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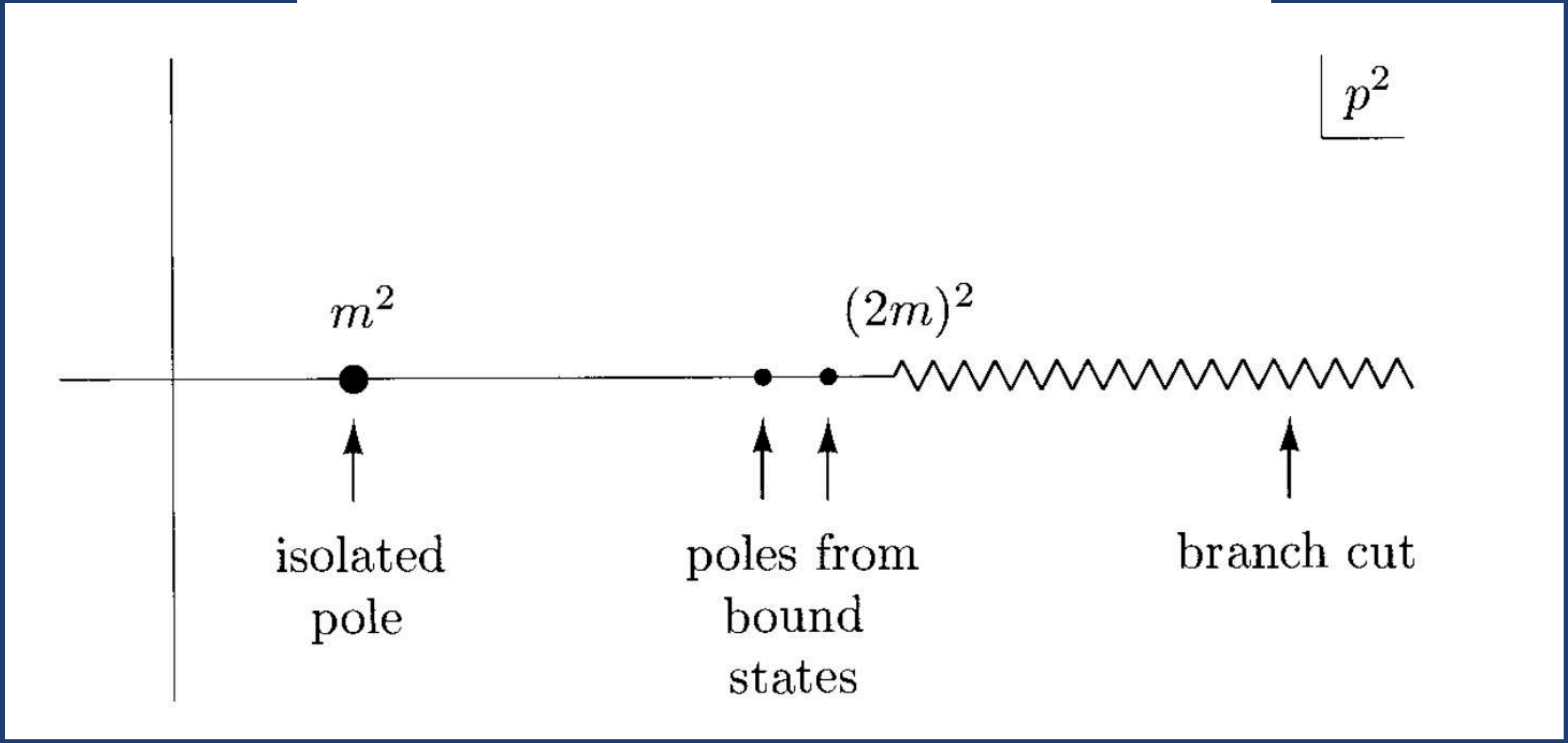
HOW MUCH INFORMATION IS REQUIRED TO RECONSTRUCT A QFT SPECTRUM?

LJUBLJANA, 2.3.2023

ANALYTIC STRUCTURE OF CORRELATORS

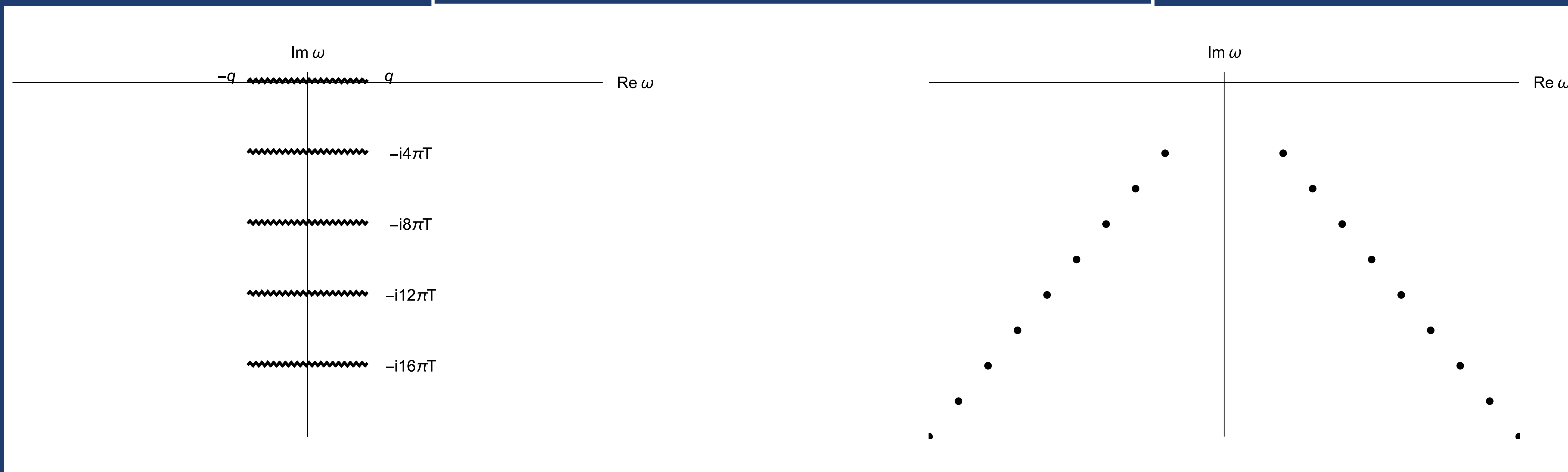
$$\langle \phi\phi \rangle_{p^2=-\omega^2+\mathbf{q}^2}$$



QFT at zero temperature

ANALYTIC STRUCTURE OF CORRELATORS

$$\langle T_{\mu\nu}(-\omega,-q), T_{\rho\sigma}(\omega,q)\rangle_R$$

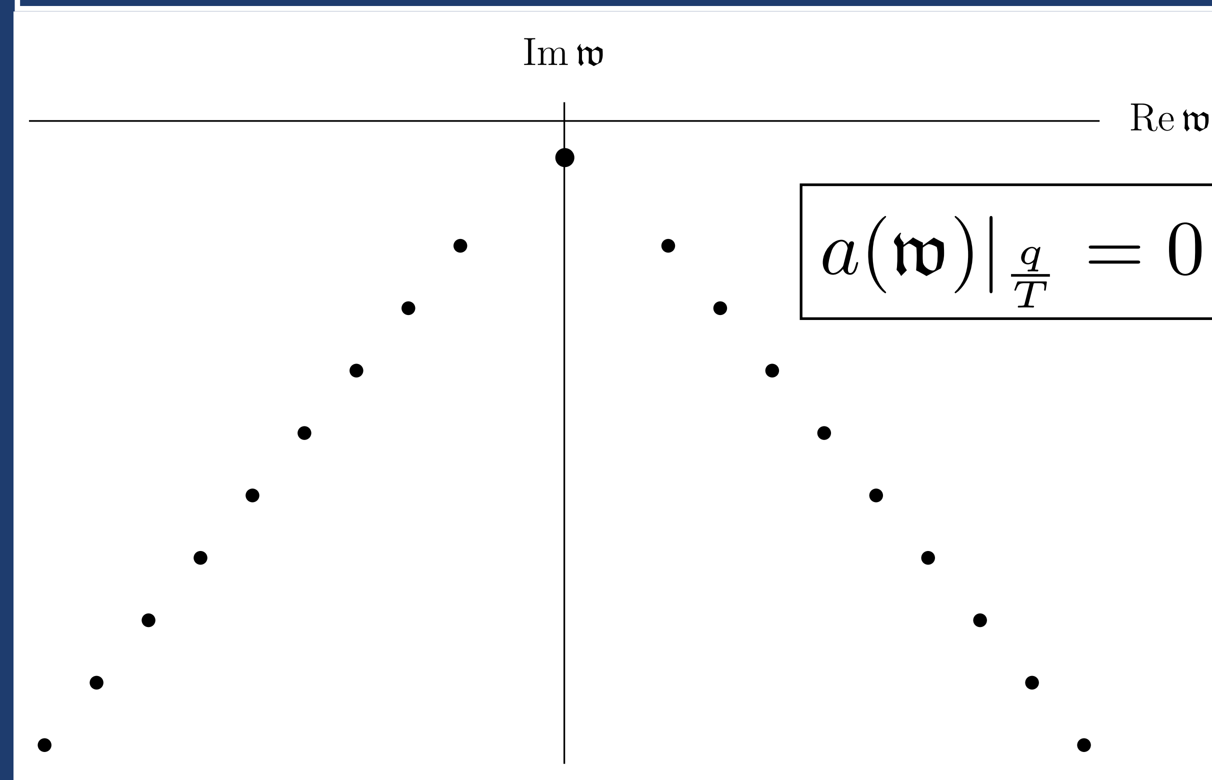


$\lambda \to 0$ $\lambda \to \infty$

'large-N' QFT at finite temperature

ANALYTIC STRUCTURE OF CORRELATORS

$$\langle T_{\mu\nu}(-\omega,-q), T_{\rho\sigma}(\omega,q)\rangle_R \sim \frac{b(\omega,q)}{a(\omega,q)} = \frac{B(\omega,q)}{\prod_{i=0}^{\infty}(\omega-\omega_i(q))}$$



'large-N' QFT at finite temperature

quasinormal mode spectrum of black holes

spectra of linear non-Hermitian operators

zeros of (algebraic) equations

OUTLINE

- analytic structure of classical hydrodynamics: holomorphicity
- reconstruction of spectra beyond hydrodynamics
- summary and future directions

ANALYTIC STRUCTURE OF CLASSICAL HYDRODYNAMICS: HOLOMORPHICITY

HYDRODYNAMICS

- low-energy limit of QFTs – a Schwinger-Keldysh effective field theory
  [SG, Polonyi (2013); Crossley, Glorioso, Liu (2015); Haehl, Loganayagam, Rangamani (2015); …]

- globally conserved operators

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu J^\mu = 0 \qquad \ldots\ \nabla_\mu J^{\mu\nu} = 0$$

higher-form currents in MHD
[SG, Hofman, Iqbal,
PRD (2017)]

- tensor structures (symmetries, gradient expansion, EFT) and transport coefficients (QFT)

$$T^{\mu\nu} = \sum_{n=0}^{\infty} \left[\sum_i^N \lambda_i^{(n)} \mathcal{T}^{\mu\nu}_{(n)} \right] \xrightarrow[u^\mu \sim T \sim e^{-i\omega t + iqz}]{\nabla_\mu T^{\mu\nu} = 0} \omega(q) = \sum_{n=1}^{\infty} \alpha_n q^n$$

$$\partial u^\mu \sim \partial T \ll 1 \longrightarrow \omega / T \sim q/T \ll 1$$

equilibrium
temperature

$q = \sqrt{\mathbf{q}^2}$

- dispersion relations:

shear diffusion	sound
$\omega = -iDq^2$	$\omega = \pm v_s q - i\Gamma q^2$

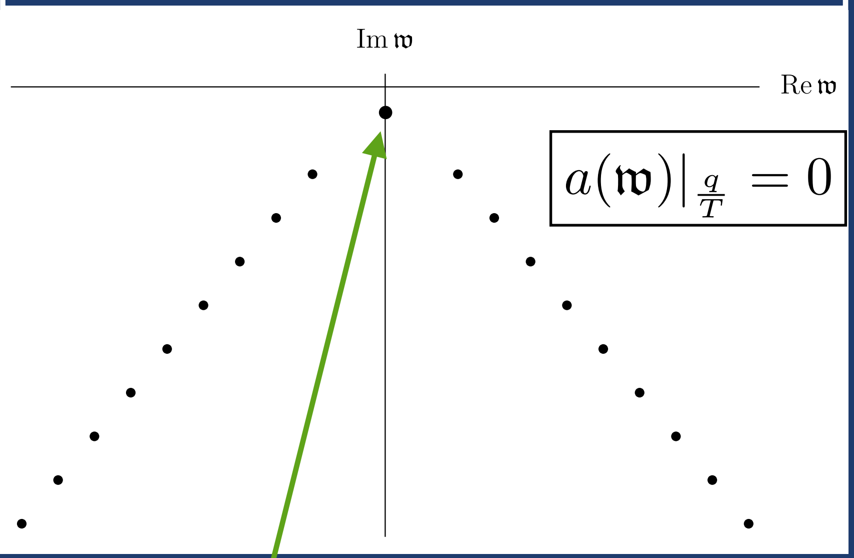
HYDRODYNAMICS FROM HOLOGRAPHY

- duality: theory A = theory B
- a result of string theory (quantum gravity) [Maldacena (1997)]

strongly coupled quantum theory (extremely hard) = *weakly coupled gravity* (much easier)

- perturbations of black holes (quasinormal modes) give spectra of QFT operators for $\mathfrak{w} \equiv \frac{\omega}{2\pi T} \in \mathbb{C}$
- invaluable explicit (toy) models: the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory [SG, Kovtun, Starinets, Tadić, JHEP (2019)]

$$\langle T_{\mu\nu}(-\omega,-q), T_{\rho\sigma}(\omega,q)\rangle_R \sim \frac{b(\omega,q)}{a(\omega,q)}$$



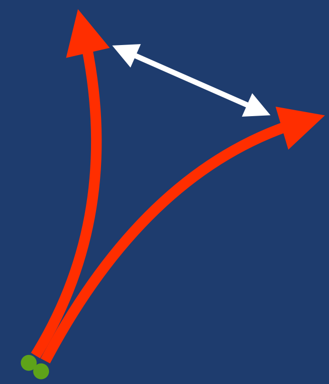
sound:

$$\omega = \pm\frac{1}{\sqrt{3}}q - \frac{i}{6\pi T}q^2 \pm \frac{3-2\ln 2}{24\sqrt{3}\pi^2 T^2}q^3 - \frac{i\left(\pi^2 - 24 + 24\ln 2 - 12\ln^2 2\right)}{864\pi^3 T^3}q^4 \pm \cdots$$

shear diffusion:

$$\omega = -\frac{i}{4\pi T}q^2 - \frac{i(1-\ln 2)}{32\pi^3 T^3}q^4 - \frac{i(24\ln^2 2 - \pi^2)}{96\,(2\pi T)^5}q^6 - \frac{i\left[2\pi^2(\ln 32 - 1) - 21\zeta(3) - 24\ln 2(1 + \ln 2(\ln 32 - 3))\right]}{384\,(2\pi T)^7}q^8 + \cdots$$

CHAOS



$t = t_i$

Lyapunov exponent → butterfly velocity ↓

$$|\Delta Z(t,\mathbf{x})| \approx |\Delta Z(t_i,\mathbf{x}_i)|\, e^{\lambda_L (t-|\mathbf{x}|/v_B)}$$

- classical chaos means extreme sensitivity to initial conditions

- "what is quantum chaos?"
  a measure: "out-of-time-ordered" correlation functions
  [Larkin, Ovchinnikov; Kitaev]

$$C(t,\mathbf{x}) = \left\langle [W(t,\mathbf{x}), V(0,\mathbf{0})]^\dagger [W(t,\mathbf{x}), V(0,\mathbf{0})] \right\rangle_T \sim \epsilon\, e^{\lambda_L (t-|\mathbf{x}|/v_B)}$$

'quantum' Lyapunov exponent

butterfly velocity

- the Maldacena-Shenker-Stanford bound on exponential Lyapunov chaos

OTOC of $\mathcal{O}(t,x)$

$$C(t,x) \sim \epsilon\, e^{\lambda_L (t-x/v_B)}$$

$$\lambda_L \leq 2\pi T/\hbar$$

holomorphicity

CHAOS FROM HYDRODYNAMICS: POLE-SKIPPING

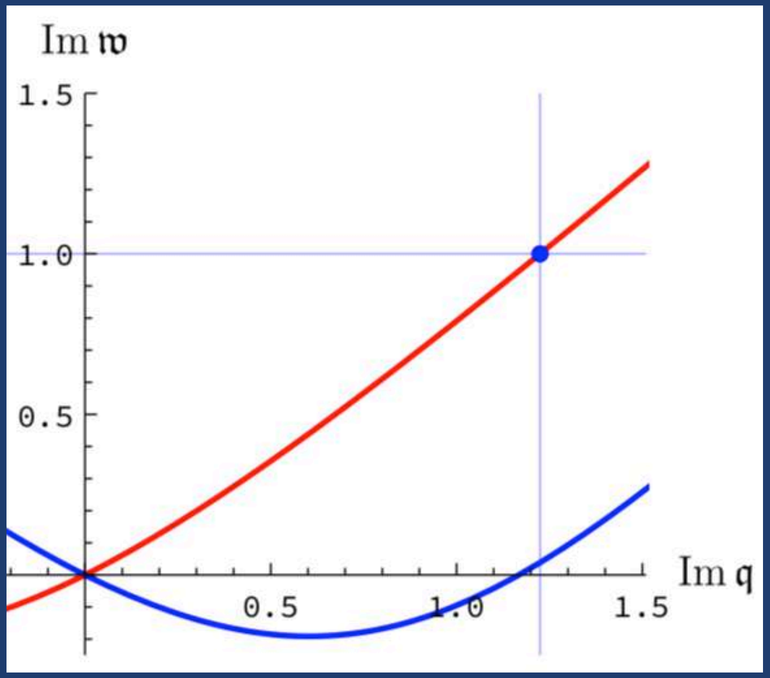
- precise analytic connection between 'low-energy' hydrodynamics and quantum chaos
  [SG, Schalm, Scopelliti, PRL (2017); Blake, Lee, Liu, JHEP (2018); Blake, Davison, SG, Liu, JHEP (2018); SG, JHEP (2019)]

- resumed all-order hydrodynamic series (e.g. sound)

$$\omega(q) = \sum_{n=1}^{\infty} \alpha_n\left(T, \mu_i, \langle \mathcal{O}_j \rangle, \lambda\right) q^n$$

  passes through a "chaos point" at imaginary momentum

$$\omega(q = i\lambda_L / v_B) = i\lambda_L = 2\pi T i$$



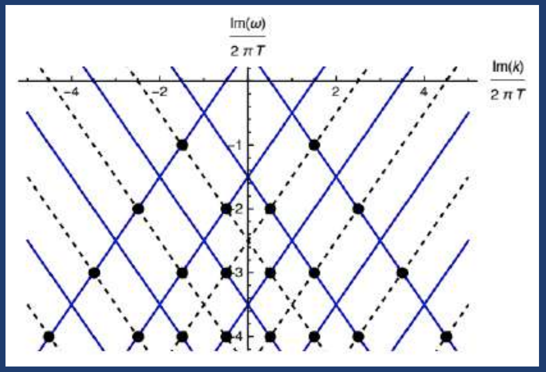
  where the associated *2*-pt function is "0/0":

$$\mathrm{Res}\, G_R^{\varepsilon\varepsilon}\left(\omega = i\lambda_L, q = i\lambda_L / v_B\right) = 0$$

- triviality of Einstein's equations at the horizon [Blake, Davison, SG, Liu, JHEP (2018)]

- infinite constraints on correlators
  [SG, Kovtun, Starinets, Tadić, JHEP (2019); Blake, Davison, Vegh, JHEP (2019)]

$$\omega_n(q_n) = -2\pi T i n$$



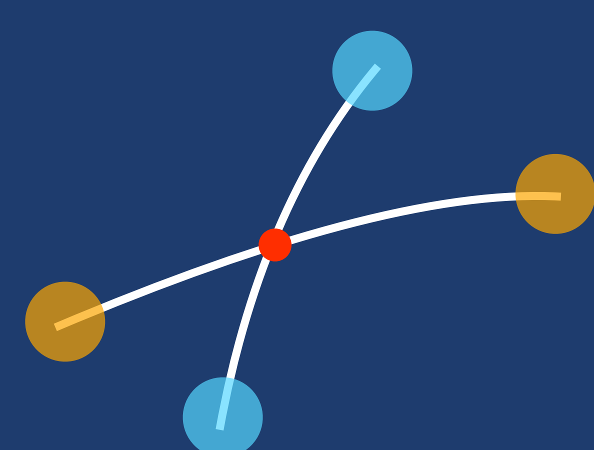
[from Blake, Davison, Vegh, JHEP (2019)]

COMPLEX SPECTRAL CURVES

- spectral curves are solutions to $$P(x,y)=0 \implies y(x);\; x,y\in\mathbb{C}$$
- simple example: $P(x,y)=x^2+y^2-1=0$
- local analysis
  - regular point $P(x_r,y_r)=0,\; \partial_y P(x_r,y_r)\neq 0$
    Taylor series around $(x_r,y_r)=(0,1)$ $\quad y=y^{(T)}(x)=1-\frac{x^2}{2}-\frac{x^4}{8}+\cdots$
  - critical point (order 2) $P(x_*,y_*)=0,\; \partial_y P(x_*,y_*)=0,\; \partial_y^2 P(x_*,y_*)\neq 0$
    Puiseux series around each $(x_*,y_*)=(\pm 1,0)$ has 2 branches

$$\text{at } (x_*,y_*)=(1,0): \qquad y=y_1^{(P)}(x)=i\sqrt{2}(x-1)^{\frac{1}{2}}+i2^{-\frac{3}{2}}(x-1)^{\frac{3}{2}}+\cdots$$
$$y=y_2^{(P)}(x)=-i\sqrt{2}(x-1)^{\frac{1}{2}}-i2^{-\frac{3}{2}}(x-1)^{\frac{3}{2}}+\cdots$$

  - convergence *at least* up to nearest critical point (branch point): $R_x^{(T)}=1,\; R_x^{(P)}=2$
- level-crossing vs. level-touching



HYDRODYNAMICS FROM COMPLEX SPECTRAL CURVE

- hydrodynamic modes as complex spectral curves
  [SG, Kovtun, Starinets, Tadić, PRL (2019) and JHEP (2019)]

hydro: $\det \mathcal{L}(\mathbf{q}^2, \omega) = 0$

QNM: $a(\mathbf{q}^2, \omega) = 0$

$\longrightarrow$ $\boxed{P(\mathbf{q}^2, \omega) = 0} \implies \boxed{\omega_i(\mathbf{q}^2)}$

$$\mathfrak{w} = \frac{\omega}{2\pi T}, \; \mathfrak{q} = \frac{|\mathbf{q}|}{2\pi T} \in \mathbb{C}$$

- e.g., first-order hydrodynamics: $P_1(\mathbf{q}^2, \omega) = \left(\omega + iD\mathbf{q}^2\right)^2 \left(\omega^2 + i\Gamma\omega\mathbf{q}^2 - v_s^2\mathbf{q}^2\right) = 0$ (factorisation)
- Puiseux theorem: there exists a convergent series around a critical point of any order

$$P(\mathbf{q}_*^2, \omega_*) = 0, \; \partial_\omega P(\mathbf{q}_*^2, \omega_*) = 0, \; \dots, \; \partial_\omega^p P(\mathbf{q}_*^2, \omega_*) \neq 0$$

- convergence guaranteed up to the nearest level-crossing critical point (branch point)
  cf., the Newton polygon or the Darboux theorem [SG, Starinets, Tadić, JHEP (2021)]

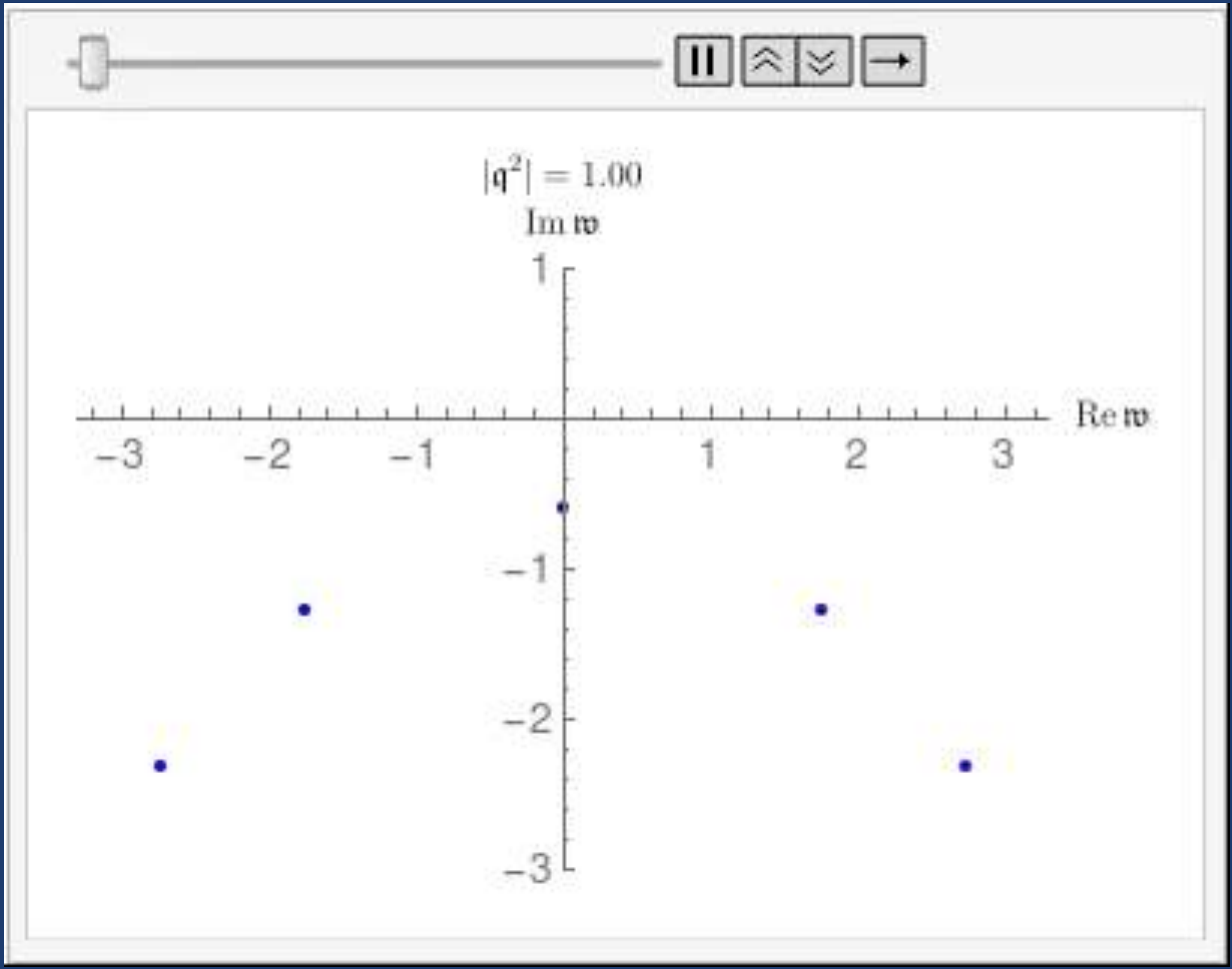
$$f(z) \sim r(z)(z - z_1)^{-\nu}, \;\; z \to z_1$$

$$\nu = \lim_{n\to\infty} \left[z_1 (n+1) \frac{a_{n+1}}{a_n} - n \right]$$

next critical point

HYDRODYNAMICS FROM COMPLEX SPECTRAL CURVE

- radius of convergence of $\mathfrak{w}(\mathfrak{q}) = \sum_{i=1}^{\infty} c_n \mathfrak{q}^n$, $|\mathfrak{q}| < \mathfrak{q}_*$, is set by the lowest momentum at which the hydro pole collides (level-crossing): $\boxed{\mathfrak{q}_* = \min\left[|\mathfrak{q}_{\text{collision}}|\right]}$



$\boxed{\mathfrak{q}^2 = |\mathfrak{q}^2|\, e^{i\theta}}$

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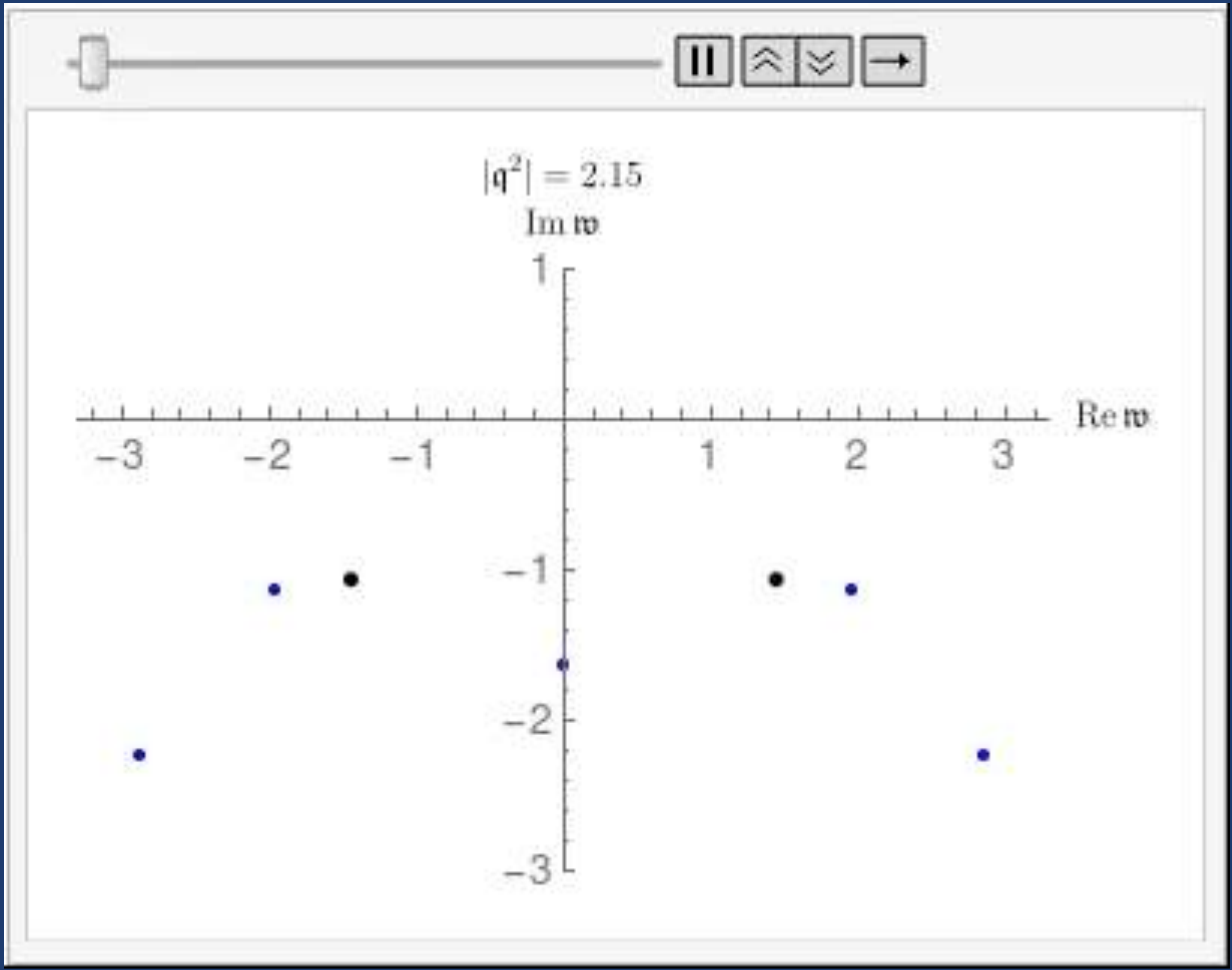

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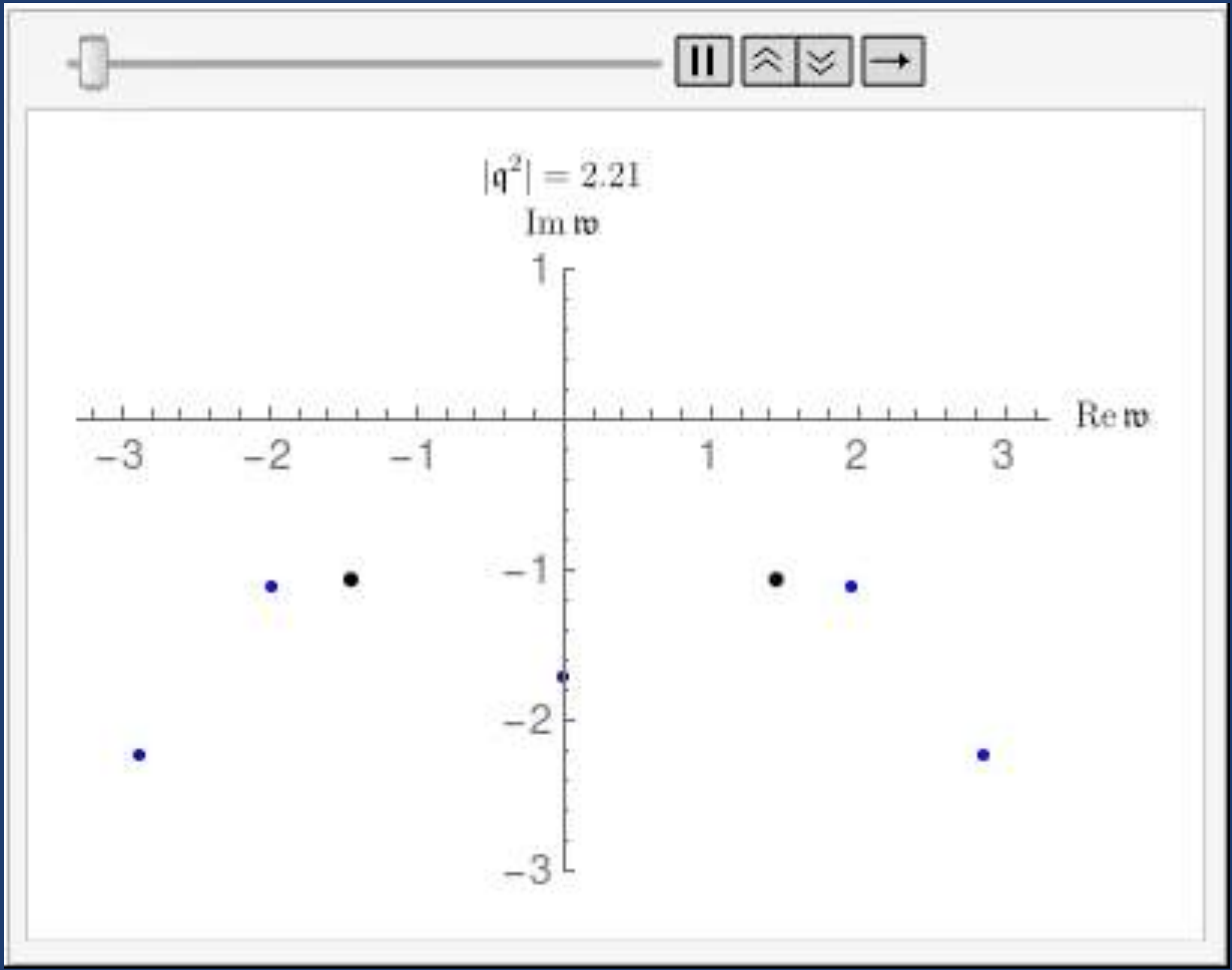

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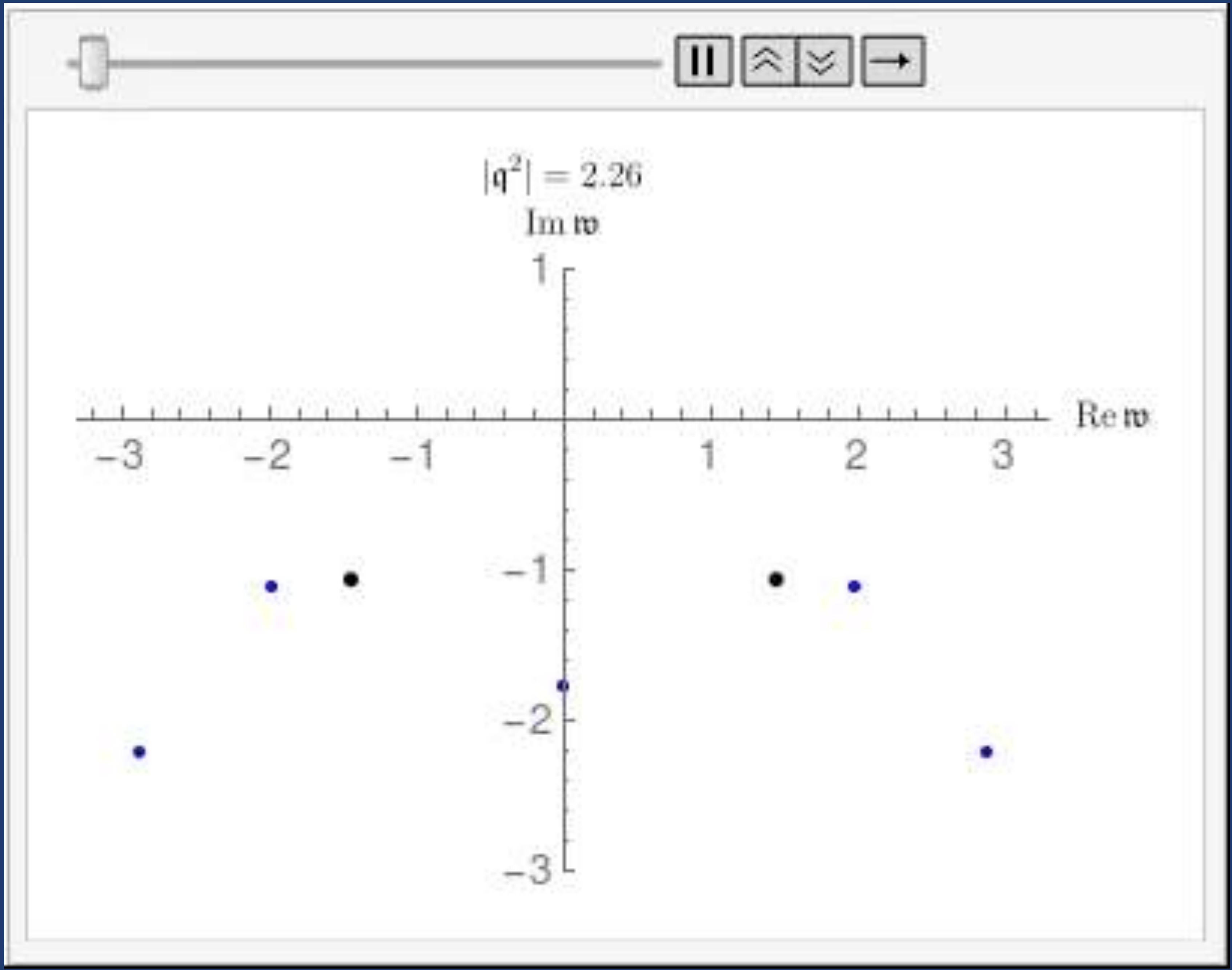

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HYDRODYNAMICS FROM COMPLEX SPECTRAL CURVE

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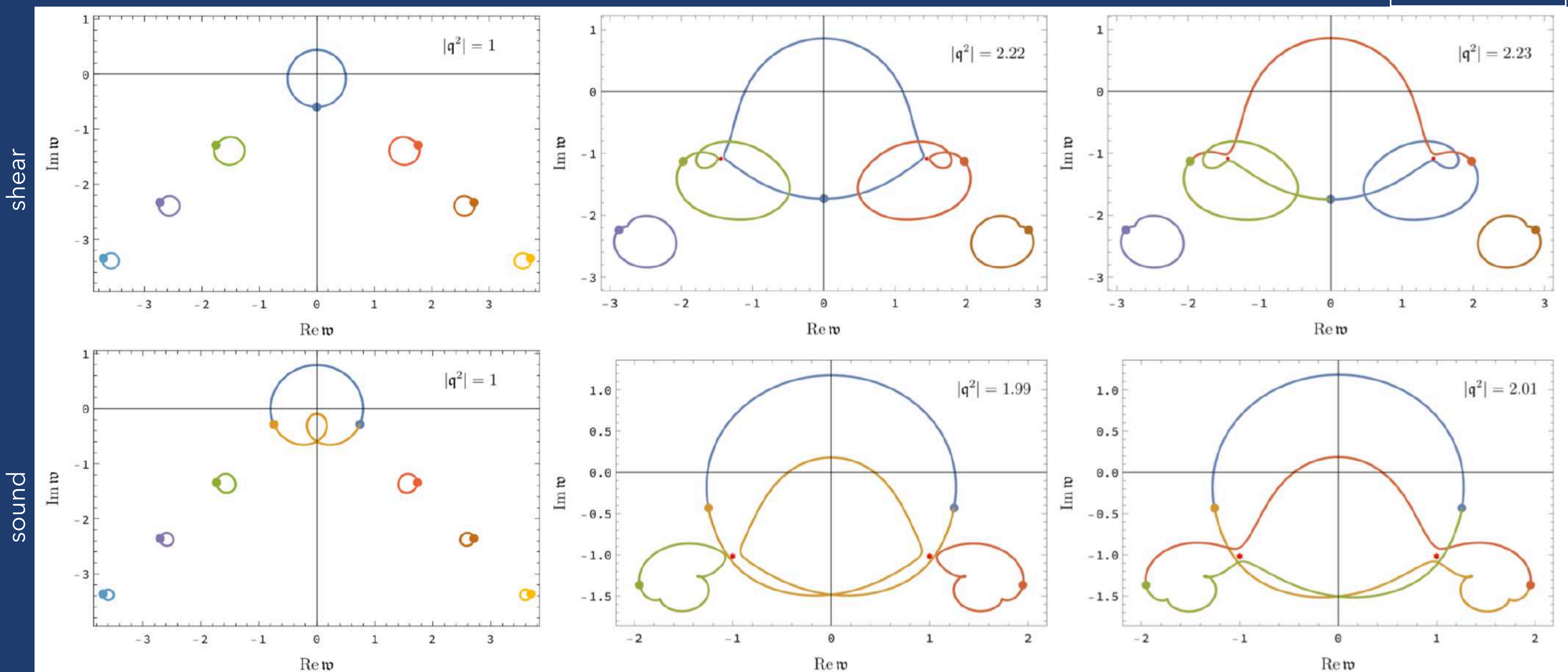
$\mathfrak{q}^2 = |\mathfrak{q}^2|\, e^{i\theta}$



shear:	$\mathfrak{q}_* \approx 1.49131$ $\mathfrak{w}(\mathfrak{q}_*) \approx \pm 1.4436414 - 1.0692250i$	$\mathcal{N} = 4$ SYM	sound:	$\mathfrak{q}_* = \sqrt{2} \approx 1.41421$ $\mathfrak{w}(\mathfrak{q}_*) = \pm 1 - i$

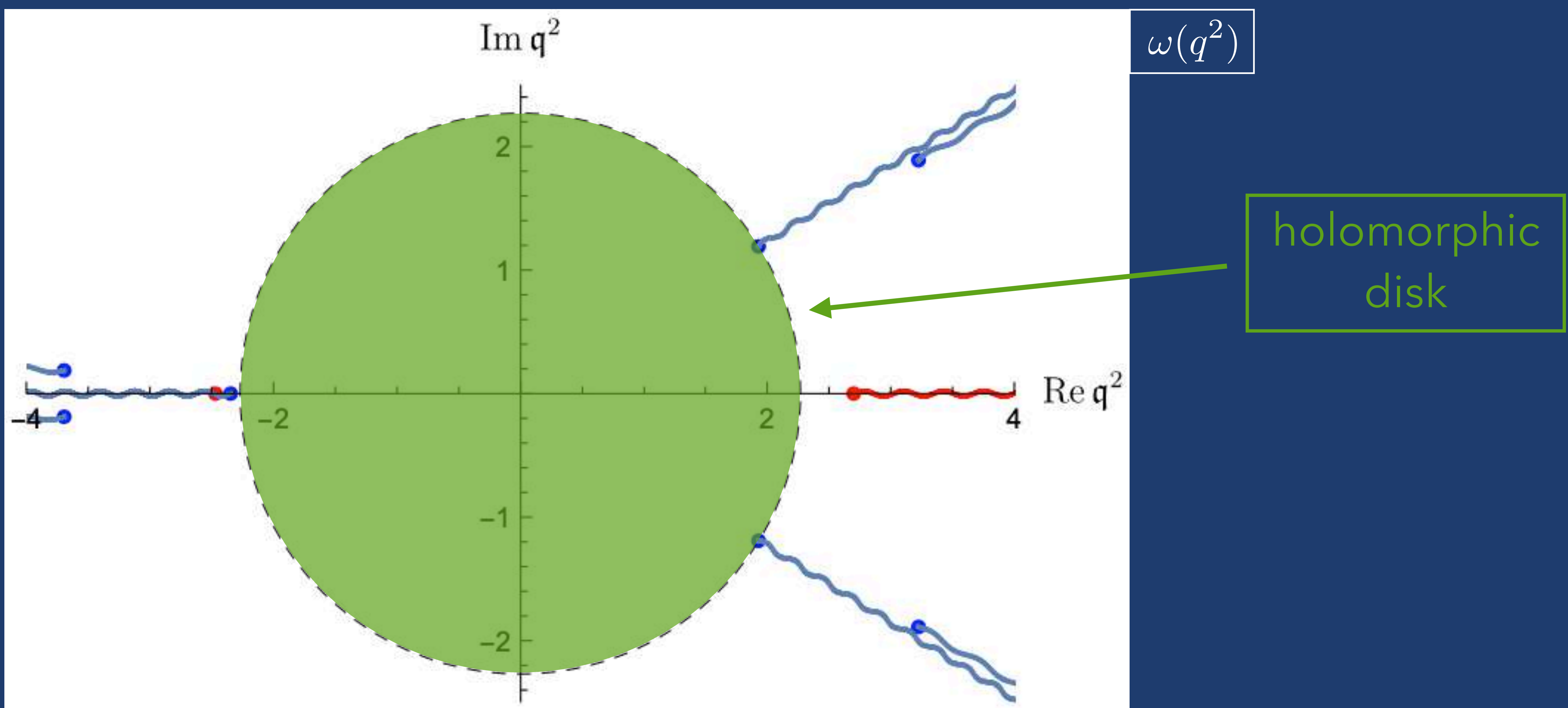
HYDRODYNAMICS FROM COMPLEX SPECTRAL CURVE

- hydrodynamic series are convergent Puiseux series (shear *p=1*, sound *p=2*)
  [SG, Kovtun, Starinets, Tadić, PRL (2019); … ; see also Withers; JHEP (2018); Heller, et.al. (2020, …)]

$$\mathfrak{w}_{\rm shear} = -i\sum_{n=1}^{\infty} c_n \left(\mathfrak{q}^2\right)^n = -i\mathfrak{D}\mathfrak{q}^2 + \ldots$$

$$\mathfrak{w}_{\rm sound} = -i\sum_{n=1}^{\infty} a_n e^{\pm\frac{i\pi n}{2}} \left(\mathfrak{q}^2\right)^{n/2} = \pm v_s \mathfrak{q} - \frac{i}{2}\mathfrak{G}\mathfrak{q}^2 + \ldots$$

- dispersion relations are holomorphic in a disk



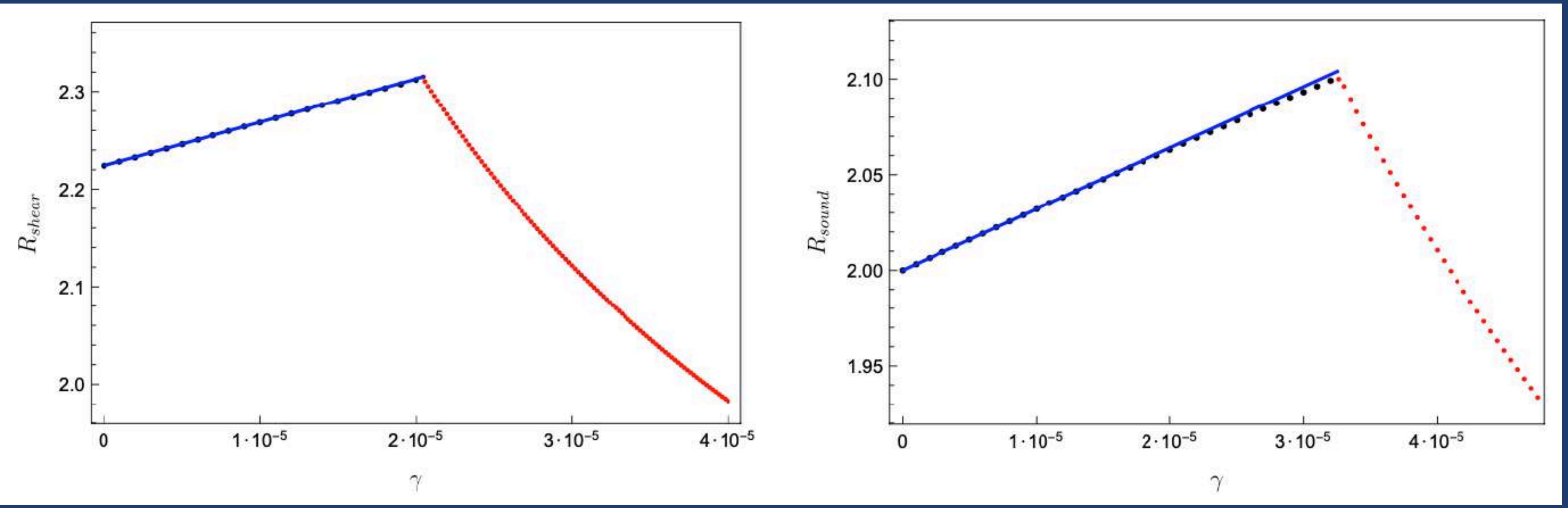
HYDRODYNAMICS FROM COMPLEX SPECTRAL CURVE

- hydrodynamic series are convergent Puiseux series (shear *p=1*, sound *p=2*)

$$\mathfrak{w}_{\text{shear}} = -i\sum_{n=1}^{\infty} c_n \left(\mathfrak{q}^2\right)^n = -i\mathfrak{D}\mathfrak{q}^2 + \ldots$$

$$\mathfrak{w}_{\text{sound}} = -i\sum_{n=1}^{\infty} a_n e^{\pm\frac{i\pi n}{2}} \left(\mathfrak{q}^2\right)^{n/2} = \pm v_s\mathfrak{q} - \frac{i}{2}\mathfrak{G}\mathfrak{q}^2 + \ldots$$

- coupling dependence of in $\mathcal{N}=4$ SYM [SG, Starinets, Tadić, JHEP (2021)]



$$R_{\text{shear}}(\lambda) = 2.22\left(1 + 674.15\,\lambda^{-3/2} + \cdots\right)$$

$$R_{\text{sound}}(\lambda) = 2\left(1 + 481.68\,\lambda^{-3/2} + \cdots\right)$$

→ $q/T \sim O(10)$

orders of magnitude larger radius of convergence than naive $q/T \ll 1$ –
this is a precise incarnation of the "unreasonable effectiveness of hydrodynamics"

RECONSTRUCTION OF SPECTRA BEYOND HYDRODYNAMICS

[SG, Lemut, JHEP (2023)]

PUISEUX AND DARBOUX THEOREMS

- *Puiseux theorem*
  Around a critical point of order *p*, we expect *p* branches of solutions

$$f(x_* = 0, y_* = 0) = 0,\ \partial_y f(0,0) = 0,\ \ \ldots,\ \partial_y^p f(0,0) \neq 0$$

$$y = Y_j(x) = \sum_{k \geq k_0}^{\infty} a_k x^{k/m_j}, \quad j = 1, \ldots, p$$

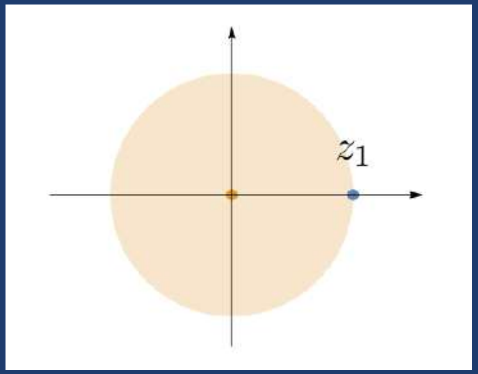
If some $m_j > 1$, we necessarily have a family of m_j solutions

$$y = Y_l(x) = \sum_{k \geq k_0}^{\infty} a_k \left(e^{\frac{2\pi i l}{m_j}} \right)^k x^{k/m_j}, \quad l = 0, 1, \ldots, m_j - 1$$

- recall: sound

$$\mathfrak{w}_{\text{sound}} = -i \sum_{n=1}^{\infty} a_n e^{\pm \frac{i\pi n}{2}} \left(\mathfrak{q}^2\right)^{n/2} = \pm v_s \mathfrak{q} - \frac{i}{2} \mathfrak{G} \mathfrak{q}^2 + \ldots$$

PUISEUX AND DARBOUX THEOREMS



- *Darboux theorem*
  Consider a power series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

that converges up to a critical point of order $\nu\,[= -1/p]$, which can be computed

$$f(z) \sim (z - z_1)^{-\nu\,[=1/2]}\, r(z) + q(z)$$

$$\nu = \lim_{n\to\infty} \left[z_1 (n+1) \frac{a_{n+1}}{a_n} - n \right]$$

as well as all coefficients in the expansion and subleading (non-singular) terms

$$r(z) = \sum_{m=0}^{\infty} r_m \,(z - z_1)^m$$

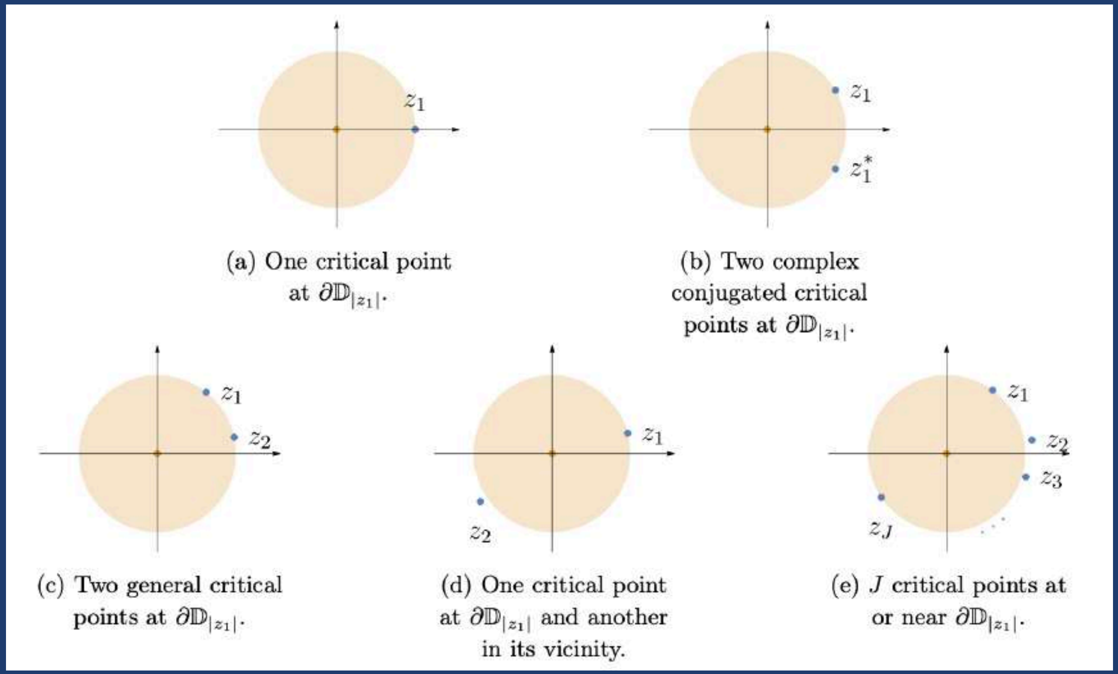
$$r_m = \lim_{n\to\infty} \left[\frac{(-1)^{m-\nu} n! z_1^{n-m+\nu} a_n}{(\nu - m)_n} - \sum_{k=0}^{m-1} \frac{(-1)^{m-k} (\nu - k)_n r_k}{(\nu - m)_n z_1^{m-k}} \right]$$

$$q_m = \lim_{n\to\infty} \left[\sum_{k=0}^{n} \frac{(-1)^{n+m-k} n! (\nu)_{n-k} a_k}{(-\nu - m)_n (n-k)! z_1^{m-k}} - \sum_{k=0}^{m-1} \frac{(-1)^{k-m} (-\nu - k)_n q_k}{(-\nu - m)_n z_1^{m-k}} \right]$$

PUISEUX AND DARBOUX THEOREMS

- *Darboux theorem*

  Need generalisation to different configurations of critical points



- Potential problem: need to know either location of the critical point or exponent… but this is resolved by following Hunter and Guerrieri (1980), which we generalise

- Moreover, assume we only know a finite number of coefficients: $a_n, \quad n = 0, \ldots, N$

$$X_n^0(\nu, z_1) = a_n$$

$$X_n^{m+1}(\nu, z_1) = X_n^m(\nu, z_1) - \frac{(n+\nu-2m-1)}{n z_1} X_{n-1}^m(\nu, z_1), \quad \text{for } m \geq 0$$

$$X_n^m(\nu, z_1) \sim \sum_{k=m}^{\infty} \frac{(-1)^{k+m-\nu} k!(\nu-k)_{n-m} r_k}{n!(k-m)! z_1^{n+\nu-k}} \sim O(n^{\nu-2m-1})$$

$$X_N^1 = 0, \; X_{N-1}^1 = 0 \overset{\text{iteration}}{\longrightarrow} X_N^m = 0, \; X_{N-1}^m = 0 \quad \longrightarrow \quad z_1, \; \nu$$

PUISEUX AND DARBOUX THEOREMS

- *Darboux theorem*
- Similarly, define

$$Y^0_{\ell,n}(\nu, z_1) = a_n$$

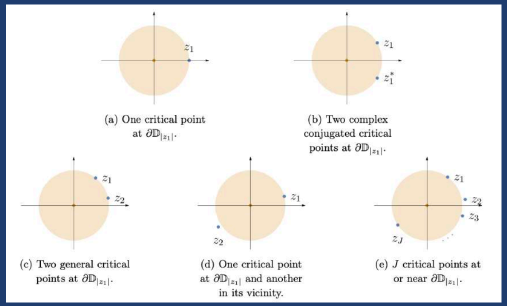
$$Y^{m+1}_{\ell,n}(\nu, z_1) = Y^m_{\ell,n}(\nu, z_1) - \frac{(n+\nu-2m-\ell-2)}{n z_1} Y^m_{\ell,n-1}(\nu, z_1), \quad \text{for } m \geq 0$$

$$Y^m_{\ell,n} \sim \sum_{k=0}^{\ell} \frac{(-1)^{k-\nu}(m+\ell-k)!(\nu-k)_{n-m} r_k}{n!(\ell-k)! z_1^{n+\nu-k}} + \mathcal{O}(n^{\nu-2m-\ell-2})$$

$$r_\ell = \lim_{n\to\infty} \left[\frac{(-1)^{\ell-\nu} n! z_1^{n+\nu-\ell}}{m!(\nu-\ell)_{n-m}} Y^m_{\ell,n} - \sum_{k=0}^{\ell-1} \binom{m+\ell-k}{m} \frac{(-1)^{\ell-k}(\nu-k)_{n-m} r_k}{(\nu-\ell)_{n-m} z_1^{\ell-k}} \right]$$

subleading parts of the function (recall: *q*) follow in an analogous way

- We also extended this algorithm to several critical points in different configurations



RECONSTRUCTION OF '*ALL*' UV MODES

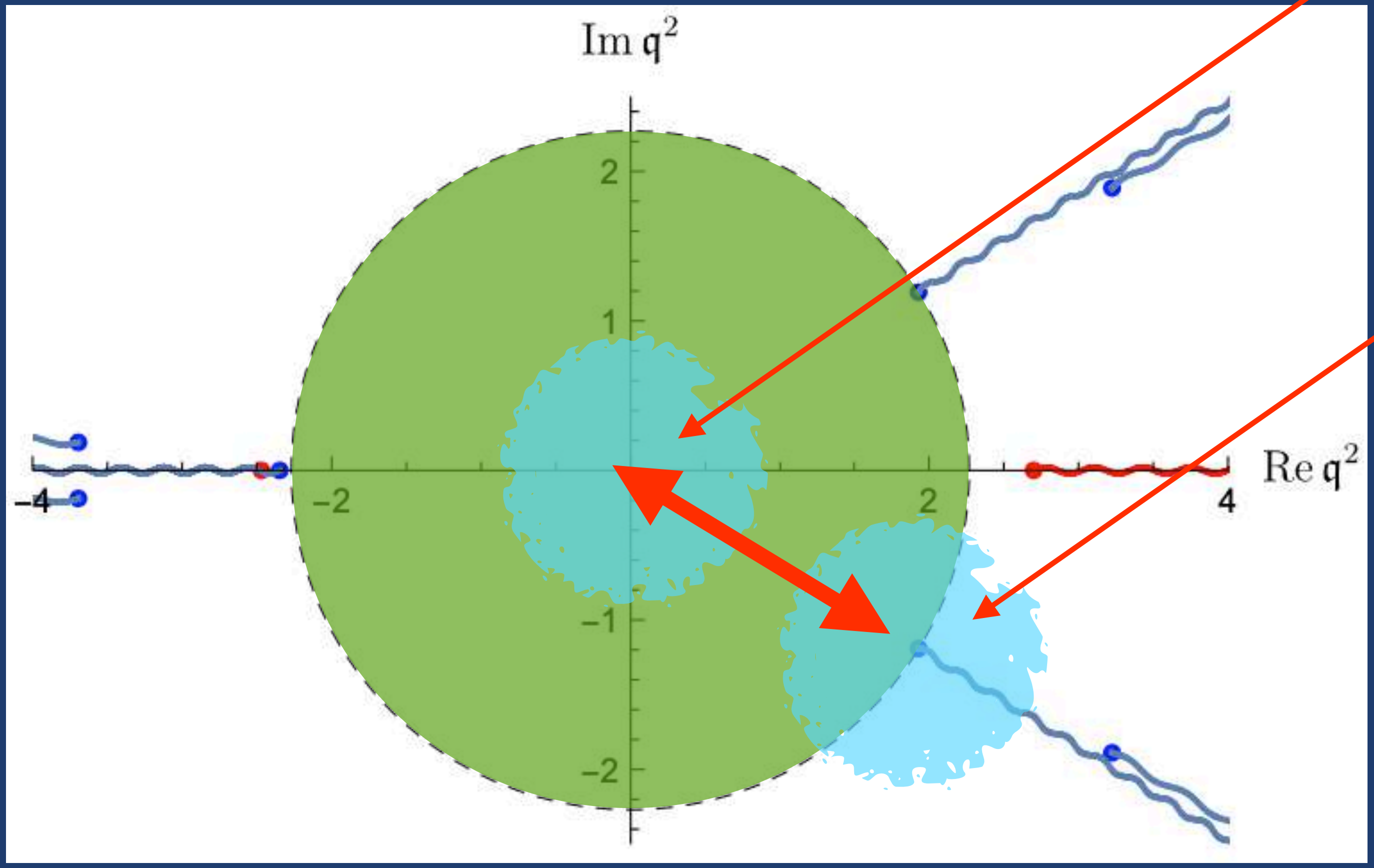
claim: systematic reconstruction of *all* modes connected via *level-crossing* is possible by exploration (analytic continuations) of the Riemann surface connecting physical modes

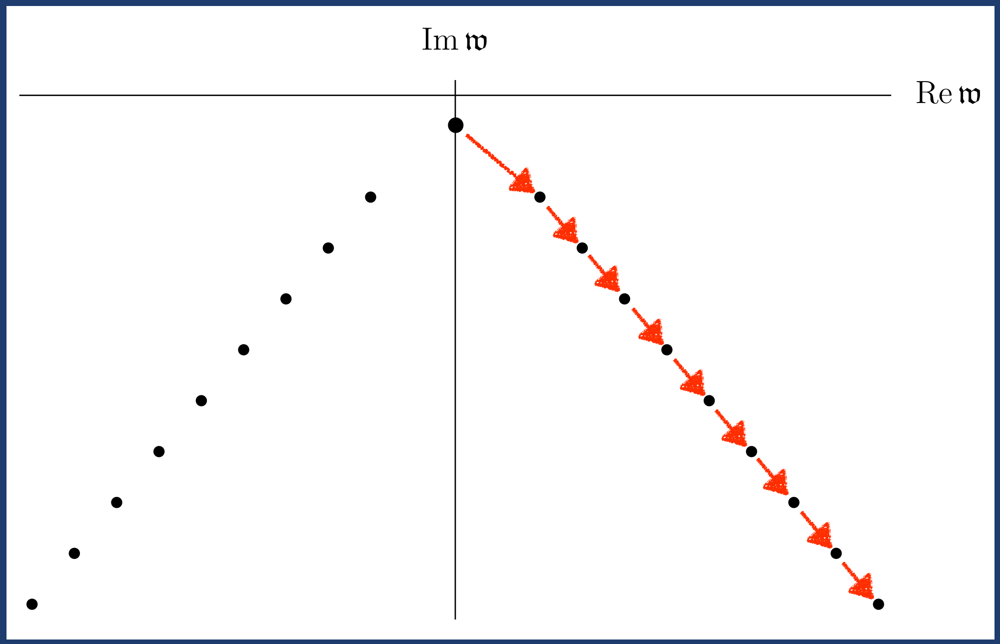
- momentum space analogue of resurgence [*this workshop*] – everything is convergent!
- see related papers by Bender, et.al; Dunne, et.al.; Withers, JHEP (2019); …

$$\omega_0(z) = \sum_{n=0}^{\infty} a_n z^n$$

$$\omega_0(z) = -i \sum_{n=0}^{\infty} e^{\frac{i\pi n}{2}} b_n (z - z_1)^{n/2}$$

$$\omega_1(z) = -i \sum_{n=0}^{\infty} e^{-\frac{i\pi n}{2}} b_n (z - z_1)^{n/2}$$





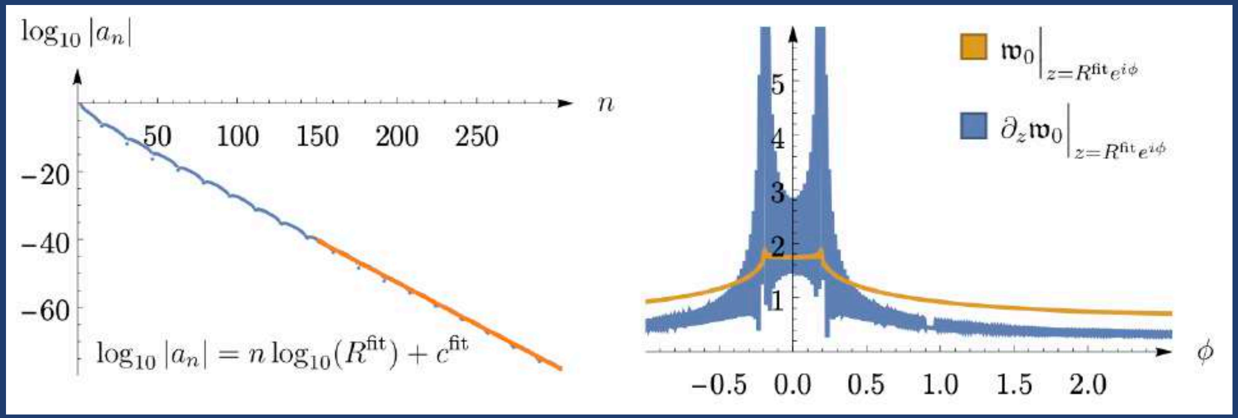
conceptually fascinating! all UV modes from one IR mode

EXAMPLE: MOMENTUM DIFFUSION OF M2 BRANES

- start from 300 coefficients

$$\mathfrak{w}_0(z) = \sum_{n=1}^{N_0=300} a_n z^n, \quad z \equiv \mathfrak{q}^2 \equiv q^2/4\pi^2 T^2$$

- analyse convergence and get a non-rigorous hint for the number of critical points

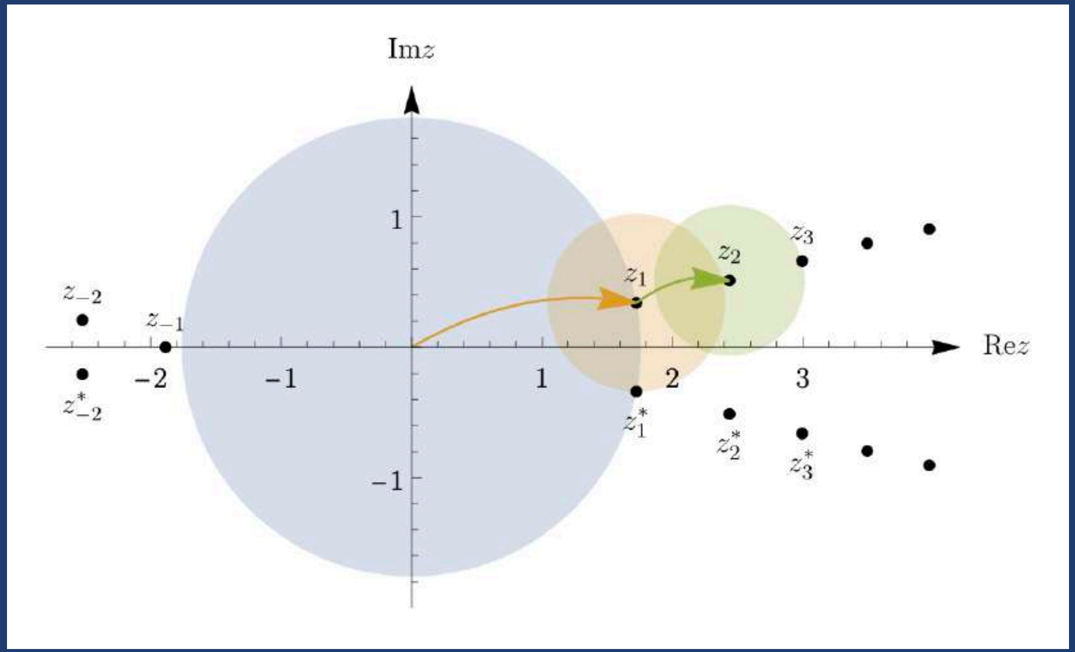


- use algorithm with 2 complex conjugate critical points and 'recover' 12 coefficients

$$\mathfrak{w}_1(z) = \sum_{n=0}^{(N_1=12)-1} b_n (z - z_1)^{n/2}$$

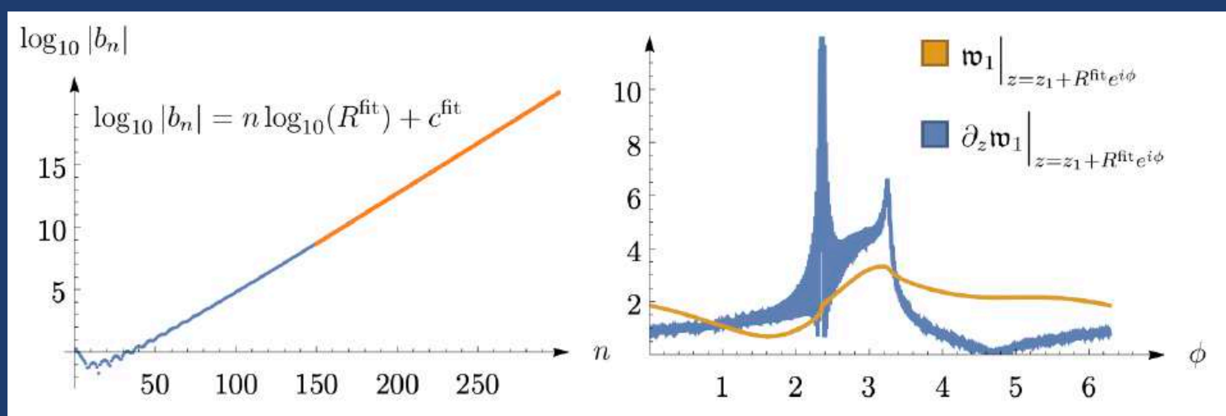
- the gap: analytic continuation within the same sheet (e.g. Padé approximant, conformal maps…)

$$\mathfrak{w}_1^{\rm calc}(0) = 1.23506 - 1.76338i$$
$$\mathfrak{w}(0) = 1.23455 - 1.77586i$$



EXAMPLE: MOMENTUM DIFFUSION OF M2 BRANES

- this is *not* good enough to continue;
  as a proof of principle, we (re)compute the first 300 coefficients b_n
- analyse convergence and get a non-rigorous hint for the number of critical points



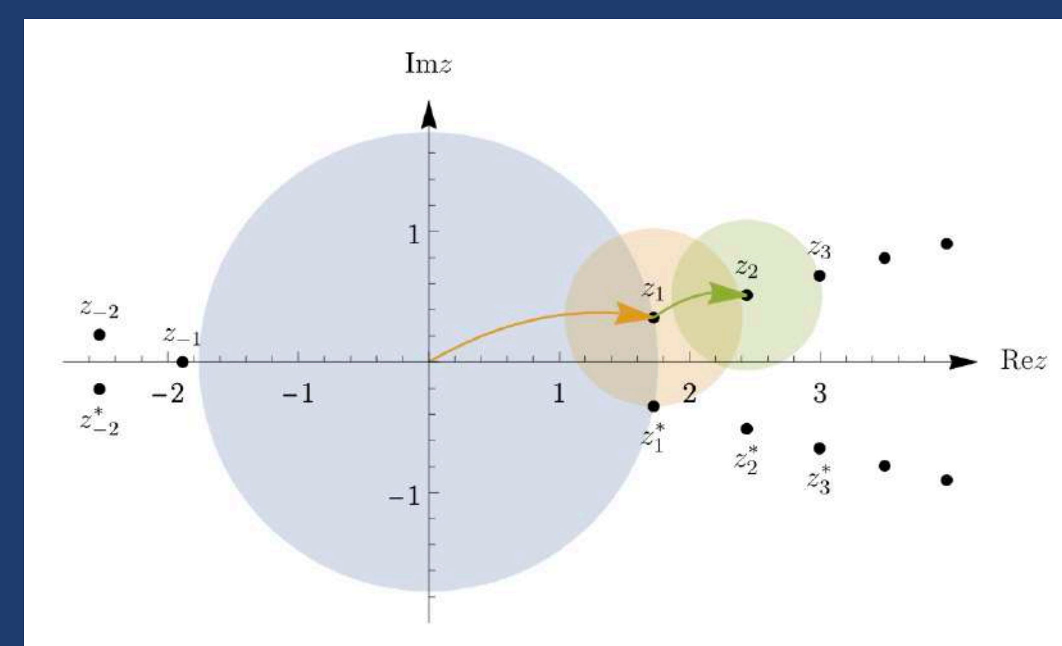
- using algorithm with 2 general critical points and 'recover' 12 coefficients

$$\mathfrak{w}_2(z) = \sum_{n=0}^{(N_2=12)-1} c_n (z-z_2)^{n/2}$$

- the gap: analytic continuation within the same sheet

$$\mathfrak{w}_2^{\text{calc}}(0) = 2.16275 - 3.25341i$$
$$\mathfrak{w}_2(0) = 2.12981 - 3.28100i$$

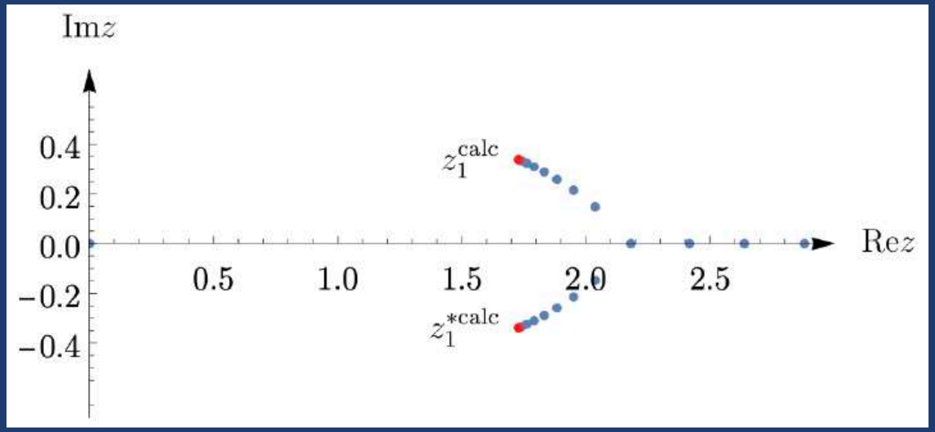
- … exploration continues …



EXAMPLE: MOMENTUM DIFFUSION OF M2 BRANES

- comparison with a Padé approximant from 300 coefficients [see also Withers, JHEP (2019)]
- Darboux appears to be superior in recovering the location of critical points and subsequent expansions

Darboux:	z_1	to 18 significant figures
Padé:	z_1	to 3 significant figures



- Padé appears to be superior in recovering the location of the gap

Darboux:	$\mathfrak{w}_1(0)$	to 2 significant figures
Padé:	$\mathfrak{w}_1(0)$	to 17 significant figures

Note: we used Padé within the same sheet

- if exact critical point is used, then Padé works spectacularly

Padé: $\mathfrak{w}_1(0)$ to 26 significant figures and 80 coefficients b_n to at least 10 significant figures

- unsurprising conclusion: combination of numerical methods is best
- is this useful for a reconstruction? conceptually yes, practically not quite (yet)…

SUMMARY AND
FUTURE DIRECTIONS

SUMMARY AND FUTURE DIRECTIONS

- complex analytic structures of transport are a powerful tool for exploring physics
- classical hydrodynamic dispersion relations are convergent in momentum space

- in some QFTs reconstruction of a spectrum should be possible all the way from IR to UV
- useful not only in QFTs but also for QNM reconstructions and other similar problems
- improve practical aspects of reconstructions given a limited number of known coefficient
- large-*d* story still has a few open questions…

- can these techniques be used in realistic QFTs (Euler-Heisenberg, chiral Lagrangian)?

THANK YOU!