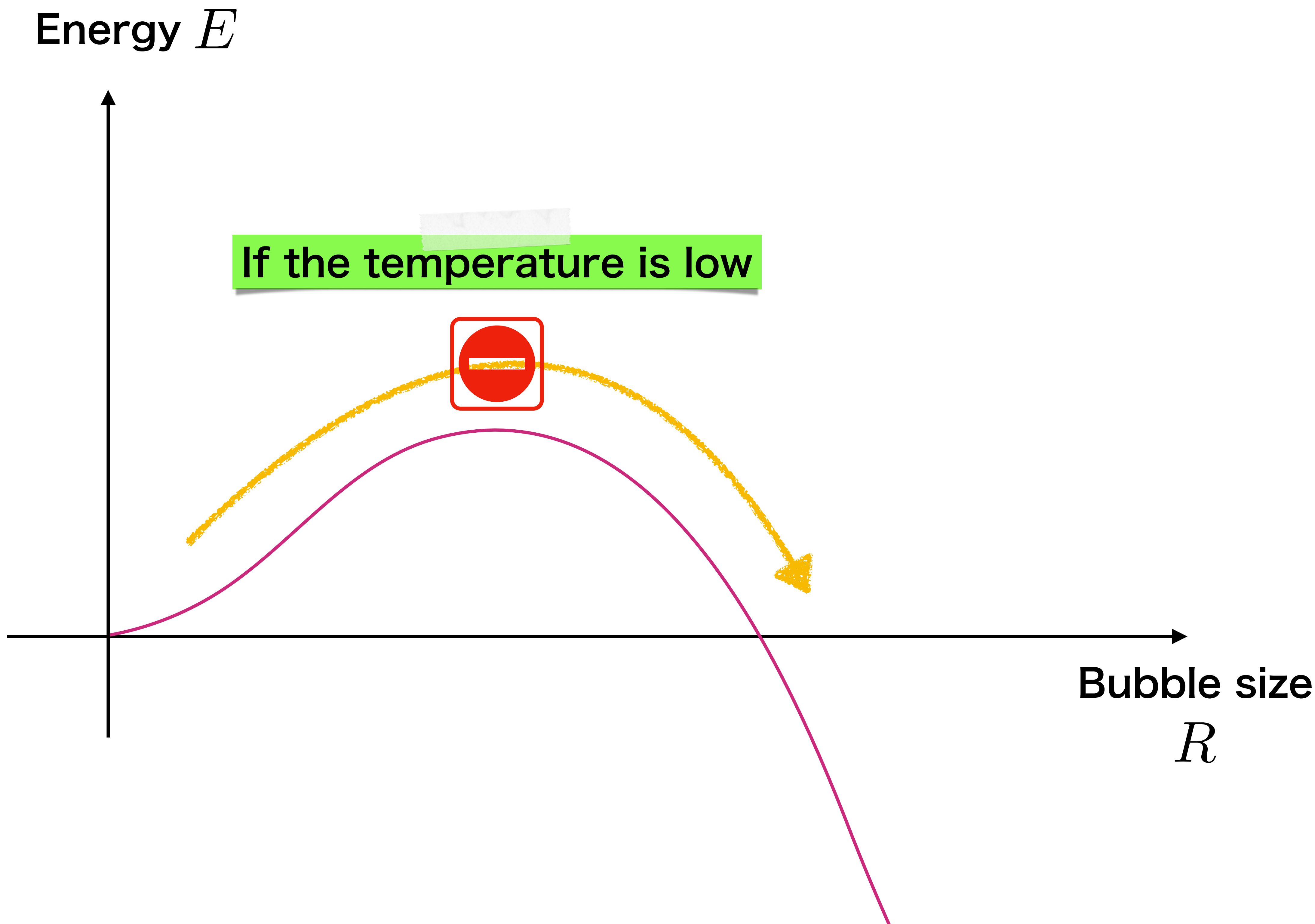
Vacuum decay starts with the nucleation of a “bubble” due to fluctuations of scalar fields



Energy barrier



Bubble nucleation rate



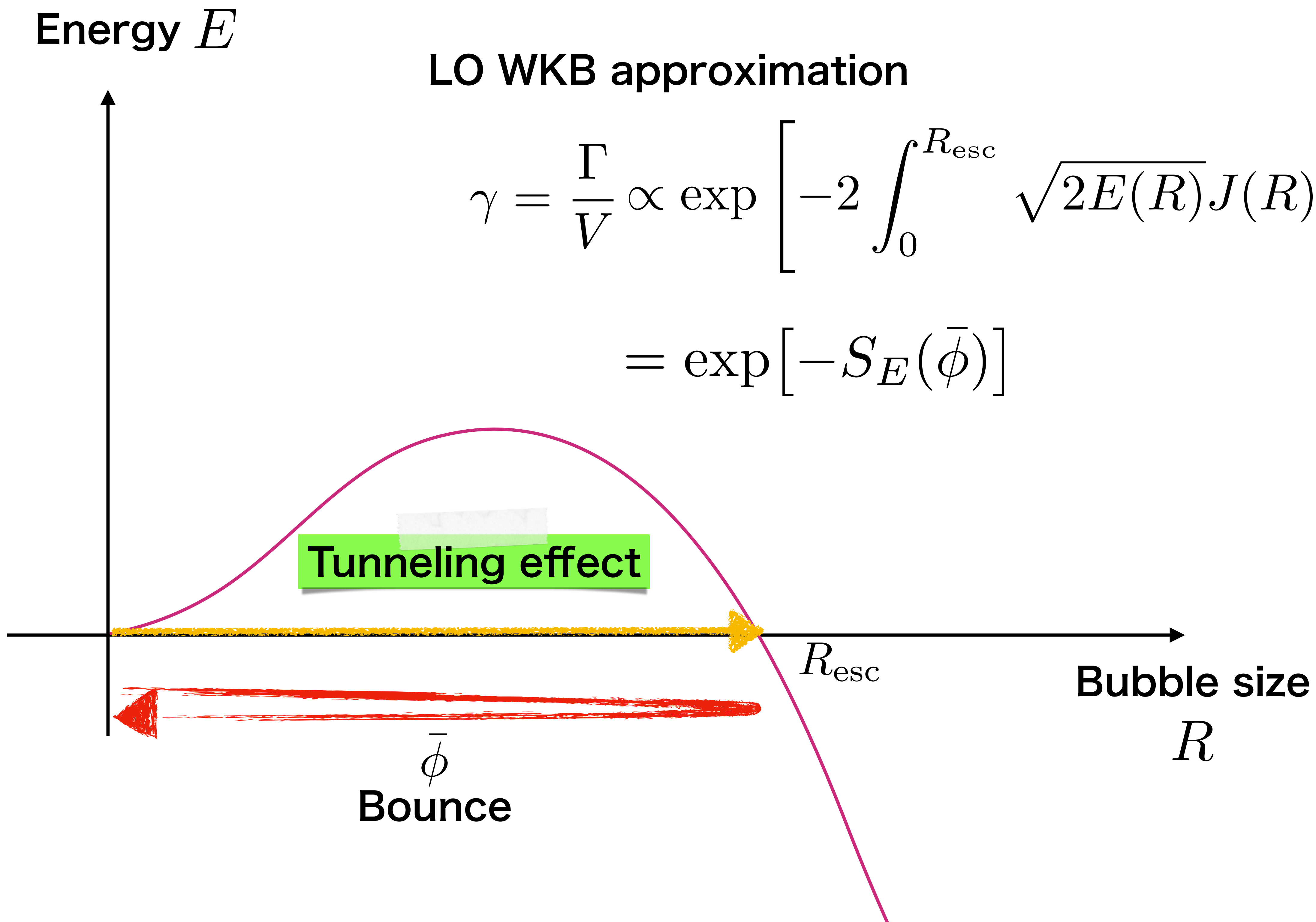
Bubble nucleation rate

[T. Banks, C. M. Bender, T. T. Wu, ’73; S. R. Coleman, ’77]

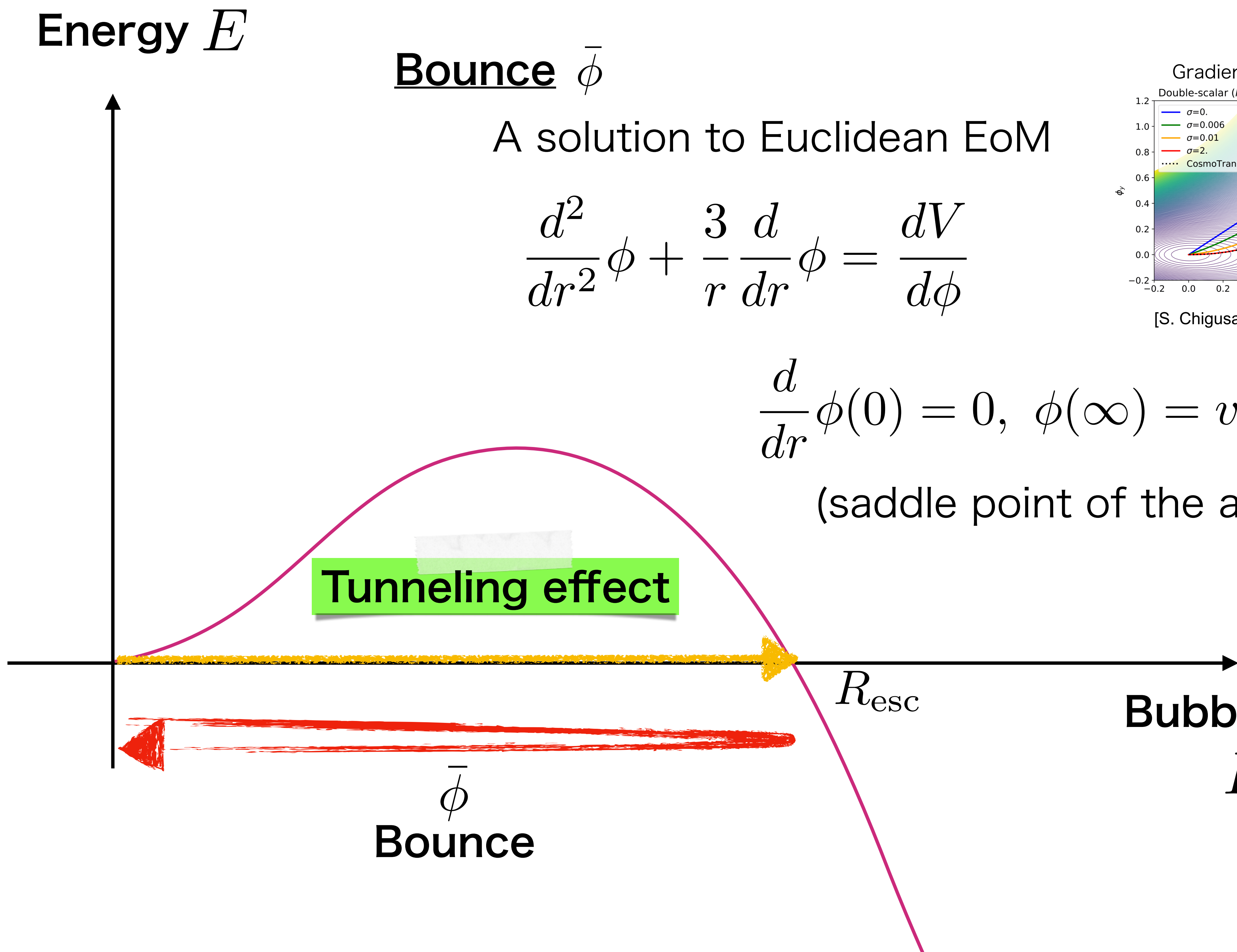
LO WKB approximation

$$\gamma = \frac{\Gamma}{V} \propto \exp\left[-2\int_0^{R_{\rm esc}} \sqrt{2E(R)}J(R)dR\right]$$

$$= \exp\left[-S_E(\bar{\phi})\right]$$



Bubble nucleation rate



Bounce $\bar{\phi}$

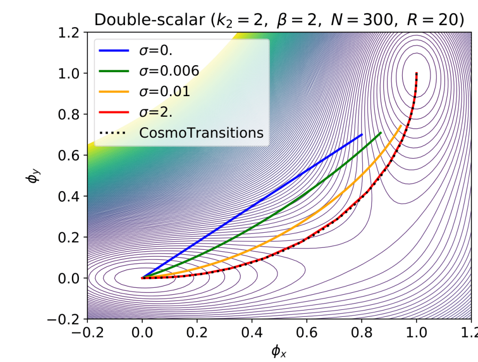
A solution to Euclidean EoM

$$\frac{d^2}{dr^2}\phi + \frac{3}{r}\frac{d}{dr}\phi = \frac{dV}{d\phi}$$

$$\frac{d}{dr}\phi(0) = 0, \ \ \phi(\infty) = v \text{ (false vacuum)}$$

(saddle point of the action)

Gradient flow method



[S. Chigusa, T. Moroi and YS, '20]

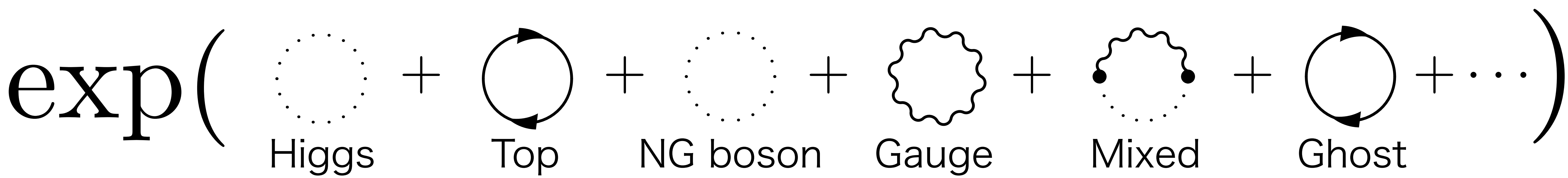
Prefactor

[C. G. Callan, S. R. Coleman, '77]

$$\gamma = Ae^{-B}$$

$$B = S_E(\bar{\phi})$$

Quantum correction to the action



$$A \sim \left(\frac{\det S''_E|_{\rm bounce}}{\det S''_E|_{\rm false}} \right)^{-1/2}$$

Dimensional analysis

$$A \sim m^4$$

Typical scale

Renormalization scale uncertainty

[M. Endo, T. Moroi, M. M. Nojiri, YS; '16]

$$V = \frac{m^2}{2}\phi^2 - \frac{A_T}{2}\phi^3 + \frac{\alpha}{8}\phi^4 + \xi\phi$$

Renormalization scale (Q) dependent

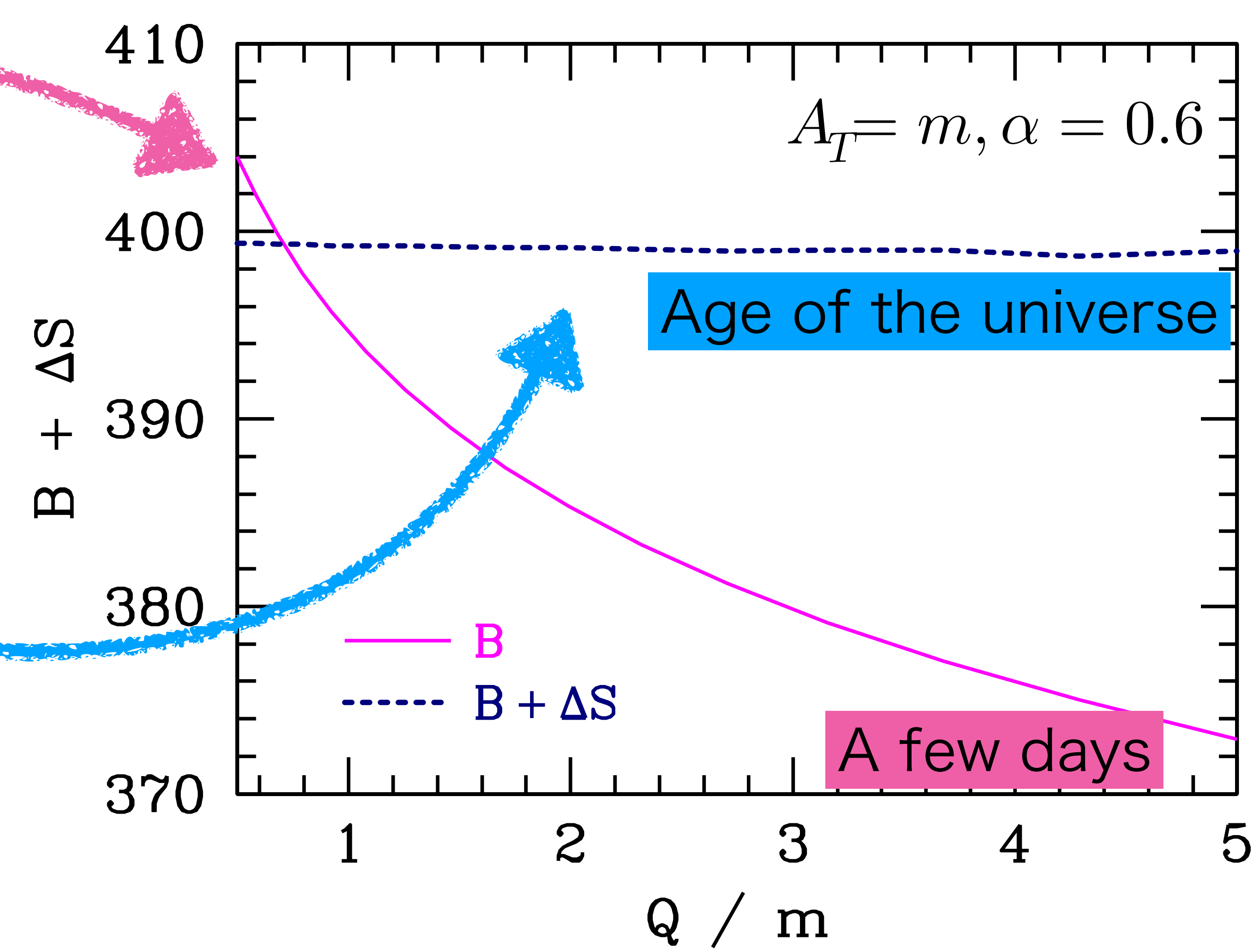
"Tree"

$$\gamma \simeq m^4 e^{-B}$$

Typical scale=m

1-loop

$$\gamma = A e^{-B} = m^4 e^{-B-\Delta S}$$

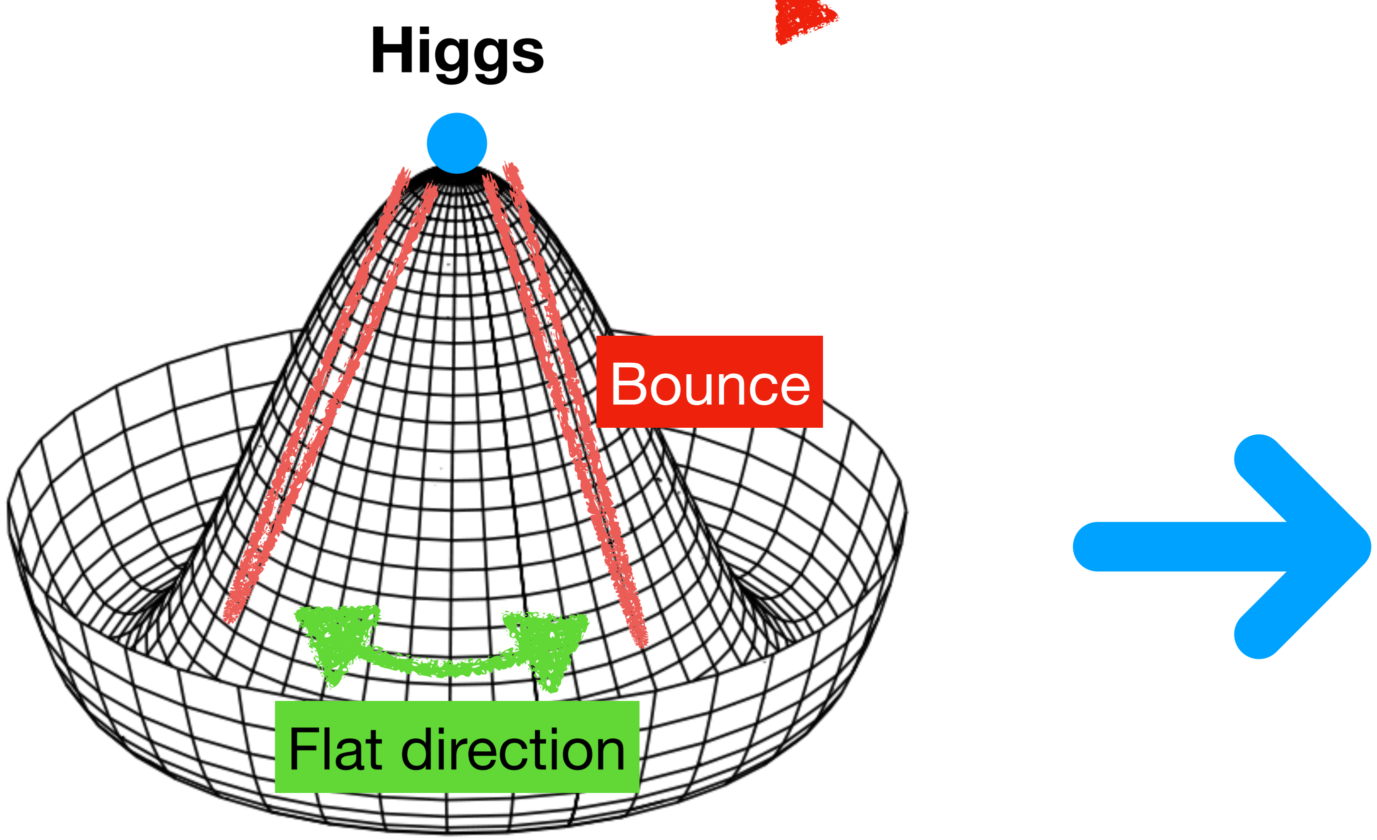


Gauge zero mode

[A. Kusenko, K. M. Lee, E. J. Weinberg; '97]

If we try to calculate the prefactor, ...

There appears gauge zero modes when a gauge symmetry is broken (only) by the bounce



$$\det S''_E|_{\text{bounce}} = 0$$

The treatment of the gauge zero modes had not been established for a long time

Gauge fixing

Bounce solution → $\bar{\phi}$ NG boson → χ

$$\mathcal{L}_{\rm GF} = (\partial_\mu A_\mu - g\bar{\phi}\chi)^2 \quad \rightarrow \quad \mathcal{L}_{\rm GF} = \frac{1}{\xi}(\partial_\mu A_\mu)^2$$

	$\mathcal{L}_{\rm GF} = (\partial_\mu A_\mu - g\bar{\phi}\chi)^2$	$\mathcal{L}_{\rm GF} = \frac{1}{\xi}(\partial_\mu A_\mu)^2$
Numerical Evaluation	✓	✗
Treatment of gauge zero modes	✗	✓
Semi-analytical Evaluation	✓	✓
	Phys. Lett. B771(2017)281 (Single-field)	JHEP11(2017)074 (Single-field)

Gauge fixing

Bounce solution → $\bar{\phi}$; NG boson → χ

$$\mathcal{L}_{\rm GF} = (\partial_\mu A_\mu - g\bar{\phi}\chi)^2 \quad \longrightarrow \quad \mathcal{L}_{\rm GF} = \frac{1}{\xi}(\partial_\mu A_\mu)^2$$

	$\mathcal{L}_{\rm GF} = (\partial_\mu A_\mu - g\bar{\phi}\chi)^2$	$\mathcal{L}_{\rm GF} = \frac{1}{\xi}(\partial_\mu A_\mu)^2$
Numerical Evaluation	✓	✗
Treatment of gauge zero modes	✗	✓
Semi-analytical Evaluation	✓	✓

✓ Much more stable and easier →

We have obtained the conversion relation
(Multi-field: JHEP 11 (2020) 006)

Phys. Lett. B771(2017)281
(Single-field)

JHEP11(2017)074
(Single-field)

Contents

Semi-analytic formula (background gauge)

Phys. Lett. B771(2017) 281

Background gauge

(ξ=1 of this gauge is used in the previous studies)

Gauge fixing function

$$\mathcal{F} = \partial_\mu A_\mu - 2\xi g(\mathrm{Re}\Phi)(\mathrm{Im}\Phi)$$

U(1)

Lagrangian

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + [(\partial_\mu + igA_\mu)\Phi^\dagger][(\partial_\mu - igA_\mu)\Phi] + V + \mathcal{L}_{\mathrm{G.F.}} + \mathcal{L}_{\mathrm{ghost}},$$

$$\mathcal{L}_{\mathrm{G.F.}} = \frac{1}{2\xi}\mathcal{F}^2, \quad \mathcal{L}_{\mathrm{ghost}} = \bar{c}\left[-\partial_\mu\partial_\mu + \xi g^2(\Phi^2 + {\Phi^\dagger}^2)\right] c.$$

$$\Phi = \frac{1}{\sqrt{2}}(\bar{\phi} + h + i\varphi)$$

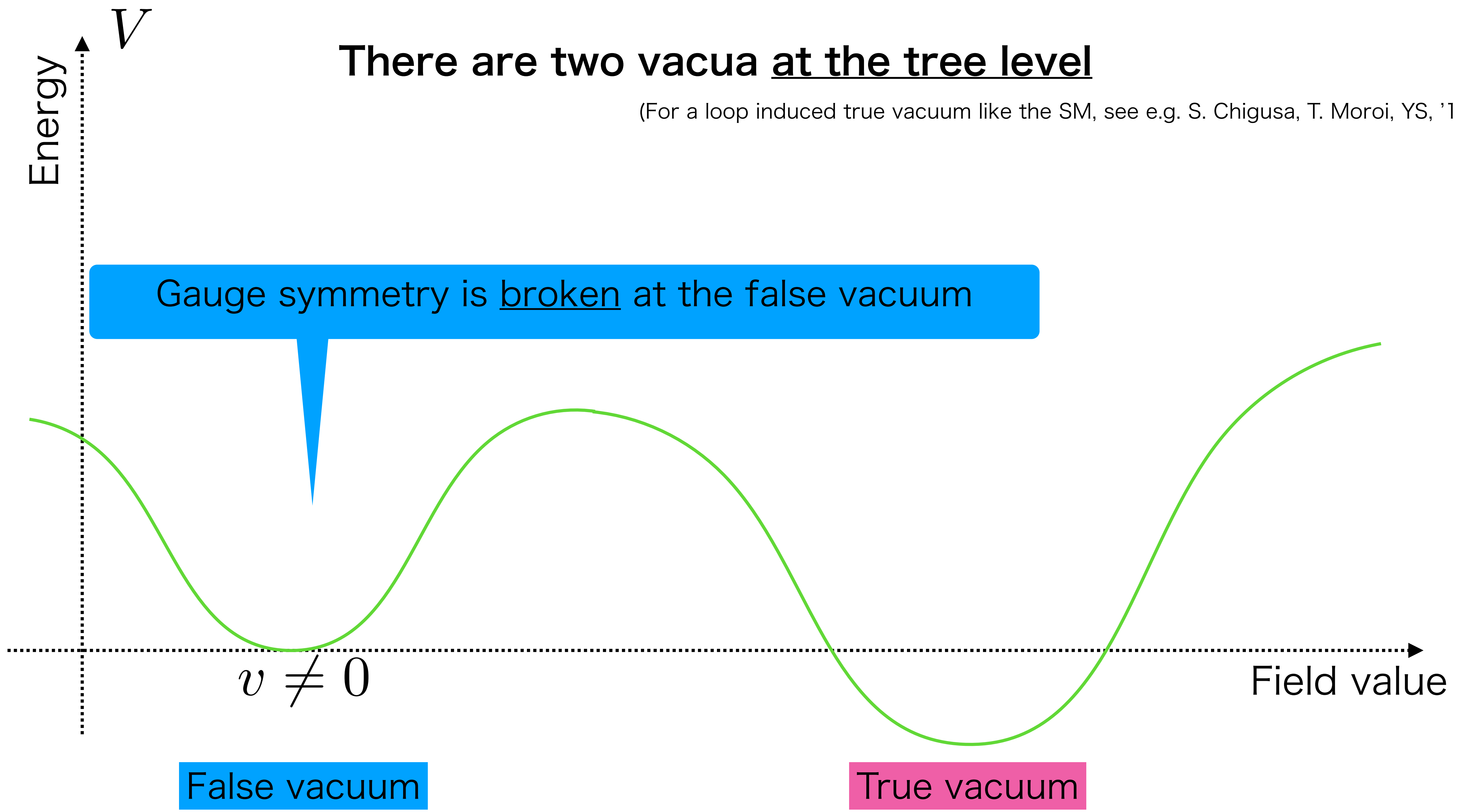
$\bar{\phi}$: bounce solution (real function)

V : potential of Φ

Potential

There are two vacua at the tree level

(For a loop induced true vacuum like the SM, see e.g. S. Chigusa, T. Moroi, YS, '17, '18)



Pre-factor

$$A = A^{(A_\mu, \varphi)} A^{(c,\bar{c})} A^{(\text{extra})}$$

Gauge boson
NG boson

FP ghost

Other scalars
Fermions

include ξ parameter

Mixing appears from

$$\mathcal{L} \supset 2g(\partial_\mu \bar{\phi}) A_\mu \varphi$$

We also show the ξ-independence

FP ghost

$$A = A^{(A_\mu,\varphi)} A^{(c,\bar{c})} A^{(\mathrm{extra})}$$

$$A^{(c,\bar{c})} = \frac{\det[-\partial^2 + \xi g^2 \bar{\phi}^2]}{\det[-\partial^2 + \xi g^2 v^2]}$$

Since bounce is O(4) symmetric, we can use the partial wave expansion.

$$= \prod_{J=0}^{\infty} \left(\frac{\det[-\Delta_J + \xi g^2 \bar{\phi}^2]}{\det[-\Delta_J + \xi g^2 v^2]} \right)^{(2J+1)^2}$$

ξ -dependent

$$\Delta_J = \partial_r^2 + \frac{3}{r}\partial_r - \frac{L^2}{r^2} \qquad L = \sqrt{4J(J+1)}$$

Gauge-NG

$$A = A^{(A_\mu,\varphi)} A^{(c,\bar{c})} A^{(\mathrm{extra})}$$

$$A^{(A_\mu,\varphi)} = A^{(S,L,\varphi)} A^{(T)}$$

$$A^{(T)} = \prod_{J=1/2}^{\infty} \left(\frac{\det[-\Delta_J + g^2 \bar{\phi}^2]}{\det[-\Delta_J + g^2 v^2]} \right)^{-(2J+1)^2}$$

ξ -independent

$$A^{(S,L,\varphi)} = \prod_{J=0}^{\infty} \left(\frac{\det \mathcal{M}_J^{(S,L,\varphi)}}{\det \widehat{\mathcal{M}}_J^{(S,L,\varphi)}} \right)^{-(2J+1)^2/2}$$

ξ -dependent

Gauge-NG

$$A^{(S,L,\varphi)} = \prod_{J=0}^{\infty} \left(\frac{\det \mathcal{M}_J^{(S,L,\varphi)}}{\det \widehat{\mathcal{M}}_J^{(S,L,\varphi)}} \right)^{-(2J+1)^2/2}$$

Gauge boson

NG boson

$$\mathcal{M}_J^{(S,L,\varphi)} \equiv \begin{pmatrix} -\Delta_J + \frac{3}{r^2} + g^2\bar{\phi}^2 & -\frac{2L}{r^2} & 2g\bar{\phi}' \\ -\frac{2L}{r^2} & -\Delta_J - \frac{1}{r^2} + g^2\bar{\phi}^2 & 0 \\ 2g\bar{\phi}' & 0 & -\Delta_J + \frac{(\Delta_0\bar{\phi})}{\bar{\phi}} + \xi g^2\bar{\phi}^2 \end{pmatrix}$$

$$+ \left(1 - \frac{1}{\xi}\right) \begin{pmatrix} \partial_r^2 + \frac{3}{r}\partial_r - \frac{3}{r^2} & -L\left(\frac{1}{r}\partial_r - \frac{1}{r^2}\right) & 0 \\ L\left(\frac{1}{r}\partial_r + \frac{3}{r^2}\right) & -\frac{L^2}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

(The middle column and row elements are absent when J=0)

Functional determinant

Theorem

[J. H. van Vleck, ’28; R. H. Cameron and W. T. Martin, ’45; …]

We give a proof for our case in JHEP11(2017)074

$$\frac{\det \mathcal{M}}{\det \widehat{\mathcal{M}}} = \left(\lim_{r \to \infty} \frac{\det[\psi_1(r) \cdots \psi_n(r)]}{\det[\hat{\psi}_1(r) \cdots \hat{\psi}_n(r)]} \right) \left(\lim_{r \to 0} \frac{\det[\psi_1(r) \cdots \psi_n(r)]}{\det[\hat{\psi}_1(r) \cdots \hat{\psi}_n(r)]} \right)^{-1}$$

$\mathcal{M}, \widehat{\mathcal{M}}$: (n x n) radial fluctuation operators

$\mathcal{M}\psi_i = 0,\ \widehat{\mathcal{M}}\hat{\psi}_i = 0$: independent solutions (regular at r=0)

FP ghost

$$A^{(c,\bar{c})} = \prod_{J=0}^{\infty} \left(\frac{\det[-\Delta_J + \xi g^2 \bar{\phi}^2]}{\det[-\Delta_J + \xi g^2 v^2]} \right)^{(2J+1)^2}$$

$$= \prod_{J=0}^{\infty} \left(\lim_{r\to\infty} \frac{f_J^{(\mathrm{FP})}(r)}{\hat{f}_J^{(\mathrm{FP})}(r)} \right)^{(2J+1)^2}$$

$$[-\Delta_J + \xi g^2 \varphi^2] f_J^{(\mathrm{FP})} = 0,$$

$$[-\Delta_J + \xi g^2 v^2] \hat{f}_J^{(\mathrm{FP})} = 0$$

with $\lim_{r\to 0} \frac{f_J^{(\mathrm{FP})}(r)}{r^{2J}} = \lim_{r\to 0} \frac{\hat{f}_J^{(\mathrm{FP})}(r)}{r^{2J}} = 1$

Gauge-NG

$$A^{(S,L,\varphi)} = \prod_{J=0}^{\infty} \left(\frac{\det \mathcal{M}_J^{(S,L,\varphi)}}{\det \widehat{\mathcal{M}}_J^{(S,L,\varphi)}} \right)^{-(2J+1)^2/2}$$

$$\frac{\det \mathcal{M}_J^{(S,L,\varphi)}}{\det \widehat{\mathcal{M}}_J^{(S,L,\varphi)}} = \left(\lim_{r\to\infty} \frac{\det[\Psi_1(r)\Psi_2(r)\Psi_3(r)]}{\det[\hat{\Psi}_1(r)\hat{\Psi}_2(r)\hat{\Psi}_3(r)]} \right) \left(\lim_{r\to 0} \frac{\det[\Psi_1(r)\Psi_2(r)\Psi_3(r)]}{\det[\hat{\Psi}_1(r)\hat{\Psi}_2(r)\hat{\Psi}_3(r)]} \right)^{-1}$$

$$\mathcal{M}_J^{(S,L,\varphi)} \Psi_i = 0$$

$$\widehat{\mathcal{M}}_J^{(S,L,\varphi)} \hat{\Psi}_i = 0$$

: independent (regular) solutions

Solutions to differential equations

Differential equations

We want to find solutions to

$$\mathcal{M}_J^{(S,L,\varphi)}\Psi_i = 0$$

$$\mathcal{M}_J^{(S,L,\varphi)} \equiv \begin{pmatrix} -\Delta_J + \frac{3}{r^2} + g^2\bar{\phi}^2 & -\frac{2L}{r^2} & 2g\bar{\phi}' \\ -\frac{2L}{r^2} & -\Delta_J - \frac{1}{r^2} + g^2\bar{\phi}^2 & 0 \\ 2g\bar{\phi}' & 0 & -\Delta_J + \frac{(\Delta_0\bar{\phi})}{\bar{\phi}} + \xi g^2\bar{\phi}^2 \end{pmatrix}$$

$$+\left(1-\frac{1}{\xi}\right)\begin{pmatrix} \partial_r^2 + \frac{3}{r}\partial_r - \frac{3}{r^2} & -L\left(\frac{1}{r}\partial_r - \frac{1}{r^2}\right) & 0 \\ L\left(\frac{1}{r}\partial_r + \frac{3}{r^2}\right) & -\frac{L^2}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

with using $f_J^{(\mathrm{FP})},\ \hat{f}_J^{(\mathrm{FP})}$

Solutions

$$\mathcal{M}_J^{(S,L,\varphi)}\Psi_i = 0$$

$$\Psi = \begin{pmatrix} \partial_r \chi \\ \frac{L}{r}\chi \\ g\bar{\phi}\chi \end{pmatrix} + \begin{pmatrix} \frac{1}{rg^2\bar{\phi}^2}\eta \\ \frac{1}{Lr^2g^2\bar{\phi}^2}\partial_r(r^2\eta) \\ 0 \end{pmatrix} + \begin{pmatrix} -2\frac{\bar{\phi}'}{g^2\bar{\phi}^3}\zeta \\ 0 \\ \frac{1}{g\bar{\phi}}\zeta \end{pmatrix},$$

$$\left(\Delta_J - \xi g^2\bar{\phi}^2\right)\chi = \frac{2\bar{\phi}'}{rg^2\bar{\phi}^3}\eta + \frac{2}{r^3}\partial_r\left(\frac{r^3\bar{\phi}'}{g^2\bar{\phi}^3}\zeta\right),$$

FP

$$\left(\Delta_J - g^2\bar{\phi}^2 - 2\frac{\bar{\phi}'}{\bar{\phi}}\frac{1}{r^2}\partial_r r^2\right)\eta = -\frac{2L^2\bar{\phi}'}{r\bar{\phi}}\zeta,$$

$$\left(\Delta_J - \xi g^2\bar{\phi}^2\right)\zeta = 0$$

FP

Solution 1

$$\chi = f_J^{(\mathrm{FP})},\ \eta = 0,\ \zeta = 0$$

0

$$\left(\Delta_J - \xi g^2 \bar{\phi}^2\right) \chi = \frac{2\bar{\phi}'}{r g^2 \bar{\phi}^3}\eta + \frac{2}{r^3}\partial_r\left(\frac{r^3\bar{\phi}'}{g^2\bar{\phi}^3}\zeta\right),$$

FP

$$\left(\Delta_J - g^2\bar{\phi}^2 - 2\frac{\bar{\phi}'}{\bar{\phi}}\frac{1}{r^2}\partial_r r^2\right)\eta = -\frac{2L^2\bar{\phi}'}{r\bar{\phi}}\zeta,$$

$$\left(\Delta_J - \xi g^2\bar{\phi}^2\right)\zeta = 0$$

Solution 2

$$\eta = f_J^{(\eta)}, \ \zeta = 0$$

$$(\Delta_J - \xi g^2 \bar{\phi}^2)\chi = \frac{2\bar{\phi}'}{r g^2 \bar{\phi}^3}\eta$$

$$\left(\Delta_J - \xi g^2 \bar{\phi}^2\right)\chi = \frac{2\bar{\phi}'}{r g^2 \bar{\phi}^3}\eta \cancel{+ \frac{2}{r^3}\partial_r\left(\frac{r^3\bar{\phi}'}{g^2\bar{\phi}^3}\zeta\right)},$$

$$\left(\Delta_J - g^2\bar{\phi}^2 - 2\frac{\bar{\phi}'}{\bar{\phi}}\frac{1}{r^2}\partial_r r^2\right)\eta = \cancel{-\frac{2L^2\bar{\phi}'}{r\bar{\phi}}\zeta},$$ 0

$$\cancel{\left(\Delta_J - \xi g^2 \bar{\phi}^2\right)\zeta = 0}$$

Solution 3

$$\zeta = f_J^{(\mathrm{FP})}$$

$$(\Delta_J - g^2\bar{\phi}^2)\eta - \frac{2\bar{\phi}'}{r^2\bar{\phi}}\partial_r\left(r^2\eta\right) = -\frac{2L^2\bar{\phi}'}{r\bar{\phi}}\zeta,$$

$$(\Delta_J - \xi g^2\bar{\phi}^2)\chi = \frac{2\bar{\phi}'}{rg^2\bar{\phi}^3}\eta + \frac{2}{r^3}\partial_r\left(\frac{r^3\bar{\phi}'}{g^2\bar{\phi}^3}\zeta\right),$$

$$\left(\Delta_J - \xi g^2\bar{\phi}^2\right)\chi = \frac{2\bar{\phi}'}{rg^2\bar{\phi}^3}\eta + \frac{2}{r^3}\partial_r\left(\frac{r^3\bar{\phi}'}{g^2\bar{\phi}^3}\zeta\right),$$

$$\left(\Delta_J - g^2\bar{\phi}^2 - 2\frac{\bar{\phi}'}{\bar{\phi}}\frac{1}{r^2}\partial_r r^2\right)\eta = -\frac{2L^2\bar{\phi}'}{r\bar{\phi}}\zeta,$$

$$\left(\underline{\Delta_J - \xi g^2\bar{\phi}^2}\right)\zeta = 0$$

FP

Result

$J \neq 0$

$$\frac{\det \mathcal{M}_J^{(S,L,\varphi)}}{\det \widehat{\mathcal{M}}_J^{(S,L,\varphi)}} = \left(\lim_{r\to\infty} \frac{\det[\Psi_1(r)\Psi_2(r)\Psi_3(r)]}{\det[\hat{\Psi}_1(r)\hat{\Psi}_2(r)\hat{\Psi}_3(r)]} \right) \left(\lim_{r\to 0} \frac{\det[\Psi_1(r)\Psi_2(r)\Psi_3(r)]}{\det[\hat{\Psi}_1(r)\hat{\Psi}_2(r)\hat{\Psi}_3(r)]} \right)^{-1}$$

= (a large part of our paper is devoted to this calculation)

$$= \frac{\bar{\phi}(0)}{v} \lim_{r\to\infty} \frac{f_J^{(\eta)}(r)\left[f_J^{(\mathrm{FP})}(r)\right]^2}{\hat{f}_J^{(\eta)}(r)\left[\hat{f}_J^{(\mathrm{FP})}(r)\right]^2} \left[1 + O\left(\frac{1}{r}\right)\right]$$

Dominates the result for large r

Non-tribial gauge dependence disappears

Result

$$A^{(S,L,\varphi)} = \prod_{J=0}^{\infty} \left(\frac{\det \mathcal{M}_J^{(S,L,\varphi)}}{\det \widehat{\mathcal{M}}_J^{(S,L,\varphi)}} \right)^{-(2J+1)^2/2}$$

$$= \cdots$$

$$= \left[A^{(c,\bar{c})} \right]^{-1} \left(\frac{v}{\bar{\phi}(0)} \right)^{-1/2} \prod_{J \geq 1/2} \left(\lim_{r\to\infty} \frac{\bar{\phi}(0) f_J^{(\eta)}(r)}{v \hat{f}_J^{(\eta)}(r)} \right)^{-(2J+1)^2/2}$$

ξ -independent

$$\left[\Delta_J - g^2 \bar{\phi}^2 - 2 \frac{\bar{\phi}'}{\bar{\phi}} \frac{1}{r^2} \partial_r r^2 \right] f_J^{(\eta)} = 0$$

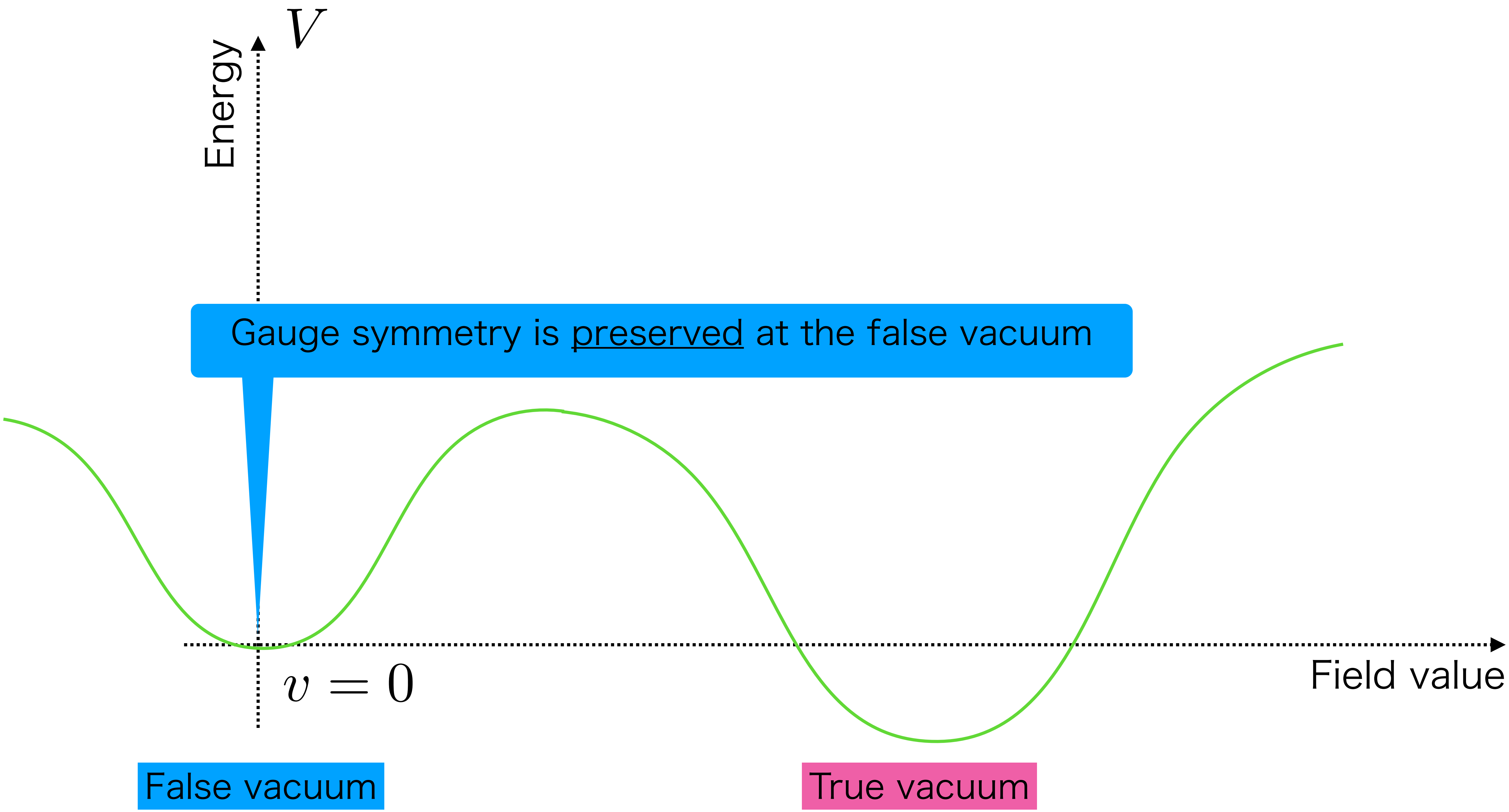
$$\left[\Delta_J - g^2 v^2 \right] \hat{f}_J^{(\eta)} = 0$$

$$\lim_{r\to 0} \frac{f_J^{(\eta)}(r)}{r^{2J}} = \lim_{r\to 0} \frac{\hat{f}_J^{(\eta)}(r)}{r^{2J}} = 1$$

Gauge zero mode

$(v = 0)$

Potential


V
Energy
Gauge symmetry is preserved at the false vacuum
$v = 0$
Field value
False vacuum
True vacuum

Gauge zero mode in the background gauge

Gauge boson NG boson

$$\mathcal{M}_{J=0}^{(S,\varphi)} \equiv \begin{pmatrix} \frac{1}{\xi}\left(-\Delta_0 + \frac{3}{r^2} + \xi g^2 \bar{\phi}^2\right) & 2g\bar{\phi}' \\ 2g\bar{\phi}' & -\Delta_0 + \frac{(\Delta_0 \bar{\phi})}{\bar{\phi}} + \xi g^2 \bar{\phi}^2 \end{pmatrix}.$$

Zero mode

$$\delta\Psi_0 = \begin{pmatrix} \partial_r f(r) \\ g\bar{\phi}(r) f(r) \end{pmatrix} \qquad [\Delta_0 - \xi g^2 \bar{\phi}^2] f(r) = 0$$

has a zero eigenvalue $\mathcal{M}_{J=0}^{(S,\varphi)} \delta\Psi_0 = 0$

Gauge zero mode in the background gauge

However, if we “rotate” the VEV toward the zero mode,

$$\delta\Psi_0 = \begin{pmatrix} \partial_r f(r) \\ g\bar{\phi}(r) f(r) \end{pmatrix}$$

Gauge boson

NG boson

Gauge boson gets a VEV

Bounce has a position dependent phase

The “global symmetry” is highly non-trivial

It is difficult to integrate over the moduli space

Semi-analytic formula (Fermi gauge)

JHEP11(2017)074

Fermi gauge

Gauge fixing function

$$\mathcal{F} = \partial_\mu A_\mu \cancel{- 2\xi g (\mathrm{Re}\Phi)(\mathrm{Im}\Phi)}$$

The gauge fixing term does not break the naive global symmetry

Lagrangian

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + [(\partial_\mu + igA_\mu)\Phi^\dagger][(\partial_\mu - igA_\mu)\Phi] + V + \mathcal{L}_{\mathrm{G.F.}} + \mathcal{L}_{\mathrm{ghost}},$$

$$\mathcal{L}_{\mathrm{G.F.}} = \frac{1}{2\xi}\mathcal{F}^2, \quad \mathcal{L}_{\mathrm{ghost}} = \bar{c}\left[-\partial_\mu\partial_\mu \cancel{+ \xi g^2(\Phi^2 + \Phi^{\dagger 2})}\right] c.$$

$$\Phi = \frac{1}{\sqrt{2}}(\bar{\phi} + h + i\varphi)$$

$\bar{\phi}$: bounce solution (real function)

V : potential of Φ

FP ghost

$$A = A^{(A_\mu,\varphi)} A^{(c,\bar{c})} A^{(\text{extra})}$$

$$A^{(c,\bar{c})} = \frac{\det[-\partial^2]}{\det[-\partial^2]} = 1$$

ξ -independent

Gauge-NG

$$A = A^{(A_\mu,\varphi)} A^{(c,\bar{c})} A^{(\mathrm{extra})}$$

$$A^{(A_\mu,\varphi)} = A^{(S,L,\varphi)} A^{(T)}$$

The same as in the BG gauge

$$A^{(T)} = \prod_{J=1/2}^{\infty} \left(\frac{\det[-\Delta_J + g^2 \bar{\phi}^2]}{\det[-\Delta_J + g^2 v^2]} \right)^{-(2J+1)^2}$$

ξ -independent

$$A^{(S,L,\varphi)} = \prod_{J=0}^{\infty} \left(\frac{\det \mathcal{M}_J^{(S,L,\varphi)}}{\det \widehat{\mathcal{M}}_J^{(S,L,\varphi)}} \right)^{-(2J+1)^2/2}$$

Includes ξ -parameter

Gauge-NG

$$A^{(S,L,\varphi)} = \prod_{J=0}^{\infty} \left(\frac{\det \mathcal{M}_J^{(S,L,\varphi)}}{\det \widehat{\mathcal{M}}_J^{(S,L,\varphi)}} \right)^{-(2J+1)^2/2}$$

Gauge boson

NG boson

$$\mathcal{M}_J^{(S,L,\varphi)} \equiv \begin{pmatrix} -\Delta_J + \frac{3}{r^2} + g^2\bar{\phi}^2 & -\frac{2L}{r^2} & g\bar{\phi}' - g\bar{\phi}\partial_r \\ -\frac{2L}{r^2} & -\Delta_J - \frac{1}{r^2} + g^2\bar{\phi}^2 & -\frac{L}{r} g\bar{\phi} \\ 2g\bar{\phi}' + g\bar{\phi}\partial_r + \frac{3}{r} g\bar{\phi} & -\frac{L}{r} g\bar{\phi} & -\Delta_J + \frac{(\Delta_0 \bar{\phi})}{\bar{\phi}} \end{pmatrix}$$

different from BG gauge

$$+ \left(1 - \frac{1}{\xi}\right) \begin{pmatrix} \partial_r^2 + \frac{3}{r}\partial_r - \frac{3}{r^2} & -L\left(\frac{1}{r}\partial_r - \frac{1}{r^2}\right) & 0 \\ L\left(\frac{1}{r}\partial_r + \frac{3}{r^2}\right) & -\frac{L^2}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

(The middle column and row elements are absent when J=0)

Solutions

$$\mathcal{M}_J^{(S,L,\varphi)}\Psi_i = 0$$

$$\Psi = \begin{pmatrix} \partial_r \chi \\ \frac{L}{r}\chi \\ g\bar{\phi}\chi \end{pmatrix} + \begin{pmatrix} \frac{1}{rg^2\bar{\phi}^2}\eta \\ \frac{1}{Lr^2g^2\bar{\phi}^2}\partial_r(r^2\eta) \\ 0 \end{pmatrix} + \begin{pmatrix} -2\frac{\bar{\phi}'}{g^2\bar{\phi}^3}\zeta \\ 0 \\ \frac{1}{g\bar{\phi}}\zeta \end{pmatrix},$$

$$\left(\Delta_J \cancel{- \xi g^2\bar{\phi}^2}\right)\chi = \frac{2\bar{\phi}'}{rg^2\bar{\phi}^3}\eta + \frac{2}{r^3}\partial_r\left(\frac{r^3\bar{\phi}'}{g^2\bar{\phi}^3}\zeta\right) \cancel{,} - \xi\zeta$$

$$\left(\Delta_J - g^2\bar{\phi}^2 - 2\frac{\bar{\phi}'}{\bar{\phi}}\frac{1}{r^2}\partial_r r^2\right)\eta = -\frac{2L^2\bar{\phi}'}{r\bar{\phi}}\zeta,$$

The same as in the BG gauge!

$$\left(\Delta_J \cancel{- \xi g^2\bar{\phi}^2}\right)\zeta = 0$$

Gauge zero mode

$(v = 0)$

Gauge zero mode in the Fermi gauge

Gauge boson

NG boson

$$\mathcal{M}^{(S,\varphi)}_{J=0} \equiv \begin{pmatrix} \frac{1}{\xi}\left(-\Delta_0 + \frac{3}{r^2} + \xi g^2 \bar{\phi}^2\right) & g\bar{\phi}' - g\bar{\phi}\partial_r \\ 2g\bar{\phi}' + g\bar{\phi}\partial_r + \frac{3}{r} g\bar{\phi} & -\Delta_0 + \frac{(\Delta_0 \bar{\phi})}{\bar{\phi}} \end{pmatrix} .$$

Zero mode

$$\delta\Psi_0 = \begin{pmatrix} 0 \\ g\bar{\phi}(r) \end{pmatrix}$$

Naive phase rotation

$$\Phi = \frac{1}{\sqrt{2}}(\bar{\phi} + h + i\varphi) \to e^{i\theta}\Phi$$

The rotation toward the zero mode does not change the fluctuation matrix

Zero mode subtraction

$$\mathcal{M}^{(S,\varphi)}_{J=0}\delta\Psi_0 = 0$$

$$\det \mathcal{M}^{(S,\varphi)}_{J=0} = 0 \cdot \lambda_1 \cdot \lambda_2 \cdots$$

Add a “mass” term

$$\det[\mathcal{M}^{(S,\varphi)}_{J=0} + \mathrm{diag}(\nu,\nu)] = \nu \cdot (\nu + \lambda_1) \cdot (\nu + \lambda_2) \cdots$$

Thus, the determinant after the zero mode subtraction is

$$\det{}' \mathcal{M}^{(S,\varphi)}_{J=0} = \lim_{\nu \to 0} \frac{1}{\nu} \det[\mathcal{M}^{(S,\varphi)}_{J=0} + \mathrm{diag}(\nu,\nu)] = \lambda_1 \cdot \lambda_2 \cdots$$

Zero mode integration

$$\left[\det S_E''(\bar{\phi})\right]^{-1/2} \simeq \int_{\Phi\sim\bar{\phi}} \mathcal{D}\Phi e^{-S_E} = \int_0^{2\pi} d\theta J \int \prod_{i\neq 0} dc_i e^{-S_E}$$

Non-zero modes

Jacobian

(From the normalization of the path integral)

$$\left(\frac{\det \mathcal{M}_0^{(S,\varphi)}}{\det \widehat{\mathcal{M}}_0^{(S,\varphi)}}\right)^{-1/2} \to 2\pi\left(\lim_{r\to\infty} 2\pi m_\phi \bar{\phi}(0) r^3 \bar{\phi}(r) \hat{f}_0^{(\sigma)}(r)\right)^{1/2}$$

$$(\Delta_J - m_\phi^2)\hat{f}_J^{(\sigma)} = 0$$

m_ϕ : scalar mass at the false vacuum

Conversion relation

Single-field bounce

[M. Endo, T. Moroi, M. M. Nojiri, YS, '17]

<u>Without gauge zero modes</u>

$$A'_{\rm Fermi} = A'_{\rm BG}$$

Numerical evaluation

<u>With gauge zero modes</u>

$$A'_{\rm Fermi} = \sqrt{\frac{1}{g^2 \int dr r^3 \bar{\phi}^2(r)} \lim_{r\to\infty} r^3 \frac{\partial_r f_{\rm FP}(r)}{f_{\rm FP}(r)}} A'_{\rm BG}$$

(Zero modes are subtracted similarly as in the Fermi gauge)

Integration over the moduli space

$$\left[-\partial_r^2 - \frac{3}{r}\partial_r + g^2\bar{\phi}^2\right] f_{\rm FP} = 0$$

Multi-field bounce

[S. Chigusa, T. Moroi, YS, '20]

Without gauge zero modes

$$A'_{\mathrm{Fermi}} = A'_{\mathrm{BG}}$$

With gauge zero modes

Numerical evaluation

$$A'_{\mathrm{Fermi}} = \sqrt{\frac{\det \mathcal{K}}{\det \mathcal{Z}}} A'_{\mathrm{BG}}$$

Integration over the moduli space

$$\mathcal{K} = \lim_{r\to\infty} r^3 \mathcal{U}^T (\partial_r f_{\mathrm{FP}})(f_{\mathrm{FP}})^{-1}\mathcal{U}$$

$$\mathcal{Z} = \int dr r^3 \mathcal{U}^T M^T M \mathcal{U}$$

$$M_{ia}(r) = -g_a T^a_{ik}\bar{\phi}_k(r) \qquad \lim_{r\to\infty} M(r)\mathcal{U} = 0$$

$$\left[-\partial_r^2 - \frac{3}{r}\partial_r + M^T M\right] f_{\mathrm{FP}} = 0 \qquad \mathcal{U}^T\mathcal{U} = \mathbb{1}_{n_{\mathrm{zero}}}$$

Summary

- The lifetime of a metastable vacuum is a fundamental and interesting quantity and its precise determination is very important
- In the calculation of the prefactor, gauge zero modes can appear and their correct treatment had been unknown
- We proposed a way to treat the gauge zero modes and enabled the full one-loop calculation of the pre-factor for generic models
- We also showed the gauge parameter independence for both the background gauge and the fermi gauge