Effective field theory for cosmological phase transitions

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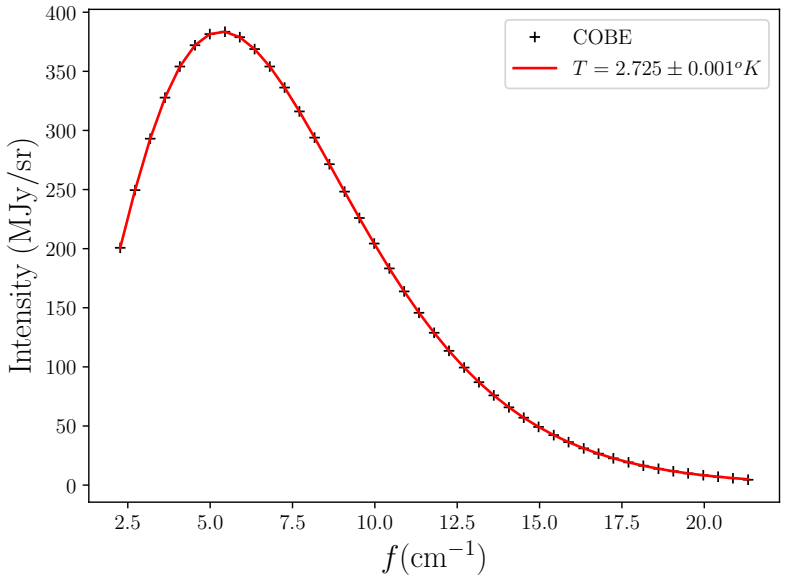
2 February 2023

Hot Big Bang



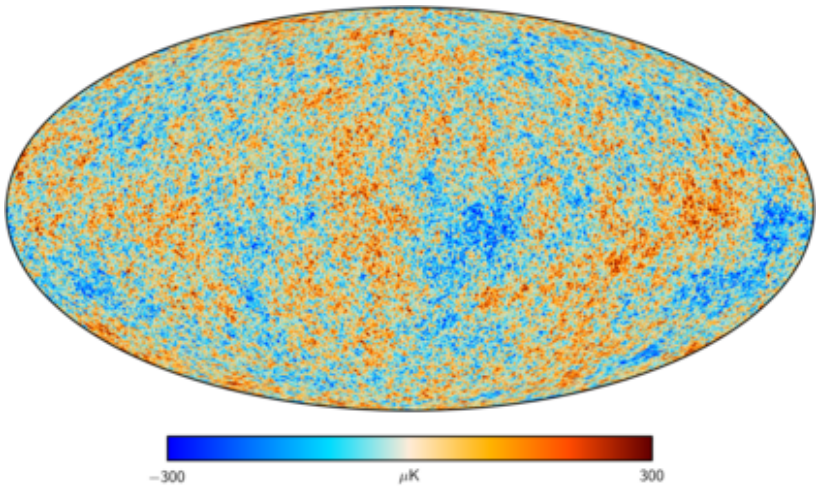
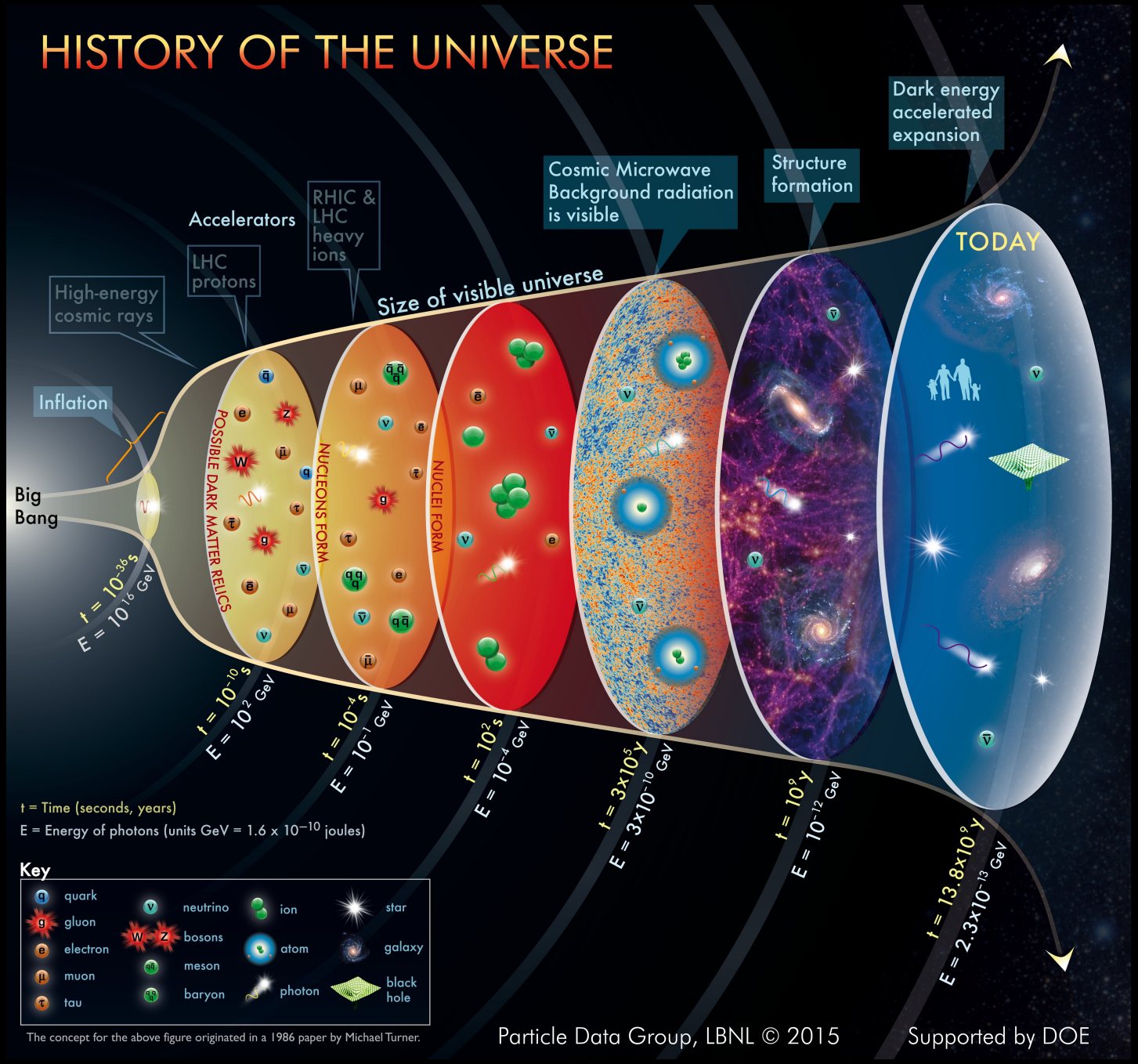


Figure: Blackbody spectrum of cosmic microwave background (COBE), and temperature anisotropies (Planck).

- Matter was very close to thermal in the early universe.
- Lots of interesting thermal physics.


HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
Accelerators
RHIC & LHC heavy ions
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
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t = Time (seconds, years)
E = Energy of photons (units GeV = 1.6 x 10^-10 joules)
Key
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electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
The concept for the above figure originated in a 1986 paper by Michael Turner.
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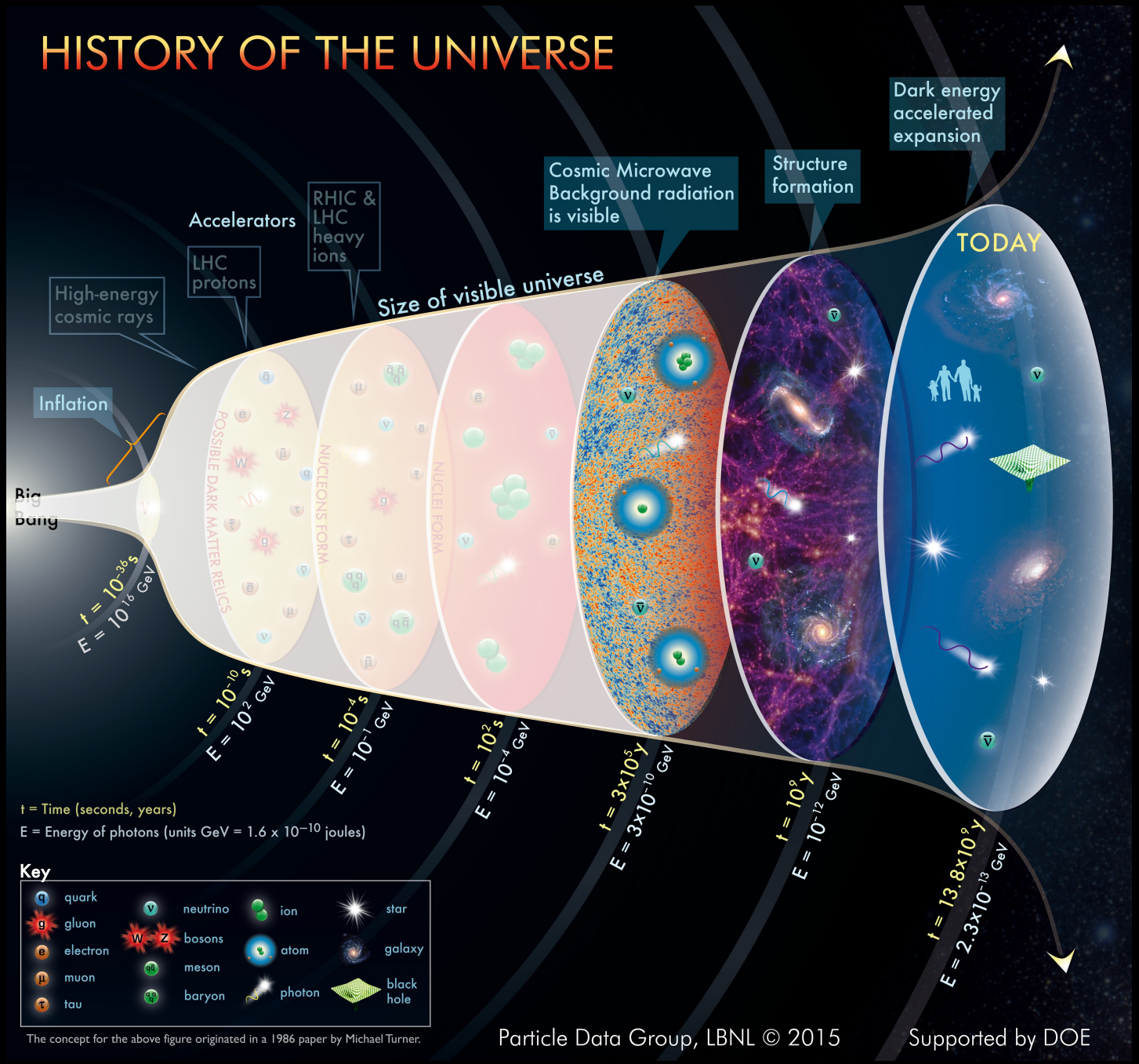
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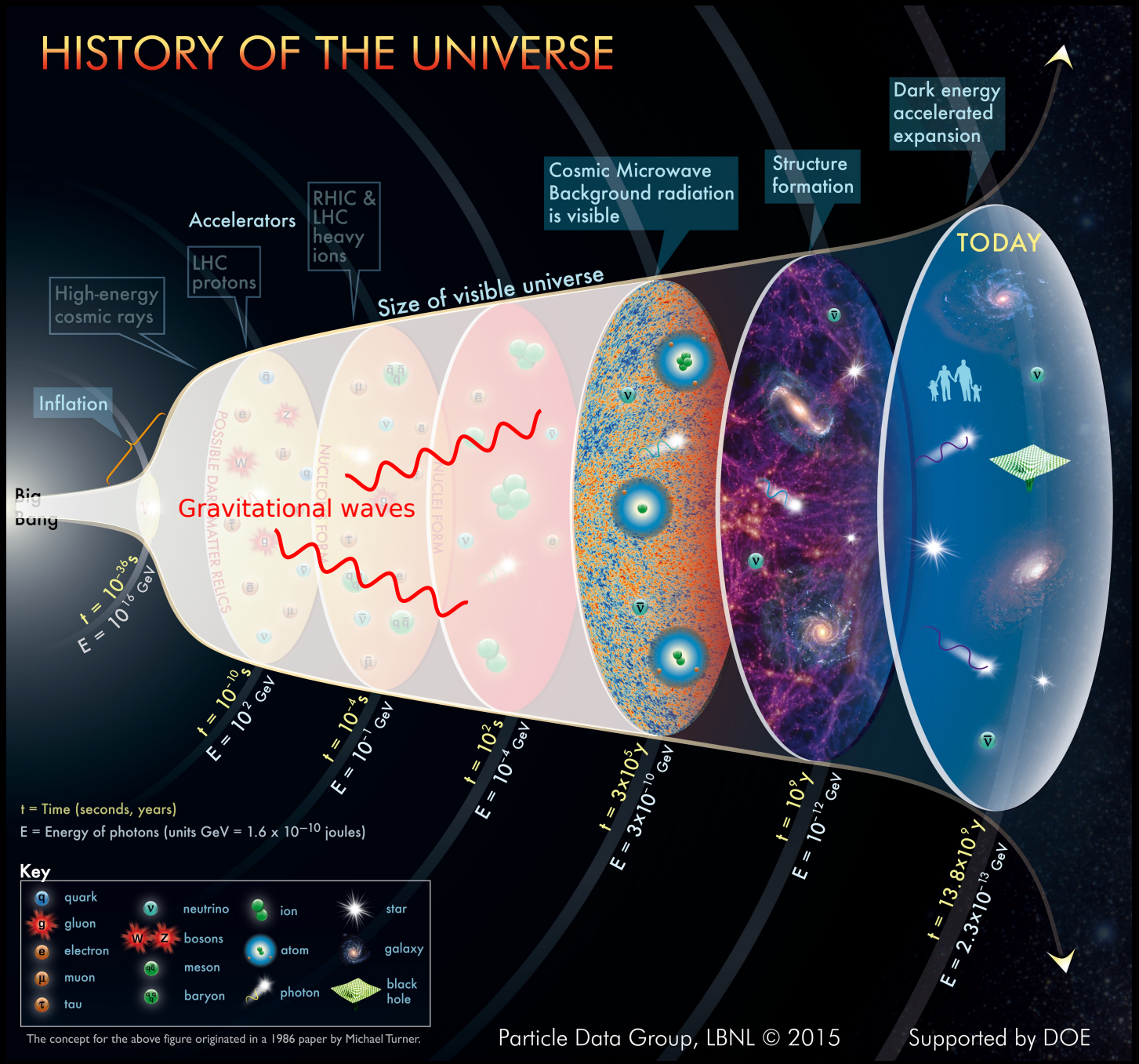
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Gravitational waves

- Gravitational waves directly observed by LIGO/Virgo →
- Future experiments will extend sensitivity ↓

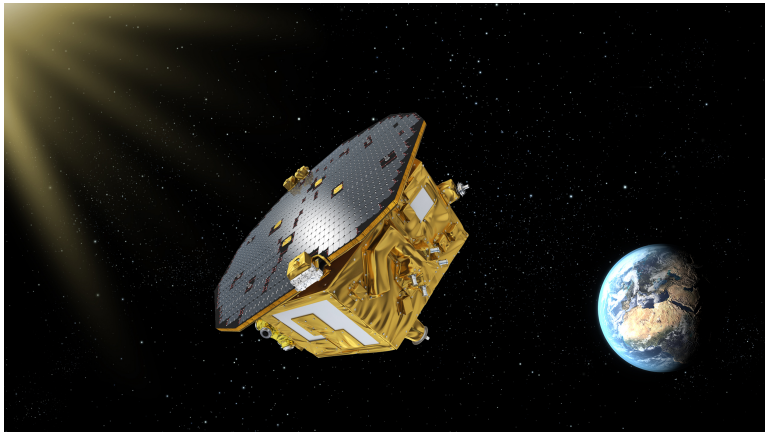


Figure: LISA Pathfinder

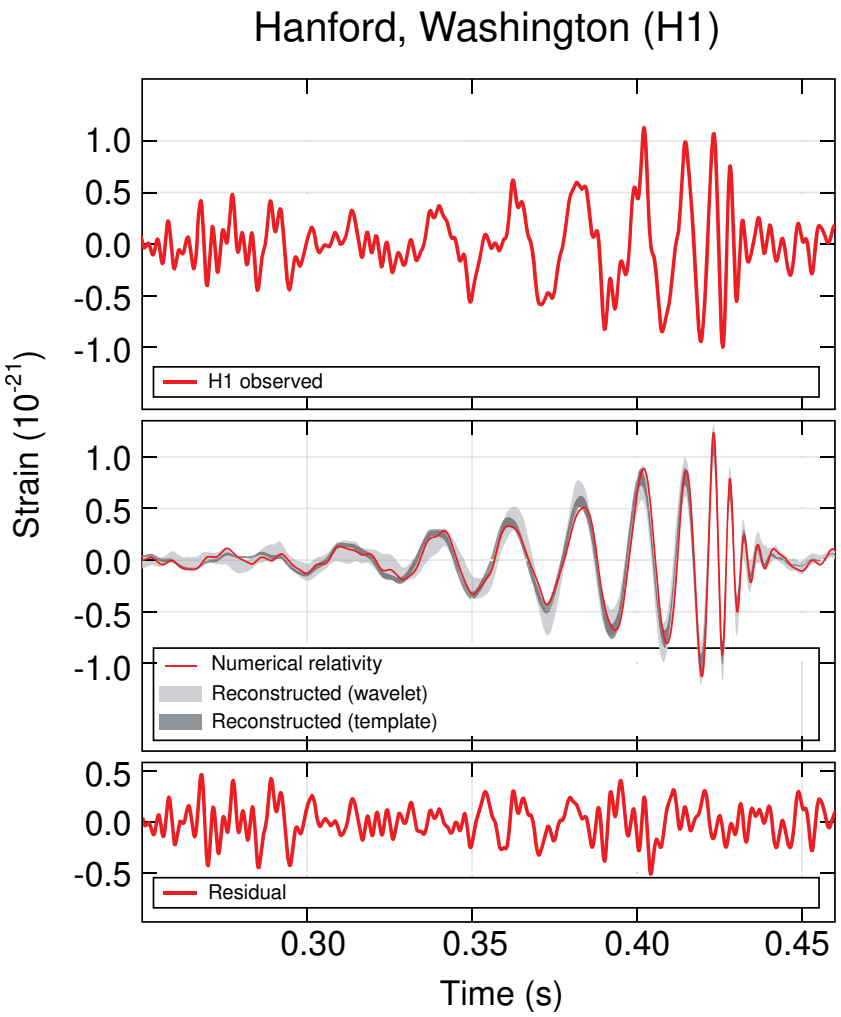
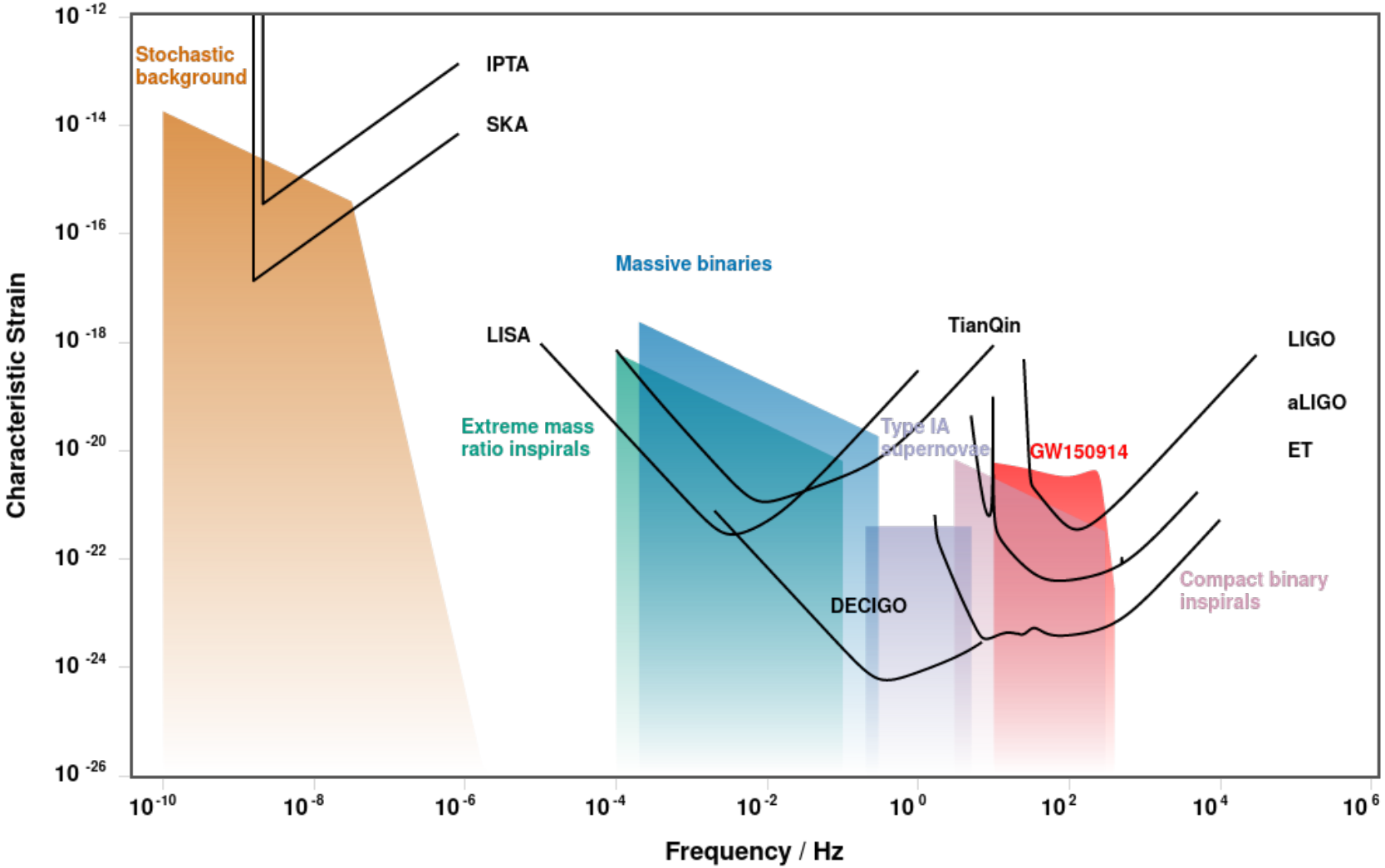


Figure: GW150914 1602.03837

The gravitational wave spectrum



gwplotter.com

Cosmological 1st-order phase transitions

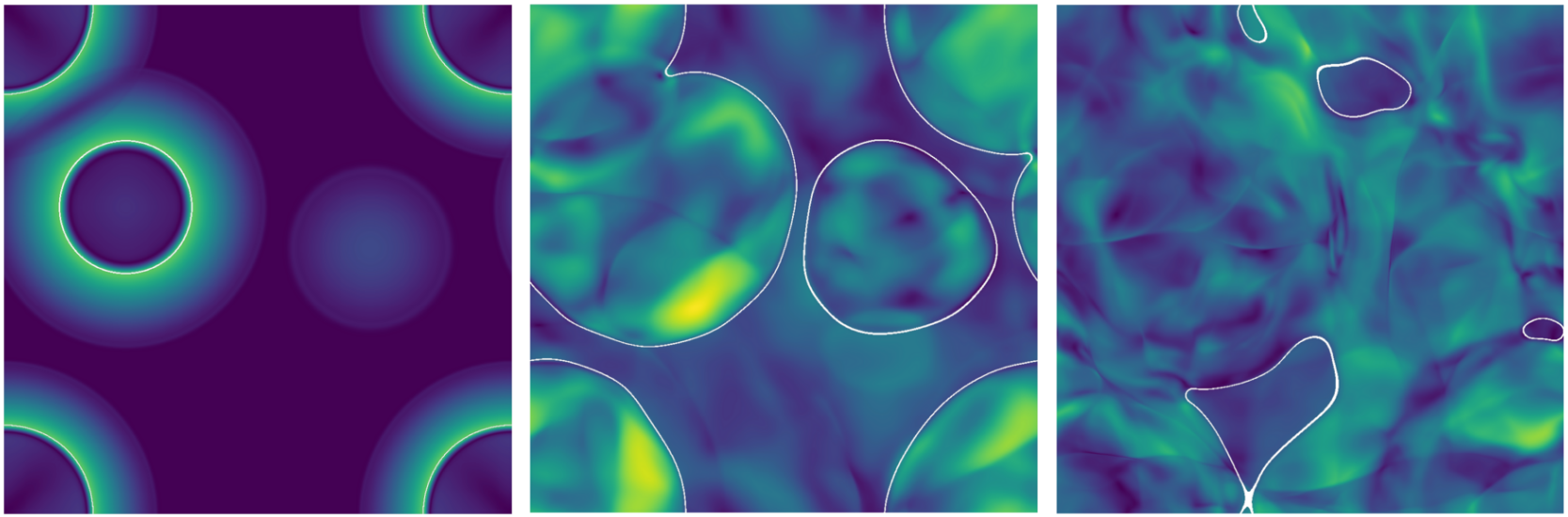


Figure: Cutting et al. arXiv:1906.00480.

- Bubbles nucleate, expand and collide
- This creates long-lived fluid flows
- And creates gravitational waves

Gravitational waves from phase transitions: the pipeline

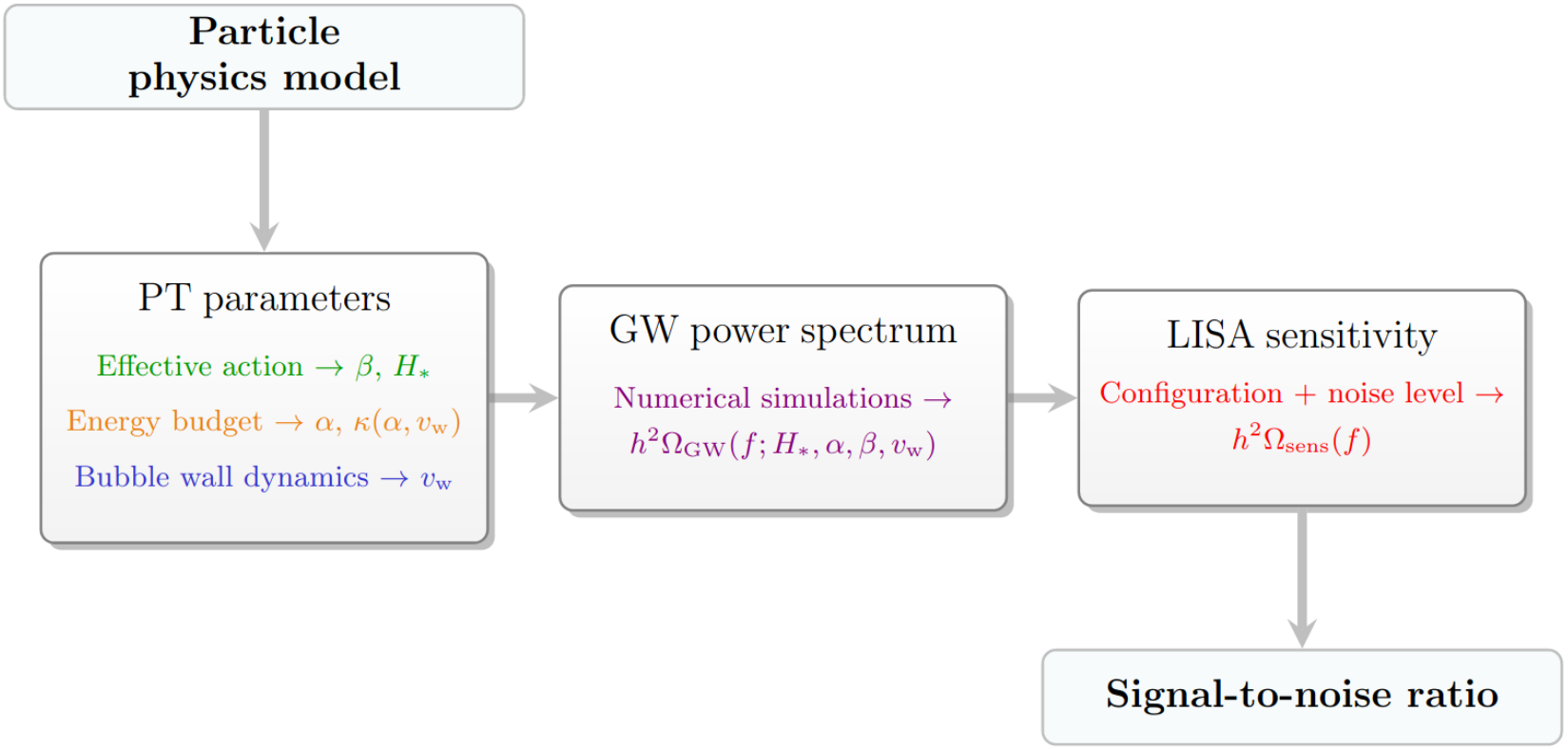
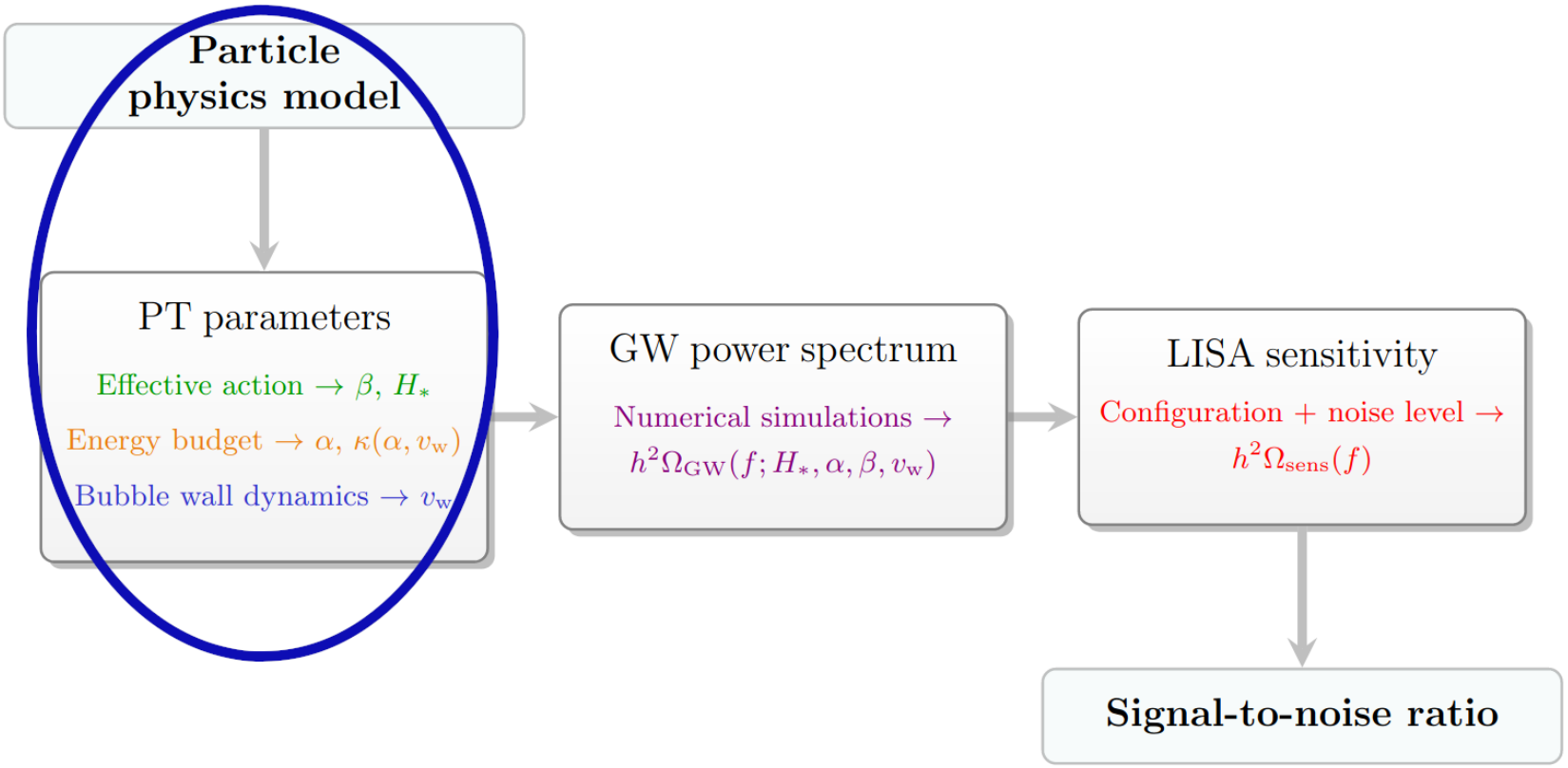


Figure: The Light Interferometer Space Antenna (LISA) pipeline $\mathscr{L} \to \mathrm{SNR}(f)$, Caprini et al. 1910.13125.

Gravitational waves from phase transitions: the pipeline



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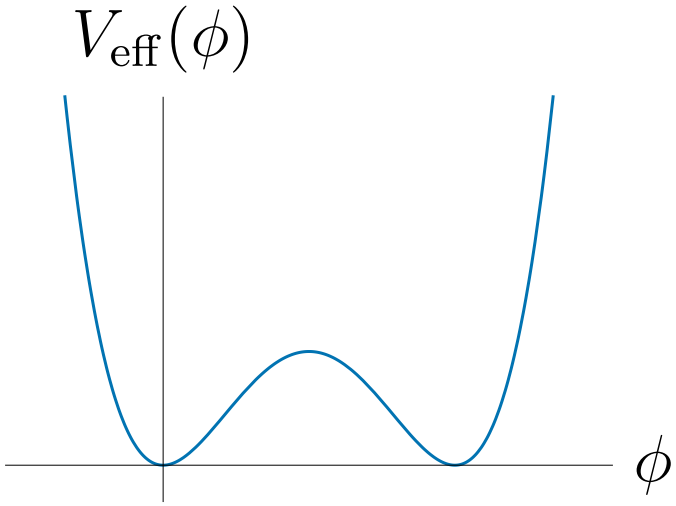
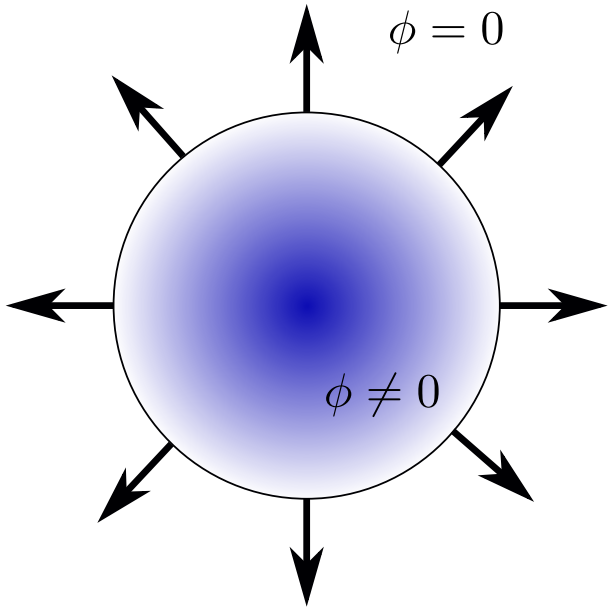
Phase transition parameters





Equilibrium (hom.)

- order of transition
- T_c, critical temperature
- $\Delta\theta_c$, latent heat
- c_s^2, sound speed

Near-equilibrium

- Γ, bubble nucleation rate $\Rightarrow T_*$, $\Delta\theta_*$, α_*, β/H_*

Nonequilibrium

- v_w, bubble wall speed

Standard approach to computing parameters

1-loop resummed approximation is based on

$$V_{\text{eff}}(\phi, T) = V_{\text{tree}}(\phi) + \underbrace{\frac{1}{2}\sum\!\!\!\!\!\!\int_P \log(P^2 + V''_{\text{tree}})}_{\text{1-loop}}$$

$$\underbrace{-\frac{T}{12\pi}\left((V''_{\text{tree}} + \Pi_T)^{3/2} - (V''_{\text{tree}})^{3/2}\right)}_{\text{daisy correction}}.$$

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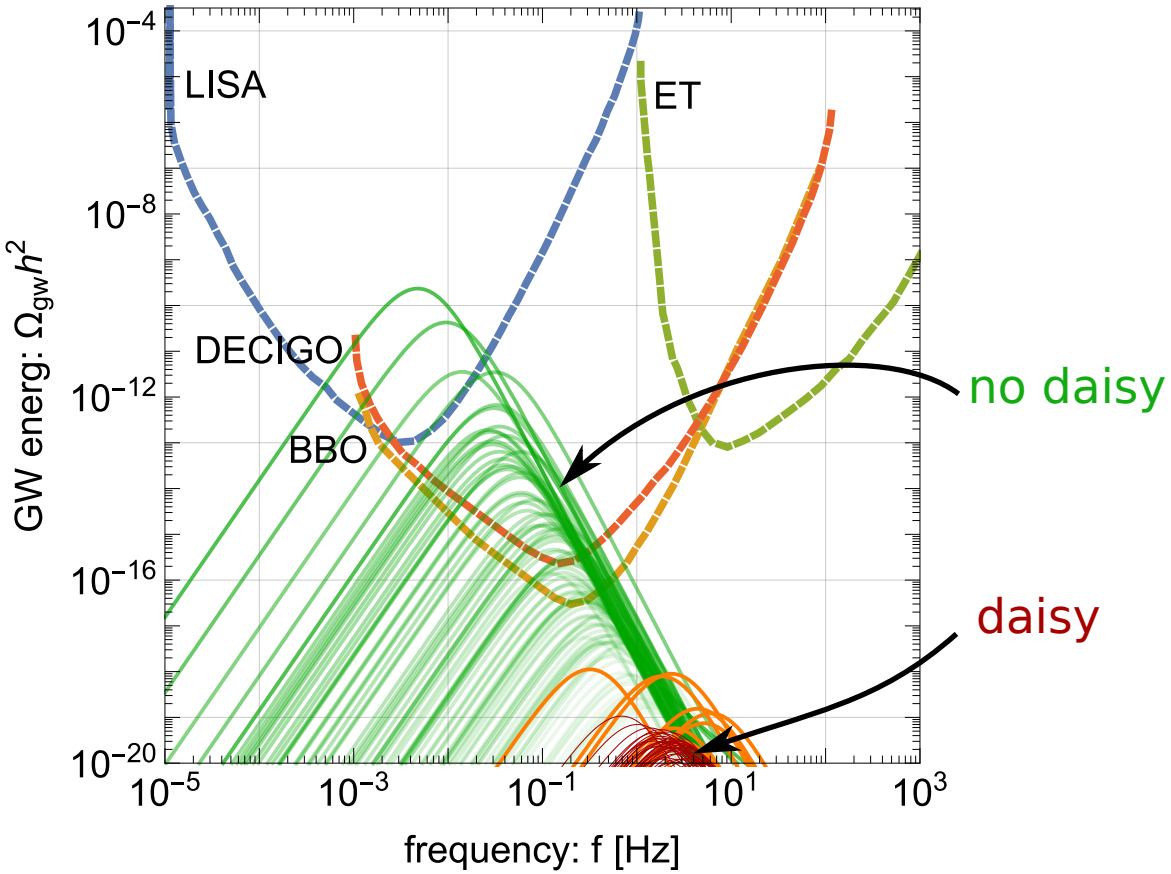
Solve:

- $\Re V'_{\text{eff}}(\phi, T) = 0 \Rightarrow$ phases
- $-\partial_r^2\phi - 2\partial_r\phi + \Re V'_{\text{eff}}(\phi, T) = 0 \Rightarrow$ critical bubble
- $\partial_t^2\phi - \partial_z^2\phi + \Re V'_{\text{eff}}(\phi, T) + \sum_i (m_i^2)'(\phi) \int_p \delta f_i(p, z) = 0 \Rightarrow v_w$

Theoretical uncertainties

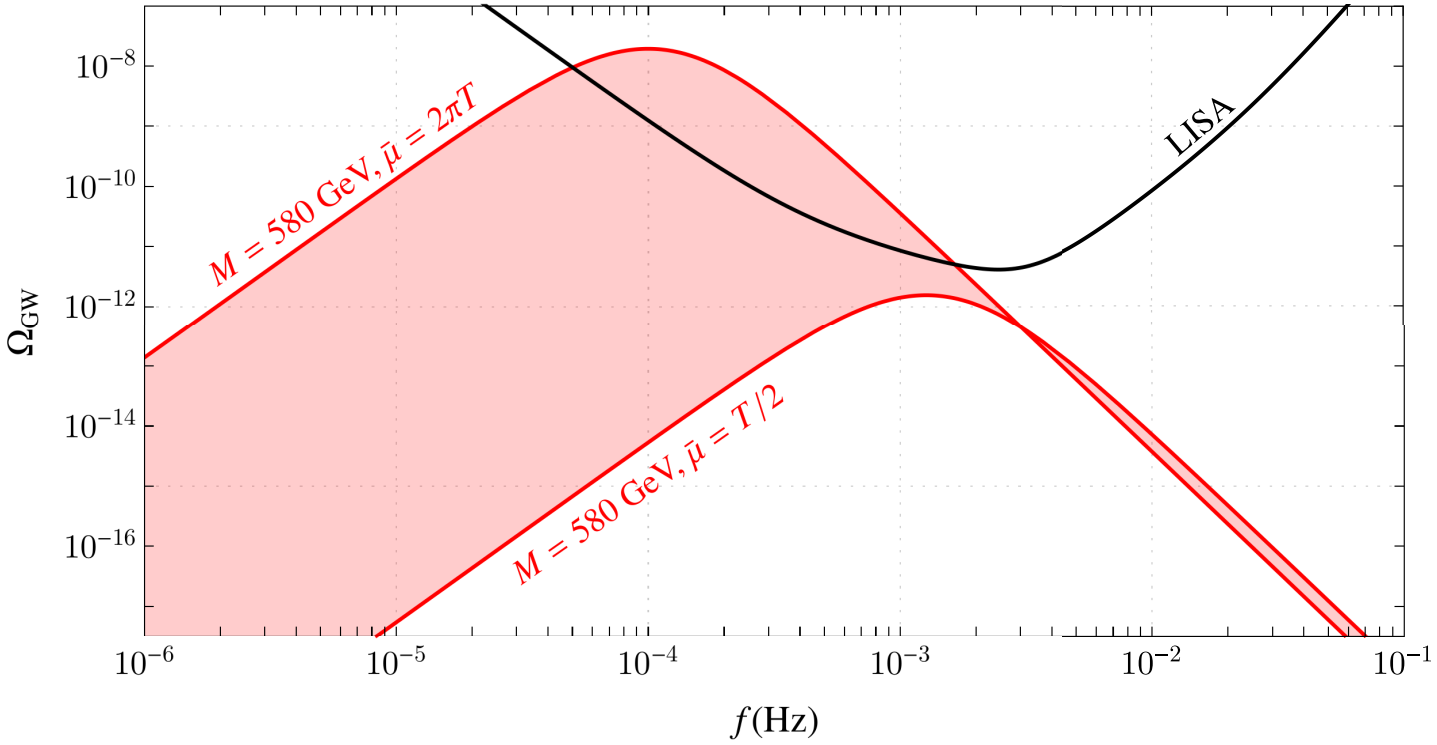


GW signals in two different 1-loop approximations for

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \frac{a_2}{2}(\Phi^\dagger\Phi)\sigma^2 + \frac{1}{2}(\partial\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{b_4}{4}\sigma^4$$

Carena, Liu & Wang 1911.10206

Theoretical uncertainties

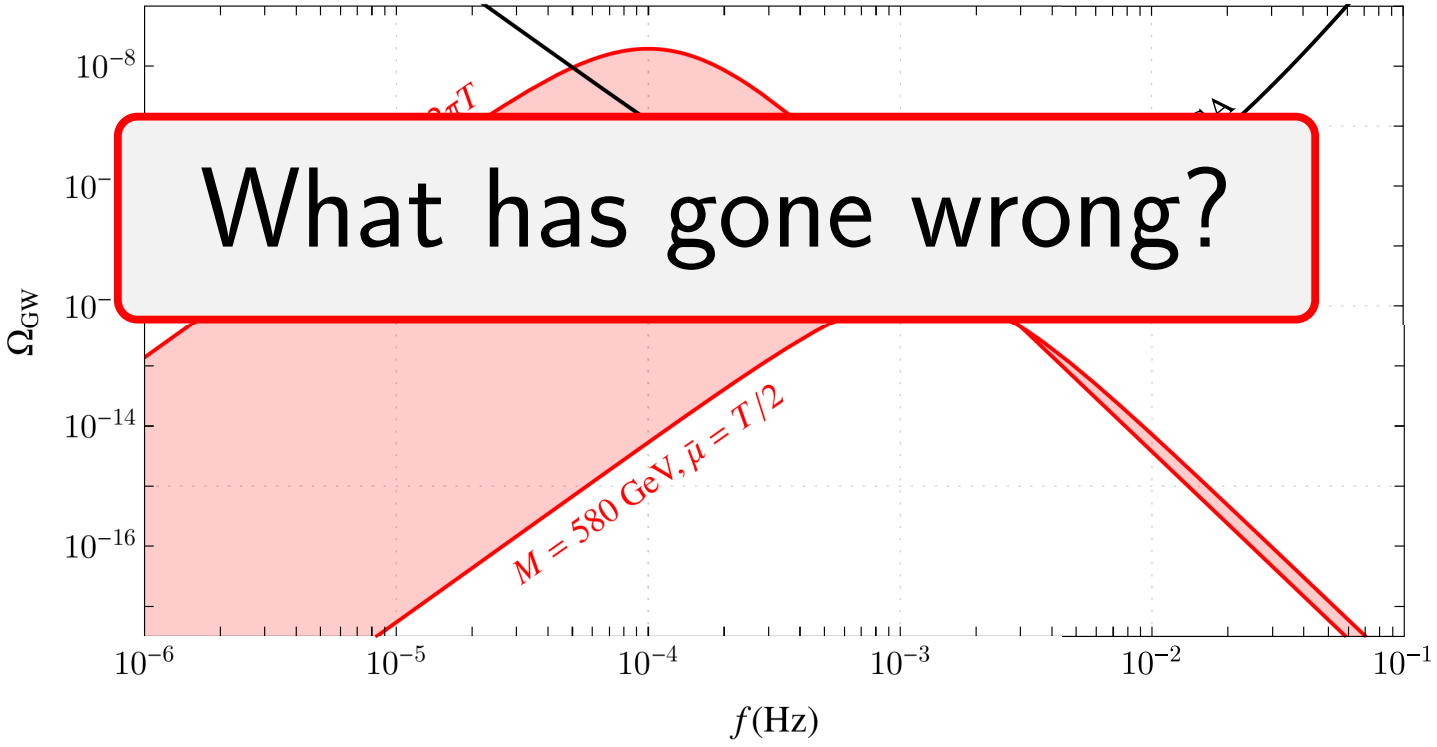


Renormalisation scale dependence of GW spectrum at one physical parameter point for

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \frac{1}{M^2}(\Phi^\dagger\Phi)^3.$$

Croon, OG, Schicho, Tenkanen & White 2009.10080

Theoretical uncertainties



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What has gone wrong?

Possible sources of theoretical uncertainties:

- nonperturbativity? Linde '80
- inconsistencies? E. Weinberg & Wu '87, E. Weinberg '92
- higher order perturbative corrections? Arnold & Espinosa '92
- gauge dependence or infrared divergences? Laine '94
- renormalisation scale dependence? Farakos et al. '94
- ...

Overview

1. Motivation
2. Scale hierarchies in phase transitions
3. EFT for equilibrium physics
4. EFT for bubble nucleation
5. Conclusions

Scale hierarchies in phase transitions

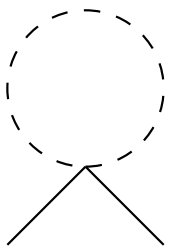
A hierarchy problem

Let's assume there is some very massive particle χ, $M_\chi \gg m_H$, coupled to the Standard Model Higgs Φ like

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + g^2 \Phi^\dagger \Phi \chi^\dagger \chi + \mathscr{L}_\chi .$$

If we integrate out χ, we find that the Higgs mass parameter gets a correction of the form

$$(\Delta m_H^2)\Phi^\dagger\Phi \; =$$

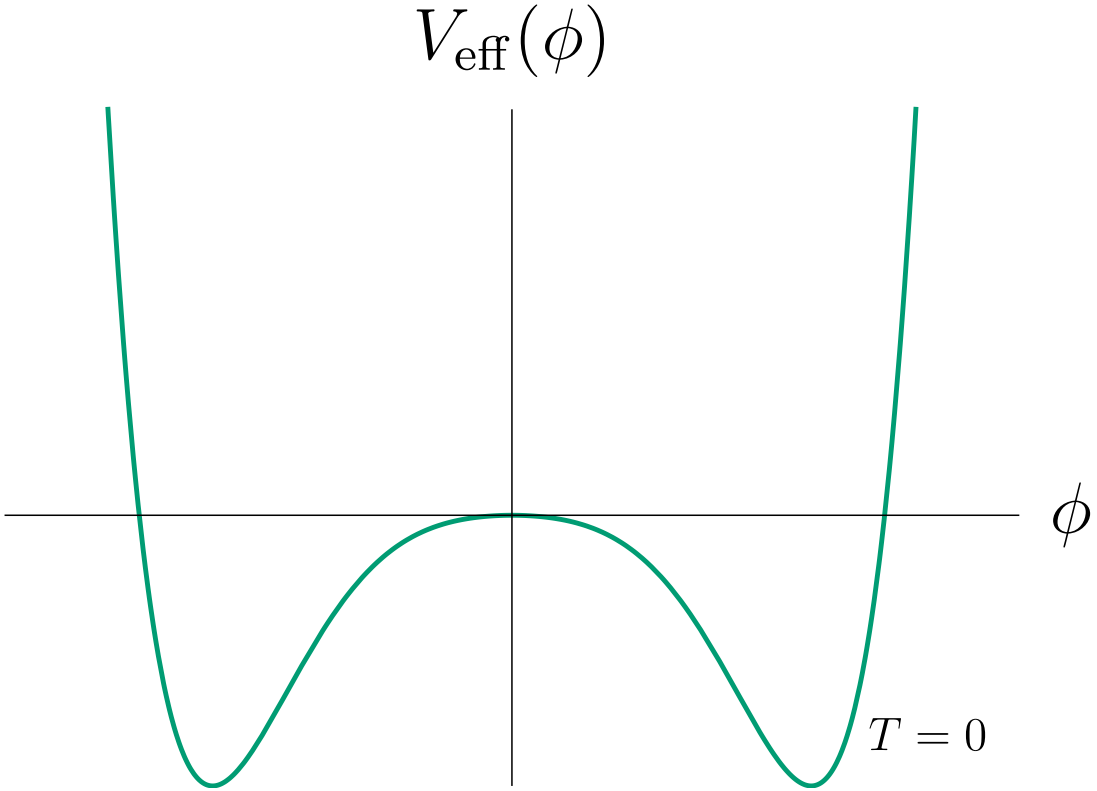


$$\sim g^2 M_\chi^2 \Phi^\dagger \Phi \; .$$

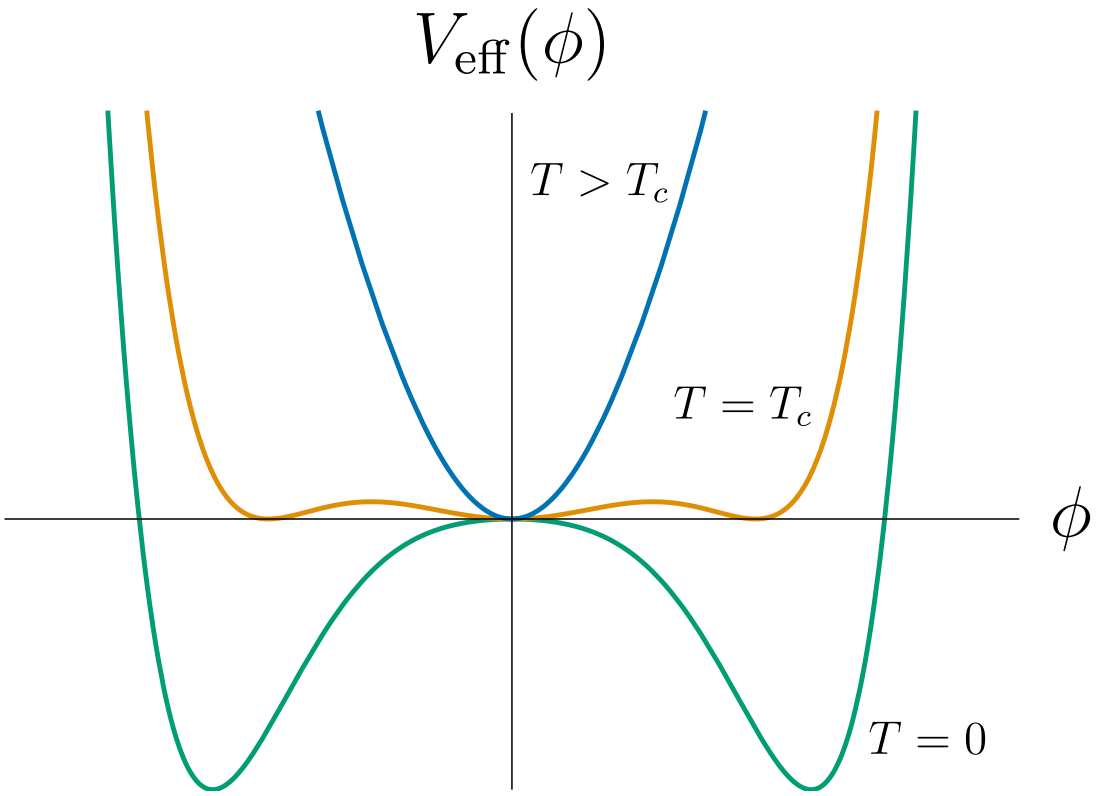
Relevant operators in the IR get large contributions from the UV,

$$\frac{\Delta m_H^2}{m_H^2} \sim g^2 \left(\frac{M_\chi}{m_H} \right)^2 .$$

Phase transitions



Phase transitions



For there to be a phase transition, thermal/quantum fluctuations should modify the potential at leading order,

$$V_{\rm eff} = V_{\rm tree} + \Delta V_{\rm fluct}.$$

Hierarchies in phase transitions

So, for there to be a phase transition, we need

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim g^2 N \left(\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}}\right)^{\sigma} \overset{!}{\sim} 1,$$

where $\sigma > 0$ for relevant operators.

Hierarchies in phase transitions

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$\Rightarrow$ either:

(i) $g^2 N \gtrsim 1$, i.e. strong coupling

(ii) $\Lambda_{\text{fluct}} \gg \Lambda_{\text{tree}}$, i.e. scale hierarchy

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Perturbative phase transitions require scale hierarchies!

Imaginary time formalism

- Thermodynamics formulated in $\mathbb{R}^3 \times S^1$.

- Fields are expanded into Fourier (Matsubara) modes:

$$\Phi(\mathbf{x},\tau) = \sum_{n \text{ even}} \phi_n(\mathbf{x}) e^{i(n\pi T)\tau} \leftarrow \text{boson}$$

$$\Psi(\mathbf{x},\tau) = \sum_{n \text{ odd}} \psi_n(\mathbf{x}) e^{i(n\pi T)\tau} \leftarrow \text{fermion}$$

- Masses of Matsubara modes are

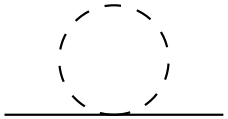
$$m_n^2 = m^2 + (n\pi T)^2$$

Introducing thermal scales

hard: $E \sim \pi T$ (nonzero Matsubara modes)

$$\phi_n, \psi_n, \text{ with } n \neq 0$$

soft: $E \sim gT$ (Debye screened)

$m_{\text{eff}}^2 \sim$  $\sim g^2 T^2$

supersoft: $E \sim g^{3/2} T$ (symmetry breaking, $\phi \sim T$)

$$V_{\text{eff}} \approx \frac{1}{2} m_{\text{eff}}^2 \phi^2 - \frac{T}{16\pi} (g^2 \phi^2)^{3/2} + \frac{\lambda}{4} \phi^4$$

ultrasoft: $E \sim g^2 T$ (non-Abelian gauge bosons)

$$A_i^a$$

Infrared strong coupling

Infrared bosons are highly occupied; the effective expansion parameter $\alpha_{\rm eff}$ grows

$$\alpha_{\rm eff} \sim g^2 \frac{1}{1 - e^{E/T}} \approx g^2 \frac{T}{E}$$

Softer modes are classically occupied and more strongly coupled:

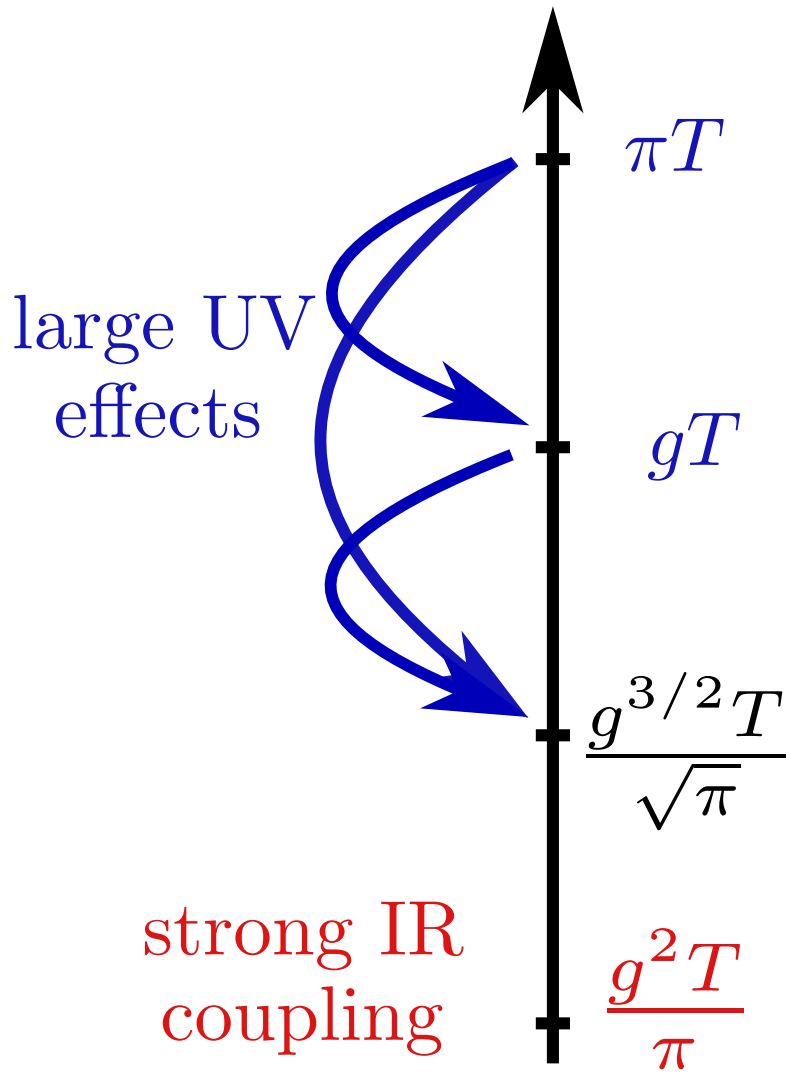
$$\begin{aligned}
\text{hard}: &\quad E \sim \pi T \Rightarrow \alpha_{\rm eff} \sim g^2 \sim 0.03, \\
\text{soft}: &\quad E \sim gT \Rightarrow \alpha_{\rm eff} \sim g \sim 0.18, \\
\text{supersoft}: &\quad E \sim g^{3/2} T \Rightarrow \alpha_{\rm eff} \sim g^{1/2} \sim 0.42, \\
\text{ultrasoft}: &\quad E \sim g^2 T \Rightarrow \alpha_{\rm eff} \sim g^0 \sim 1.
\end{aligned}$$

UV and IR problems

There are two main difficulties

- large UV effects break loop expansion
- IR becomes more strongly coupled

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim \alpha_{\text{eff}} \left(\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}} \right)^{\sigma}$$



EFT for equilibrium physics

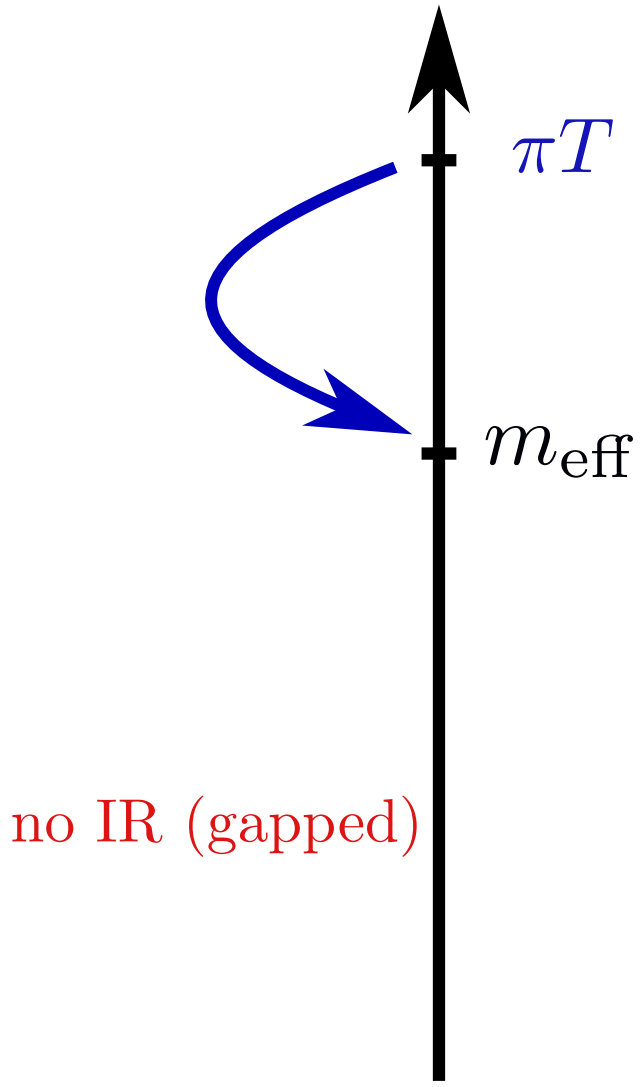
Real scalar model

A simple model,

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \sigma\phi + \frac{m^2}{2}\phi^2 + \frac{\kappa}{3!}\phi^2 + \frac{g^2}{4!}\phi^4 + J_1\phi + J_2\phi^2,$$

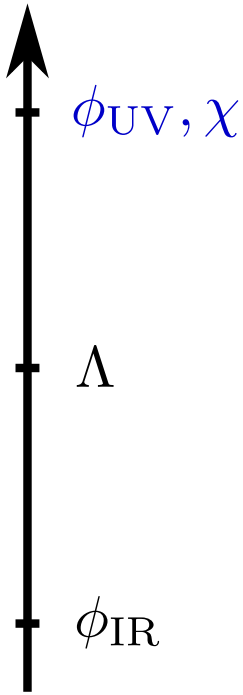
with only two scales: πT, $m_{\text{eff}} \sim gT$.

- large UV effects $(\pi T/m_{\text{eff}}) \sim \pi/g$
- IR coupling $\alpha_{\text{eff}} \sim g$



Wilsonian EFT

- Split degrees of freedom $\{\phi, \chi\}$ based on energy $\rightarrow$



- Integrate out the UV modes:

$$\int \mathcal{D}\phi \int \mathcal{D}\chi \; e^{-S[\phi,\chi]} = \int \mathcal{D}\phi_{\rm IR} \left(\int \mathcal{D}\phi_{\rm UV} \mathcal{D}\chi \; e^{-S[\phi,\chi]} \right)$$
$$= \int \mathcal{D}\phi_{\rm IR} \; e^{-S_{\rm eff}[\phi_{\rm IR}]}$$

- Careful power-counting cancels dependence on Λ.
- $S_{\rm eff}[\phi_{\rm IR}]$ must be expansion in ∂^2 (but not in $\phi_{\rm IR}$).

Burgess '21, Hirvonen '22

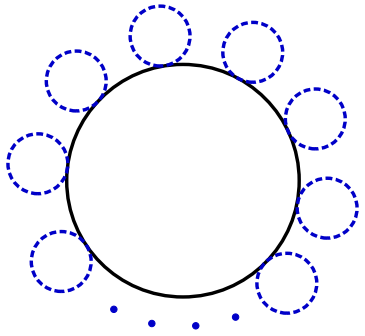
Resummations with EFT

By first integrating out the UV modes

$$S_{\text{eff}}[\phi_{\text{IR}}] = S_\phi[\phi_{\text{IR}}] - \log \int \mathcal{D}\phi_{\text{UV}} \mathcal{D}\chi \; e^{-S[\phi_{\text{IR}}+\phi_{\text{UV}},\chi]+S_\phi[\phi_{\text{IR}}]},$$

$$\approx S_\phi[\phi_{\text{IR}}] + \int_x \left[(\sigma_{\text{eff}} - \sigma)\phi_{\text{IR}} + \frac{1}{2}(m_{\text{eff}}^2 - m^2)\phi_{\text{IR}}^2 \right],$$

the daisy resummations arise naturally.



So do all other necessary resummations, order by order.

$\Rightarrow$ Solves UV problems

EFT factorisation

Contributions to physical quantities factorise

$$L = \underbrace{\frac{d\sigma_{\text{eff}}}{d\log T}}_{\text{hard modes}} \underbrace{\Delta\langle\bar{\phi}_{\text{IR}}\rangle}_{\text{soft modes}},$$

$$= \underbrace{(A + Bg^2 + O(g^4))}_{\text{hard modes}} \times \underbrace{(a + bg + cg^2 + dg^3 + O(g^4))}_{\text{soft modes}}.$$

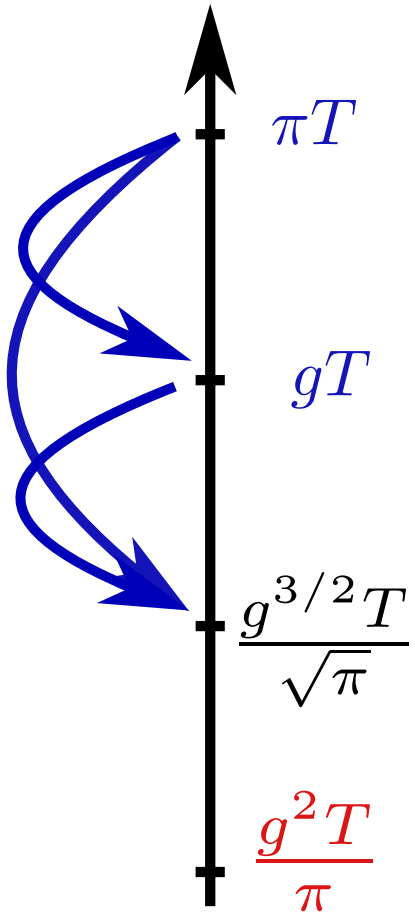
One must work harder for the soft modes.

Scalar triplet extension of the Standard Model

A more complicated model with all the scales

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \frac{a_2}{2}\Phi^\dagger\Phi\Sigma^a\Sigma^a + \frac{1}{2}D_\mu\Sigma^a D_\mu\Sigma^a + \frac{m_\Sigma^2}{2}\Sigma^a\Sigma^a + \frac{b_4}{4}(\Sigma^a\Sigma^a)^2.$$

- large UV effects
- strongly coupled IR



UV and IR in concert

For some observable $\mathcal{O}$ at $T = 0$

$$\mathcal{O}_0 = \underbrace{A}_{\text{0-loop}} + \underbrace{Bg^2}_{\text{1-loop}} + \underbrace{Cg^4}_{\text{2-loop}} + \underbrace{Dg^6}_{\text{3-loop}} + \underbrace{Eg^8}_{\text{4-loop}} + \dots$$

UV and IR in concert

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$$\mathcal{O}_0 = \underbrace{A}_{\text{0-loop}} + \underbrace{Bg^2}_{\text{1-loop}} + \underbrace{Cg^4}_{\text{2-loop}} + \underbrace{Dg^6}_{\text{3-loop}} + \underbrace{Eg^8}_{\text{4-loop}} + \dots$$

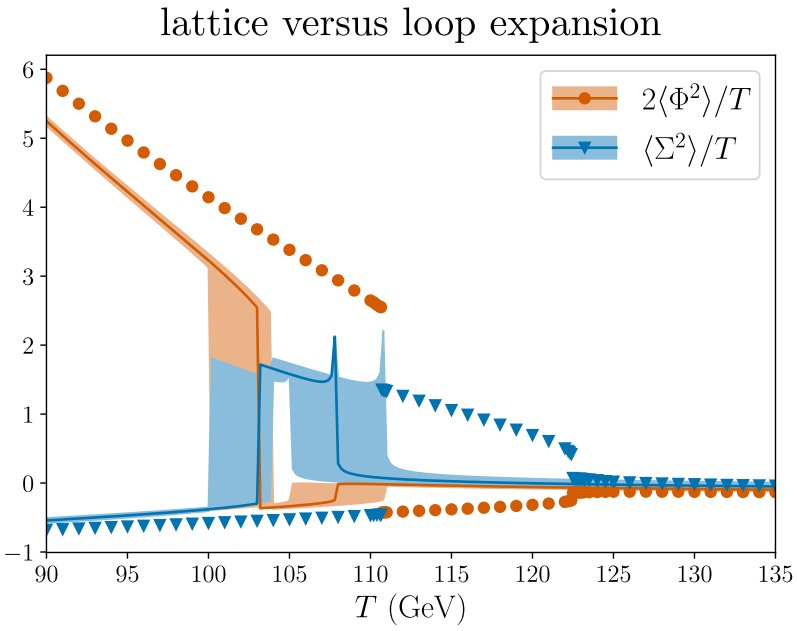
At a Higgs-like first-order phase transition, instead

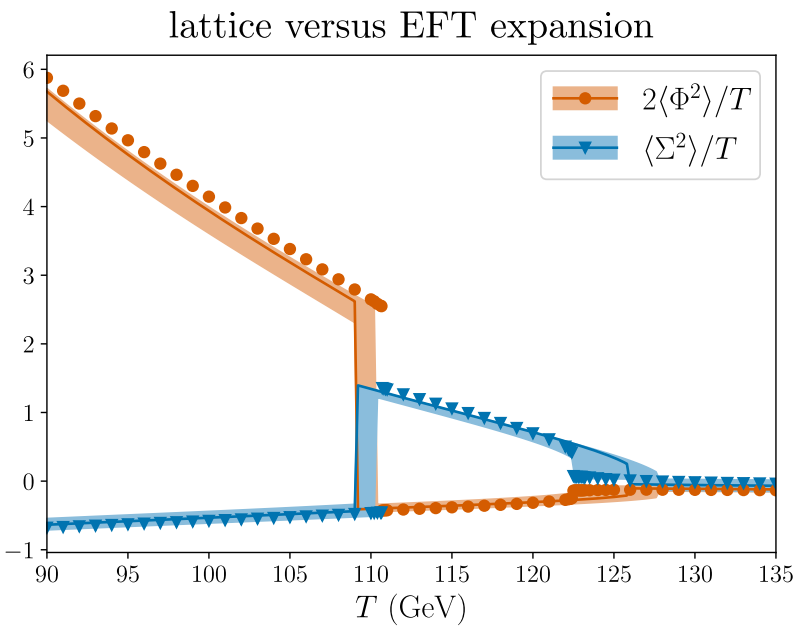
$$\mathcal{O}_T = \underbrace{a}_{\text{1-loop}^+} + \underbrace{bg}_{\text{2-loop}^+} + \underbrace{cg^{3/2}}_{\text{1-loop}^\dagger} + \underbrace{dg^2}_{\text{3-loop}^+} + \underbrace{eg^{5/2}}_{\text{3-loop}^\dagger} + \underbrace{fg^3}_{\infty\text{-loop}} + \dots$$

where $^+$ and † refer to different resummations of infinite classes of diagrams.

Ekstedt, OG & Löfgren 2205.07241

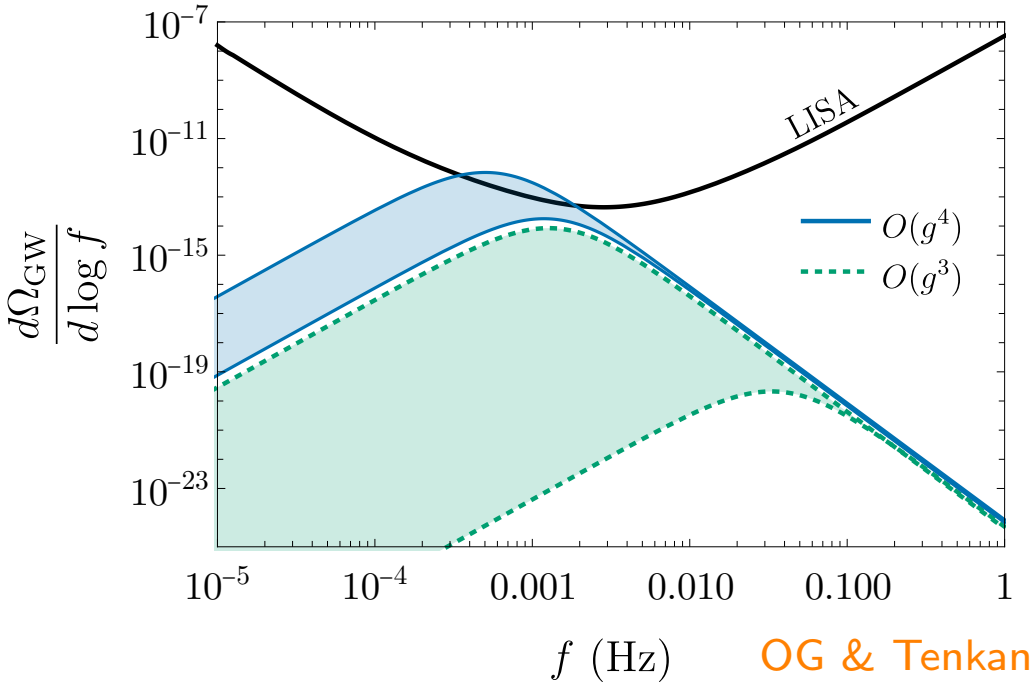
Lattice versus perturbation theory





Niemi et al. 2005.11332, OG & Tenkanen forthcoming

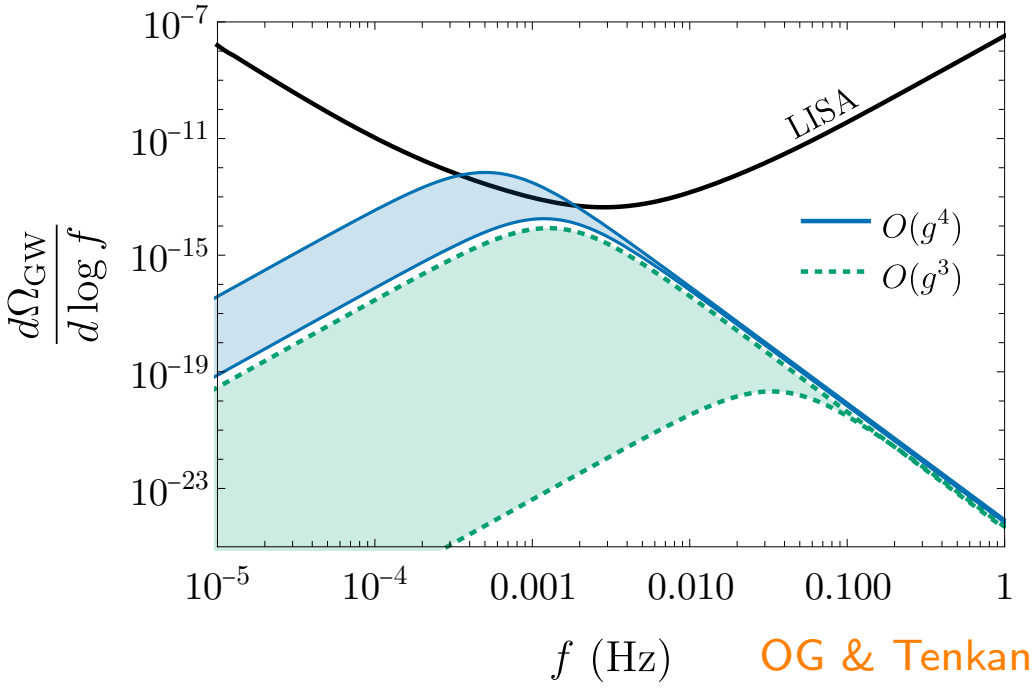
Conclusions



OG & Tenkanen 2104.04399

- Phase transitions may produce observable gravitational waves
- Large theoretical uncertainties in standard computations
  - UV "hierarchy" problems
  - IR strong-coupling problems
  - Consistency problems for bubble nucleation
- EFT solves UV problems, and gives definition of nucleation rate
- Higher orders (or lattice) solves problems from IR

Conclusions



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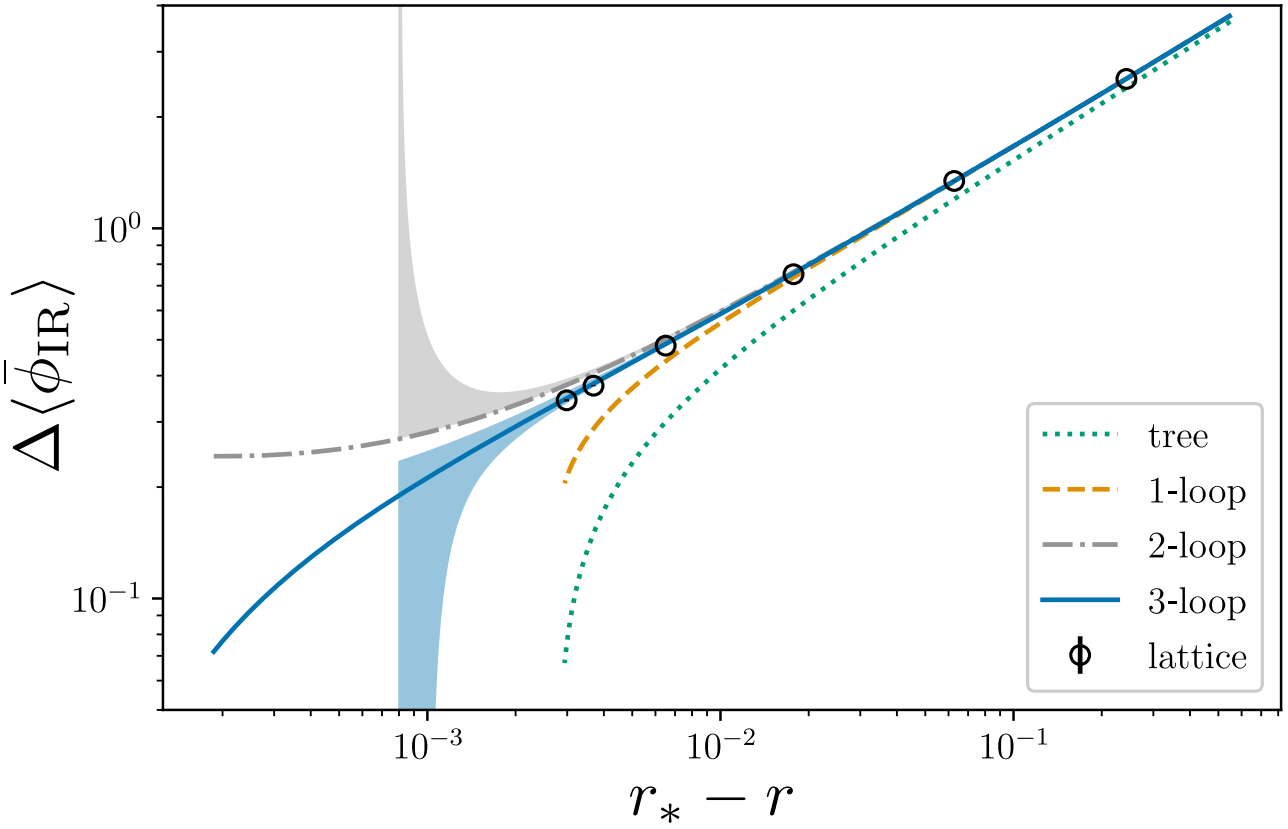
Thanks for listening!

Backup slides

Lattice vs perturbation theory: real scalar model



$$\Delta\langle\bar{\phi}_{\rm IR}\rangle = \frac{1}{4\pi\alpha_{\rm eff}}\Big[\,2+\sqrt{3}\,\alpha_{\rm eff}+\frac{1}{2}\left(1+2\log\tilde{\mu}_3\right)\alpha_{\rm eff}^2$$
$$+\sqrt{3}\left(-\frac{3}{8\sqrt{2}}\xi+\frac{21}{32}\mathrm{Li}_2\frac{1}{4}-\frac{7\pi^2}{128}-\frac{1}{2}+\frac{21}{64}\log^2\frac{4}{3}+\frac{5}{8}\log\frac{4}{3}\right)\alpha_{\rm eff}^3$$
$$+\,O\left(\alpha_{\rm eff}^4\right)\Big]$$

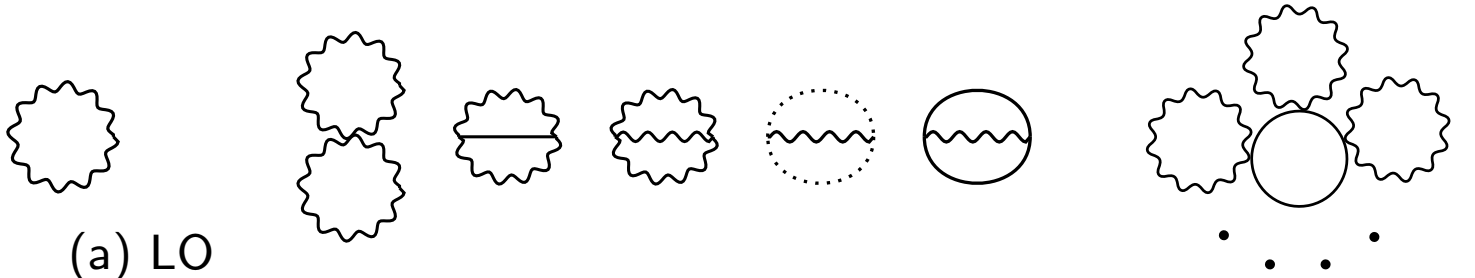
OG 2101.05528

Supersoft scale EFT

Integrating out the scales πT and gT gives

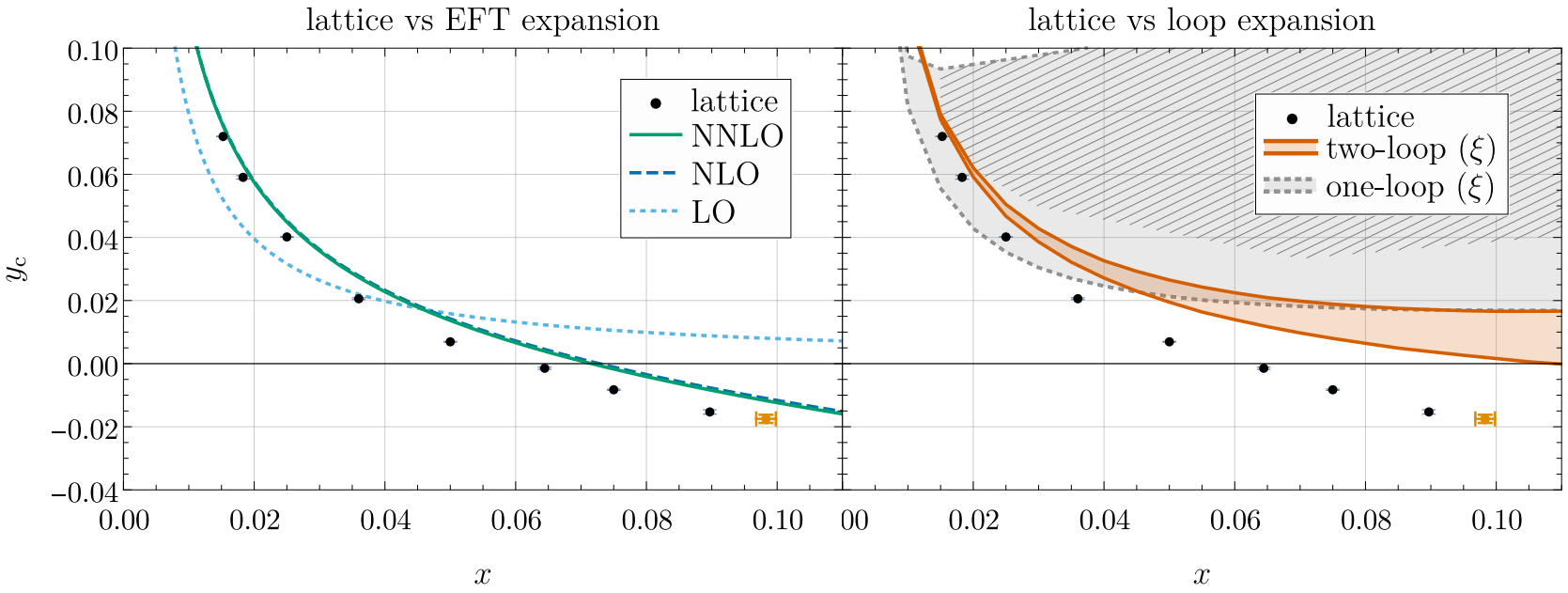
$$\mathscr{L}_{\text{eff}} = \frac{1}{2}\partial_i\varphi^\dagger\partial_i\varphi + \frac{m_3^2}{2}\varphi^\dagger\varphi - \frac{g_3^3}{4(4\pi)}(\varphi^\dagger\varphi)^{3/2} + \frac{\lambda_3}{4}(\varphi^\dagger\varphi)^2$$
$$- \frac{11g_3}{8(4\pi)}\frac{\partial_i\varphi^\dagger\partial_i\varphi}{(\varphi^\dagger\varphi)^{1/2}} - \frac{51}{64}\frac{g_3^4}{(4\pi)^2}\varphi^\dagger\varphi\log\frac{g_3^2\varphi^\dagger\varphi}{\tilde{\mu}_3^2}$$

After integrating out the scale πT, the relevant diagrams are



(a) LO (b) NLO (c) NNLO

EFT solution: gauge independence



EFT approach provides exact order-by-order gauge invariance.

Ekstedt, OG & Löfgren 2205.07241

(OG & Hirvonen 2108.04377, Löfgren et al. 2112.05472, Hirvonen et al. 2112.08912)