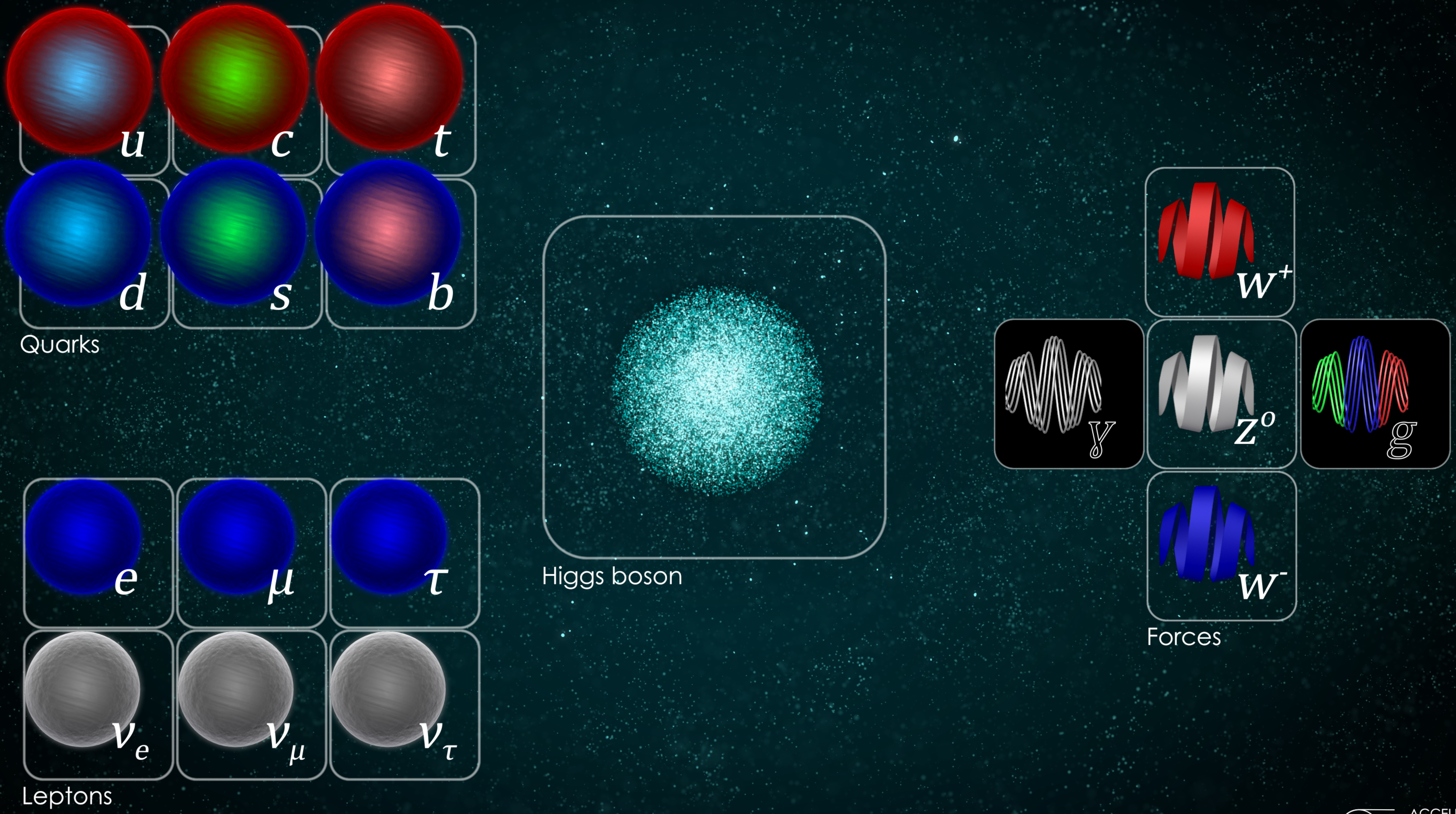
Luka Leskovec

Jožef Stefan Institute

Transition amplitudes from lattice QCD

IJS–FMF High-Energy Physics Seminars

Thursday, December 9th 2021



ACCELERATING SCIENCE

u
c
t
d
s
b
Quarks
e
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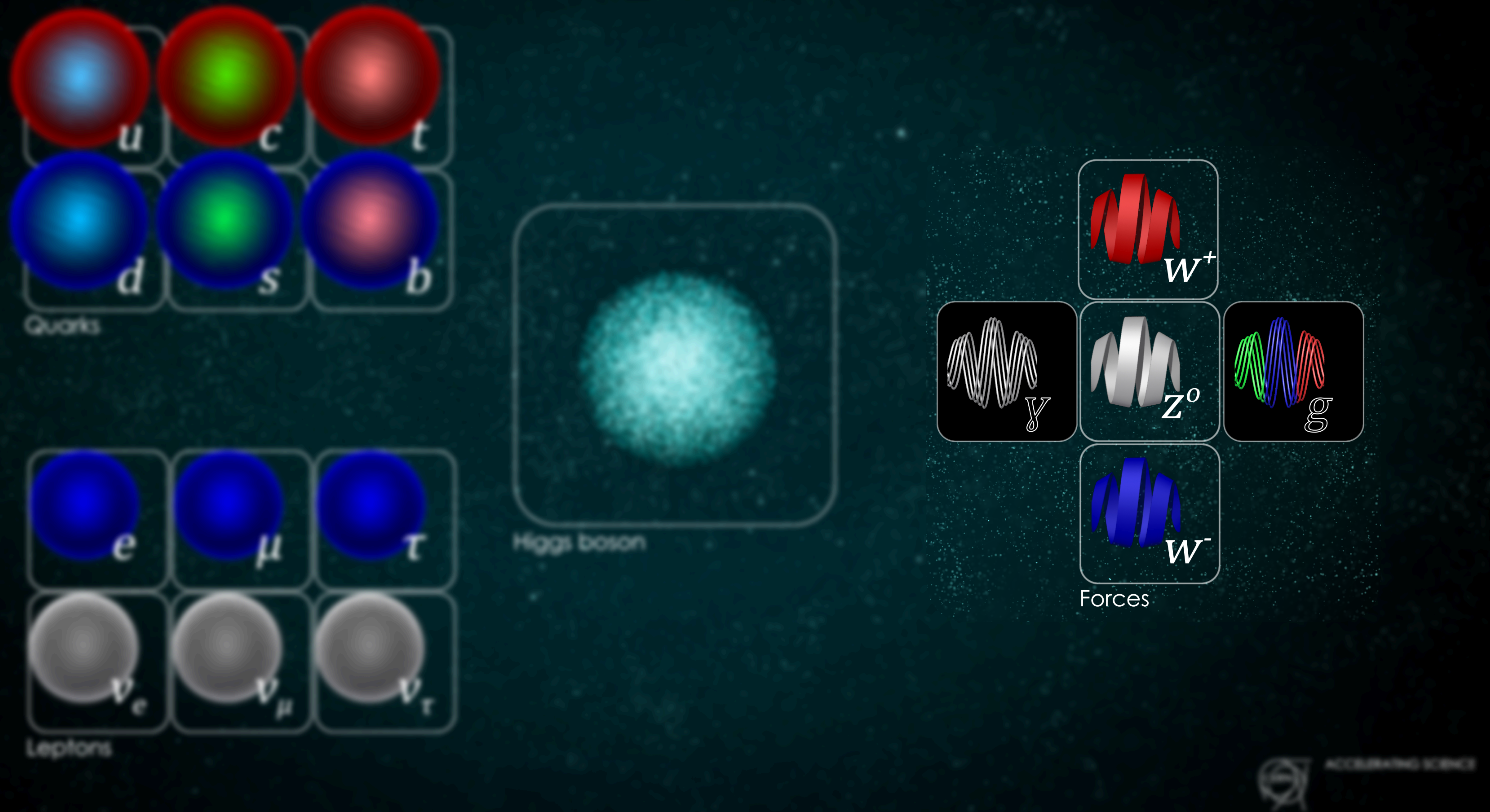
ν_e
ν_μ
ν_τ
Leptons
Higgs boson
W+
γ
Z0
g
W-
Forces


u
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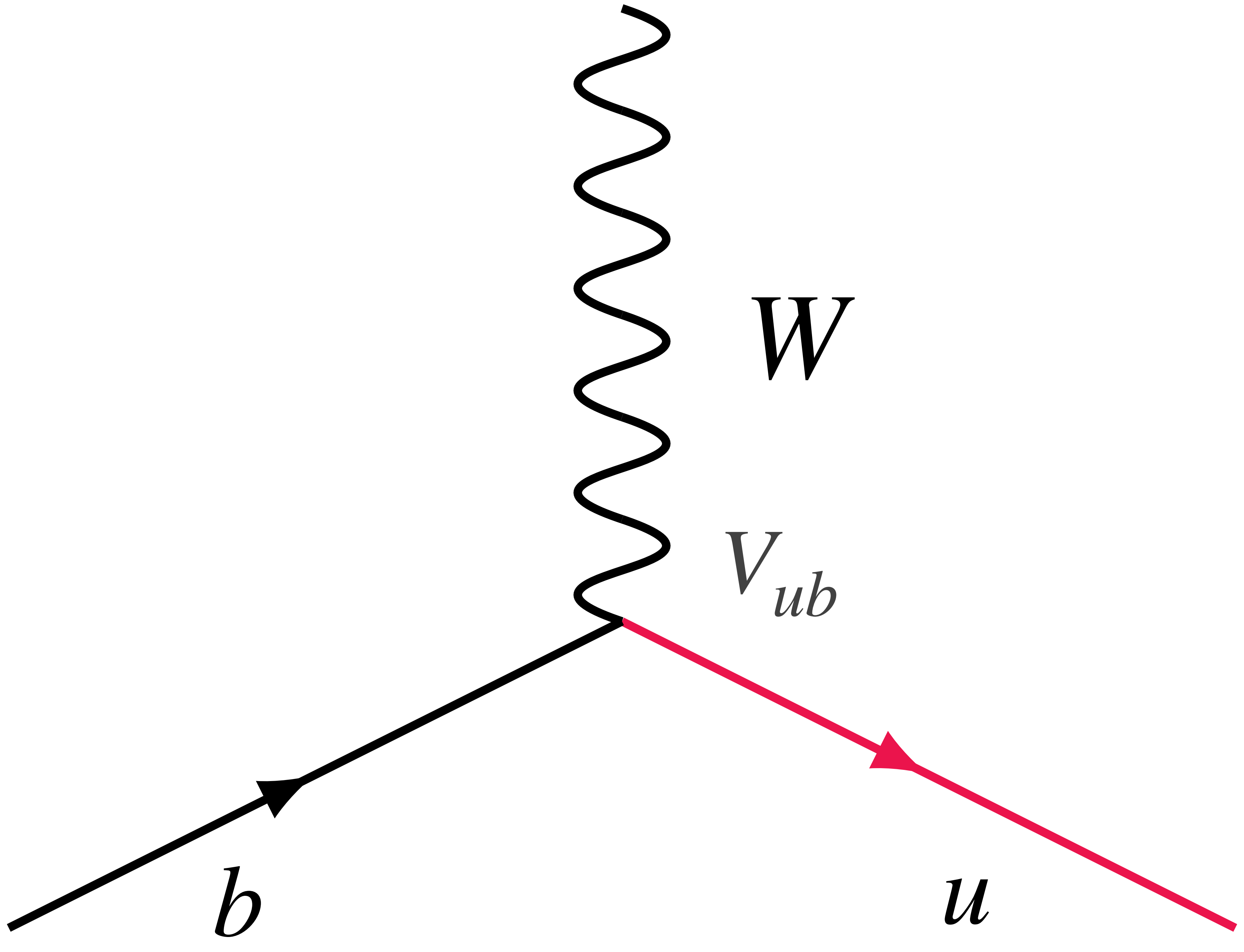
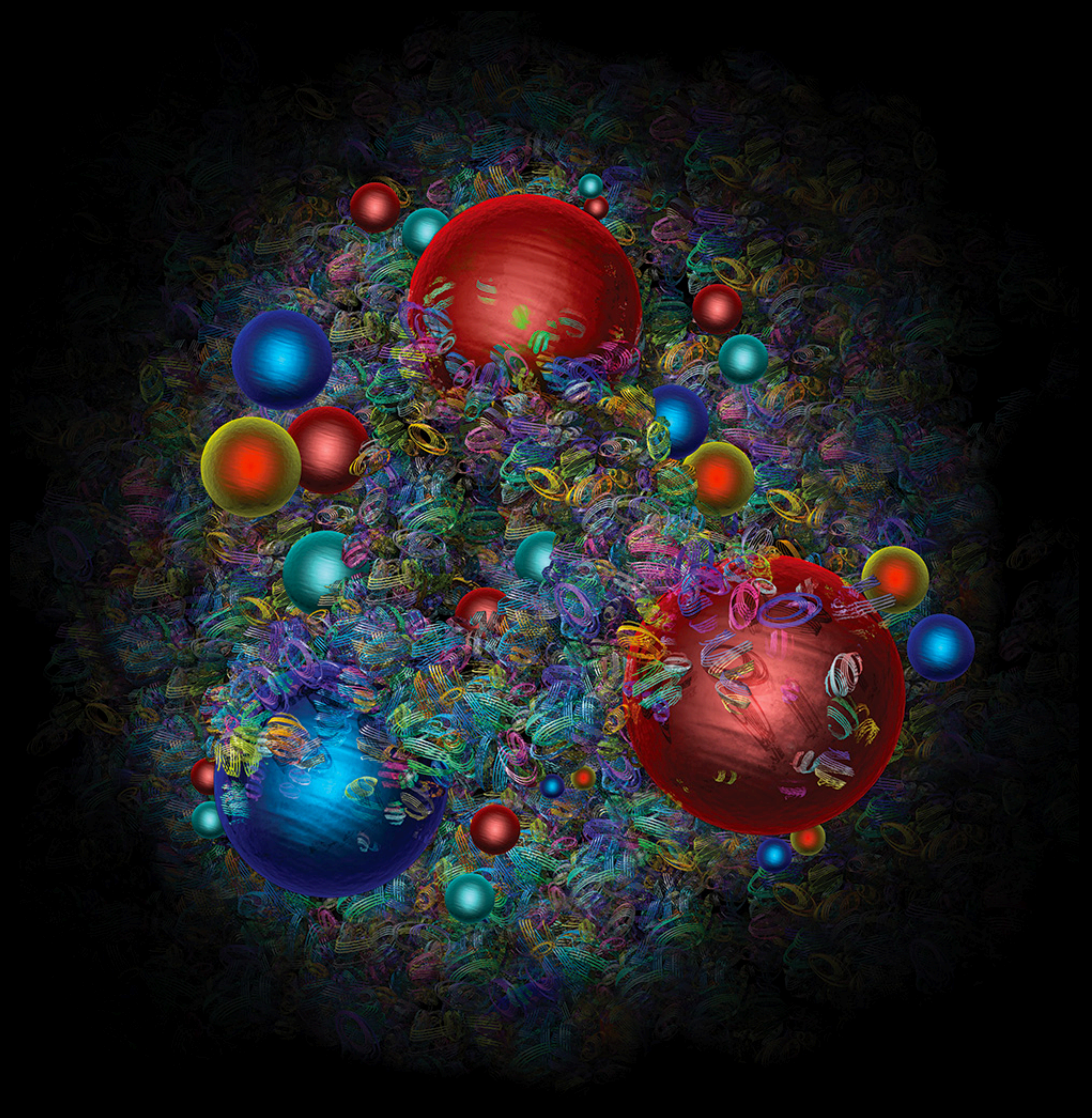
Leptons
Higgs boson
W+
γ
Z0
g
W-
Forces

transitions between quarks

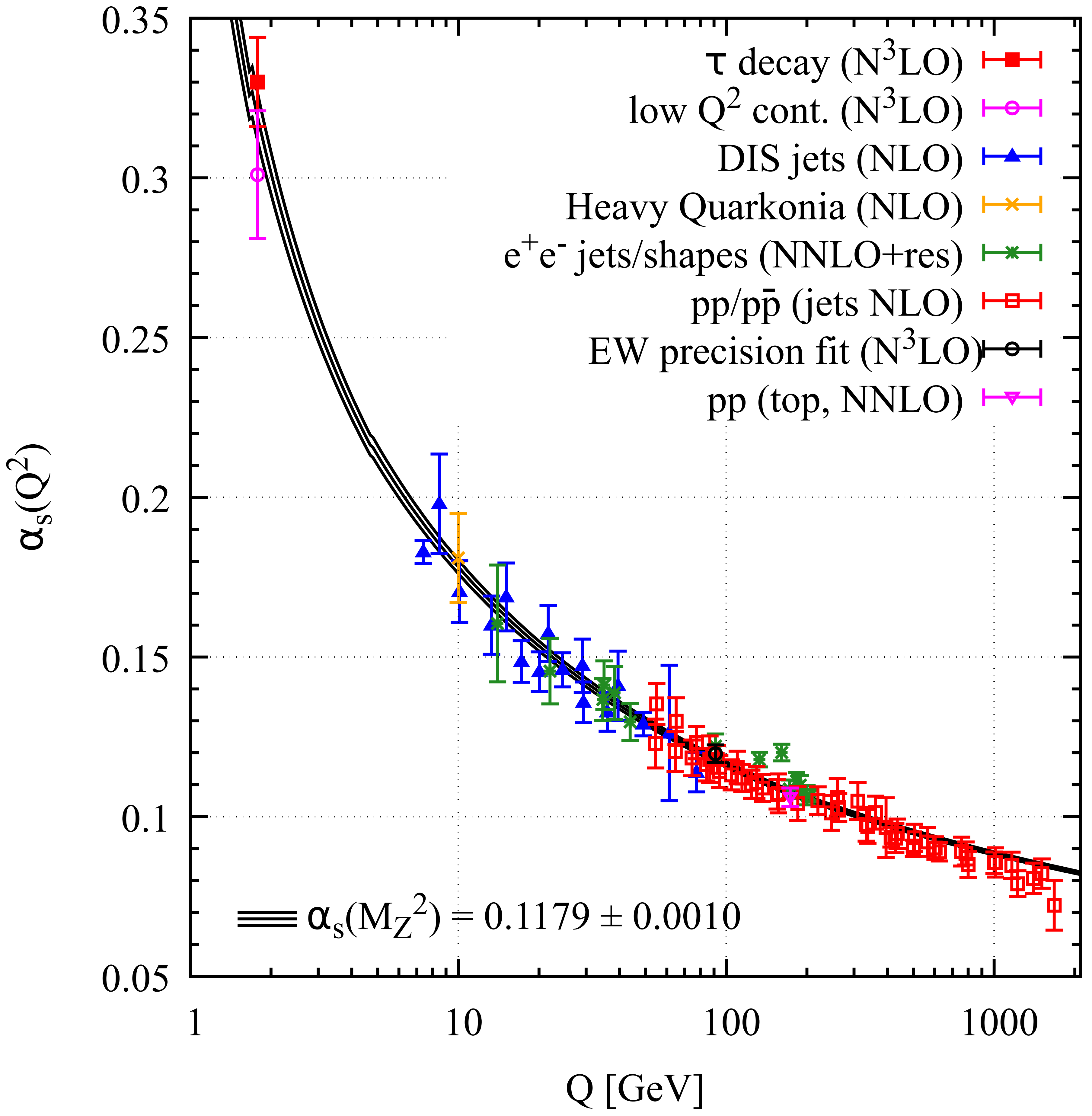
- mass basis vs interaction basis

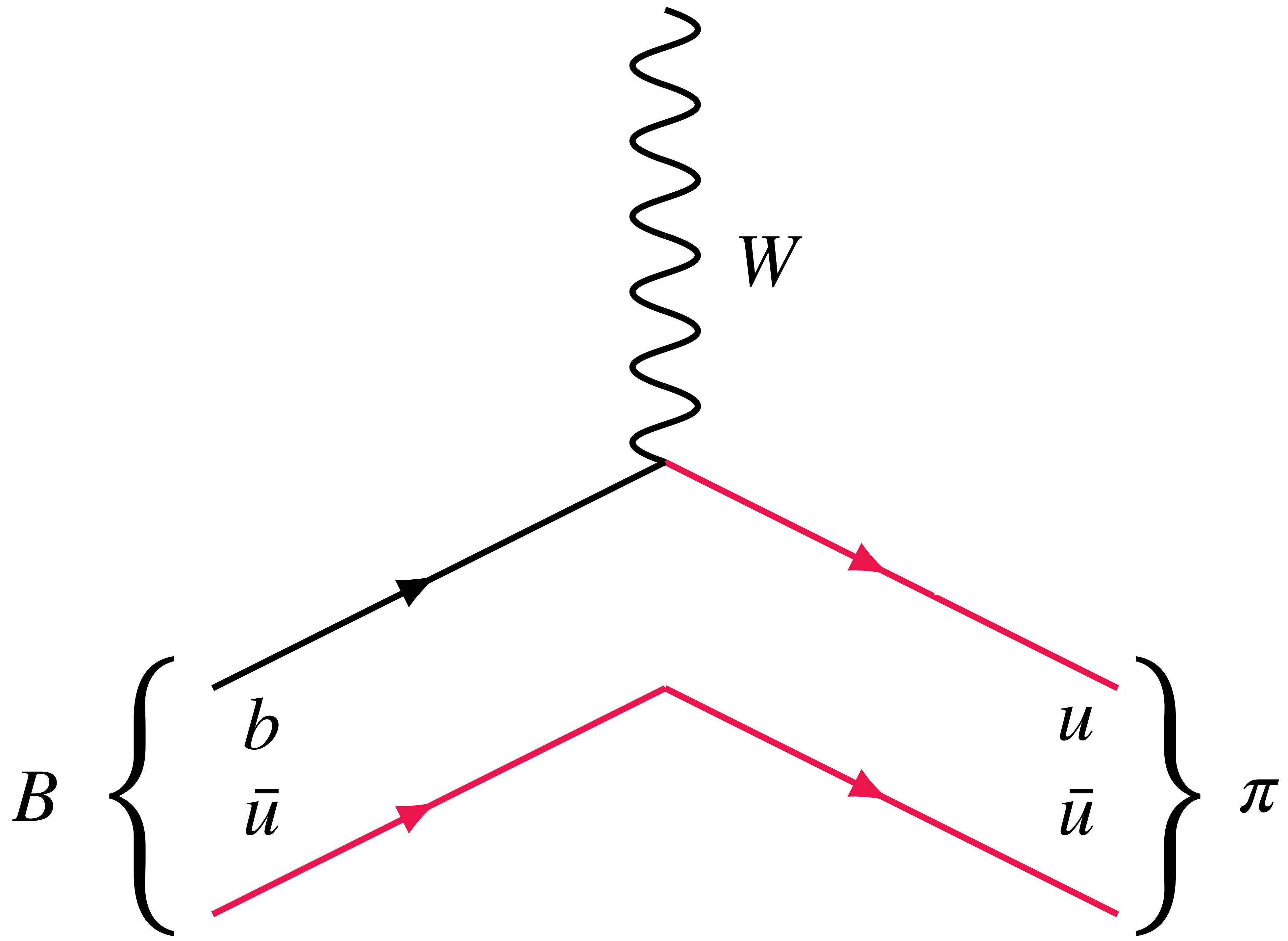
$$\begin{bmatrix} d^W \\ s^W \\ b^W \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$





but it is not that simple…

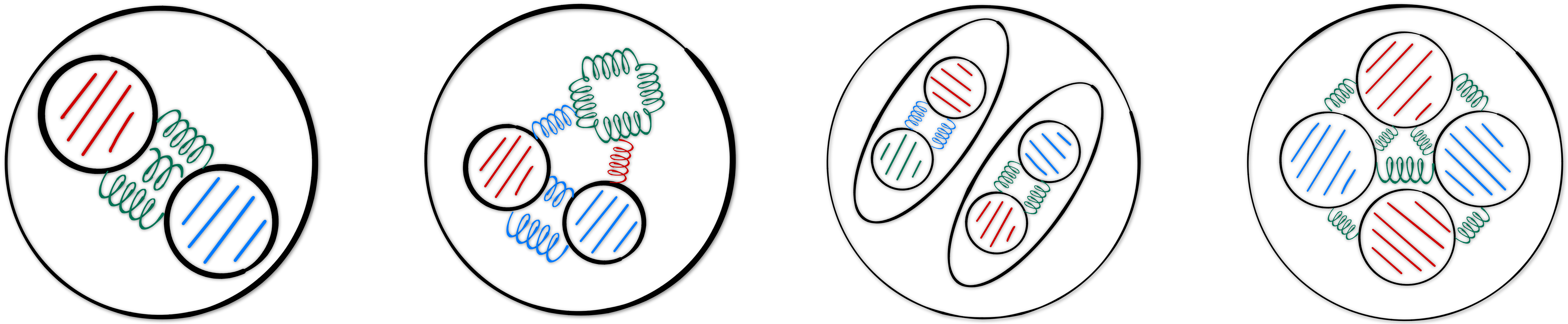




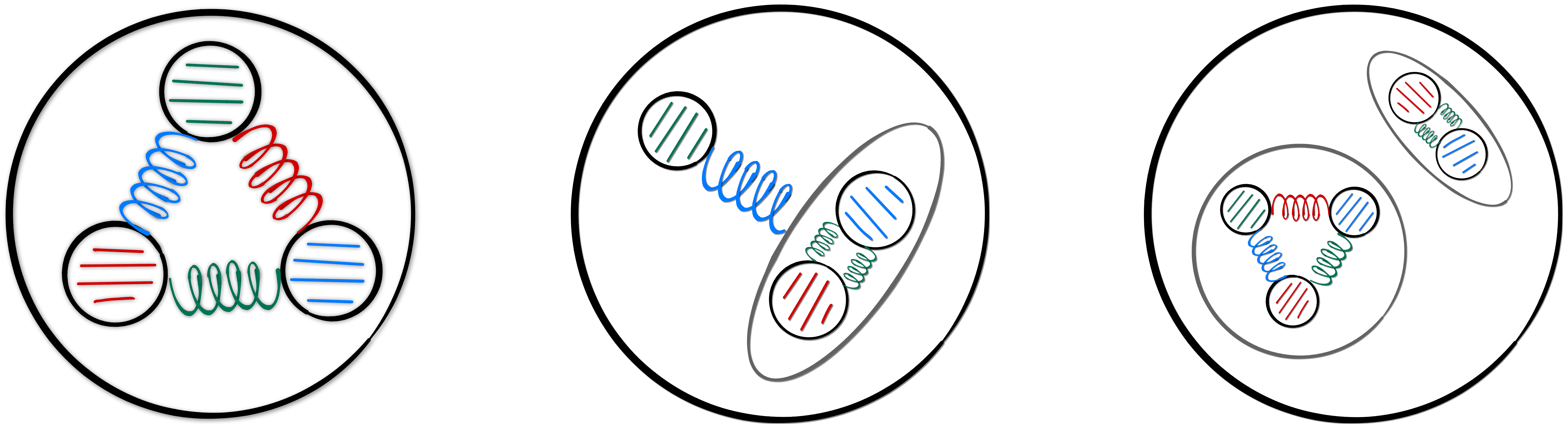
PDG

hadrons…

- mesons

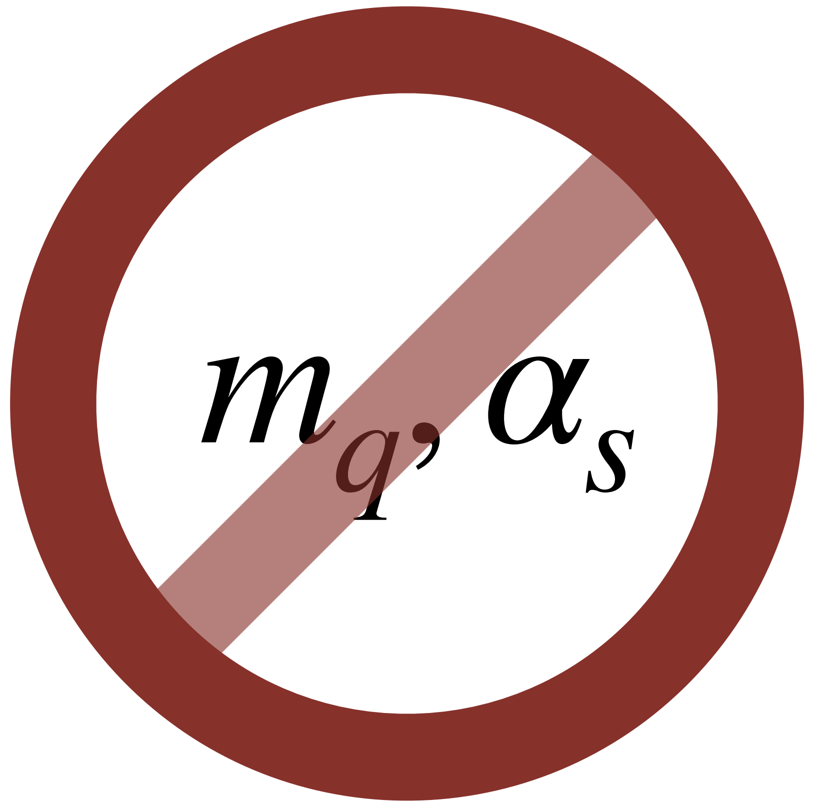


- baryons



how to QCD?

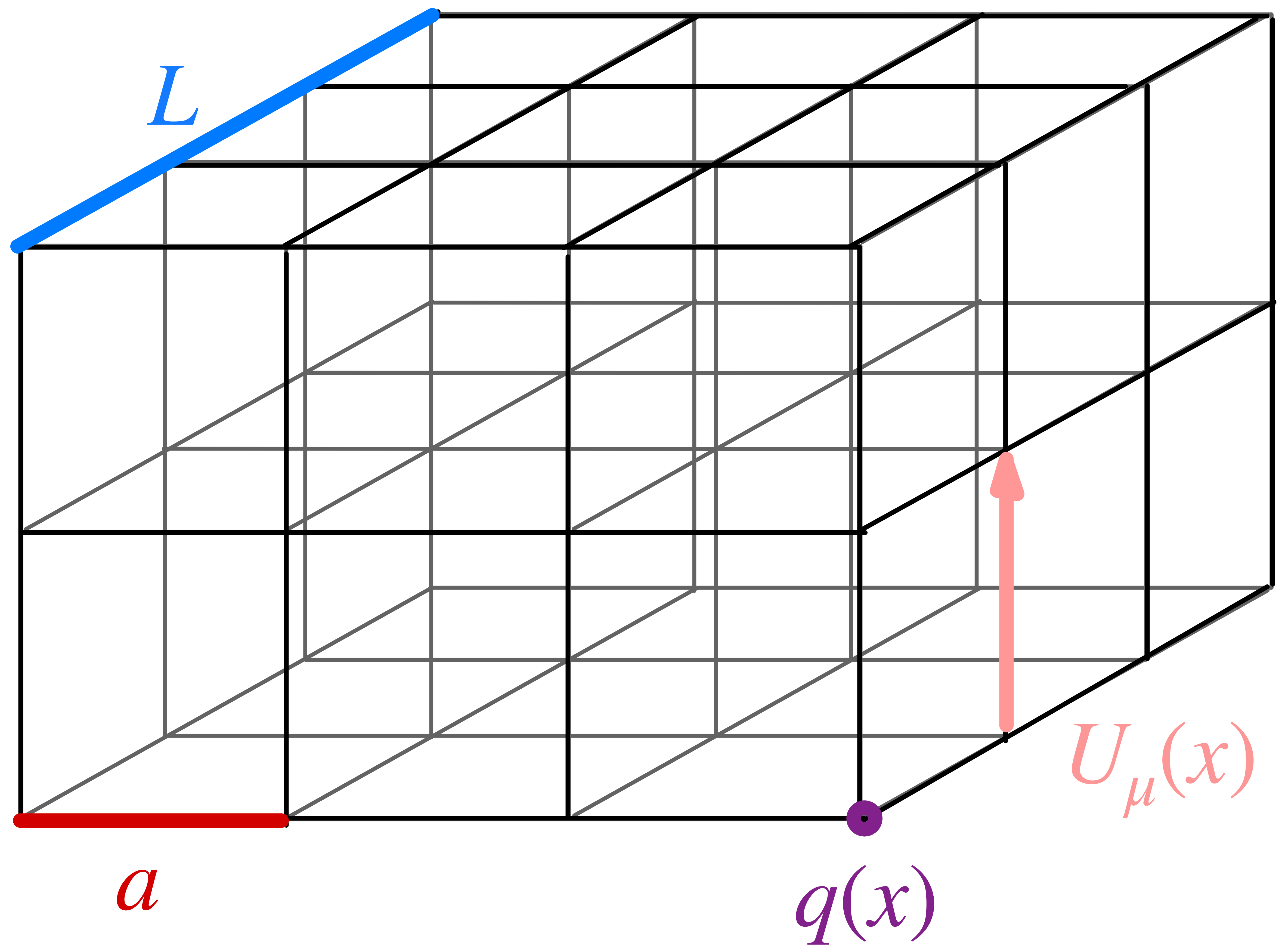
- there are many methods to study QCD:
  - effective field theories
  - Chiral Perturbation Theory
  - quark models
  - AdS/QCD duality



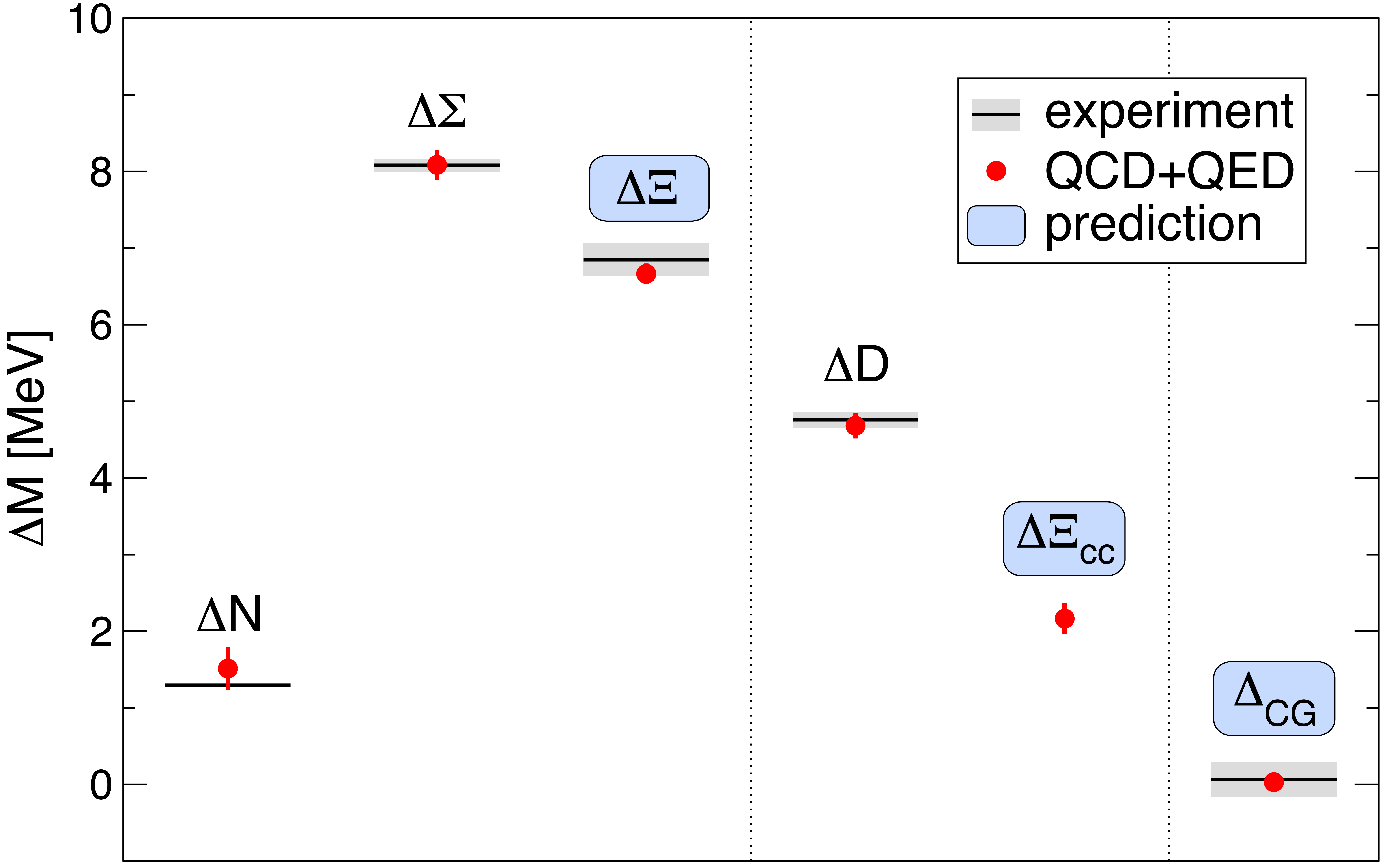
- lattice QCD:
  - m_q, α_s
  - QFT
  - systematically improvable

lattice QCD

- QFT
- UV regulated (a)
- IR regulated (L)
- quark fields ($q(x)$)
- gauge fields ($U_\mu(x)$)



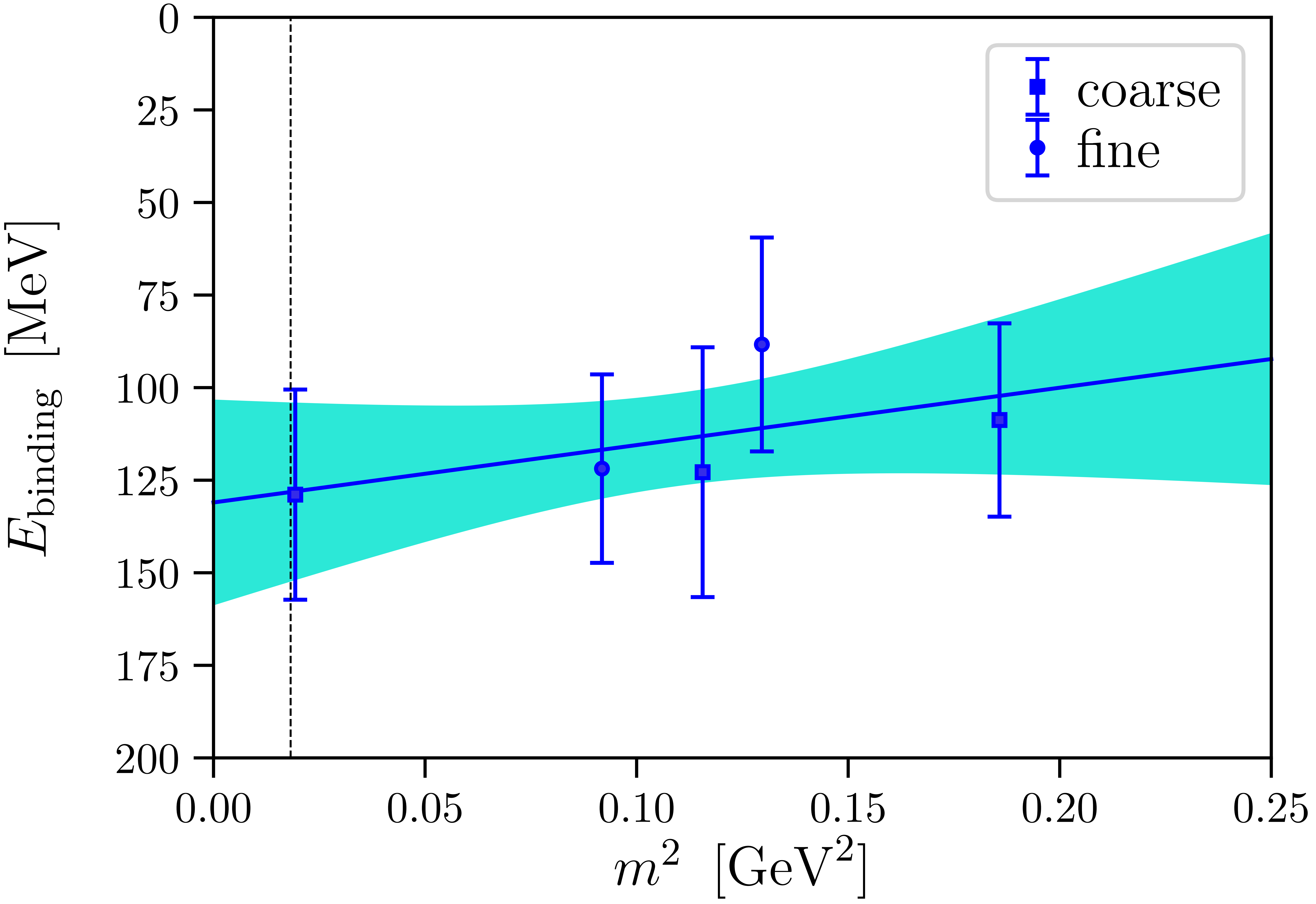
lattice QCD so far



BMW 2014

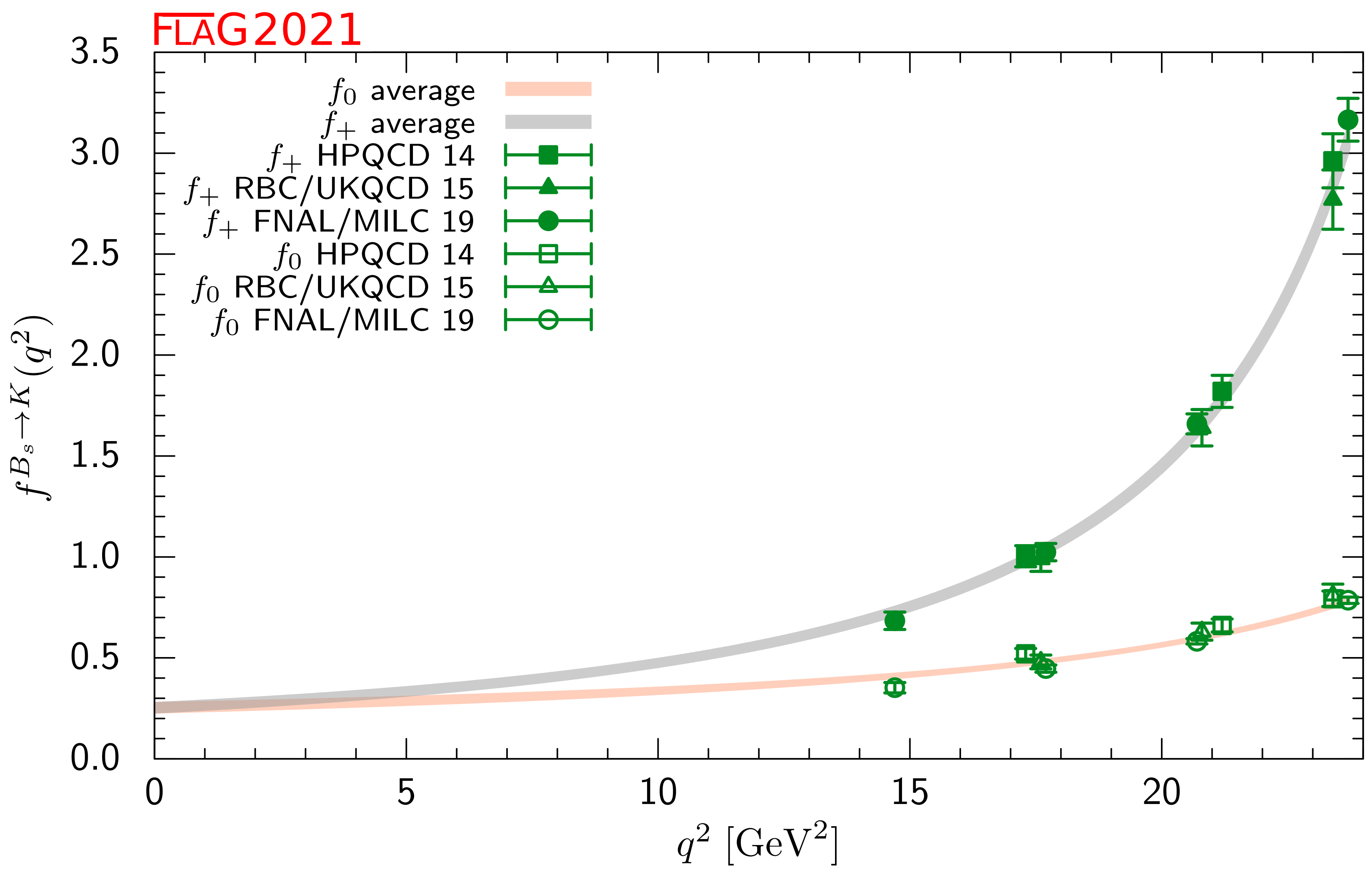
lattice QCD so far

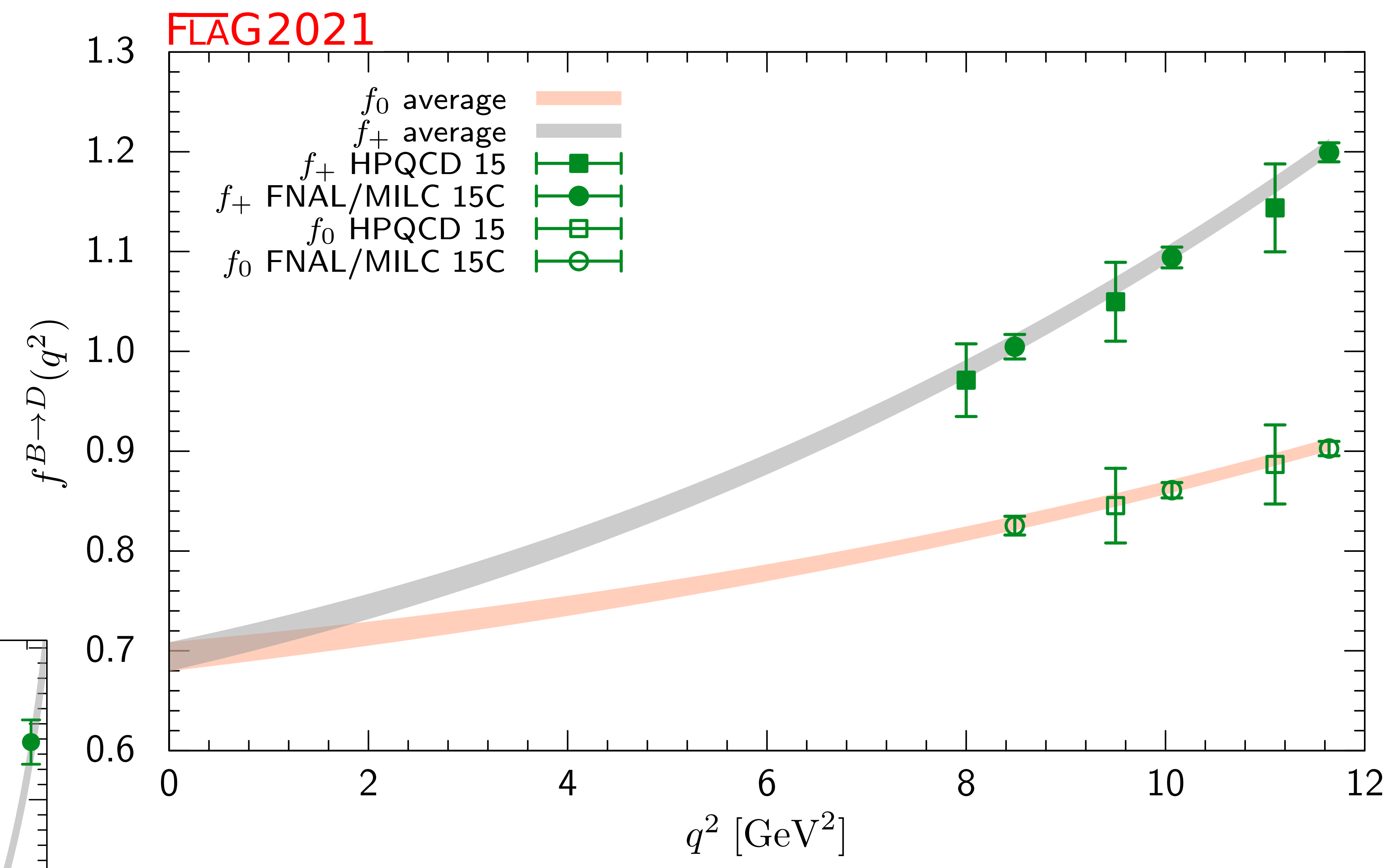
- $I = 0$
- $J^P = 1^+$
- $B = 2$
- $\bar{b}\bar{b}ud$
- tetraquark

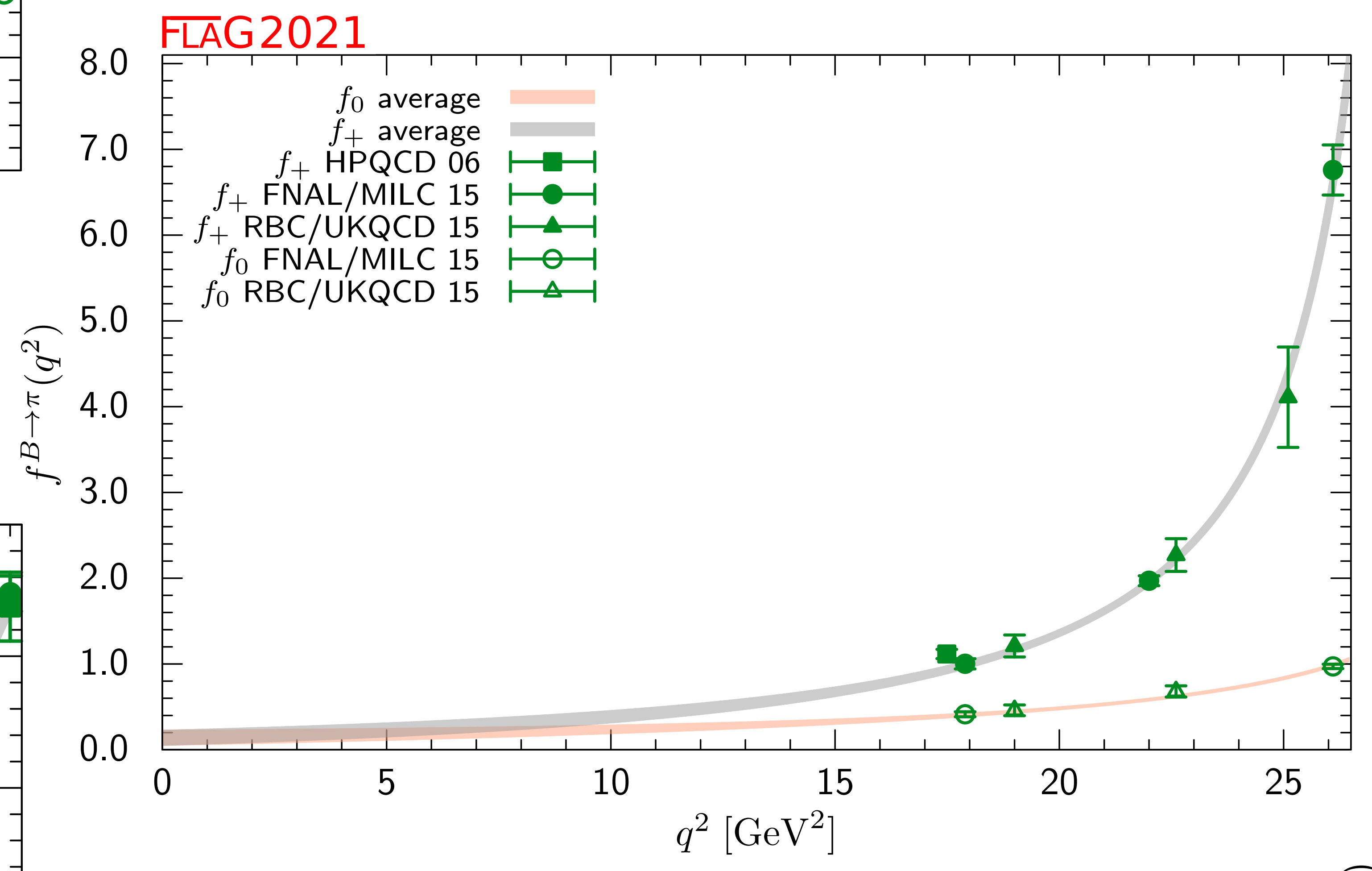


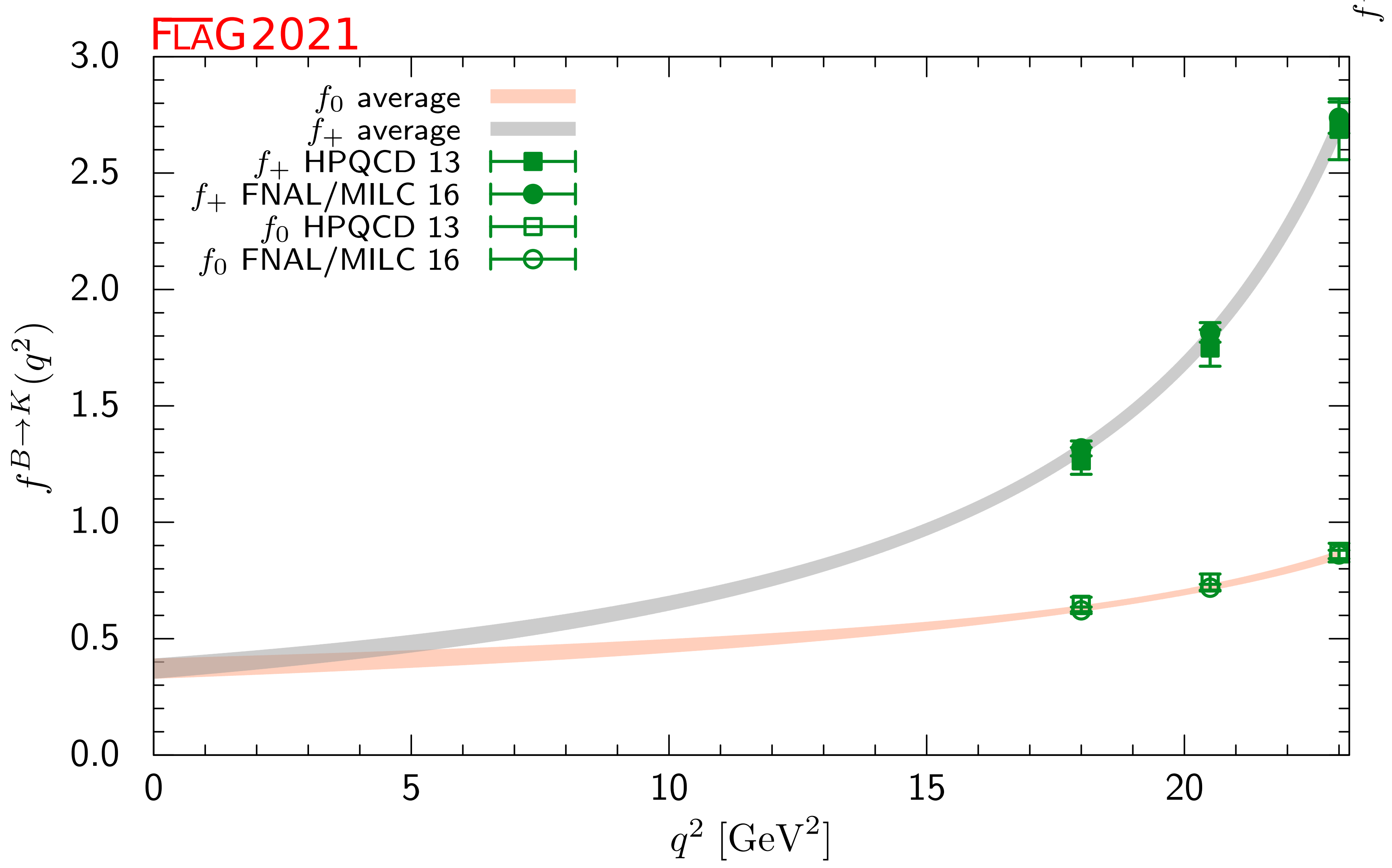
LL et al. 2019

lattice QCD so far

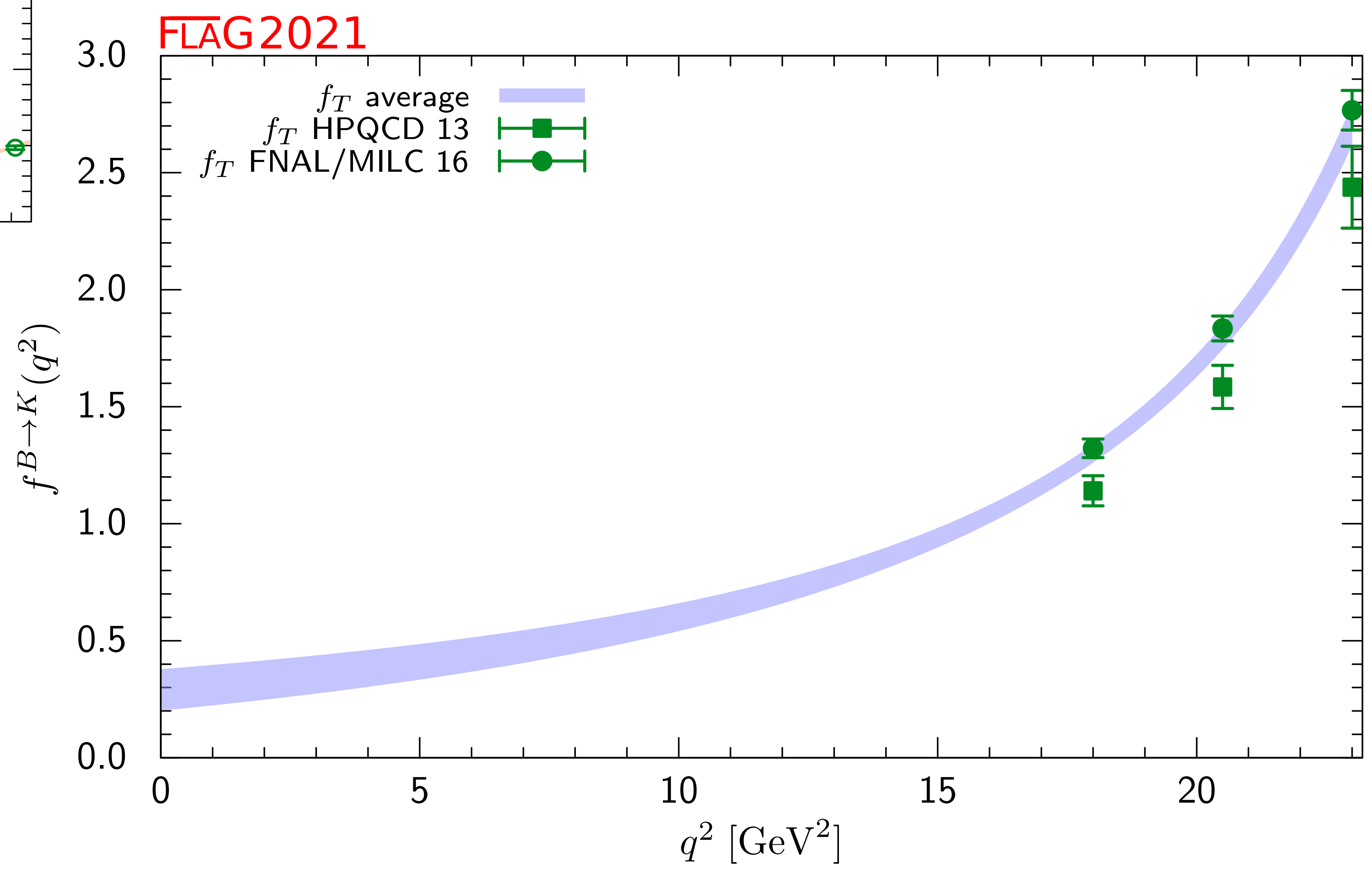








and so much more….



but most hadrons are resonances

$f_0(500)$ aka σ; was $f_0(600)$	$0^+(0^{++})$
$\rho(770)$	$1^+(1^{--})$
$\omega(782)$	$0^-(1^{--})$
$\eta'(958)$	$0^+(0^{-+})$
$f_0(980)$	$0^+(0^{++})$
$a_0(980)$	$1^-(0^{++})$
$\phi(1020)$	$0^-(1^{--})$
$h_1(1170)$	$0^-(1^{+-})$
$b_1(1235)$	$1^+(1^{+-})$
$a_1(1260)$	$1^-(1^{++})$
$f_2(1270)$	$0^+(2^{++})$
$f_1(1285)$	$0^+(1^{++})$
$\eta(1295)$	$0^+(0^{-+})$

$\pi(1300)$	$1^-(0^{-+})$
$a_2(1320)$	$1^-(2^{++})$
$f_0(1370)$	$0^+(0^{++})$
$\pi_1(1400)$	$1^-(1^{-+})$
$\eta(1405)$	$0^+(0^{-+})$
$h_1(1415)$ was $h_1(1380)$	$0^-(1^{+-})$
$f_1(1420)$	$0^+(1^{++})$
$\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
$a_0(1450)$	$1^-(0^{++})$
$\rho(1450)$	$1^+(1^{--})$
$\eta(1475)$	$0^+(0^{-+})$
$f_0(1500)$	$0^+(0^{++})$

…

how do we study resonances?

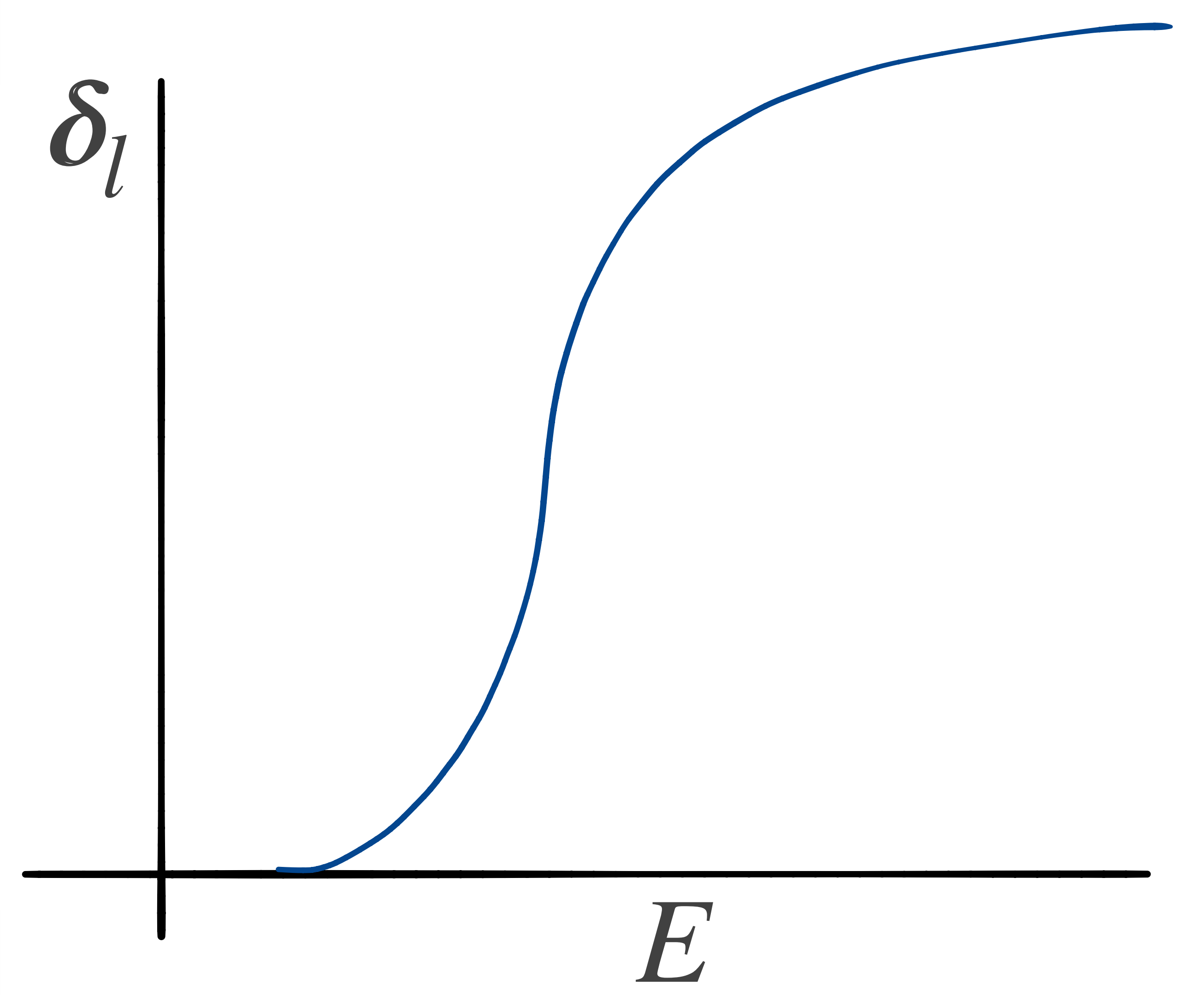
unitary!
$1 = SS^\dagger$

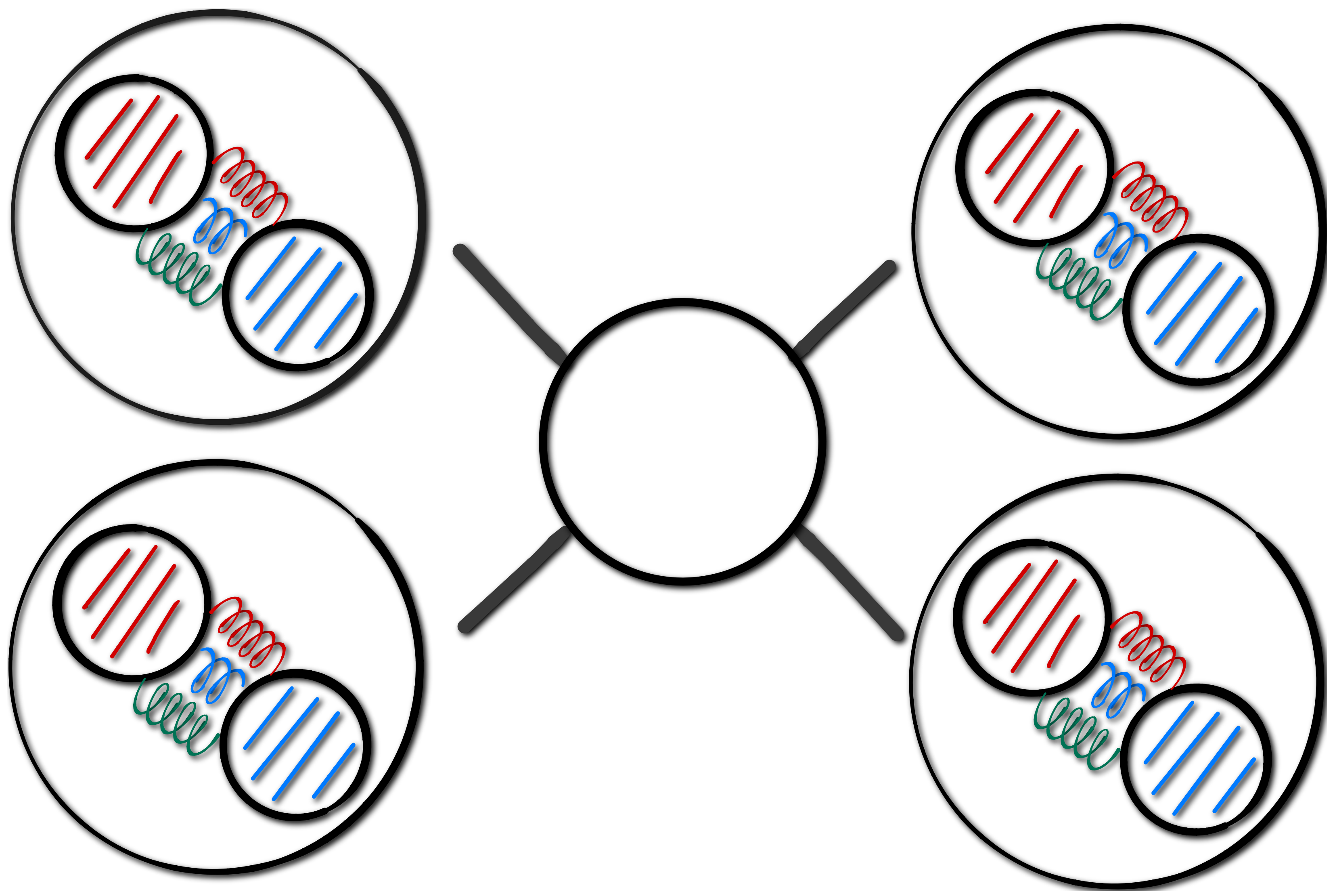
$$S = \langle out | \hat{S} | in \rangle$$

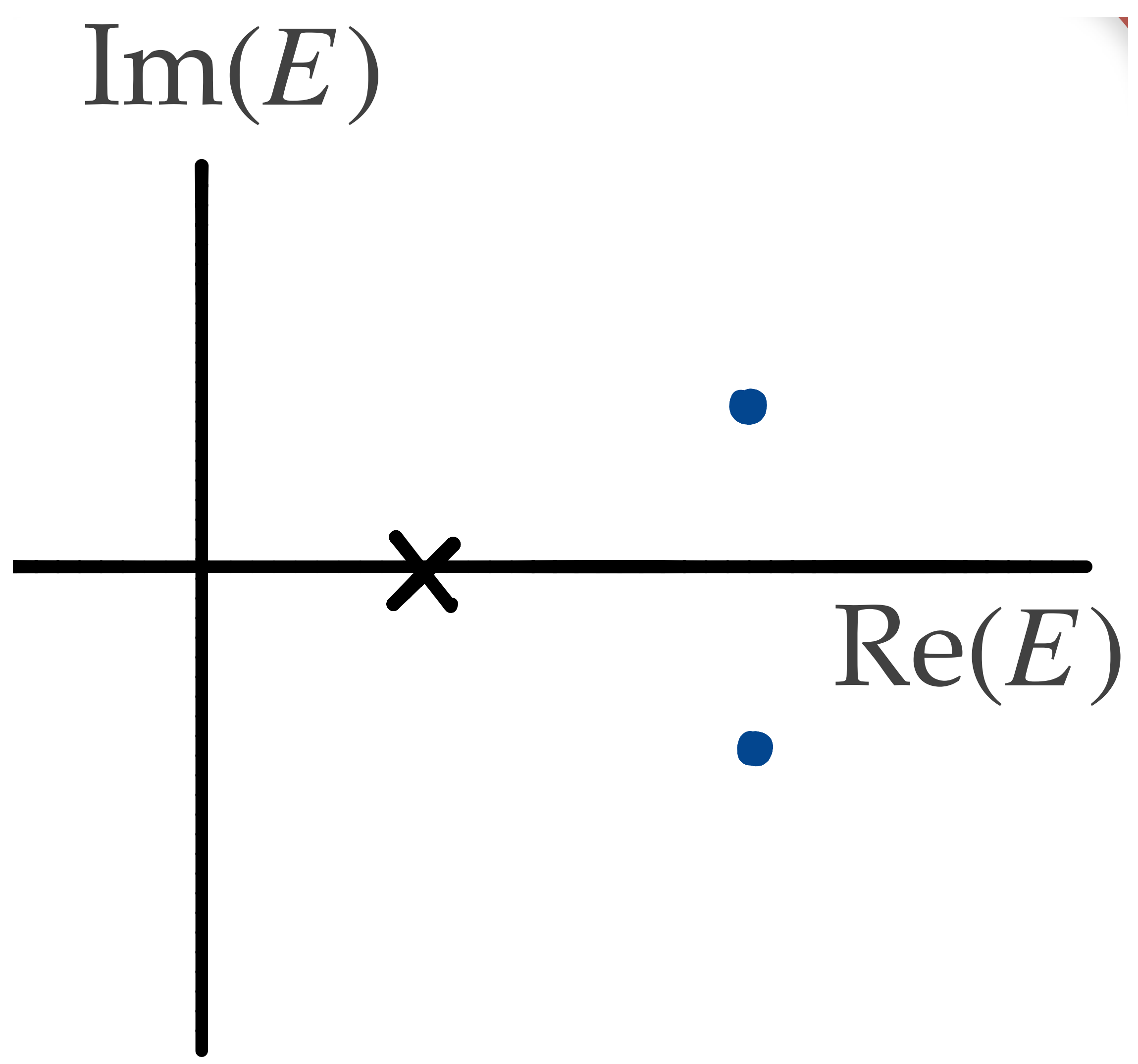
$$S = 1 + 2iT$$

$$T \propto \frac{1}{\cot\delta - i}$$

analytic!
(poles and residues)







and resonances on the lattice?

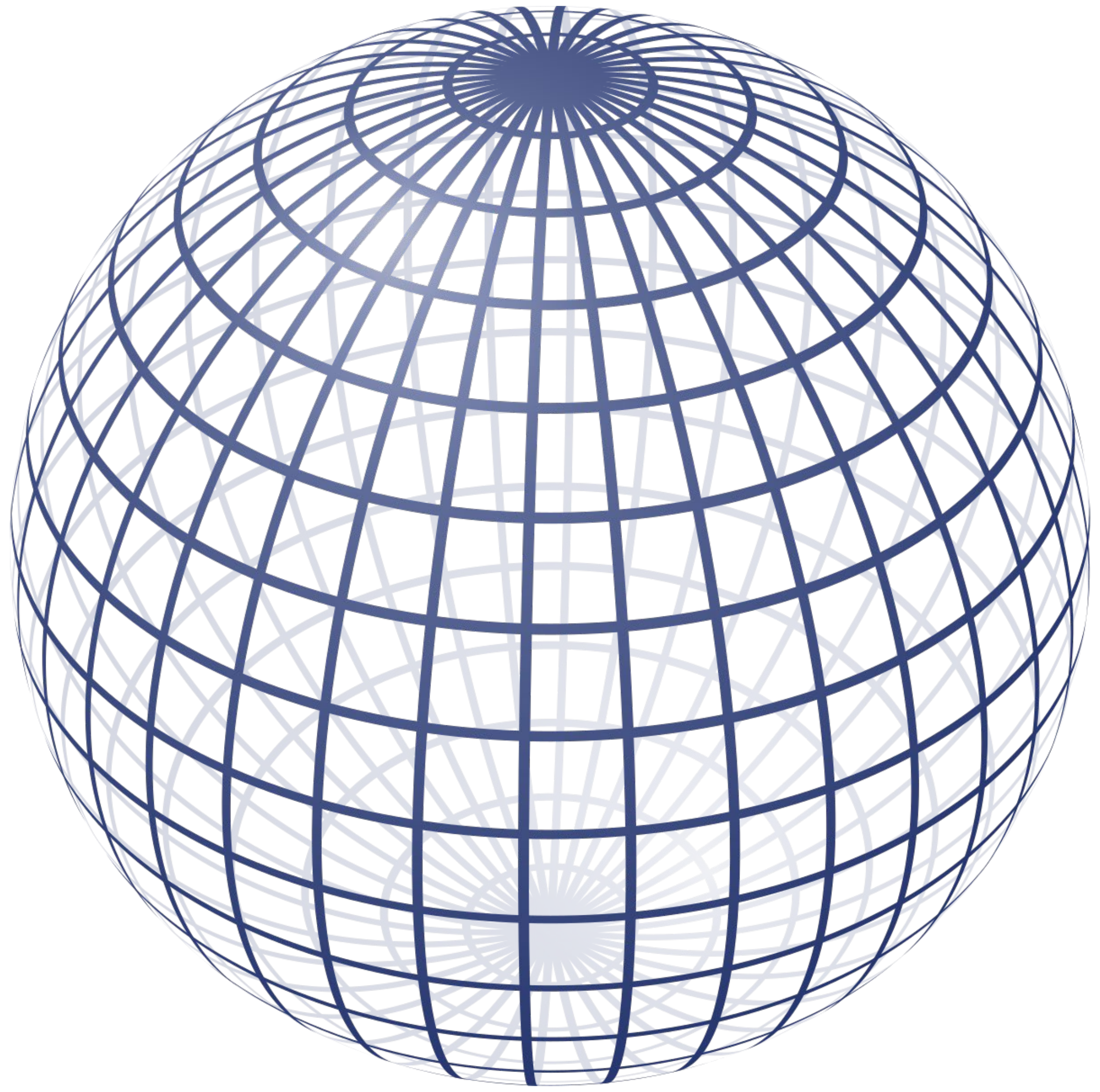
infinite volume:

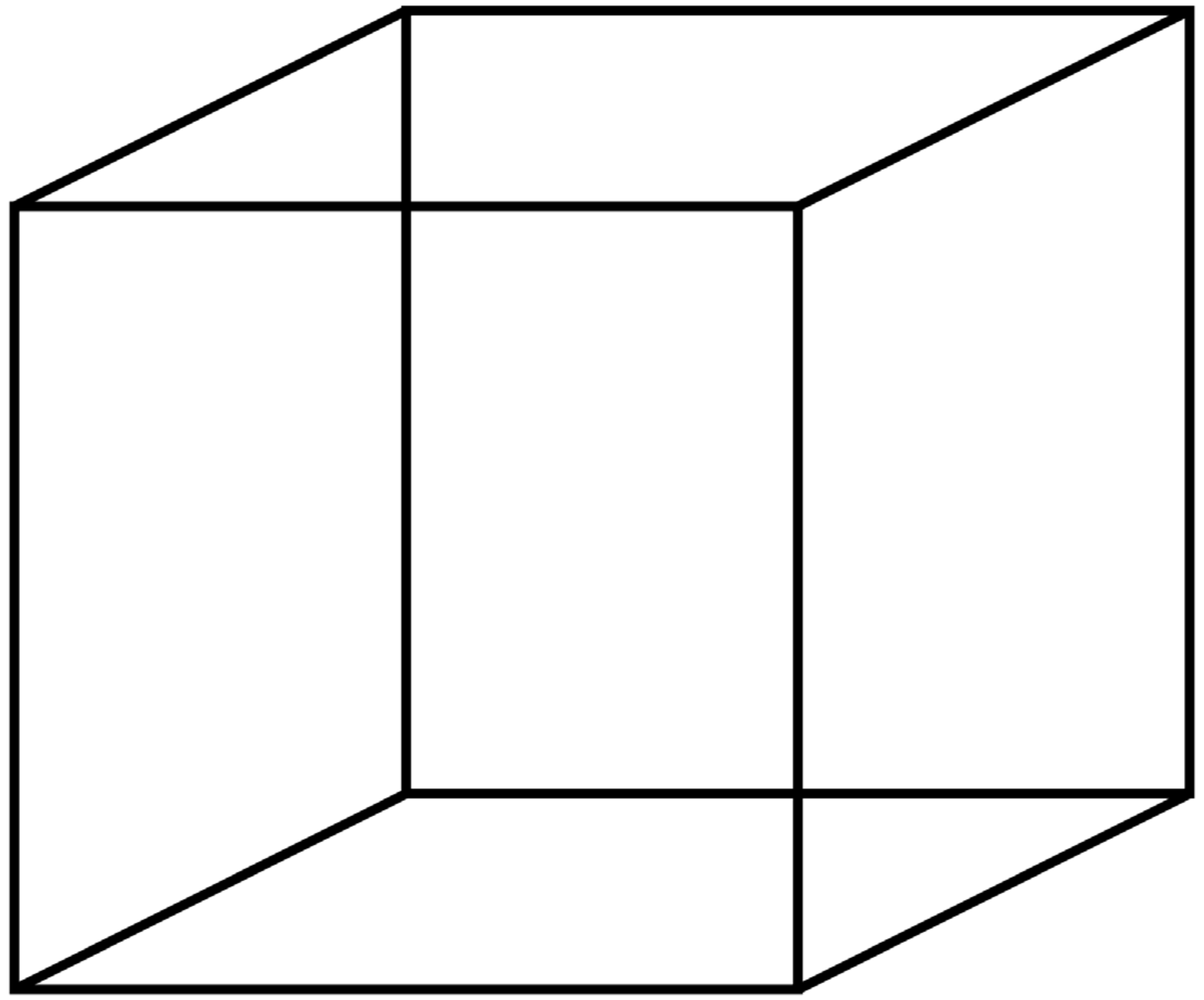
- $O(3)$ symmetry
- infinite irreps (J^P)

many-to-one mapping

finite volume:

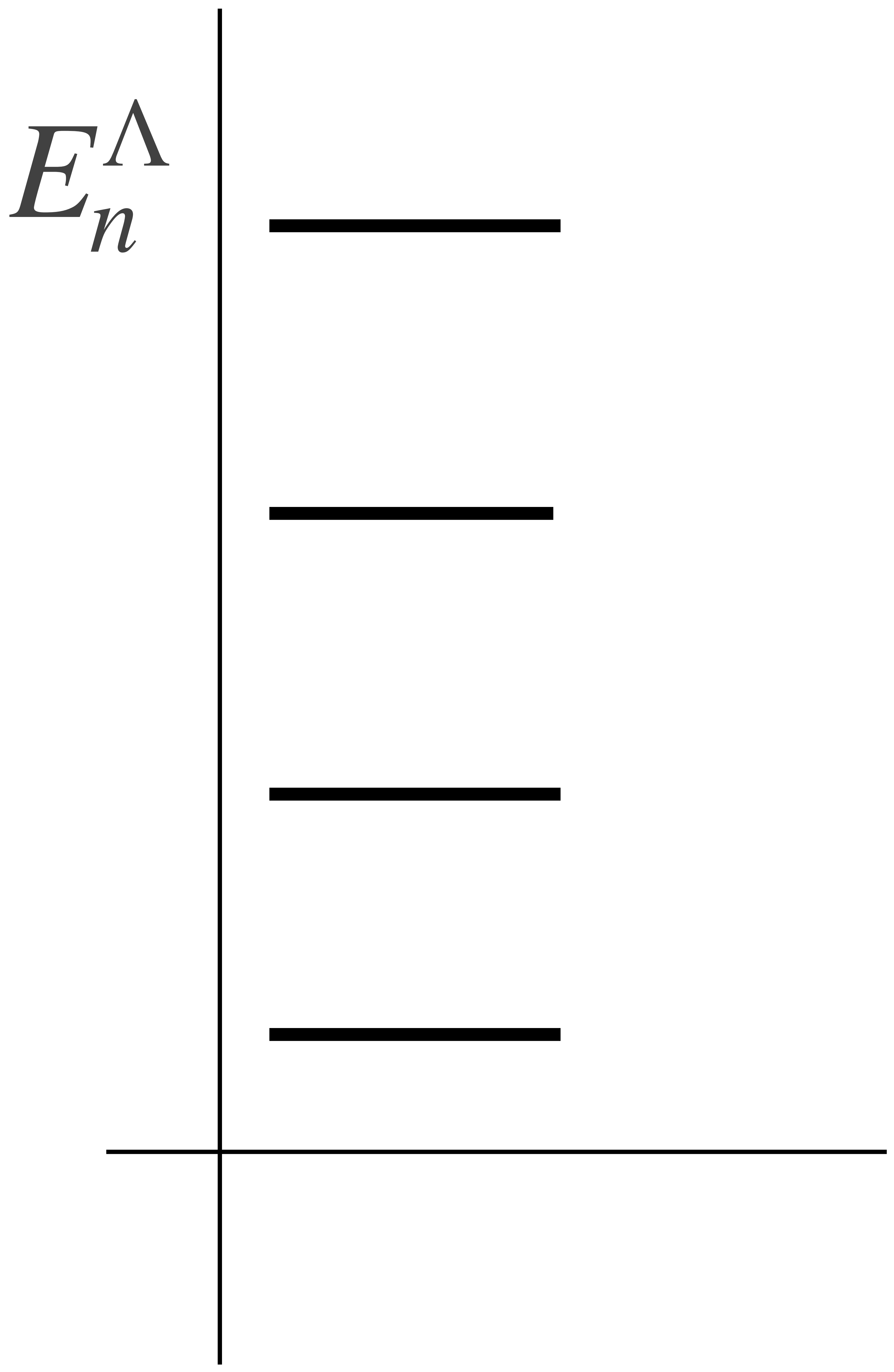
- O_h symmetry
- 10 irreps Λ





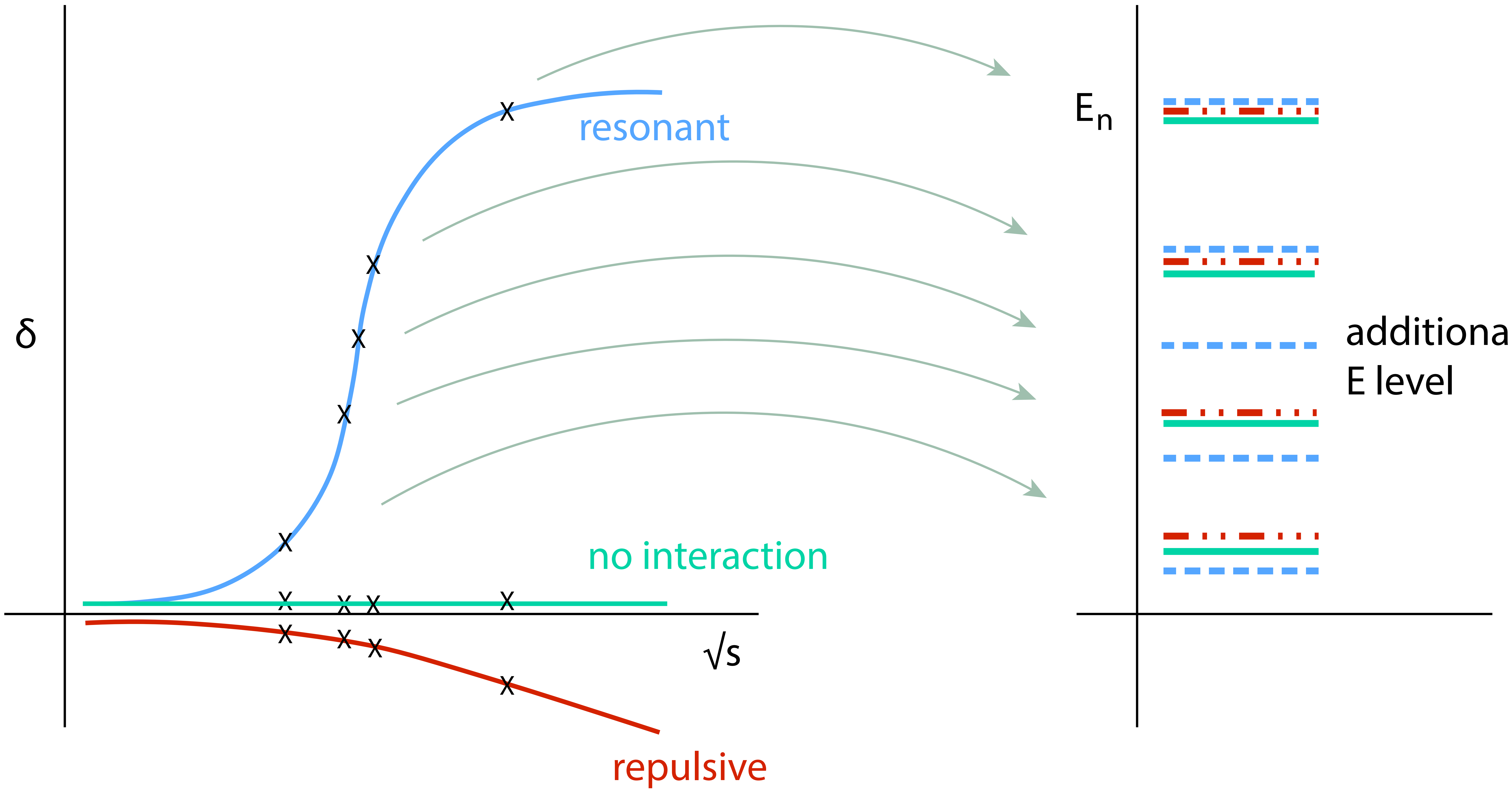
spectrum in a finite volume

- QCD
- flavor quantum numbers
- irreducible representation



and resonances on the lattice?

$$\det\left[F^{-1}(E) + T(E)\right]\Big|_{E=E_n} = 0$$



and resonances on the lattice

$$\det\left[F^{-1}(E)+T(E)\right]\Big|_{E=E_n} = 0$$

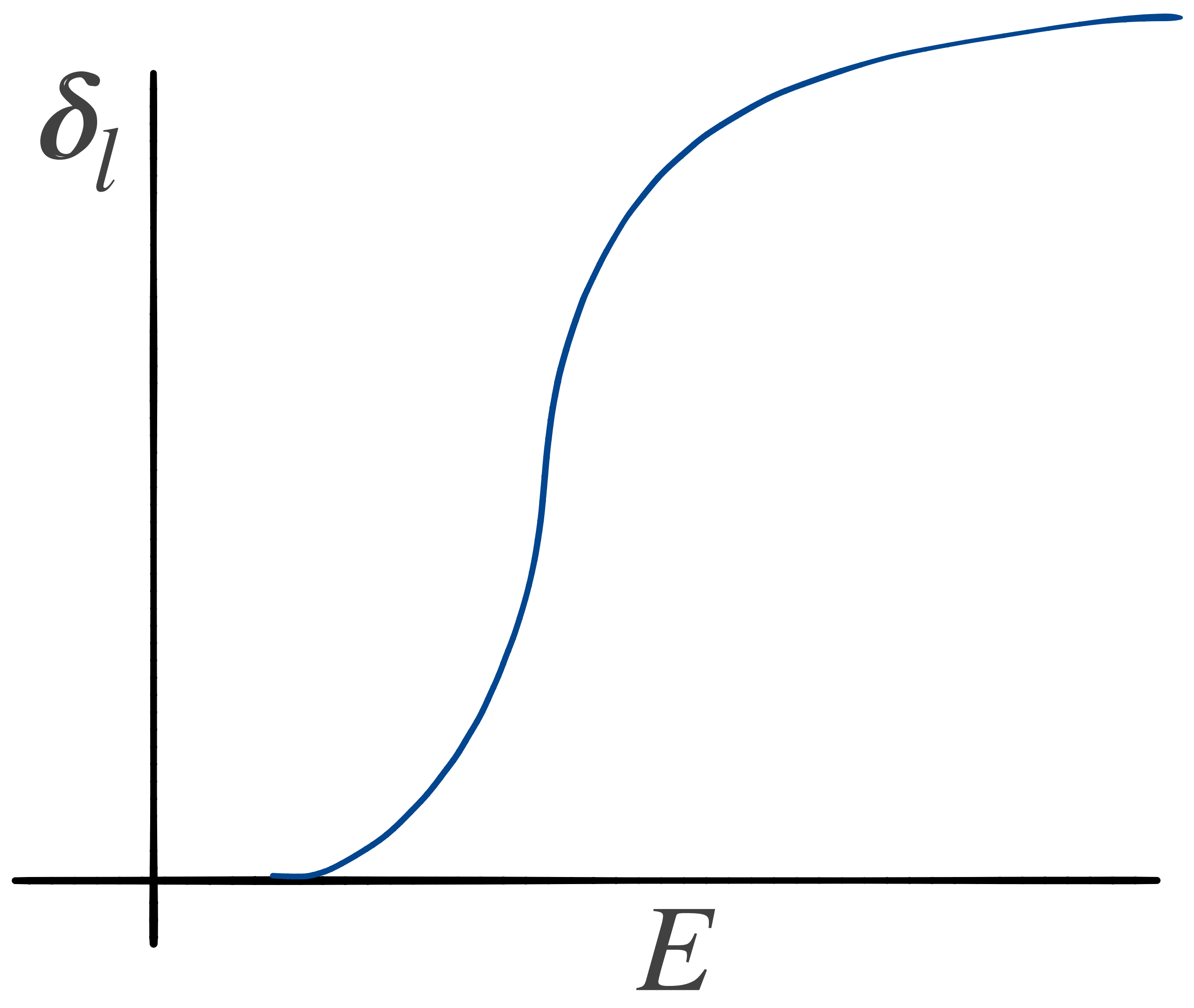
$$F_{lM,l'M'}(E) = \frac{ik}{8\pi E}\left[\delta_{MM'}\delta_{ll'} + i\sum_{\bar{l},\bar{m}}\sqrt{\frac{(2l+1)(2\bar{l}+1)}{4\pi(2l'+1)}}\langle lM,\bar{l}\bar{m}_l\,|\,l'M'\rangle\langle l0,\bar{l}0\,|\,l'0\rangle\,\frac{(4\pi)^2}{\gamma L k^{\bar{l}+1}}\left(\frac{2\pi}{L}\right)^{\bar{l}-2} Z_{\bar{l}\bar{m}}(k^2)\right]$$

Clebsch-Gordan coefficients

the Lüscher Zeta function

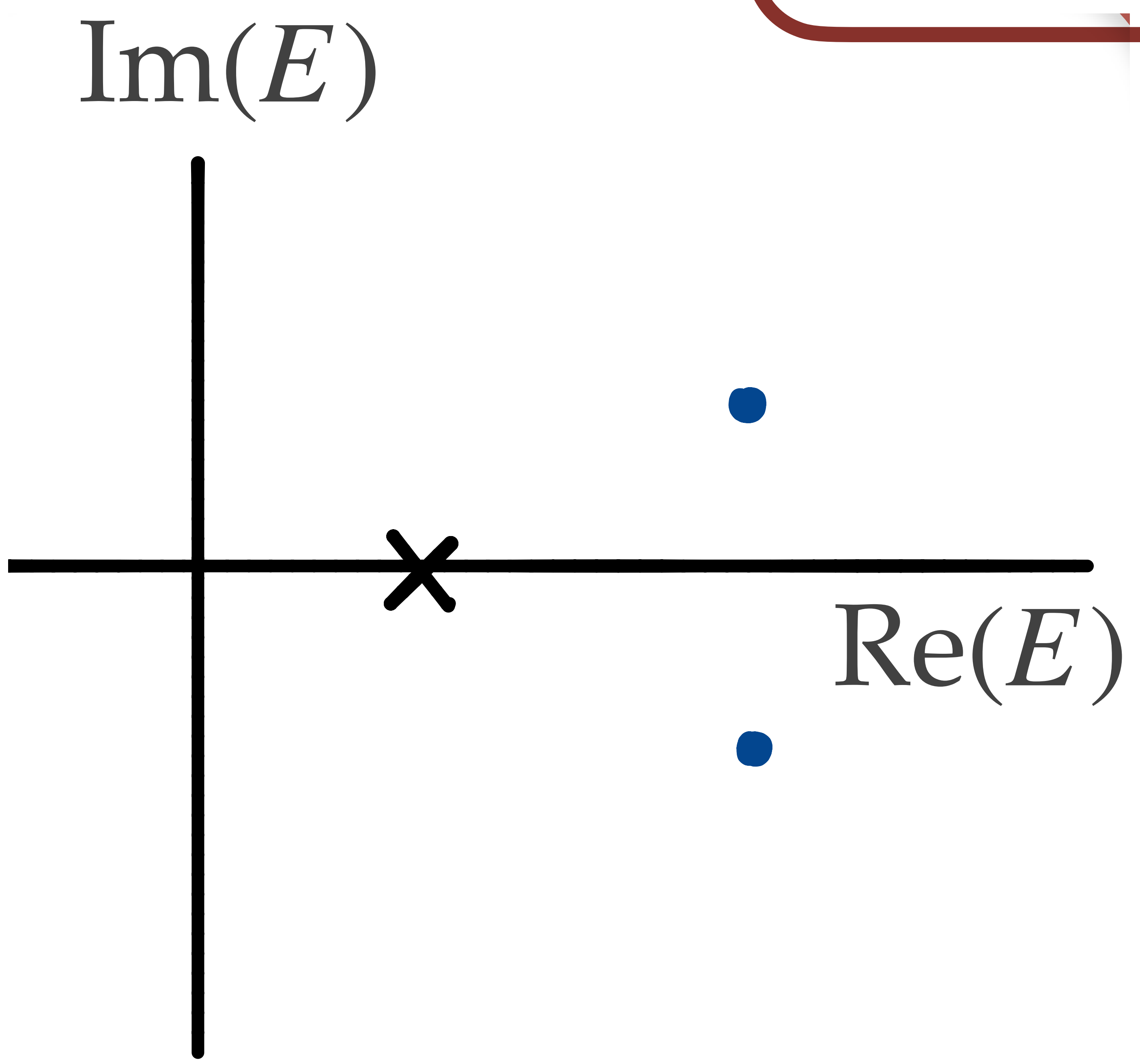
Briceño et al. 2017 (but really a whole bunch of people)

and resonances on the lattice

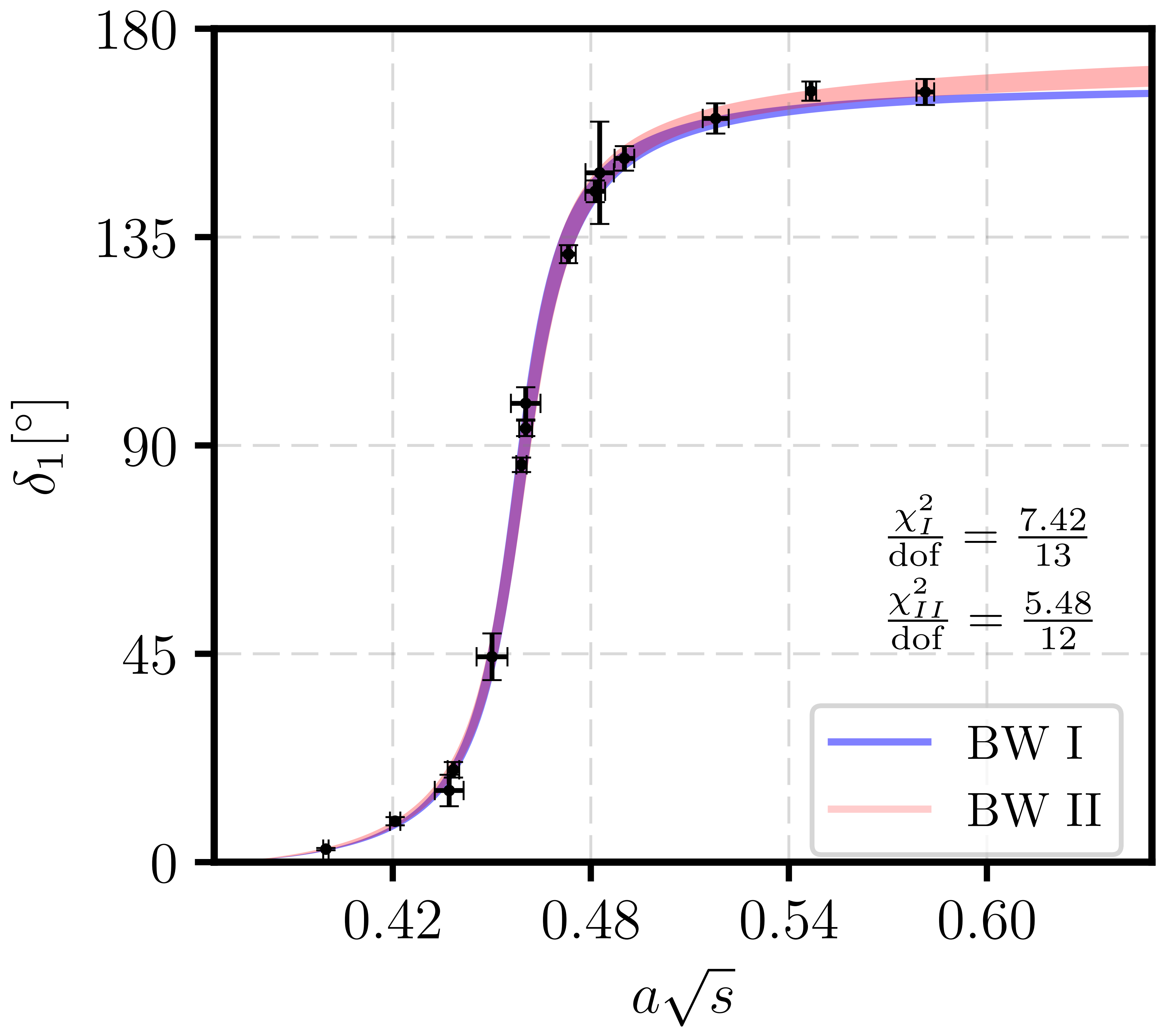


$$\det\left[F^{-1}(E)+T(E)\right]\Big|_{E=E_n} = 0$$

$$T_{lM,l'M'}(E) = \delta_{ll'}\delta_{MM'}\frac{8\pi E}{ik}\frac{1}{\cot\delta_l(E) - i}$$



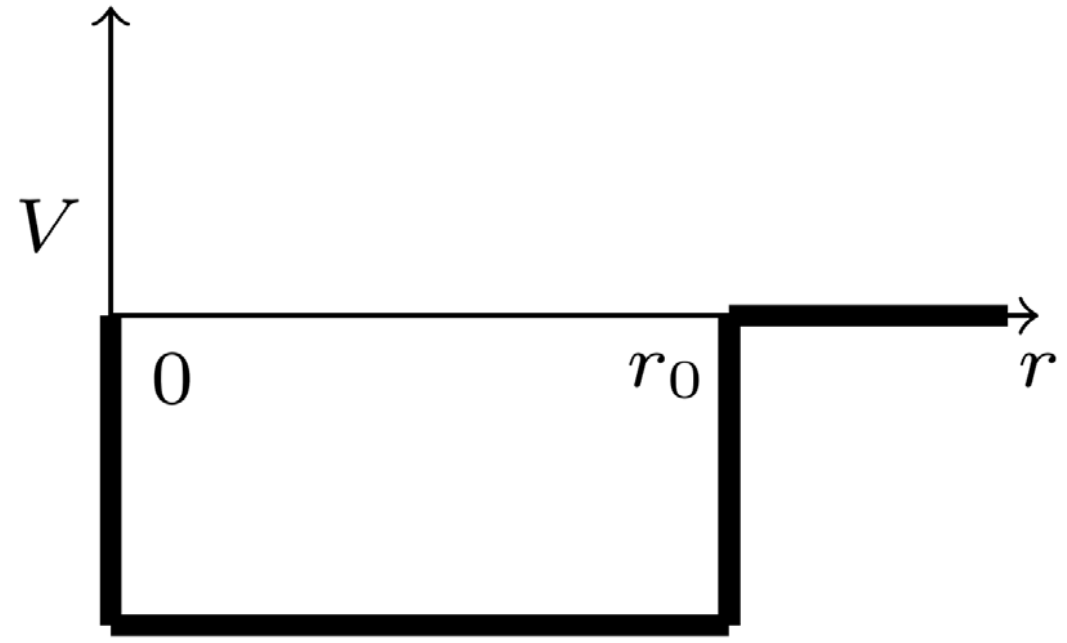
the $\rho(770)$ resonance



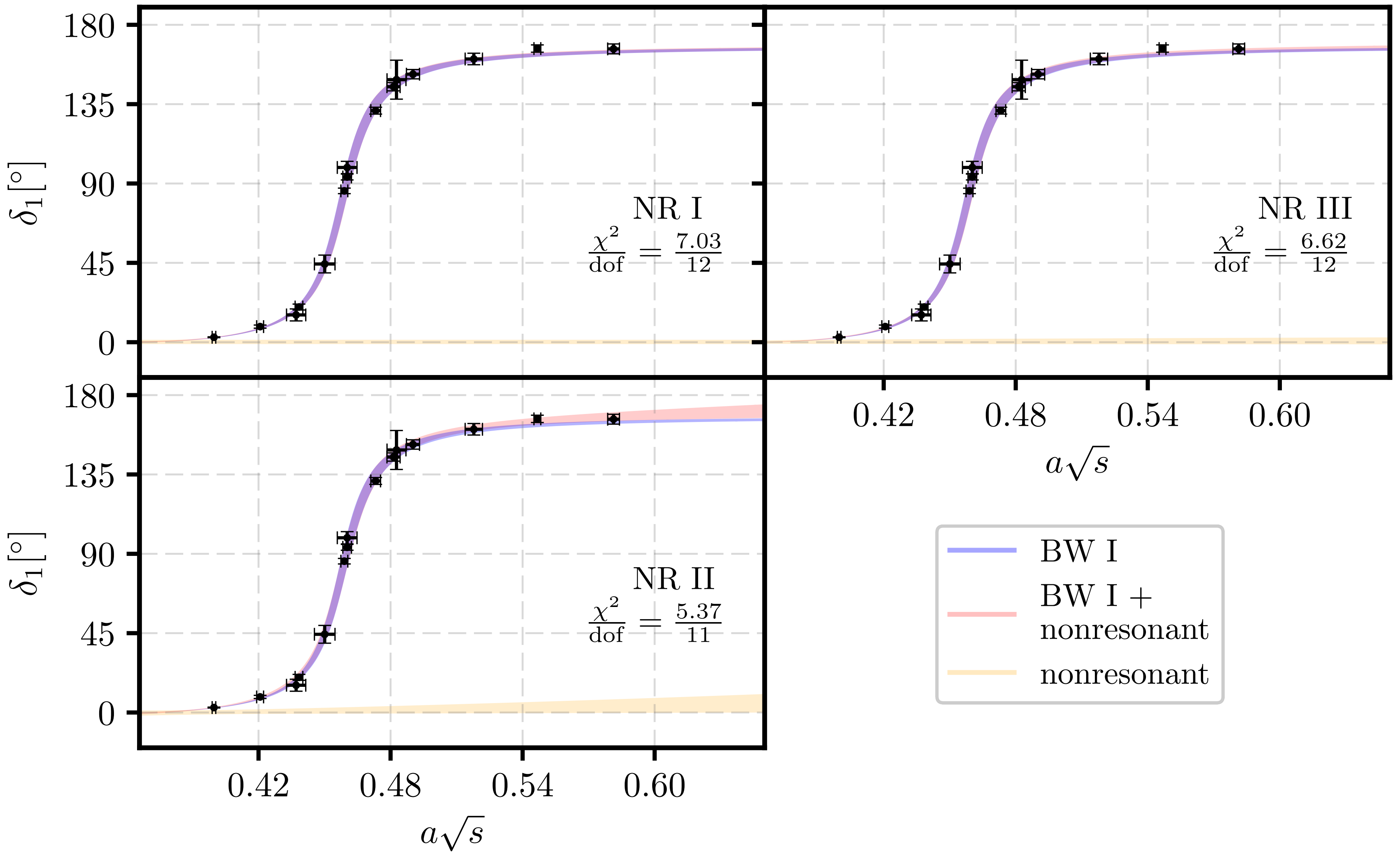
$m_\pi \approx 320$ MeV
$L \approx 3.6$ fm
$a \approx 0.11$ fm

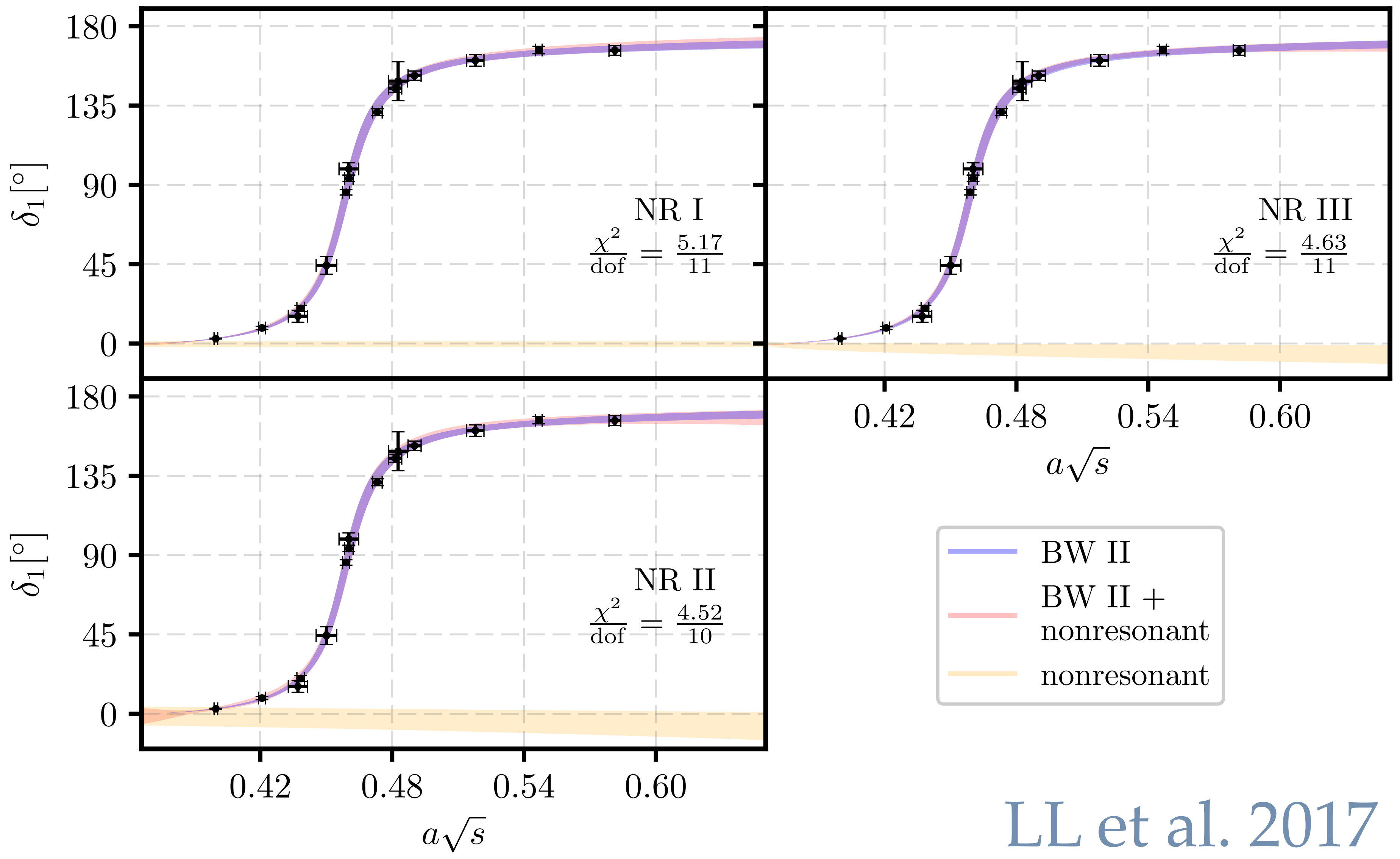
$$T = \frac{\sqrt{s}\Gamma_i}{m_R^2 - s - i\sqrt{s}\Gamma_i}$$

$$\Gamma_I = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{p^3}{s}$$



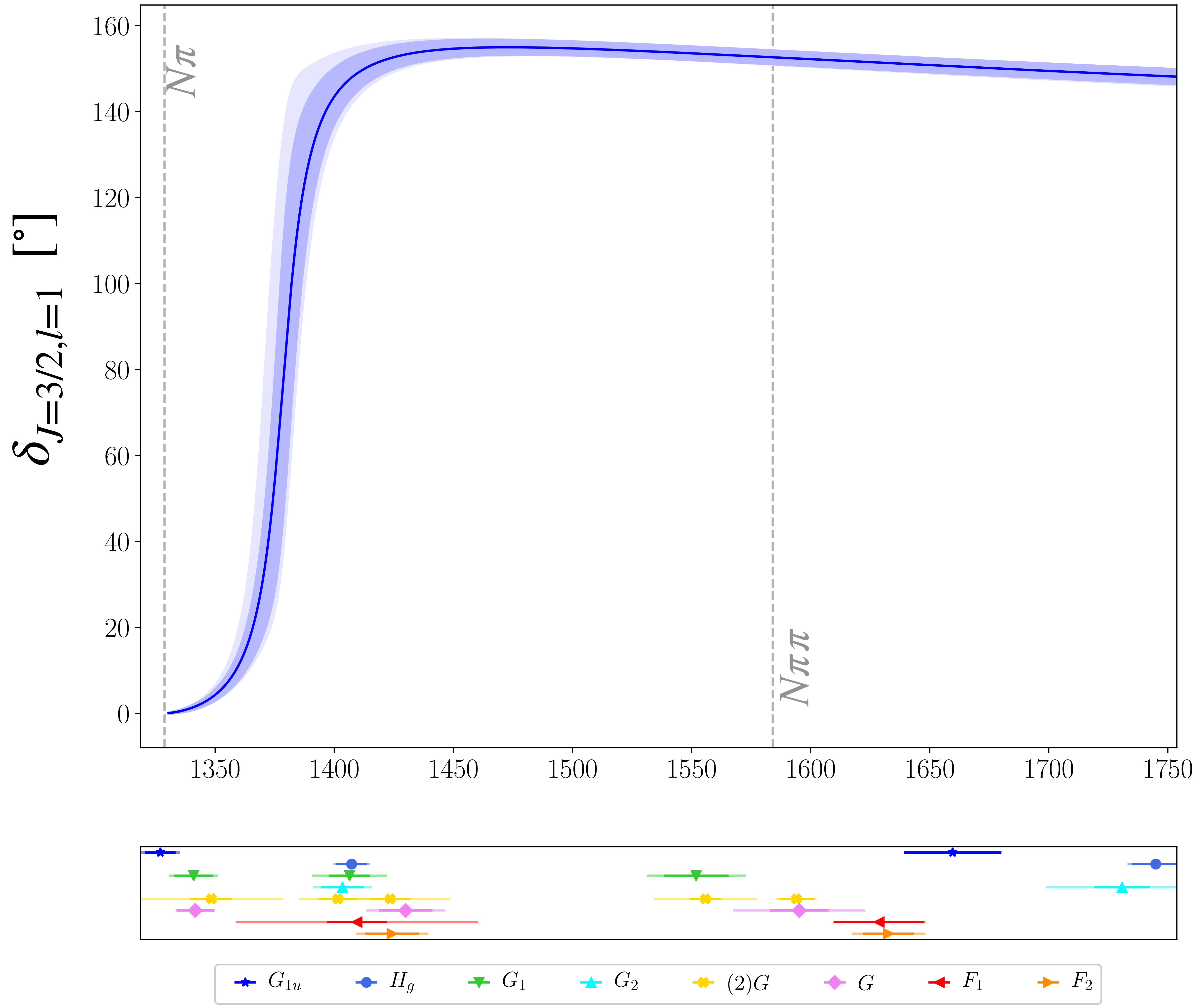
$$\Gamma_{II} = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{p^3}{s}\frac{1+(k_R r_0)^2}{1+(kr_0)^2}$$





LL et al. 2017

the Δ(1232) resonance



amplitude pole position

$$m_\Delta = (1378.3 \pm 6.6 \pm 9.0)\ \text{MeV}$$

$$\Gamma/2 = (8.2 \pm 1.0 \pm 1.4)\ \text{MeV}$$

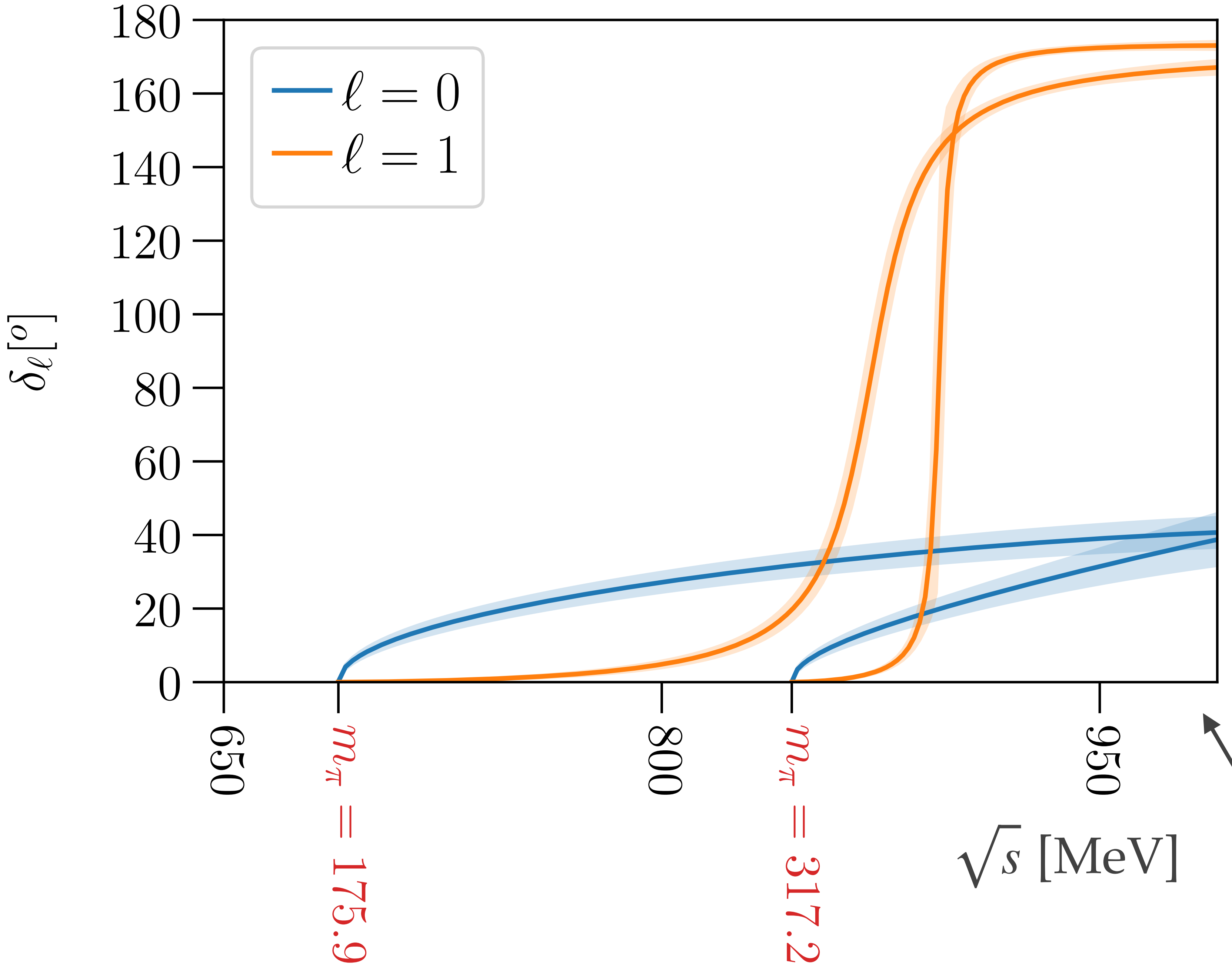
$m_\pi \approx 250$ MeV
$L \approx 2.8$ fm
$a \approx 0.11$ fm

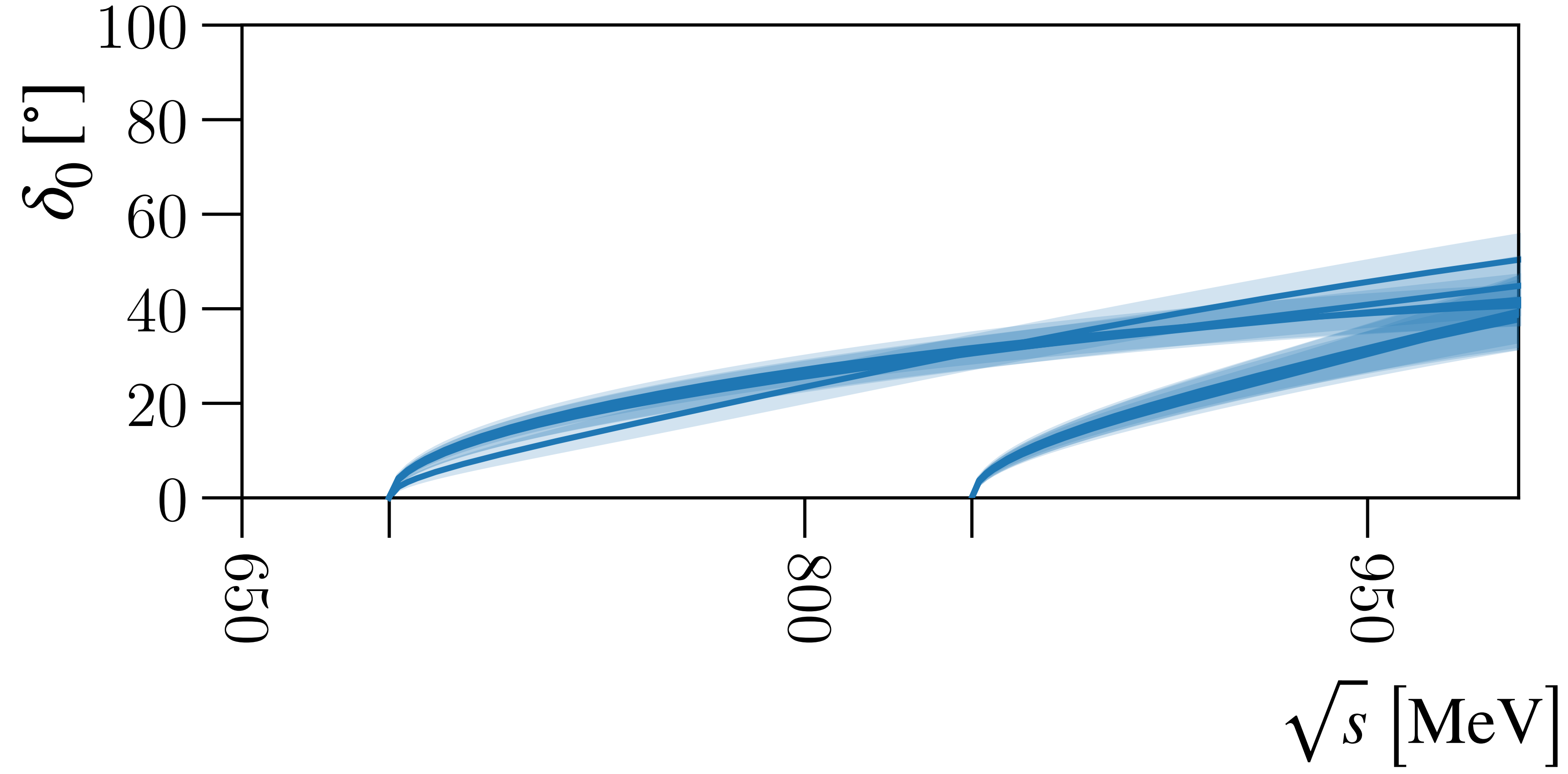
LL et al. 2021

the $K_0^\star(700)$ and $K^\star(892)$ resonances

$T_l^{-1} = K_l^{-1} - \Theta(s - s_{thr})\,\delta_{ij}$

- $K^\star(892)$:
  - Breit-Wigner
- $K_0^\star(700)$
  - 4 different K parameterizations
  - 2 with Adler zero
  - 2 without



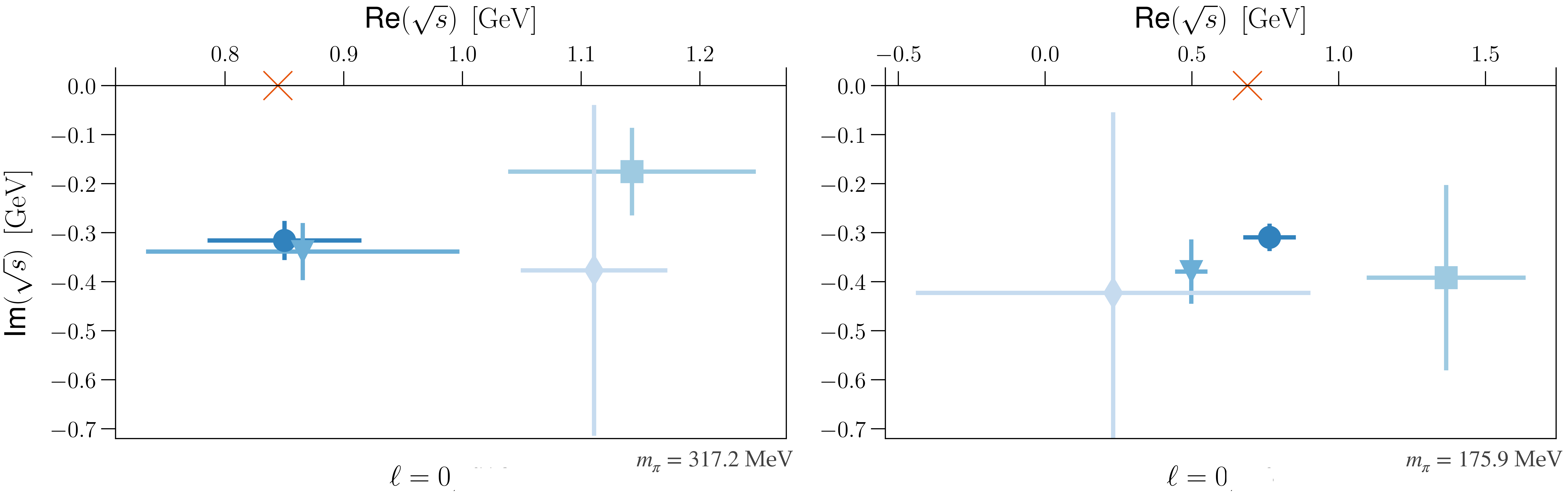


$L \approx 3.6, 4.2$ fm

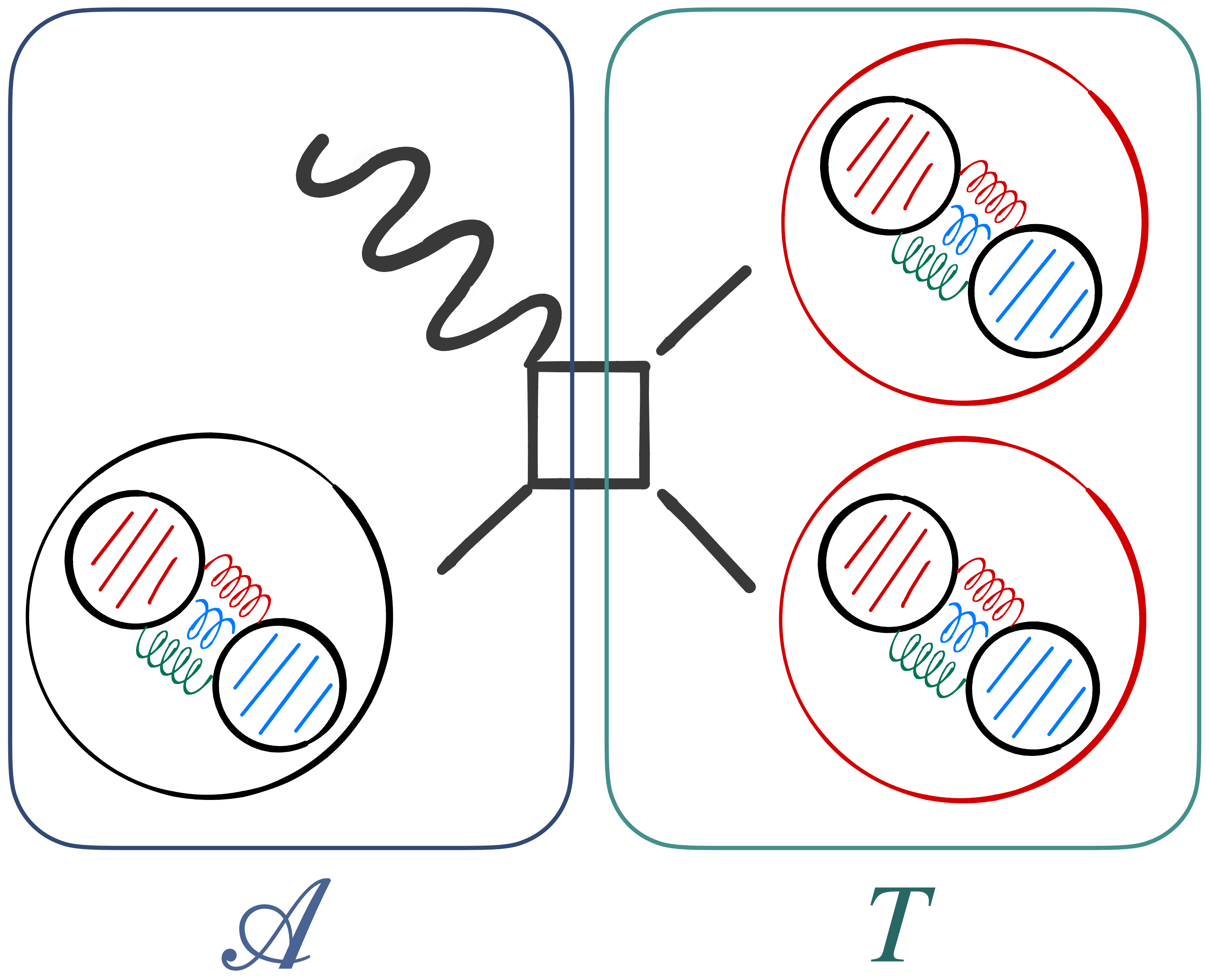
$a \approx 0.11, 0.088$ fm

LL et al. 2020

the $K_0^\star(700)$ and $K^\star(892)$ resonances



how do we study transitions?



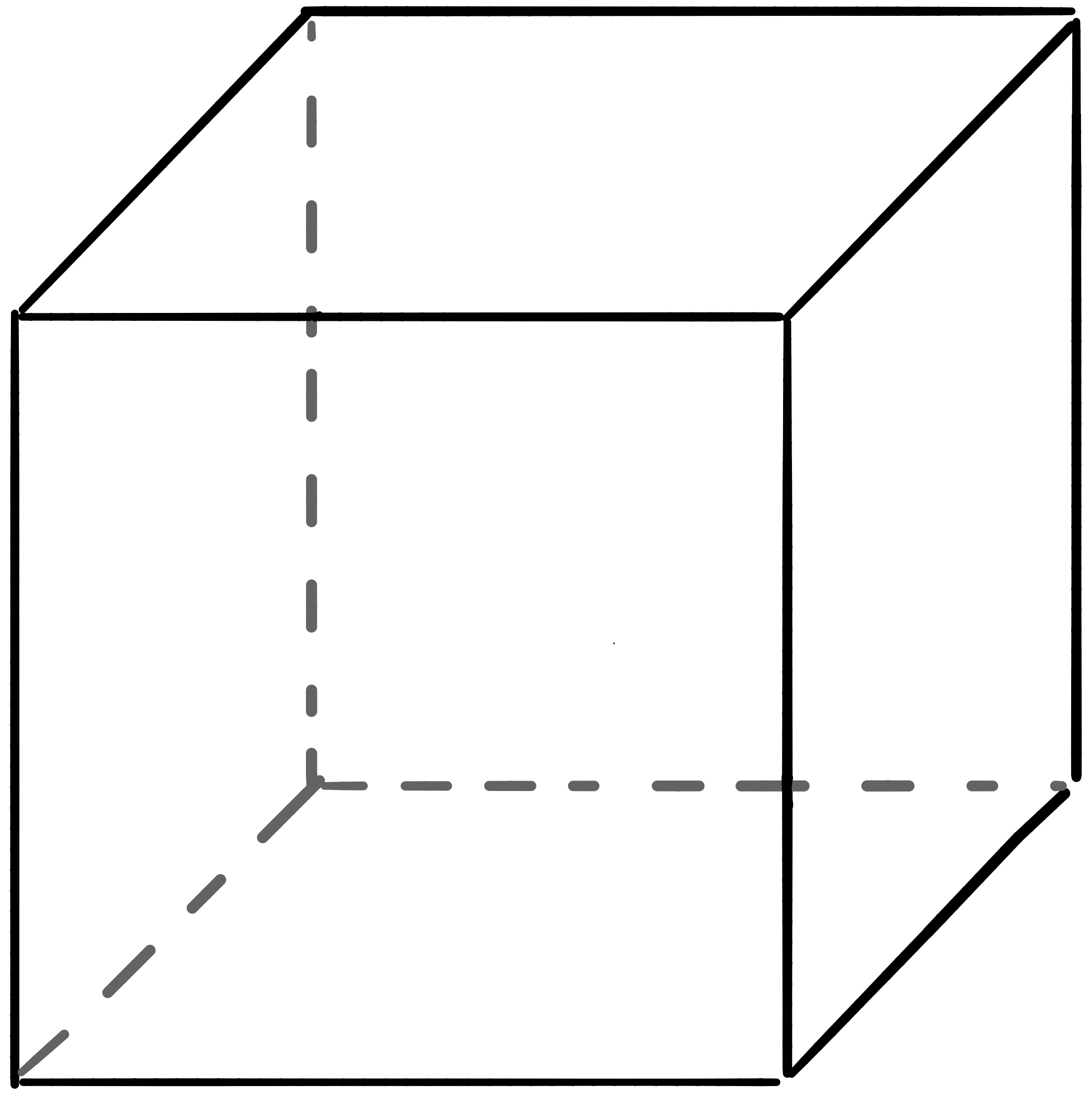
unitarity!

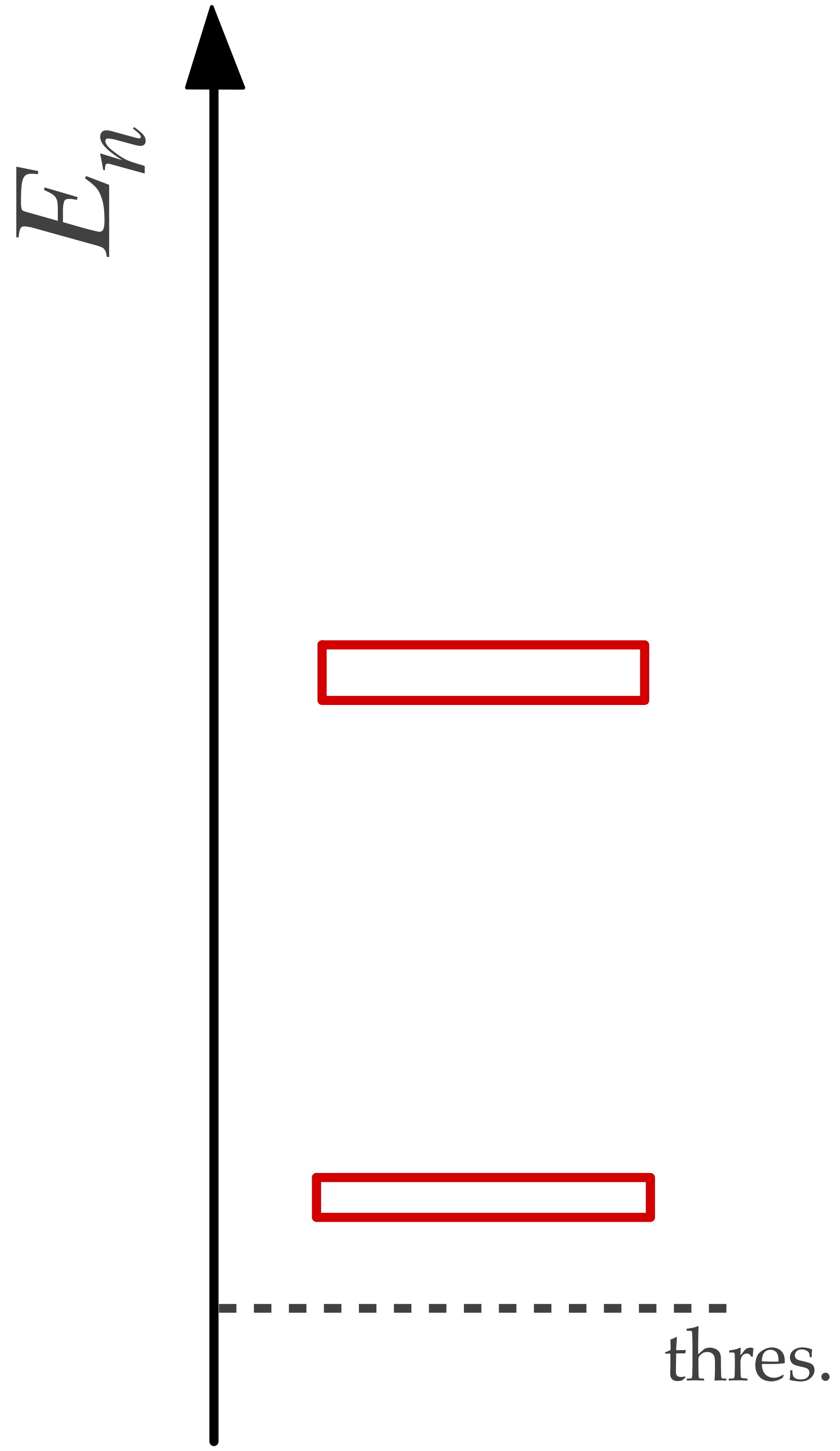
analyticity!

$$\mathscr{H}(q^2, E) = \frac{\mathscr{A}(q^2, E) \cdot T_l(E)}{k^l}$$

fixes threshold behavior mismatch!

transitions on the lattice?

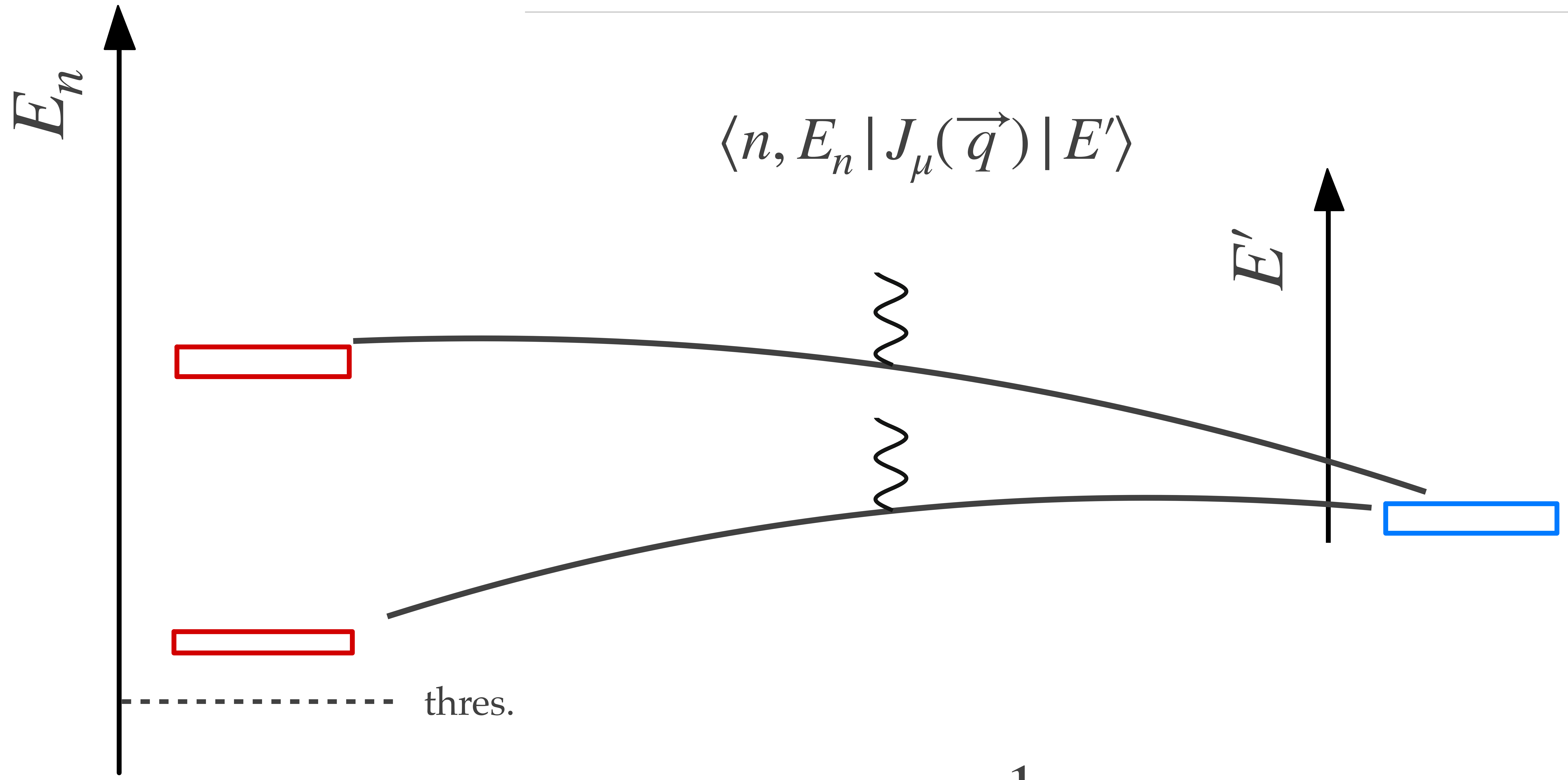




$$|E_n\rangle_L \sim \sqrt{\mathcal{R}_n}\,|\varphi\varphi(E = E_n)\rangle_\infty$$

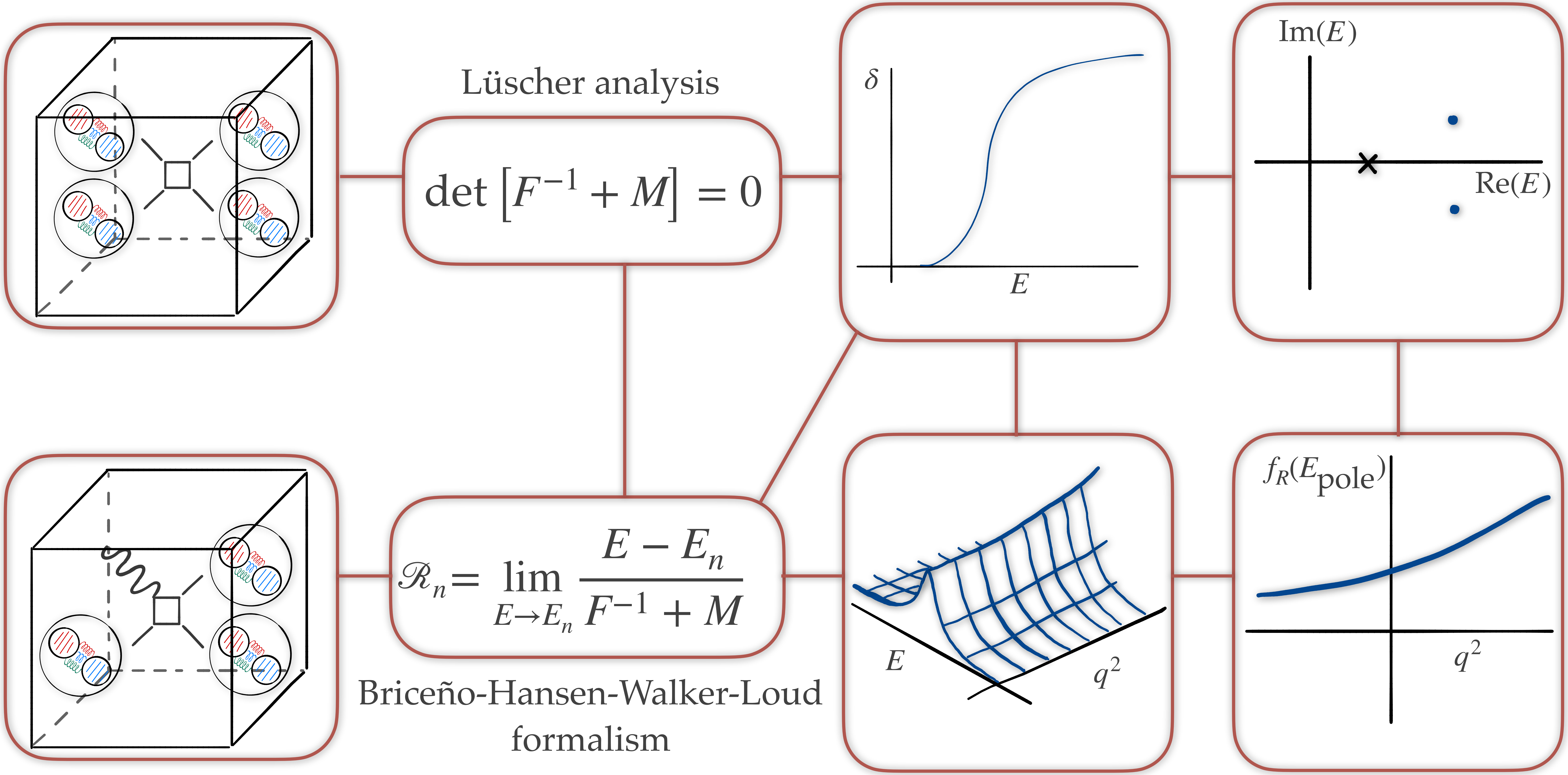
$$\mathcal{R}_n = \lim_{E \to E_n} \frac{E - E_n}{F^{-1} + T}$$

transitions on the lattice?

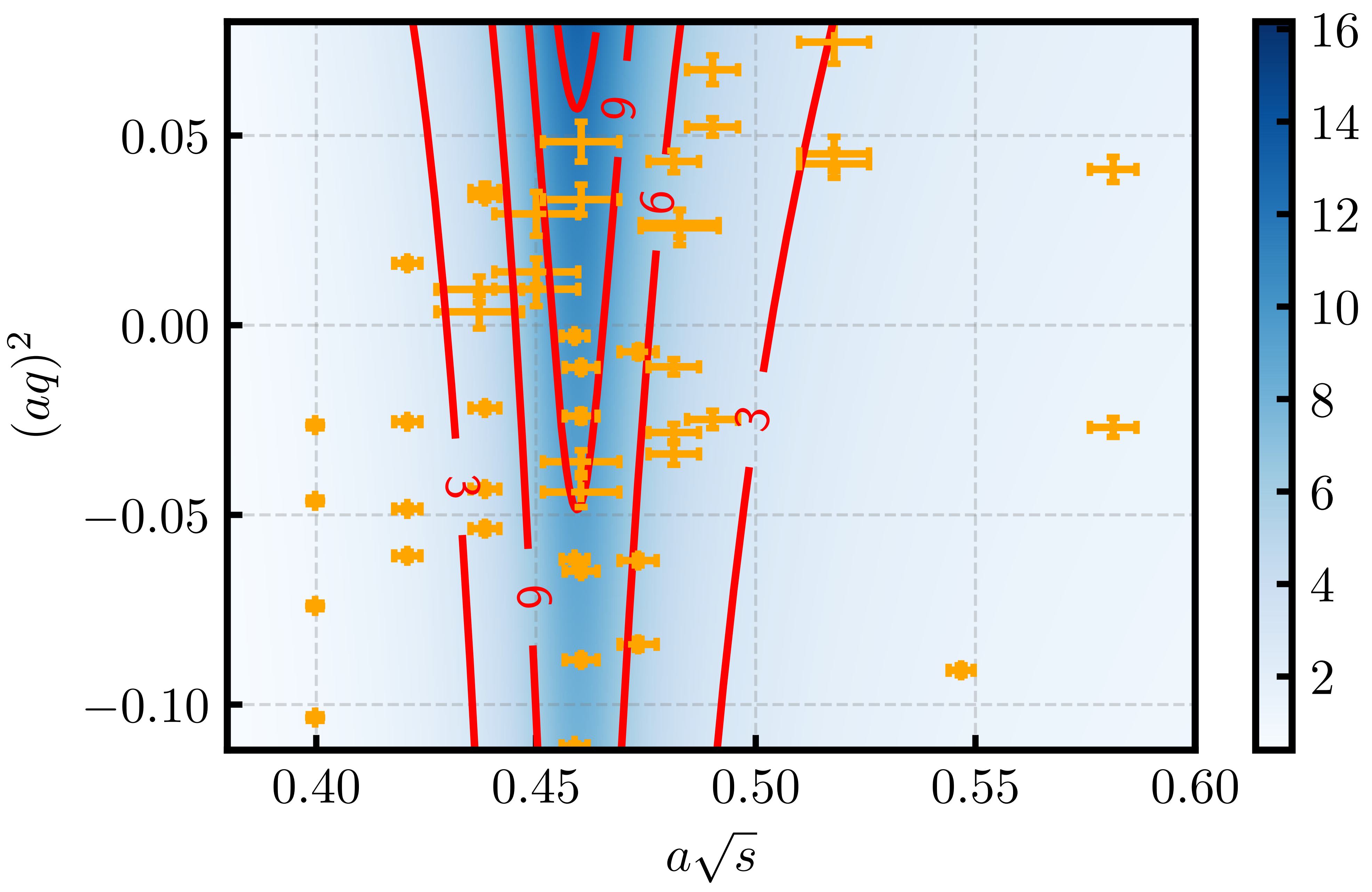


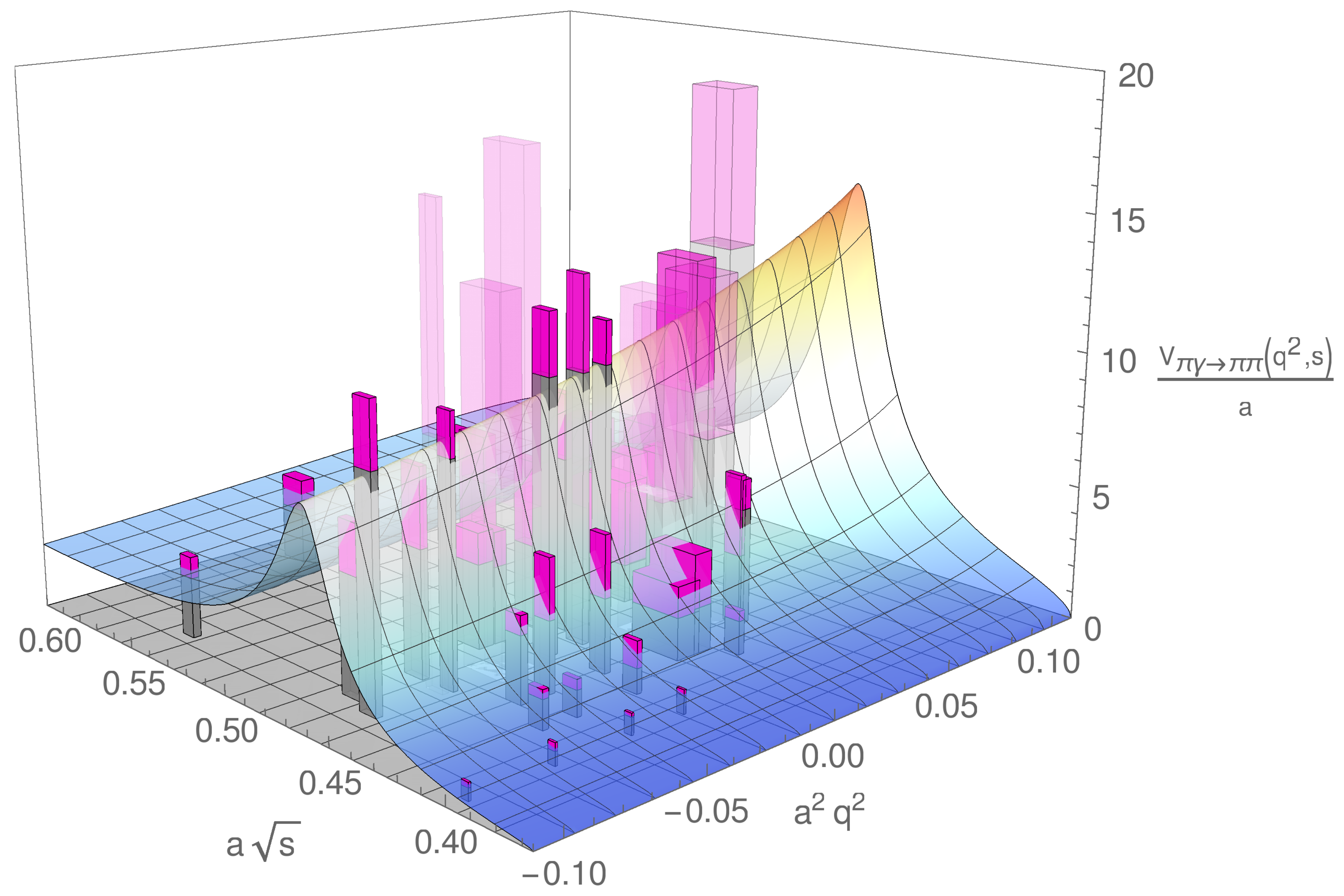
$$|\langle n, E_n | J_\mu(\vec{q}) | E' \rangle| = \frac{1}{L^3\, 2E_n\, 2E'} \left[\mathcal{H}\, \mathcal{R}_n \mathcal{H}\right]^{1/2}$$

transitions on the lattice?



proof of concept: $\pi\gamma \to \pi\pi$



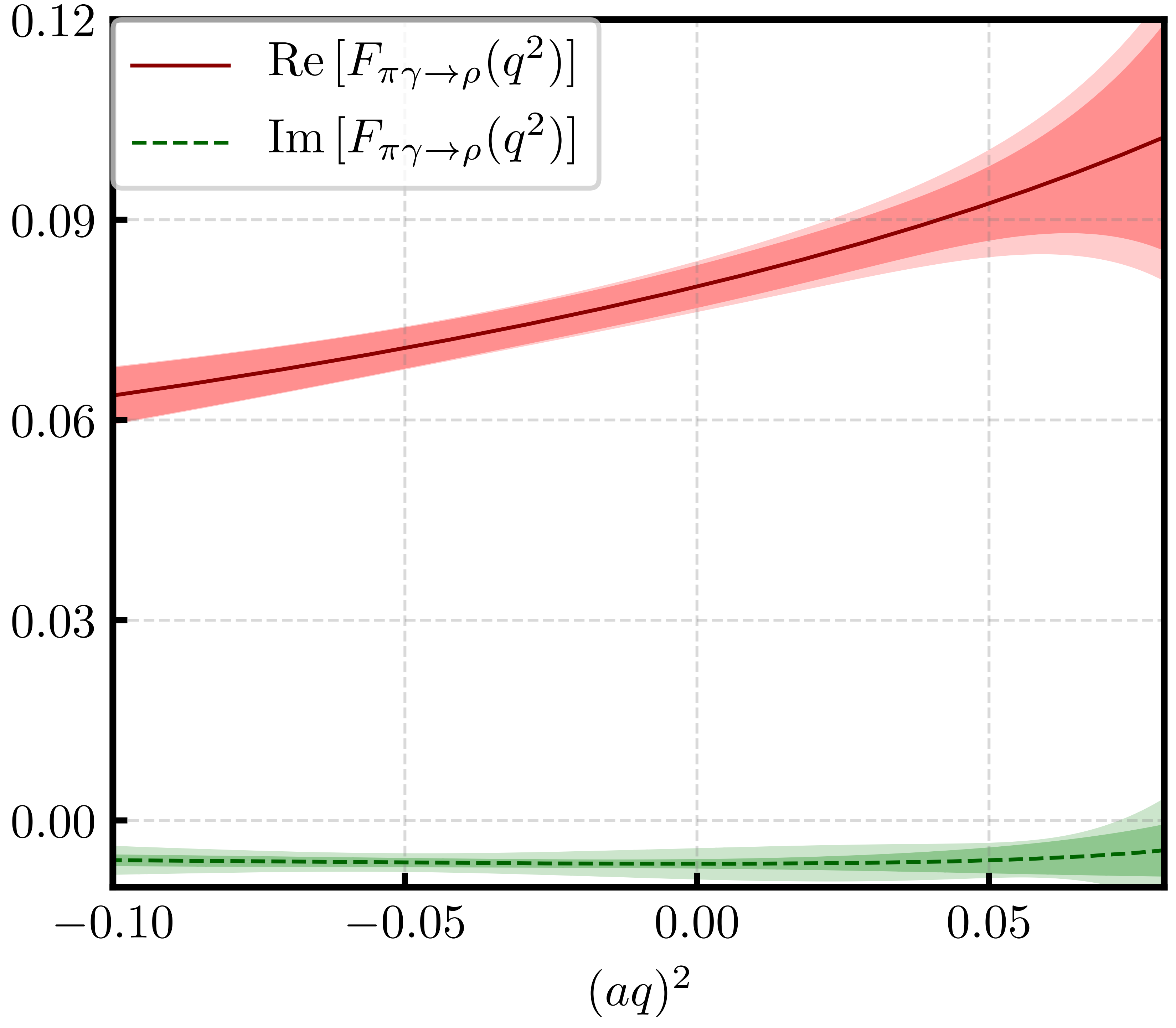


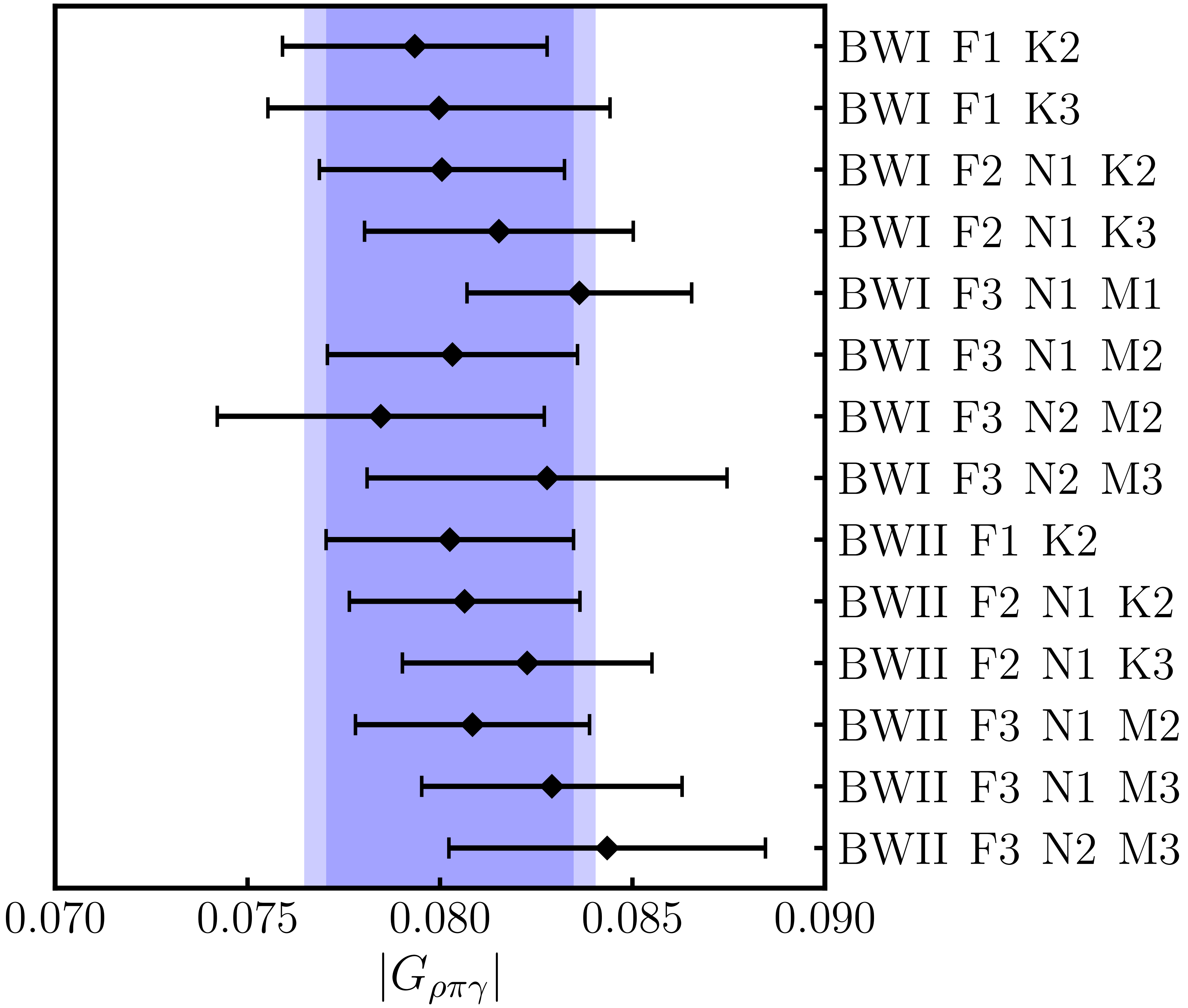
$$\mathscr{A}(q^2, s) = \frac{1}{1 - \frac{q^2}{m_P^2}} \sum_{n,m} A_{n,m}\, s^m\, z^n$$

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

LL et al. 2018

proof of concept: $\pi\gamma \to \pi\pi$





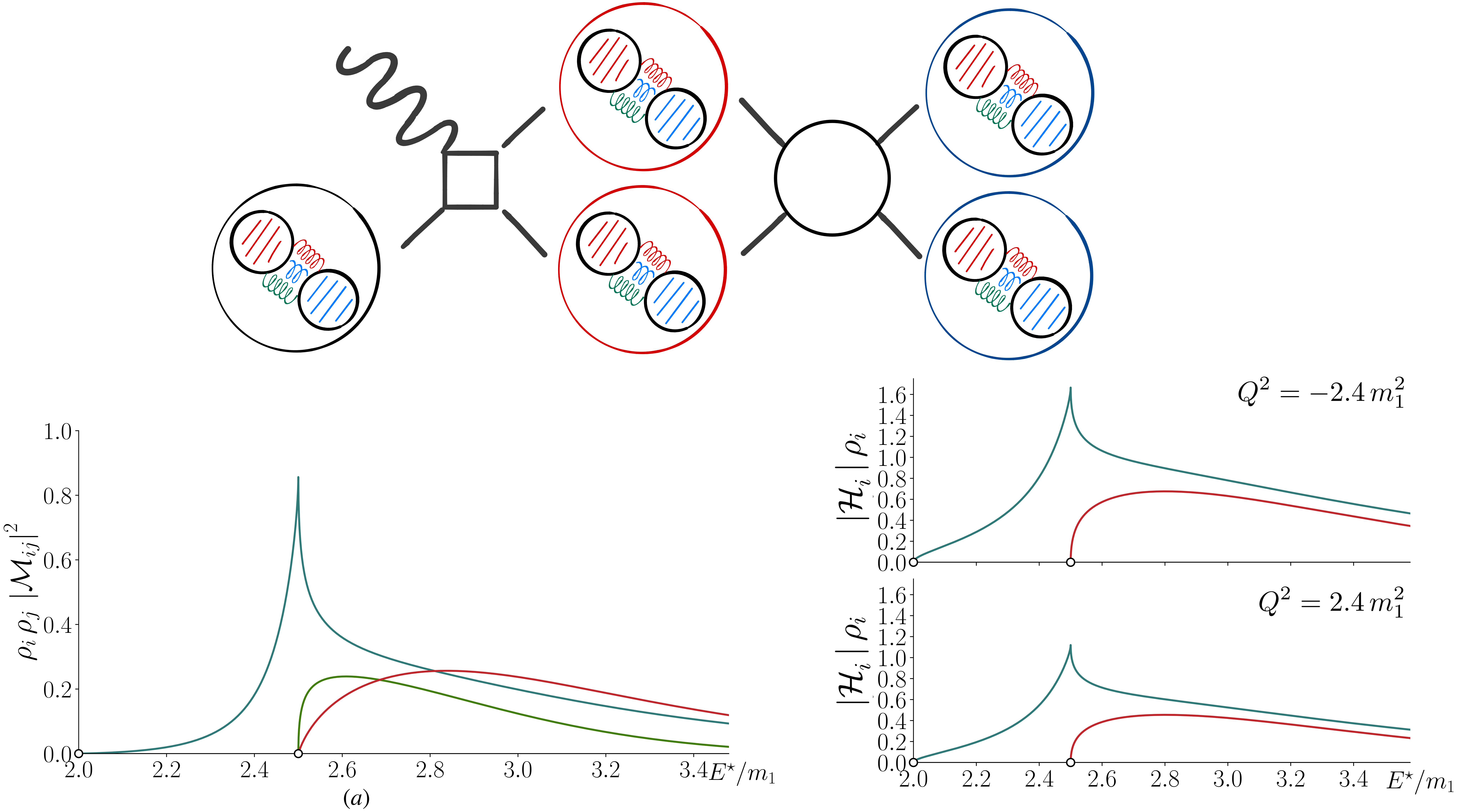
$$\mathscr{H} \approx \frac{G_{\rho\pi\pi} F_{\pi\gamma\to\rho}(q^2)}{s_P - s}$$

$$G_{\rho\pi\gamma} = F_{\pi\gamma\to\rho}(q^2 = 0)$$

$$|G_{\rho\pi\gamma}| = 0.0802(32)(20)$$

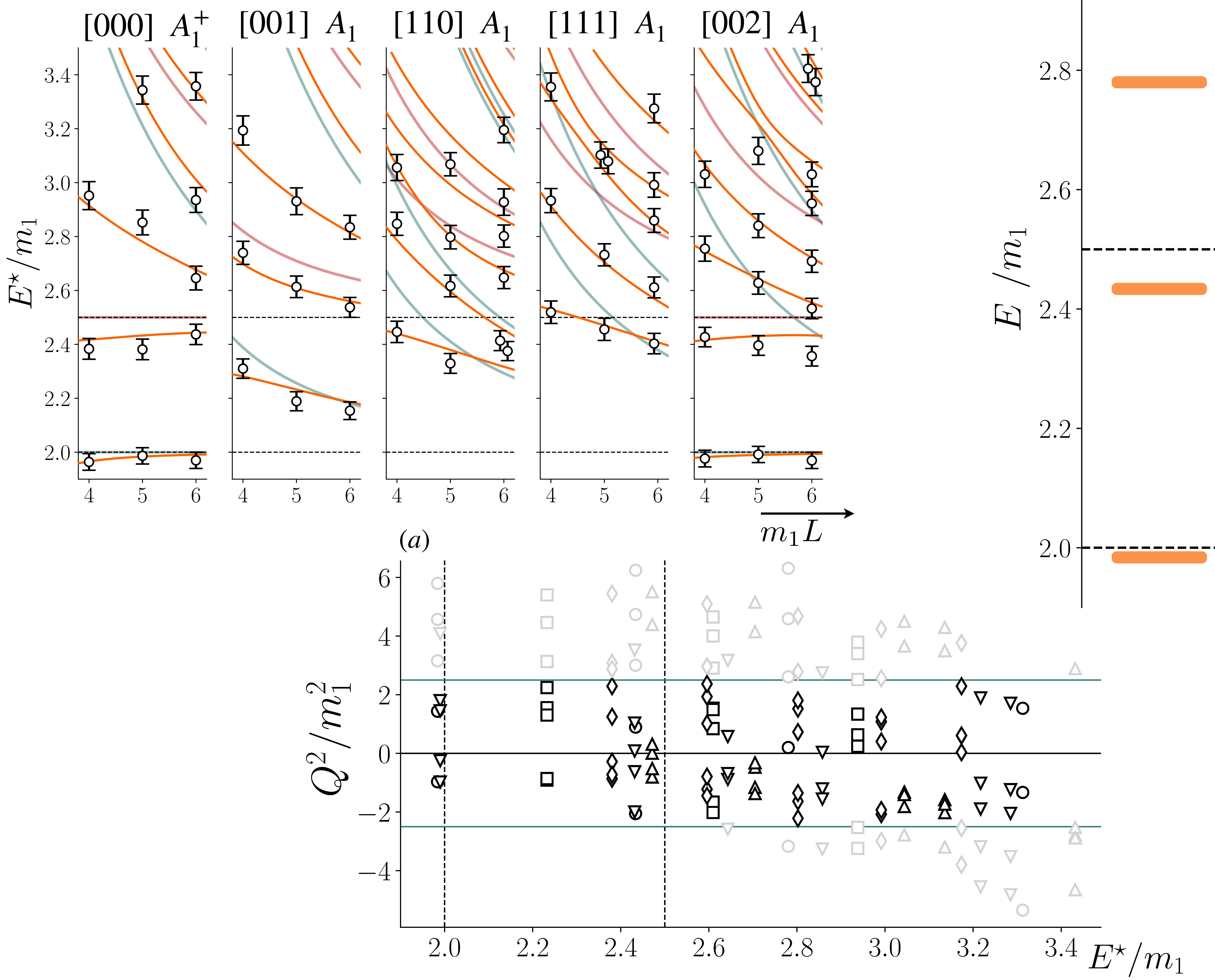
4.0 % 2.5 %

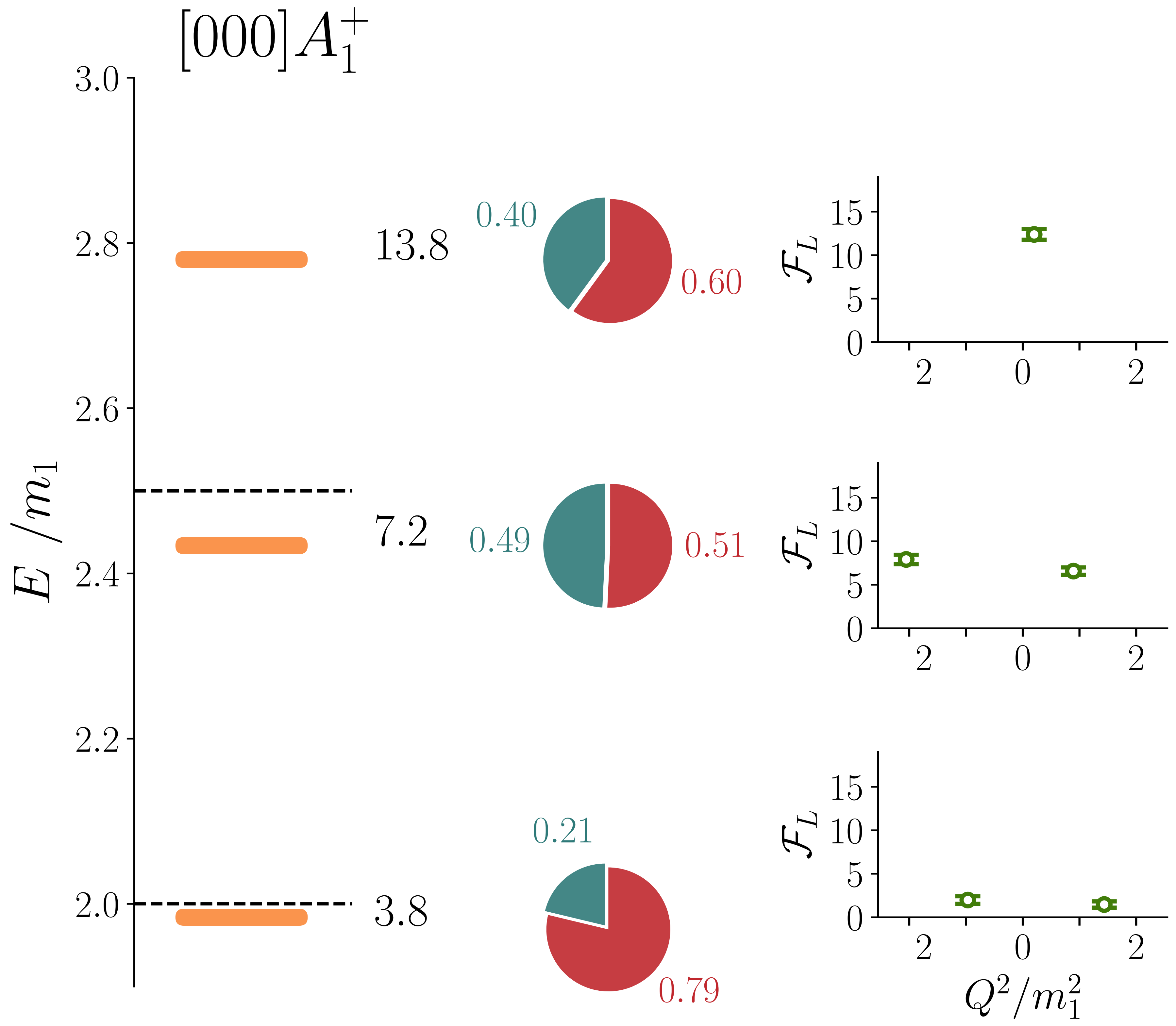
what about more complicated systems?



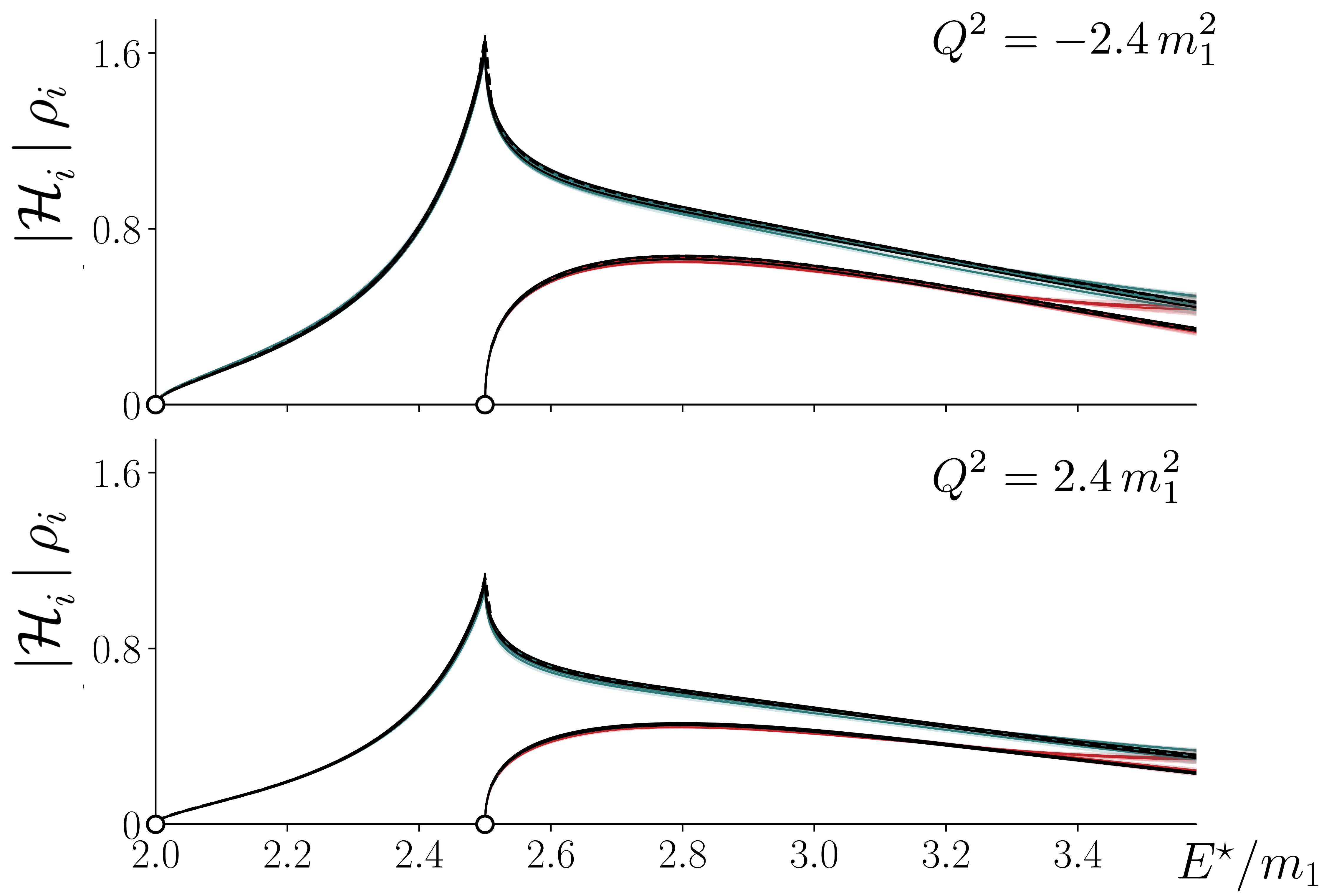
LL et al. 2021

what about more complicated systems?

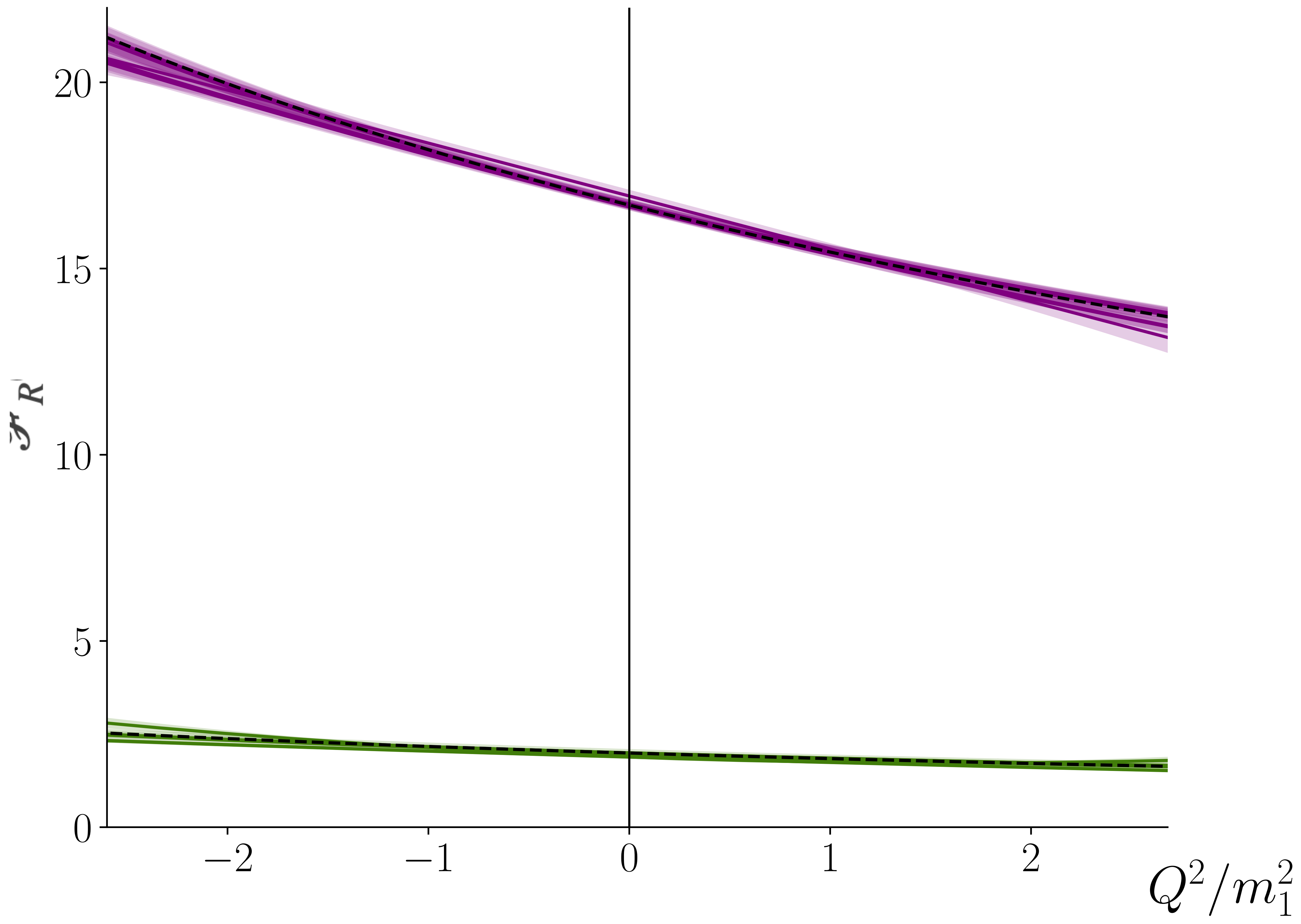


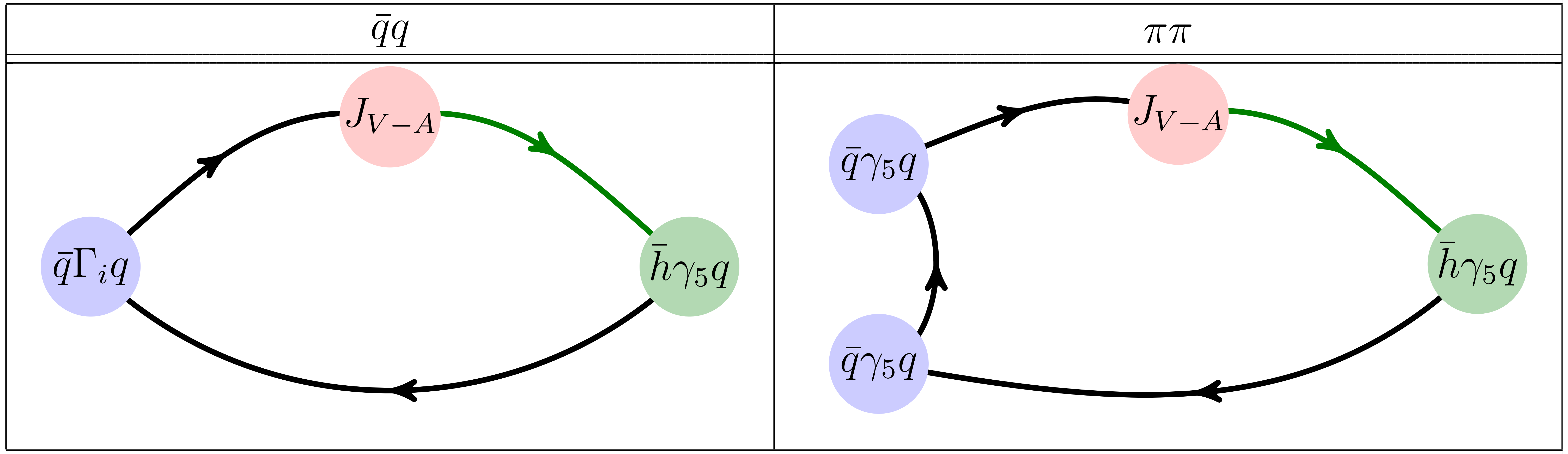


what about more complicated systems?



$$\mathscr{H}_i \approx \frac{c_i\,\mathscr{F}_R(Q^2)}{s_P - s}$$

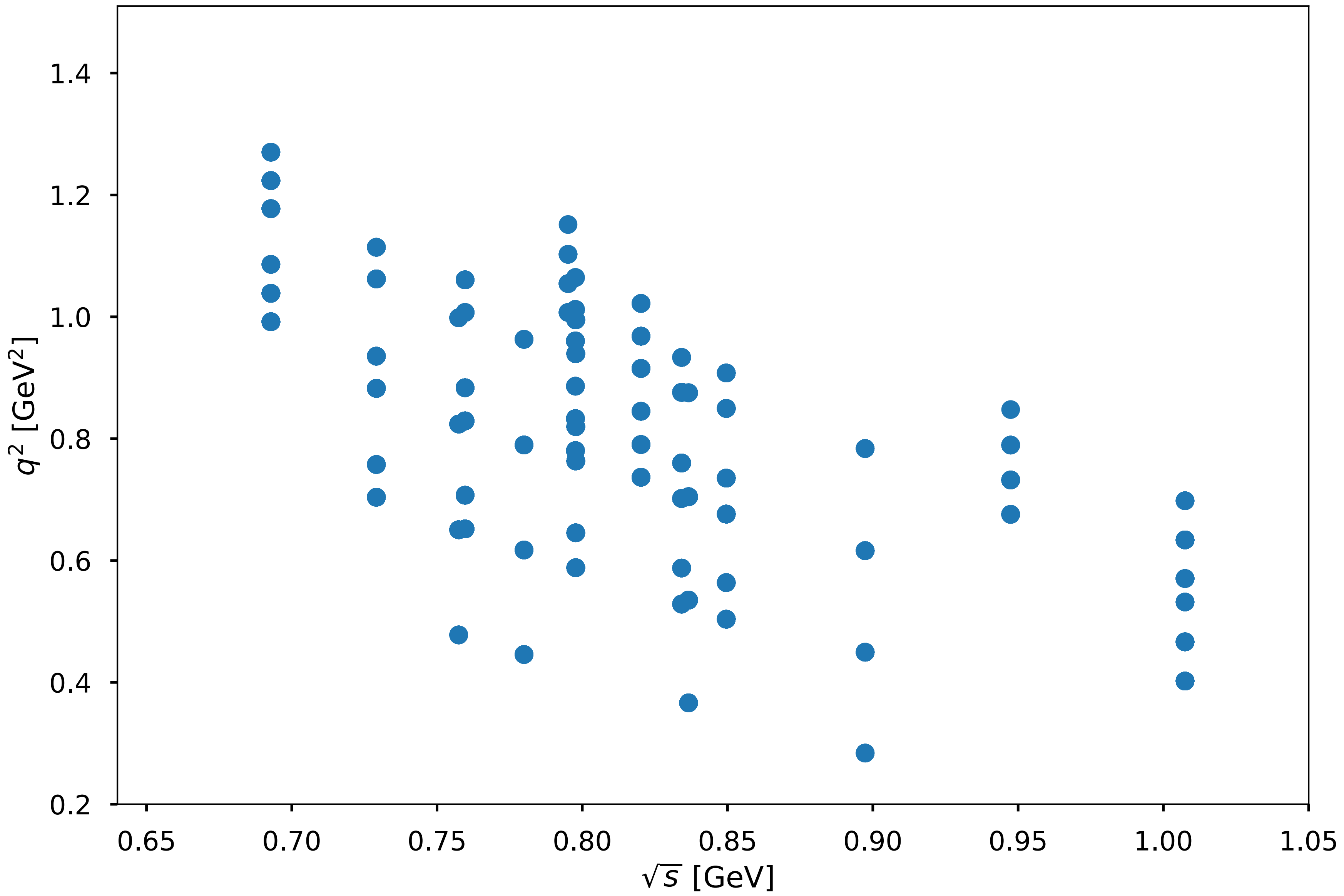




outlook

what is in the pipeline?

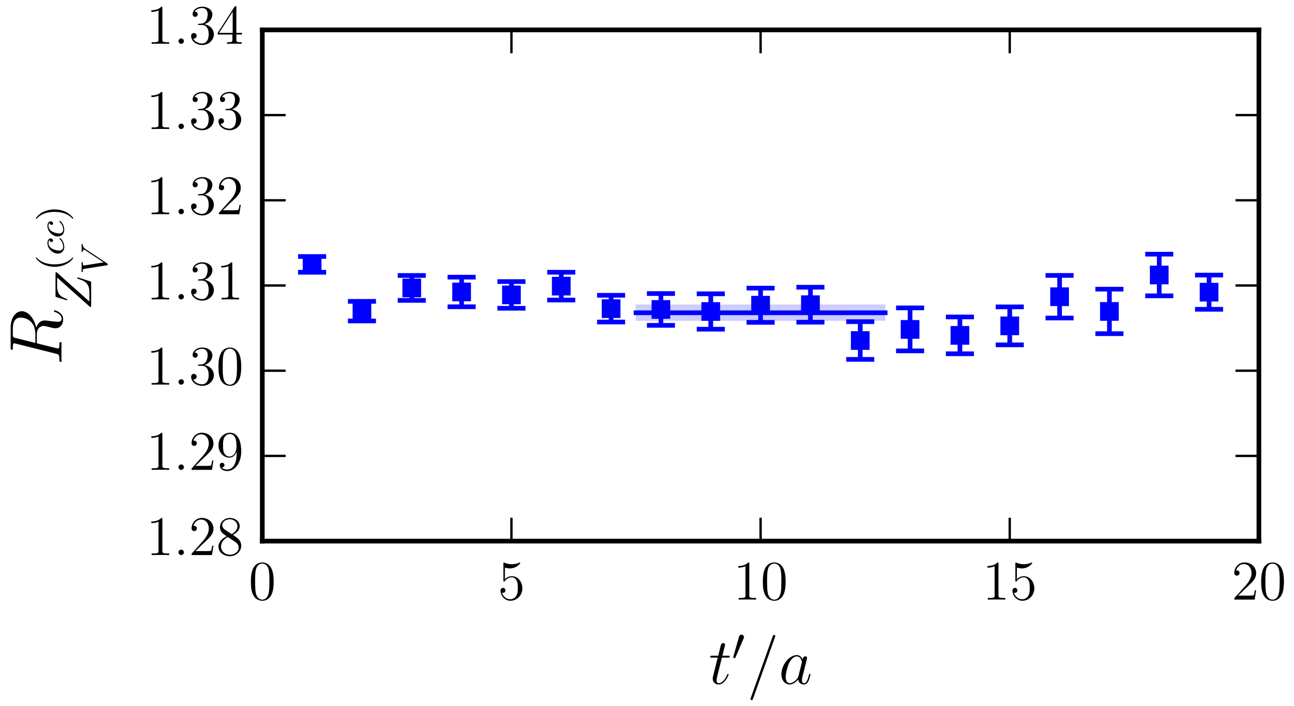
outlook: $D \to \rho(\to \pi\pi)\ell\nu$



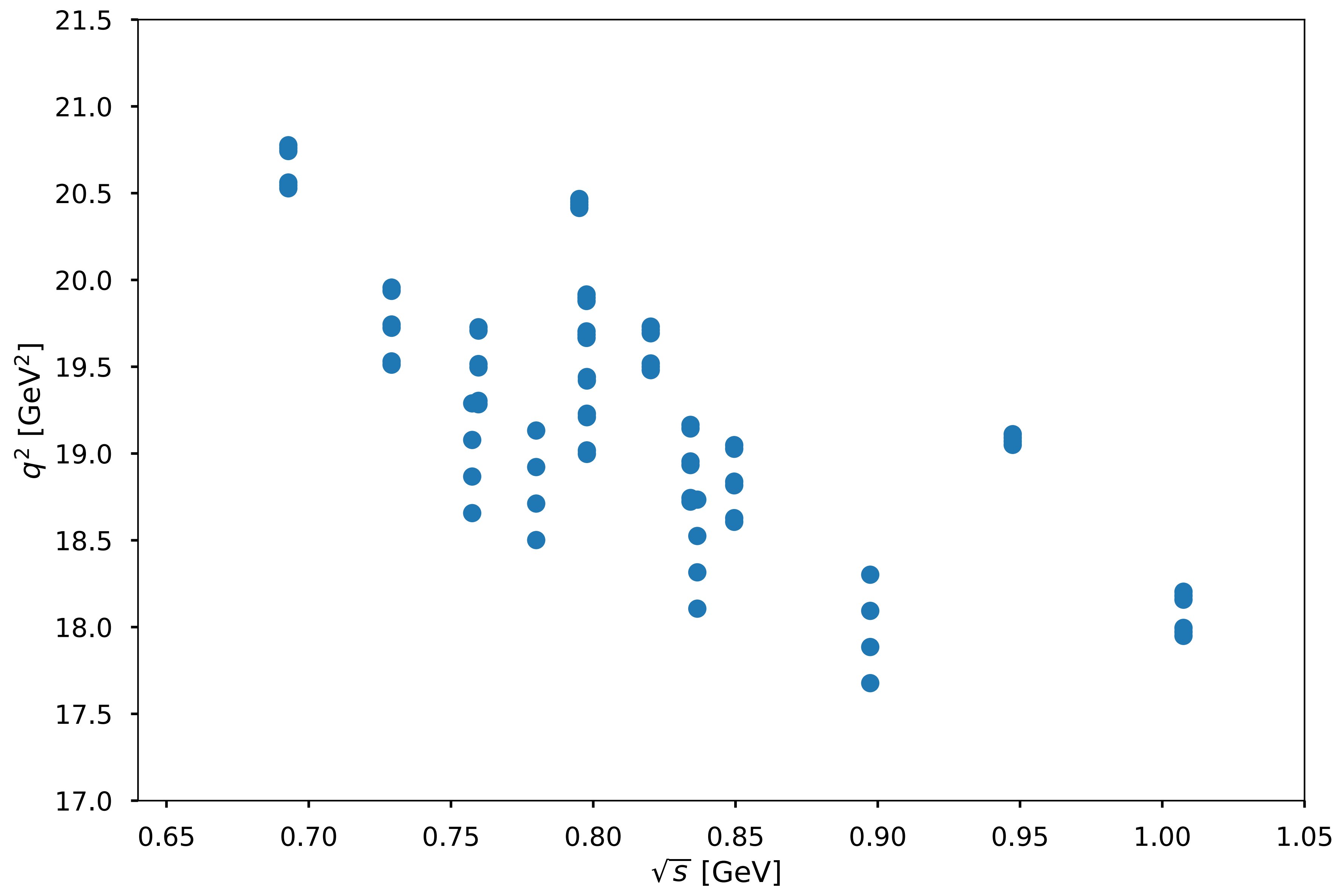
~80 points

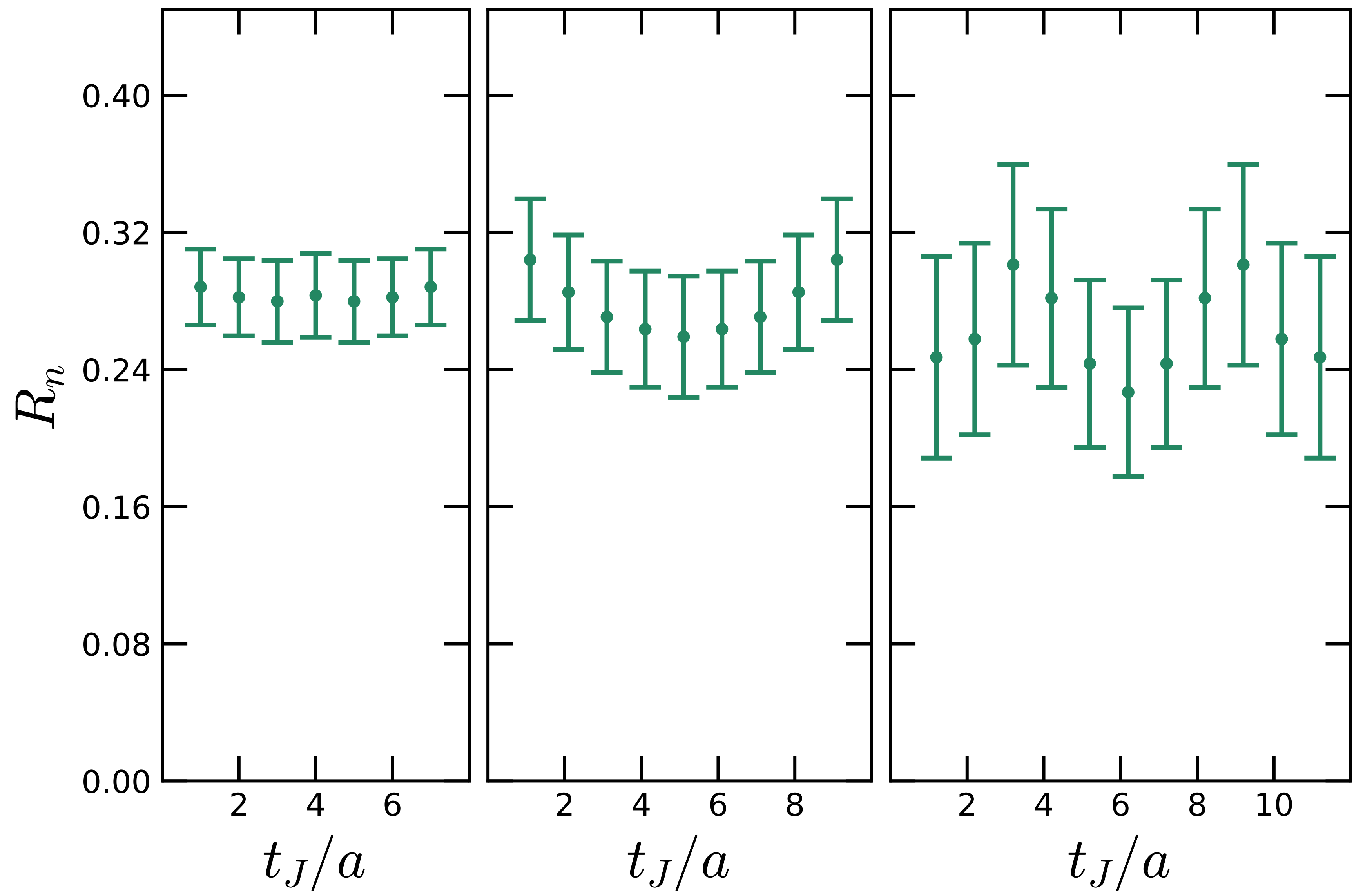
$$\langle \pi\pi(\varepsilon, p_f)\,|\,\bar{q}\gamma^\mu Q\,|\,D(p_i)\rangle = \frac{2iV(q^2,s)}{m_D + 2m_\pi}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{\nu *}p_i^\alpha p_f^\beta$$

$$\langle \pi\pi(\varepsilon, p_f)\,|\,\bar{q}\gamma^\mu\gamma^5 Q\,|\,D(p_f)\rangle = \varepsilon^* \cdot q\frac{4m_\pi}{q^2}q^\mu A_0(q^2,s)$$
$$+(m_D + 2m_\pi)\left[\varepsilon^{\mu *} - \frac{\varepsilon^* \cdot q}{q^2}q^\mu\right] A_1(q^2,s)$$
$$-\frac{\varepsilon^* \cdot q}{(m_D + 2m_\pi)}\left[(p_i + p_f)^\mu - \frac{m_D^2 - 4m_\pi^2}{q^2}q^\mu\right] A_2(q^2,s)$$



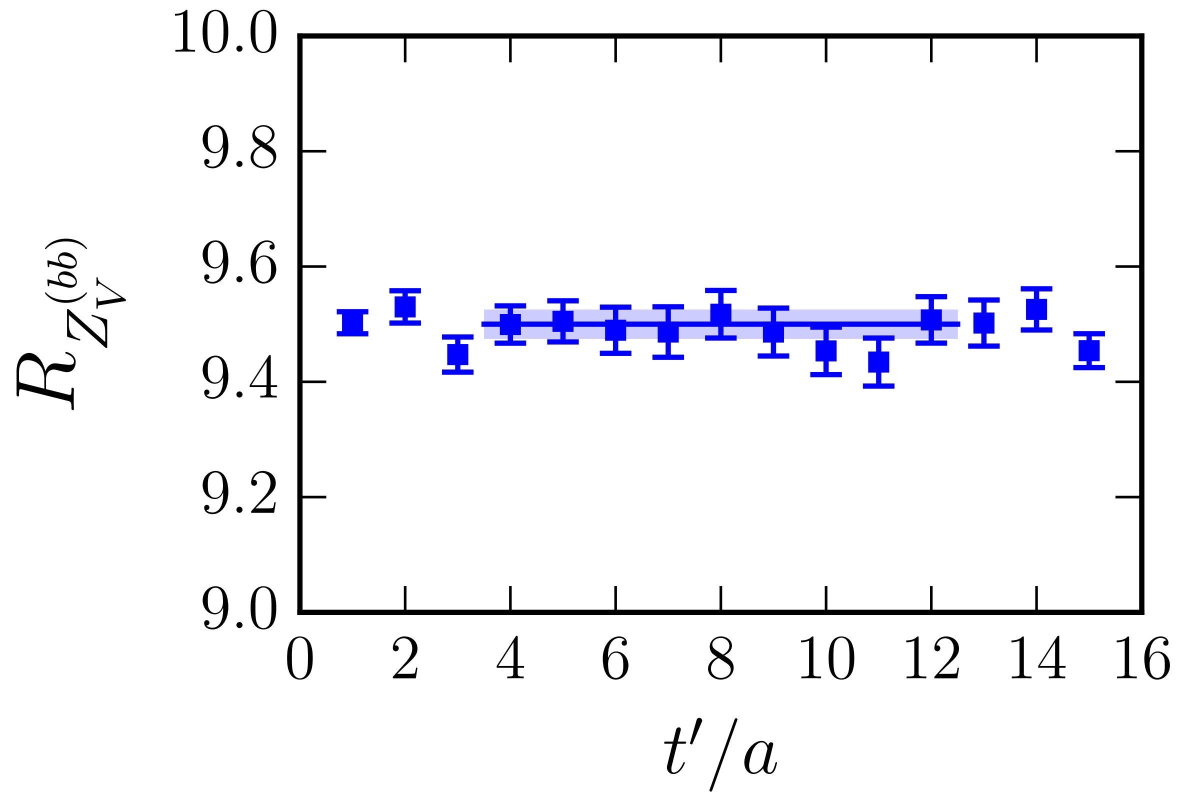
outlook: $B \to \rho(\to \pi\pi)\ell\nu$

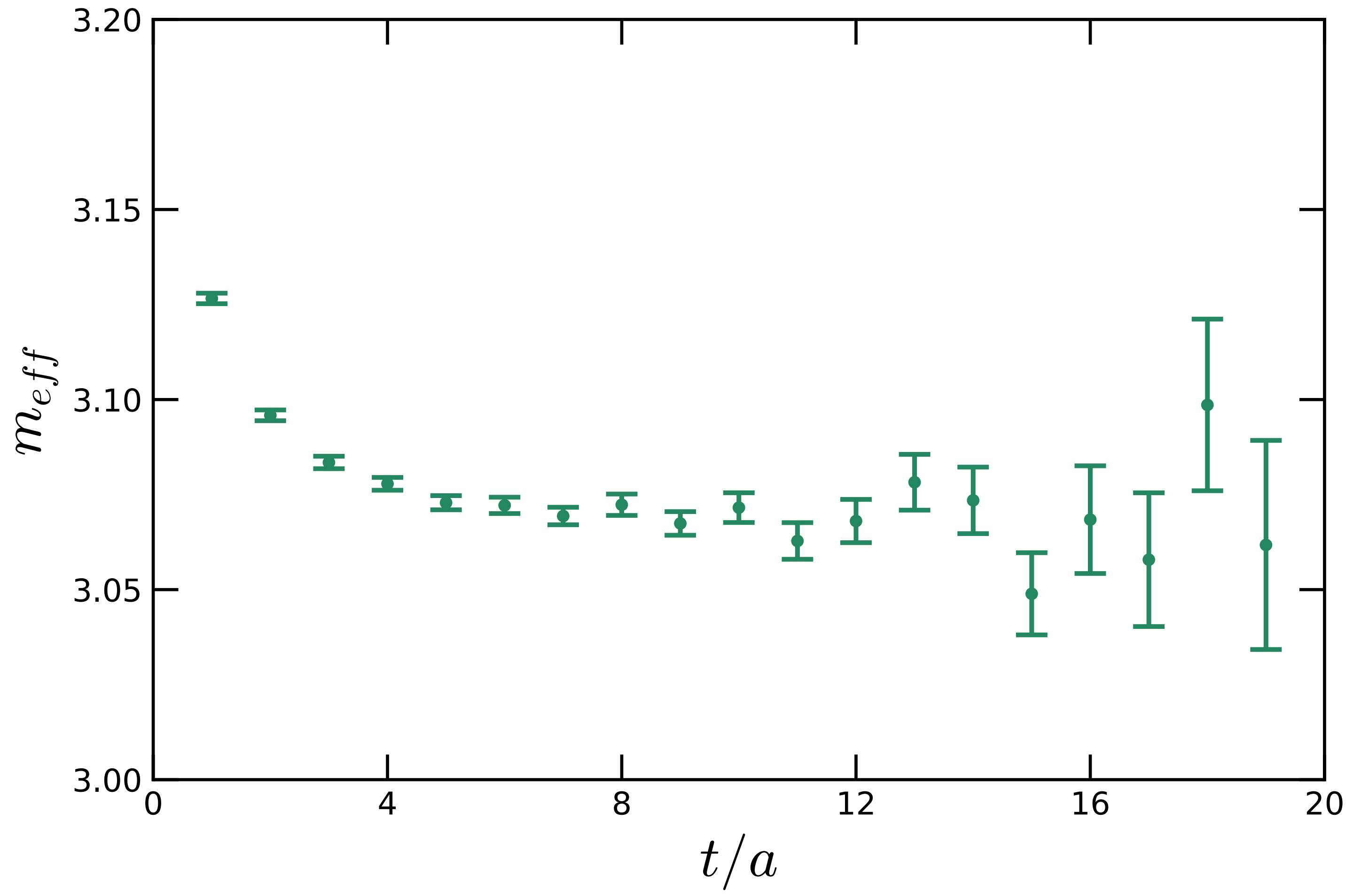


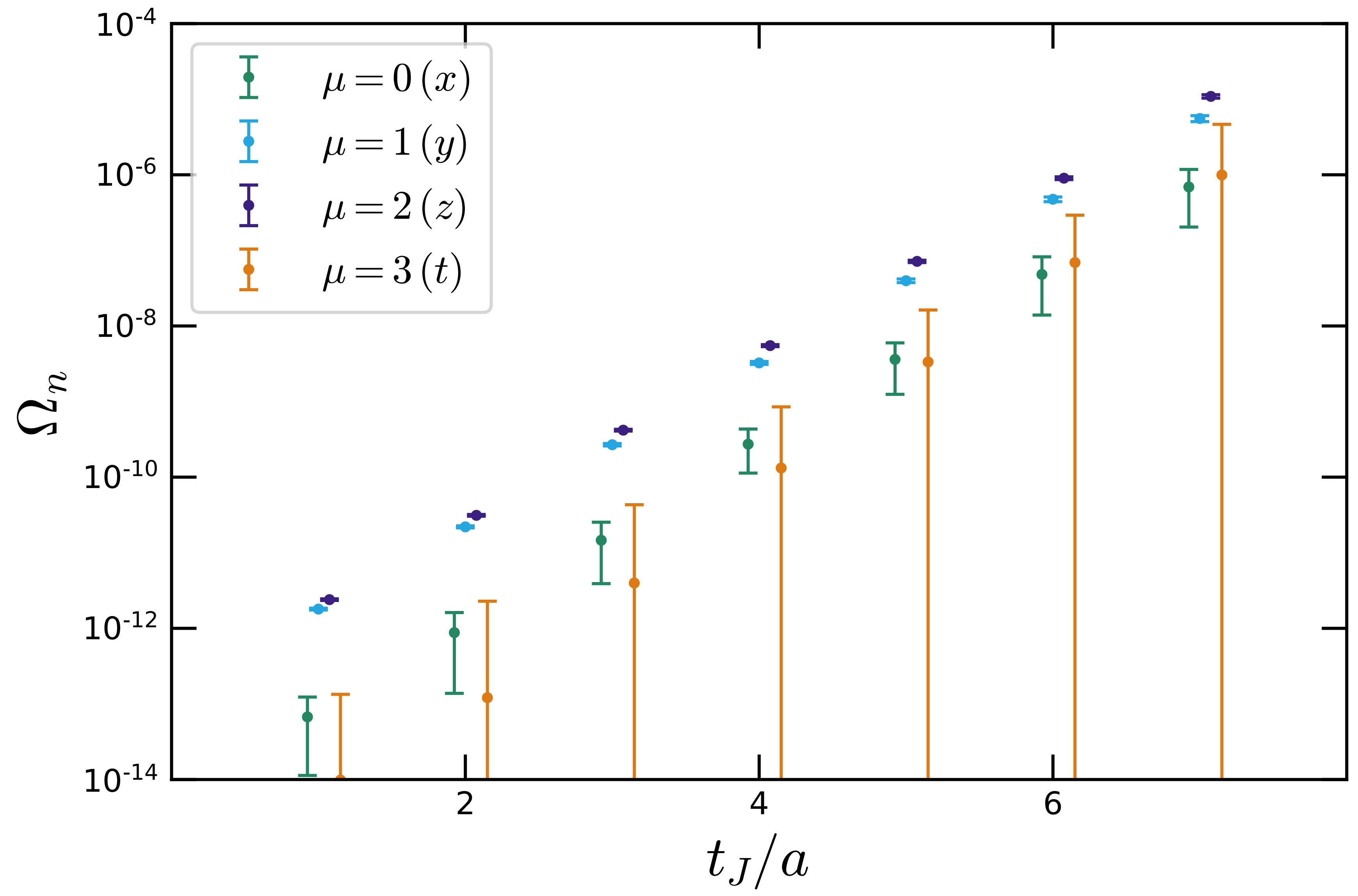


square of matrix element

~80 points

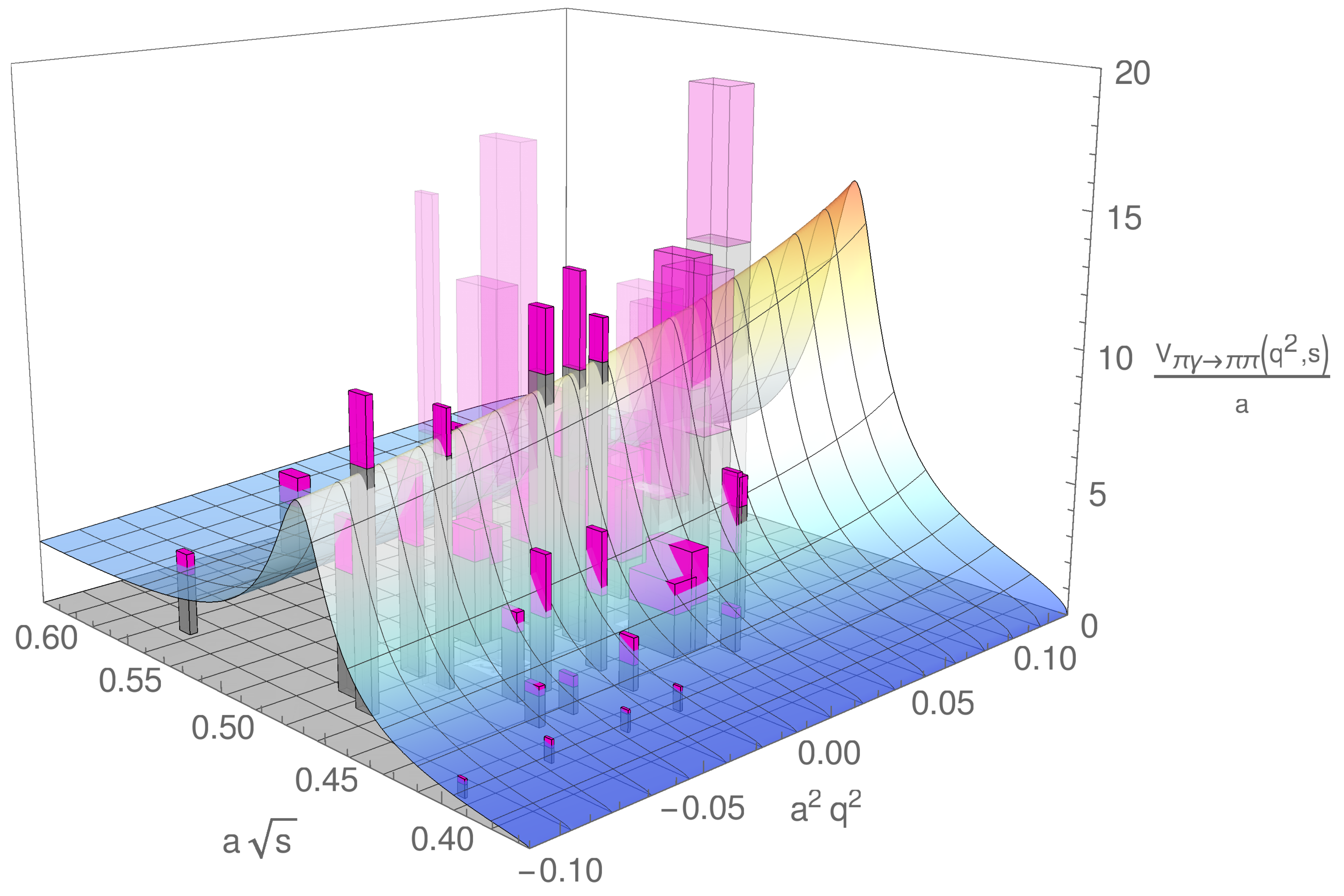






summary

- $bb\bar{u}\bar{d}$ tetraquark
- $\pi\pi$ scattering and the $\rho(770)$
- $K\pi$ scattering and the $K_0^\star(700)$ and $K^\star(892)$
- $N\pi$ scattering and the $\Delta(1232)$
- transitions $\pi\gamma \to \pi\pi$, toys
- **heavy meson transitions**
- **chiral, continuum extrapolations**



thank you :)