

Relativistic expansion of bubbles in the early universe and baryogenesis

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Production of heavy states with relativistic bubble walls

Outline

- ▶ Introduction of phase transitions in the early universe
- ▶ Production of heavy particles: [\[arXiv:2010.02590\]](#) with *Aleksandr Azatov*
- ▶ Baryogenesis with *relativistic* bubble walls: [\[arXiv:2106.14913\]](#) with *Aleksandr Azatov* and *Wen Yin*
- ▶ Singlet extension of EWPT with relativistic walls: [\[arXiv:2205.xxxxx\]](#) with *Aleksandr Azatov*, *Wen Yin*, *Sabyasachi Chakraborty* and *Giulio Barni*

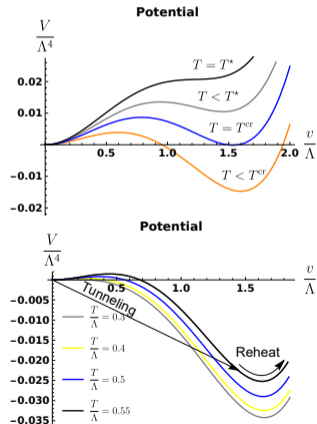
Optimistically, we can make all of that!

Phase transitions in the early universe

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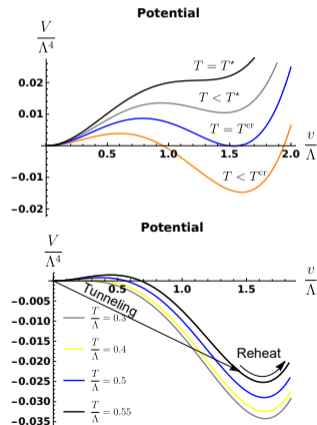
First order phase transition (FOPT) in the early universe

► QFT = landscape of minima \Rightarrow PT



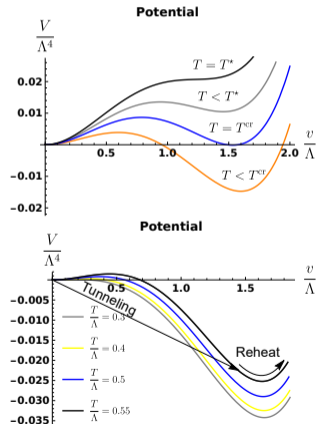
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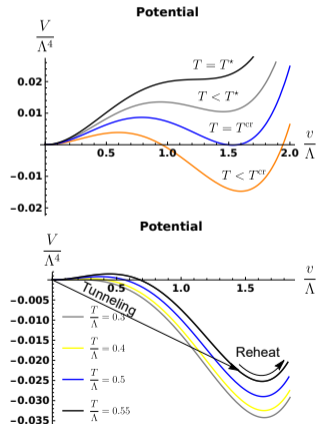
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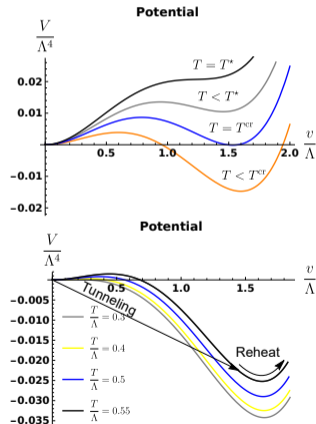
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- ▶ FOPT feature a barrier between two vacua and cooling ($T_n < T_c$)
- ▶ Nucleation controlled by bounce solution

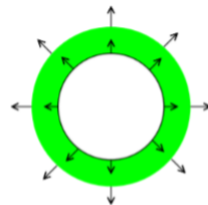
$$\Gamma \sim T^4 \text{Exp} \left[-\frac{S_3}{T} \right]$$



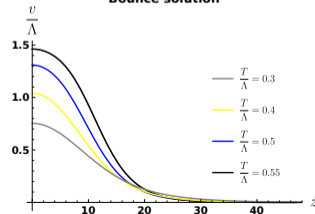
Nucleation and early expansion

- ▶ Energy released $\Delta V \Rightarrow$ Driving energy:

$$E_{\text{driving}} = -\frac{4}{3}\pi\Delta VR^3$$



Bounce solution



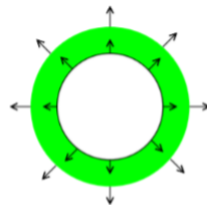
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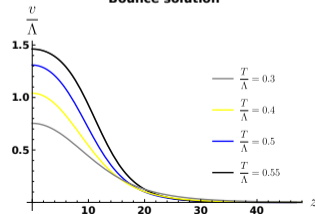
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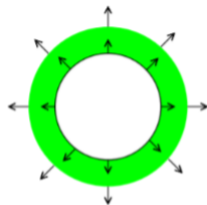
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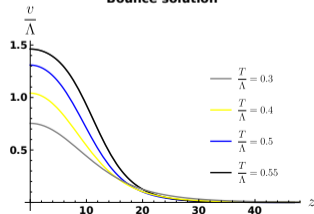
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- ▶ Expansion when $R_{\text{initial}} > R_c \sim \sigma/\Delta V$



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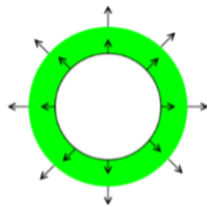
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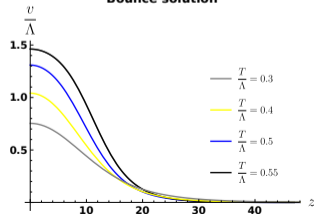
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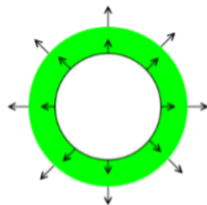
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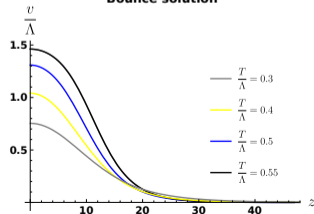
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- ▶ Expansion when $R_{\text{initial}} > R_c \sim \sigma/\Delta V$
- ▶ Transition starts approximately when one bubble per Hubble volume, $T = T_{\text{nuc}}$
- ▶ Reheat behind the wall

$$T_r \approx (1 + \alpha)^{1/4} T_{\text{nuc}}, \quad \alpha \equiv \frac{\Delta V}{\rho_r}$$



Bounce solution



Production of heavy particles

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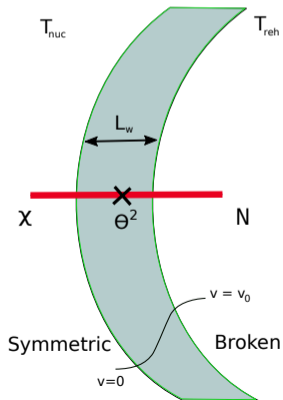
[arXiv:2010.02590] with *Aleksandr Azatov*

Production of heavy states and wall suppression [2010.02590]: Idea

Scale of the transition and particles involved

CLAIM: transition is dictated by fields $M \lesssim T_{\text{nuc}} \sim v_\phi$ because $n_{\text{heavy}} \propto e^{-M/T_{\text{nuc}}}$

► ϕ scalar, χ light, N heavy: $\mathcal{L}_{\text{int}} = Y\phi\bar{\chi}N + M\bar{N}N, \quad M \gg T_{\text{nuc}}$



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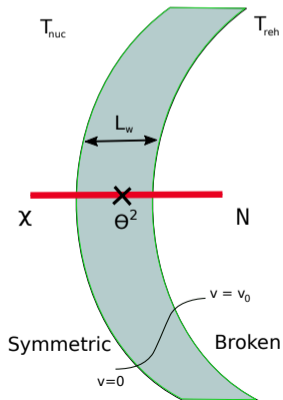
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▶ Conservation of momentum: *Origins*

$$\text{No wall: } \int d^4x e^{ip \cdot x} \propto (2\pi)^4 \delta^4(p), \quad p = p_N - p_\chi$$

$$p_\chi = (E, 0, 0, E) \quad p_N = (E, 0, 0, \sqrt{E^2 - M^2})$$



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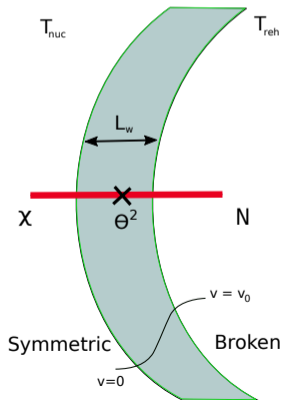
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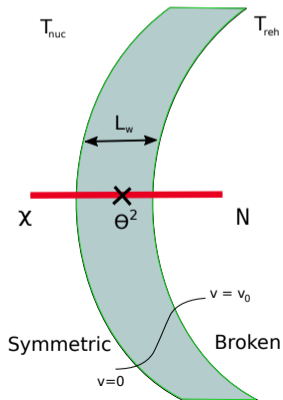
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▶ With wall: $p = p_N - p_\chi$ not conserved: if $E > M$, $\chi \rightarrow N$ allowed

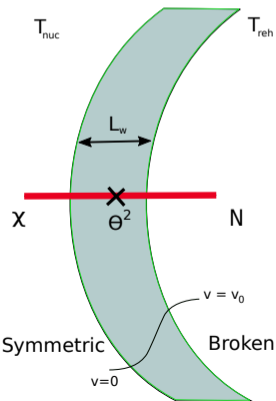
$$\int d^3x_\perp e^{ip_\perp \cdot x_\perp} \int \langle \phi \rangle(z) e^{izp_z} dz \propto (2\pi)^3 \delta^3(p_\perp) \frac{\sin \Delta p_z L_w}{\Delta p_z L_w}$$



Production of heavy states and wall suppression[2010.02590]: computation

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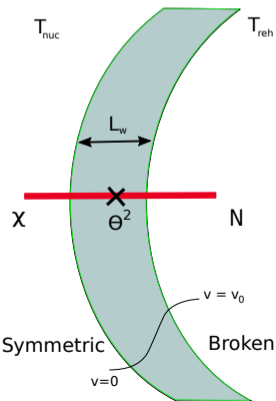


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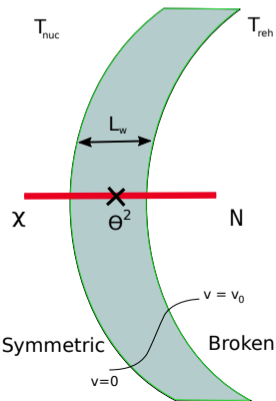
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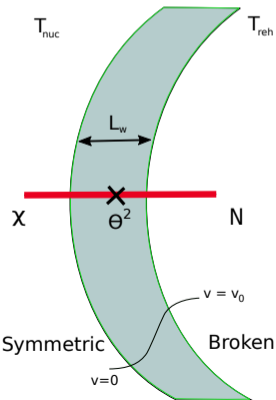
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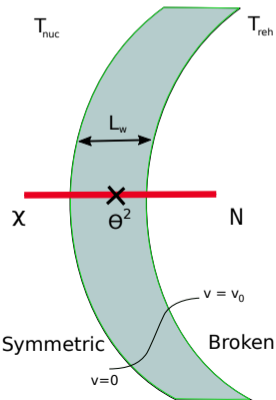
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$$\mathcal{P}(\chi \rightarrow N) \approx \theta^2 \times \Theta(\gamma_{wp} T_{\text{nuc}} - M^2 L_w), \quad \theta \equiv \frac{Y v_\phi}{M}$$

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▶ Claim WRONG: states with $M \gg T_{\text{nuc}}$ can become dynamical during PT

Idea: production of heavy states

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$\chi \rightarrow N$ allowed if enough energy in wall frame: $\Delta p_z L_w < 1 \Rightarrow \gamma_{wp} T_{\text{nuc}} > M^2 L_w$

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- ▶ Cosmological consequences: 1) Pressure on the bubble wall, 2) Non-thermal DM (1 to 2 splittings), 3) Baryogenesis ...

We focus on Baryogenesis

Baryogenesis with relativistic walls

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[[arXiv:2106.14913](https://arxiv.org/abs/2106.14913)] with *Aleksandr Azatov* and *Wen Yin*

Baryogenesis

- ▶ **B-number violation**; B-violating interactions, sphalerons
- ▶ **CP-violation**; *physical* phase into the yukawa matrix
- ▶ **Out-of-equilibrium situation**; expansion of the universe, first-order phase transition

Electroweak baryogenesis

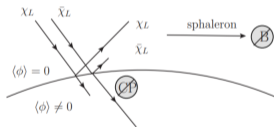


Figure: Credit: T. Konstandin [1302.6713]

Scattering of quarks with CP-violating yukawas off the *slow* bubble wall.
B-violation via *sphalerons*

Leptogenesis



Figure: Credit: T. Konstandin [1302.6713]

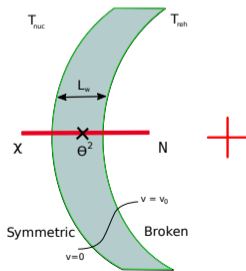
Out-of-equilibrium decay of heavy L-violating RH neutrinos. B-violation via sphalerons. CP violation via *loops*

VS

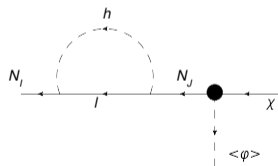
CP violation inside the bubble wall

Ingredients: Higgs field H , φ scalar, 2 heavy N_I , SM $SU(2)_L$ -fermions L_α , and χ_i light fermions

$$\mathcal{L} = i\bar{\chi}_i P_R \not{\partial} \chi_i + i\bar{N}_I \not{\partial} N_I - M_I \bar{N}_I N_I - Y_{iI} \varphi \bar{N}_I P_R \chi_i - y_{I\alpha} (H \bar{L}_\alpha) P_R N_I + h.c.$$



$$A(\chi_i \rightarrow N_I)_{\text{tree}} \propto Y_{iI}$$



$$A(\chi_i \rightarrow N_I)_{1\text{-loop}} \propto + \sum_{\alpha, J} Y_{iJ} Y_{\alpha J}^* Y_{\alpha I} \times f_{IJ}^{(hl)}$$



$$\frac{\Gamma(\chi \rightarrow N_I) - \Gamma(\bar{\chi} \rightarrow \bar{N}_I)}{\Gamma(\chi \rightarrow N_I) + \Gamma(\bar{\chi} \rightarrow \bar{N}_I)} =$$

$$\frac{2 \sum_{\alpha, J, i} \text{Im}(Y_{iI} Y_{iJ}^* y_{\alpha J} y_{\alpha I}^*) \text{Im} f_{IJ}^{(hl)}}{\sum_i |Y_{iI}|^2}.$$

and

$$\text{Im}[f_{IJ}^{(hl)}(x)] = \frac{1}{16\pi} \frac{\sqrt{x}}{1-x}, \quad x = \frac{M_J^2}{M_I^2}$$

Possible baryogenesis in our production setting?

Idea of baryogenesis with relativistic bubble walls

- ▶ **Out-of-equilibrium situation**; Automatically embedded via the process of production

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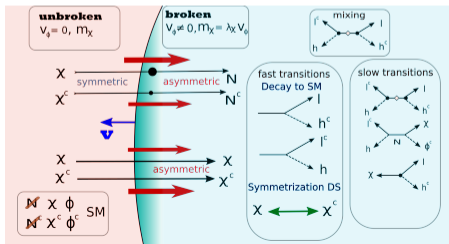
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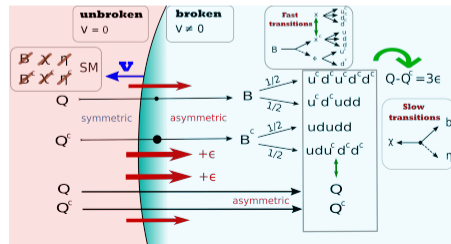
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phase transition induced leptogenesis

EWPT Baryogenesis with relativistic walls



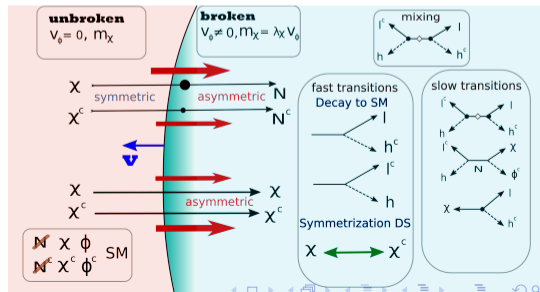
Or



Two models; phase transition induced leptogenesis

$$\sum_{il} \underbrace{\left(Y_{il}(\phi \bar{\chi}_i) P_L N_l + Y_{il}^* \bar{N}_l P_R(\phi^\dagger \chi_i) \right)}_{\text{production sector}} - V(\phi) + \underbrace{\sum_i \lambda_\chi \phi \bar{\chi}_i^c \chi_i}_{\text{Majorana mass}} + \sum_l M_l \bar{N}_l N_l + \underbrace{\sum_{\alpha l} y_{\alpha l} (H \bar{L}_{\alpha, SM}) P_R N_l}_{\text{CP-violation}}$$

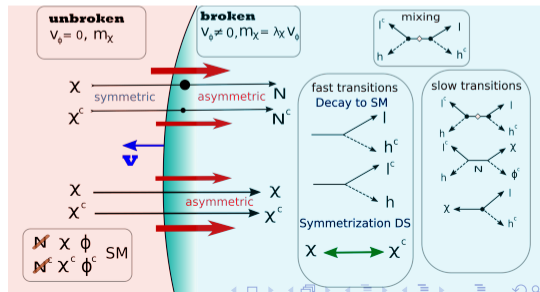
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 $U_L(1) : L(\chi) = -1, L(N) = 1, L(\phi) = 2.$



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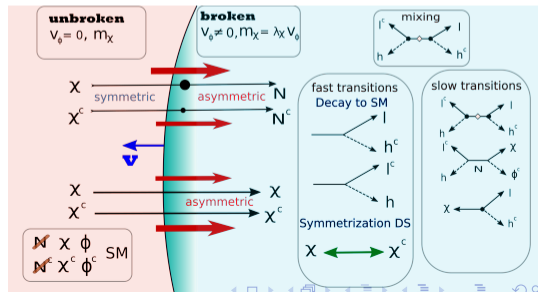
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Two models; phase transition induced leptogenesis

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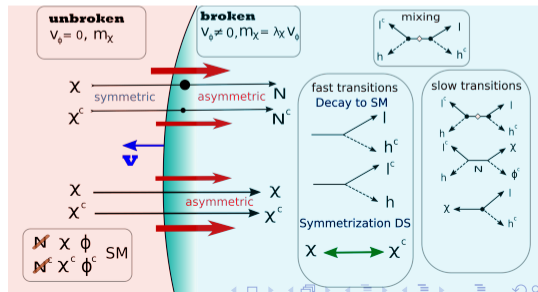
- ▶ χ_i Majorana, ϕ scalar, N_l Heavy Dirac RHN:
 $U_L(1) : L(\chi) = -1, L(N) = 1, L(\phi) = 2.$
- ▶ Production $\mathcal{P}(\chi_i \rightarrow N_l) \neq \mathcal{P}(\chi_i^c \rightarrow N_l^c) \propto \theta_{il}^2$
- ▶ Separation in light and heavy sector
 $\sum_l \Delta n_{N_l} = -\sum_i \Delta n_{\chi_i} \propto \sum_{il} \epsilon_{il} \theta_{il}^2$



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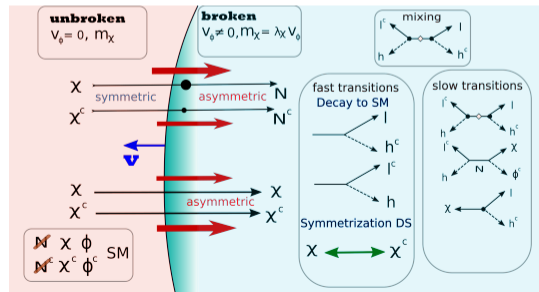
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 $\sum_l \Delta n_{N_l} = -\sum_i \Delta n_{\chi_i} \propto \sum_{il} \epsilon_{il} \theta_{il}^2$
- ▶ **Fast interactions;**
 1. $\chi \leftrightarrow \chi^c$: *Annihilation* in light sector
 2. $N_l \rightarrow LH$: *Transfer* to SM
 3. $N_l \rightarrow \phi \chi$: *Wash-out*



Two models; phase transition induced leptogenesis

$$\text{Lepton asym produced} \approx \frac{\Delta n_L}{s} \sim \sum_{il\alpha} \epsilon_{il} \times \frac{\theta_{il}^2}{g_*} \left(\frac{T_{nuc}}{T_{reh}} \right)^3 \times \frac{|y_{l\alpha}|^2}{|y_{l\alpha}|^2 + |Y_{il}|^2}$$

- **Slow interactions:** sphalerons $\Delta L \rightarrow \Delta B$



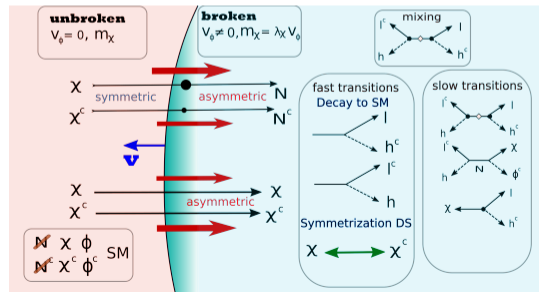
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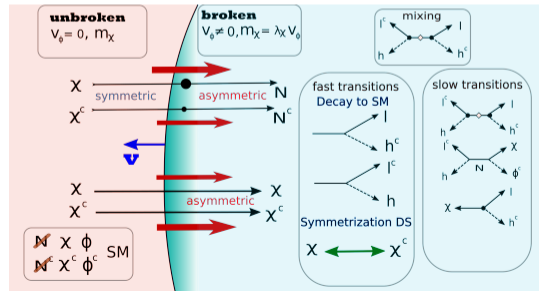
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▶ **But...** Wash-outs from L -violating;

$$H^c L \rightarrow H L^c \Rightarrow T_{reh} < 10^{13} \text{ GeV}$$

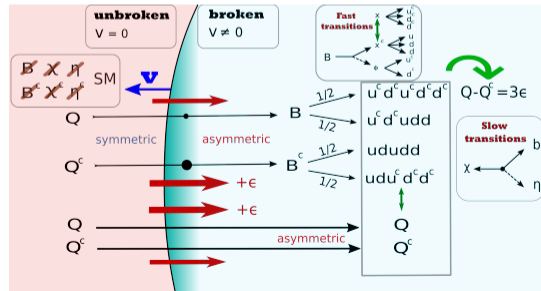
$$H L \rightarrow \chi \Rightarrow \frac{m_\chi}{T_{reh}} > 15$$



Two models; low energy baryogenesis

$$\mathcal{L}_{SM} + \sum_{I=1,2} \underbrace{Y_I(\bar{B}_I H) P_L Q}_{\text{production}} + M_I \bar{B}_I B_I + \underbrace{y_I \eta \chi^c P_L B_I + \kappa \eta^c du}_{\text{decay dark sector}} + \underbrace{\frac{1}{2} m_\chi \bar{\chi}^c \chi + m_\eta^2 |\eta|^2}_{\text{B-violating}}$$

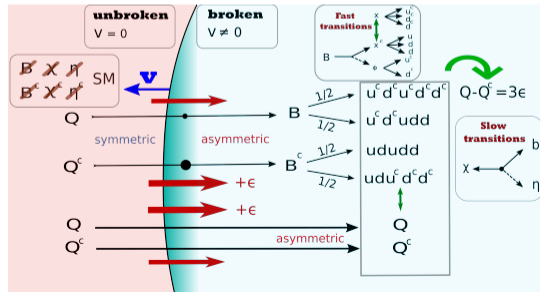
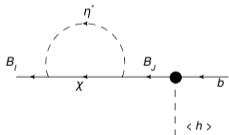
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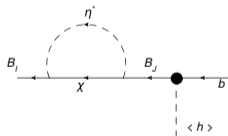
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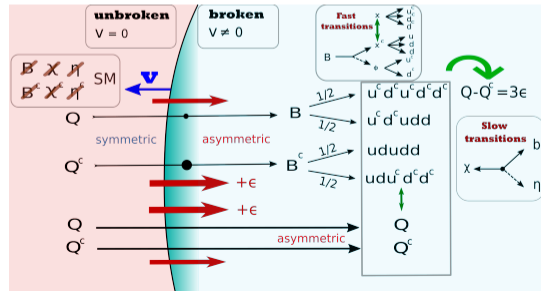
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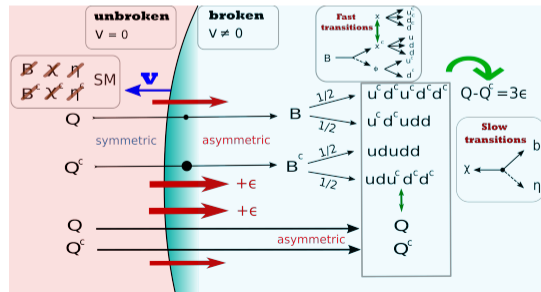
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► Fast cascades; 4 channels

wash-out : $B \rightarrow (ddud^c u^c)$, $B^c \rightarrow (d^c d^c u^c du)$

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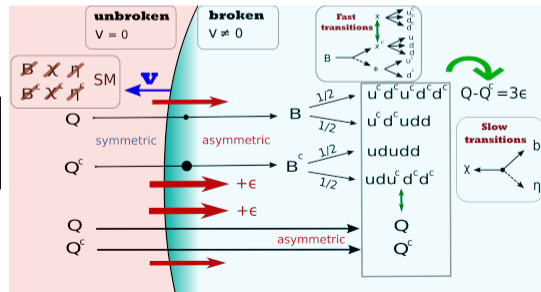
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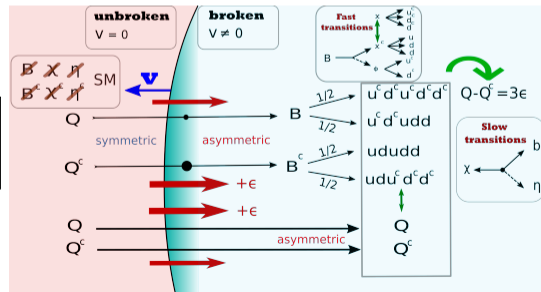
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- Experimental signatures: $N \leftrightarrow \bar{N}$, Flavor,
collider push: $m_\chi \sim m_\eta \sim M_B \gtrsim 2 \text{ TeV}$ and
 $d = b, u = t$



Conclusion baryogenesis

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- ▶ Question: How can we make EWPT with ultra-fast bubble walls ?

EWPT with relativistic walls

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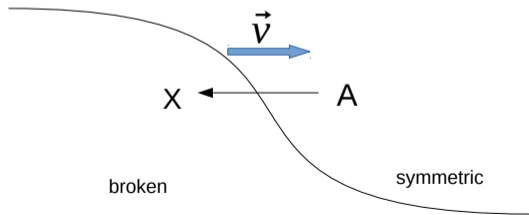
[[arXiv:2205.xxxxx](#)] with *Aleksandr Azatov, Wen Yin, Sabyasachi Chakraborty and Giulio Barni*

Velocity

Final velocity $\gamma_{wp}^{MAX} = \frac{1}{\sqrt{1-v_{MAX}^2}}$ of the wall set by

$$\Delta V = \Delta \mathcal{P}(\gamma_{wp}^{MAX}) \Rightarrow \text{determination } \gamma_{wp}^{MAX}$$

- ▶ ΔV independent of the velocity of the wall

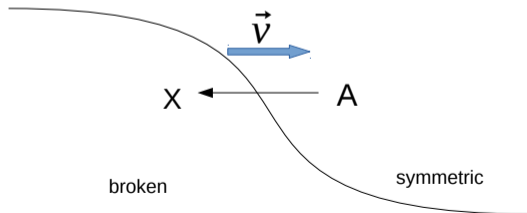


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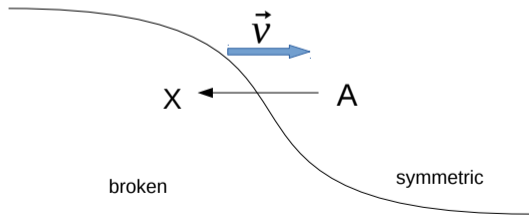
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- ▶ ΔV independent of the velocity of the wall
- ▶ $\Delta \mathcal{P}(\gamma_{wp}^{MAX})$ very difficult to compute in general and depends on the velocity
- ▶ Generic method: solve the full coupled system of Boltzmann equations

$$p^\mu \partial_\mu f_i + \frac{1}{2} \partial_z m_i[\phi] \partial_{p_z} f_i = \mathcal{C}[f_i, \phi]$$

$$\square \phi + \frac{dV}{d\phi} + \sum_i \frac{dm_i^2[\phi]}{d\phi} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_i} f_i = 0$$



Primer on pressure in relativistic regime

$$\Delta P_{A \rightarrow X} = \int \frac{p_z d^3 p}{p_0 (2\pi)^3} f_A(p) \times \sum_X \int d\Delta P_{A \rightarrow X} \Delta p$$

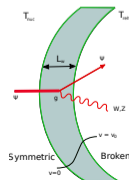
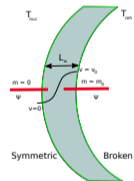
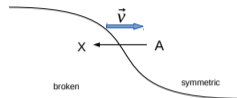
LO pressure [0903.4099]

$$\Delta P_{LO} \approx \sum_i \frac{\Delta m_i^2 T^2}{24}, \quad \Delta m_i^2 \equiv m_{bro,i}^2 - m_{sym,i}^2$$

Condition for relativistic wall $\Delta V > P_{LO}$

Theories with gauge bosons V [1703.08215]

$$\Delta P_{NLO} \sim \sum_i g_i \frac{g_{\text{gauge}}^3 v}{16\pi^2} \gamma_{wp} T^3$$



Primer on pressure in relativistic regime: case of the EWPT

LO pressure:

$$\Delta \mathcal{P}_{\text{LO}}^{\text{SM}} \approx T_{\text{nuc}}^2 v_{\text{EW}}^2 \left(\frac{y_t^2}{8} + \frac{g^2 + g'^2}{32} + \frac{g^2}{16} \right) \approx 0.17 T_{\text{nuc}}^2 v_{\text{EW}}^2.$$

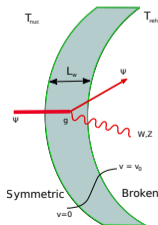
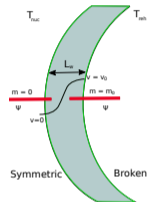
Condition for relativistic wall $\Delta V > 0.17 T_{\text{nuc}}^2 v_{\text{EW}}^2$

NLO pressure: [\[arXiv:2112.07686\]](https://arxiv.org/abs/2112.07686): Gouttenoire, Jinno, Sala

$$\Delta \mathcal{P}_{\text{NLO}}^{\text{SM}} \approx \left[\sum_{abc} \nu_a g_a \beta_c C_{abc} \right] \frac{\kappa \zeta(3)}{\pi^3} \times \alpha M_Z(v_{\text{EW}}) \gamma_{\text{wp}} T_{\text{nuc}}^3$$

Terminal velocity

$$\Delta V - \Delta \mathcal{P}_{\text{LO}}^{\text{SM}} = \Delta \mathcal{P}_{\text{NLO}}^{\text{SM}} \Rightarrow \gamma_{\text{wp}}^{\text{terminal}} \approx 50 \times \left(\frac{40 \text{ GeV}}{T_{\text{nuc}}} \right)^3 \left(\frac{\Delta V}{v_{\text{EW}}^4} \right)^{1/4}$$



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- ▶ With a singlet S with a Z_2 symmetry ?

$$V(h, S) = -\frac{m_h^2}{2}h^2 + \frac{\lambda}{4}h^4 - \frac{m_s^2}{4}S^2 + \frac{\lambda_s}{4}S^4 + \frac{\lambda_{hs}}{2}S^2h^2,$$

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Criterion for relativistic: $\underbrace{\Delta V}_{\text{E released}} > \underbrace{\Delta \mathcal{P}_{\text{rel}}(T_{\text{nuc}})}_{\text{pressure at LO}}, \quad \mathcal{P}_{\text{rel}}(T_{\text{nuc}}) \approx C \times T_{\text{nuc}}^2 v_{EW}^2$

$$m_{\text{eff}}^2(T) = -\frac{m_h^2}{2} + C \times T^2 \quad \Rightarrow \quad T_{\text{min}}^2 = \frac{m_h^2}{2C} \quad \mathcal{P}_{\text{rel}}^{\text{min}} = \frac{m_h^2 v_{EW}^2}{4} > \Delta V$$

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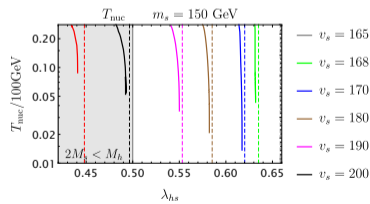
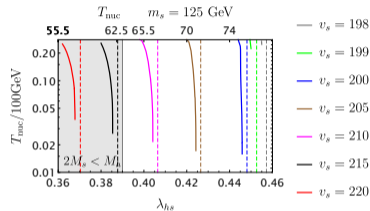
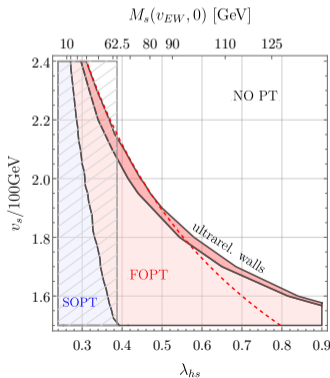
- ▶ **Two-steps PT:** $(0, 0) \xrightarrow{SOPT} (0, v_s) \xrightarrow{FOPT} (v_{EW}, 0)$

In the second PT:

$$m_{\text{eff}}^2(T) = -\frac{m_h^2}{2} + \frac{\lambda_{hs}}{2} v_s^2 + C \times T^2$$

Scan of parameter space

$$M^{MAX} \approx \sqrt{v_{EW} T_{nuc} \gamma_w} \approx 700 \text{ GeV} \times \left(\frac{40 \text{ GeV}}{T_{nuc}} \right) \left(\frac{\Delta V}{v_{EW}^4} \right)^{1/8}$$



$$M^{MAX} \in [1, 20] \text{ TeV}$$

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- ▶ Second transition is FOPT and has relativistic walls in a sizable region of parameter space
- ▶ Ultra-relativistic walls exist in a tuned region of parameter space when $M_s \sim 70 - 100$ GeV

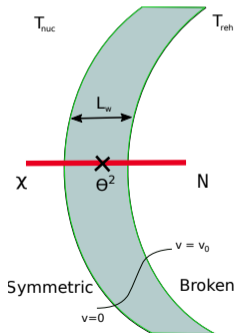
Back-up

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Production of heavy states and wall suppression [2010.02590]

Scale of the transition and particles involved

Claim: *transition is dictated by fields* $M \lesssim T_{\text{nuc}} \sim v_{\text{trans}}$ because $n_{\text{heavy}} \propto e^{-M/T_{\text{nuc}}}$



► **Adiabatic transition** $\Delta p L > 1$

$$\frac{M}{T} < \gamma_{wp} < \frac{M^2 L_w}{T}$$

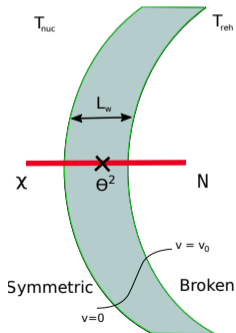
light to light $\chi \rightarrow \chi'$ only

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- ▶ **Adiabatic transition** $\Delta pL > 1$

$$\frac{M}{T} < \gamma_{wp} < \frac{M^2 L_w}{T}$$

light to light $\chi \rightarrow \chi'$ only

- ▶ **non-adiabatic transition** $\Delta pL < 1$

$$\gamma_{wp} > \frac{M^2 L_w}{T}$$

light to heavy $\chi \rightarrow N$ unsuppressed

Falkowski and No bubble wall production

Production of heavy states during the collision of bubbles. [arXiv:1211.5615](#)

- ▶ Can be non thermal DM: [arXiv:1211.5615](#)
- ▶ Or make a baryogenesis mechanism: [arXiv 1608.00583](#)

Necessary ingredients

- ▶ Portal coupling similar to ours.
- ▶ Runaway bubble (otherwise, energy dissipated in the plasma): not operative in EWPT.
- ▶ Elastic collision (restoration of the false vacuum in between the bubble)

Constraints and experimental signatures on the EWBG proposed

1. **Neutron-anti-neutron oscillations:** baryon number violation by 2 units

$$\frac{1}{\Lambda_{n\bar{n}}^5} \overline{u^c d^c d^c} udd \equiv \frac{(\sum \kappa \theta_I y_I)^2}{M_\eta^4 m_\chi} \overline{u^c d^c d^c} udd \quad \Rightarrow \quad \delta m_{\bar{n}-n} \sim \frac{\Lambda_{QCD}^6}{M_\eta^4 m_\chi} (\sum \kappa \theta_I y_I)^2$$

Current bounds on this mixing mass are of order $\delta m_{\bar{n}-n} \lesssim 10^{-33}$

$$\Lambda_{n\bar{n}} \gtrsim 10^6 \text{ GeV} \quad (M_\eta, m_\chi) \gtrsim 10^5 \text{ GeV}$$

2. **Flavor violation:** Need to couple strongly only to t_R, b_R
3. **Contribution to electron EDM:**

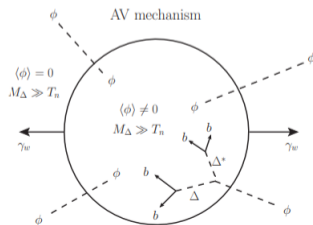
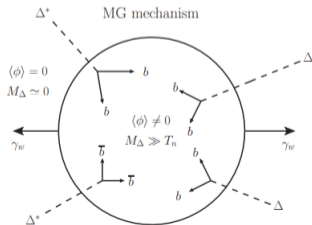
$$\frac{d_e}{e} \sim \frac{m_e (y Y_e)^2}{(4\pi)^6} \left(\frac{1}{\Lambda_{EDM}^2} \right) \sim 3 \times 10^{-33} \times \left(\frac{10 \text{ TeV}}{\Lambda_{EDM}} \right)^2 \text{ cm}$$

while experimental bound is $|d_e| < 1.1 \times 10^{-29} \text{ cm} \cdot e$

Comparison with proposal in arXiv:2106.15602

Baryogenesis with relativistic walls by Baldes et al. arXiv:2106.15602

- ▶ Relativistic walls $\gamma_{wp} \gg 1$
- ▶ scalar model $\Delta\mathcal{L} = -\frac{\lambda}{2}\phi^2 h^2 + \frac{M_\phi^2}{2}\phi^2$ with production of heavy scalar ϕ
- ▶ ϕ in $(3, 1, 2/3)$ of the SM and $\Delta\mathcal{L} = y_{di}\phi_i \bar{d}_R d'_R + y_{ui}\phi_i \bar{N}_R u'_R$ with physics phase in y'
- ▶ CP and B violation in decay $\phi \rightarrow bb$



Full expression

PT leptogenesis: CP violation in production+decay

$$\frac{n_B - n_{\bar{B}}}{s} \simeq -\frac{28}{79} \times \frac{135\zeta(3)g_\chi}{8\pi^4 g_*} \times \sum_I \theta_I^2 \sum_{\alpha, J} \text{Im}(Y_I Y_J^* y_{\alpha J} y_{\alpha I}^*) \text{Im} f_{IJ}^{(hl)}$$

$$\times \left(\frac{2}{|Y_I|^2} - \frac{1}{\sum_\alpha |y_{\alpha I}|^2} \right) \left(\frac{T_{nuc}}{T_{reh}} \right)^3 \frac{\sum_\alpha |y_{\alpha I}|^2}{\sum_\alpha |y_{\alpha I}|^2 + |Y_I|^2}$$

EWPT baryogenesis: CP violation in production+decay

$$\frac{\Delta n_{Baryon}}{s} \approx \frac{135\zeta(3)}{8\pi^4} \sum_{I, J} \theta_I^2 \frac{|y_I|^2}{|y_I|^2 + |Y_I|^2} \times \frac{g_b}{g_*} \left(\frac{T_{nuc}}{T_{reh}} \right)^3$$

$$\times \text{Im}(Y_I Y_J^* y_I^* y_J) \left(-\frac{2\text{Im}[f_B^{IJ}]}{|Y_I|^2} + \frac{4\text{Im}[f_B^{IJ}]|_{m_{\chi, \eta} \rightarrow 0}}{|y_I|^2} \right).$$