(Gauge independent) Bubble nucleation rate at finite temperature[*]

Philipp Schicho
philipp.schicho@helsinki.fi

Helsinki Institute of Physics, University of Helsinki
Institute for Theoretical Physics, Goethe University Frankfurt

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[*] J. Hirvonen, J. Löfgren, M. J. Ramsey-Musolf, P. Schicho, and T. V. I. Tenkanen, *Computing the gauge-invariant bubble nucleation rate in finite temperature effective field theory,* JHEP **07** (2022) 135 [2112.08912], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, DRalgo*: a package for effective field theory approach for thermal phase transitions,* [2205.08815]

Motivation

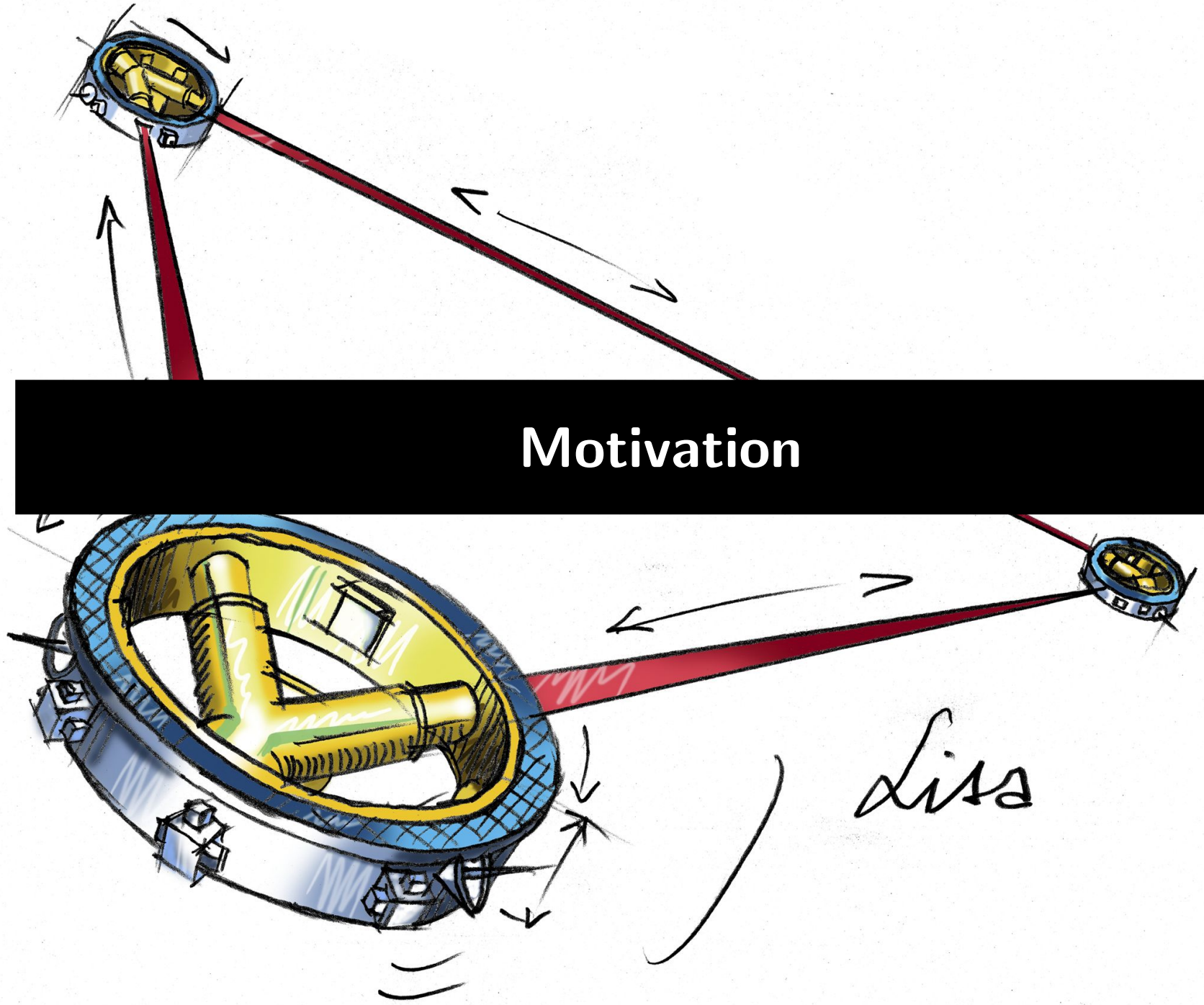

figure by ESA-C. Vijoux

The thermal history of electroweak symmetry breaking

Strong first-order cosmic phase transition at EW scale $T_c \sim 100$ GeV:

- Baryogenesis
- Colliding bubbles $\rightarrow$ Gravitational wave production

In Standard Model EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions.

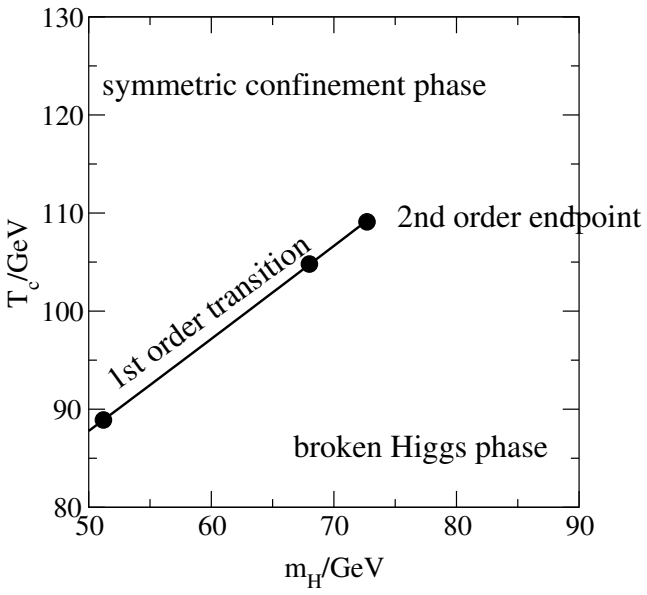

figure by M. Laine, *Electroweak phase transition beyond the standard model*, in *4th International Conference on Strong and Electroweak Matter*, pp. 58–69, 6, 2000 [hep-ph/0010275]

The thermal history of electroweak symmetry breaking

Strong first-order cosmic phase transition at EW scale $T_c \sim 100$ GeV:

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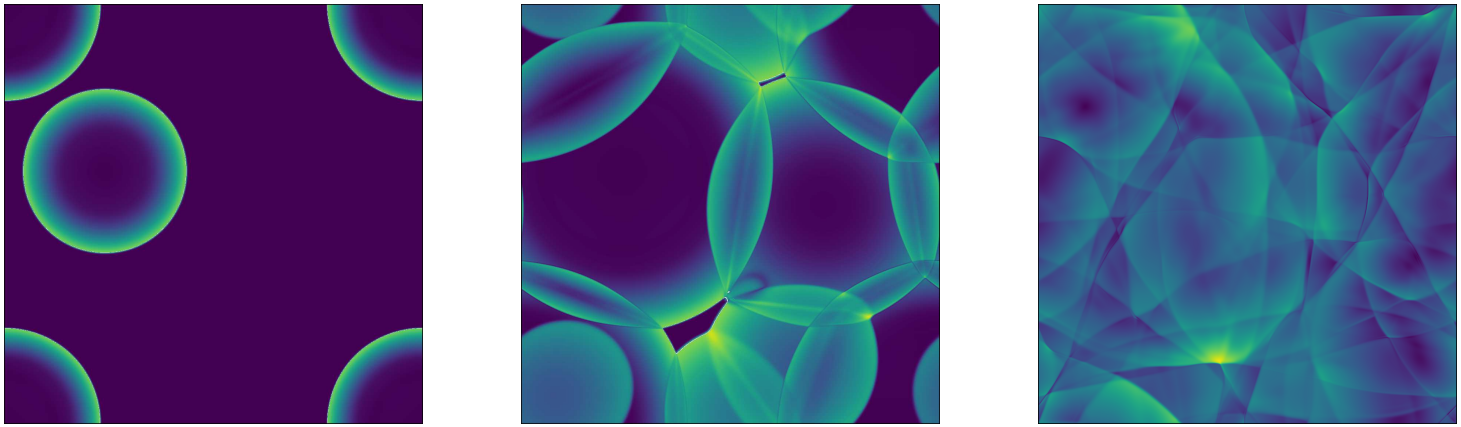
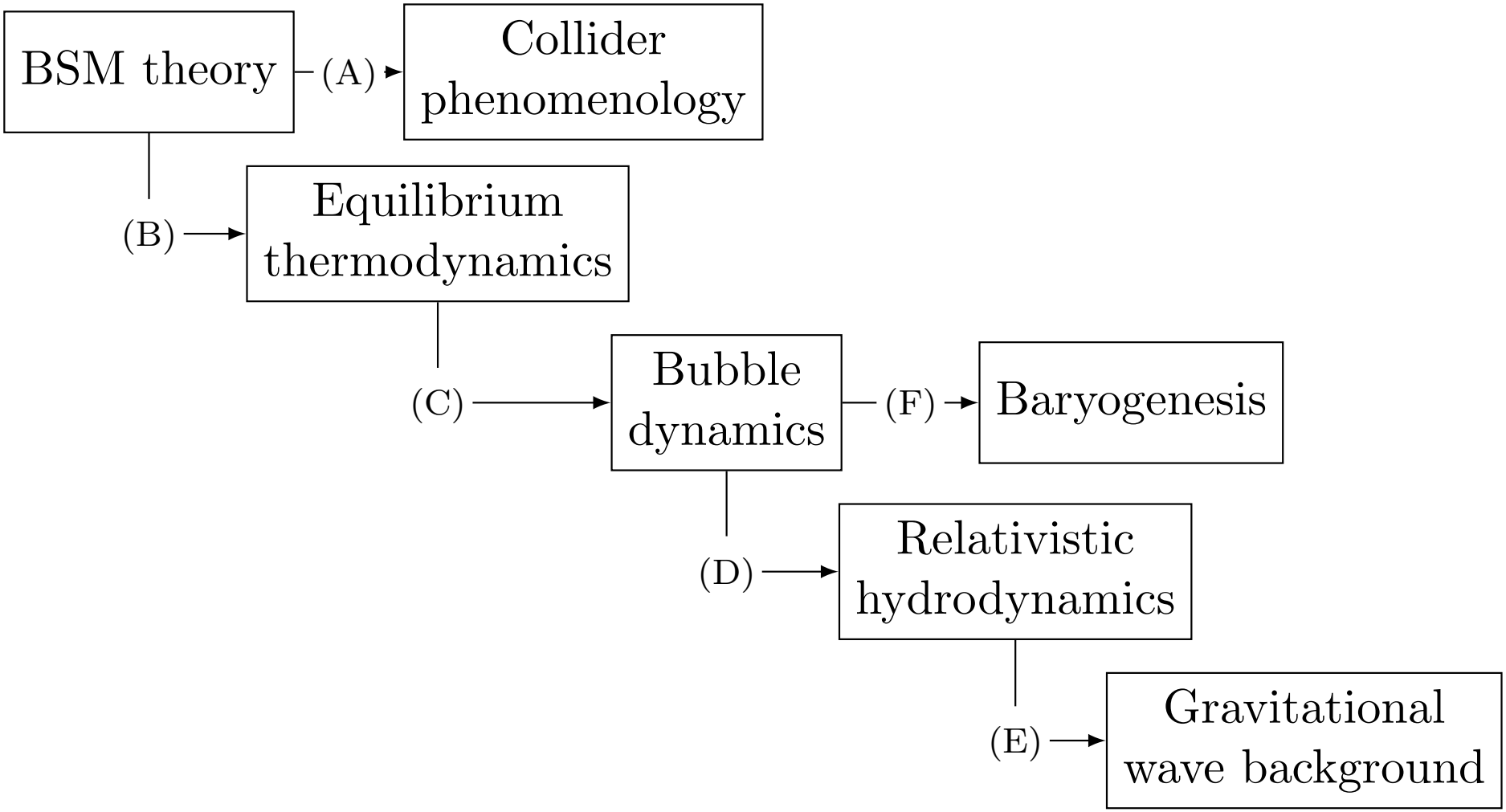
In Standard Model EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions. Study BSM physics near EW scale in context of phase transitions:

- Light fields strongly coupled to Higgs
- Collider targets. BSM testing pipeline: Collider phenomenology



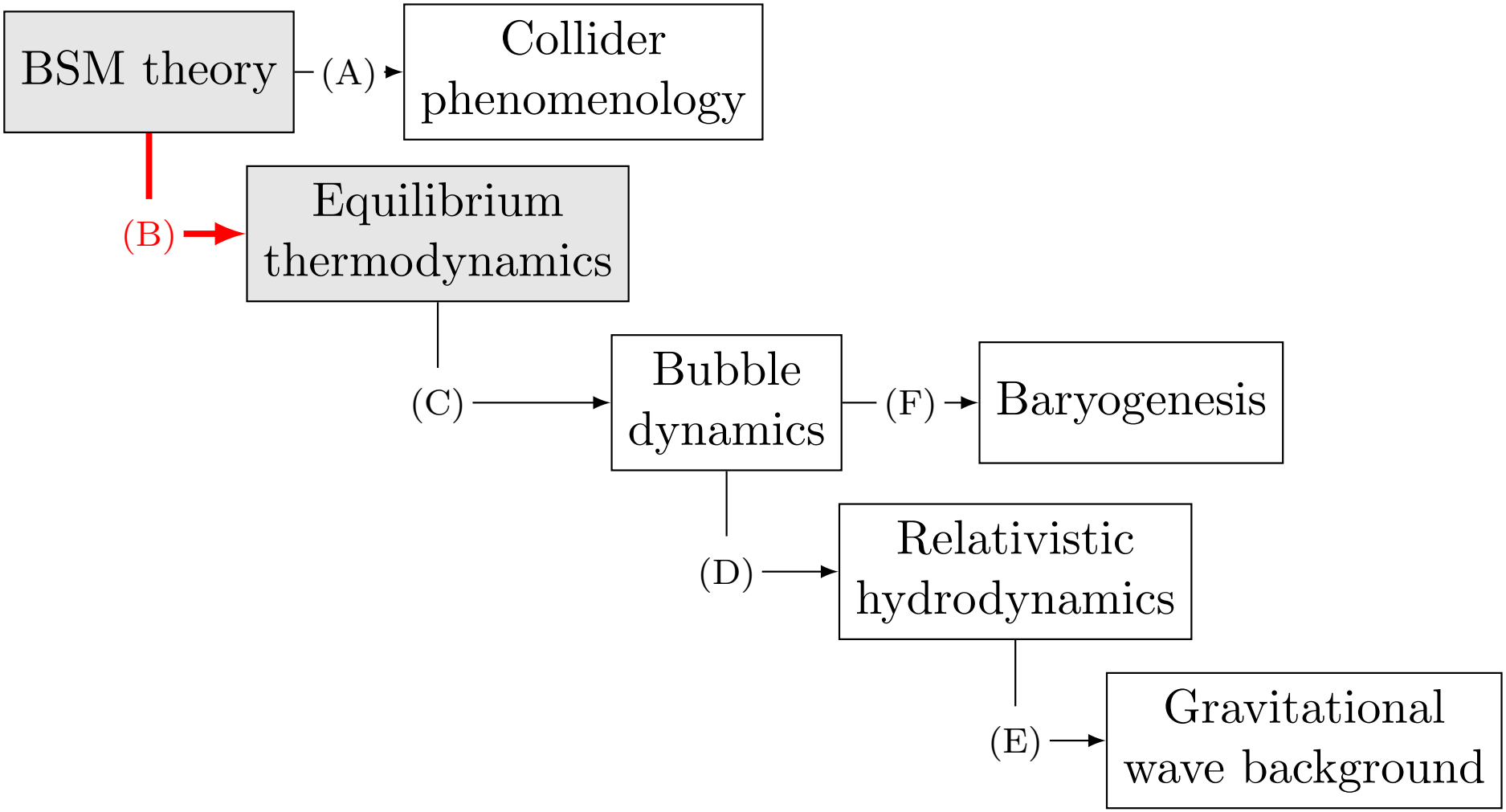
figures by D. Cutting, M. Hindmarsh, and D. J. Weir, *Vorticity, kinetic energy, and suppressed gravitational wave production in strong first order phase transitions*, Phys. Rev. Lett. **125** (2020) 021302 [1906.00480]

The gravitational wave pipeline



(B): Equilibrium thermodynamics as a function of BSM theory parameters. Encoded in effective potential, V^{eff}. Source of uncertainty.

The gravitational wave pipeline



(B): Equilibrium thermodynamics as a function of BSM theory parameters. Encoded in effective potential, V^{eff}. Source of uncertainty.

Theoretical predictions

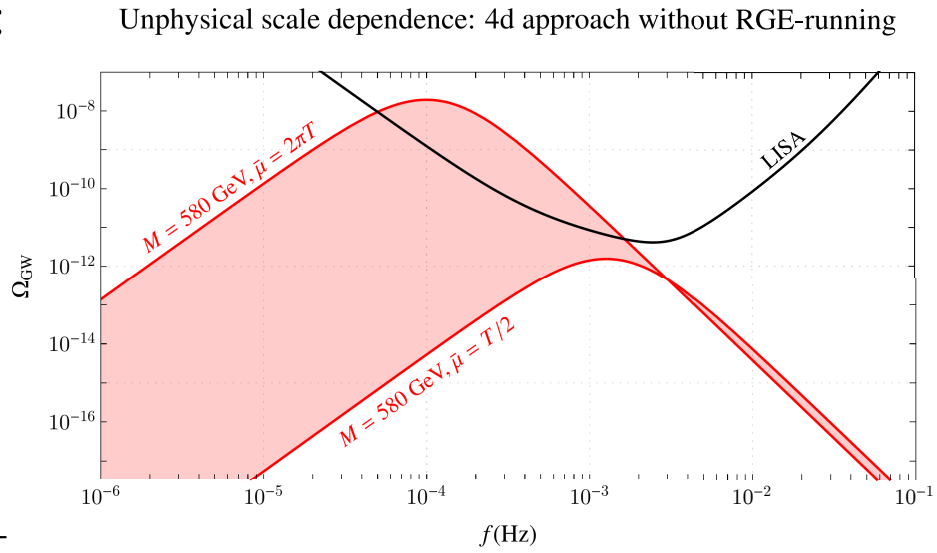
Problem: They are not robust.
$\mathcal{O}(10^4)$ uncertainty even for purely perturbative regimes[1] as $\Omega_{\rm GW}$ depends very strongly on the temperature of the transition:

$$\Omega_{\rm GW} \propto \frac{(\Delta V_*)^2}{T_*^8}$$

- Ensure (improve) quantitative precision at finite T?

Minimal SM extensions e.g.:

- SMEFT



[1] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions*, JHEP **04** (2021) 055 [2009.10080], O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

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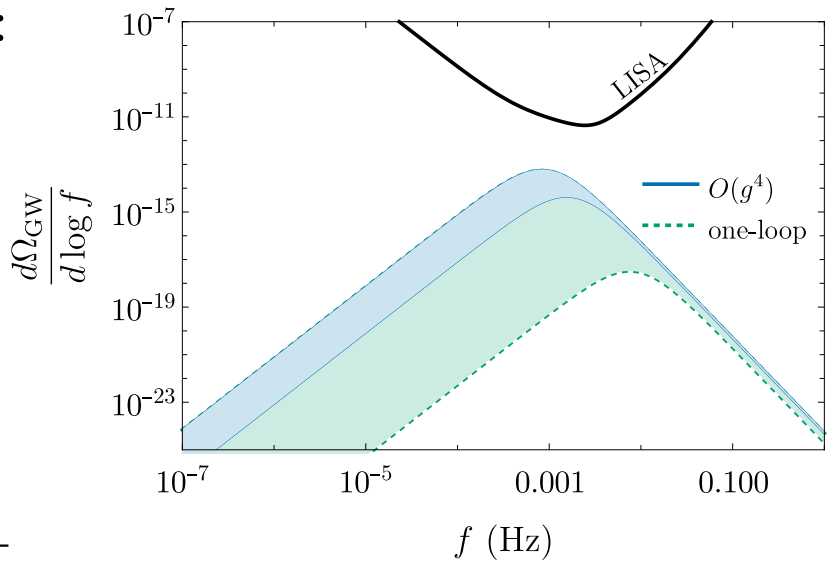
- Ensure (improve) quantitative precision at finite T?

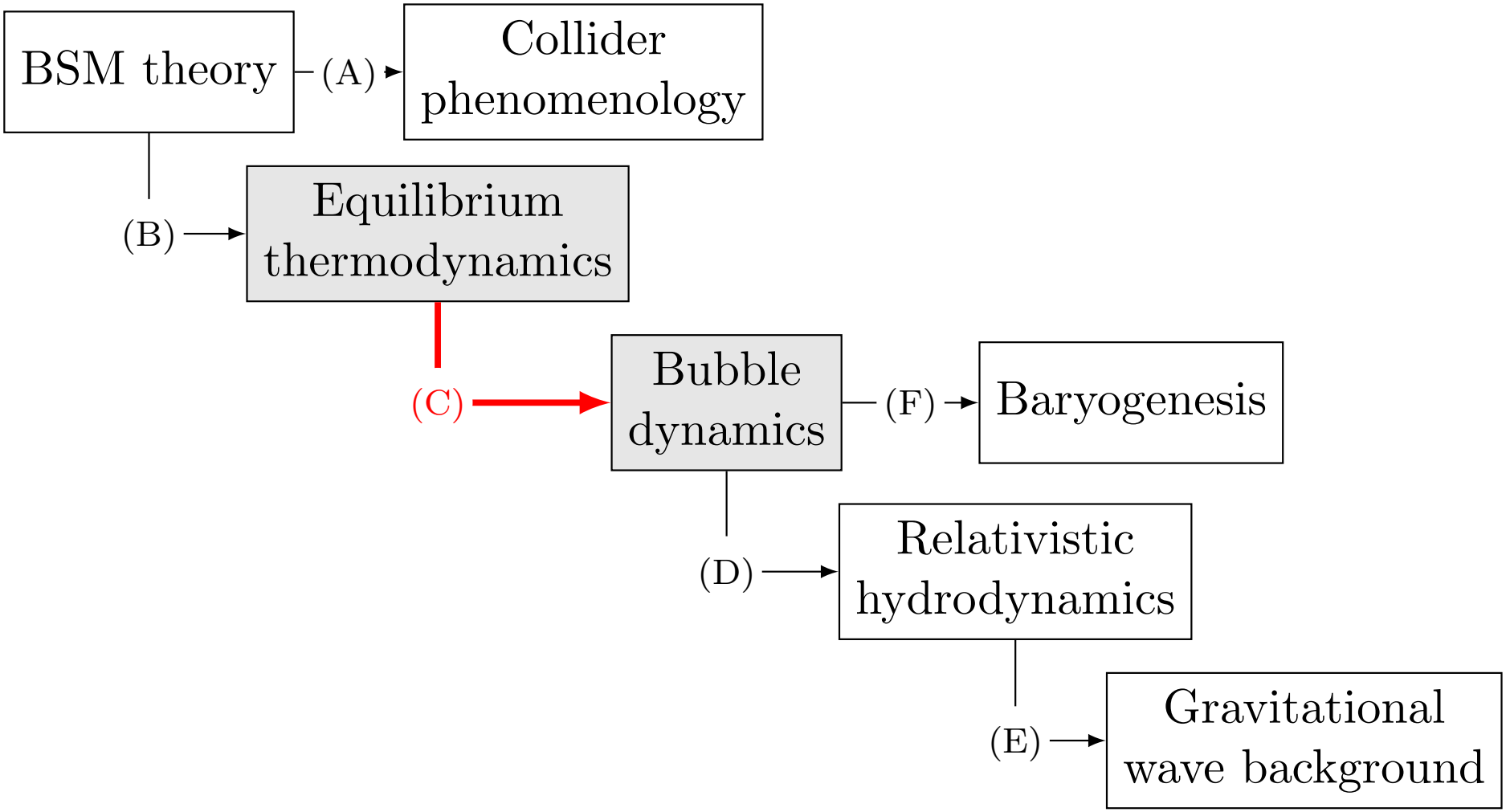
Minimal SM extensions e.g.:

- SMEFT
- SM + singlet (xSM)



[1] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions*, JHEP **04** (2021) 055 [2009.10080], O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

The gravitational wave pipeline



(C): Semi-classically approximate bubble nucleation rate by effective action, S^{eff}, incl. quantum/thermal corrections. Source of uncertainty.

Thermal escape: Nucleation rate

Problem: Does the existence of EWPT mean it happened?

Transition out of the false vacuum (FV) via large nucleation rate Γ.
Nucleation rate per unit volume schematically

$$\Gamma = \Gamma_{\rm dyn} \times \Gamma_{\rm stat} \ , \qquad \Gamma_{\rm stat} \simeq A\, e^{-\mathcal{B}} \ ,$$

$\mathcal{B}$ is the Euclidean action of the bounce solution and $A \sim \mathcal{O}(1) \times \sigma$.

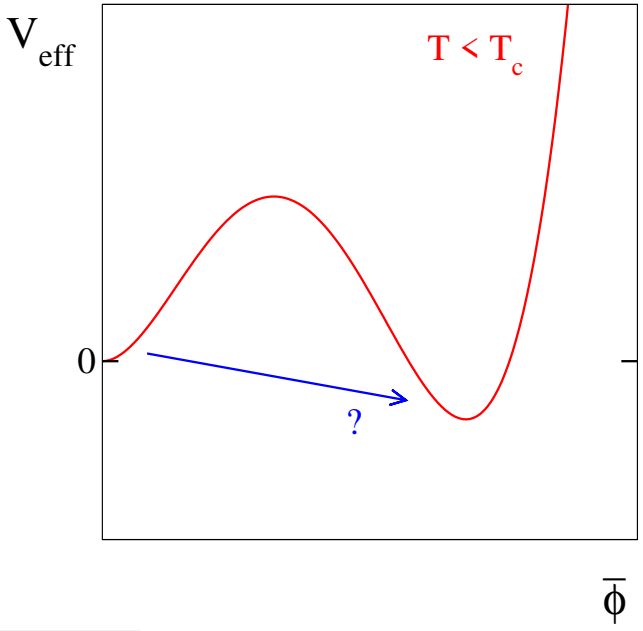


figure by M. Laine and A. Vuorinen, *Basics of Thermal Field Theory*, vol. 925 of *Lecture Notes in Physics*. Springer International Publishing, Cham, Jan, 2016, [1701.01554]

Nucleation rate at zero temperature

A minimal EW sector

Abelian Higgs (AH) model[2] with complex singlet (Φ).

$$\mathcal{L}_{\text{AH}} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + (D_\mu \Phi)^* (D_\mu \Phi) + V(\Phi) \ ,$$
$$V(\Phi) = \mu^2 \Phi^* \Phi + \lambda (\Phi^* \Phi)^2 \ .$$

$D_\mu = \partial_\mu - igB_\mu$. Expand in real fields $\Phi = \frac{1}{\sqrt{2}} (\phi + H + i\chi)$.

Phases at finite T

- Φ condenses, SM-like Higgs regime
- Gauge fields strongly coupled in IR
- Background $\phi \to \phi(r)$

[2] D. Metaxas and E. J. Weinberg, *Gauge independence of the bubble nucleation rate in theories with radiative symmetry breaking*, Phys. Rev. D **53** (1996) 836 [hep-ph/9507381]

Gauge invariance = gauge parameter independence

Gauge fixing functional induces ghost Lagrangian

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi} F^2 \,,$$
$$\mathcal{L}_{\mathrm{FP}} = -\bar{c}\, \delta_g F c \,,$$

- General covariant (Fermi) gauge: $F = \partial_\mu B^\mu$,
- Feynman gauge: $\xi = 1$,
- Unitary gauge: $\xi \to \infty$,
- Landau gauge $\xi \to 0$.

Fermi gauge introduces mixing between GB and B_μ after SSB for $\xi \neq 0$. Special choice R_ξ-gauge[3]:

$$F = -\big(\partial_i B_i + ig\xi(\tilde{\phi}^* \Phi - \Phi^* \tilde{\phi})\big) \,.$$

ξ-dependence cancels duly for (IR) physical observables $\to$ crosscheck.

[3] R. Kobes, G. Kunstatter, and A. Rebhan, *Gauge dependence identities and their application at finite temperature*, Nucl. Phys. B **355** (1991) 1

Standard vacuum bounce formalism

Find solution to bounce equation

$$\Box\phi = \left.\frac{\partial V}{\partial \phi}\right|_{\phi_b} \xrightarrow{V \to V^{\text{eff}}} \Box\phi = \left.\frac{\partial V^{\text{eff}}}{\partial \phi}\right|_{\phi_b} .$$

If radiative corrections induce SB, a bounce solution is non-existent (reliable). Replace: $V \to V^{\text{eff}}$.

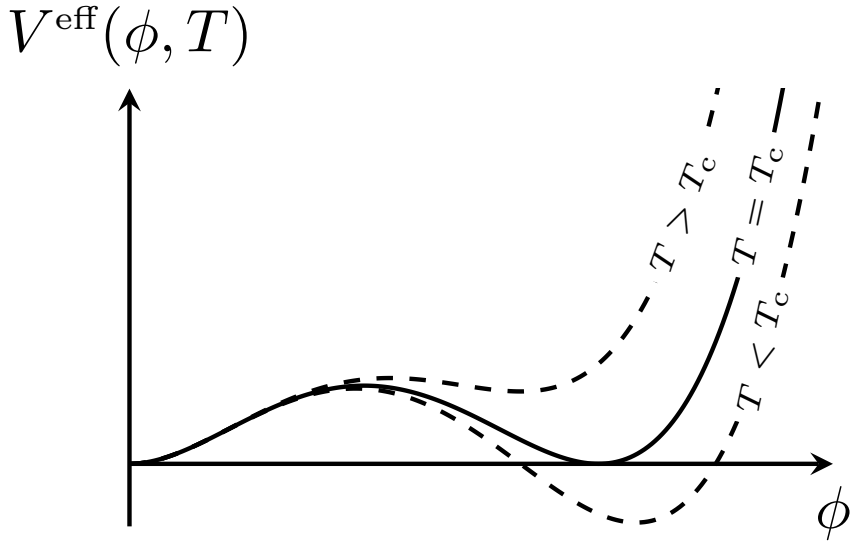
One option: Derivative expansion of effective action:

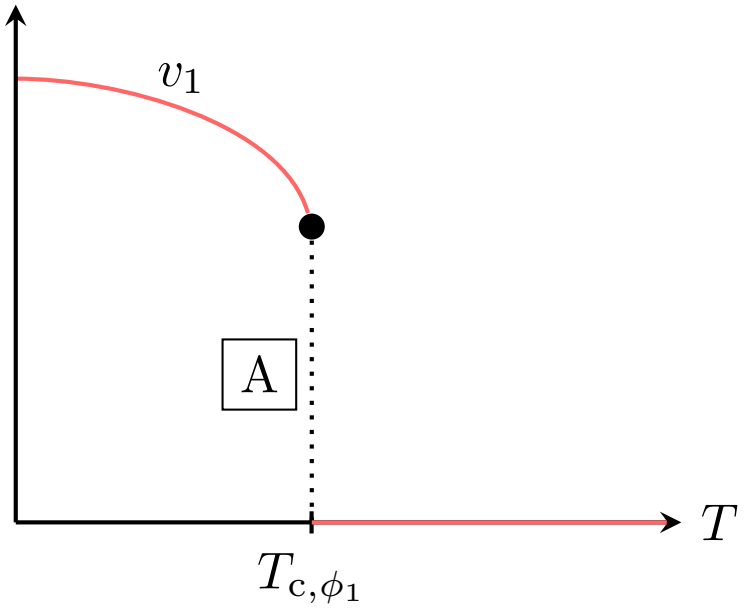
$$S^{\text{eff}} = \int \mathrm{d}^4 x \left[V^{\text{eff}}(\phi) + \frac{1}{2} Z(\phi) \left(\partial_\mu \phi\right)^2 + \ldots \right]$$

Perturbatively expand in weak coupling g

$$V^{\text{eff}} = V^{\text{eff}}_{g^4} + V^{\text{eff}}_{g^6} + V^{\text{eff}}_{g^8} + \ldots ,$$
$$Z = 1 + Z_{g^2} + Z_{g^4} + \ldots .$$

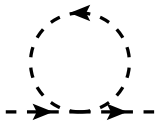
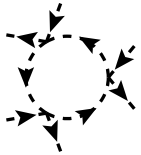
The effective potential in perturbation theory





$$V^{\rm eff} \simeq \frac{1}{2}(-\mu^2 + \Pi)\phi^2 + \frac{1}{2}\lambda\phi^4 + \#\phi^3 + \dots \ .$$

receives quantum (thermal) corrections $\Pi \sim g^n$ where $V^{\rm eff} = V^{\rm eff}_{\rm tree} + V^{\rm eff}_{1\ell}$. At 1-loop sum over n-point functions at $Q_i = 0$ external momenta

$V^{\rm eff}_{1\ell} =$  $+ \frac{1}{2}$  $+ \frac{1}{3}$  $+ \dots \Big|_{Q_i=0} \sim \frac{1}{2}\int_P \ln\left(P^2 + m_\alpha^2\right)$

The effective potential

at 1-loop for $\alpha = \{H, \chi, B, c\}$ has closed form:

$$V_{1\ell}^{\text{eff}} = \frac{1}{4(4\pi)^2} \sum_\alpha N_\alpha m_\alpha^4 \left(\ln\left[\frac{m_\alpha^2}{\Lambda^2}\right] - C_\alpha \right) .$$

The LO effective potential of AH is gauge independent ($m_B^2 \sim g^2\phi^2$)

$$V_{g^4}^{\text{eff}} = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \frac{3}{4(4\pi)^2}(g^2\phi^2)^2 \left(\ln\left[\frac{g^2\phi^2}{\Lambda^2}\right] - \frac{5}{6} \right) .$$

Barrier is realised for V^{eff} if Loop effects $\sim$ Tree-level effects

$$\mu^2\phi^2 \sim \lambda\phi^4 \sim g^4\phi^4 .$$

Expect breakdown of the loop expansion. However, perturbative expansion in g remains valid[4] for $\lambda \sim g^4$.
Determine scale at which nucleation occurs via LO bounce solution

$$\Box\phi = \left.\frac{\partial V_{g^4}^{\text{eff}}}{\partial\phi}\right|_{\phi_b} \implies \mu_{\text{eff}} = m_\chi = \mathcal{O}(g^2\sigma)$$

[4] S. Coleman and E. Weinberg, *Radiative Corrections as the Origin of Spontaneous Symmetry Breaking*, Phys. Rev. D **7** (1973) 1888 [0507214]

Nielsen identities[5]: A useful tool to show gauge invariance

Vary effective action with gauge parameter, $\int_{\mathbf{x}} \equiv \int \mathrm{d}^4 x$

$$\xi \frac{\partial S^{\mathrm{eff}}}{\partial \xi} = - \int_{\mathbf{x}} \frac{\delta S^{\mathrm{eff}}}{\delta \phi(x)} \mathcal{C}(x) \, .$$

Derivative expansion of Nielsen functional $\mathcal{C}(x) \sim \int_{\mathbf{y}} \langle c(x) \bar{c}(y) \Delta F(y) \rangle$:

$$\mathcal{C}(x) = C(\phi) + D(\phi)(\partial_\mu \phi)^2 - \partial_\mu \big(\tilde{D}(\phi) \partial_\mu \phi \big) + \mathcal{O}(\partial^4)$$

results in Nielsen identities

$$\xi \frac{\partial}{\partial \xi} V^{\mathrm{eff}} = -C \frac{\partial}{\partial \phi} V^{\mathrm{eff}} \, , \tag{1}$$

$$\xi \frac{\partial}{\partial \xi} Z = -C \frac{\partial}{\partial \phi} Z - 2Z \frac{\partial}{\partial \phi} C - 2D \frac{\partial}{\partial \phi} V^{\mathrm{eff}} - 2\tilde{D} \frac{\partial^2}{\partial \phi^2} V^{\mathrm{eff}} \, . \tag{2}$$

Identity (1): EWSB is gauge invariant.

[5] N. Nielsen, *On the gauge dependence of spontaneous symmetry breaking in gauge theories*, Nucl. Phys. B **101** (1975) 173

LO and NLO nucleation rate

Up to NLO relate $S^{\text{eff}} \to \mathcal{B}_{0,1}$

$$\Gamma = A' e^{-(\mathcal{B}_0 + \mathcal{B}_1)} \ ,$$

$$\mathcal{B}_0 = \int_{\mathbf{x}} \left[V^{\text{eff}}_{g^4}(\phi_b) + \frac{1}{2} Z_{g^0} \left(\partial_\mu \phi_b \right)^2 \right] \sim \mathcal{O}(g^{-4}) \ ,$$

$$\mathcal{B}_1 = \int_{\mathbf{x}} \left[V^{\text{eff}}_{g^6}(\phi_b) + \frac{1}{2} Z_{g^2} \left(\partial_\mu \phi_b \right)^2 \right] \sim \mathcal{O}(g^{-2}) \ .$$

Effective mass scale $\mu_{\text{eff}} \sim \mathcal{O}(g^2 \sigma)$ from V^{eff} at LO implies $\mathcal{B}_{0,1}$ scaling.

Perturbatively expand Nielsen identities e.g. $(C = C_{g^2} + \dots)$ to verify gauge independence i.e. $\xi \frac{\partial \mathcal{B}_1}{\partial \xi} = 0$

$$\xi \frac{\partial}{\partial \xi} V^{\text{eff}}_{g^6} = -C_{g^2} \frac{\partial}{\partial \phi} V^{\text{eff}}_{g^4} \ ,$$

$$\xi \frac{\partial}{\partial \xi} Z_{g^2} = -2 \frac{\partial}{\partial \phi} C_{g^2} \ .$$

Aiming higher

Diagrams contributing to $\mathcal{B}_{0,1}$ contain only propagating gauge fields. Consider higher-order contributions of

$$\Gamma = A'' e^{-(\mathcal{B}_0+\mathcal{B}_1+\mathcal{B}_2)} \,,$$
$$\mathcal{B}_2 = \int_{\mathbf{x}} \left[V_{g^8}^{\text{eff}}(\phi_b) + \frac{1}{2} Z_{g^4} \left(\partial_\mu \phi_b\right)^2 + \ldots \right] \sim \mathcal{O}(g^0) \,.$$

$\mathcal{B}_2$ contains propagating scalar fields with characterstic $P \sim m_H \sim m_\chi$. The effective expansion parameter $P/\Lambda = P/m_H \sim \mathcal{O}(1) \implies$ breakdown.

Consistently improve statisical Γ_{stat}[6] and dynamical Γ_{dyn}[7] prefactor.

[6] A. Ekstedt, *Higher-order corrections to the bubble-nucleation rate at finite temperature*, Eur. Phys. J. C **82** (2022) 173 [2104.11804], I. M. Gelfand and A. M. Yaglom, *Integration in functional spaces and it applications in quantum physics*, J. Math. Phys. **1** (1960) 48

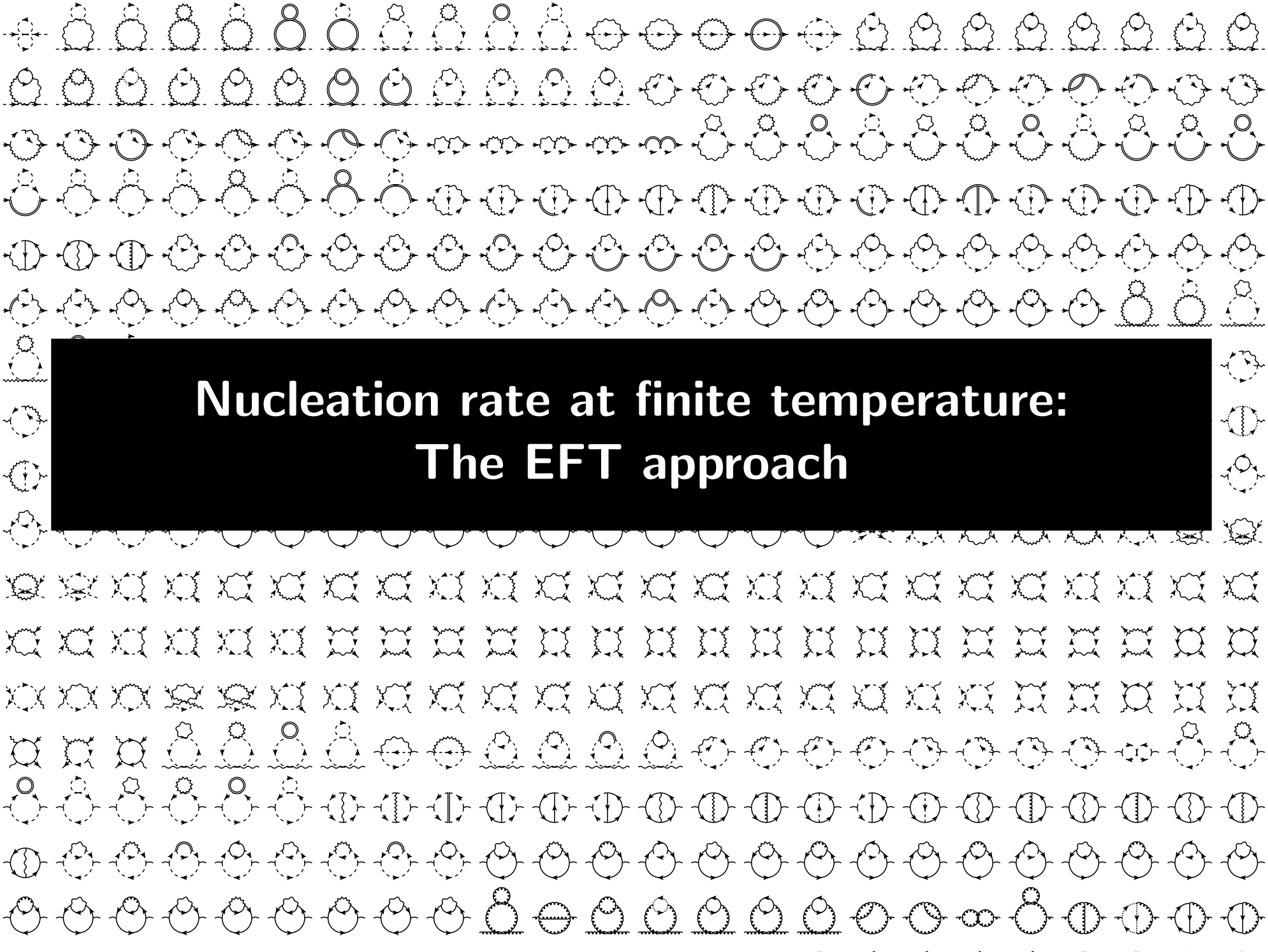
[7] A. Ekstedt, *Bubble nucleation to all orders*, JHEP **08** (2022) 115 [2201.07331]

Gauge invariance at zero T

LO contributions to $\mathcal{B}_0$ are already gauge-independent.

NLO contribution $\mathcal{B}_1$ require Nielsen identities:

$$
\begin{aligned}
\xi \frac{\partial}{\partial \xi} \mathcal{B}_1 &= \xi \frac{\partial}{\partial \xi} \int_{\mathbf{x}} \left[V_{g^6}^{\text{eff}}(\phi_b) + \frac{1}{2} Z_{g^2} \left(\partial_\mu \phi_b\right)^2 \right] && \text{(Nielsen identity)} \\
&= \int_{\mathbf{x}} \left[-C_{g^2} \frac{\partial}{\partial \phi} V_{g^4}^{\text{eff}}(\phi_b) - \frac{\partial C_{g^2}}{\partial \phi} (\partial_\mu \phi_b)^2 \right] && \text{(chain rule)} \\
&= -\int_{\mathbf{x}} \left[C_{g^2} \frac{\partial}{\partial \phi} V_{g^4}^{\text{eff}}(\phi_b) + \partial^\mu C_{g^2} (\partial_\mu \phi_b) \right] && \text{(IBP)} \\
&= -\int_{\mathbf{x}} C_{g^2} \left[\frac{\partial}{\partial \phi} V_{g^4}^{\text{eff}}(\phi_b) - \Box \phi_b \right] && \text{(EOM)} \\
&= 0 && .
\end{aligned}
$$



Nucleation rate at finite temperature: The EFT approach

Equilibrium Thermodynamics: Imaginary Time Formalism

$\rho(\beta) = e^{-\beta\mathcal{H}} \to \mathcal{U}(t) = e^{-i\mathcal{H}t}$. Relating density operator to time evolution corresponds to path integral over imaginary-time $t \to -i\tau$,

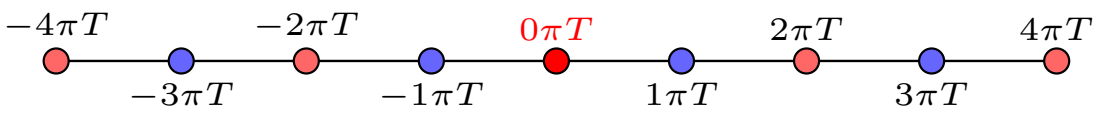
$$\mathcal{Z} = C \int_{\text{b.c.}} \mathcal{D}\phi \, \exp\left[- \int_0^{\beta=1/T} \mathrm{d}\tau \int_{\mathbf{x}} \mathcal{L}_{\mathrm{E}} \right] , \quad \phi(0,\mathbf{x}) = \pm\phi(\beta,\mathbf{x}) \, .$$

(Anti-)periodic bosonic(fermionic) fields at boundaries $\to$ compactified time direction: $\mathbb{R}^3 \times S^1_\beta$.

Finite-τ and (b.c.) induce a discrete Fourier sum for time component $P = (\omega_n, \mathbf{p})$ with Matsubara frequencies

$$\omega_n = \begin{cases} 2n\pi T & \text{bosonic} \\ (2n+1)\pi T & \text{fermionic} \end{cases}$$

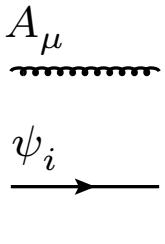
Absent zero mode $\omega_{n=0}$ for fermions:



Differences to zero temperature

$(d+1) \to d$-dimensional theory with infinite tower of massive modes.

Euclidean free-particle propagator for bosonic(fermionic) fields:



$$\propto \frac{1}{P^2+m^2} = \frac{1}{\mathbf{p}^2+\omega_n^2+m^2}\,, \qquad P=(\omega_n,\mathbf{p})\,.$$

Same diagrams as in zero temperature QFT, go over to Euclidean space, and substitute Euclidean frequency integrals by sums

$$\int \frac{\mathrm{d}^{d+1}p}{(2\pi)^{d+1}} f(p) \to T\sum_n \int \frac{\mathrm{d}^d p}{(2\pi)^d} f(\omega_n,\mathbf{p}) = \sum\!\!\!\!\!\!\int_P f(\omega_n,\mathbf{p})\,.$$

- Ultraviolet (UV) contained at $T=0$
- Infrared (IR) sensitivity worsened $\to$ field in reduced spacetime dimension

Multi-scale Hierarchy in hot gauge theories

Evaluate Matsubara sums yielding Bose(Fermi) distribution. At asymptotically high-T and weak $g \ll 1$ the effective expansion parameter

$$\epsilon_{\mathrm{B}} \equiv g^2 n_{\mathrm{B}}(|p|) = \frac{g^2}{e^{|p|/T} - 1} \approx \frac{g^2 T}{|p|}$$

differs from the weak coupling g^2. Fermions are IR-safe $g^2 n_{\mathrm{F}} |p| \sim g^2/2$.

Theory separates scales rigorously:

$$|p| \sim \begin{cases} \pi T & \textit{hard} \text{ scale} \\ gT & \textit{soft} \text{ scale} \\ \mu_{\mathrm{nucl}} & \textit{(softer) nucleation} \text{ scale} \\ g^2 T/\pi & \textit{ultrasoft} \text{ scale} \end{cases}$$

Limit: Confinement-like behavior in ultrasoft sector $g^2 n_{\mathrm{B}}(g^2 T) \sim \mathcal{O}(1)$. Light bosons are non-perturbative at finite T: Linde's IR problem[8].

[8] A. Linde, *Infrared problem in the thermodynamics of the Yang-Mills gas*, Phys. Lett. B **96** (1980) 289

Resummation

Dynamically generated masses through collective plasma effects

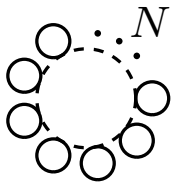
$$m_T = g^n T + m \ .$$

Evaluate Matsubara sums yielding Bose(Fermi) distribution. At asymptotically high-T and weak $g \ll 1$ the effective expansion parameter

$$\epsilon_{\rm B} \equiv g^2 n_{\rm B}(|p|) = \frac{g^2}{e^{|p|/T} - 1} \approx \frac{g^2 T}{|p|} \geq \frac{g^2 T}{m}$$

differs from the weak coupling g^2. Fermions are IR-safe $g^2 n_{\rm F}|p| \sim g^2/2$.

Cure IR sensittive contributions at $m_T \sim gT$ by thermal resummation:



$$\propto \quad g^{2N}\left[{m_T}^{3-2N} T\right]\left[\frac{T^2}{12}\right]^N \propto m^3 T\left[\frac{gT}{m_T}\right]^{2N}$$

For $m_T \leq g^2 T$ weak expansion breaks down. At finite T light bosons are non-perturbative .

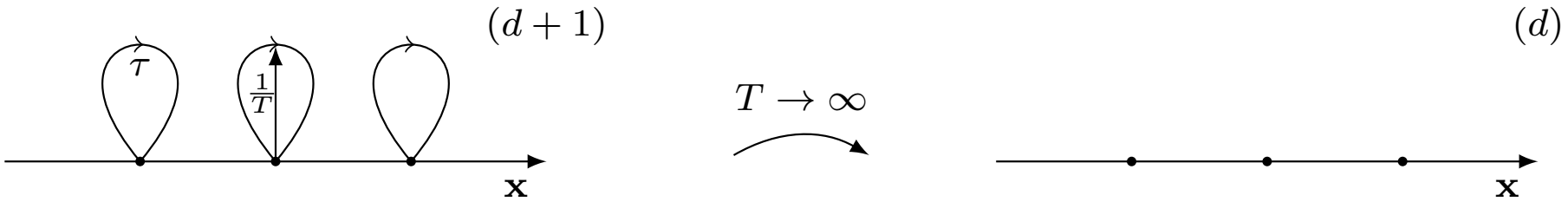
Effective Theory (EFT): Definition

Framework to describe theory with scale hierarchy: Effective Field Theory.

1. Identify soft degrees of freedom.
2. Construct most general low-energy Lagrangian.
3. *Match* Green's functions $\rightarrow$ determine EFT coupling constants.

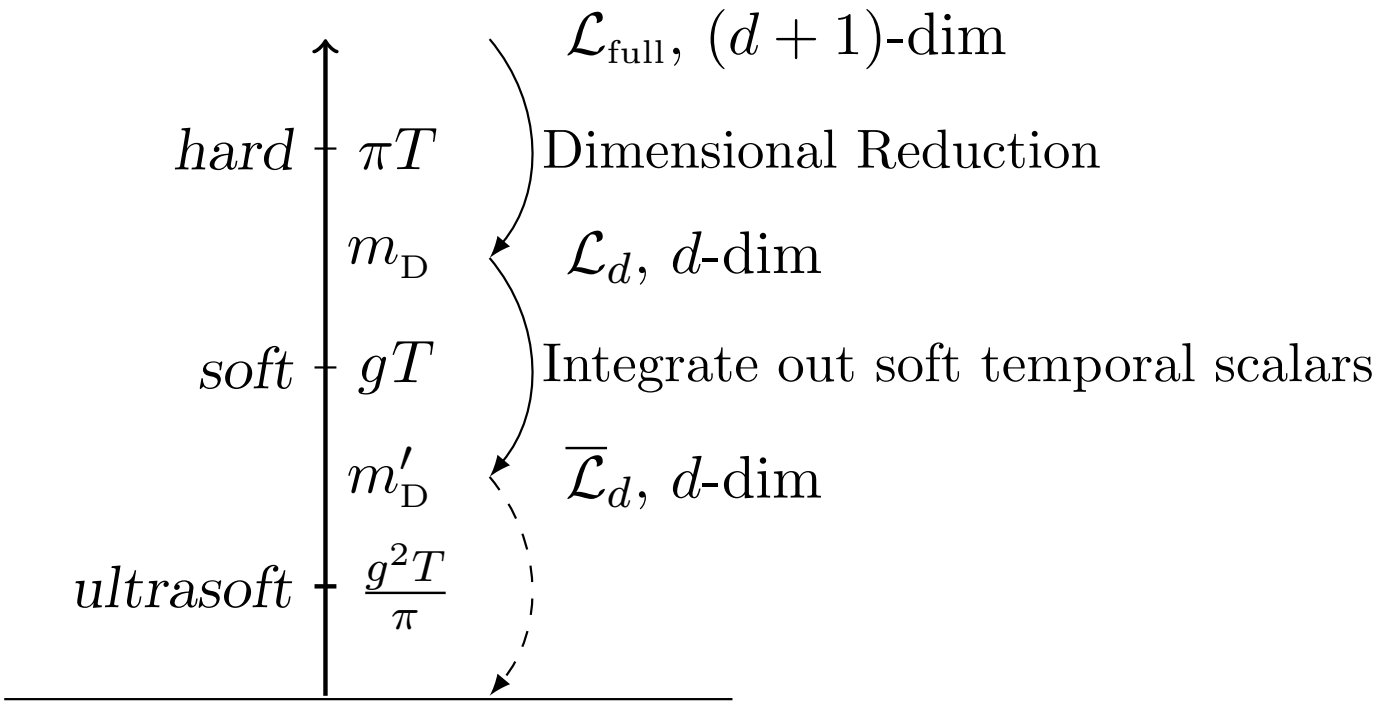
Perturbative and IR safe: Matching in the IR (regime of mutual validity), UV incorporated in EFT coefficients.

Modes with wavelengths $|\mathbf{x}|, |x_0| \gg \beta$ or $\omega_n^2 + m^2 \ll T^2$ *effectively* live in 3-dimensions.



Dimensional Reduction (DR)

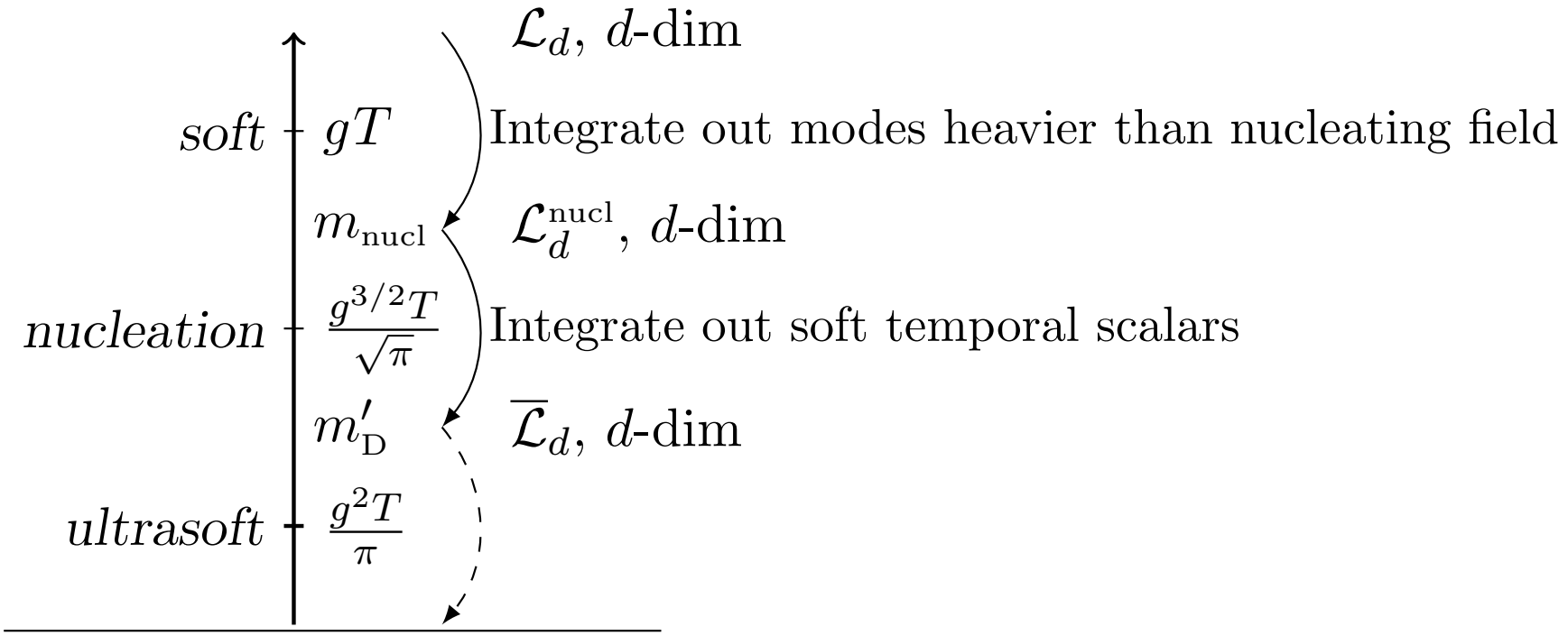
Integrate out hard modes perturbatively → EFT for static modes.
Incorporates an all order thermal resummation to by-pass IR problem.
Applications for thermodynamics of non-Abelian gauge theories such as (EW) phase transitions[9] and QCD.



[9] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379], K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020], T. Brauner, T. V. I. Tenkanen, A. Tranberg, A. Vuorinen, and D. J. Weir, *Dimensional reduction of the Standard Model coupled to a new singlet scalar field,* JHEP **2017** (2016) 7 [1609.06230]

Dimensional Reduction (DR)

Integrate out hard modes perturbatively → EFT for static modes.
Incorporates an all order thermal resummation to by-pass IR problem.
Applications for thermodynamics of non-Abelian gauge theories such as (EW) phase transitions[9] and QCD.



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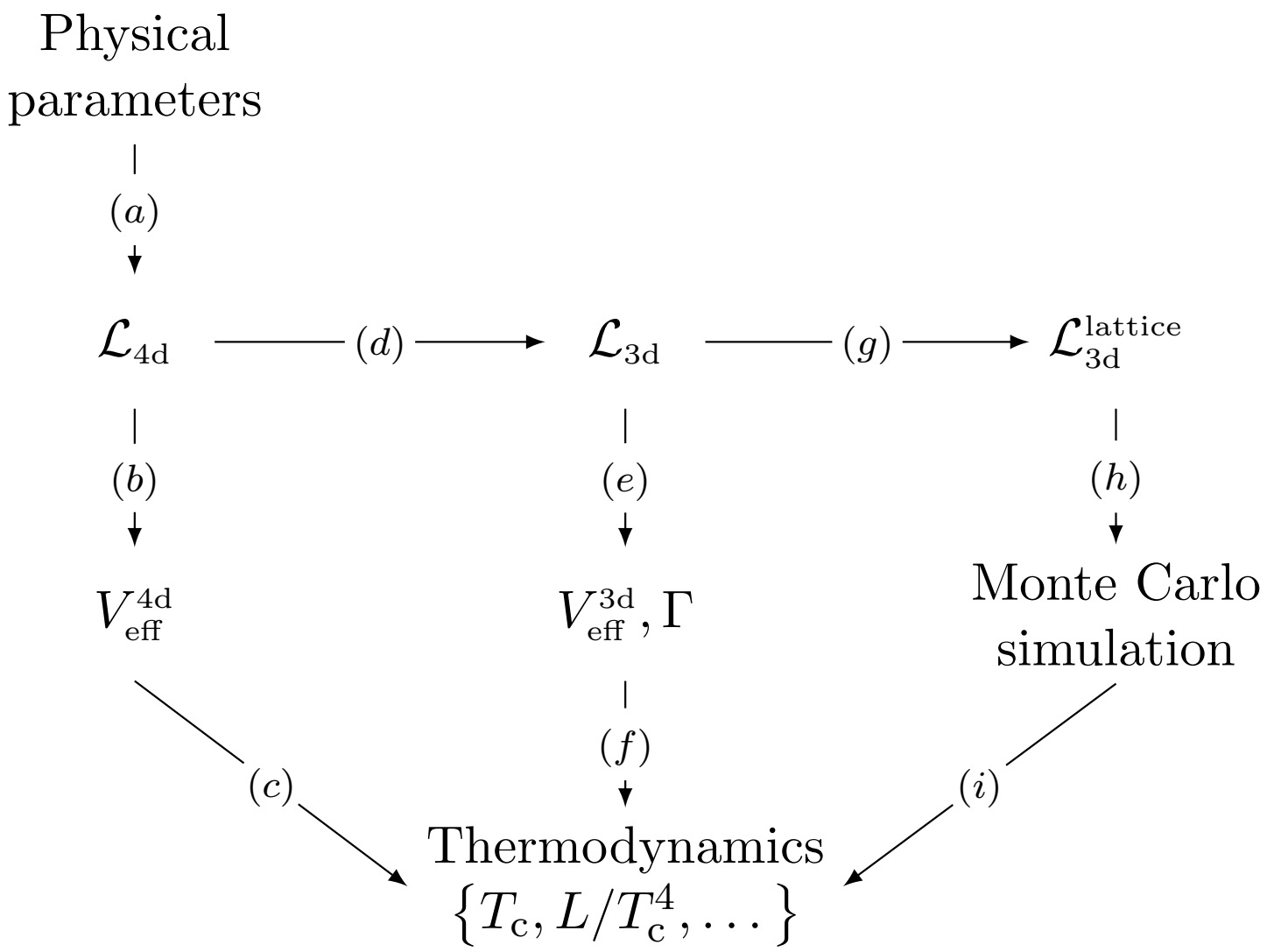
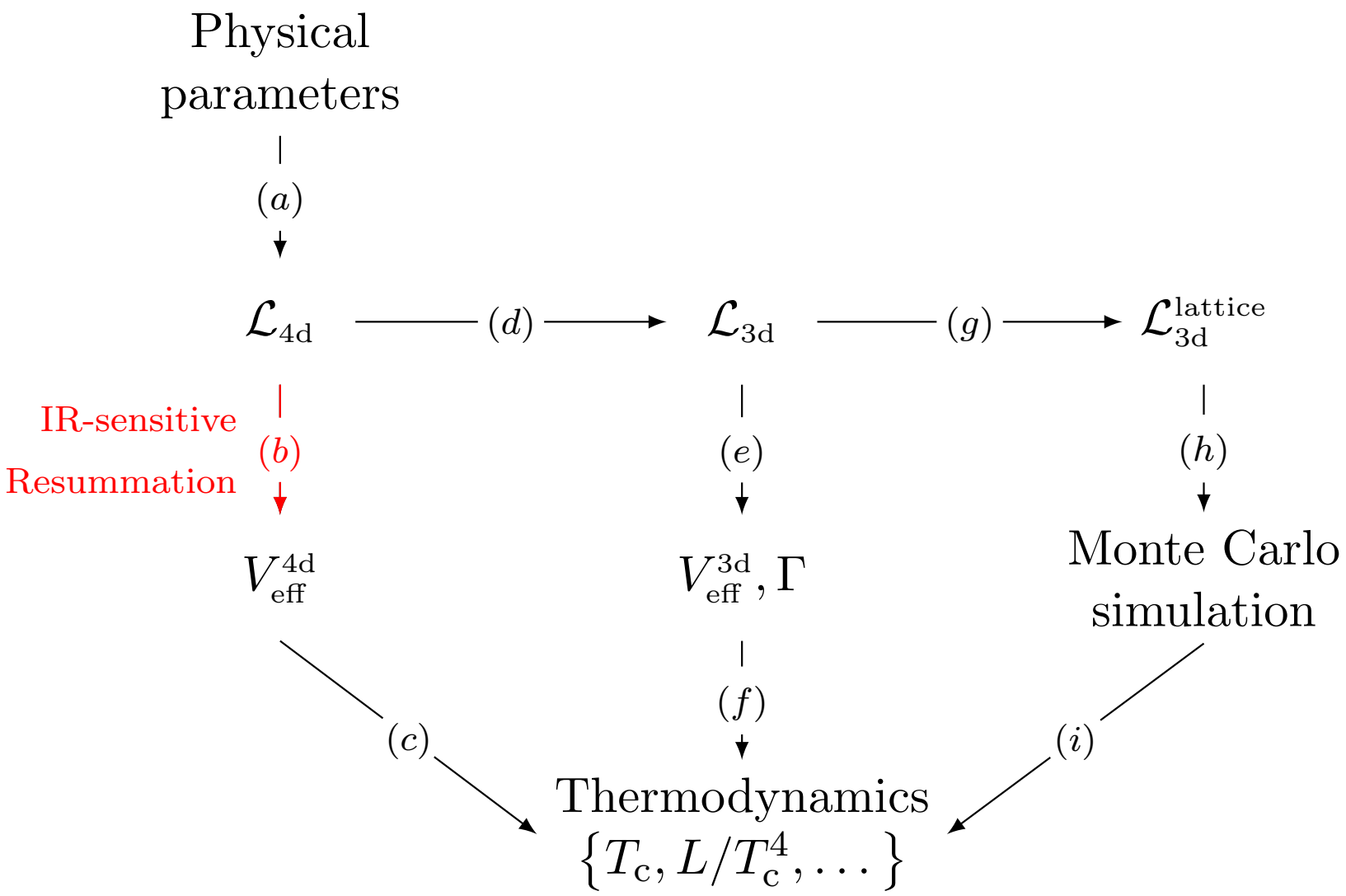
Thermodynamics of electroweak phase transition

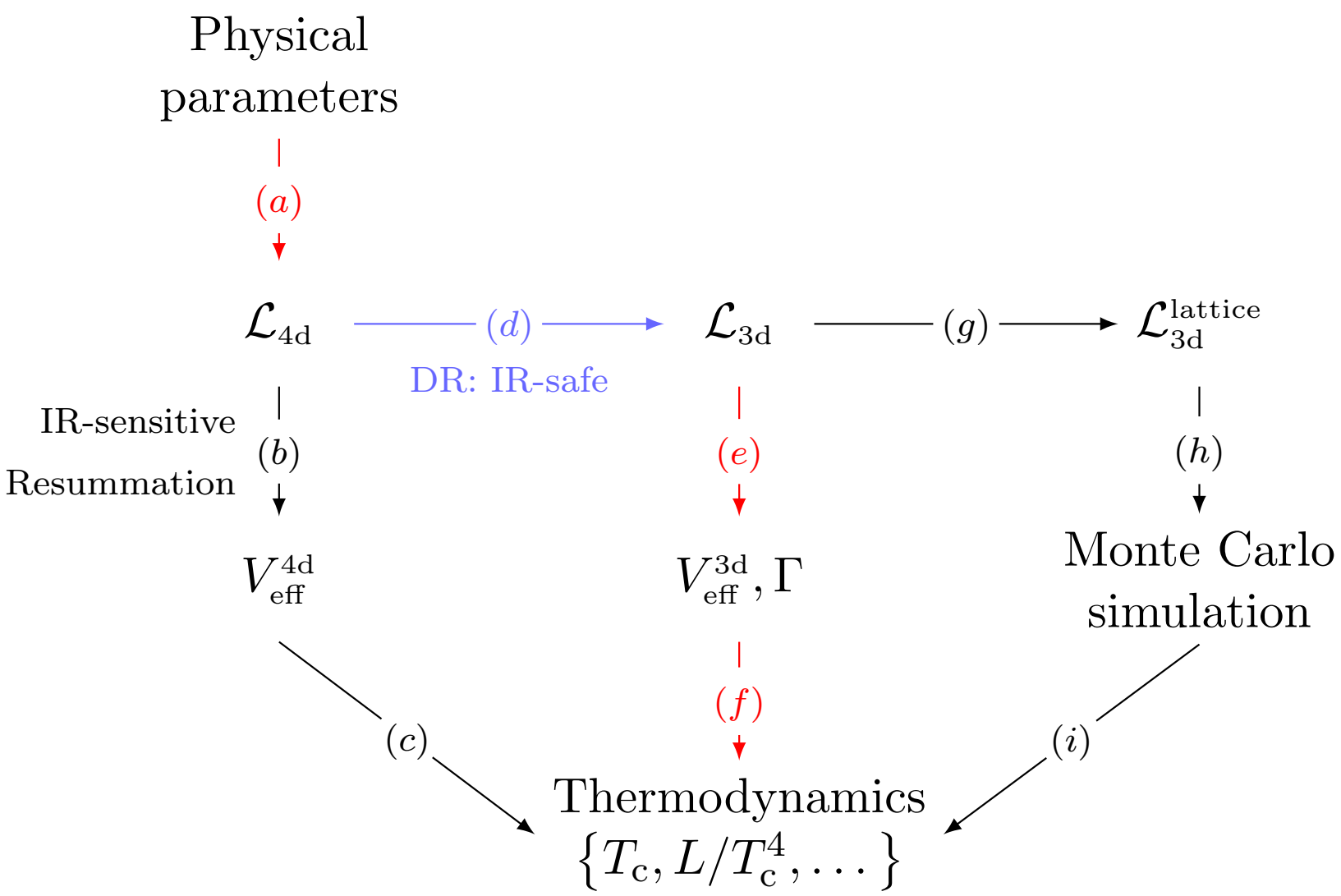


Thermodynamics of electroweak phase transition



- $4d$ approach: $(a) \to (b) \to (c)$

Thermodynamics of electroweak phase transition



- $4d$ approach: $(a) \to (b) \to (c)$
- Perturbative $3d$ approach: $(a) \to (d) \to (e) \to (f)$

Step 1: Dimensionally reduced effective theory

Describe theory by $3d$ EFT[10]. Super-renormalisable "Electrostatic AH" (E-AH) to study high-T thermodynamics with UV dynamics inside matching coefficients:

$$\mathcal{L}_{\rm AH}^{\rm 3d} = \frac{1}{4}F_{3,ij}F_{3,ij} + (D_i\Phi_3)^*(D_i\Phi_3) + V(\Phi_3) + \mathcal{L}_{\rm temp}^{\rm 3d} \,,$$
$$V^{\rm 3d}(\Phi_3) = \mu_3^2\Phi_3^*\Phi_3 + \lambda_3(\Phi_3^*\Phi_3)^2 \,.$$

Broken Lorentz symmetry induces temporal-scalars coupling to the complex singlet

$$\mathcal{L}_{\rm temp}^{\rm 3d} = \frac{1}{2}(\partial_r B_0)^2 + \frac{1}{2}m_{\rm D}^2(B_0)^2 + \frac{1}{4}\kappa_3(B_0)^4 + h_3\Phi_3^*\Phi_3(B_0)^2 \,.$$

Truncate operators at high T:

$$S_{\rm AH}^{\rm 3d} = \frac{1}{T}\int_{\mathbf{x}}\left\{\mathcal{L}_{\rm AH}^{\rm 3d} + \sum_{n\geq 5}\frac{\mathcal{O}_n}{(\pi T)^n}\right\} \,.$$

[10] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model*, Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

Step 1: AH $\to$ E-AH

Inspect Higgs potential: $V(\phi) \supset \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$.

DR step 1 fixes high-T E-AH. EFT for Electrostatic modes ($D_i = \partial_i - ig_3A_i - ig_3'B_i$). Describes AH IR dynamics and contains UV in matching coefficients:

$$\mu_3^2 = \underbrace{\boxed{\underset{\text{tree-level}}{\mu^2} + \underset{\text{1-loop}}{\#g^2T^2}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\underset{\text{1-loop}}{\#g^2\mu^2} + \underset{\text{2-loop}}{\#g^4T^2}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6)\ ,$$

$$\lambda_3 = \underbrace{\boxed{\underset{\text{tree-level}}{T\lambda}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\underset{\text{1-loop}}{\#g^4}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6)\ .$$

Dimensional Reduction automated (DRalgo)

State-of-the-art Mathematica package DRalgo.[11]
Supply model Lagrangian $\mathcal{L}_{4\mathrm{d}}$:

$\mathcal{L}_{4\mathrm{d}}$ —❶→ Feynman Rules

↓❷

QGRAF[12] → n-point diagrams

↓❸

Canonization
$q_i \sim \mathcal{O}(gT)$

↓❹

Algebra:
Group, Lorentz, Dirac

↓❺

$\int \mathcal{L}_K \partial_{k_i} f_i(k) = 0$ → IBP[13] —❻→ Masters —❼→ Match: $\mathcal{L}_{3\mathrm{d}}, V_{3\mathrm{d}}^{\mathrm{eff}}$

[11] A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, DRalgo*: a package for effective field theory approach for thermal phase transitions,* [2205.08815]

[12] P. Nogueira, *Automatic Feynman Graph Generation,* J. Comput. Phys. **105** (1993) 279

[13] S. Laporta, *High precision calculation of multiloop Feynman integrals by difference equations,* Int. J. Mod. Phys. A **15** (2000) 5087 [hep-ph/0102033]

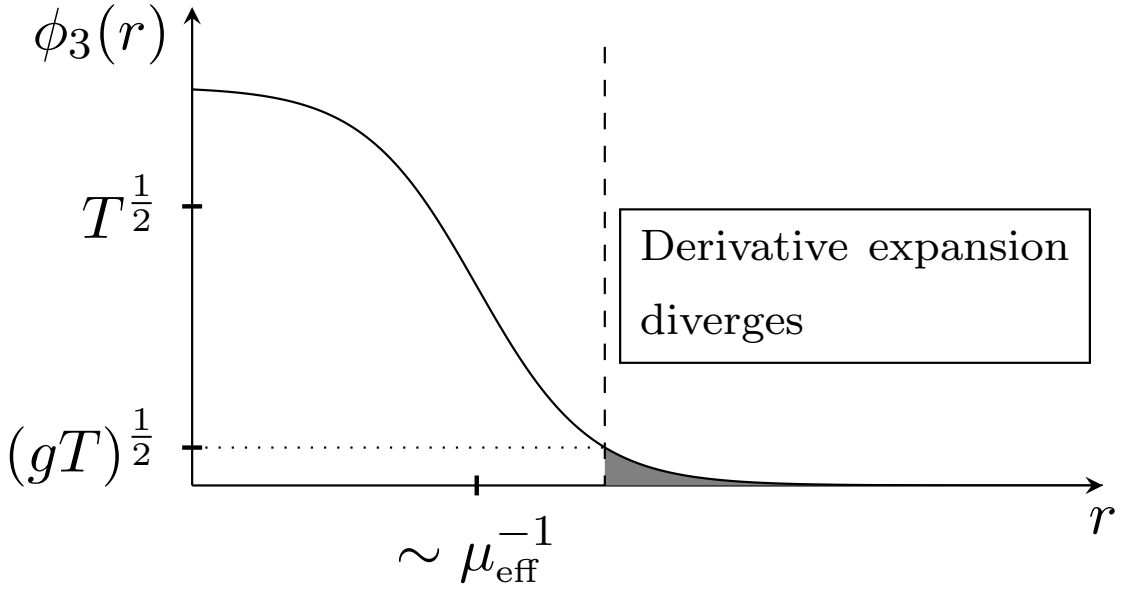
Step 2: Nucleation scale effective theory

$3d$ EFT[14] at the nucleation scale set by mass of the nucleating d.o.f.

$$\Lambda_{\rm nucl} = \mu_{\rm eff} \ll (\pi T) \ .$$

Two gauge-independent steps in EFT matching. Possible remaining gauge dependence from within $T = 0$ and d-dimensional nucleation EFT $\implies$ vacuum nucleation theory.

Scale-shifters: ultrasoft in the symmetric, soft in the broken phase.



[14] O. Gould and J. Hirvonen, *Effective field theory approach to thermal bubble nucleation*, Phys. Rev. D **104** (2021) 096015 [2108.04377]

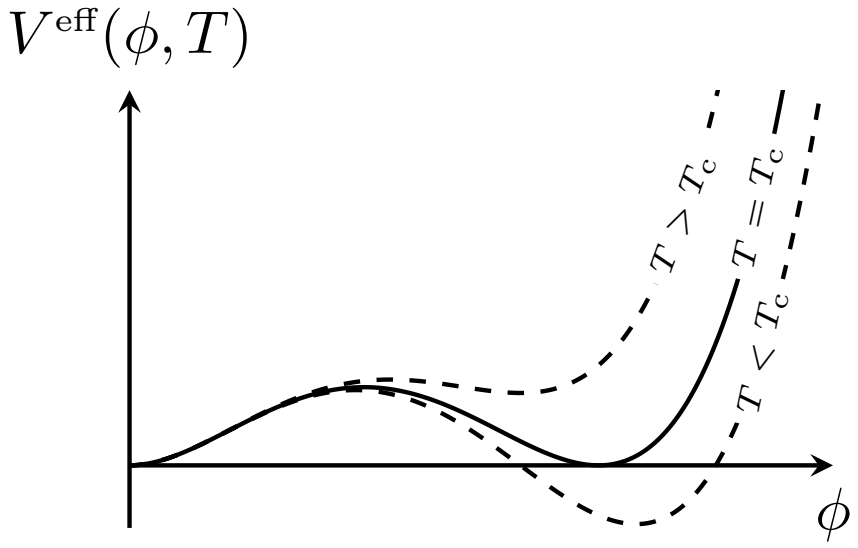
The effective potential in perturbation theory

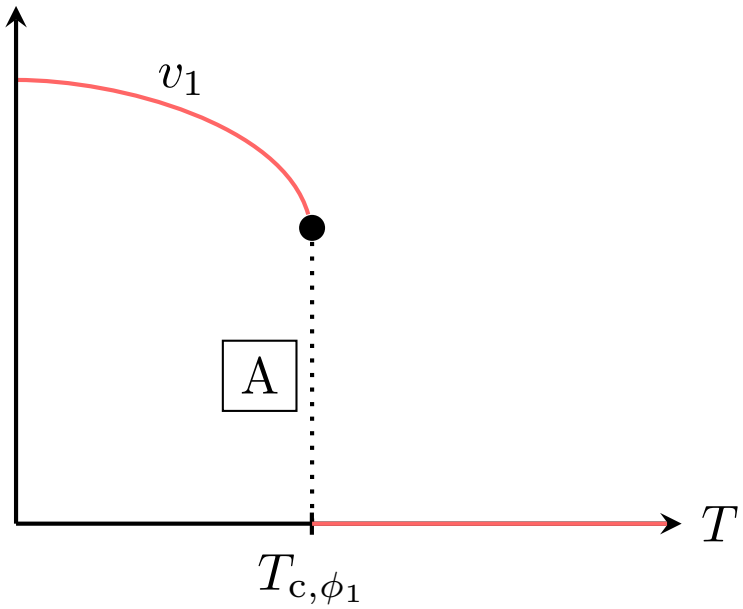
receives thermal corrections $\Pi_T \sim \gamma T^2$ with $\gamma \sim g^n$:

$$V^{\text{eff}} \simeq \frac{1}{2}(-\mu^2 + \Pi_T)\phi^2 + \frac{1}{2}\lambda\phi^4 + \#\phi^3 + \dots$$

Close to critical temperature T_{c}:

$$(-\mu^2 + \gamma T^2) \sim 0 \times (gT)^2 + (g^2 T)^2 \ .$$





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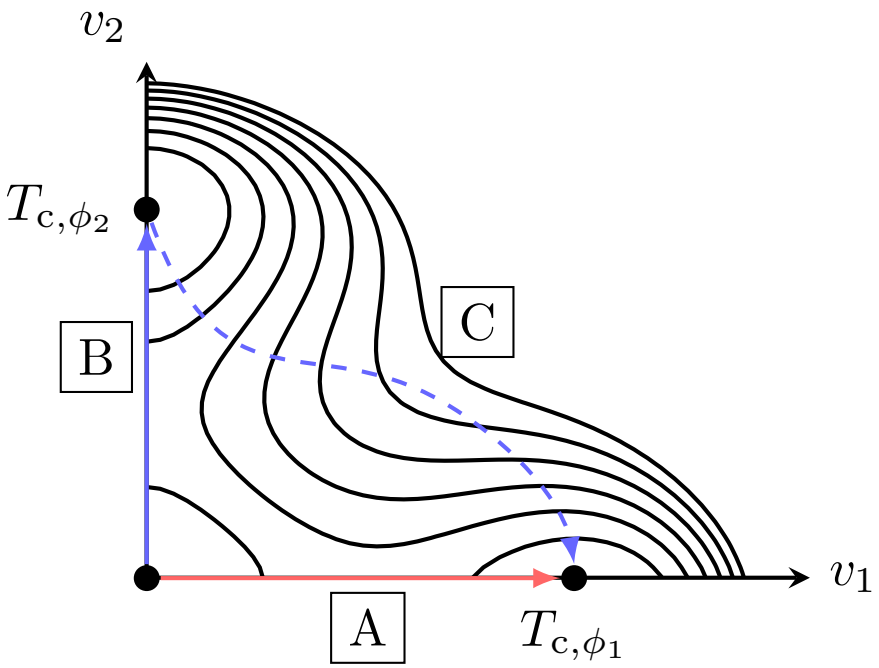
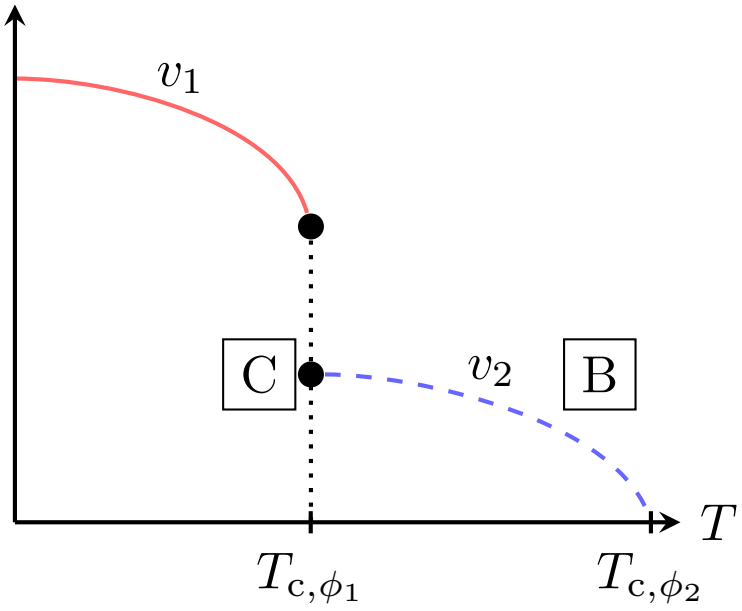
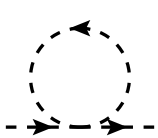
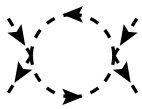
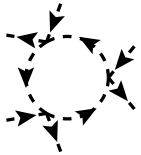




The thermal effective potential at LO

$$V^{\text{eff}} = V^{\text{eff}}_{\text{tree}} + V^{\text{eff}}_{1\ell} \, .$$

At 1-loop sum over n-point functions at $Q_i = 0$ external momenta

$V^{\text{eff}}_{1\ell} =$  $+ \frac{1}{2}$  $+ \frac{1}{3}$  $+ \ldots \Big|_{Q_i=0}$

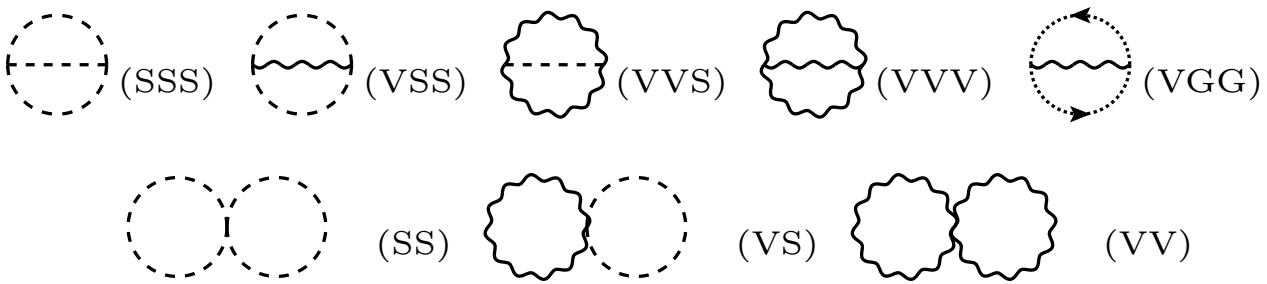
$$= \frac{1}{2} \sum\!\!\!\!\!\!\int_P \ln\left(P^2 + m^2\right)$$

$$V^{\text{eff}}_{1\ell} = \underbrace{\frac{1}{2} \int_P \ln(P^2 + m^2)}_{\equiv V_{\text{CW}}(m)} \underbrace{- \int_p T \ln\Big(1 \mp n_{\text{B/F}}(E_p, T)\Big)}_{V_{T,b/f}\left(\frac{m^2}{T^2}\right)}$$

$$= \underbrace{\frac{T}{2} \int_p \ln(p^2 + m^2)}_{\equiv T V_{\text{soft}}(m)} + \underbrace{\frac{1}{2} \sum\!\!\!\!\!\!\int_{P/\{P\}}{}' \ln(P^2 + m^2)}_{\equiv V_{\text{hard}}(m)} \, .$$

The effective potential at NLO

Computation up to 2-loop[15] V^{eff} straightforward with vacuum integrals in $3d$ theory:



[15] K. Farakos, K. Kajantie, K. Rummukainen, and M. E. Shaposhnikov, *3-D physics and the electroweak phase transition: Perturbation theory,* Nucl. Phys. B **425** (1994) 67 [hep-ph/9404201], M. Laine, *The Two loop effective potential of the 3-d SU(2) Higgs model in a general covariant gauge,* Phys. Lett. B **335** (1994) 173 [hep-ph/9406268], L. Niemi, M. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Thermodynamics of a two-step electroweak phase transition,* [2005.11332], L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops,* Phys. Rev. D **103** (2021) 115035 [2103.07467]

Gauge invariance at finite T

Analogous[16] analysis to zero-T after EFT construction:

- For radiatively induced SB in $3d$ effective potential, loop and tree-level effects at the same order
- Compute finite-T Nielsen identities
- Show that exponents $\mathcal{B}_{0,1}$ of the nucleation rate are gauge independent

[16] M. Garny and T. Konstandin, *On the gauge dependence of vacuum transitions at finite temperature*, JHEP **2012** (2012) 189 [1205.3392]

Conclusions

- ▹ Thermodynamic quantities from BSM theories essential for cosmology and gravitational wave production
  - Numerically on the lattice around $T_c \sim 100$ GeV
  - Practical approach: **Effective Theories**

- ▹ Dimensionally reduced 3-dim theories permit
  - Automatic all-order resummation at high-T
  - Analytic treatment of fermions, lattice treatment for 3-dim theory
  - Systematic higher-loop/operator improvement
  - Automation: Multi-loop sports
  - Universality
  - Description of the phase transition[17] and **Nucleation rate**

- ☆ Neither $3d$- nor $4d$-perturbative approaches "solve" the IR problem → Lattice (much more feasible now)

[17] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions,* JHEP **04** (2021) 055 [2009.10080]