Ripples in Spacetime from broken SUSY

Ljubljana-Trieste joint meeting

Thursday May 6th

Diego Redigolo

work based on

2011.13949 [hep-ph] with N. Craig, A. Mariotti and N. Levi

How we will discover SUSY?

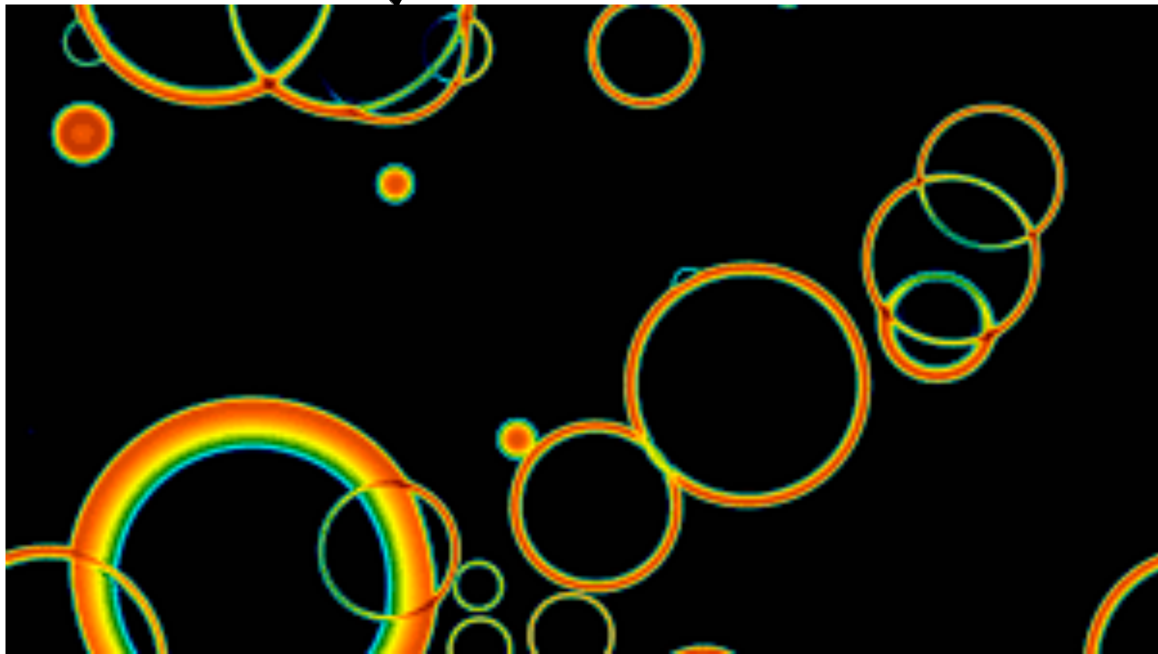
This talk is going to be a journey which starts from this first (vintage) question

How we will discover SUSY?

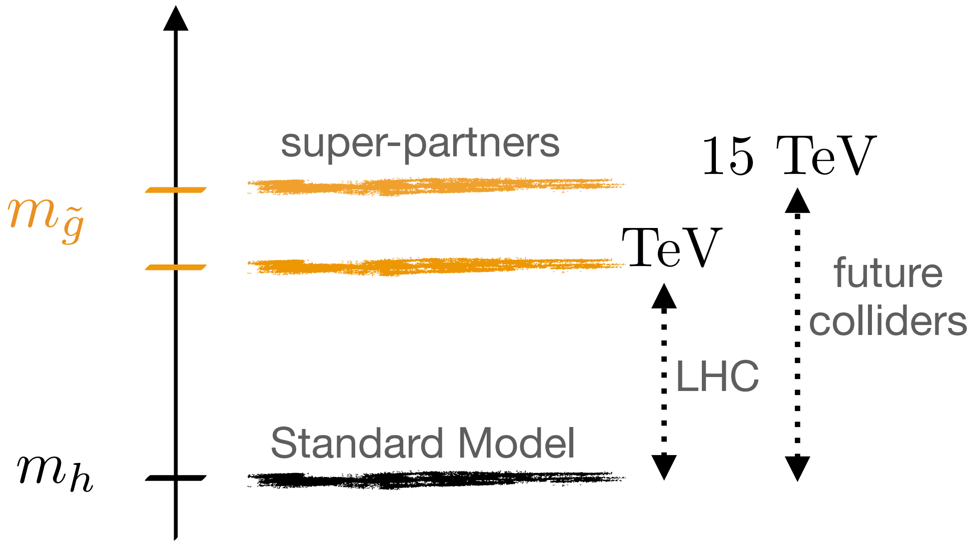
This talk is going to be a journey which starts from this first (vintage) question

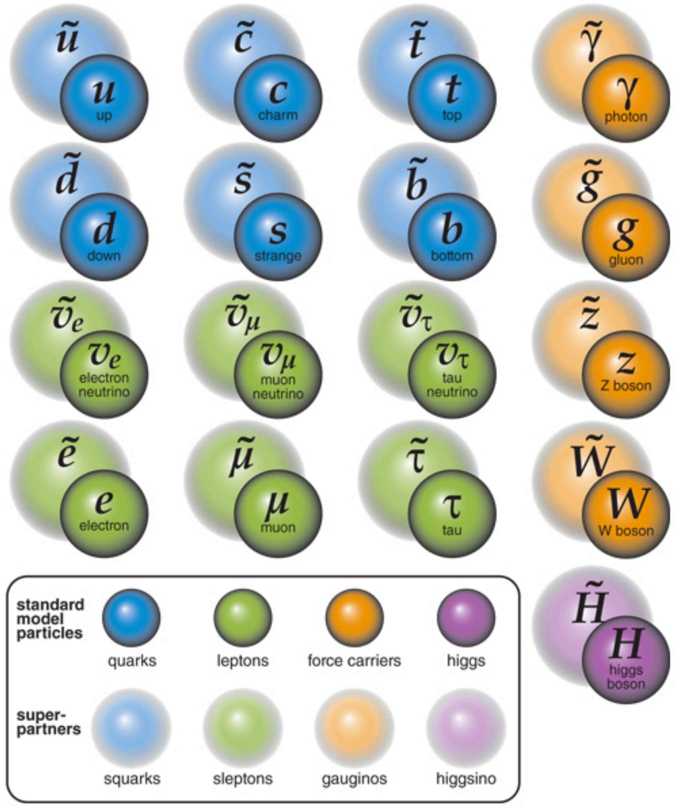
and ends up saying something new about a second (yet-to-be-defined) question

Which QFTs give rise to Gravitational Waves?



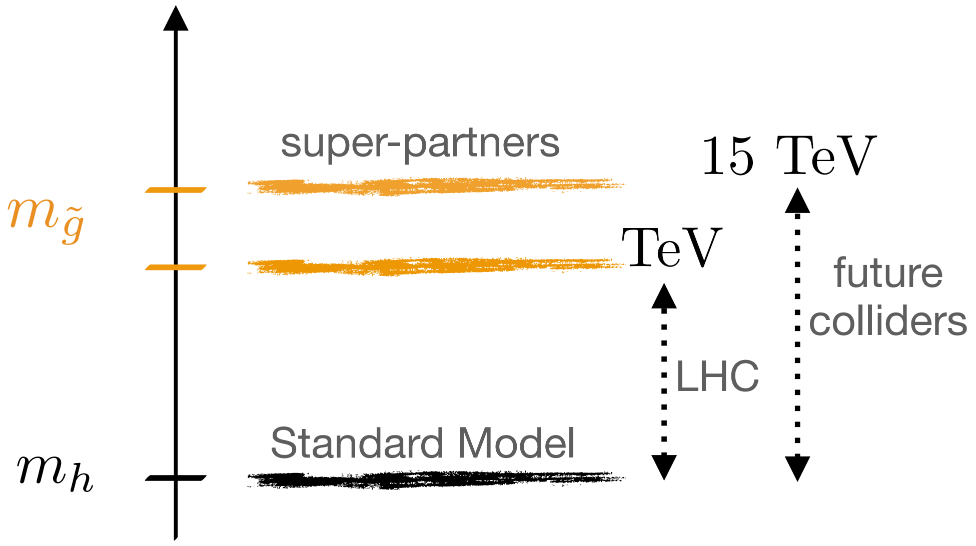
How we did not discover natural SUSY

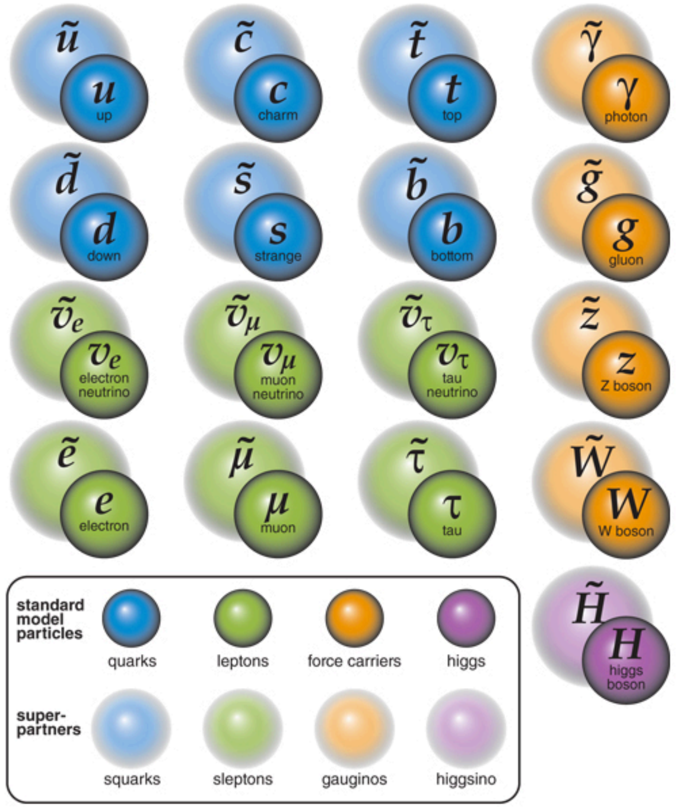




A purely collider perspective on SUSY has the super-partner scale as a target

How we did not discover natural SUSY





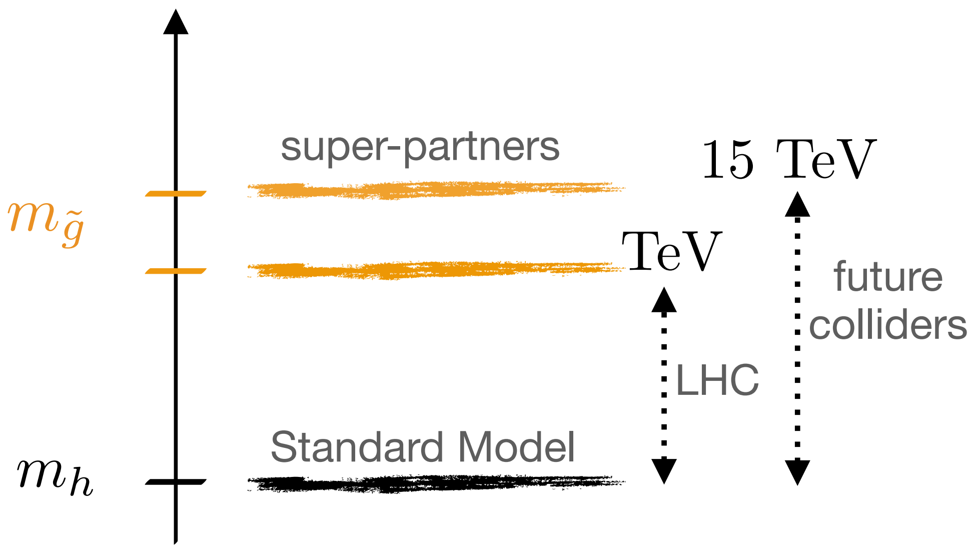
A purely collider perspective on SUSY has the super-partner scale as a target

and the naturalness of the EW scale as main motivation

$$m_{\tilde{g}} \lesssim 1\ \mathrm{TeV}\left(\frac{\Delta_h^{-1}}{1\%}\right)^{*} \dots \qquad \Delta_h \equiv \frac{\delta m_h^2}{m_h^2}$$

* The gluino mass is a robust proxy for the LHC and FCC-hh reaches

How we did not discover natural SUSY



super-partners

15 TeV

$m_{\tilde g}$

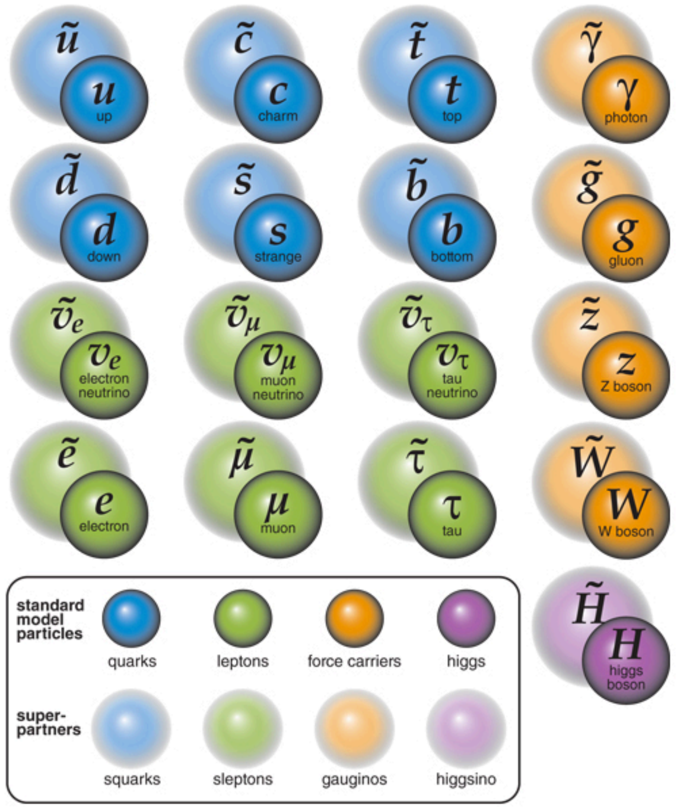
TeV

future colliders

LHC

Standard Model

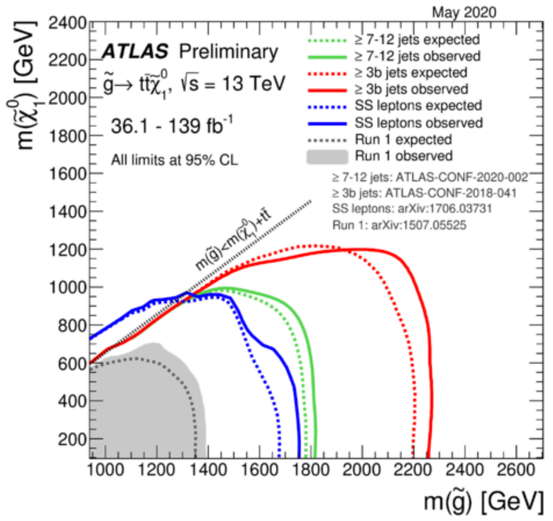
m_h



A purely collider perspective on SUSY has the super-partner scale as a target

and the naturalness of the EW scale as main motivation

$$m_{\tilde g} \lesssim 1\ \text{TeV} \left(\frac{\Delta_h^{-1}}{1\%} \right)^{*} \ldots \qquad \Delta_h \equiv \frac{\delta m_h^2}{m_h^2}$$



LHC results call into question this perspective

* The gluino mass is a robust proxy for the LHC and FCC-hh reaches

How we will discover SUSY ?

Putting the mystery of the EW scale aside

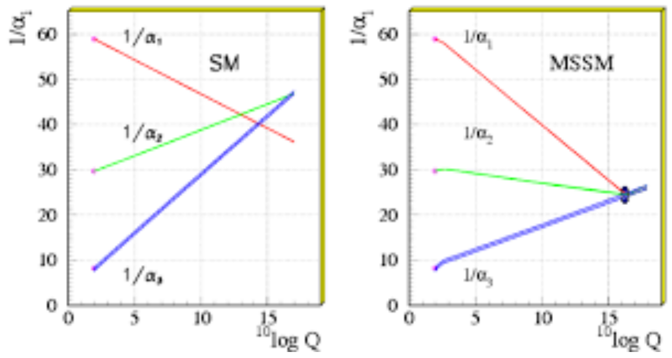
A SUSY Universe at high energies will still be welcome:

How we will discover SUSY ?

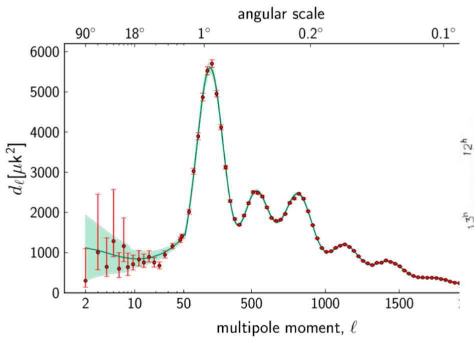
Putting the mystery of the EW scale aside

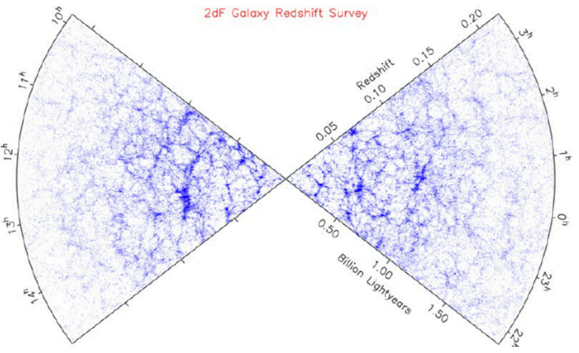
A SUSY Universe at high energies will still be welcome:

perturbative gauge coupling unification

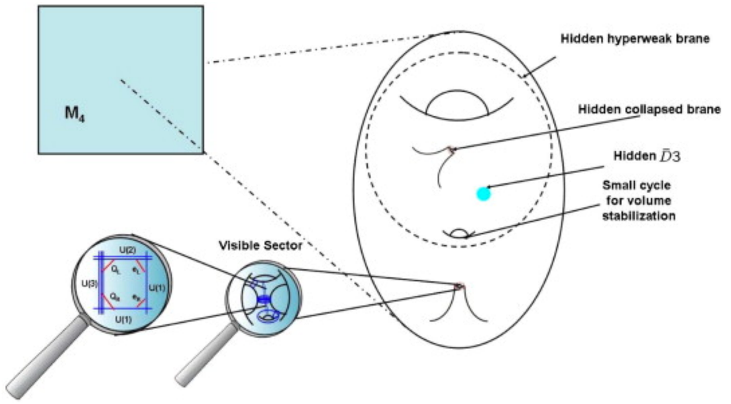


Dark Matter candidates: WIMPs, gravitino, …



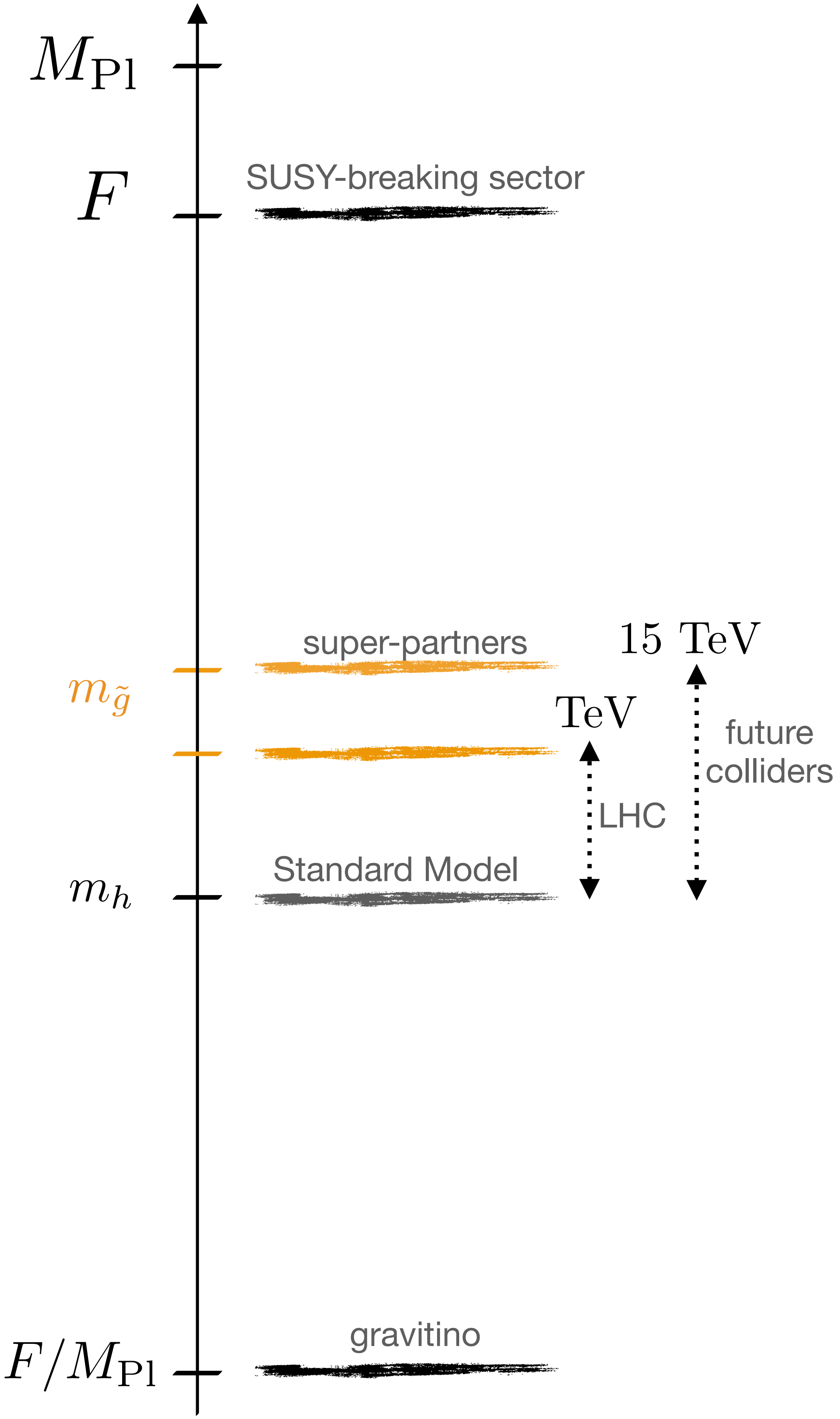


embedding in string theory



…

How we will discover unnatural SUSY?



$M_{\rm Pl}$

F

SUSY-breaking sector

super-partners

$15\ \text{TeV}$

$m_{\tilde{g}}$

TeV

future colliders

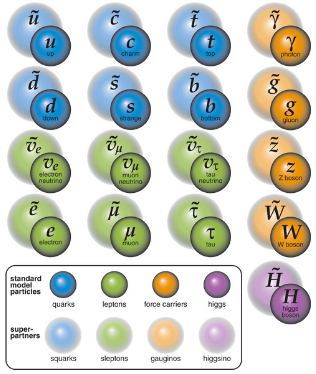
LHC

Standard Model

m_h

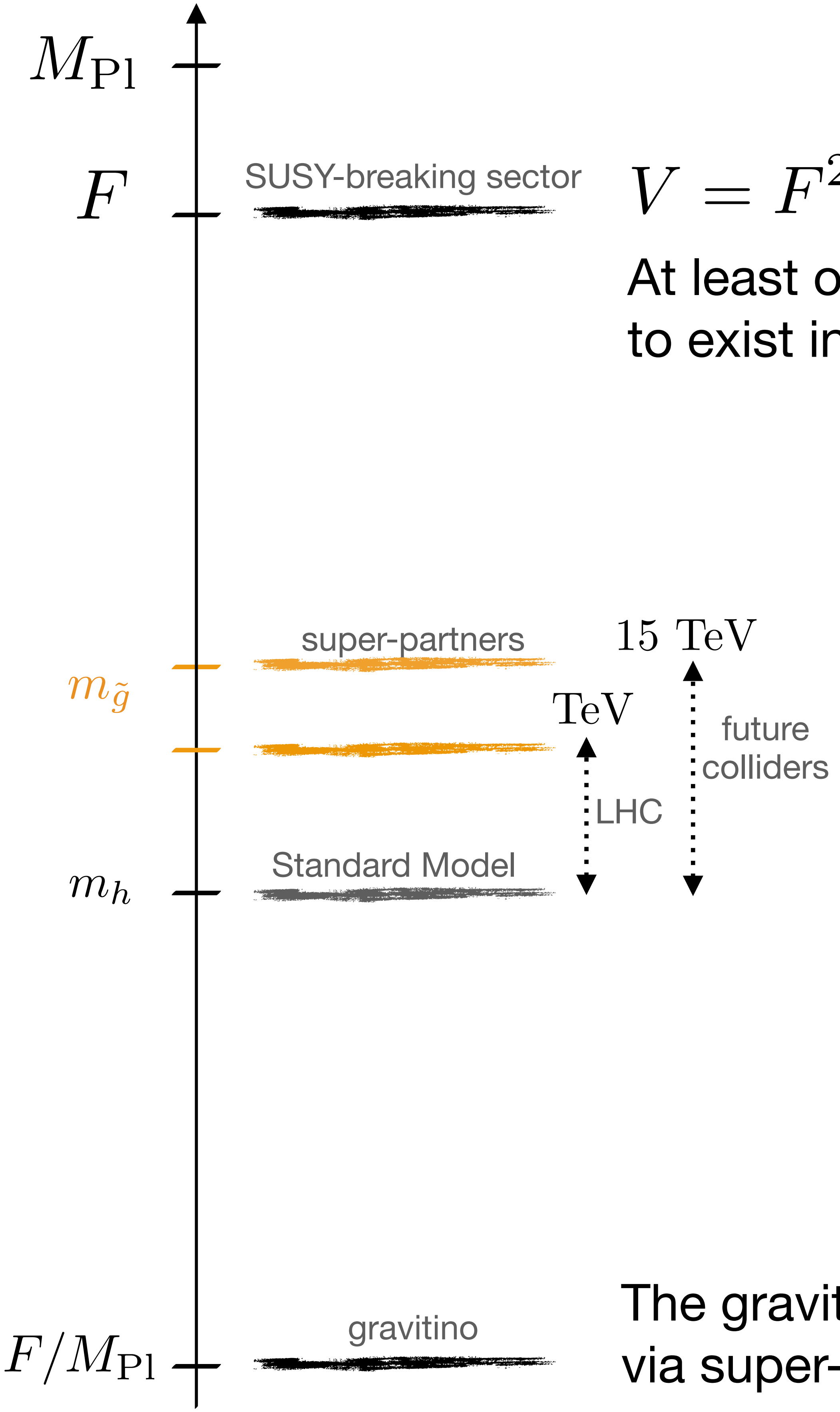
gravitino

$F/M_{\rm Pl}$



If fine-tuning is allowed the
super-partner scale
is a less robust target

How we will discover unnatural SUSY?



$M_{\rm Pl}$

F SUSY-breaking sector

$V = F^2$

At least one hidden sector breaking SUSY is guaranteed to exist in a SUSY Universe

super-partners

$m_{\tilde{g}}$

15 TeV

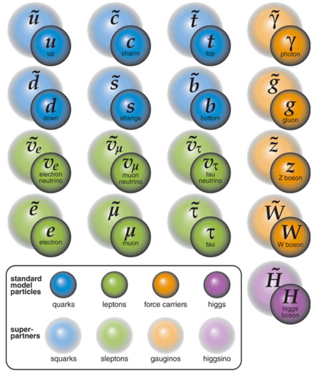
TeV

future colliders

LHC

Standard Model

m_h

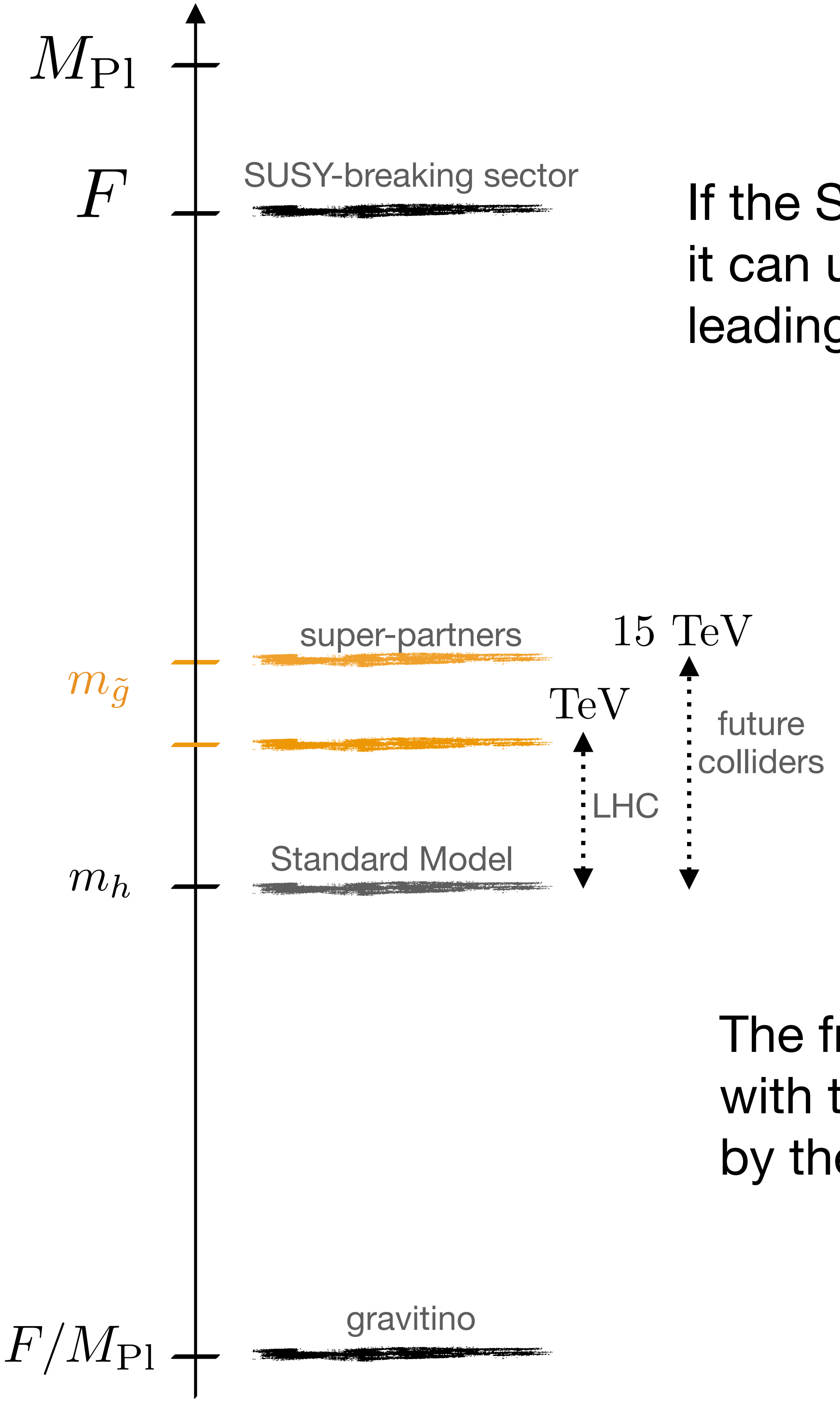


If fine-tuning is allowed the super-partner scale is a less robust target

$F/M_{\rm Pl}$ gravitino

The gravitino mass is related to SUSY-breaking scale via super-Higgs mechanism

Our goal here

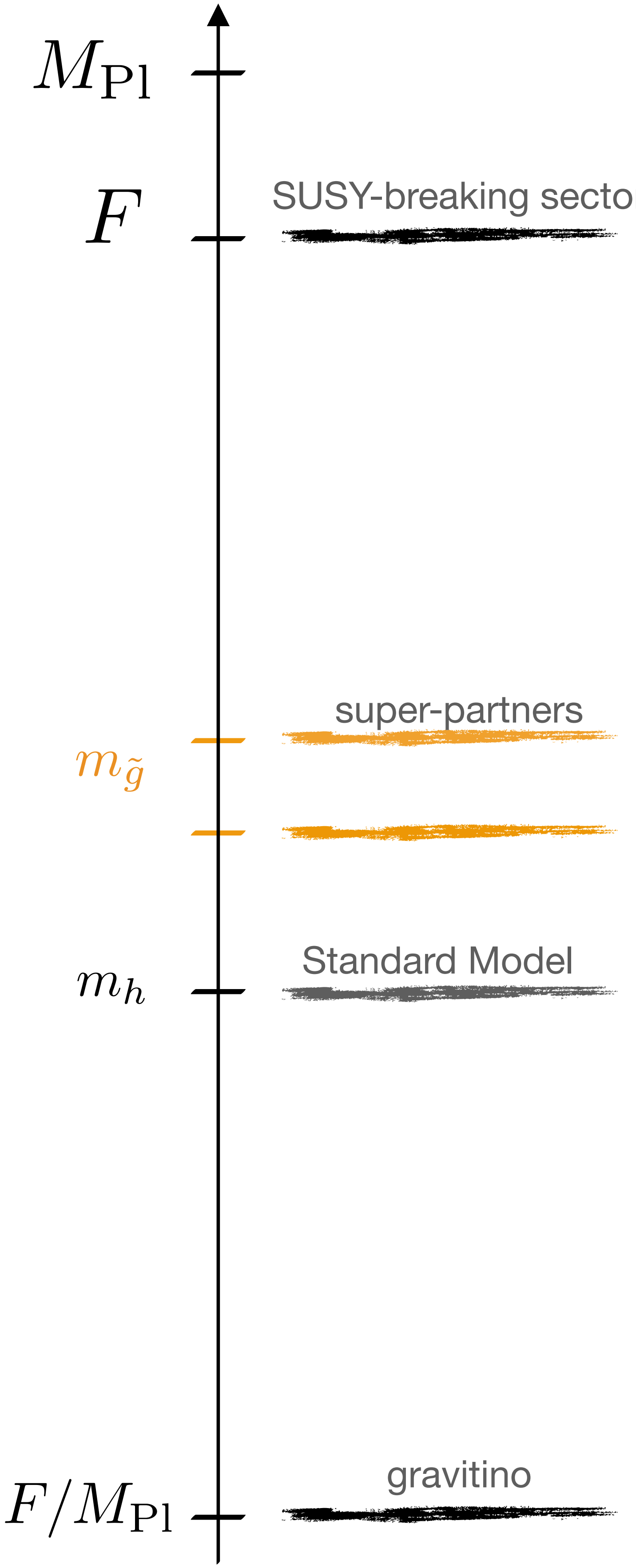


If the SUSY-breaking sector is reheated after inflation, it can undergo a 1st order PT *
leading to GW signals at future interferometers

The frequency of the signal correlates with the SUSY-spectrum and it is constrained by the requirement of a viable cosmology

* the question about the nature of this PT will be the topic of the second part of the talk

The SUSY Universe



$V = F^2$ the minimal dynamics required is a chiral field

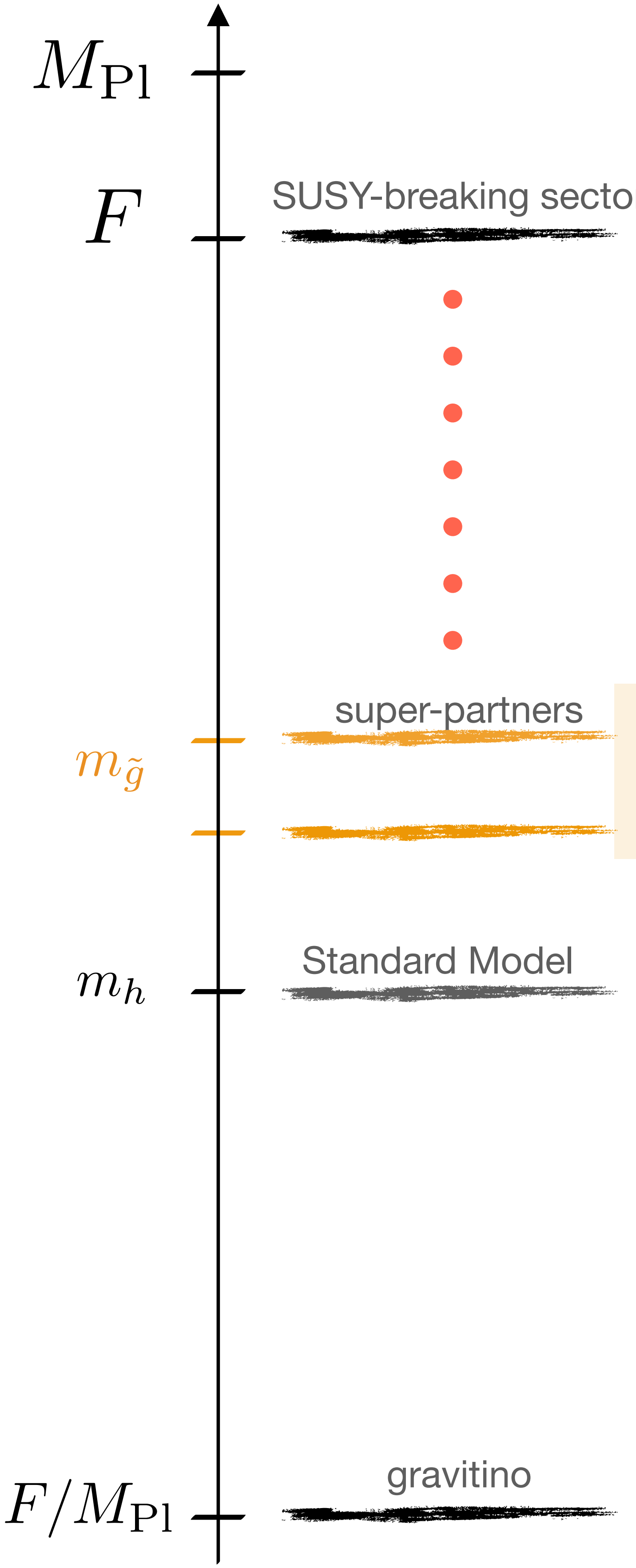
$$\langle X \rangle = \frac{f_a}{\sqrt{2}} + \sqrt{2}\theta\tilde{G} + \theta^2 F$$

R-symmetry breaking scale

Goldstino

SUSY-breaking scale

The SUSY Universe



$V = F^2$ the minimal dynamics required is a chiral field

$$\langle X \rangle = \frac{f_a}{\sqrt{2}} + \sqrt{2}\theta\tilde{G} + \theta^2 F$$

R-symmetry breaking scale

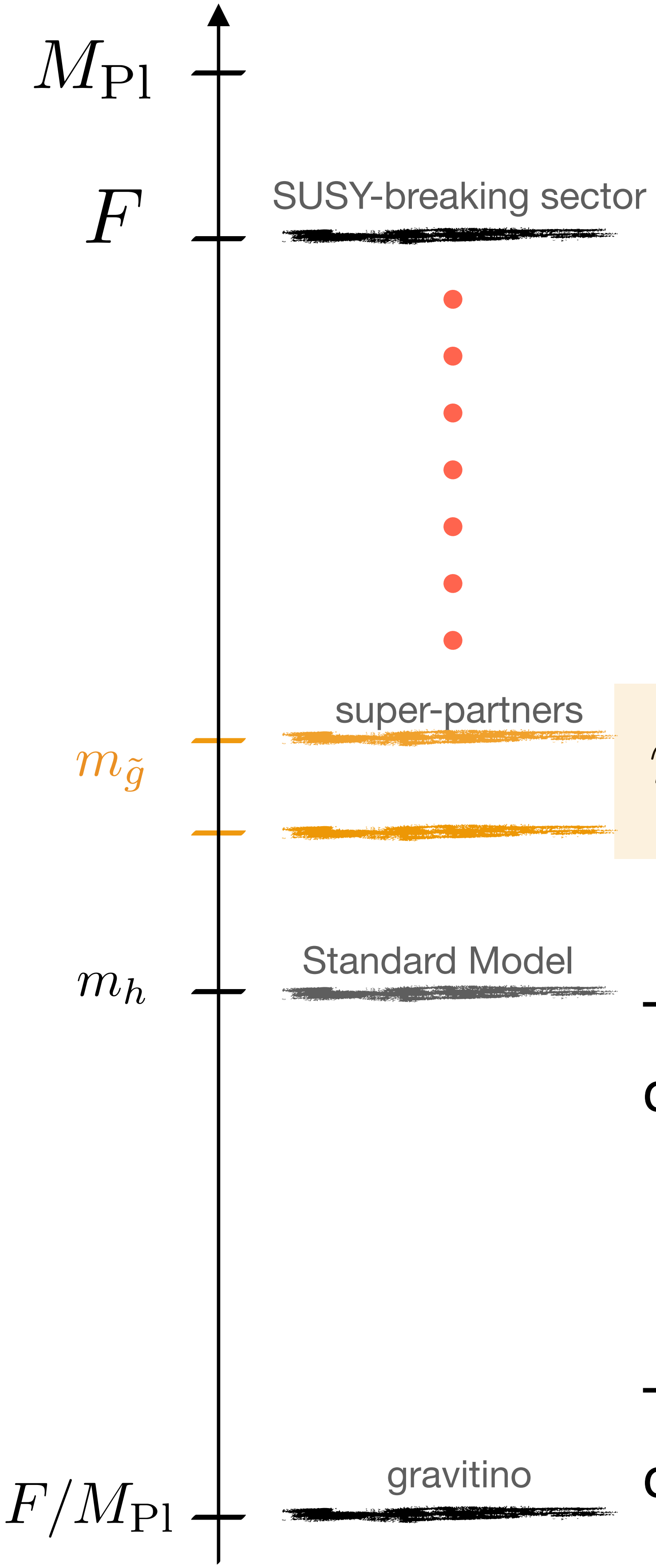
Goldstino

SUSY-breaking scale

$$m_{\tilde{g}} \sim g_M \times \sqrt{F}$$

The strength of the hidden sector coupling determines the scheme

The SUSY Universe



$V = F^2$ the minimal dynamics required is a chiral field

$$\langle X \rangle = \frac{f_a}{\sqrt{2}} + \sqrt{2}\theta\tilde{G} + \theta^2 F$$

R-symmetry breaking scale

Goldstino

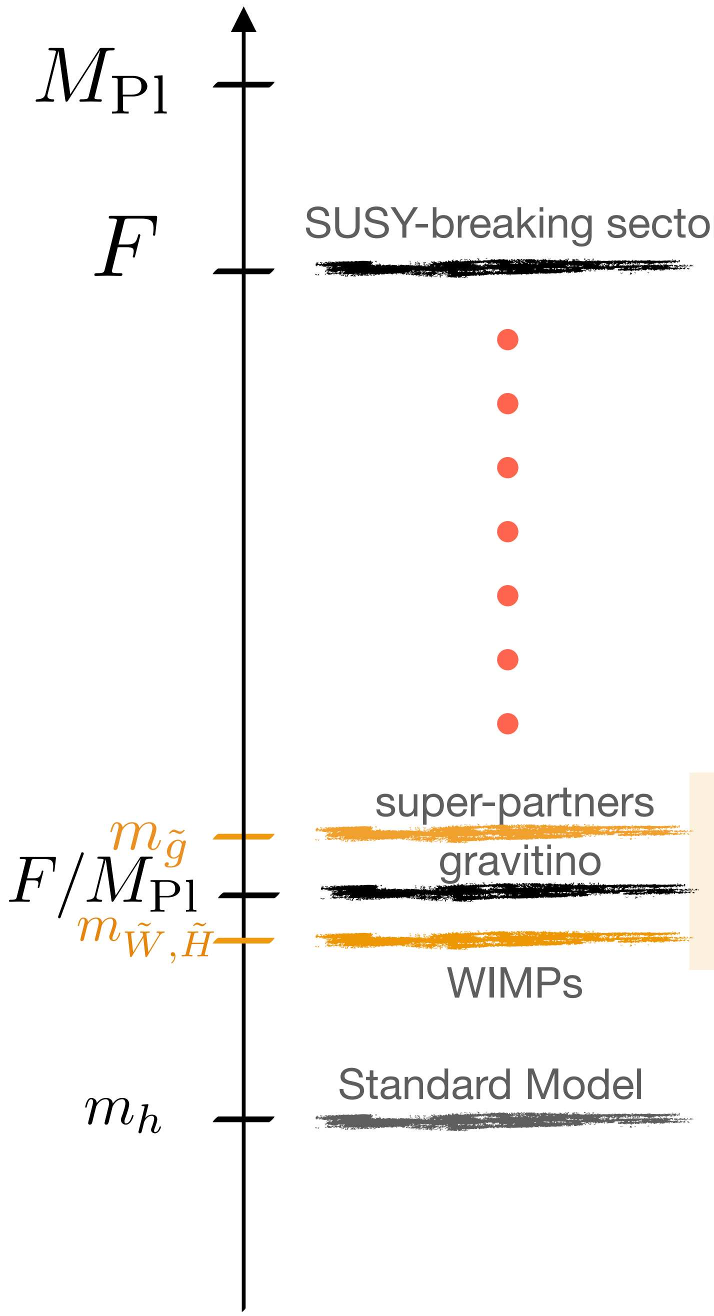
SUSY-breaking scale

$$m_{\tilde{g}} \sim g_M \times \sqrt{F}$$

The strength of the hidden sector coupling determines the scheme

The very same force has important cosmological consequences (nature of DM etc…)

Gravity Mediation



$$m_{\tilde{g}} \sim g_M \times \sqrt{F}$$

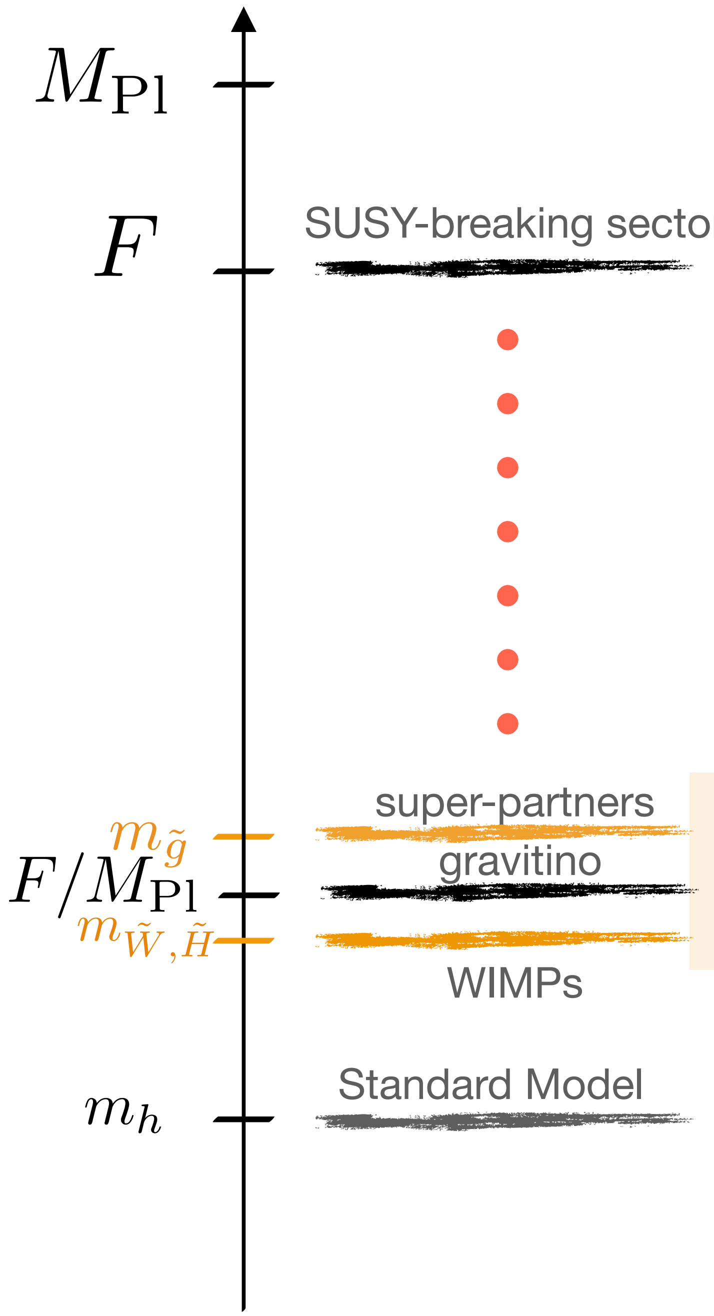
$$g_M \sim \sqrt{F}/M_{\mathrm{Pl}}$$

EWkino Lightest SUSY Particle (LSP)

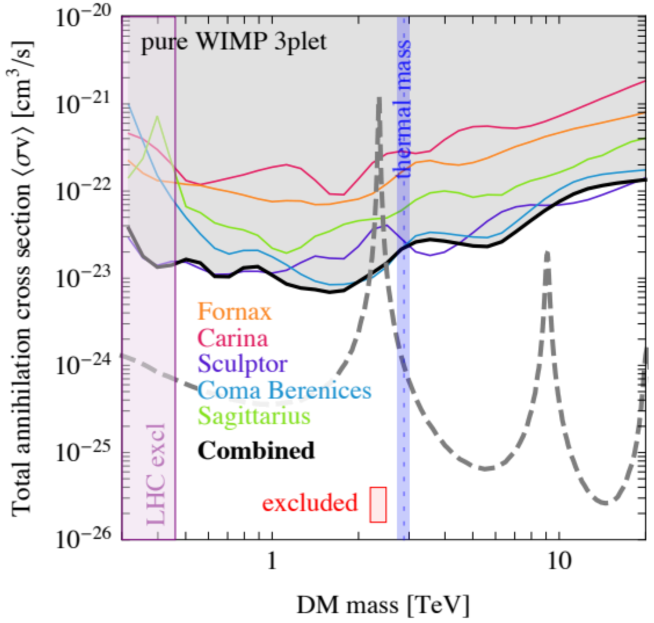
“Split SUSY”

G.F. Giudice, A. Romanino (2004)
N. Arkani-Hamed, S. Dimopoulos (2004)

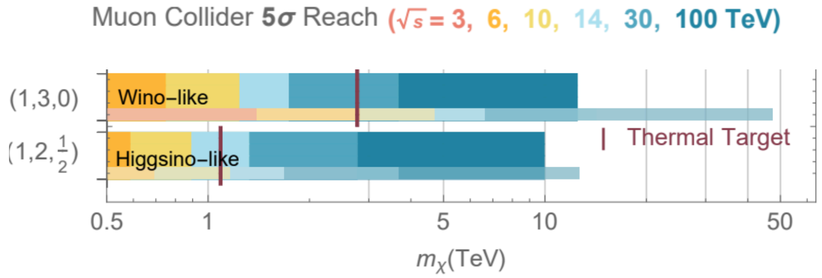
Gravity Mediation



H.E.S.S. (2018)



Han, Liu, Wang,Wang (2020)



$$m_{\tilde{g}} \sim g_M \times \sqrt{F}$$

$$g_M \sim \sqrt{F}/M_{\rm Pl}$$

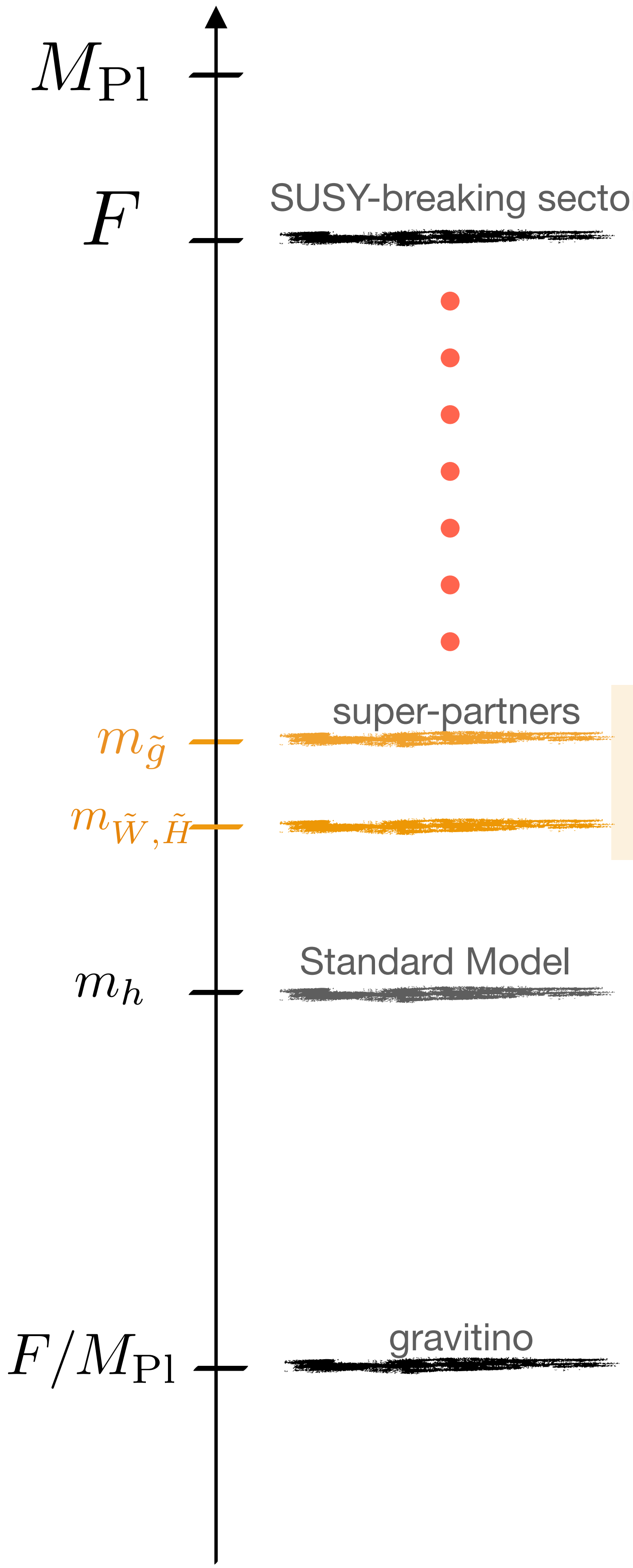
EWkino Lightest SUSY Particle (LSP)

“Split SUSY”

G.F. Giudice, A. Romanino (2004)
N. Arkani-Hamed, S. Dimopoulos (2004)

The main signal to hunt for are the WIMPs! indirect detection + future colliders

Low-energy SUSY-breaking



$$\langle X \rangle = \frac{f_a}{\sqrt{2}} + \sqrt{2}\theta\tilde{G} + \theta^2 F$$

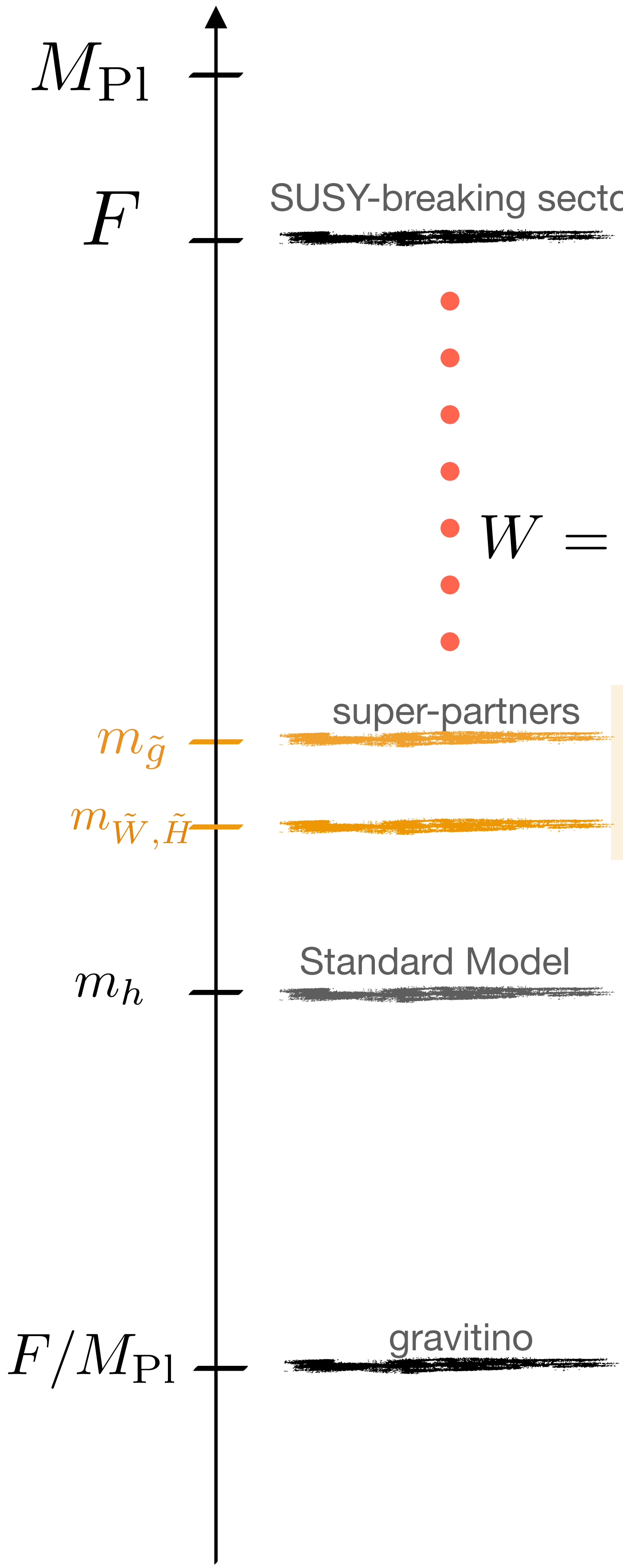
R-symmetry breaking scale

Goldstino

SUSY-breaking scale

$$m_{\tilde{g}} \sim g_M \times \sqrt{F} \quad \cdots\cdots\rightarrow \quad g_M \gg \sqrt{F}/M_{\rm Pl}$$

Low-energy SUSY-breaking



$$\langle X\rangle = \frac{f_a}{\sqrt{2}} + \sqrt{2}\theta\tilde G + \theta^2 F$$

R-symmetry breaking scale

Goldstino

SUSY-breaking scale

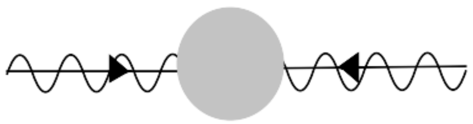
$$W = X\Phi\tilde\Phi$$

$$m_{\tilde g} \sim g_M \times \sqrt{F}$$

$\cdots\cdots\blacktriangleright$ $g_M \gg \sqrt{F}/M_{\rm Pl}$

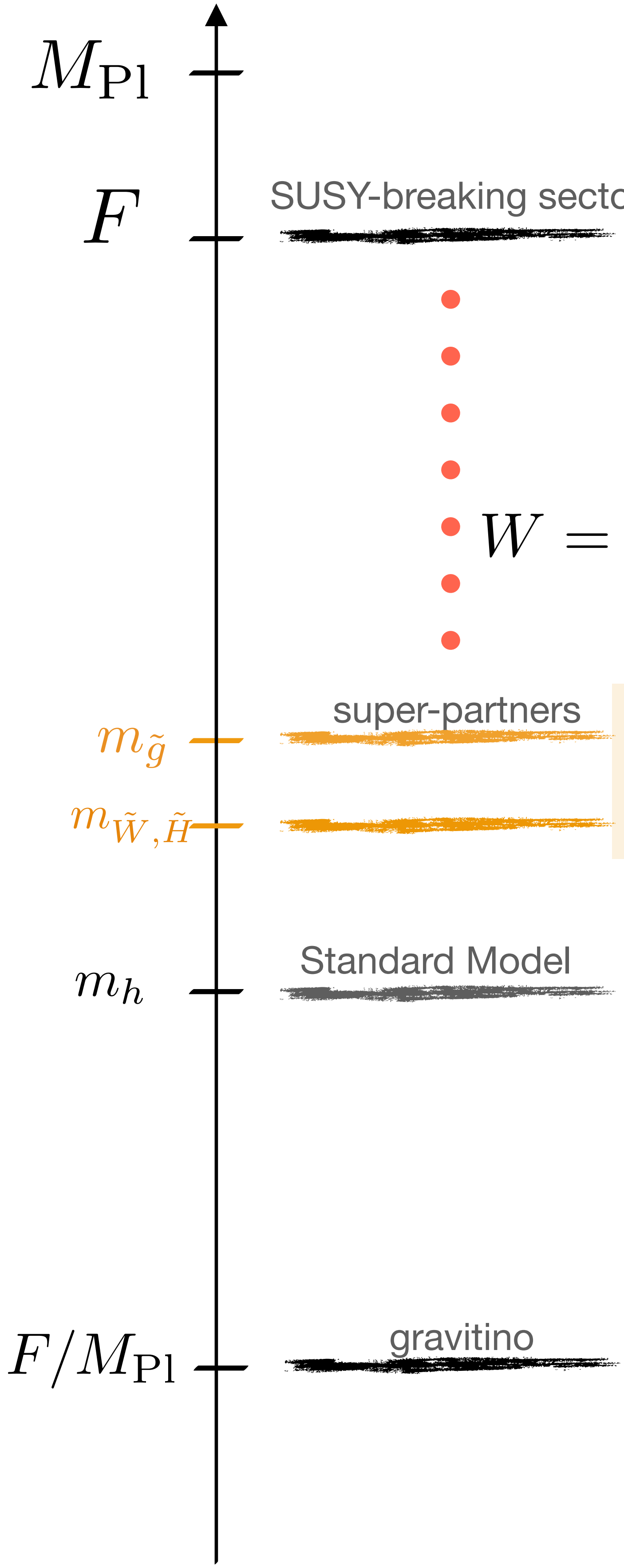
$\cdots\cdots\blacktriangleright$ ex: gauge mediation

G.F. Giudice, R. Rattazzi (1998)



$$g_M = \frac{\alpha_3 N_{\rm mess}}{4\pi} \times s_M \times \frac{\sqrt{F}}{f_a}$$

Low-energy SUSY-breaking



$$\langle X\rangle = \underbrace{\frac{f_a}{\sqrt{2}}}_{\text{R-symmetry breaking scale}} + \underbrace{\sqrt{2}\theta\tilde G}_{\text{Goldstino}} + \underbrace{\theta^2 F}_{\text{SUSY-breaking scale}}$$

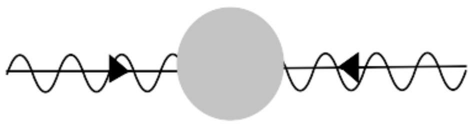
$$W = X\Phi\tilde\Phi$$

$$m_{\tilde g} \sim g_M \times \sqrt{F}$$

⋯⋯▸ $g_M \gg \sqrt{F}/M_{\rm Pl}$

⋯⋯▸ ex: gauge mediation

G.F. Giudice, R. Rattazzi (1998)



$$g_M = \frac{\alpha_3 N_{\rm mess}}{4\pi} \times s_M \times \frac{\sqrt{F}}{f_a}$$

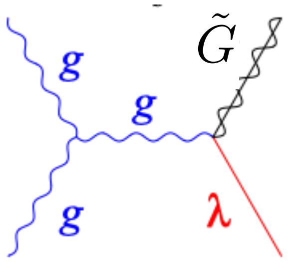
⋯⋯▸ Gravitino LSP

$$\mathcal{L}_{\tilde G} \supset \frac{1}{F}\partial^\mu \tilde G J_\mu$$

Gravitino cosmology

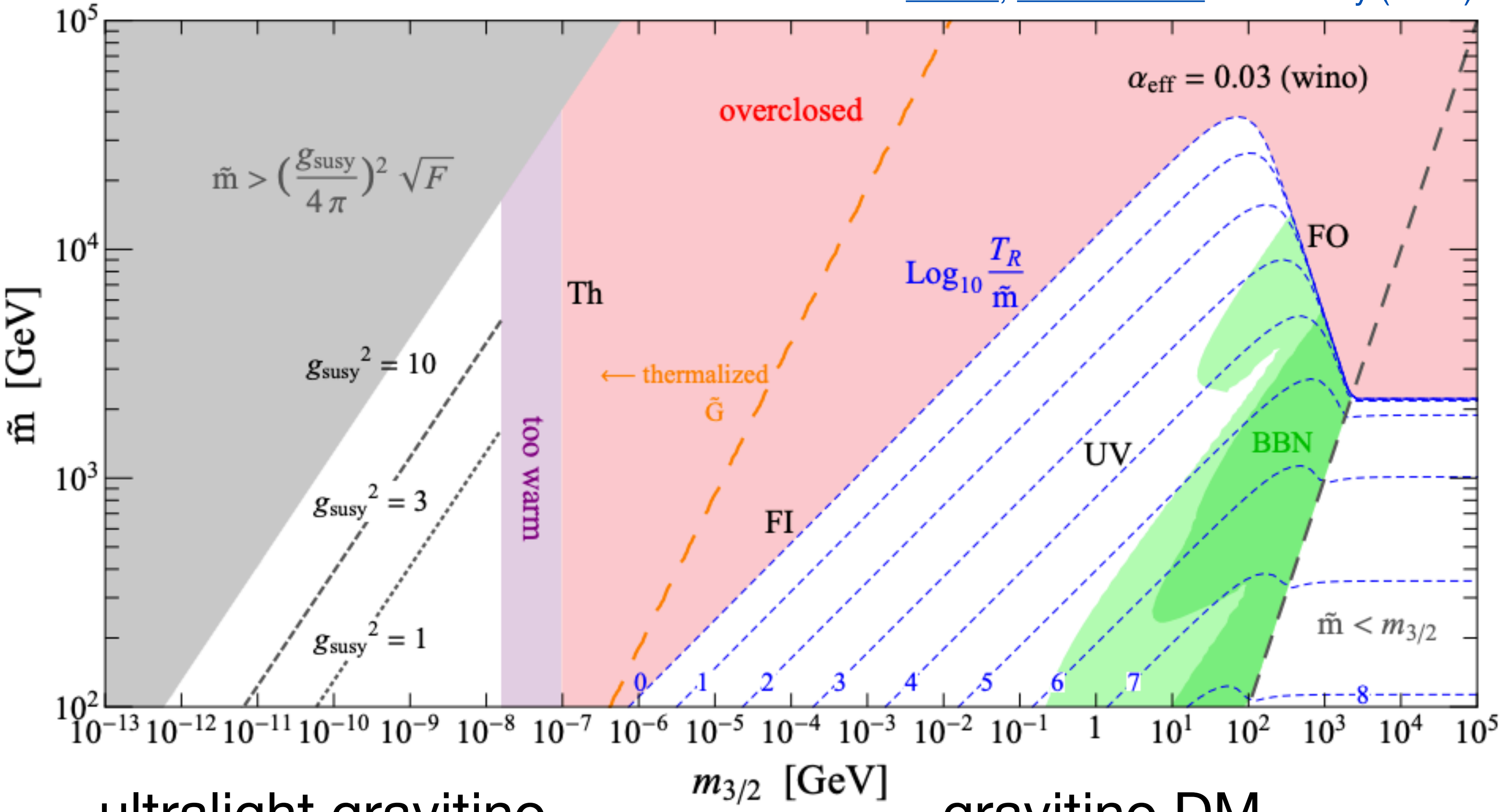
The gravitino cosmology shapes the parameter space if we require $T_{r.h.} \gtrsim \sqrt{F}$

This is known as "gravitino problem" $\mathcal{L}_{\tilde{G}} \supset \frac{1}{F}\partial^{\mu}\tilde{G}J_{\mu}$ ⋯⋯▸



S. Rychkov, A. Strumia (2007)

L. Hall, J. Ruderman T.Volansky (2013)



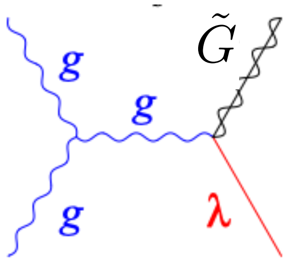
ultralight gravitino

gravitino DM

Gravitino cosmology

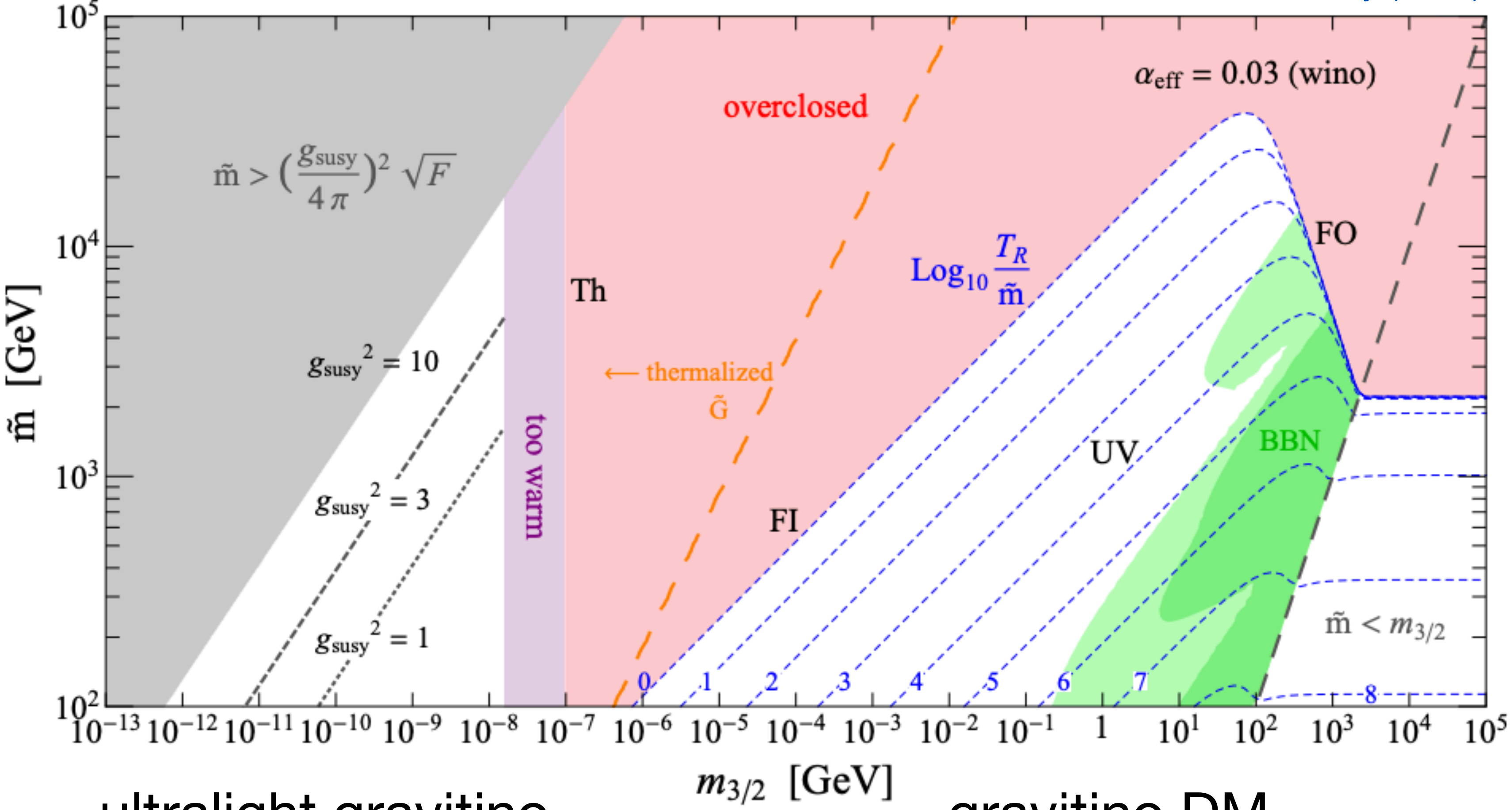
The gravitino cosmology shapes the parameter space if we require $T_{r.h.} \gtrsim \sqrt{F}$

This is known as “gravitino problem” $\mathcal{L}_{\tilde{G}} \supset \frac{1}{F} \partial^{\mu} \tilde{G} J_{\mu}$ ⋯⋯▸



S. Rychkov, A. Strumia (2007)

L. Hall, J. Ruderman T.Volansky (2013)



ultralight gravitino

gravitino DM

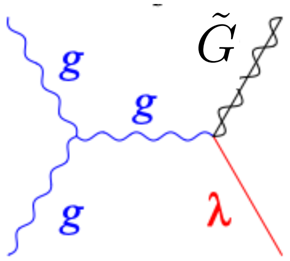
The gravitino production from the plasma is enhanced if it is light

$$Y_{3/2} \sim C_{\rm UV} \frac{M_3^2 T}{m_{3/2}^2 M_{\rm Pl}}$$

Gravitino cosmology

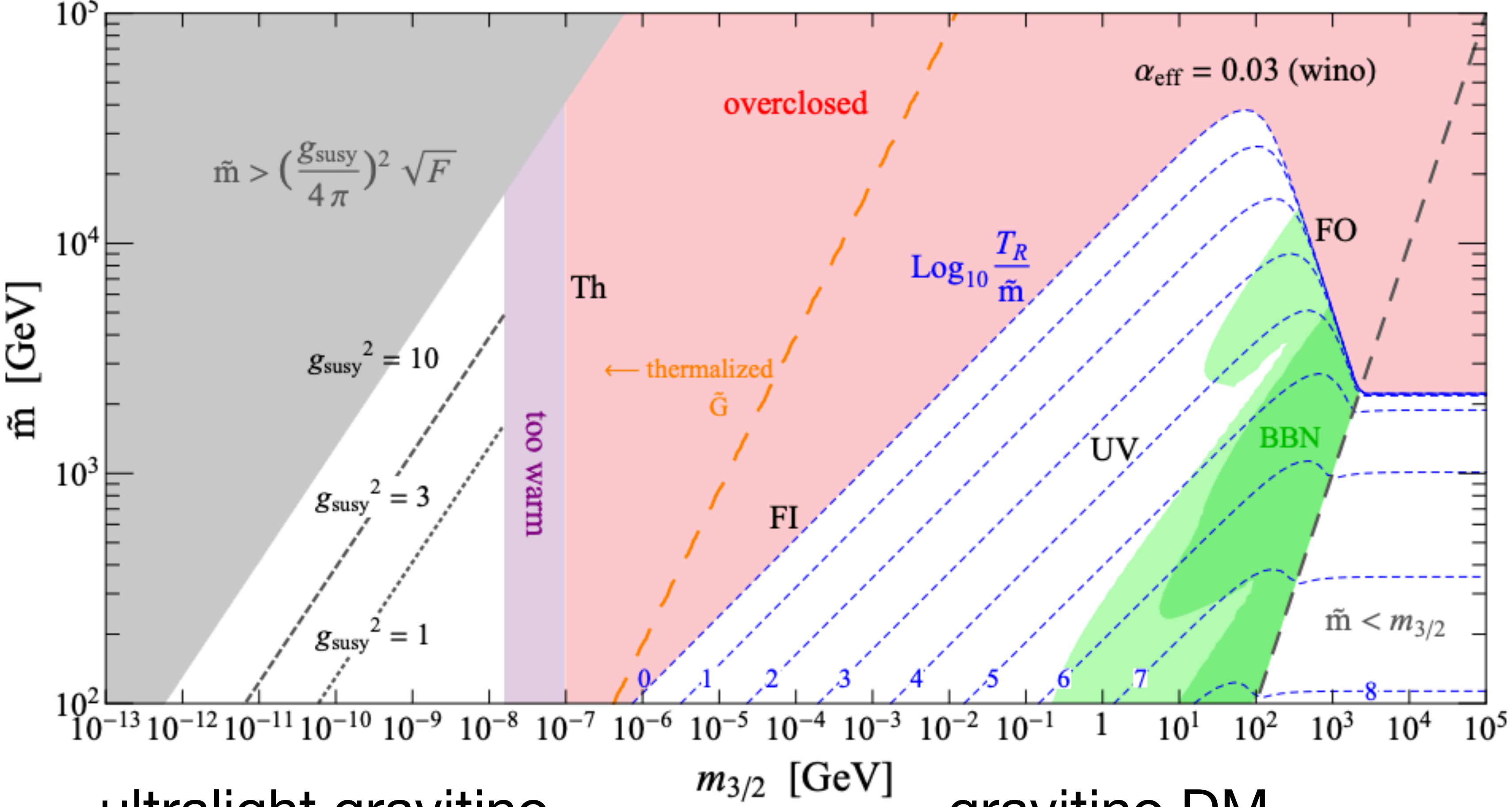
The gravitino cosmology shapes the parameter space if we require $T_{r.h.} \gtrsim \sqrt{F}$

This is known as "gravitino problem" $\mathcal{L}_{\tilde{G}} \supset \frac{1}{F}\partial^\mu \tilde{G} J_\mu$



S. Rychkov, A. Strumia (2007)

L. Hall, J. Ruderman T.Volansky (2013)



ultralight gravitino

gravitino DM

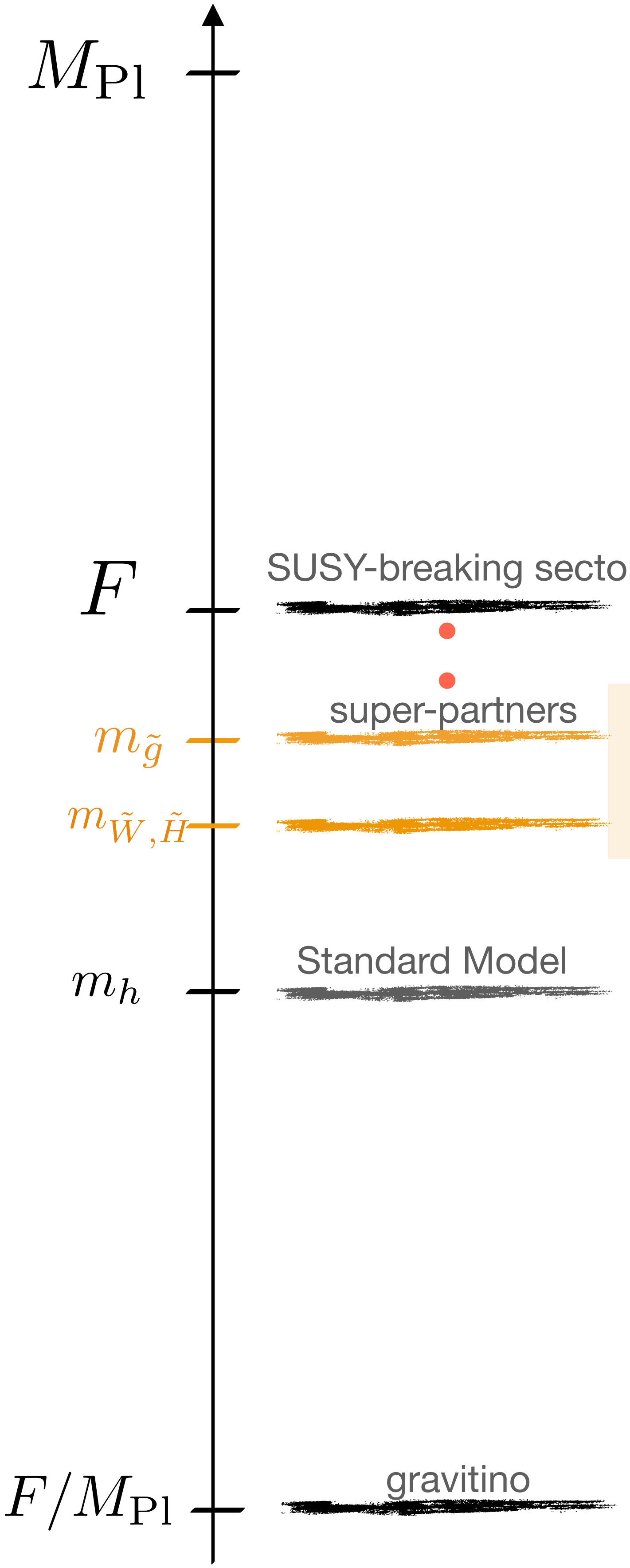
The gravitino production from the plasma is enhanced if it is light

$$Y_{3/2} \sim C_{\rm UV} \frac{M_3^2 T}{m_{3/2}^2 M_{\rm Pl}}$$

This lead generically to problems with Gravitino overabundance

$$m_{3/2} Y_{3/2} < 0.27 T_{\rm eq}$$

Ultralight Gravitino Window



$$\sqrt{F} < \left(M_{\rm Pl} m_{\tilde{g}}^2\right)^{1/3} \sim 5 \times 10^7 \ {\rm TeV}$$

the gravitino is thermal!

$$V = F^2$$

very low SUSY-breaking scale

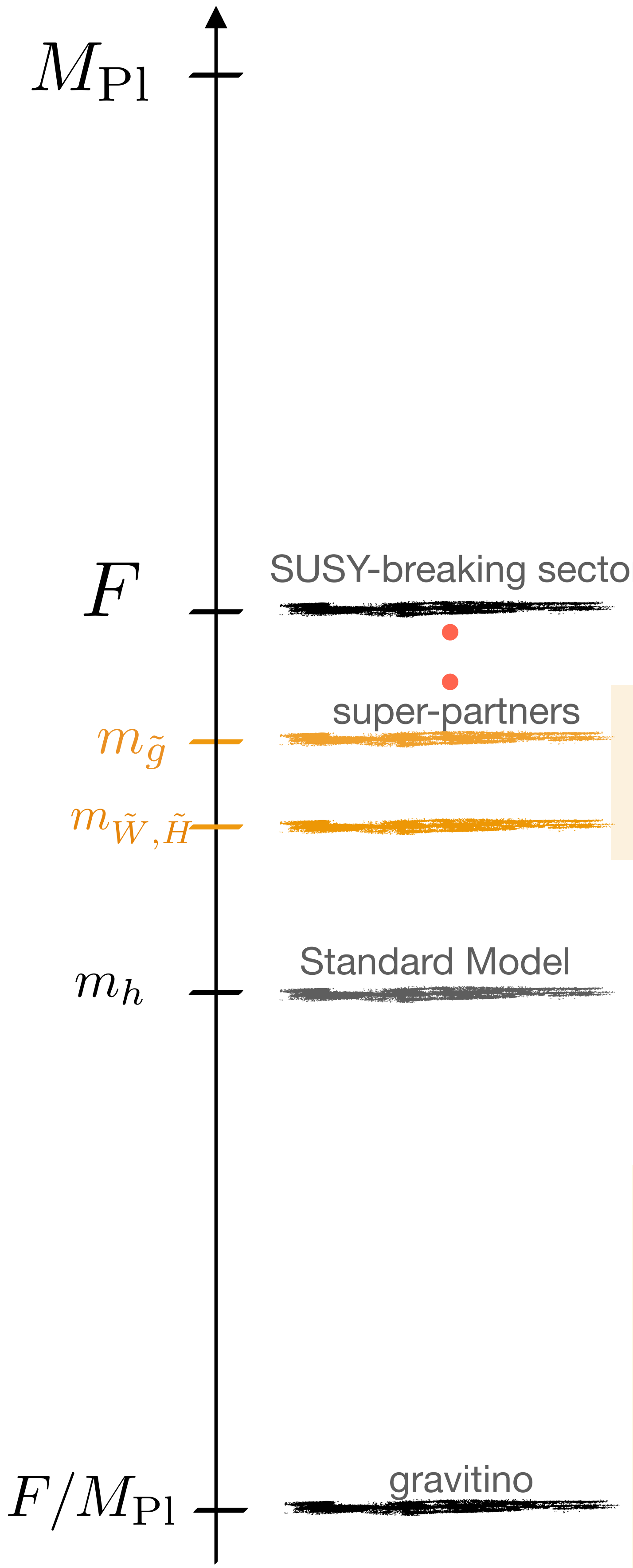
$$m_{\tilde{g}} \sim g_M \times \sqrt{F}$$

large coupling $1 \lesssim g_M \lesssim 100$

model building challenges

A. Hook, H. Murayama (2015, 2018)

Ultralight Gravitino Window



$$\sqrt{F} < \left(M_{\rm Pl} m_{\tilde g}^2\right)^{1/3} \sim 5 \times 10^7 \text{ TeV}$$

the gravitino is thermal!

$$V = F^2$$

very low SUSY-breaking scale

$$m_{\tilde g} \sim g_M \times \sqrt{F}$$

large coupling $1 \lesssim g_M \lesssim 100$

model building challenges

A. Hook, H. Murayama (2015, 2018)

bounded from above by warm DM constraints

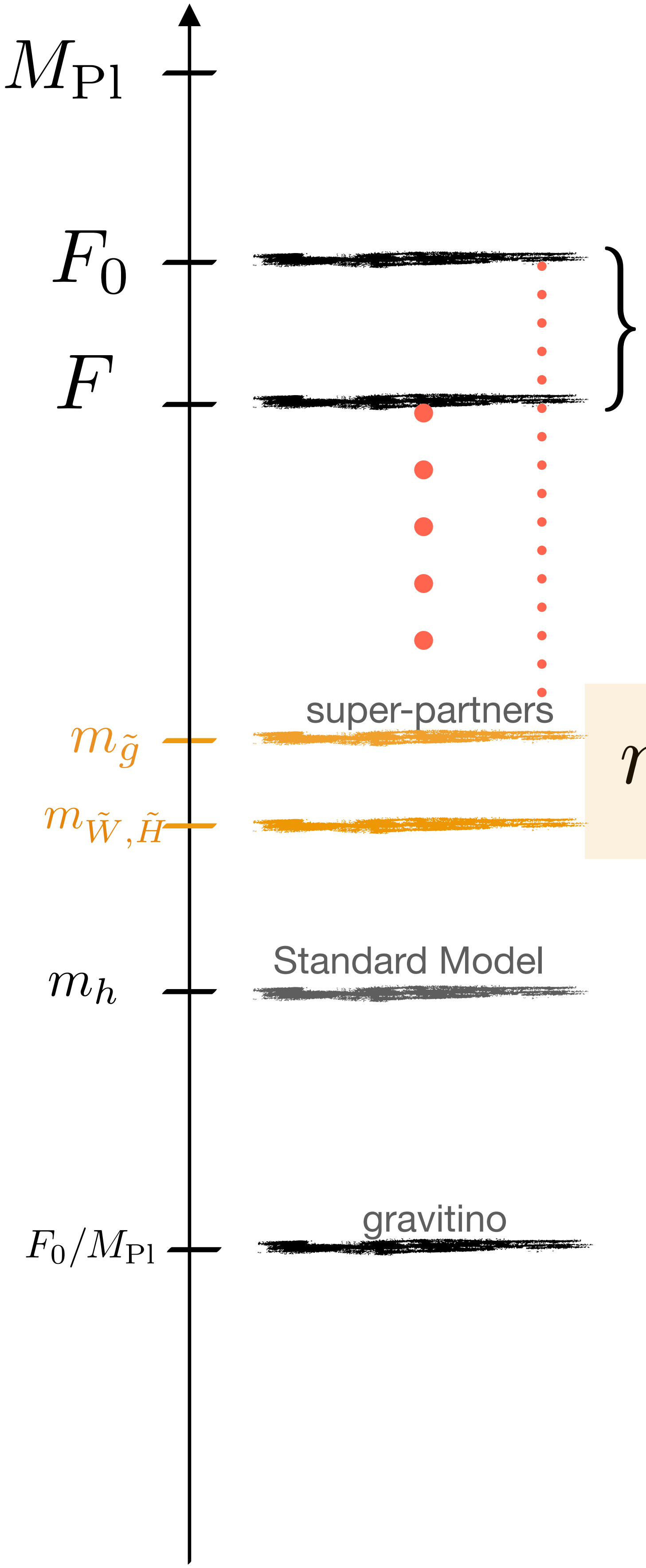
$m_{3/2} \lesssim 16 \text{ eV}$, $F \lesssim 260 \text{ TeV}$

from below by collider bounds

$F \gtrsim \text{TeV's}$

··········▸ Planck data + future colliders can close this?

Gravitino Dark Matter



SUSY-breaking hidden sectors $F/F_0 \lesssim 10^{-2} \left(\frac{F}{10^7 \text{ GeV}}\right)\left(\frac{0.1}{g_M}\right)$

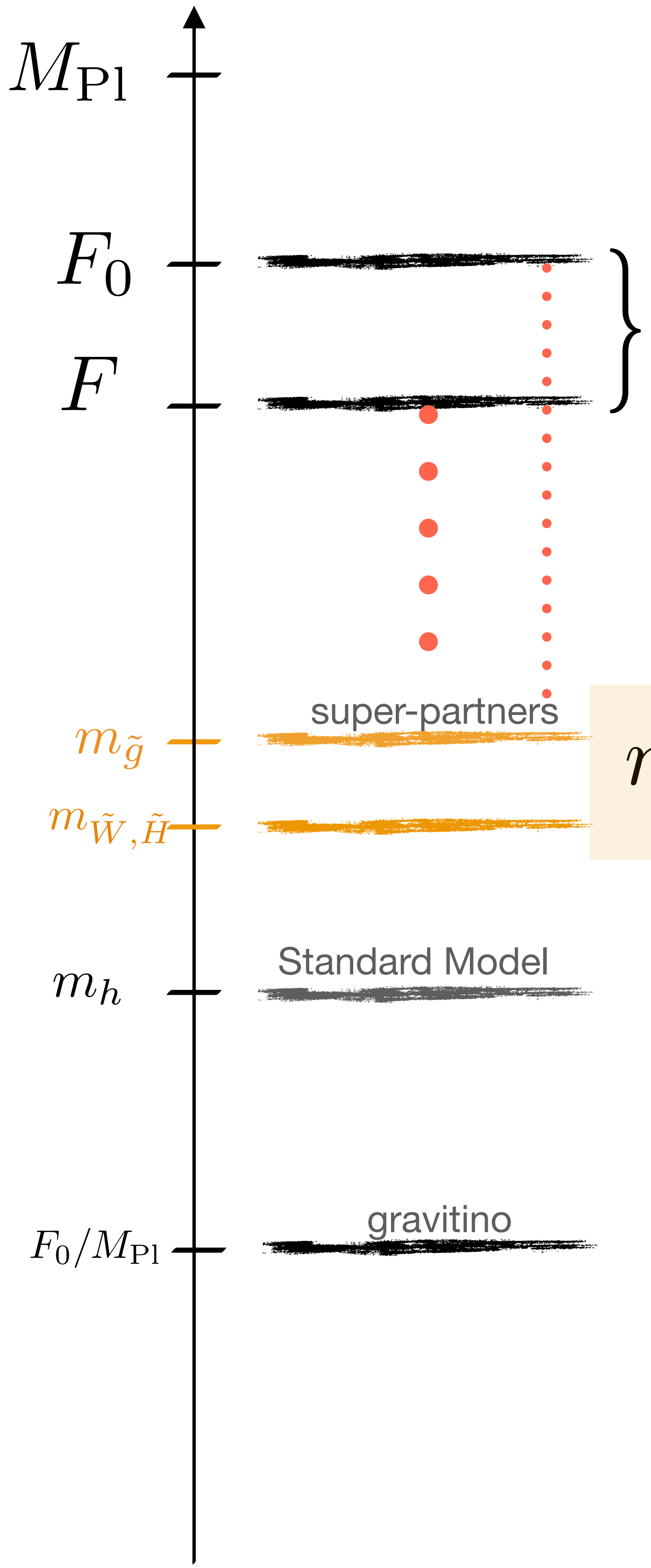
to avoid gravitino thermalization

$$m_{\tilde g} \sim g_M \times \sqrt{F}$$

$g_M \ll 1$ standard gauge mediation!

For a fixed gluino mass the UV produced gravitino can match the relic abundance today

Gravitino Dark Matter



SUSY-breaking hidden sectors

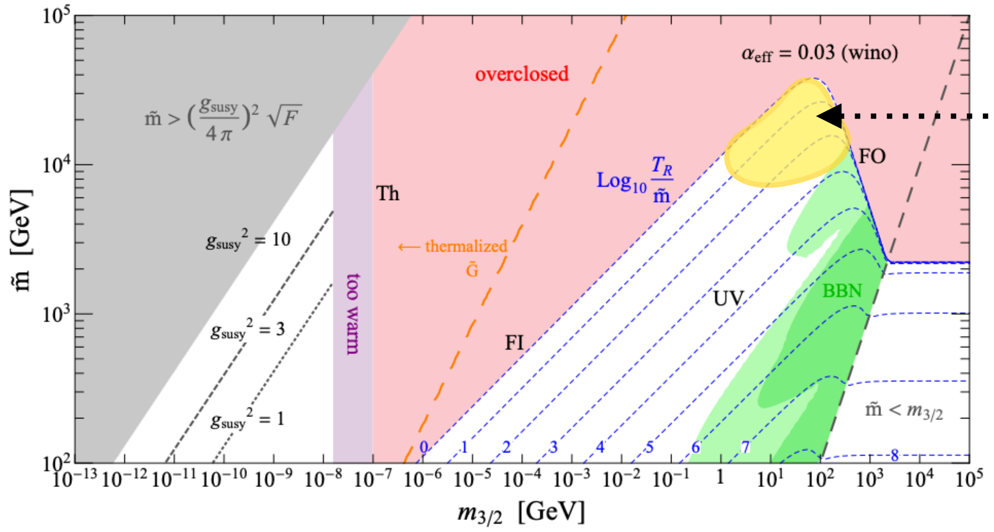
$$F/F_0 \lesssim 10^{-2}\left(\frac{F}{10^7\ \mathrm{GeV}}\right)\left(\frac{0.1}{g_M}\right)$$

to avoid gravitino thermalization

$$m_{\tilde{g}} \sim g_M \times \sqrt{F}$$

$$g_M \ll 1$$ standard gauge mediation!

For a fixed gluino mass the UV produced gravitino can match the relic abundance today



we wanna live here!

$$10^5\ \mathrm{GeV} < F < 10^8\ \mathrm{GeV}$$

The pseudomodulus phase transition

$$X = \frac{x}{\sqrt{2}} e^{2ia/f_a} + \sqrt{2}\theta\tilde{G} + \theta^2 F$$

Goldstino

SUSY-breaking scale

the complex pseudomoduls

The R-axion a is the Goldstone of R-symmetry broken spontaneously

The radial mode x is massless at tree-level $W = FX$ $\dashrightarrow$ $m_x \sim \frac{\lambda^2}{16\pi^2} F/m_*$

$m_x \ll m_*$ at weak coupling

The pseudomodulus phase transition

$$X = \frac{x}{\sqrt{2}} e^{2ia/f_a} + \sqrt{2}\theta\tilde{G} + \theta^2 F$$

the complex pseudomoduls

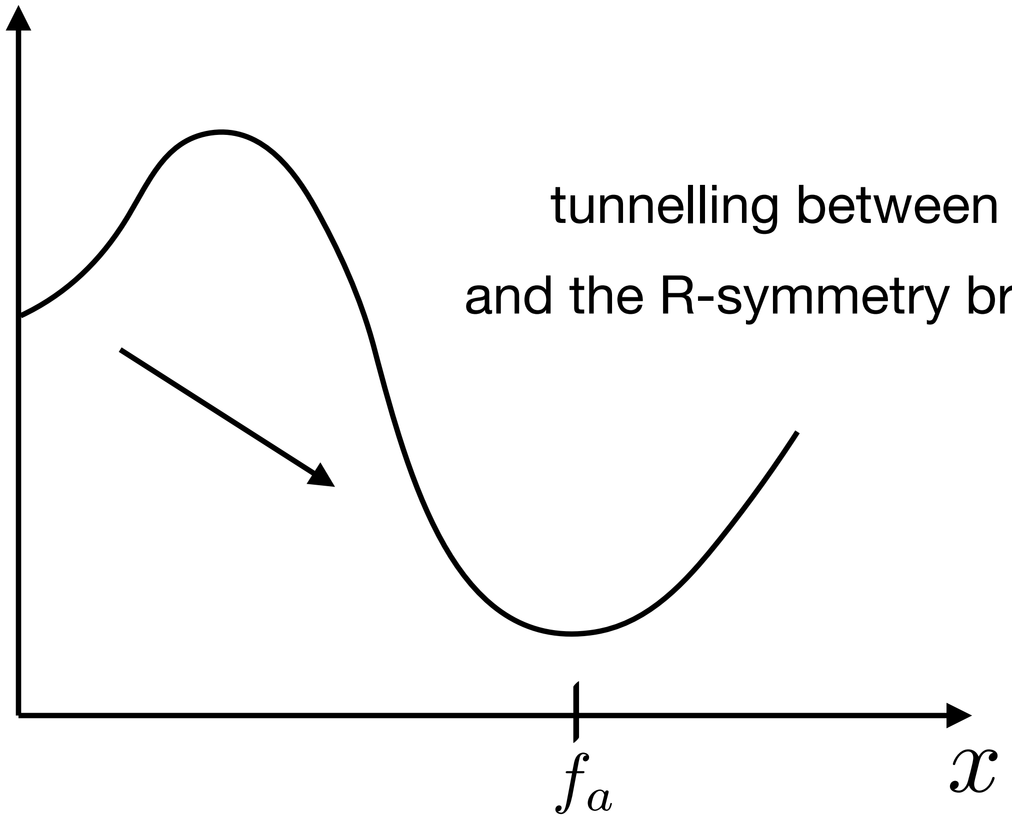
Goldstino

SUSY-breaking scale

The R-axion a is the Goldstone of R-symmetry broken spontaneously

The radial mode x is massless at tree-level $W = FX$ $\cdots\cdots\rightarrow$ $m_x \sim \frac{\lambda^2}{16\pi^2} F/m_*$

$m_x \ll m_*$ at weak coupling



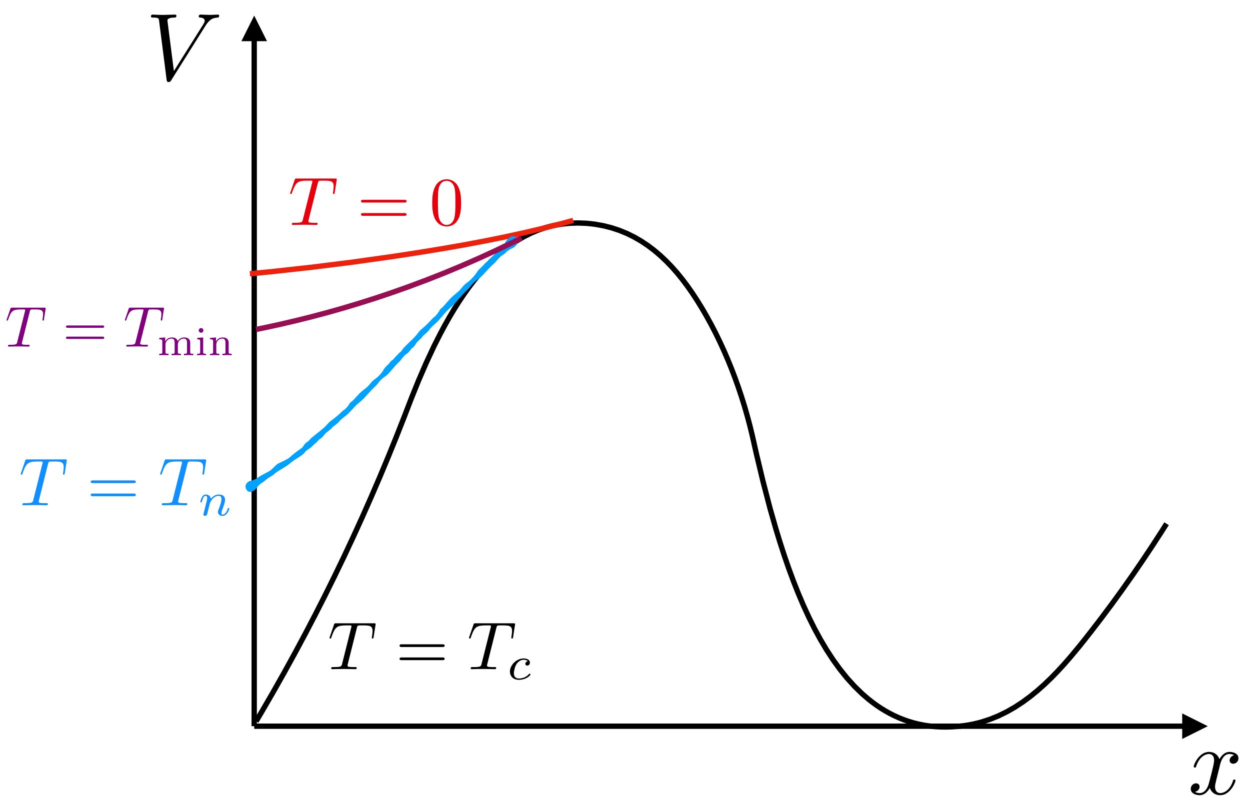
tunnelling between the origin

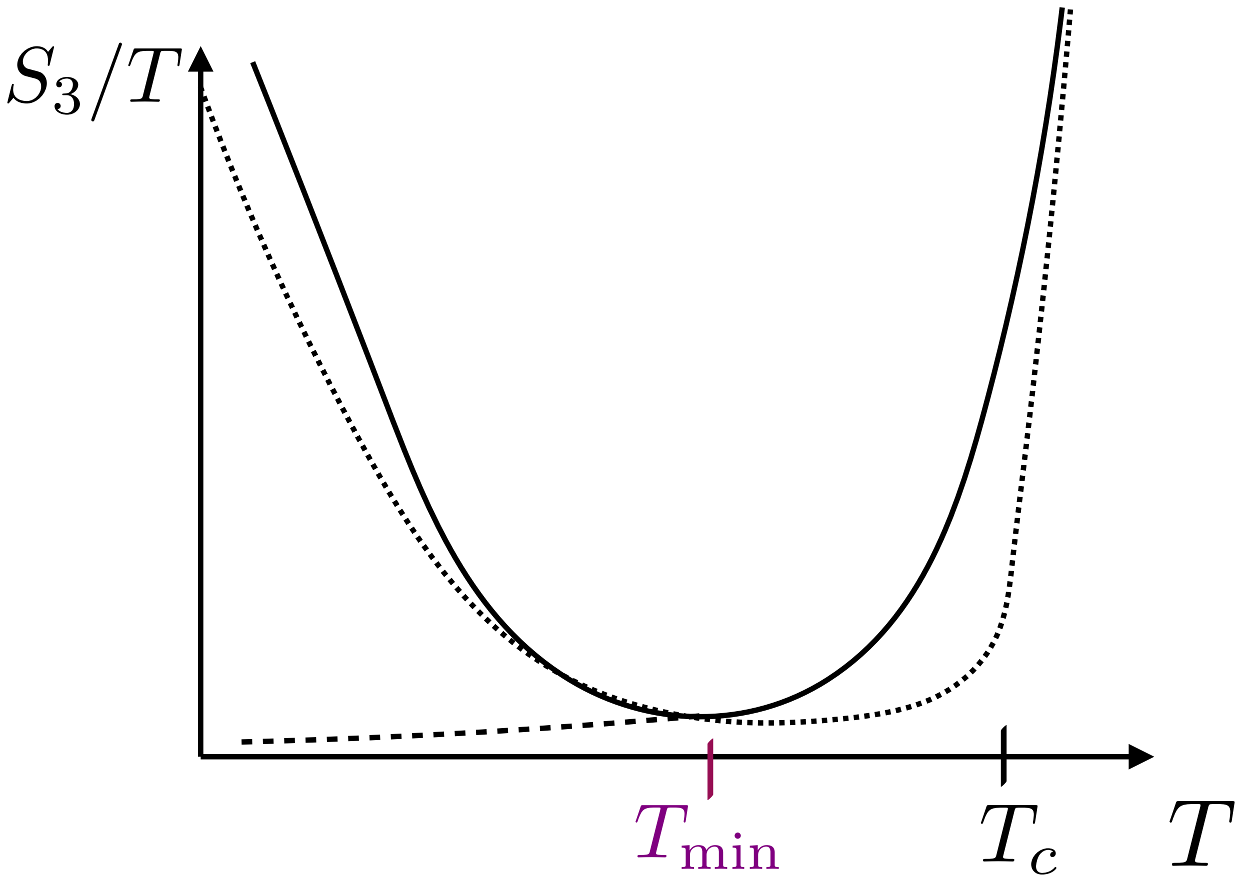
and the R-symmetry breaking vacuum*

* breaking R-symmetry is necessary to have Majorana gaugino masses

1st order Phase Transitions

Let me assume that the SUSY-breaking sector
produces a first order phase transition *(later we will see how)*



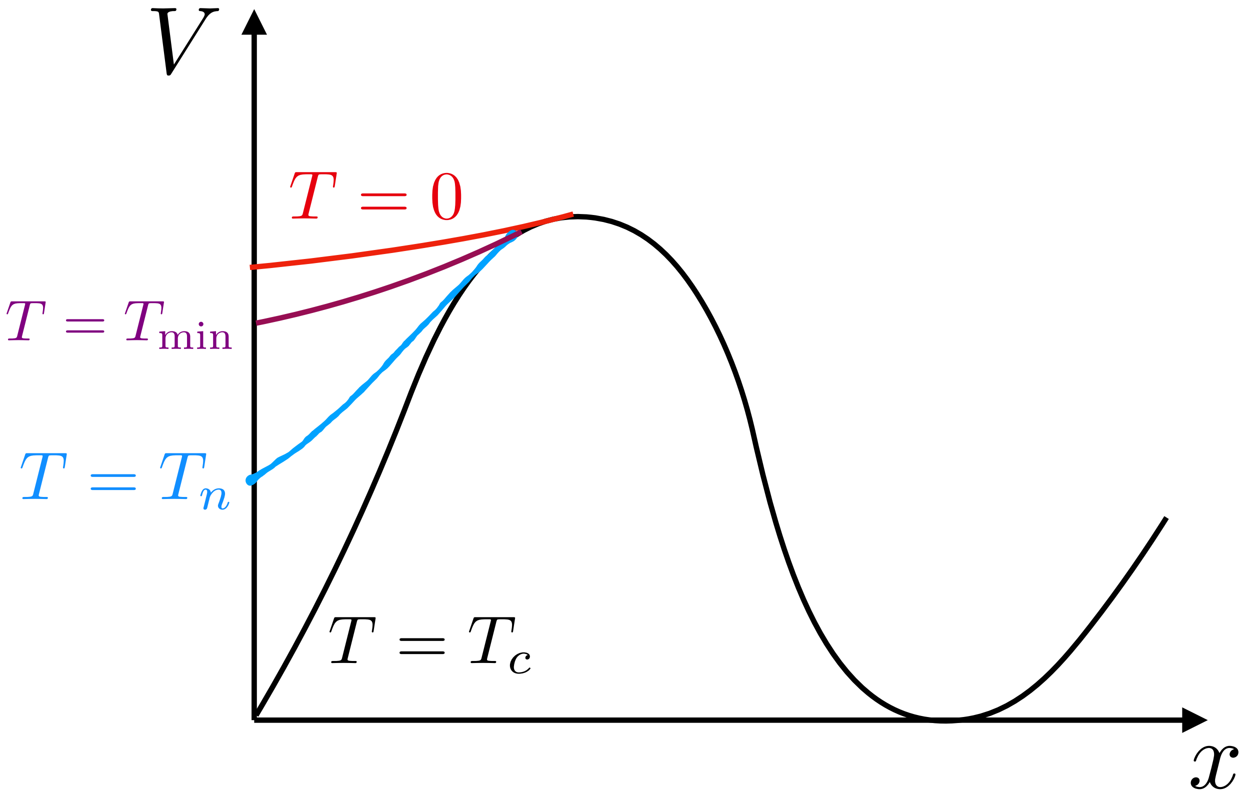


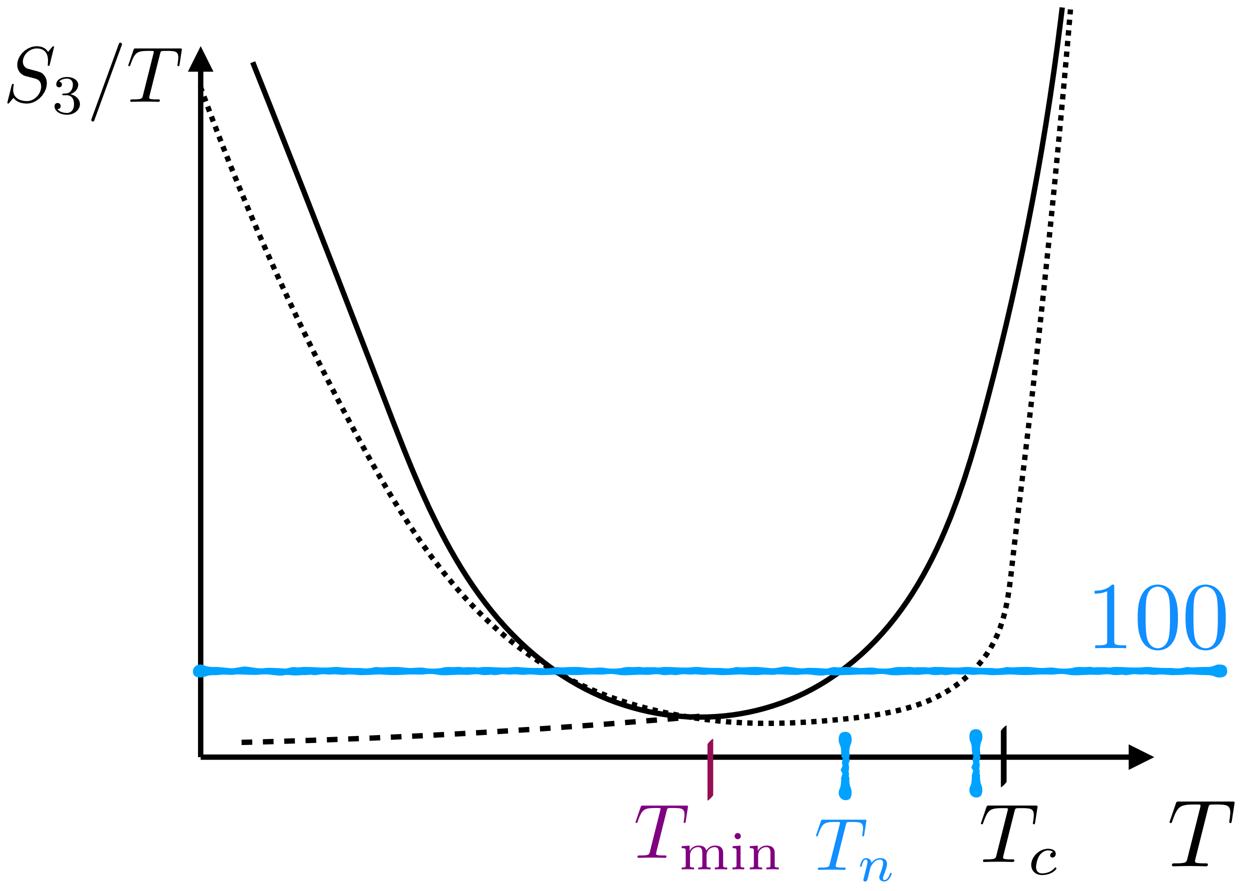
If thermal fluctuations dominate,
the tunnelling probability is encoded in

$$\Gamma(T) \simeq T^4 \left(\frac{S_3}{2\pi T}\right)^{\frac{3}{2}} \exp\left(-S_3/T\right)$$

1st order Phase Transitions

Let me assume that the SUSY-breaking sector
produces a first order phase transition *(later we will see how)*





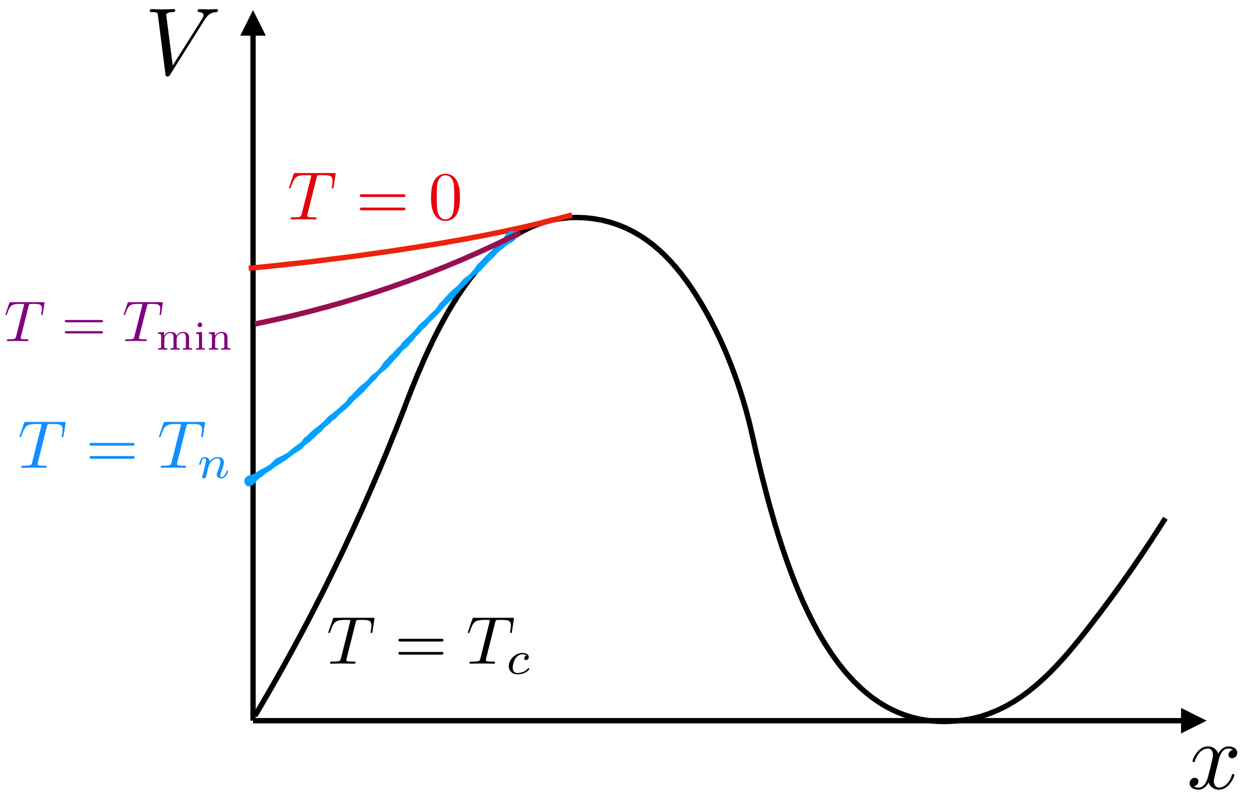
If thermal fluctuations dominate,
the tunnelling probability is encoded in

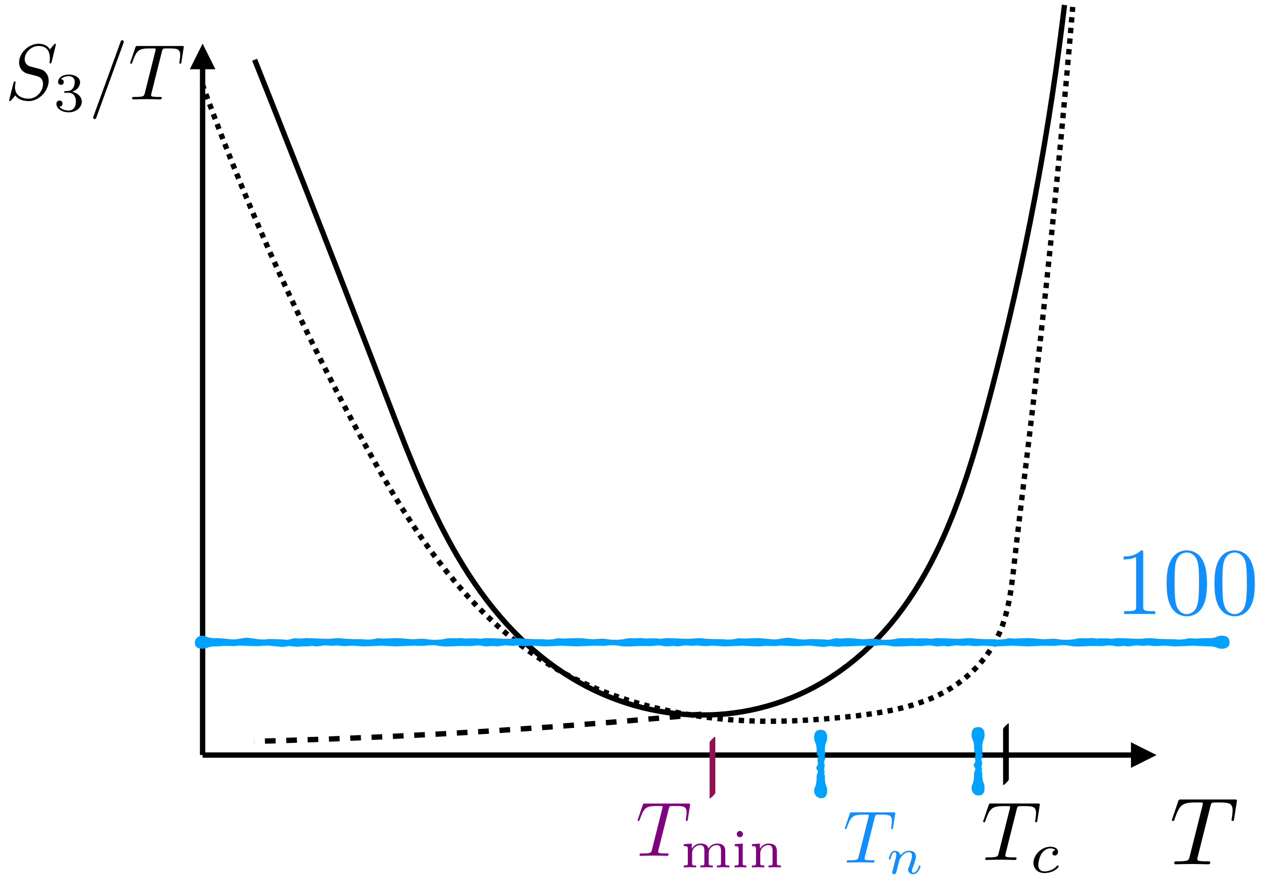
$$\Gamma(T) \simeq T^4 \left(\frac{S_3}{2\pi T}\right)^{\frac{3}{2}} \exp\left(-S_3/T\right)$$

When one bubble per Hubble volume nucleates

$$\frac{S_3(T_n)}{T_n} \simeq 100$$

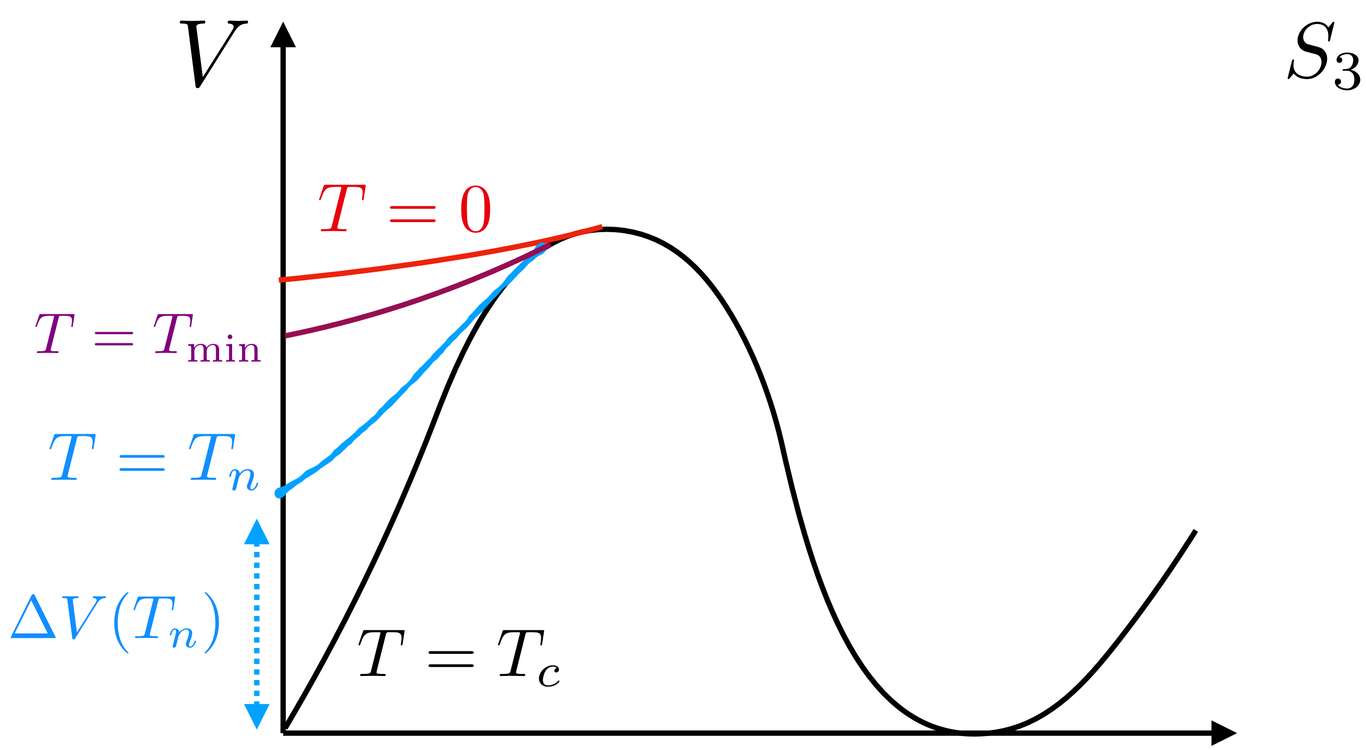
Gravitational Waves signal from 1st order Phase Transitions

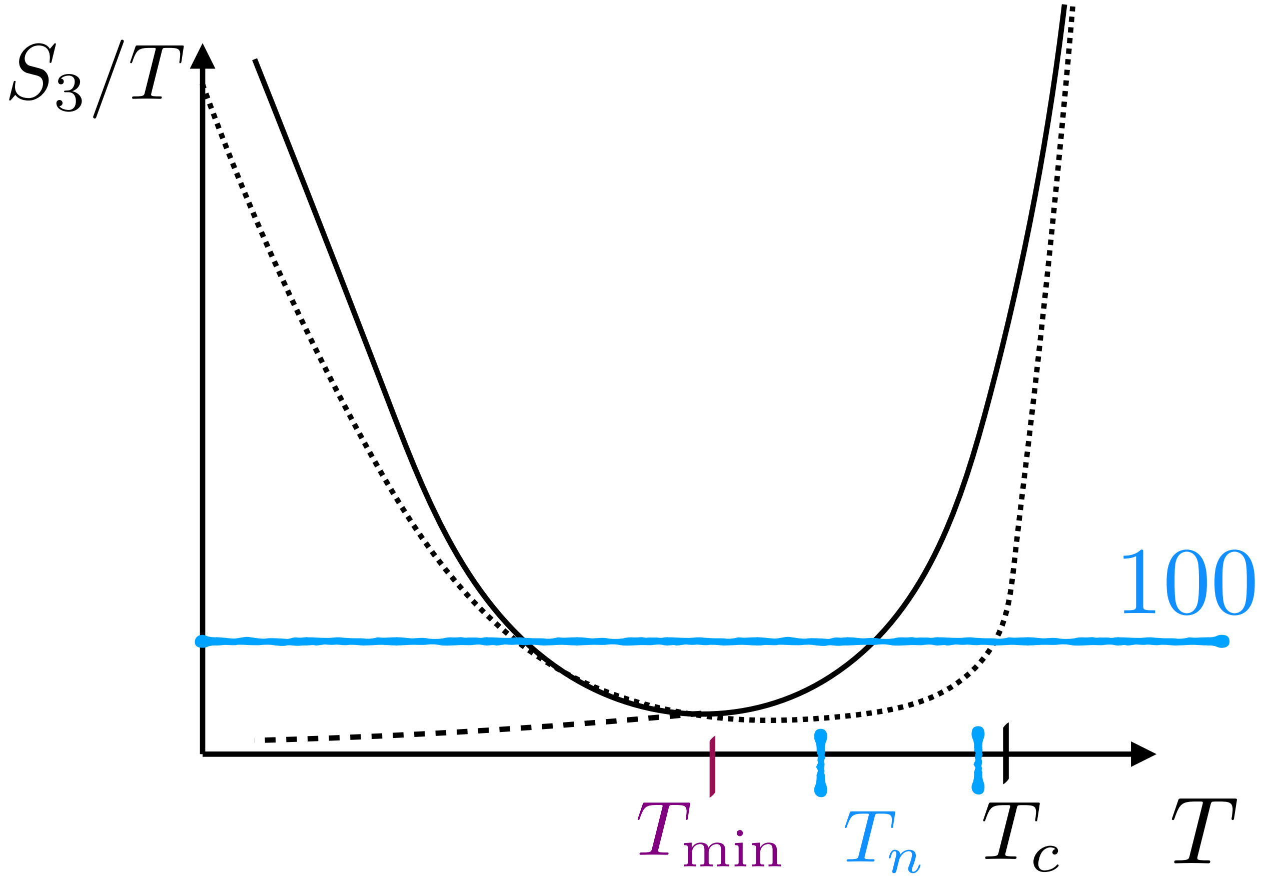




The detectability of the GWs signal depends on:

Gravitational Waves signal from 1st order Phase Transitions

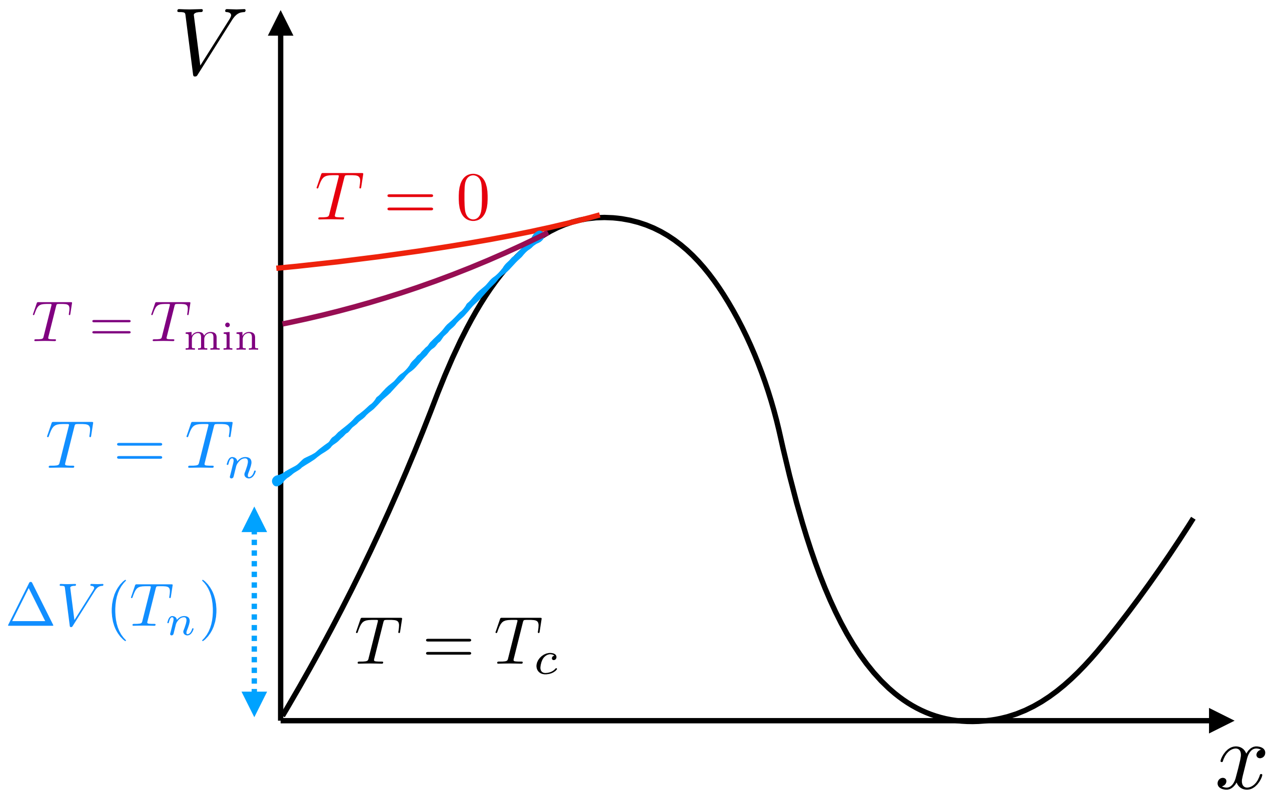


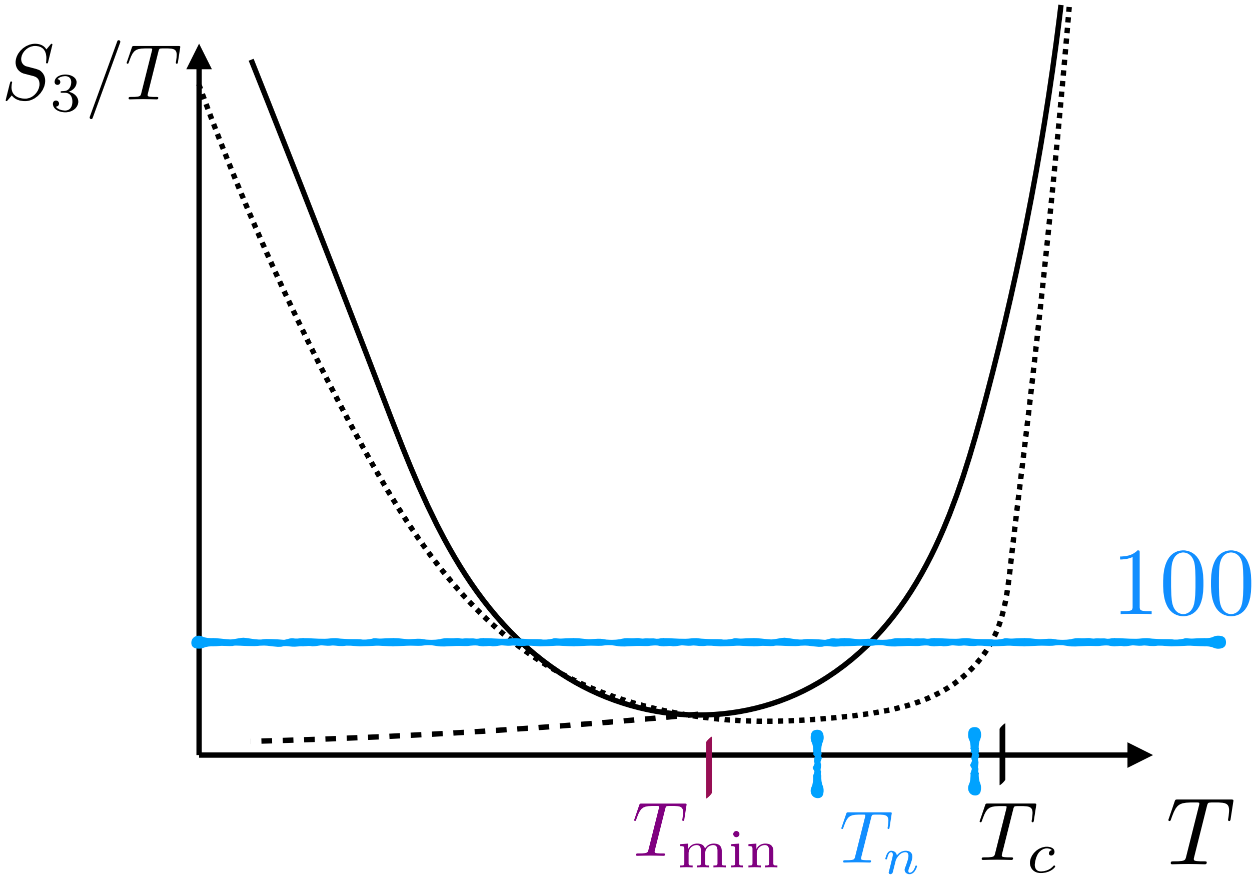


The detectability of the GWs signal depends on:

- The energy released during the PTs $\alpha(T_n) \sim \dfrac{\Delta V(T_n)}{\rho_R(T_n)}$

Gravitational Waves signal from 1st order Phase Transitions

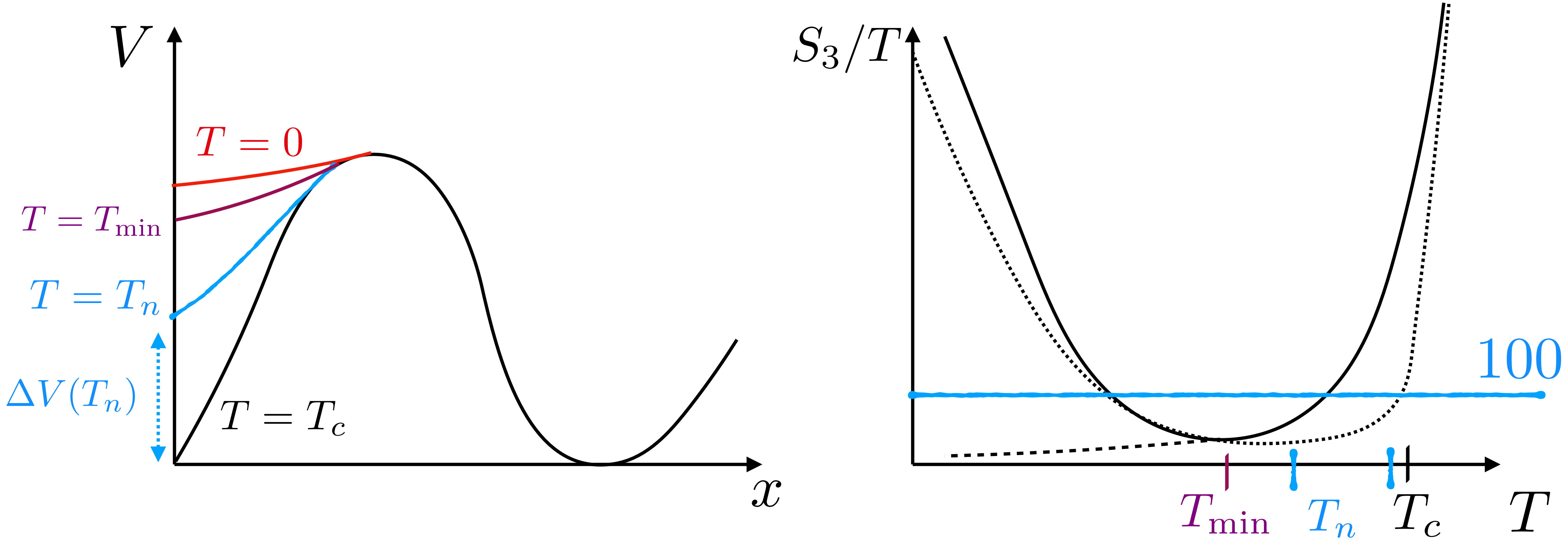




The detectability of the GWs signal depends on:

- The energy released during the PTs $\alpha(T_n) \sim \dfrac{\Delta V(T_n)}{\rho_R(T_n)}$
- The duration of the PTs $\beta_H(T_n) \overset{\text{def}}{=} \dfrac{\beta(T_n)}{H(T_n)} = T_n \dfrac{d}{dT}\left(\dfrac{S_3}{T}\right)\Big|_{T=T_n}$

Gravitational Waves signal from 1st order Phase Transitions

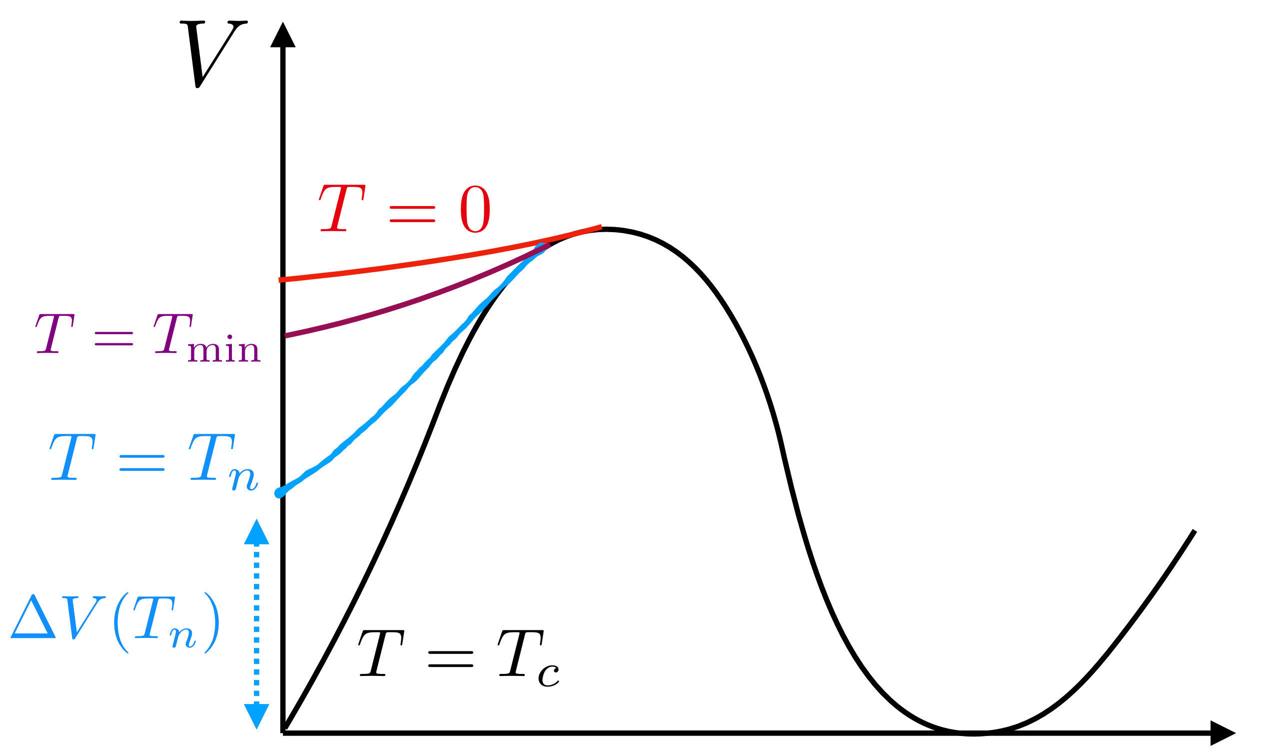


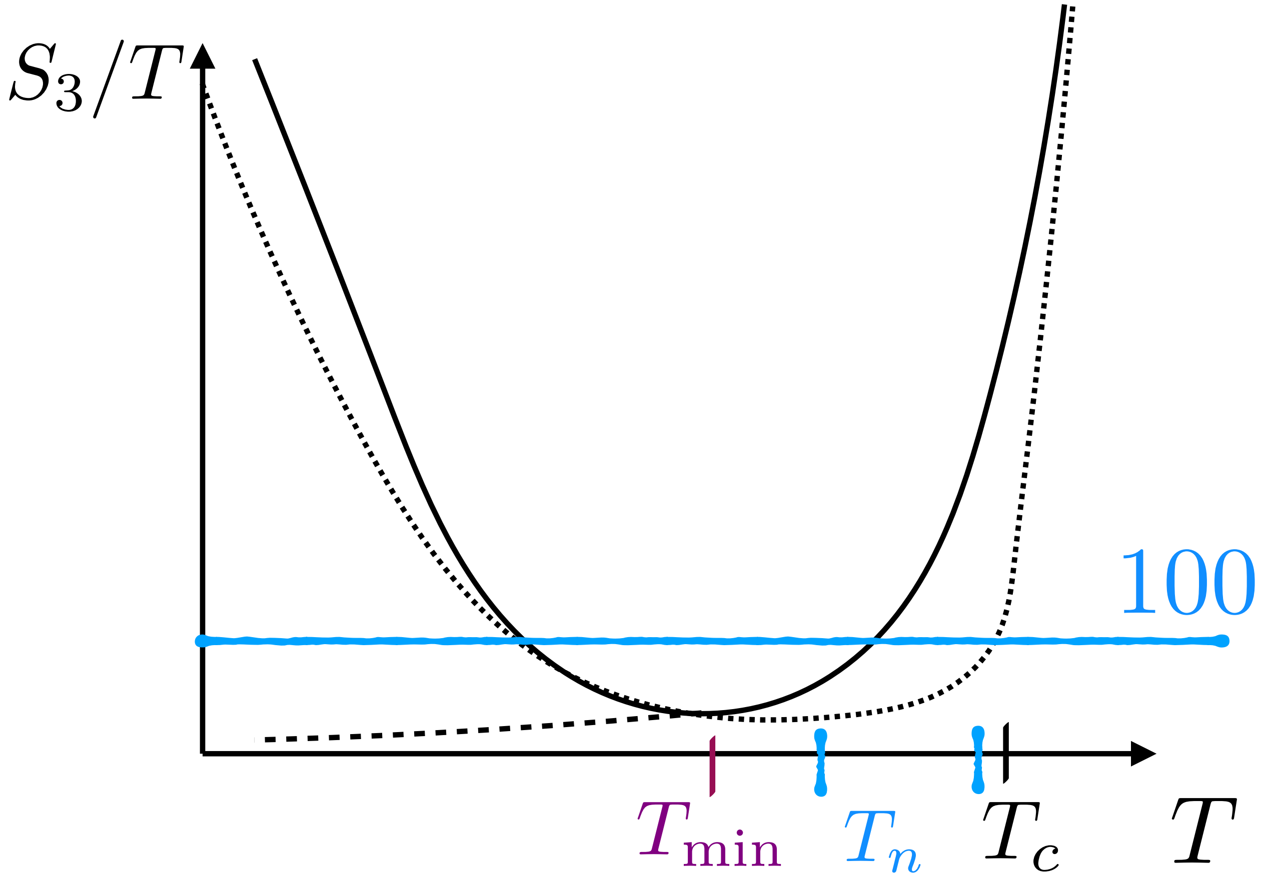
The detectability of the GWs signal depends on:

- The energy released during the PTs $\alpha(T_n) \sim \dfrac{\Delta V(T_n)}{\rho_R(T_n)}$
- The duration of the PTs $\beta_H(T_n) \overset{\text{def}}{=} \dfrac{\beta(T_n)}{H(T_n)} = T_n \dfrac{d}{dT}\left(\dfrac{S_3}{T}\right)\Bigg|^{*}_{T=T_n}$

$\star$ in PTs with a mass gap
there is always a T s.t $\beta_H(T_{\min}) = 0$

Gravitational Waves signal from 1st order Phase Transitions



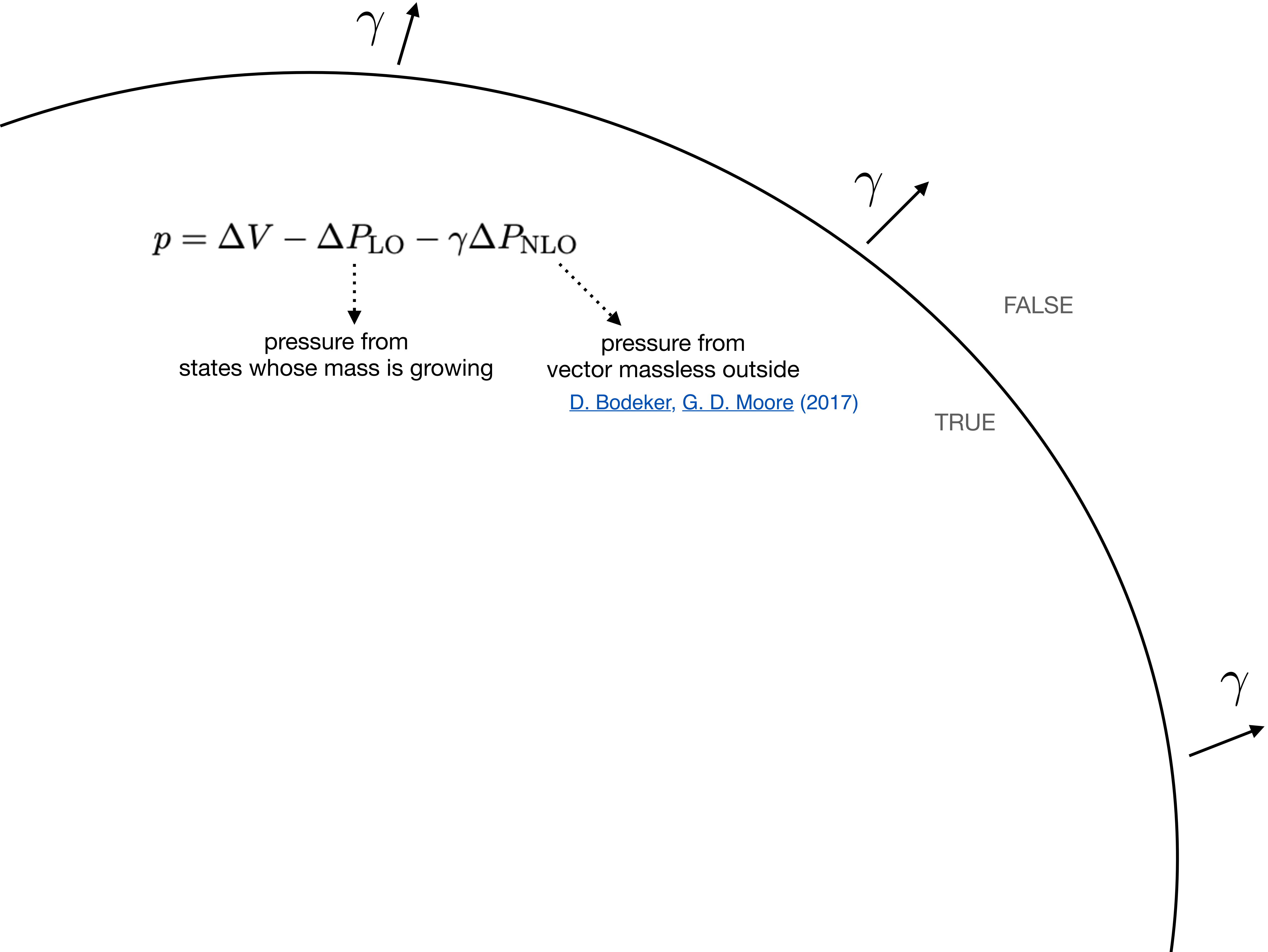


The detectability of the GWs signal depends on:

- The energy released during the PTs $\alpha(T_n) \sim \dfrac{\Delta V(T_n)}{\rho_R(T_n)}$
- The duration of the PTs $\beta_H(T_n) \overset{\text{def}}{=} \dfrac{\beta(T_n)}{H(T_n)} = T_n \dfrac{d}{dT}\left(\dfrac{S_3}{T}\right)\Bigg|^{*}_{T=T_n}$
- The behavior of the bubbles in the cosmic plasma

$_*$ in PTs with a mass gap
there is always a T s.t $\beta_H(T_{\rm min}) = 0$

Bubble frictions



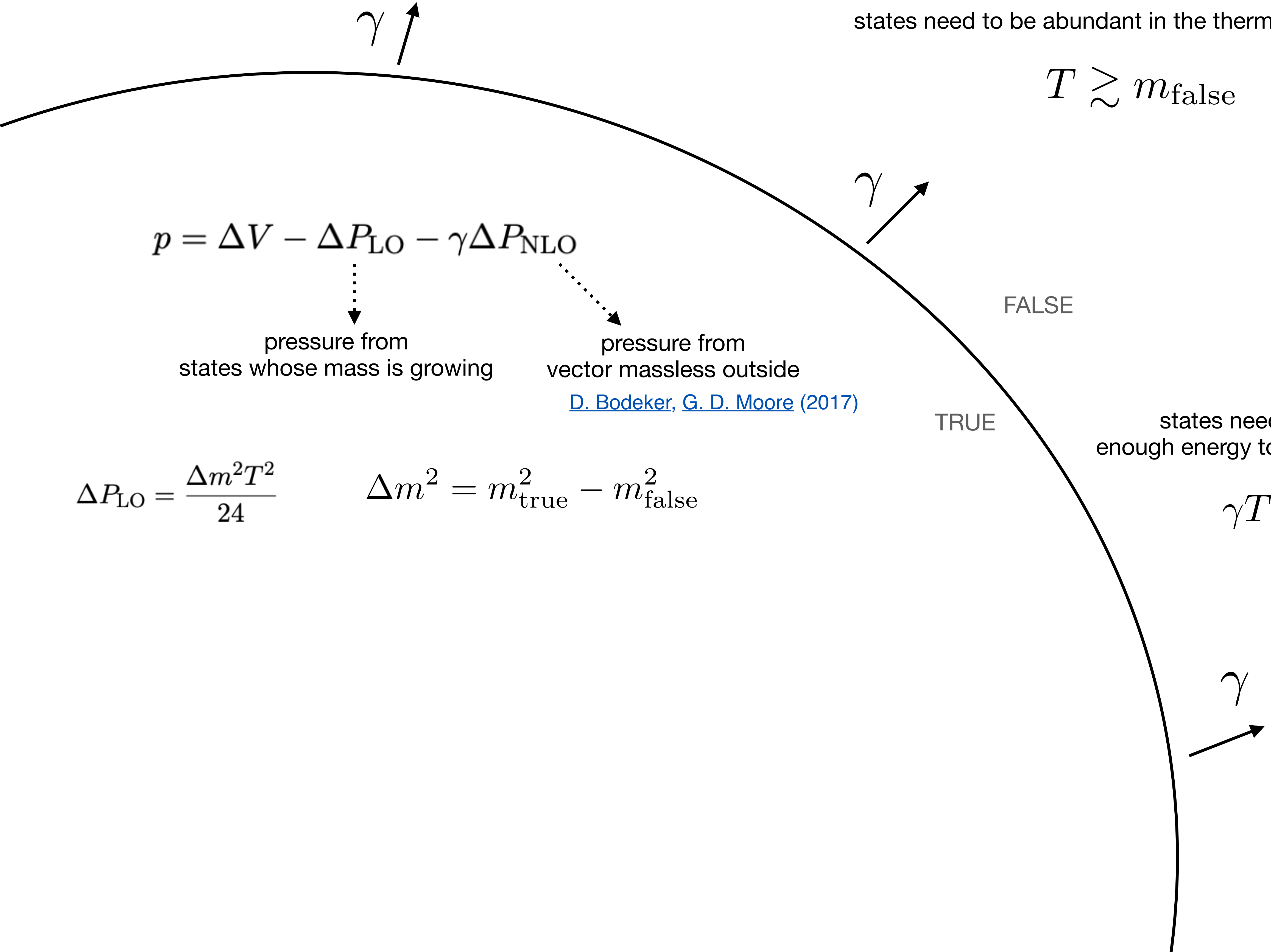
$$p = \Delta V - \Delta P_{\rm LO} - \gamma \Delta P_{\rm NLO}$$

pressure from
states whose mass is growing

pressure from
vector massless outside

D. Bodeker, G. D. Moore (2017)

Bubble frictions



states need to be abundant in the thermal bath

$$T \gtrsim m_{\text{false}}$$

$$p = \Delta V - \Delta P_{\text{LO}} - \gamma \Delta P_{\text{NLO}}$$

pressure from
states whose mass is growing

pressure from
vector massless outside

D. Bodeker, G. D. Moore (2017)

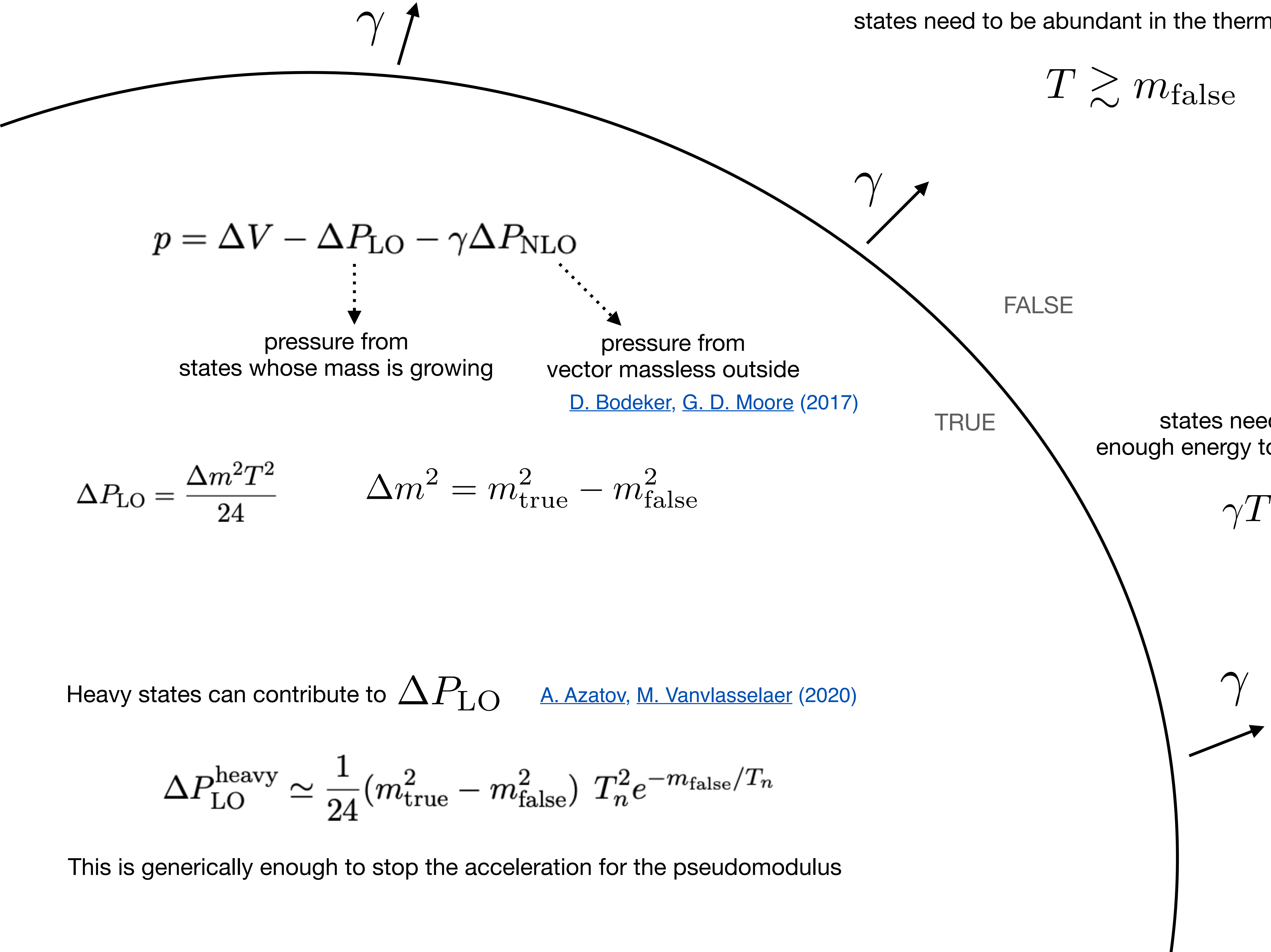
states need to have
enough energy to cross the wall

$$\gamma T \gtrsim m_{\text{true}}$$

$$\Delta P_{\text{LO}} = \frac{\Delta m^2 T^2}{24}$$

$$\Delta m^2 = m^2_{\text{true}} - m^2_{\text{false}}$$

Bubble frictions



$$p = \Delta V - \Delta P_{\rm LO} - \gamma \Delta P_{\rm NLO}$$

pressure from
states whose mass is growing

pressure from
vector massless outside

D. Bodeker, G. D. Moore (2017)

$$\Delta P_{\rm LO} = \frac{\Delta m^2 T^2}{24} \qquad \Delta m^2 = m^2_{\rm true} - m^2_{\rm false}$$

Heavy states can contribute to $\Delta P_{\rm LO}$ A. Azatov, M. Vanvlasselaer (2020)

$$\Delta P^{\rm heavy}_{\rm LO} \simeq \frac{1}{24}(m^2_{\rm true} - m^2_{\rm false})\ T_n^2 e^{-m_{\rm false}/T_n}$$

This is generically enough to stop the acceleration for the pseudomodulus

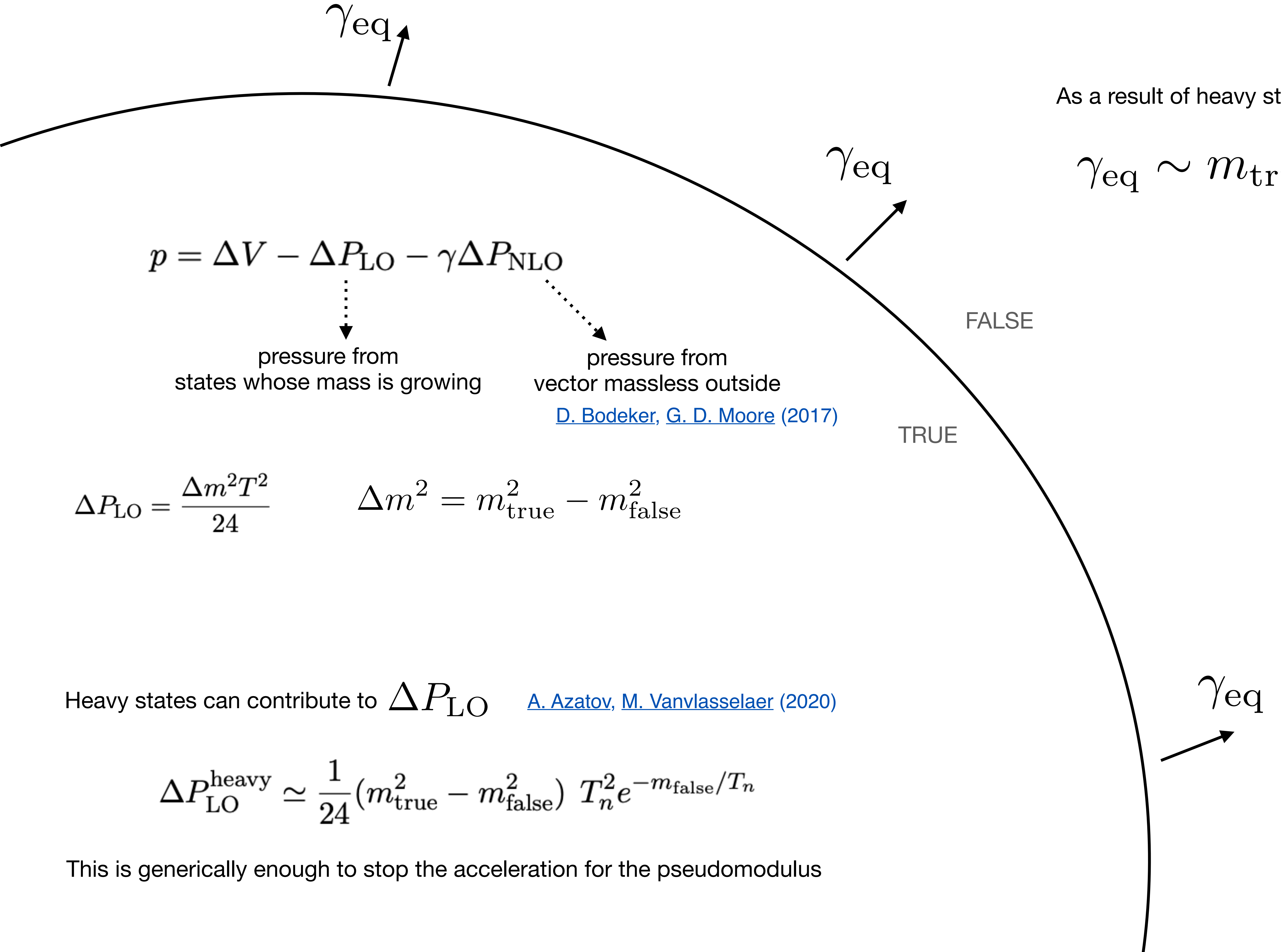
states need to be abundant in the thermal bath

$$T \gtrsim m_{\rm false}$$

states need to have
enough energy to cross the wall

$$\gamma T \gtrsim m_{\rm true}$$

Bubble frictions



As a result of heavy state friction

$$\gamma_{\rm eq} \sim m_{\rm true}/T_n$$

$$p = \Delta V - \Delta P_{\rm LO} - \gamma \Delta P_{\rm NLO}$$

pressure from
states whose mass is growing

pressure from
vector massless outside

D. Bodeker, G. D. Moore (2017)

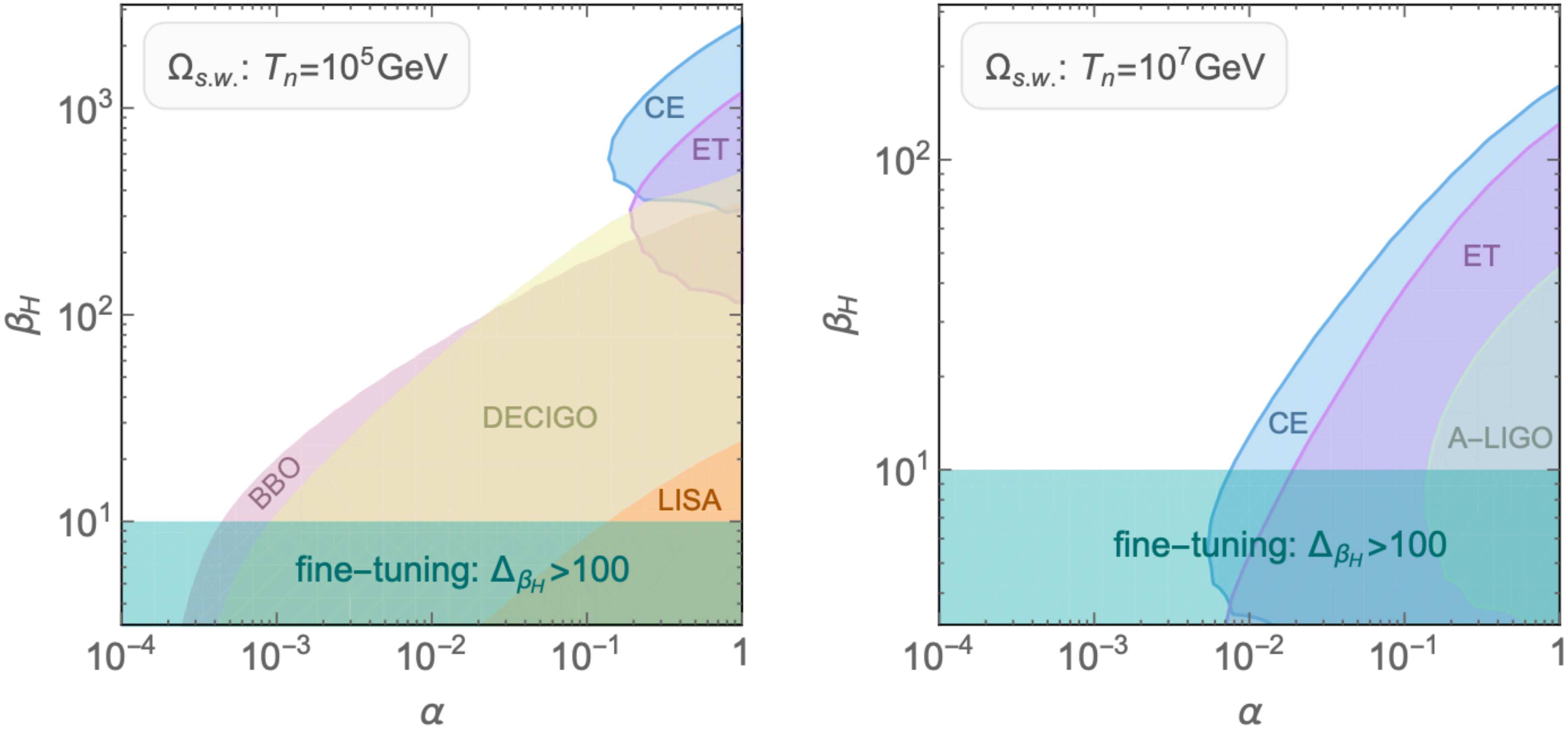
$$\Delta P_{\rm LO} = \frac{\Delta m^2 T^2}{24} \qquad \Delta m^2 = m^2_{\rm true} - m^2_{\rm false}$$

Heavy states can contribute to $\Delta P_{\rm LO}$ A. Azatov, M. Vanvlasselaer (2020)

$$\Delta P^{\rm heavy}_{\rm LO} \simeq \frac{1}{24}(m^2_{\rm true} - m^2_{\rm false})\ T^2_n e^{-m_{\rm false}/T_n}$$

This is generically enough to stop the acceleration for the pseudomodulus

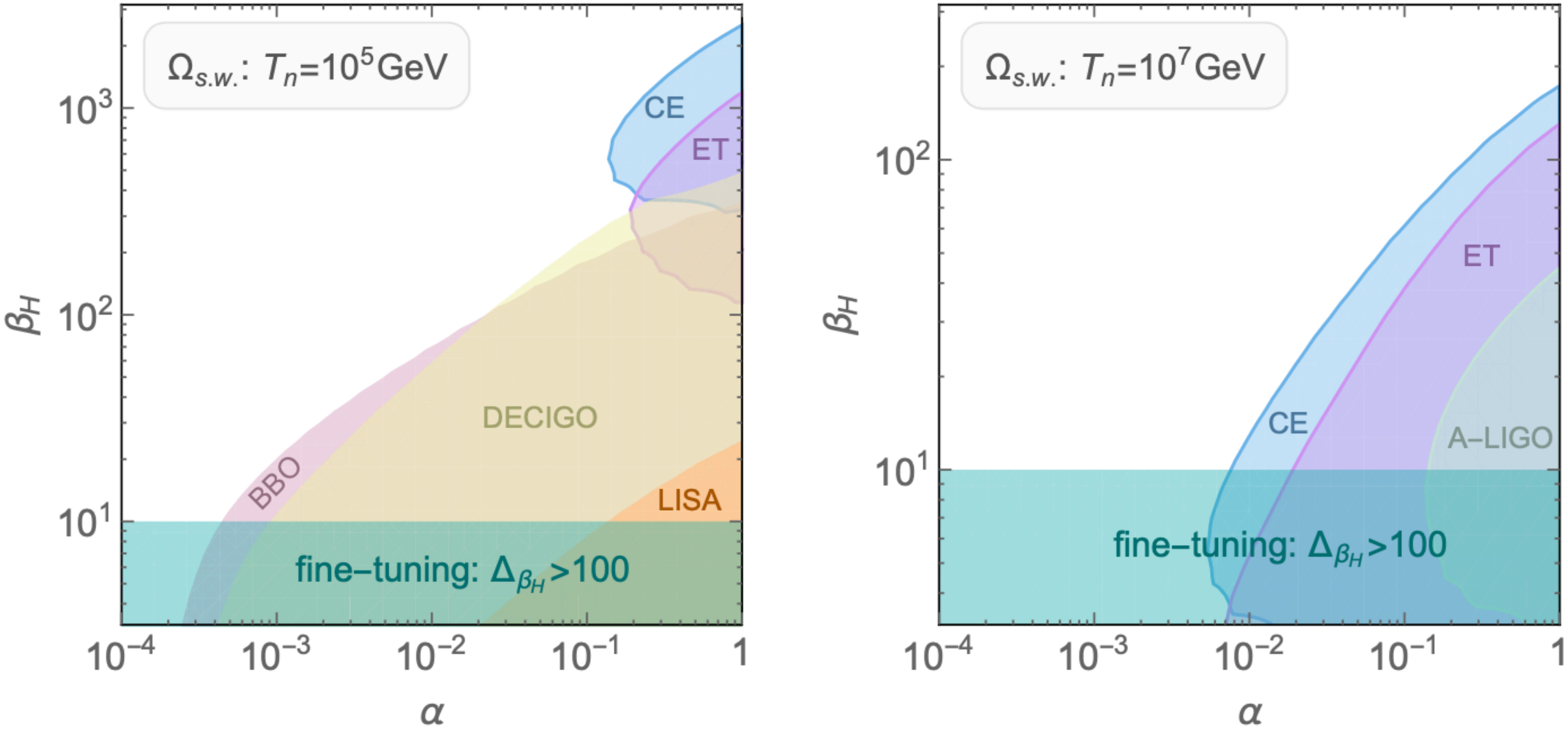
GW signal



Using the nucleation condition the duration of the PT can be written as

$$\beta_H(T_n) \simeq S'(T_n) - \mathcal{C}$$

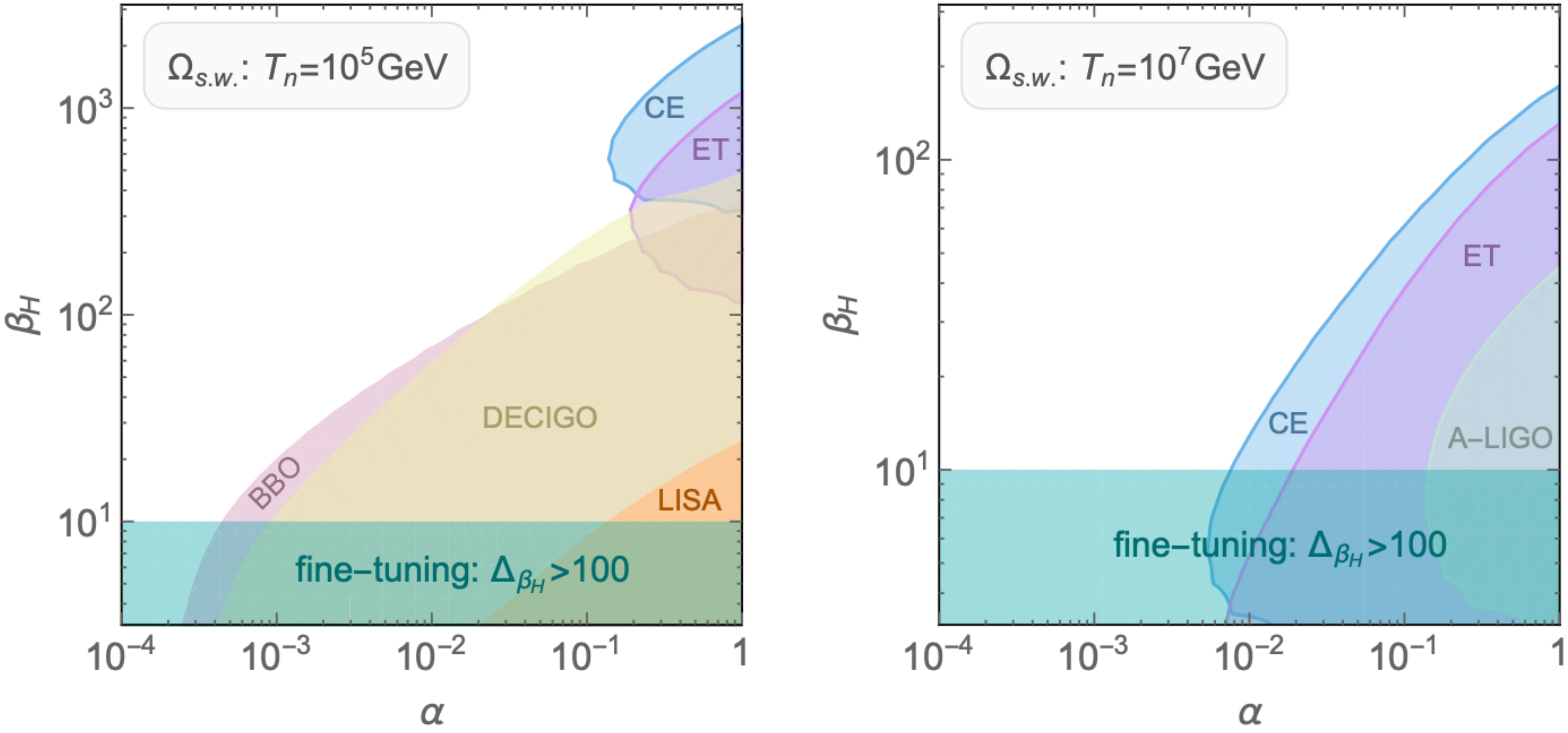
GW signal



Using the nucleation condition the duration of the PT can be written as

$\beta_H(T_n) \simeq S'(T_n) - \mathcal{C} \ \sim 100$ unless I tune the two terms to partially cancel

GW signal



Using the nucleation condition the duration of the PT can be written as

$\beta_H(T_n) \simeq S'(T_n) - \mathcal{C} \;\sim 100$ unless I tune the two terms to partially cancel

Having a very small duration seems to be a highly non-generic prediction of any PT

$$\Delta_{\beta_H} \overset{\text{def}}{=} \text{Max}_{\{p_i\}} \Delta^{p_i}_{\beta_H} = \text{Max}_{\{p_i\}} \left| \frac{d \log \beta_H}{d \log p_i} \right|$$

this “beta-tuning” can be computed
in a model a’ la Giudice-Barbieri

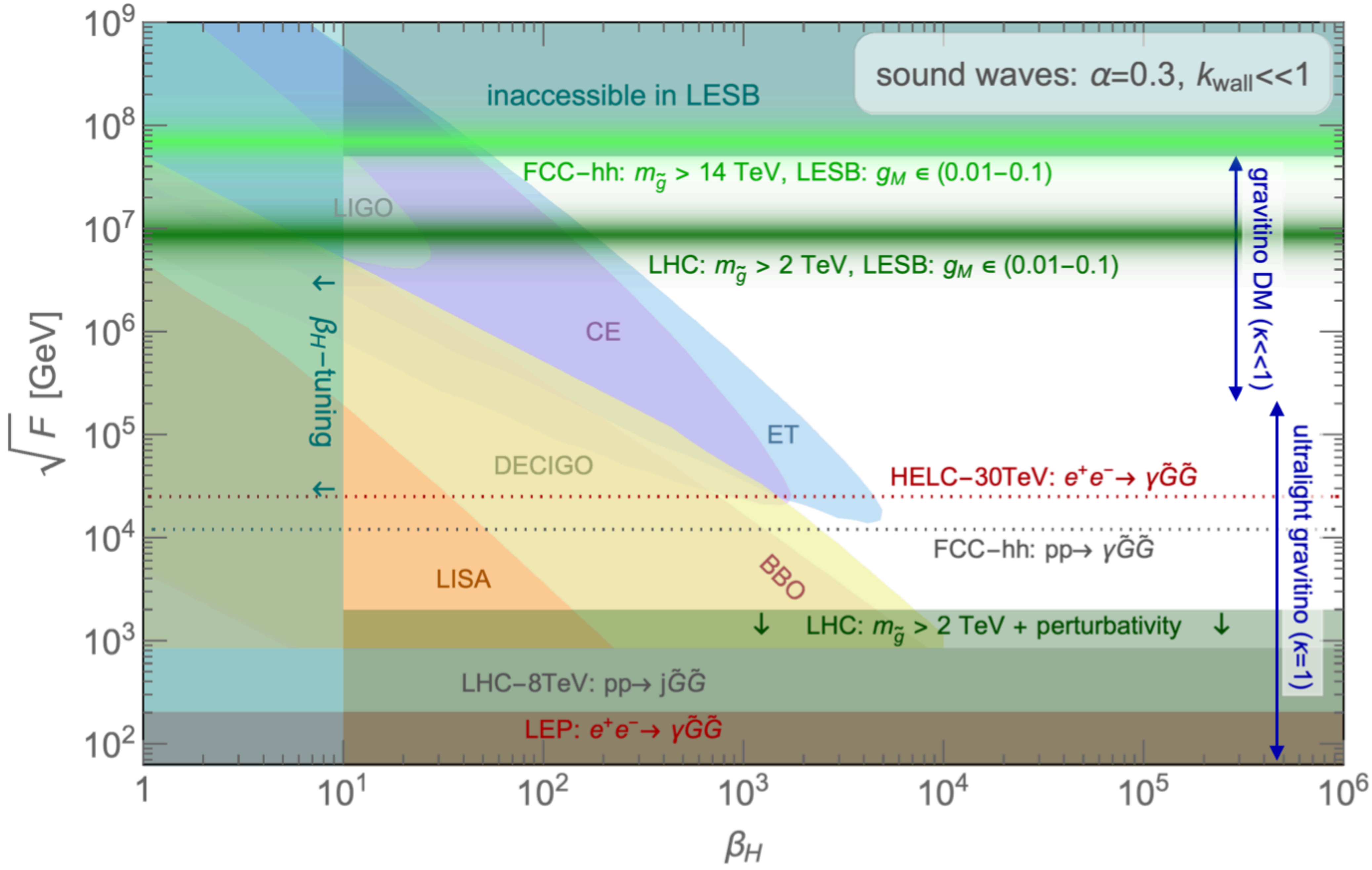
How we will discover Low Energy SUSY breaking

The pseudomodulus PTs we considered have a mass gap

The typical scale affecting the potential is the SUSY-breaking scale F

$$T_n \simeq \sqrt{F}$$

Because of efficient friction most of the energy goes into the plasma

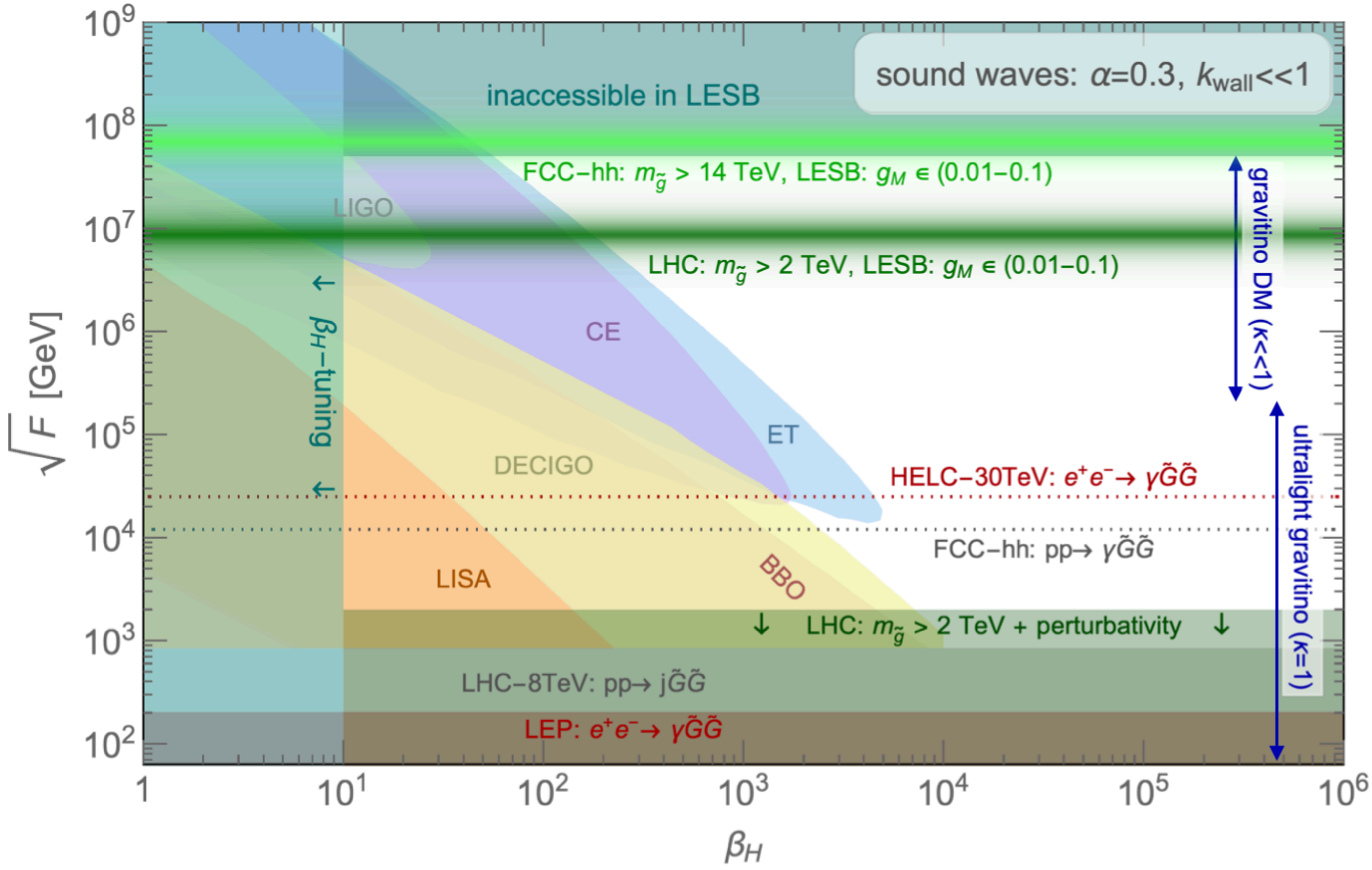


How we will discover Low Energy SUSY breaking

GRAVITINO DM: The gravitino problem bounds the GWs frequency to be always within reach of A-LIGO, CE and ET

Calculable models live at high F

They will be fully tested at FCC-hh!

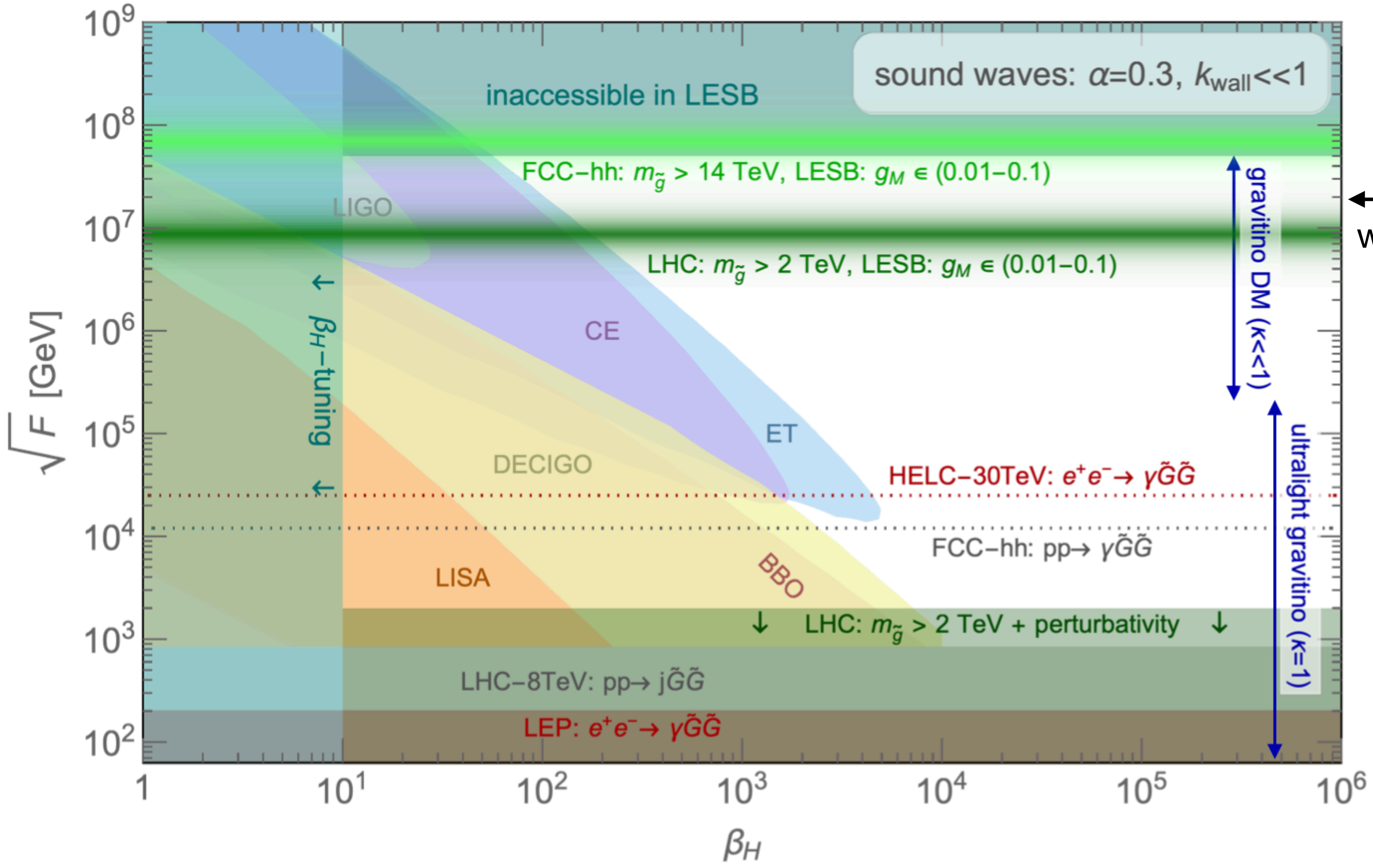


How we will discover Low Energy SUSY breaking

GRAVITINO DM: The gravitino problem bounds the GWs frequency to be always within reach of A-LIGO, CE and ET

Calculable models live at high F

They will be fully tested at FCC-hh!



here is where the calculable models I will present have to leave

The nature of the pseudomodulus PT: Goals

- How S_3/T depends qualitatively with temperature?

Higgs ⋯⋯▸ $S_3/T \sim T^\alpha$ high-T expansion

dilaton ⋯⋯▸ $S_3/T \sim 1/\log m/T$ supercooling

pseudomodulus ⋯⋯▸ $S_3/T \sim T^\alpha e^{-m/T}$ low-T expansion

- Can we extract a parametric dependence of S_3/T in terms of theory parameters?

⋯⋯▸ trying to answer the question how generic is a strong first order PT for a given class of models

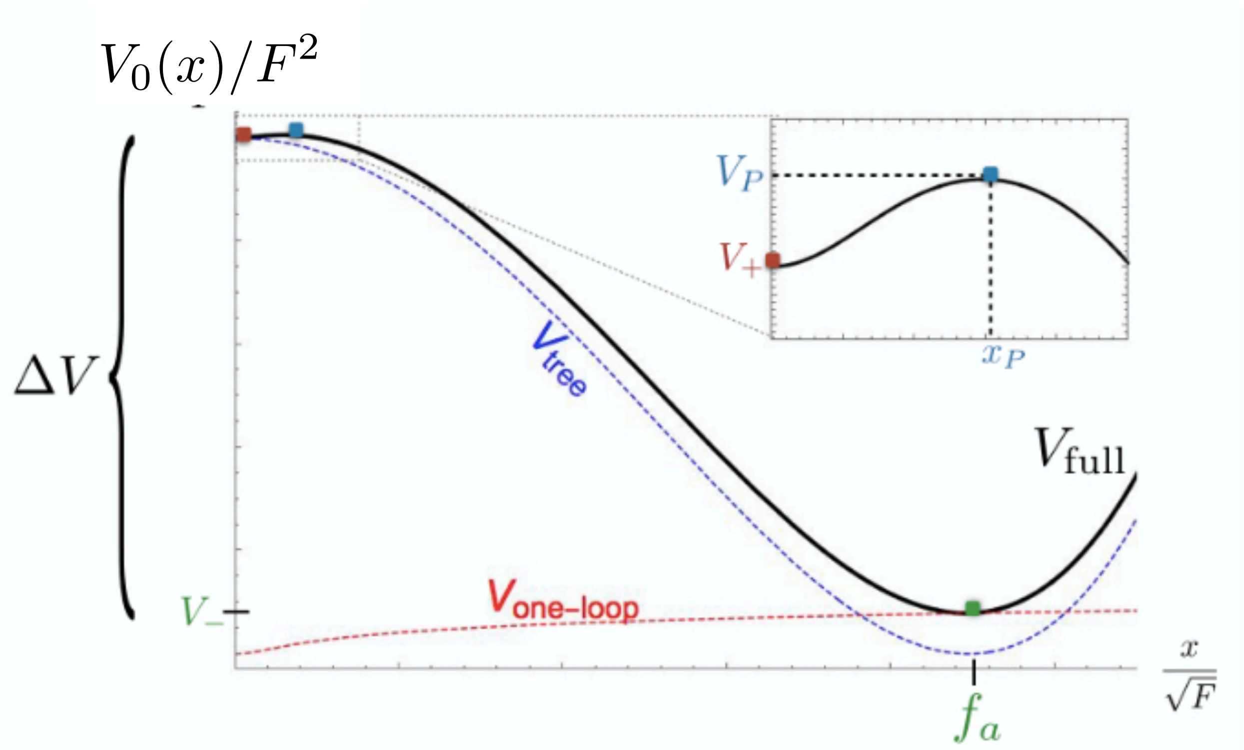
The pseudomodulus potential at T=0

$$m_x \sim \frac{\lambda^2}{16\pi^2} F/m_*$$

$$m_x \ll m_*$$

at weak coupling

$$V_{\text{eff}}(x) = V_0(x) + V_T(x)$$



- **The potential is flat** $f_a^4 \gg \Delta V$ loop corrections asymptote to a Log at large field value

- **The barrier is small** $\frac{V_P}{\Delta V} = \frac{\lambda_{\text{eff}}^2}{16\pi^2}$ $\lambda_{\text{eff}} \sim \mathcal{O}(1)$

- **The position of the barrier** does not affect the bounce much (see later)

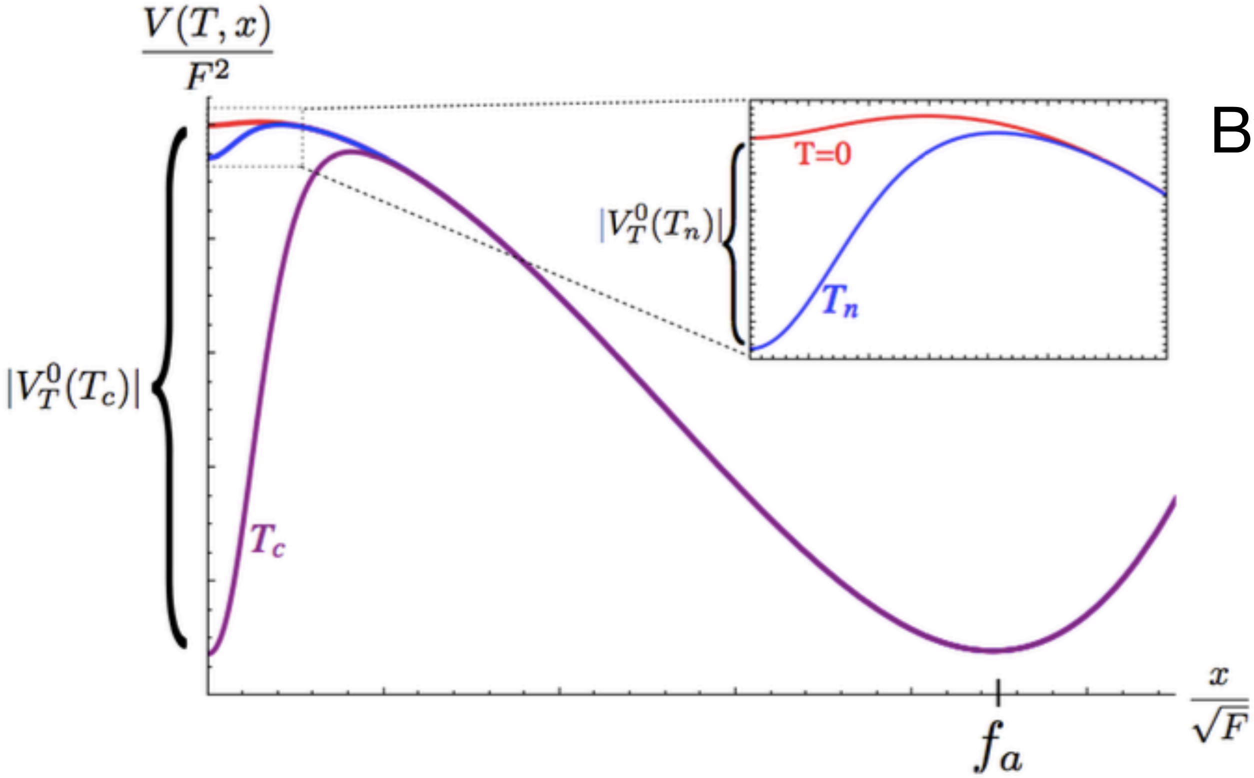
The pseudomodulus potential at finite T

$$m_x \sim \frac{\lambda^2}{16\pi^2} F/m_*$$

$$m_x \ll m_*$$

at weak coupling

$$V_{\text{eff}}(x) = V_0(x) + V_T(x)$$



By construction, the scale setting the potential is below the cutoff

$$T_n \sim \sqrt{F} \lesssim m_*$$

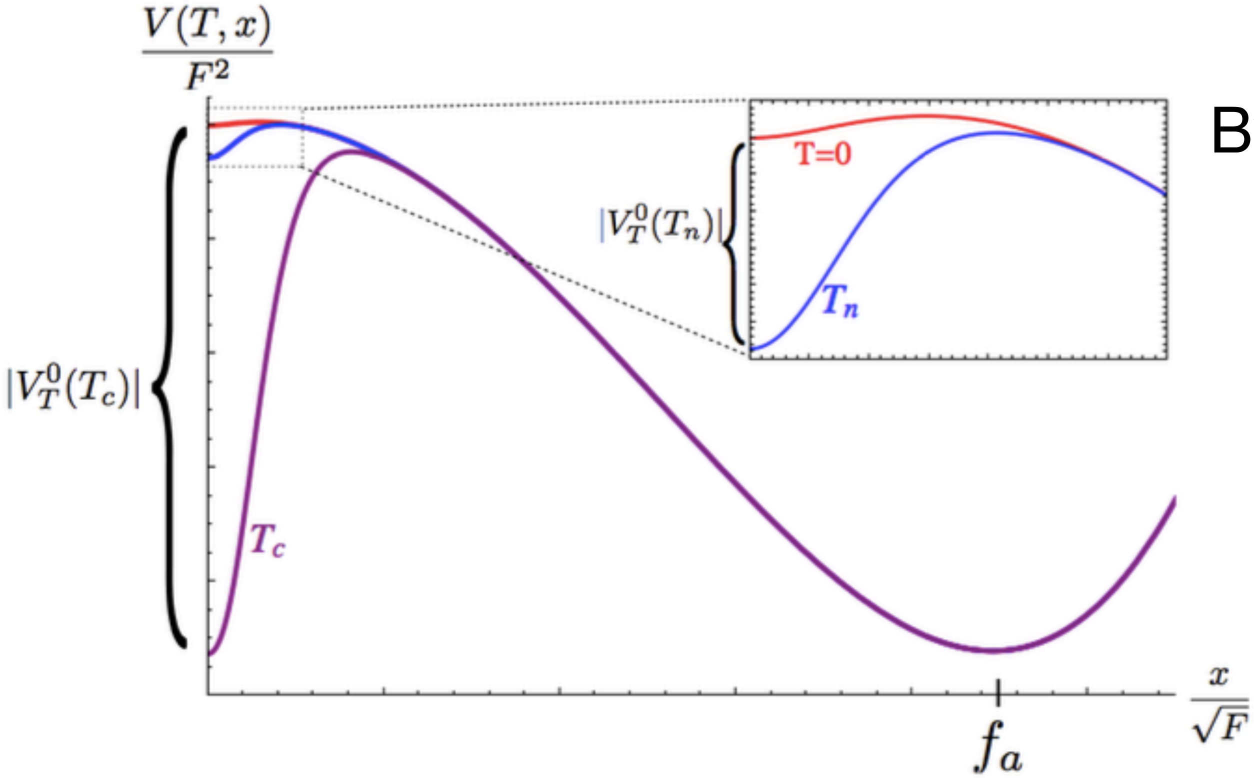
the low-T expansion applies at nucleation

The pseudomodulus potential at finite T

$$m_x \sim \frac{\lambda^2}{16\pi^2} F/m_* \qquad m_x \ll m_*$$

at weak coupling

$$V_{\text{eff}}(x) = V_0(x) + V_T(x)$$



By construction, the scale setting the potential is below the cutoff

$$T_n \sim \sqrt{F} \lesssim m_*$$

the low-T expansion applies at nucleation

$$V_T(x) \simeq -N\, T^4 \left(\frac{\lambda^2 x^2 + m_*^2}{(2\pi T)^2} \right)^{3/4} e^{-\sqrt{\frac{\lambda^2 x^2 + m_*^2}{T^2}}}$$

$$N = N_{\text{bosons}} + N_{\text{fermions}}$$

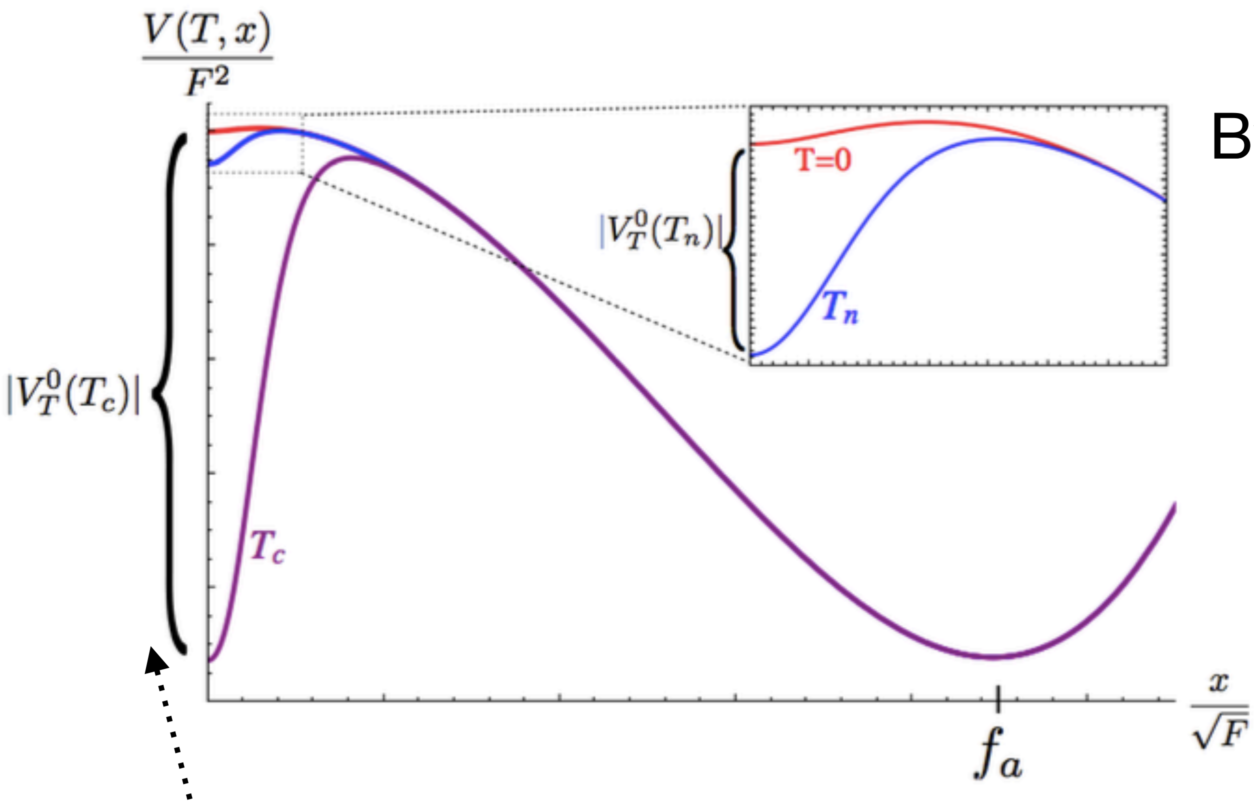
The pseudomodulus potential at finite T

$$m_x \sim \frac{\lambda^2}{16\pi^2} F/m_*$$

$$m_x \ll m_*$$

at weak coupling

$$V_{\rm eff}(x) = V_0(x) + V_T(x)$$



By construction, the scale setting the potential is below the cutoff

$$T_n \sim \sqrt{F} \lesssim m_*$$

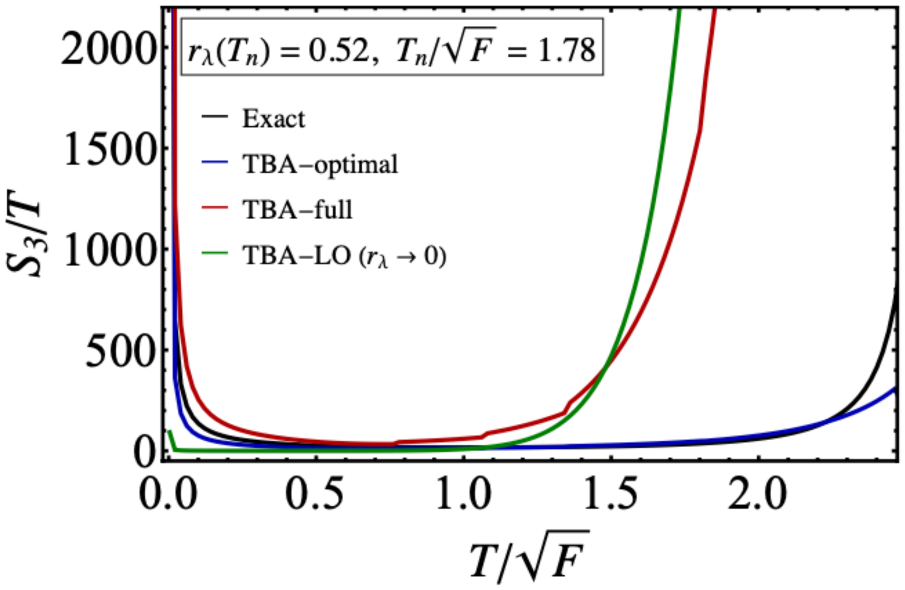
the low-T expansion applies at nucleation

Thermal corrections only make the origin deeper in this limit

$$V_T(x) \simeq -N\,T^4 \left(\frac{\lambda^2 x^2 + m_*^2}{(2\pi T)^2}\right)^{3/4} e^{-\sqrt{\frac{\lambda^2 x^2 + m_*^2}{T^2}}}$$

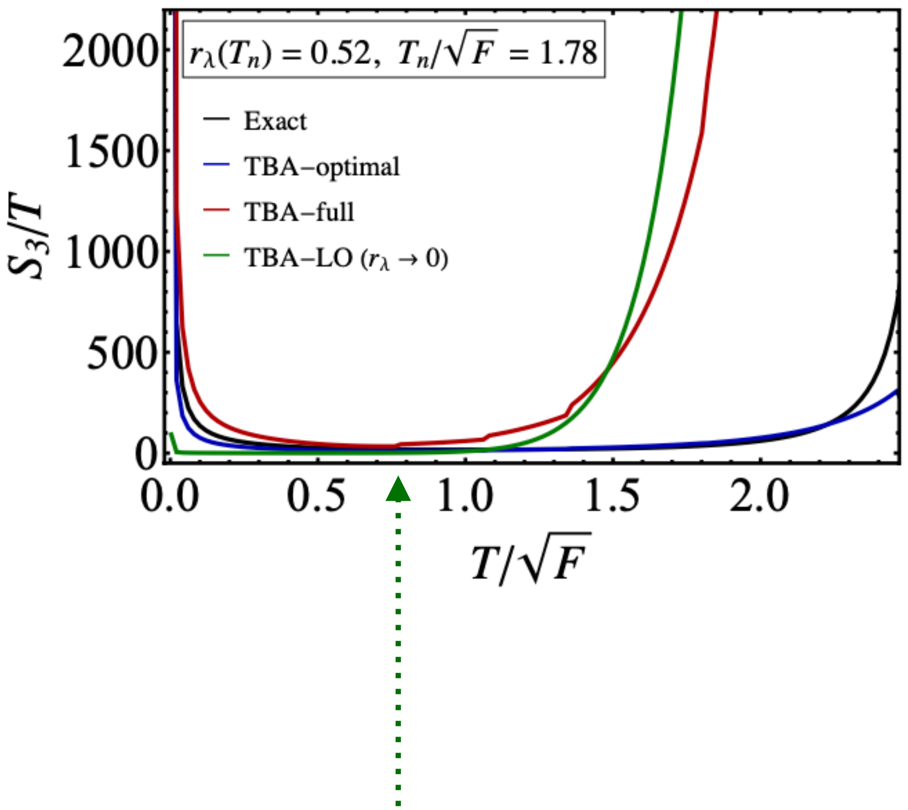
$$N = N_{\rm bosons} + N_{\rm fermions}$$

The pseudomodulus bounce action I



A simple triangular barrier approximates the full bounce quite well

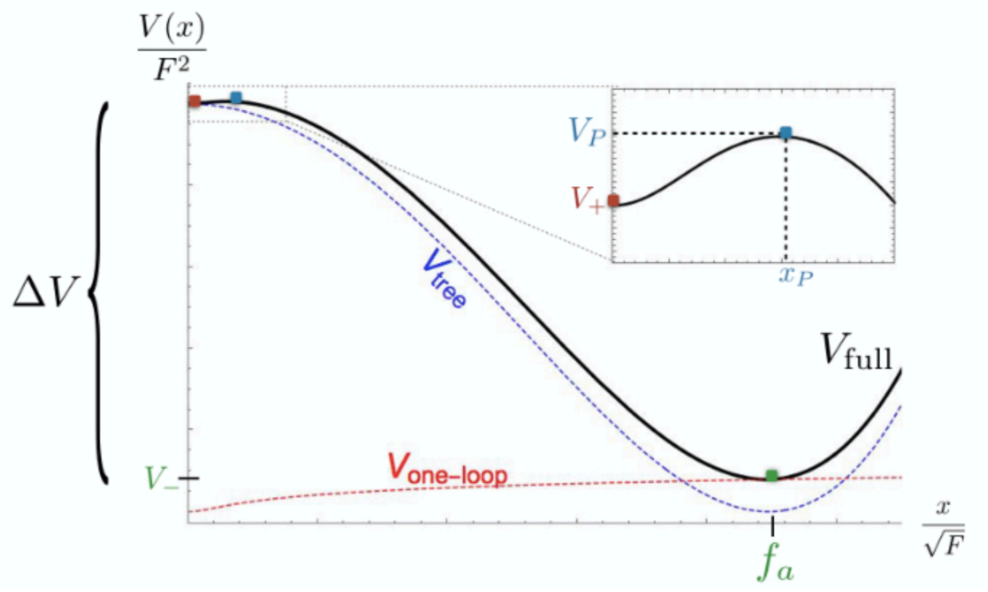
The pseudomodulus bounce action I

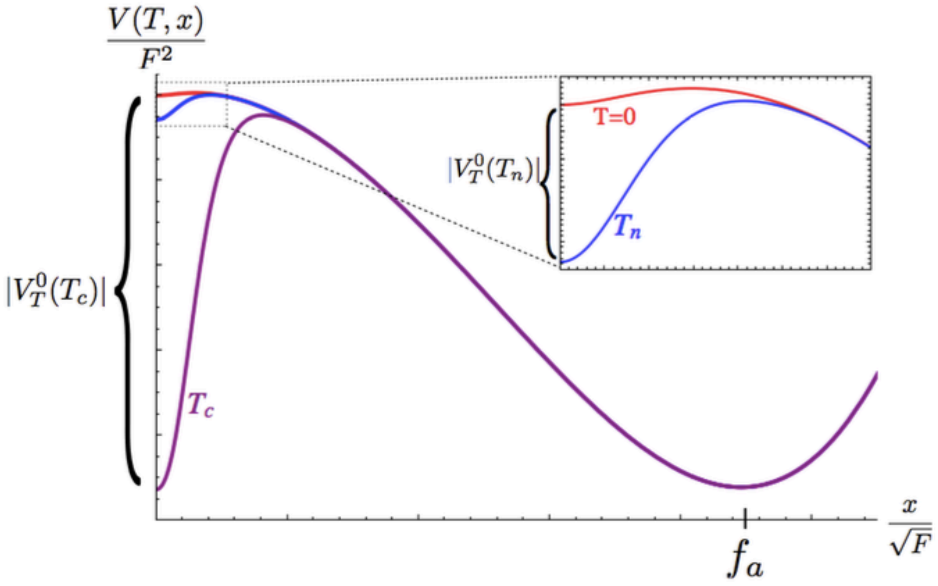


$$\frac{S_3}{T} \simeq \frac{144\sqrt{2}\pi}{5T} \frac{(V_P - V_T^0)^{5/2} f_a^3}{(\Delta V)^3}$$

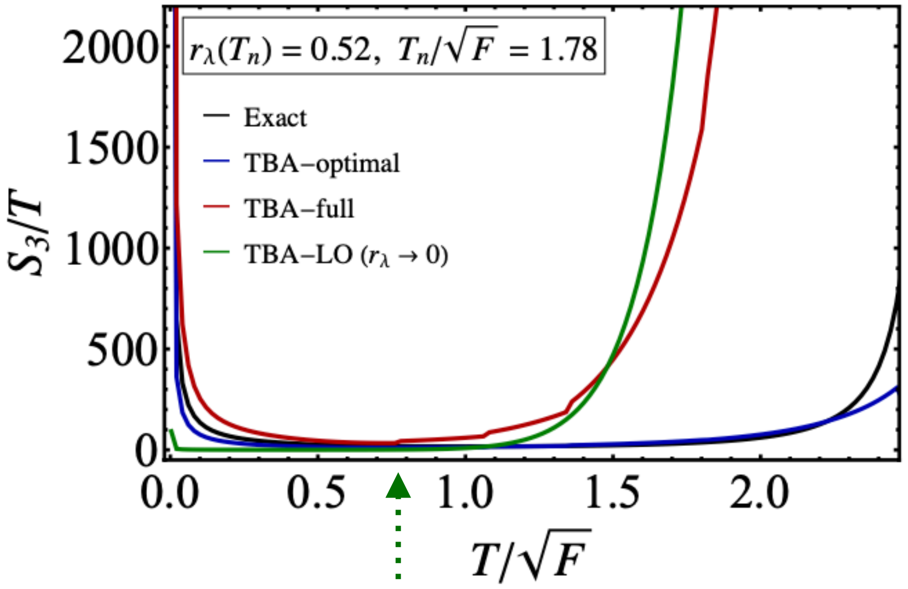
A simple triangular barrier
approximates the full bounce quite well

For a fully analytical treatment
we expand for
flat potential + small barrier





The pseudomodulus bounce action I



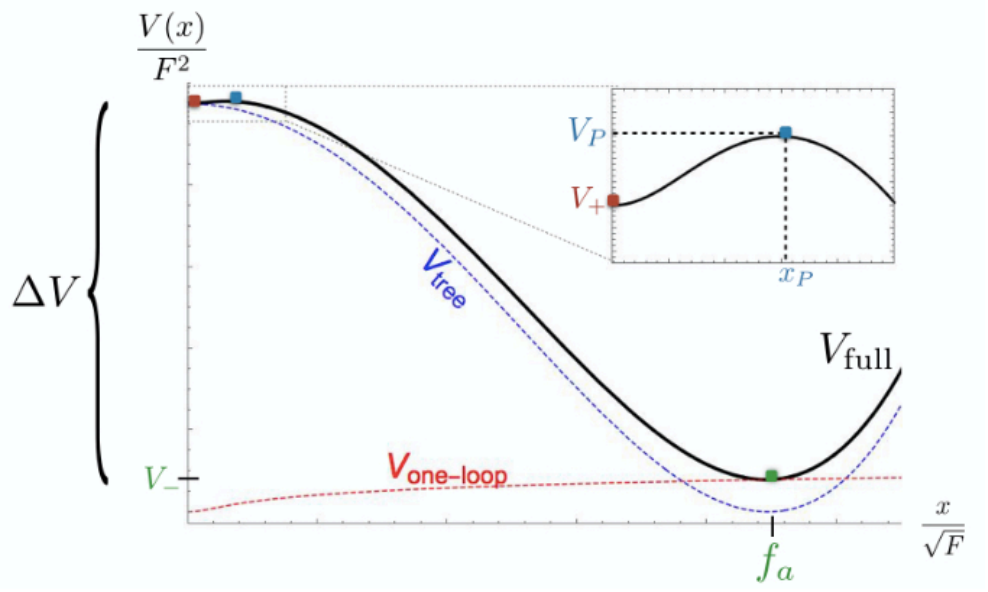
A simple triangular barrier
approximates the full bounce quite well

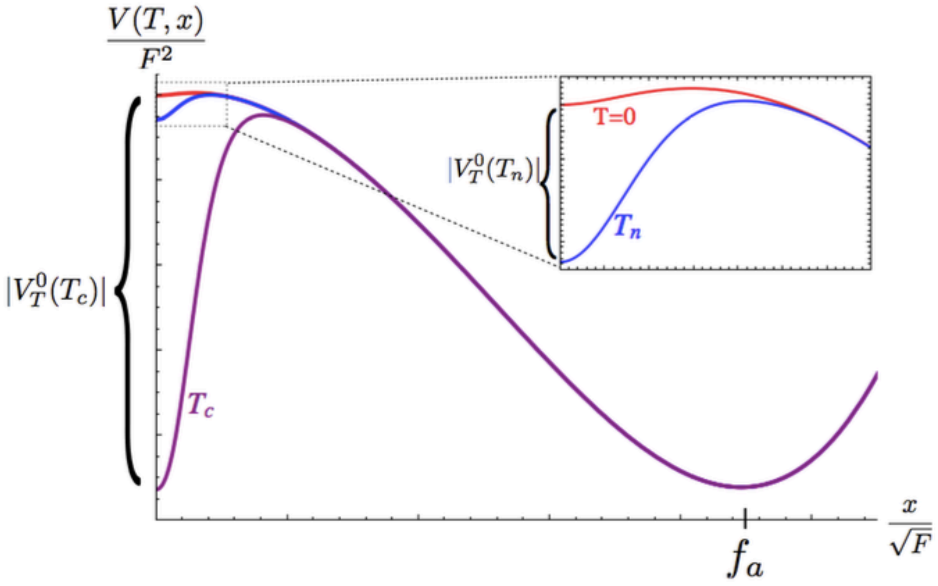
For a fully analytical treatment
we expand for
flat potential + small barrier

x_P-independent!

$$\frac{S_3}{T} \simeq \frac{144\sqrt{2}\pi}{5T} \frac{(V_P - V_T^0)^{5/2} f_a^3}{(\Delta V)^3}$$

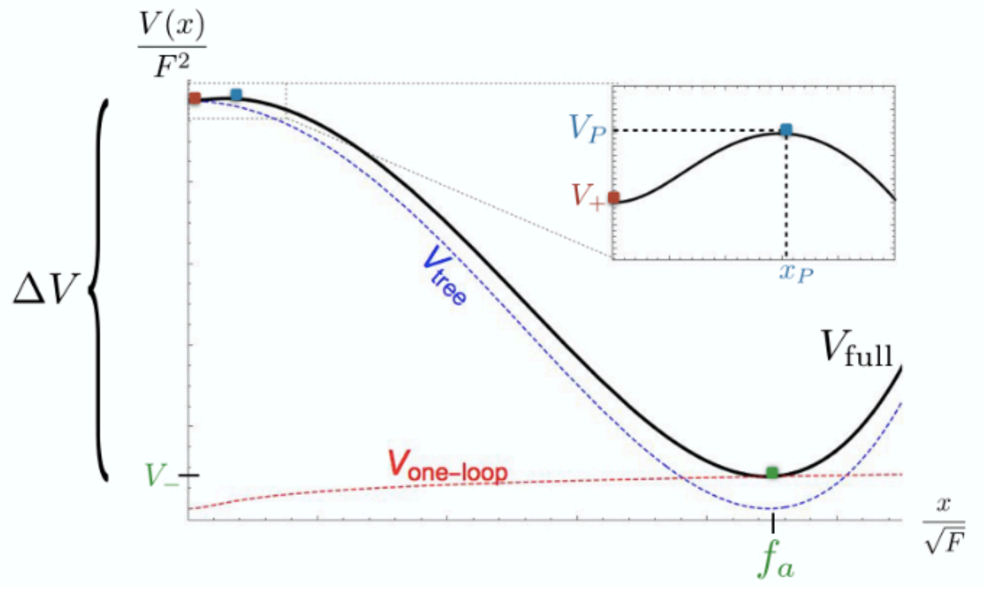
The bounce is independent on the position
of the barrier in this limit!



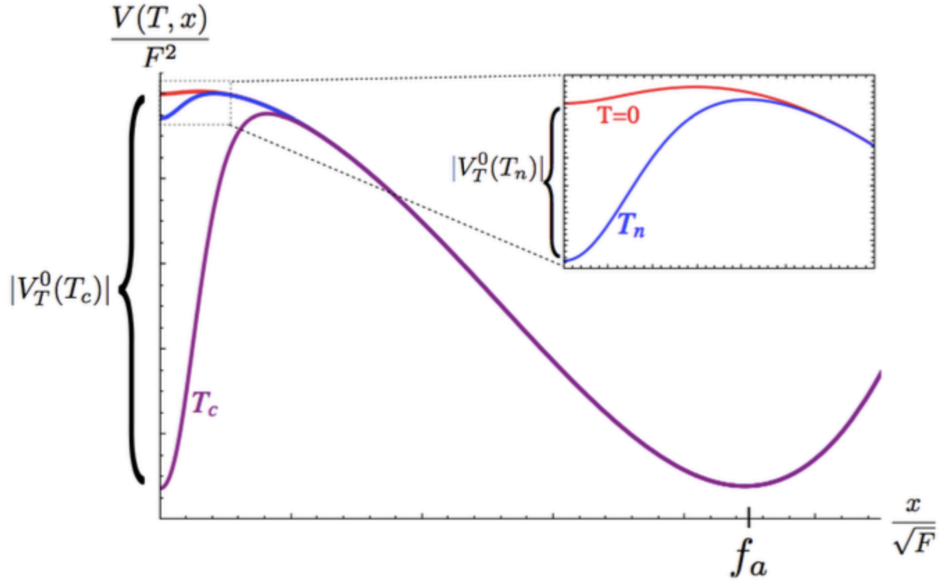


The pseudomodulus bounce action II

$$\frac{S_3}{T} \simeq \frac{144\sqrt{2}\pi}{5T} \frac{(V_P - V_T^0)^{5/2} f_a^3}{(\Delta V)^3}$$

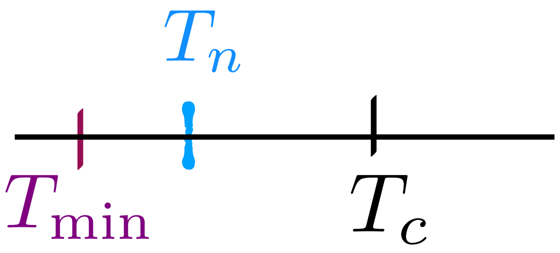


In the same approximation we can get the nucleation temperature analytically in a systematic expansion $V_P/V_T^0 \ll 1$



$$T_n \simeq T_n^0 \left(1 - \frac{7}{\mathcal{C}^{2/5}} \frac{V_P}{m_*^4} \left(\frac{T_n^0}{m_*}\right)^{3/5} \left(\frac{f_a m_*^3}{\Delta V}\right)^{6/5}\right)$$

where $T_n^0 \sim m_*/2$



The pseudomodulus bounce action II

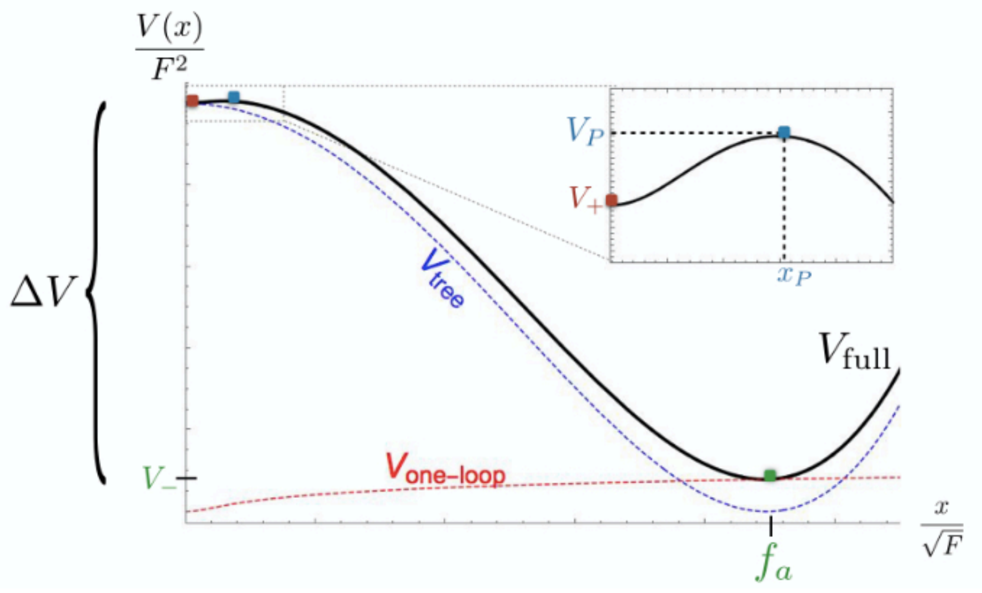
$$\frac{S_3}{T} \simeq \frac{144\sqrt{2}\pi}{5T}\frac{(V_P - V_T^0)^{5/2} f_a^3}{(\Delta V)^3}$$

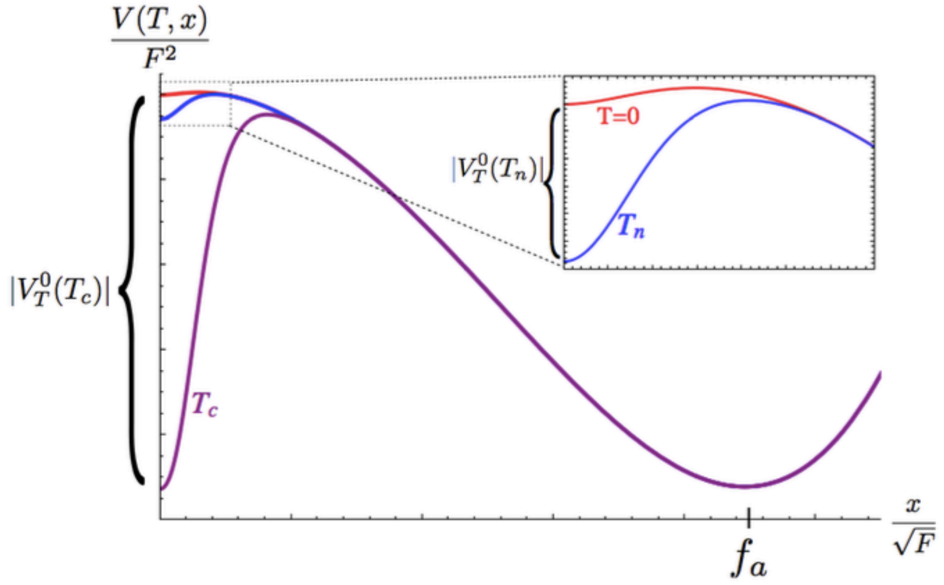
In the same approximation we can get the nucleation temperature analytically in a systematic expansion $V_P/V_T^0 \ll 1$

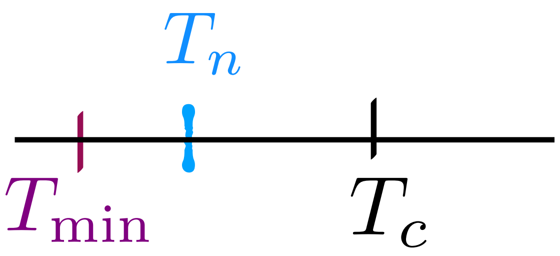
$$T_n \simeq T_n^0 \left(1 - \frac{7}{\mathcal{C}^{2/5}}\frac{V_P}{m_*^4}\left(\frac{T_n^0}{m_*}\right)^{3/5}\left(\frac{f_a m_*^3}{\Delta V}\right)^{6/5}\right)$$

where $T_n^0 \sim m_*/2$

The competition of the two terms shows that the nucleation temperature can be diminished down to the minimal temperature by increasing the barrier or the distance in field space



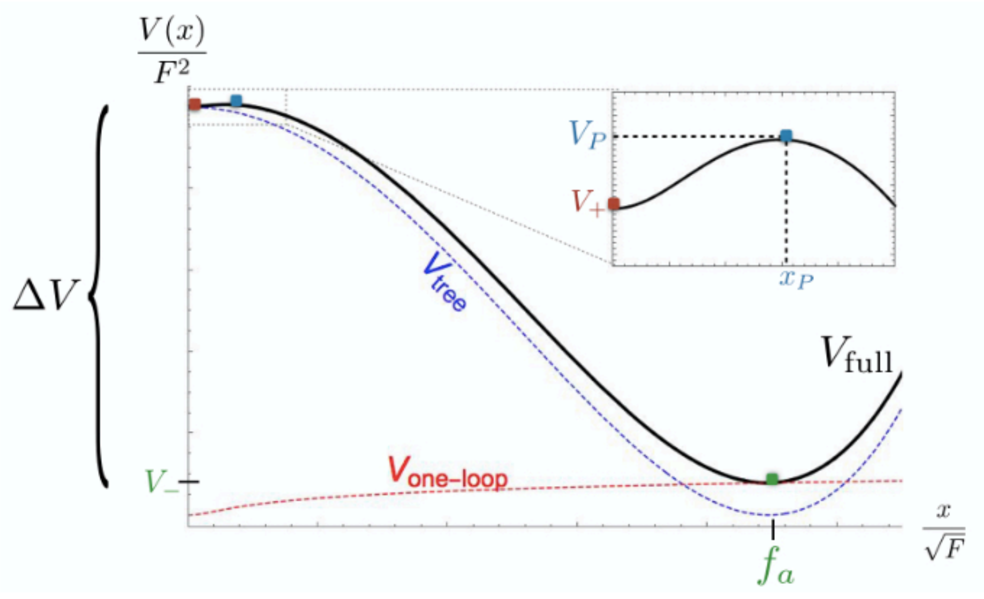




The pseudomodulus bounce action II

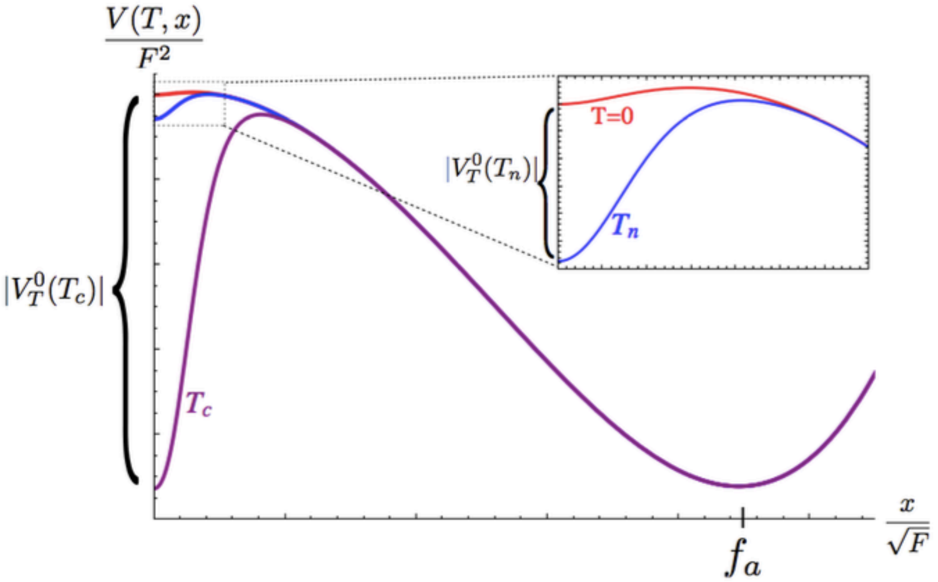
$$\frac{S_3}{T} \simeq \frac{144\sqrt{2}\pi}{5T}\frac{(V_P - V_T^0)^{5/2} f_a^3}{(\Delta V)^3}$$

In the same approximation we can
get the nucleation temperature analytically
in a systematic expansion $V_P/V_T^0 \ll 1$



$$T_n \simeq T_n^0 \left(1 - \frac{7}{\mathcal{C}^{2/5}} \frac{V_P}{m_*^4} \left(\frac{T_n^0}{m_*}\right)^{3/5} \left(\frac{f_a m_*^3}{\Delta V}\right)^{6/5}\right)$$

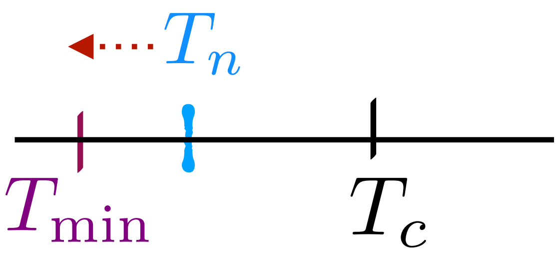
where $T_n^0 \sim m_*/2$



The competition of the two terms shows that the nucleation temperature can be diminished down to the minimal temperature by increasing the barrier or the distance in field space

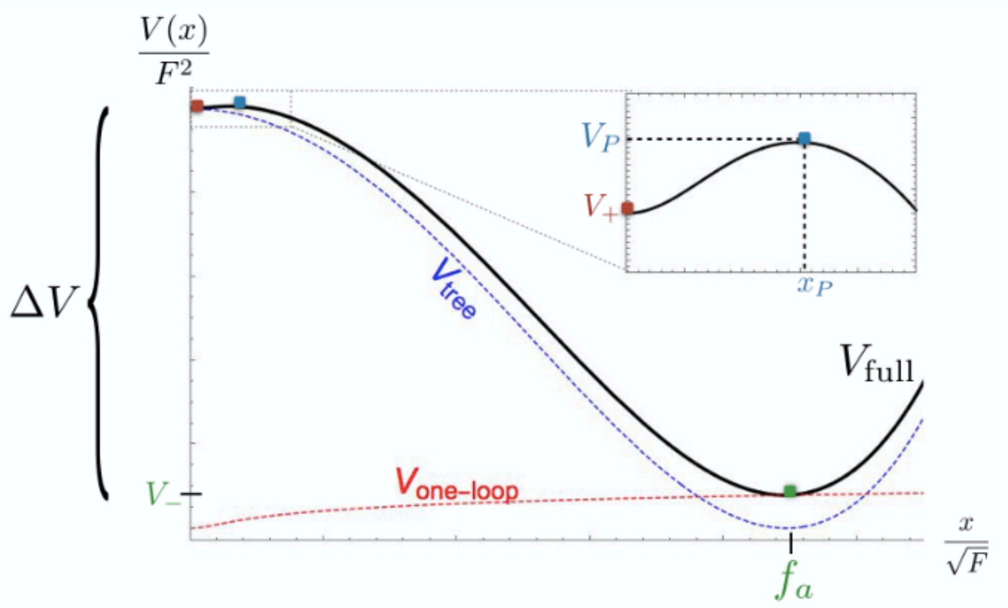
Since $\alpha \sim \Delta V/T_n^4$

at the boundary of nucleation the signal is enhanced!



A toy model

$$V_0(x) = \kappa_D^2 \left(F - \epsilon_{\not{R}} x^2\right)^2 + \frac{\lambda^2}{32\pi^2}|F|^2 \log\left(\frac{\lambda^2 x^2 + m_*^2}{m_*^2}\right)$$



$$\langle x \rangle_{\rm true} = f_a = \sqrt{\frac{F}{\epsilon_{\not{R}}}} \quad , \quad \Delta V = (\kappa_D F)^2 \ ,$$

A toy model

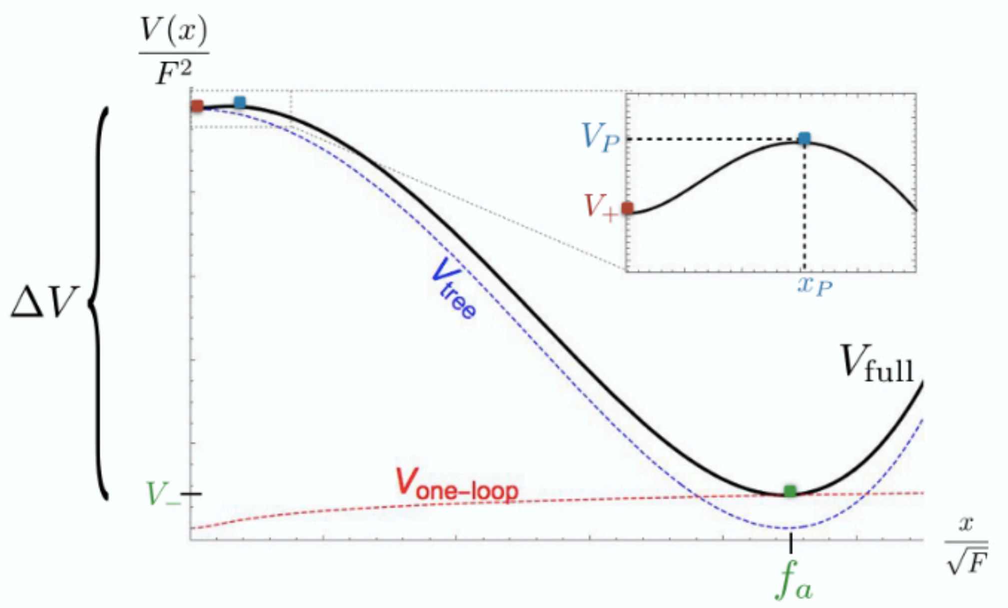
$$V_0(x) = \kappa_D^2 \left(F - \epsilon_{\not{R}} x^2\right)^2 + \frac{\lambda^2}{32\pi^2}|F|^2 \log\left(\frac{\lambda^2 x^2 + m_*^2}{m_*^2}\right)$$

* $\kappa_D = 1$
single scale
SUSY-breaking

* $\epsilon_{\not{R}} < 1/\sqrt{\kappa_D}$
to ensure flatness

* never singular = mass gap

* pure log at $x \to \infty$



$$\langle x\rangle_{\rm true} = f_a = \sqrt{\frac{F}{\epsilon_{\not{R}}}} \quad , \quad \Delta V = (\kappa_D F)^2 \ ,$$

A toy model

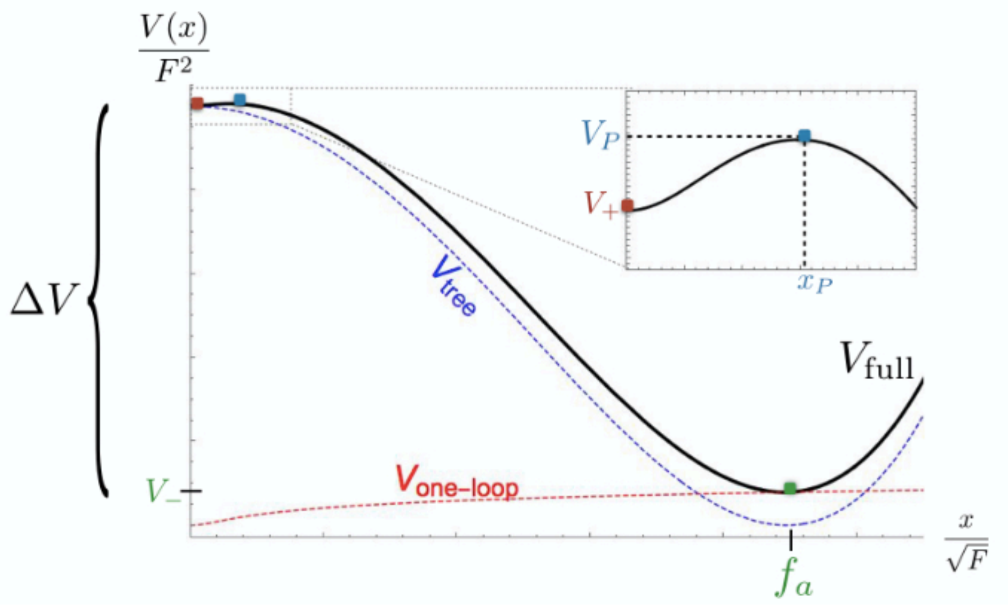
$$V_0(x) = \kappa_D^2 \left(F - \epsilon_{\not R} x^2\right)^2 + \frac{\lambda^2}{32\pi^2}|F|^2 \log\left(\frac{\lambda^2 x^2 + m_*^2}{m_*^2}\right)$$

$^*\kappa_D = 1$
single scale
SUSY-breaking

$^*\epsilon_{\not R} < 1/\sqrt{\kappa_D}$
to ensure flatness

* never singular = mass gap

* pure log at $x \to \infty$



$$\langle x \rangle_{\rm true} = f_a = \sqrt{\frac{F}{\epsilon_{\not R}}} \quad , \quad \Delta V = (\kappa_D F)^2 \ ,$$

$$\alpha = \frac{30}{g_*(T_n)\pi^2}\left(\frac{\kappa_D F}{T_n^2}\right)^2 \sim 10^{-2}\kappa_D^2 \left(\frac{F}{m_*^2}\right)^2 \left(\frac{230}{g_*(T_n)}\right)$$

A toy model

$$V_0(x) = \kappa_D^2 \left(F - \epsilon_{\not{R}} x^2\right)^2 + \frac{\lambda^2}{32\pi^2}|F|^2 \log\left(\frac{\lambda^2 x^2 + m_*^2}{m_*^2}\right)$$

* $\kappa_D = 1$

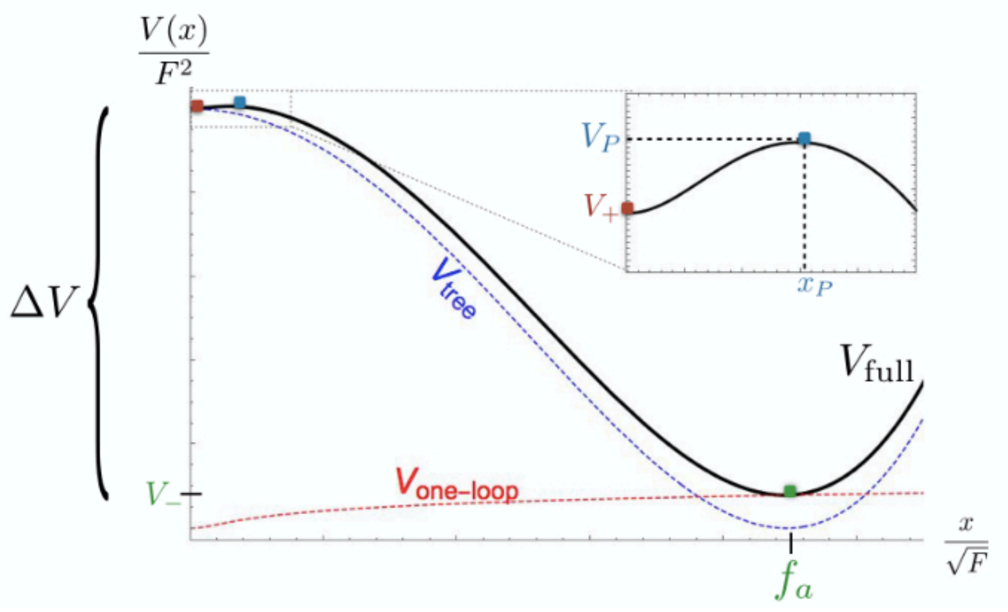
single scale
SUSY-breaking

* $\epsilon_{\not{R}} < 1/\sqrt{\kappa_D}$

to ensure flatness

* never singular = mass gap

* pure log at $x \to \infty$



$$\langle x \rangle_{\rm true} = f_a = \sqrt{\frac{F}{\epsilon_{\not{R}}}} \quad , \quad \Delta V = (\kappa_D F)^2 \ ,$$

$$\alpha = \frac{30}{g_*(T_n)\pi^2}\left(\frac{\kappa_D F}{T_n^2}\right)^2 \sim 10^{-2}\kappa_D^2 \left(\frac{F}{m_*^2}\right)^2 \left(\frac{230}{g_*(T_n)}\right)$$

no-tuning

$T_n \sim m_*/2$

2 SUSY-breaking scales are needed to have large alpha!

What did we learn

As long as the barrier is *small* and the potential is *flat*
the bounce is *independent* on the position of the barrier

At the *boundary of nucleation* the signal is *enhanced* if we allow for cancellations

For a generic nucleation temperature, 2 SUSY-breaking scales are needed
to get a sizeable GW signal

What did we learn

As long as the barrier is *small* and the potential is *flat*
the bounce is *independent* on the position of the barrier

At the *boundary of nucleation* the signal is *enhanced* if we allow for cancellations

For a generic nucleation temperature, 2 SUSY-breaking scales are needed
to get a sizeable GW signal

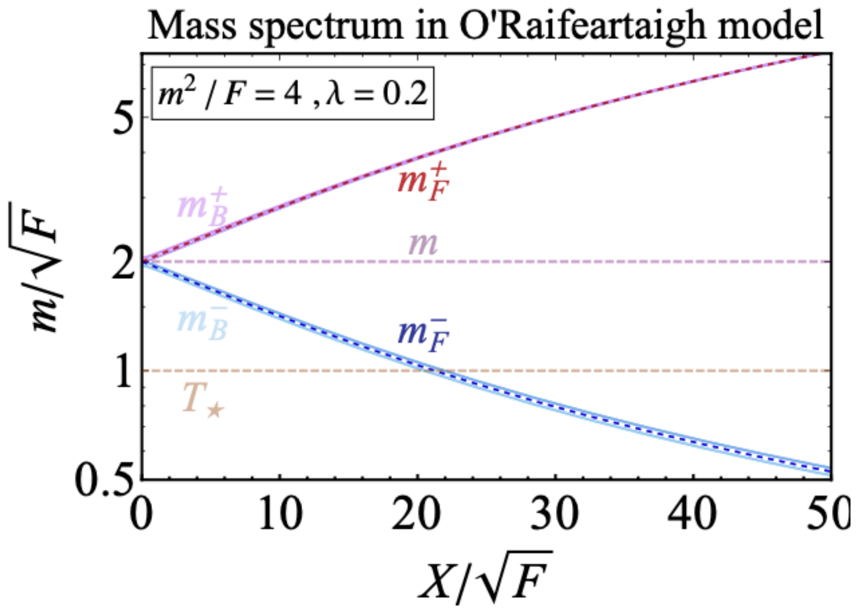
We will now see these features in explicit SUSY-breaking sectors →

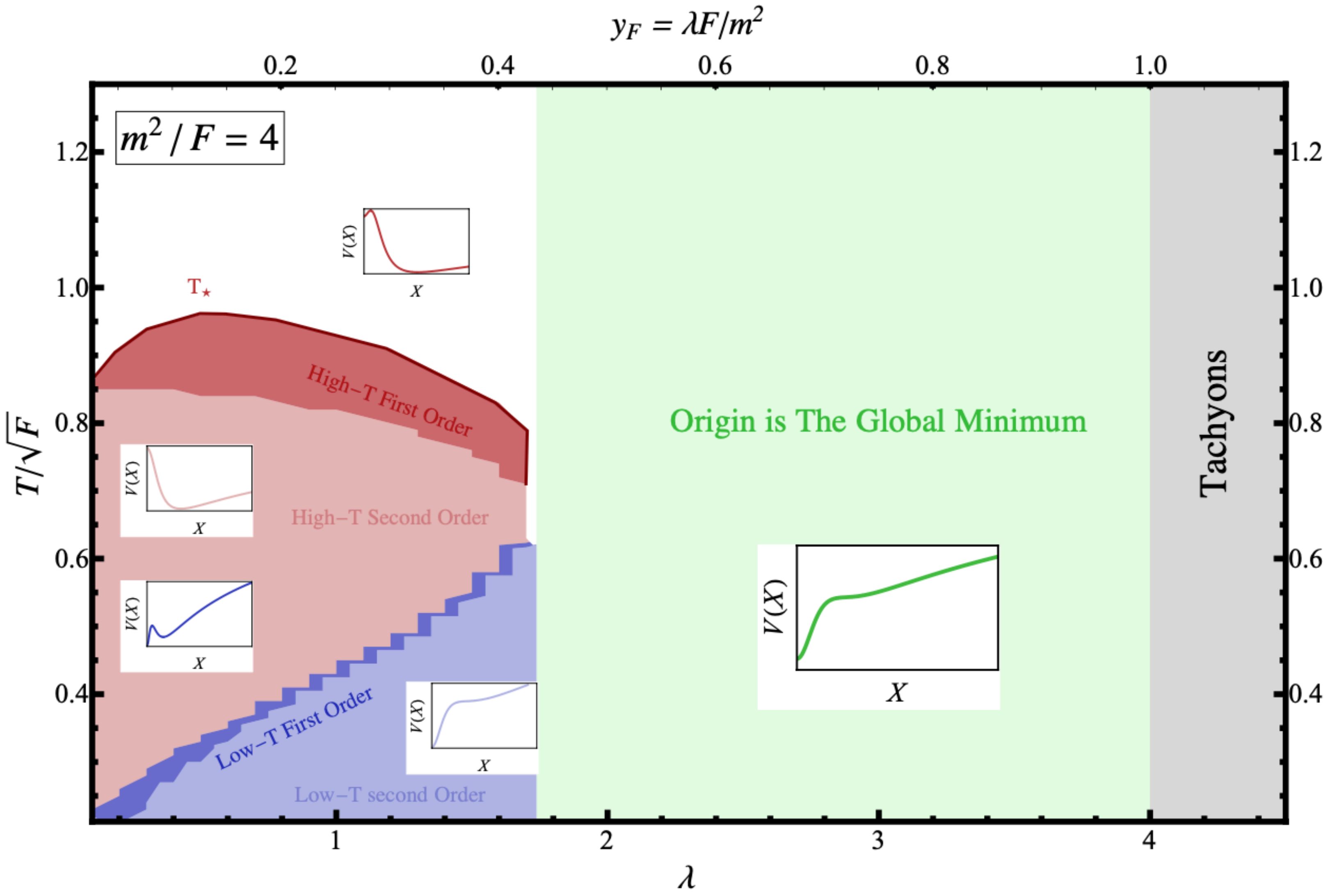
The O'Raifeartaigh phase diagram

see also A. Katz (2009)

$$W = -FX + \lambda X \Phi_1 \tilde{\Phi}_2 + m(\Phi_1 \tilde{\Phi}_1 + \Phi_2 \tilde{\Phi}_2)$$

	X	Φ_1	$\tilde{\Phi}_1$	Φ_2	$\tilde{\Phi}_2$
$U(1)_R$	2	0	2	2	0
$U(1)_D$	0	1	-1	1	-1



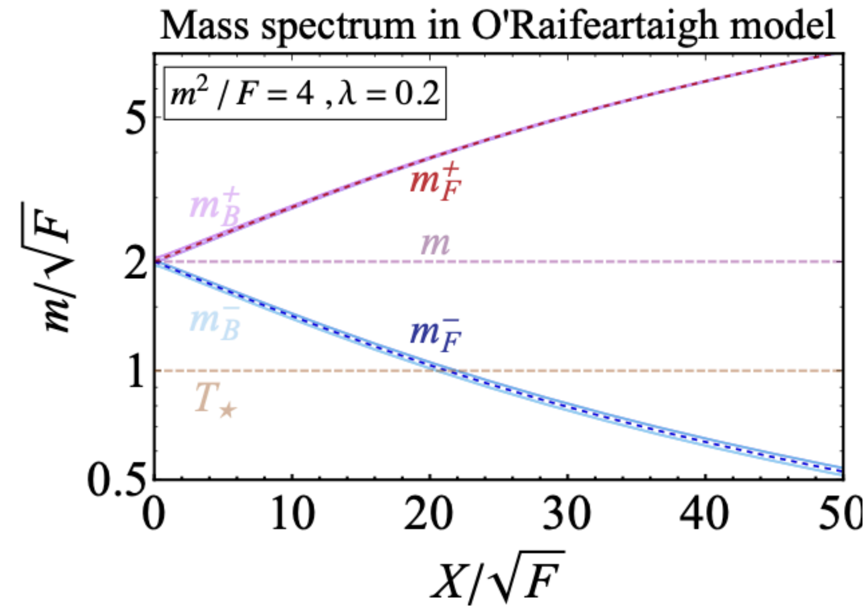


The O'Raifeartaigh phase diagram

see also A. Katz (2009)

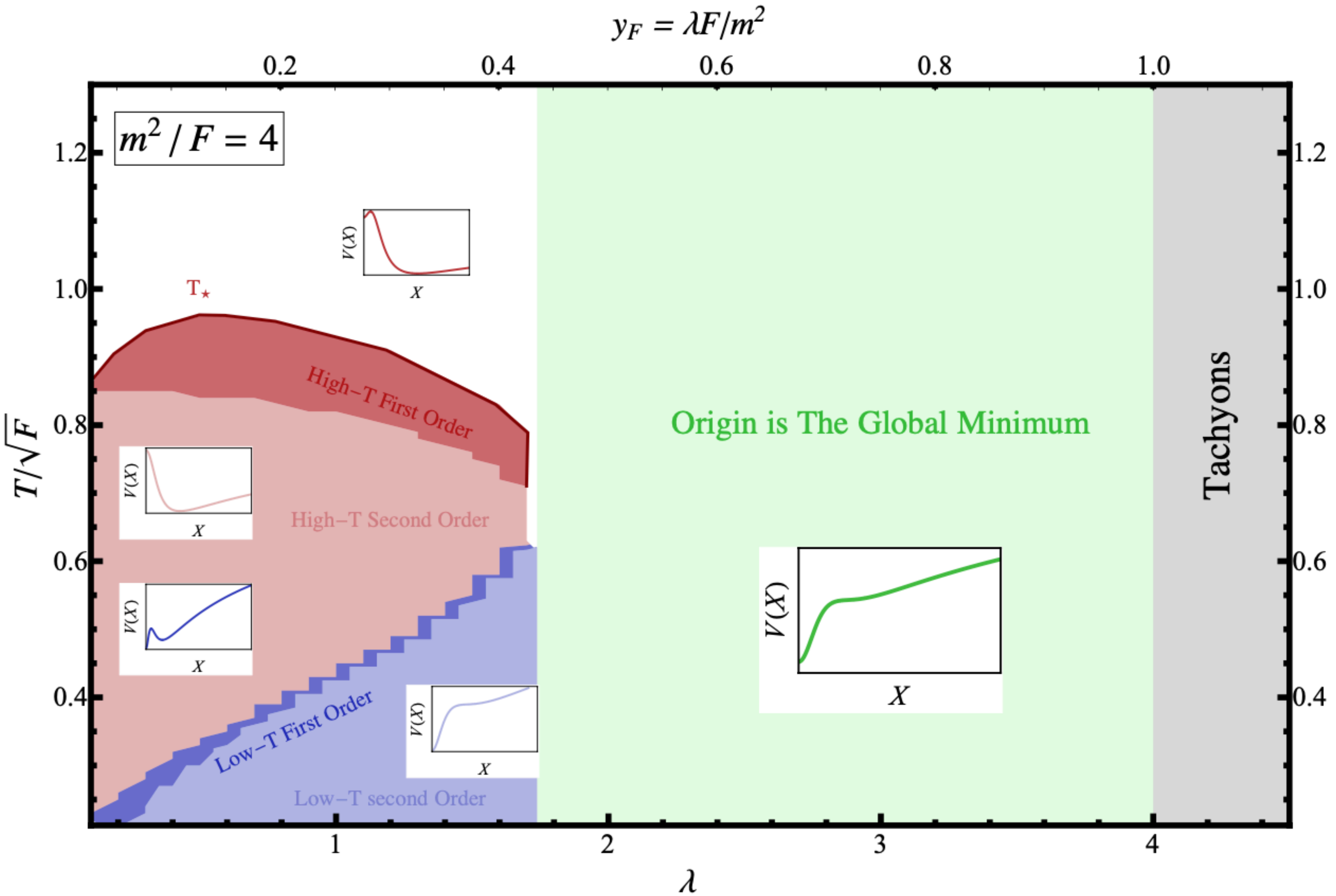
$$W = -FX + \lambda X \Phi_1 \tilde{\Phi}_2 + m(\Phi_1 \tilde{\Phi}_1 + \Phi_2 \tilde{\Phi}_2)$$

	X	Φ_1	$\tilde{\Phi}_1$	Φ_2	$\tilde{\Phi}_2$
$\mathrm{U}(1)_R$	2	0	2	2	0
$\mathrm{U}(1)_D$	0	1	-1	1	-1



$$x_\star \simeq \frac{2\sqrt{2}\pi T}{\lambda y_F} \quad , \quad T_\star \sim 0.23\sqrt{y_F} m \ ,$$

competition between thermal and loop corrections

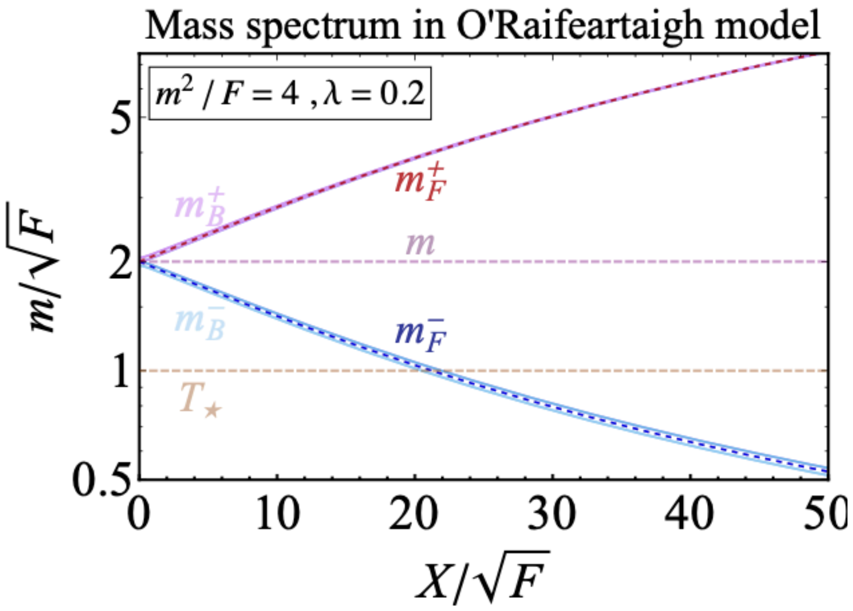


The O'Raifeartaigh phase diagram

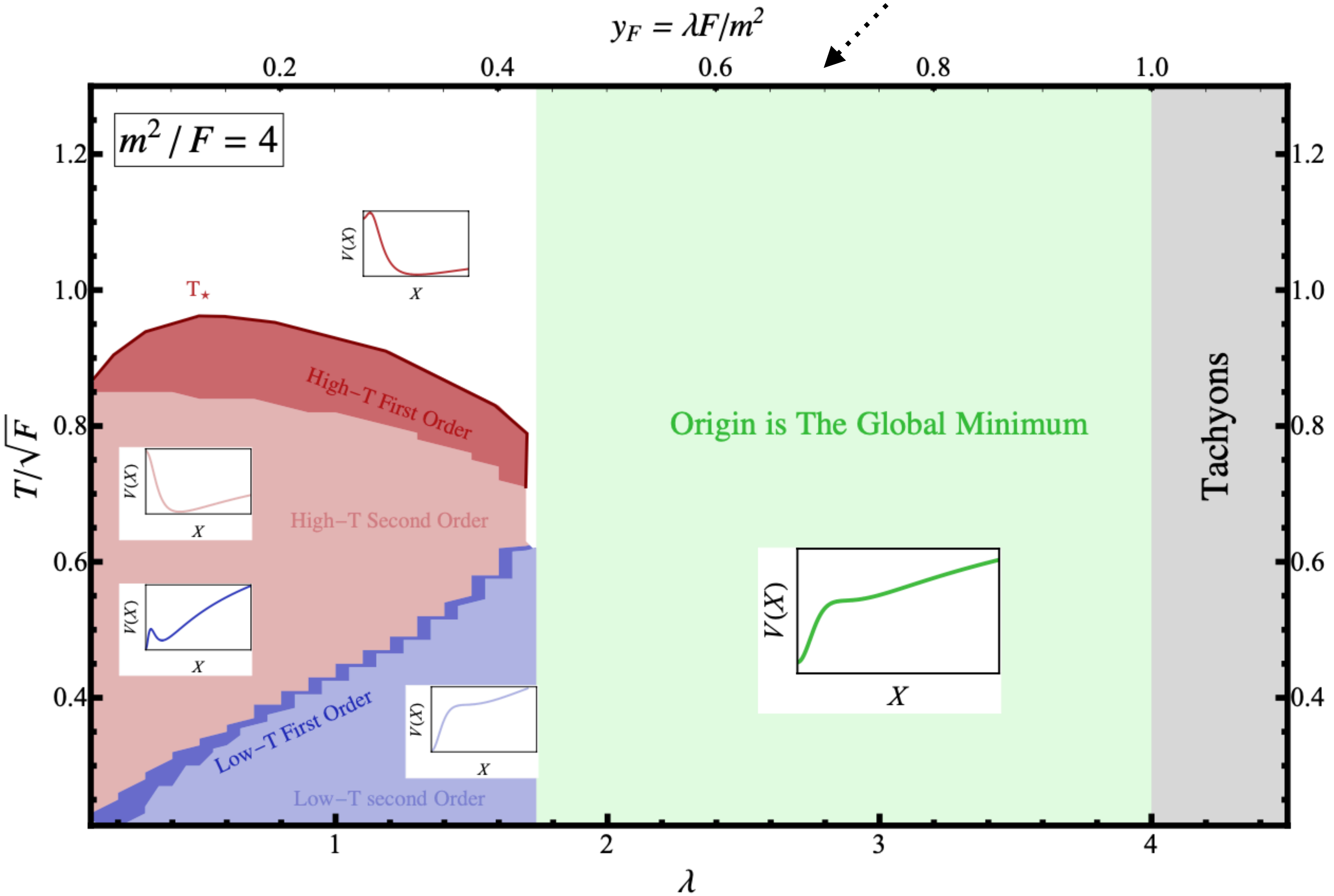
see also A. Katz (2009)

$$W = -FX + \lambda X \Phi_1 \tilde{\Phi}_2 + m(\Phi_1 \tilde{\Phi}_1 + \Phi_2 \tilde{\Phi}_2)$$

	X	Φ_1	$\tilde{\Phi}_1$	Φ_2	$\tilde{\Phi}_2$
$\mathrm{U}(1)_R$	2	0	2	2	0
$\mathrm{U}(1)_D$	0	1	-1	1	-1



we stick to this phase to avoid complications



$$x_\star \simeq \frac{2\sqrt{2}\pi T}{\lambda y_F} \quad , \quad T_\star \sim 0.23\sqrt{y_F}m \ ,$$

competition between thermal and loop corrections

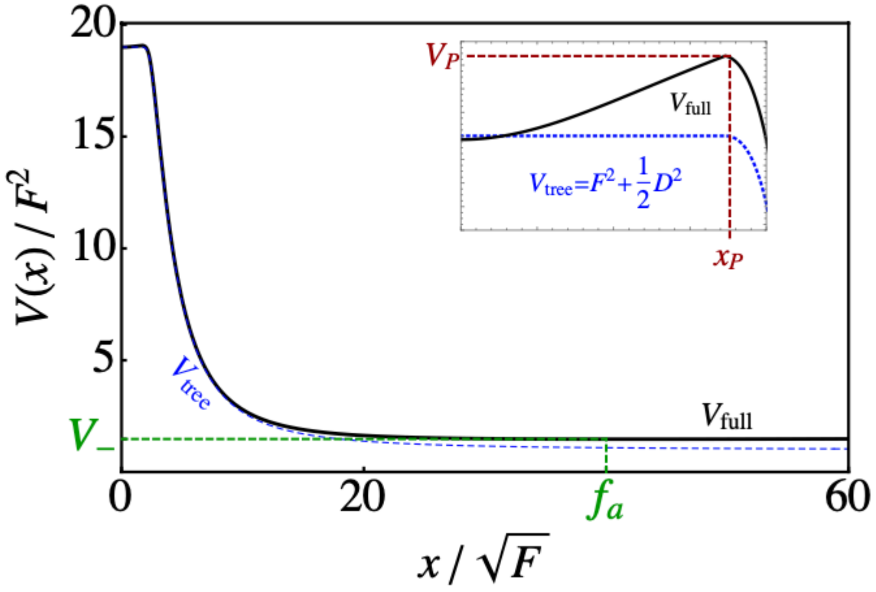
A full model of Low Energy SUSY breaking

Same field content that O'Raifeartaigh

	X	Φ_1	$\tilde{\Phi}_1$	Φ_2	$\tilde{\Phi}_2$
$U(1)_R$	2	0	2	2	0
$U(1)_D$	0	1	-1	1	-1

The flavor symmetry is gauged with an abelian gauge field with a Fayet-Iliopoulos*

$$\cdots\cdots\blacktriangleright \quad +\frac{g^2}{2}\left(\frac{D}{g}+|\phi_1|^2-|\tilde{\phi}_1|^2+|\phi_2|^2-|\tilde{\phi}_2|^2\right)^2$$



*

*A similar model can be obtained without F.I. splitting the masses of the matter fields

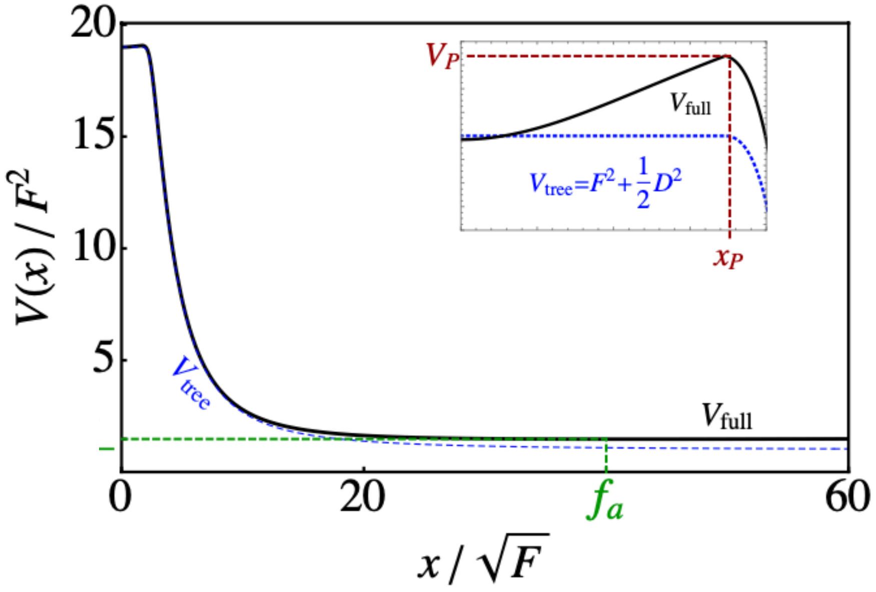
A full model of Low Energy SUSY breaking

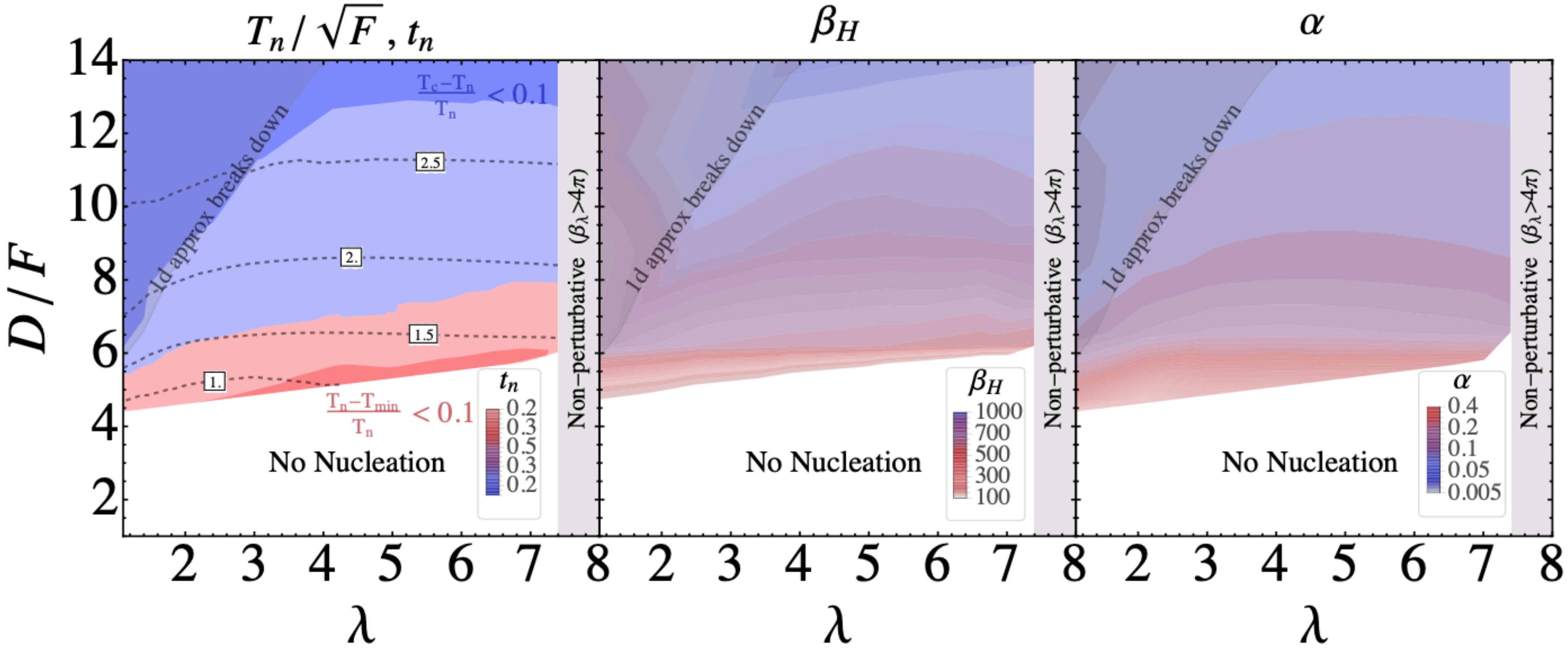
Same field content that O'Raifeartaigh

	X	Φ_1	$\tilde{\Phi}_1$	Φ_2	$\tilde{\Phi}_2$
$U(1)_R$	2	0	2	2	0
$U(1)_D$	0	1	-1	1	-1

The flavor symmetry is gauged with an abelian gauge field with a Fayet-Iliopoulos*

$$\dashrightarrow \quad +\frac{g^2}{2}\left(\frac{D}{g}+|\phi_1|^2-|\tilde{\phi}_1|^2+|\phi_2|^2-|\tilde{\phi}_2|^2\right)^2$$

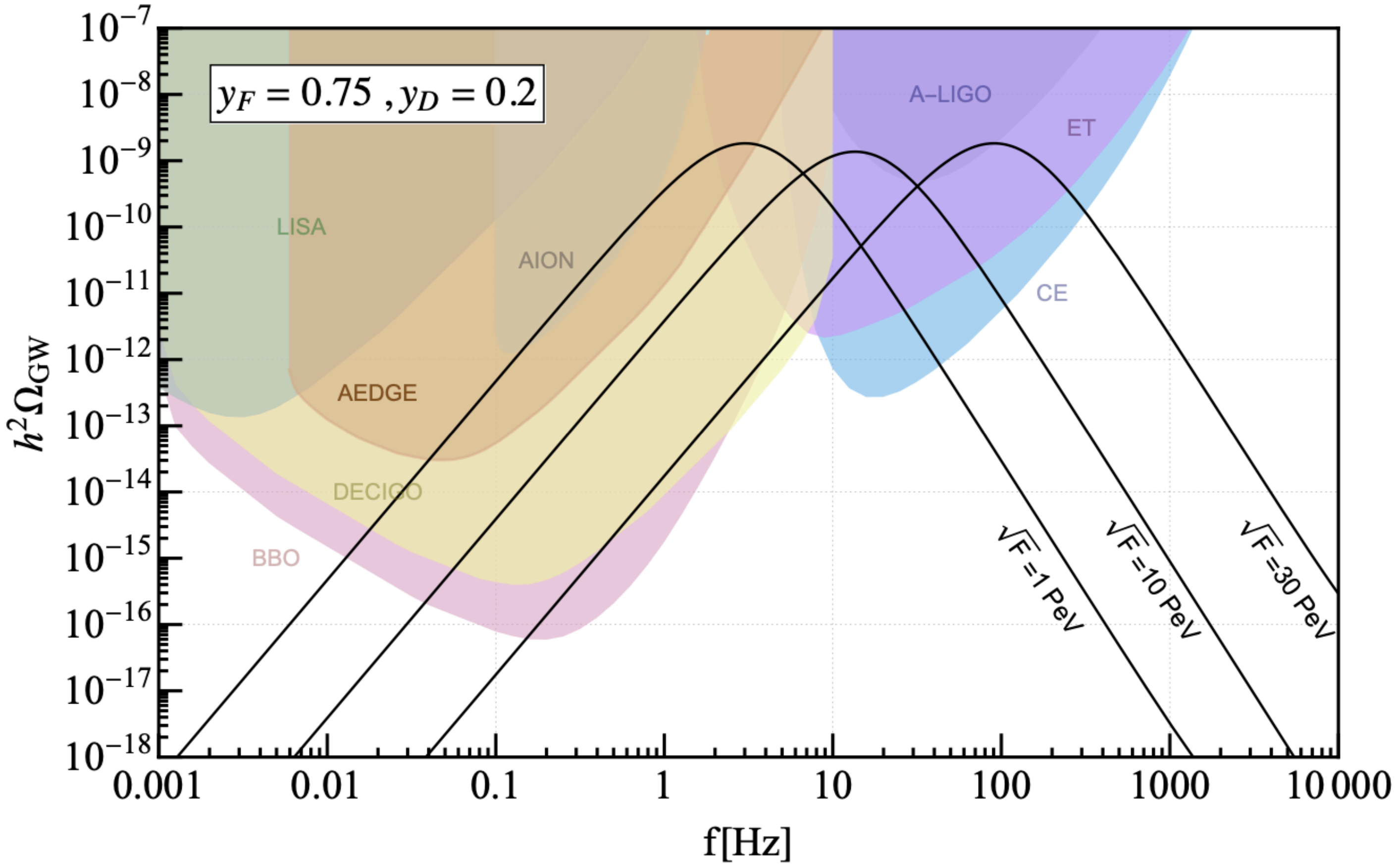




*A similar model can be obtained without F.I. splitting the masses of the matter fields

A full model of Low Energy SUSY breaking

Prediction for GWs



Prediction for the superpatner spectrum

$SU(6)/U(1)_D \supset SU(5)$ and messengers in the $5+\bar{5}$

$$\mathcal{M}_{\rm mess} = \begin{pmatrix} \frac{\lambda f_a}{\sqrt{2}} & m \\ m & 0 \end{pmatrix}^*$$

$$m_{\tilde{g}} \simeq 2\ \mathrm{TeV}\left(\frac{F}{30\ \mathrm{PeV}}\right)^{1/2}\left(\frac{y_F}{0.75}\right)^3\left(\frac{F}{2.5D}\right)^{1/2}\left(\frac{\lambda}{4}\right)\left(\frac{g}{0.4}\right)$$

* gaugino screening is unavoidable since we want to avoid massless messengers along the pseudomodulus

How we will discover SUSY?

Future colliders can possibly nail unnatural SUSY scenarios

The frequency of GWs expected from the hidden sector
correlates with the SUSY spectrum

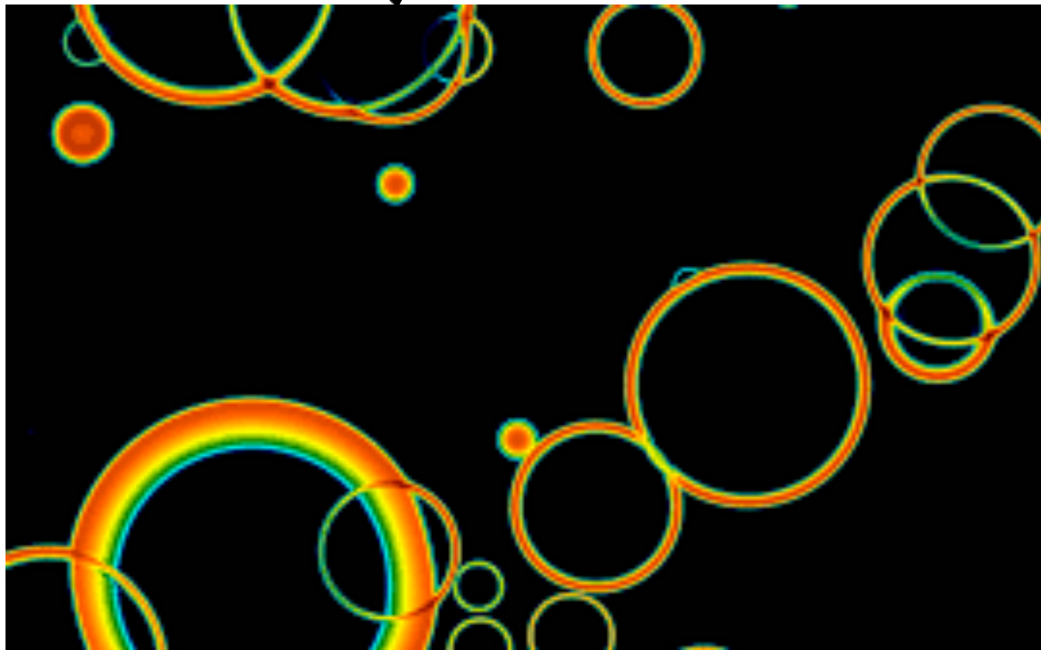
Which QFTs give rise to Gravitational Waves?

The SUSY-breaking pseudomodulus features
a new type of 1st order PTs

*low-T expansion

*pressure from Boltzmann suppressed states

*GWs are unlikely to be detectable in single scale models

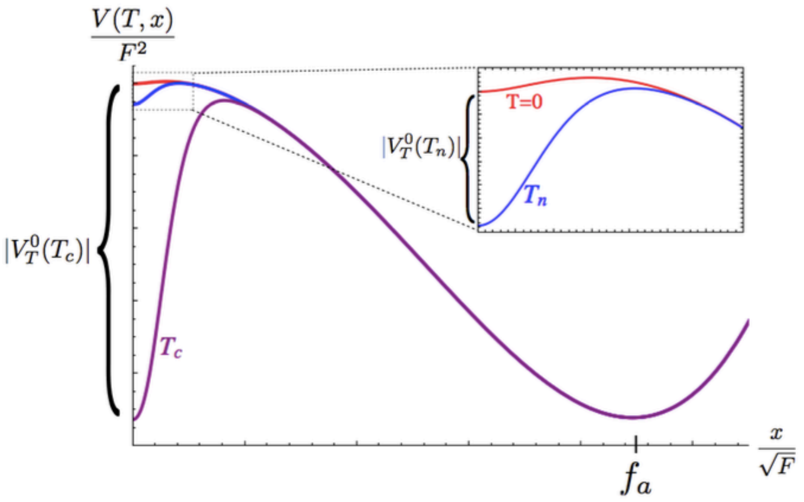


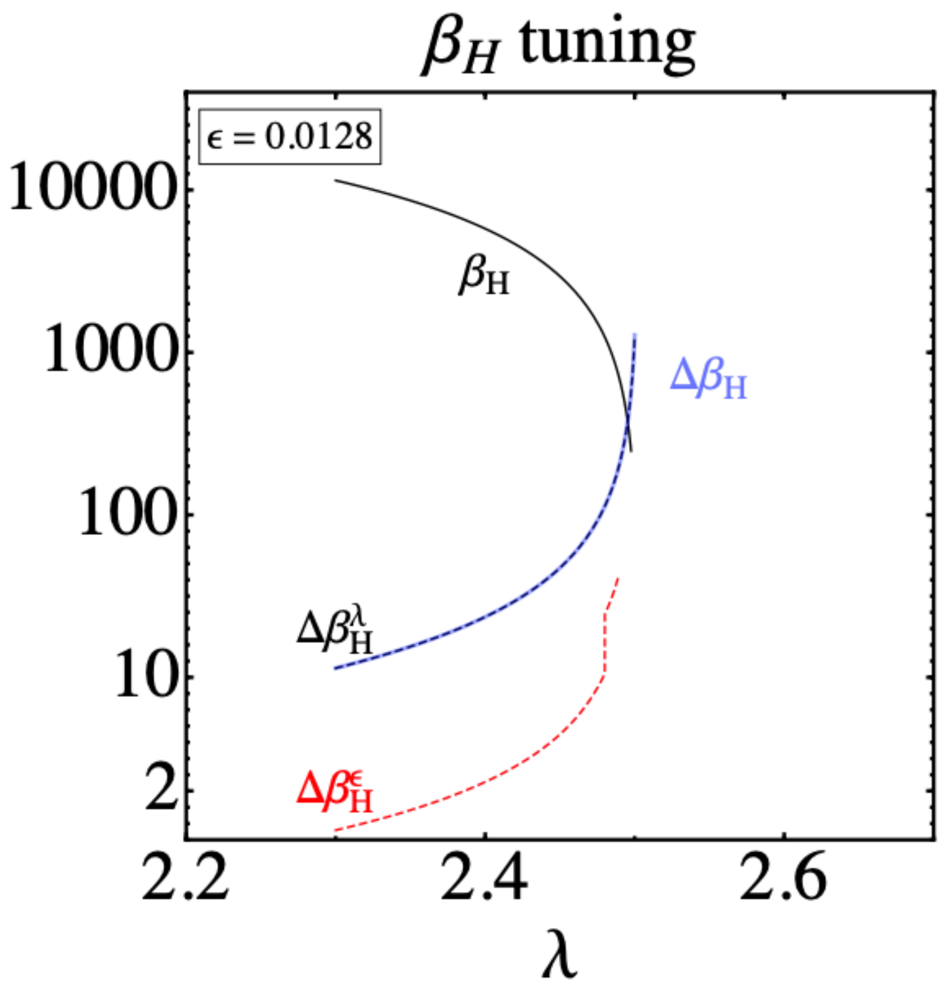
Model building extras

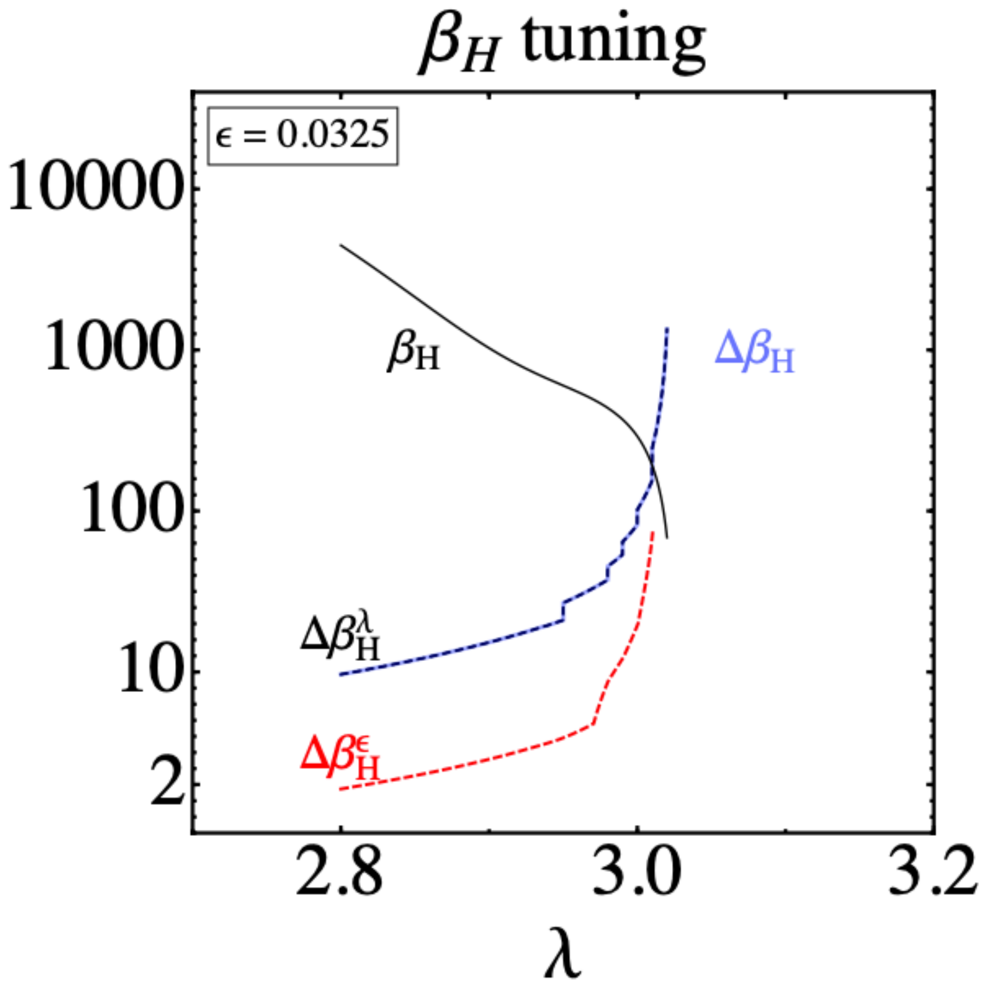
An example of beta-tuning

O'Raifeartaigh $W = -FX + \lambda X\Phi_1\tilde{\Phi}_2 + m(\Phi_1\tilde{\Phi}_1 + \Phi_2\tilde{\Phi}_2)$

+ R-breaking $W_{\cancel{R}}(X) = \frac{1}{3}\epsilon X^3$

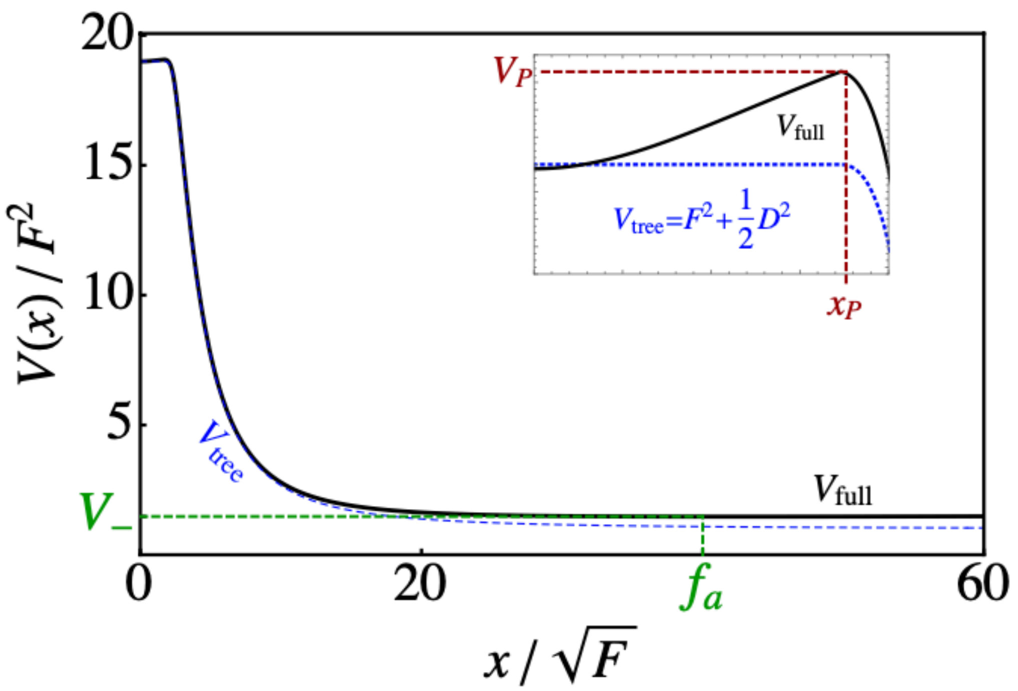




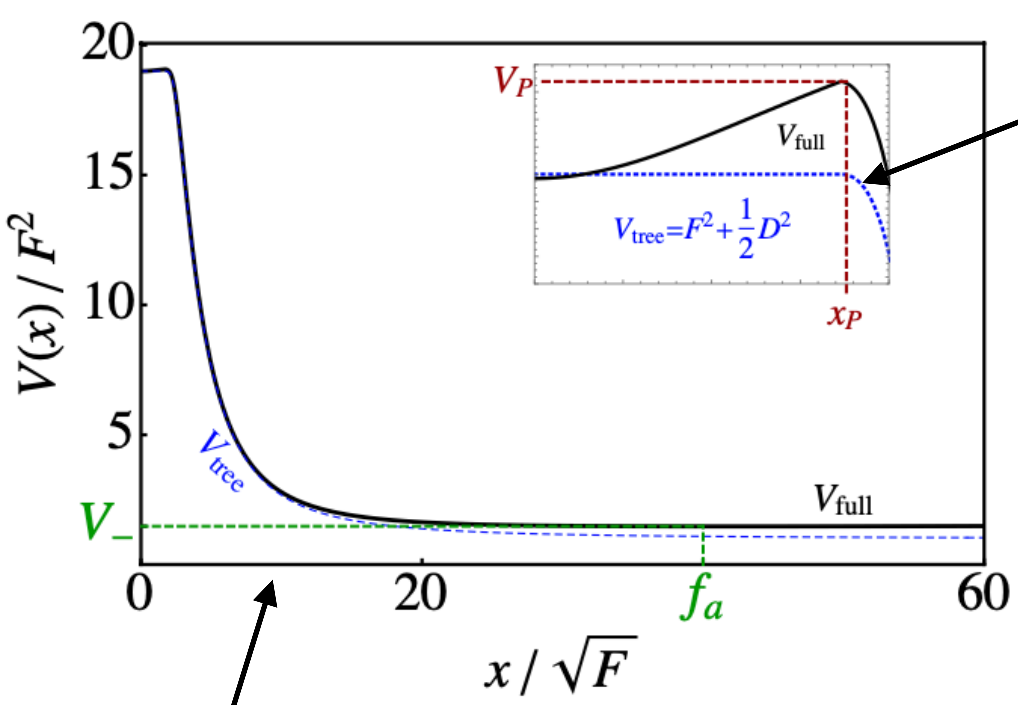


the numerically computed fine-tuning is even larger than the naive parametric we derived

The F vs D interplay



The F vs D interplay

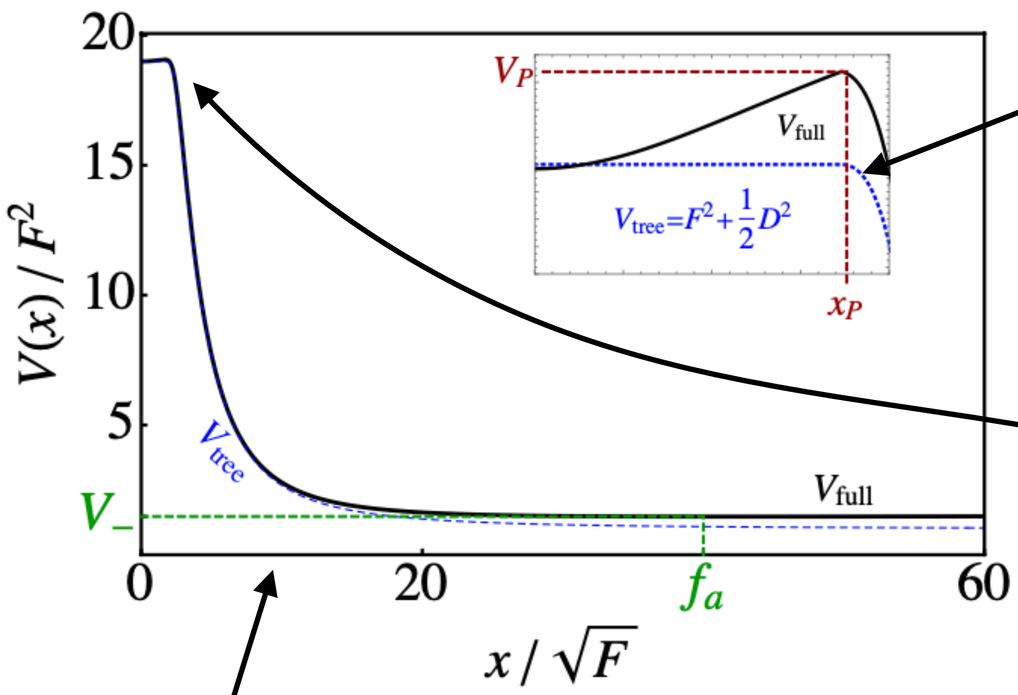


$$x^2_{\text{trans}} \simeq \frac{2m^2(m^2-gD)}{\lambda^2 gD}$$

the potentially is flat in the origin and the
it runs down into a runaway direction where $D \to 0$

$$V_{\text{tree}}(x) \simeq \begin{cases} F^2+\frac{1}{2}D^2 = V_+ & x < x_{\text{trans}} \\ F^2+\frac{1}{2}D^2 - \frac{\lambda^4 D^2\left(x^2-x^2_{\text{trans}}\right)^2}{2(2m^2+\lambda^2x^2)^2} & x > x_{\text{trans}} \end{cases}$$

The F vs D interplay



$$x^2_{\rm trans} \simeq \frac{2m^2(m^2-gD)}{\lambda^2 gD}$$

Loop corrections generate the barrier and the true vacuum

$$V_P - V_+ \simeq \frac{\lambda^2 F^2}{16\pi^2}\log\left(\frac{x^2_{\rm trans}}{m^2}\right)$$

$$\langle x\rangle_{\rm true} = f_a \simeq \frac{4\sqrt{2}\pi}{\lambda y_F}\sqrt{\frac{D}{g}}$$

the potentially is flat in the origin and the it runs down into a runaway direction where $D \to 0$

$$V_{\rm tree}(x) \simeq \begin{cases} F^2+\frac{1}{2}D^2 = V_+ & x < x_{\rm trans} \\ F^2+\frac{1}{2}D^2 - \frac{\lambda^4 D^2\left(x^2-x^2_{\rm trans}\right)^2}{2(2m^2+\lambda^2x^2)^2} & x > x_{\rm trans} \end{cases}$$