Probing the Nature of HNLs in Direct Searches and Neutrinoless Double Beta Decay

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Based on [2212.14690] with F. Deppisch, M. Rai and Z. Zhang

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Motivation

[Credit: E. Lisi]

Heavy neutral leptons (HNLs) are a well-motivated extension to the SM

- SM: Only left-handed fields $\nu_L \quad \Leftrightarrow \quad m_\nu = 0,\ \Delta L = 0$ to all orders

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$$\begin{aligned} \mathcal{L} &\supset -Y_\nu \bar{L}\tilde{H}\nu_R + \text{h.c.} \\ &\supset -\frac{Y_\nu}{\sqrt{2}}(v+h)\bar{\nu}_L\nu_R + \text{h.c.}\,, \quad m_\nu = \frac{Y_\nu v}{\sqrt{2}} \end{aligned}$$

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- Lepton number (an accidental global symmetry of the SM) forbids

$$\mathcal{L} \supset -\frac{1}{2} M_R \bar{\nu}_R^c \nu_R + \text{h.c.}$$

$\Rightarrow$ This symmetry need not hold in the UV (dim-5 SMEFT operator)

$\Rightarrow$ *A priori*, M_R of arbitrary value (high-scale/low-scale seesaw mechanisms)

$\Rightarrow$ Motivation to consider an extended neutrino SM (νSM)

Extended Neutrino Sector

Adding singlet fermion N_R to the SM (respecting $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$)

$$\mathcal{L}_{\mathrm{SMEFT}+N} = \mathcal{L}_{\mathrm{SM}} + i\bar{N}_R \partial\!\!\!/ N_R - \left[\frac{1}{2} M_R \bar{N}_R^c N_R + Y_\nu \bar{L} \tilde{H} N_R + \mathrm{h.c.}\right] + \underbrace{\sum_{d>5} \mathcal{L}^{(d)}}_{\text{Up to dim-9}}$$

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With n_S singlet states:

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Now block-diagonalise $\mathcal{M}_\nu$ as $(\frac{v}{\sqrt{2}} Y_\nu \ll M_R)$

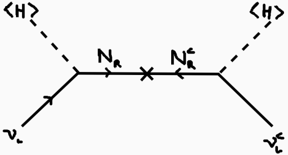
$$U^\dagger \mathcal{M}_\nu U^* = \begin{pmatrix} m_\nu & 0 \\ 0 & m_N \end{pmatrix} = \text{diag}(m_1, m_2, m_3, \underbrace{m_{N_1}, m_{N_2}, \ldots}_{\text{HNLs}})$$

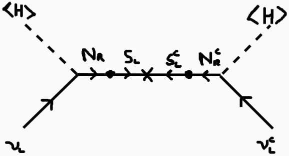
$$\text{Mixing matrix}: \quad U = \begin{pmatrix} U_\nu & U_{\nu N} \\ U_{N\nu} & U_N \end{pmatrix} = \begin{pmatrix} (1 - \frac{1}{2}\Theta\Theta^\dagger) U_{\text{PMNS}} & \Theta \\ -\Theta^\dagger U_{\text{PMNS}} & 1 - \frac{1}{2}\Theta^\dagger\Theta^* \end{pmatrix} + \mathcal{O}(\Theta^3)$$

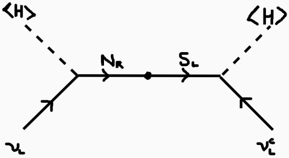
Seesaw Type

Type-I Seesaw:

$$M_\nu = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} Y_1^T & 0 \\ \frac{v}{\sqrt{2}} Y_1^T & M & 0 \\ 0 & 0 & M \end{pmatrix} \Rightarrow \begin{aligned} \Theta &= \left(\frac{v Y_1}{\sqrt{2} M}, 0 \right) \\ m_\nu &= -\frac{v^2}{2M} Y_1 Y_1^T \end{aligned}$$







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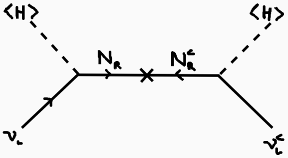
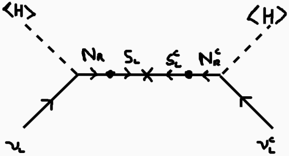
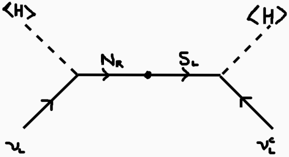


Inverse Seesaw (ISS):

$$M_\nu = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} Y_1 & 0 \\ \frac{v}{\sqrt{2}} Y_1^T & 0 & M \\ 0 & M & \mu \end{pmatrix} \Rightarrow \begin{aligned} \Theta &= \left(\frac{v Y_1}{2M}, \frac{v Y_1}{2M} \right) \\ m_\nu &= \frac{v^2}{2M^2} \mu Y_1 Y_1^T \\ \Delta m_N &= \mu \end{aligned}$$





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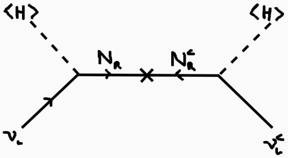

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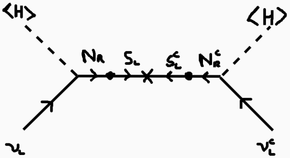

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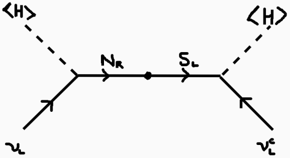

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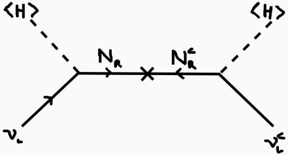

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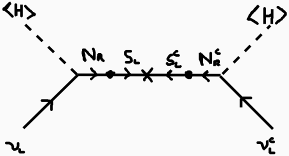

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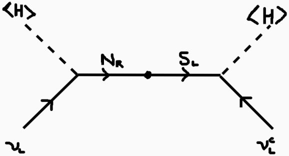

$$\Delta m_N \big|_{\text{LSS}} < \Delta m_N \big|_{\text{ISS}} < \Delta m_N \big|_{\text{ISS,1-loop}}$$

Parametrisation

Assuming general form of M_ν, we want to be compatible with the neutrino oscillation data

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$\Rightarrow$ For $n_S = 2$, only a single physical Majorana phase in light neutrino sector (α_{21})

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- Phenomenological approach: Consider instead with $(U_{\nu N})_{\alpha i} \approx \Theta_{\alpha i} = |\Theta_{\alpha i}| e^{i\phi_{\alpha i}/2}$

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where $m^\nu_{\alpha\beta} \equiv [U_\nu m_\nu U_\nu^T]_{\alpha\beta}$ and $r_\Delta \equiv (m_{N_2} - m_{N_1})/m_{N_1}$

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Solve for the mixing ratio

$$\frac{\Theta_{e2}}{\Theta_{e1}} = i\sqrt{\frac{1+x_\nu}{1+r_\Delta}}\,; \quad x_\nu = \frac{m_\nu}{m_N \Theta_{e1}^2}$$

$$\Rightarrow \quad \frac{|\Theta_{e2}|^2}{|\Theta_{e1}|^2} = \frac{|1+x_\nu|}{1+r_\Delta}\,; \quad \cos(\underbrace{\phi_{e2}-\phi_{e1}}_{\Delta\phi_e}) = -\frac{\mathrm{Re}[1+x_\nu]}{|1+x_\nu|}$$

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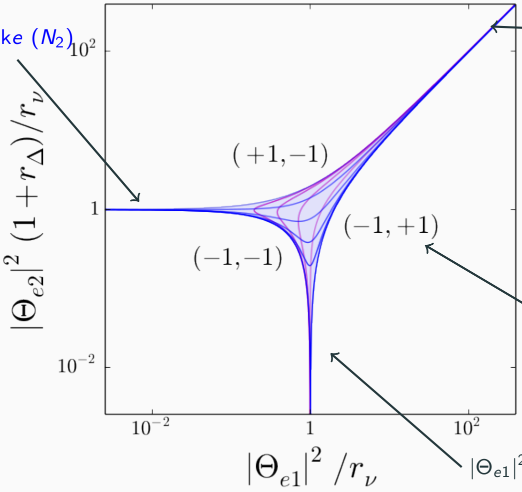
Phenomenological Parametrisation (1+2)

$$|\Theta_{e2}|^2 = \frac{m_\nu}{m_N(1+r_\Delta)}$$

$$\phi_{e2} = \pi$$

Standard Seesaw-like (N_2)



$$|\Theta_{e2}|^2 = \frac{|\Theta_{e1}|^2}{1+r_\Delta}$$

$$\Delta\phi_e = \pi$$

ISS-like

Quasi-Dirac pair

Symmetry-protected scenarios

$(\cos\phi_{e1},\ \cos\phi_{e2})$

$$|\Theta_{e1}|^2 = r_\nu \equiv \frac{m_\nu}{m_N}$$

$$\phi_{e1} = \pi$$

Standard Seesaw-like (N_1)

Phenomenological Parametrisation (3+2)

Phenomenological approach for three light neutrino flavours (3+2 model):

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Now solve as:

$$\alpha = \beta \quad \Rightarrow \quad \frac{\Theta_{\alpha 2}}{\Theta_{\alpha 1}} = i\sqrt{\frac{1 + x^\alpha_{\alpha\alpha}}{1 + r_\Delta}} \qquad\qquad x^\rho_{\alpha\beta} = \frac{m^\nu_{\alpha\beta}}{m_N \Theta^2_{\rho 1}}$$

$$\alpha \neq \beta \quad \Rightarrow \quad \frac{\Theta_{\beta 1}}{\Theta_{\alpha 1}} = y^\alpha_{\alpha\beta} \equiv \frac{x^\alpha_{\alpha\beta} + \sqrt{(x^\alpha_{\alpha\beta})^2 - x^\alpha_{\alpha\alpha} x^\alpha_{\beta\beta}}\sqrt{1 + x^\alpha_{\alpha\alpha}}}{x^\alpha_{\alpha\alpha}}$$

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$\Rightarrow$ Can express all mixings and phases in terms of $m^\nu_{\alpha\beta}$, m_N, r_Δ, $|\Theta_{e1}|$ and ϕ_{e1}

Phenomenological Parametrisation (General)

With n_A active and n_S sterile neutrinos,

$$\underbrace{M_\nu}_{\text{rank}(M_\nu)=\min(n_A,n_S)+n_S} = \begin{pmatrix} 0\big|_{n_A\times n_A} & M_D\big|_{n_A\times n_S} \\ M_D^T\big|_{n_S\times n_A} & M_R\big|_{n_S\times n_S} \end{pmatrix} = U \begin{pmatrix} m_\nu & 0 \\ 0 & m_N \end{pmatrix} U^T$$

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With n_A active and n_S sterile neutrinos,

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$$\begin{aligned} \#_{\text{params}} = \underbrace{\min(n_A, n_S)}_{\nu \text{ masses}} &+ \underbrace{\left[\min(n_A, n_S) + n_A(n_A - 2)\right]}_{U_\nu} \\ &+ \underbrace{n_S}_{N \text{ masses}} + \underbrace{\left[2n_A n_S - n_A(n_A - 1) - 2\min(n_A, n_S)\right]}_{U_{\nu N}} \end{aligned}$$

Phenomenological Parametrisation (General)

With n_A active and n_S sterile neutrinos,

$$\underbrace{M_\nu}_{\mathrm{rank}(M_\nu)=\min(n_A,n_S)+n_S} = \begin{pmatrix} 0\big|_{n_A\times n_A} & M_D\big|_{n_A\times n_S} \\ M_D^T\big|_{n_S\times n_A} & M_R\big|_{n_S\times n_S} \end{pmatrix} = U \begin{pmatrix} m_\nu & 0 \\ 0 & m_N \end{pmatrix} U^T$$

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$$\#_{\text{elim}} = n_A(n_A - 1) + 2\min(n_A, n_S) = \begin{cases} 2\,, & n_A = 1, n_S = 2 \\ 10\,, & n_A = 3, n_S = 2 \\ 12\,, & n_A = 3, n_S = 3 \end{cases}$$

Phenomenological Parametrisation (General)

With n_A active and n_S sterile neutrinos,

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Phenomenological Parametrisation (General)

With n_A active and n_S sterile neutrinos,

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$n_A = 1, n_S = 2 \rightarrow U_{\nu N} = \begin{pmatrix} \Theta_{e1} & \Theta_{e2} \end{pmatrix}$

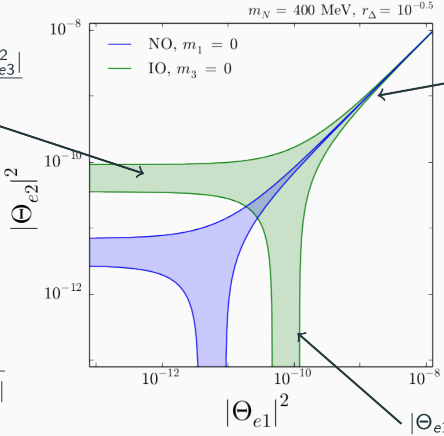
$n_A = 3, n_S = 2 \rightarrow U_{\nu N} = \begin{pmatrix} \Theta_{e1} & \Theta_{e2} \\ \Theta_{\mu 1} & \Theta_{\mu 2} \\ \Theta_{\tau 1} & \Theta_{\tau 2} \end{pmatrix}$

$$U_{\nu N} = \begin{pmatrix} \Theta_{e1} & \Theta_{e2} \\ \Theta_{\mu 1} & \Theta_{\mu 2} \\ \Theta_{\tau 1} & \Theta_{\tau 2} \end{pmatrix}$$

→ $m^{\nu}_{\alpha\beta}$: $m_{2(1)}, m_{3(2)}, \theta_{12}, \theta_{23}, \theta_{13}, \delta$ (NuFIT v5.2), α_{21}

→ $m_N, r_\Delta, |\Theta_{e1}|^2, \phi_{e1}$

$$|\Theta_{e2}|^2 = \frac{|m_2 U_{e2}^2 + m_3 U_{e3}^2|}{m_N(1+r_\Delta)}$$



$$|\Theta_{e2}|^2 = \frac{|\Theta_{e1}|^2}{1+r_\Delta}$$

NO:

$$m_2 = \sqrt{\Delta m^2_{\text{sol}}}$$

$$m_3 = \sqrt{\Delta m^2_{\text{sol}} + \Delta m^2_{\text{atm}}}$$

IO:

$$m_1 = \sqrt{|\Delta m^2_{\text{sol}} + \Delta m^2_{\text{atm}}|}$$

$$m_2 = \sqrt{|\Delta m^2_{\text{atm}}|}$$

$$|\Theta_{e1}|^2 = \frac{|m_2 U_{e2}^2 + m_3 U_{e3}^2|}{m_N}$$

$$U_{\nu N} = \begin{pmatrix} \Theta_{e1} & \Theta_{e2} \\ \Theta_{\mu 1} & \Theta_{\mu 2} \\ \Theta_{\tau 1} & \Theta_{\tau 2} \end{pmatrix}$$

→ $m_{2(1)}, m_{3(2)}, \theta_{12}, \theta_{23}, \theta_{13}, \delta$ (NuFIT v5.2), α_{21}

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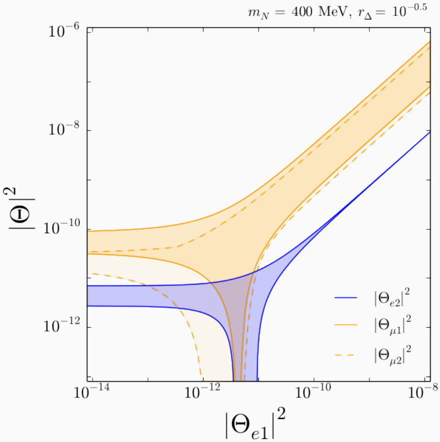
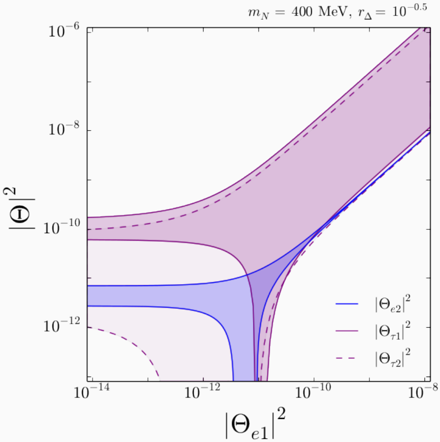
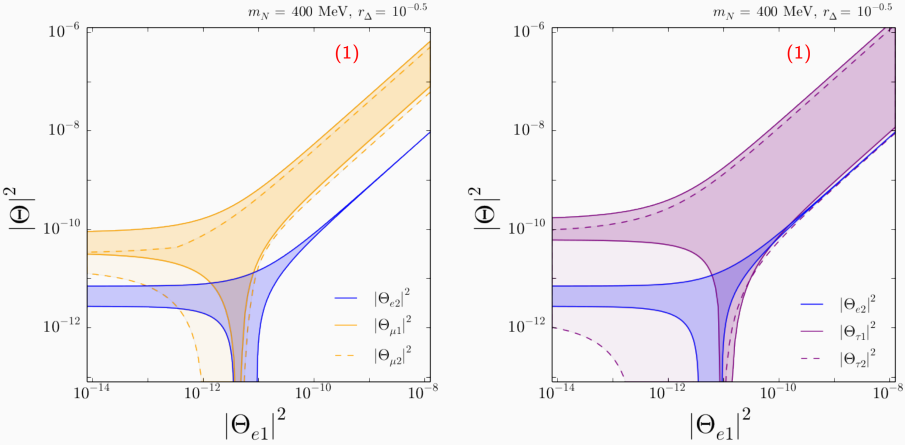


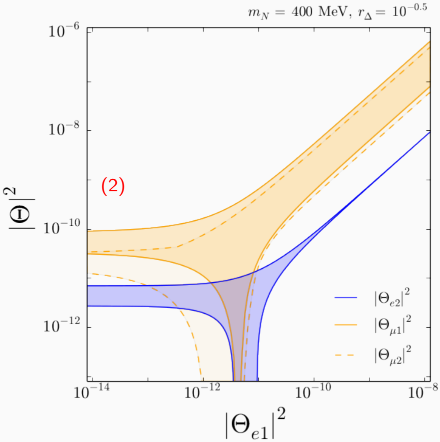


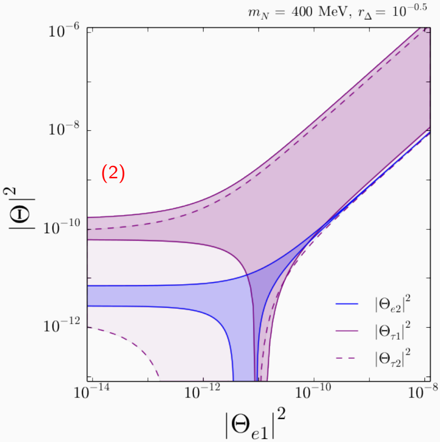
Phenomenological Parametrisation (3+2)

$$(1) \quad |\Theta_{\beta 1}|^2 = |\Theta_{e1}|^2 \left| \frac{\sqrt{m_2} U_{\beta 2} + i\sqrt{m_3} U_{\beta 3}}{\sqrt{m_2} U_{e2} + i\sqrt{m_3} U_{e3}} \right|^2, \quad |\Theta_{\beta 2}|^2 = \frac{|\Theta_{\beta 1}|^2}{1 + r_\Delta} \quad \text{(NO)}$$



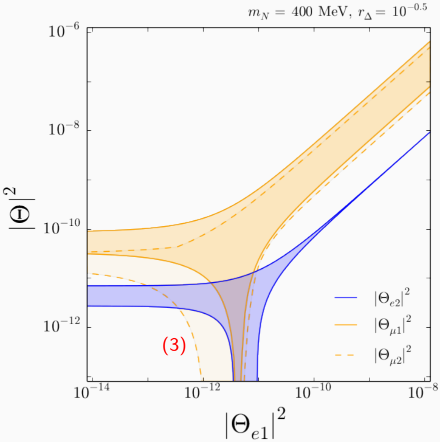
(2) $$|\Theta_{\beta 1}|^2 = \left| \frac{m_2 m_3 (U_{e2} U_{\beta 3} - U_{e3} U_{\beta 2})^2}{m_N (m_2 U_{e2}^2 + m_2 U_{e3}^2)} \right|, \quad |\Theta_{\beta 2}|^2 = \left| \frac{(m_2 U_{e2} U_{\beta 2} + m_3 U_{e3} U_{\beta 3})^2}{m_N (1 + r_\Delta)(m_2 U_{e2}^2 + m_2 U_{e3}^2)} \right| \quad \text{(NO)}$$

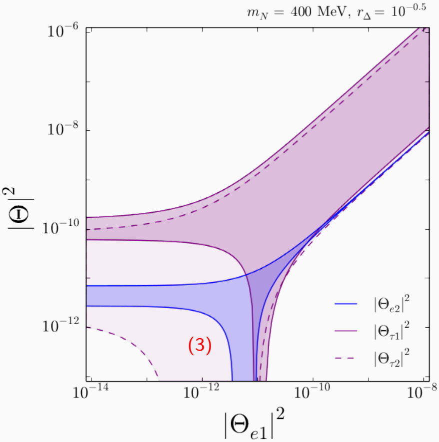




Phenomenological Parametrisation (3+2)

$$(3)\quad |\Theta_{e1}|^2 = \left| \frac{m_2 m_3 (U_{e2} U_{\beta 3} - U_{e3} U_{\beta 2})^2}{m_N (m_2 U_{\beta 2}^2 + m_2 U_{\beta 3}^2)} \right|, \quad |\Theta_{e1}|^2 = \left| \frac{(m_2 U_{e2} U_{\beta 2} + m_3 U_{e3} U_{\beta 3})^2}{m_N (m_2 U_{\beta 2}^2 + m_2 U_{\beta 3}^2)} \right| \quad \text{(NO)}$$





Mixing portal via SM weak interactions:

$$\mathcal{L}_{W^\pm} = -\frac{g}{\sqrt{2}}(\bar{\ell}_{\alpha L}\gamma^\mu \Theta_{\alpha i} N_i) W^-_\mu + \text{h.c.}$$

$$\mathcal{L}_Z = -\frac{g}{2c_W}(\bar{\nu}_{\alpha L}\gamma^\mu \Theta_{\alpha i} N_i) Z_\mu$$

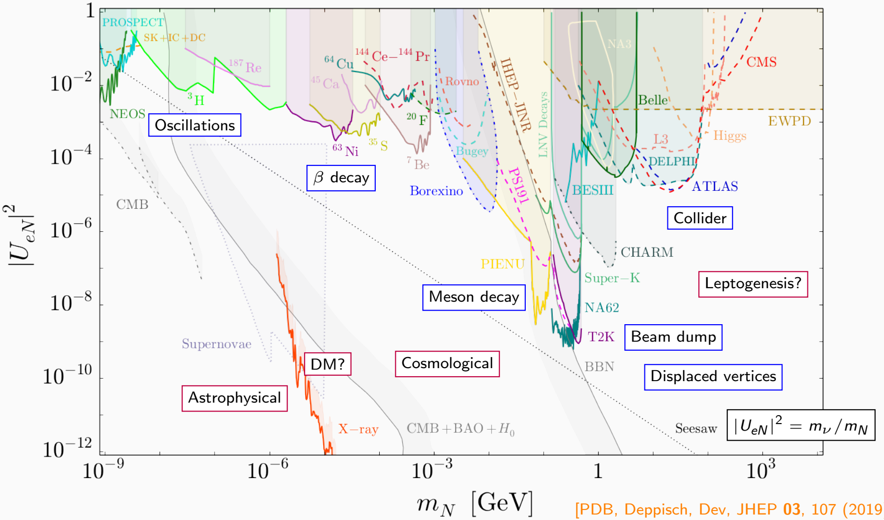
$$\mathcal{L}_h = -\frac{g}{2m_W}(\bar{\nu}_{\alpha L} m_{N_i} \Theta_{\alpha i} N_i) h$$

Phenomenology:

- Generate the light neutrino masses
- Experimental probes:
  - $0\nu\beta\beta$ decay and cLFV ($\mu \to e\gamma$, $\mu \to 3e$)
  - $3 + n$ oscillations
  - Direct searches (β decay, beam dumps, colliders)
  - Dark matter and Leptogenesis

⇒ This talk: $0\nu\beta\beta$ decay (LEGEND-1000) and direct searches at DUNE

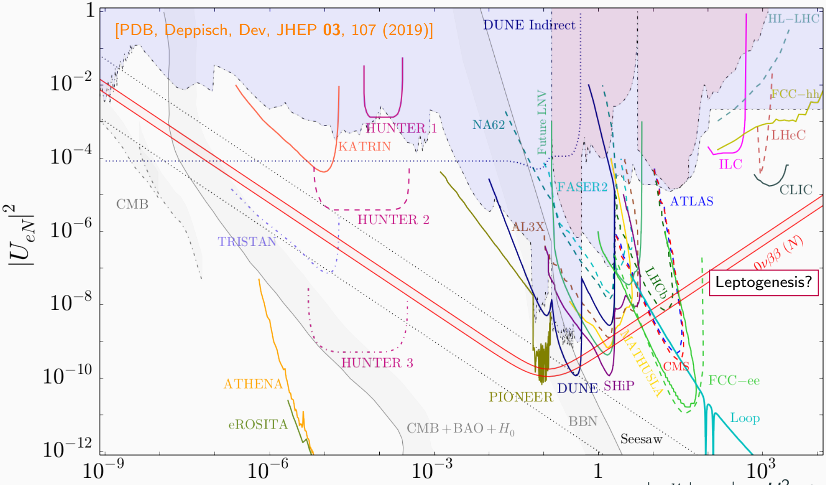
Current $|U_{eN}|^2$ Constraints



[PDB, Deppisch, Dev, JHEP **03**, 107 (2019)]

www.sterile-neutrino.org

Also see: https://github.com/mhostert/Heavy-Neutrino-Limits

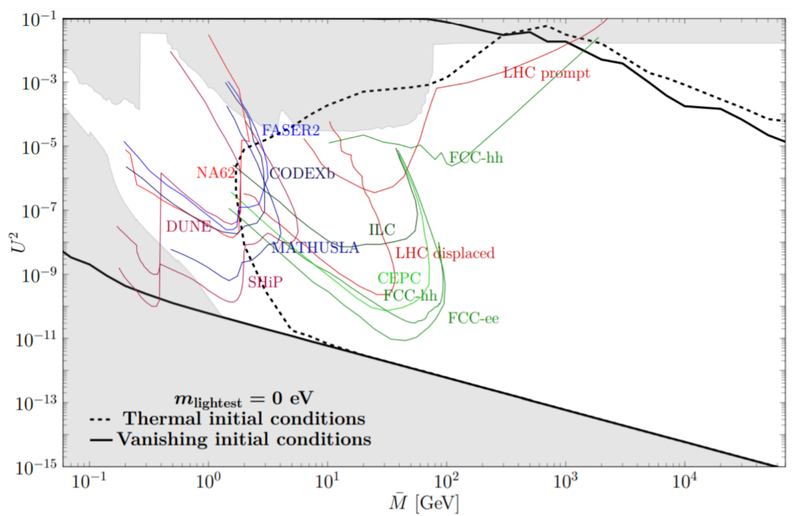


$$|\Theta_{e1}|^2 = \frac{|m_{ee}^\nu|}{m_N} = \frac{|m_2 U_{e2}^2 + m_3 U_{e3}^3|}{m_N}$$

$$1.4 \text{ meV} < |m_{ee}^\nu| < 3.7 \text{ meV} \quad \text{(NO)}$$

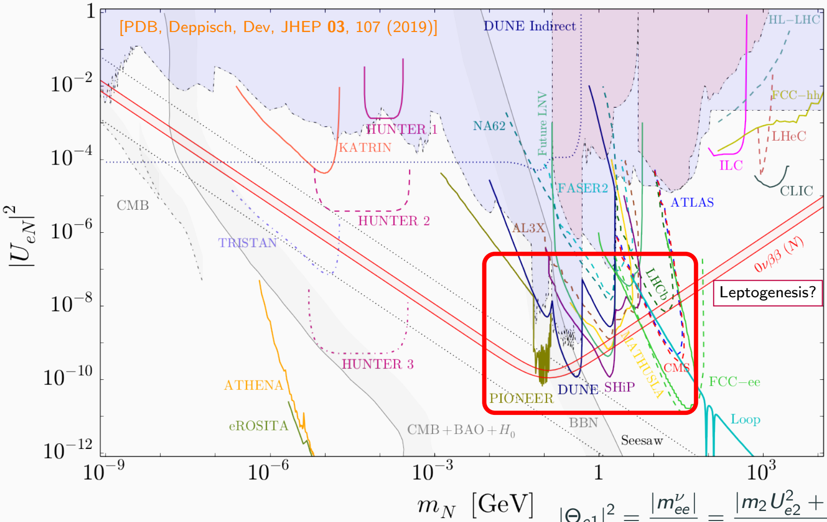
$$19 \text{ meV} < |m_{ee}^\nu| < 48 \text{ meV} \quad \text{(IO)}$$

Low-Scale Leptogenesis



[Drewes, Georis, Klarić, PRL **128** (2022)]

An Interesting Region?



$$|\Theta_{e1}|^2 = \frac{|m_{ee}^\nu|}{m_N} = \frac{|m_2 U_{e2}^2 + m_3 U_{e3}^3|}{m_N}$$

$m_N \in [100 \text{ MeV}, 2 \text{ GeV}]$

$$1.4 \text{ meV} < |m_{ee}^\nu| < 3.7 \text{ meV} \quad \text{(NO)}$$

$$19 \text{ meV} < |m_{ee}^\nu| < 48 \text{ meV} \quad \text{(IO)}$$

Neutrinoless Double Beta ($0\nu\beta\beta$) Decay

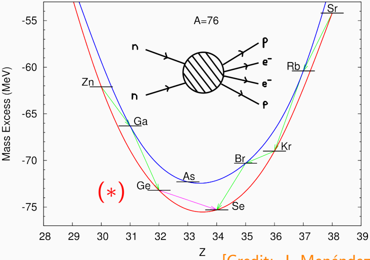
$0\nu\beta\beta$ Decay Process

When β decay is not kinematically accessable $(*)$,

$$^A_Z X \rightarrow {}_{Z+2}^{\ A}X + 2e^- + 2\bar{\nu} \quad (\Delta L = 0)$$

If lepton number is not conserved,

$$^A_Z X \rightarrow {}_{Z+2}^{\ A}X + 2e^- \quad (\Delta L = 2)$$

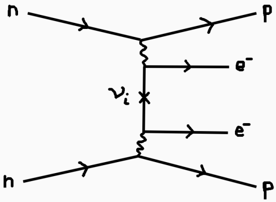


[Credit: J. Menéndez]

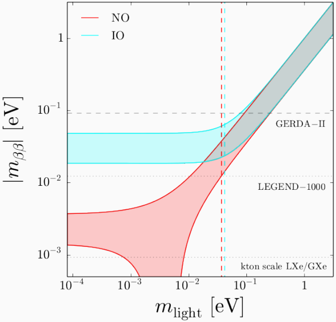
Contribution of light Majorana neutrinos:

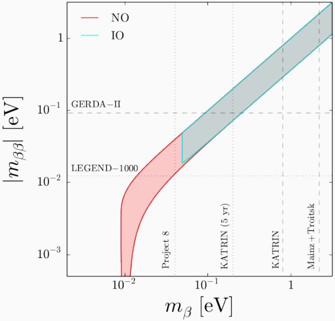
$$\frac{1}{T^{0\nu}_{1/2}} = \frac{G_{0\nu} g_A^4 |\mathcal{M}_\nu|^2}{m_e^2} |m_{\beta\beta}|^2$$

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i$$
$$= m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13} e^{i(\alpha_{31} - 2\delta)}$$



Light Neutrino Contribution

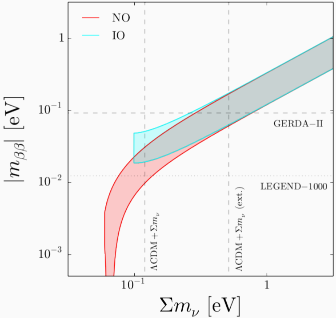




$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i$$

$$m_\beta^2 = \sum_i |U_{ei}|^2 m_i^2$$

$$\Sigma m_\nu = \sum_i m_i$$



GERDA-II (^{76}Ge):

$T_{1/2}^{0\nu} > 1.8 \times 10^{26}$ yr

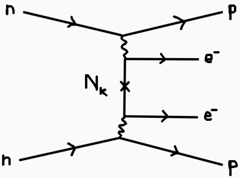
$|m_{\beta\beta}| < 92$ meV

LEGEND-1000 (*proposal*):

$T_{1/2}^{0\nu} \gtrsim 10^{28}$ yr

$|m_{\beta\beta}| \lesssim 12$ meV

Light Neutrino + HNL Contribution (3+2)



Including HNL exchange:

$$|m_{\beta\beta}^{\text{eff}}| = \left| m_{\beta\beta}^{\nu} + \sum_i U_{eN_i}^2 m_{N_i} \frac{\mathcal{M}^{0\nu}(m_{N_i})}{\mathcal{M}_\nu} \right|$$

where the nuclear matrix element (NME) naively follows

$$\lim_{m_{N_i}\to 0} \mathcal{M}^{0\nu}(m_{N_i}) = \mathcal{M}_\nu \,, \quad \lim_{m_{N_i}\to\infty} \mathcal{M}^{0\nu}(m_{N_i}) = \frac{m_e m_p}{m_{N_i}^2} \mathcal{M}_{\nu,\text{sd}}$$

So it is possible to use the interpolating formula

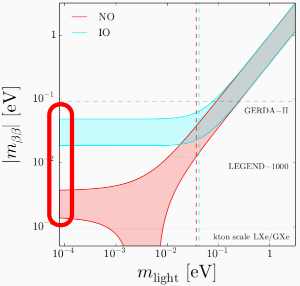
$$\mathcal{M}^{0\nu}(m_{N_i}) = \mathcal{M}_{\nu,\text{sd}} \frac{\langle \mathbf{p}^2 \rangle}{\langle \mathbf{p}^2 \rangle + m_{N_i}^2} \,; \quad \langle \mathbf{p}^2 \rangle \equiv m_e m_p \left| \frac{\mathcal{M}_{\nu,\text{sd}}}{\mathcal{M}_\nu} \right| \sim k_F$$

with $k_F \sim 100$ MeV

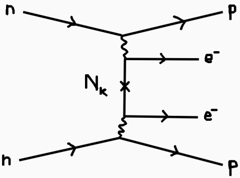
Light neutrino exchange ($m_{\beta\beta}^{\nu} \equiv m_{ee}^{\nu}$):

$$1.4 \text{ meV} < |m_{\beta\beta}^{\nu}| < 3.7 \text{ meV} \quad (\text{NO})$$

$$19 \text{ meV} < |m_{\beta\beta}^{\nu}| < 48 \text{ meV} \quad (\text{IO})$$



Light Neutrino + HNL Contribution (3+2)



With naive interpolating formula:

- $m_{N_i} \ll k_F \sim 100$ MeV

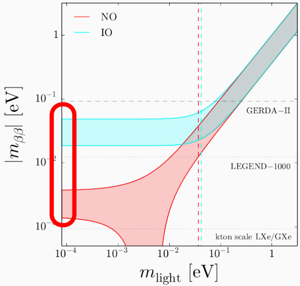
$$|m_{\beta\beta}^{\text{eff}}| = \left| m_{\beta\beta}^{\nu} + \sum_i U_{eN_i}^2 m_{N_i} \right| \approx 0$$

- $m_{N_i} \gg k_F \sim 100$ MeV

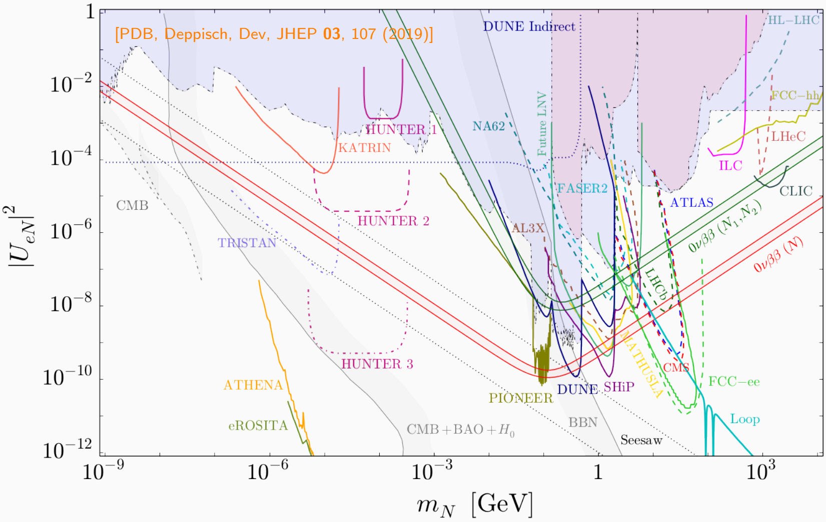
$$|m_{\beta\beta}^{\text{eff}}| = \left| m_{\beta\beta}^{\nu} + m_e m_p \sum_i \frac{U_{eN_i}^2}{m_{N_i}} \right|$$

- Can also consider N unrelated to ν masses

$$|m_{\beta\beta}^{\text{eff}}| = \left| \sum_i U_{eN_i}^2 m_{N_i} \right|$$



$0\nu\beta\beta$ Decay Constraints on $|U_{eN}|^2$



Nuclear Matrix Elements

A lot of recent progress in NME calculations in the EFT approach

[Dekens, de Vries, Fuyuto, Mereghetti, Zhou, JHEP **06** (2020)]

[Dekens, de Vries, Mereghetti, Menéndez, Soriano, Zhou (2023)]

- New leading-order contribution from *hard* light neutrino exchange ($|\mathbf{p}| \sim \Lambda_\chi$)

$$\mathcal{M}^{0\nu} = \frac{1}{g_A^2}\mathcal{M}_F - \frac{2m_e m_p g_\nu^{NN}}{g_A^2}\mathcal{M}_{F,\text{sd}} - \mathcal{M}_{GT} + \mathcal{M}_T$$

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- HNLs with $m_{N_i} \geq \Lambda_\chi$ must be integrated out $\Rightarrow$ dim-9 operators

$$\mathcal{M}^{0\nu} = \frac{m_e m_p}{m_{N_i}^2}\left[\frac{4}{g_A^2} g_1^{NN}\mathcal{M}_{F,\mathrm{sd}} - g_1^{\pi N}(\mathcal{M}_{GT,\mathrm{sd}}^{AP} + \mathcal{M}_{T,\mathrm{sd}}^{AP}) - \frac{5}{3} g_1^{\pi\pi}(\mathcal{M}_{GT,\mathrm{sd}}^{PP} + \mathcal{M}_{T,\mathrm{sd}}^{PP})\right]$$

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$\Rightarrow$ Low-energy constants (LECs) from lattice (so far, only $g_1^{\pi\pi}$)

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$\Rightarrow$ Low-energy constants (LECs) from lattice (so far, only $g_1^{\pi\pi}$)

- For light HNLs, *ultrasoft* exchange with ($|\mathbf{p}| \ll k_F$)
  $\Rightarrow$ Cannot resolve nuclear constituents; sensitive to nuclear excited states
  $\Rightarrow$ Prevents exact seesaw cancellation between ν and N

Nuclear Matrix Elements

To take these developments into account, we used

$$\mathcal{M}^{0\nu}(m_{N_i}) = \frac{1}{g_A^2}\mathcal{M}_F(m_{N_i}) - \frac{2m_e m_p g_\nu^{NN}(m_{N_i})}{g_A^2}\mathcal{M}_{F,\text{sd}}(m_{N_i}) - \mathcal{M}_{GT}(m_{N_i}) + \mathcal{M}_T(m_{N_i})$$

where

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Nuclear Matrix Elements

To take these developments into account, we used

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And match onto NME in $m_{N_i} \geq \Lambda_\chi$ limit

$$\mathcal{M}^{0\nu}(m_{N_i} \geq \Lambda_\chi) = \mathcal{M}^{0\nu}\big|_{m_{N_i}\geq\Lambda_\chi}$$

which gives $g_1^{\pi\pi} = \frac{3}{5}$, $g_1^{\pi N} = 1$, $g_1^{NN} = \frac{1}{4}(1 + g_A^2 - 2m_{N_i}^2 g_\nu^{NN}) \;\Rightarrow\; g_\nu^{NN}(m_{N_i})$

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In the end, have

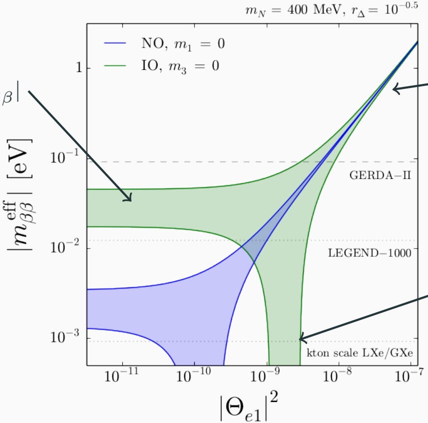
$$\mathcal{M}^{0\nu}(m_{N_i}) = \mathcal{M}_{\nu,\mathrm{sd}}\,\frac{\langle \mathbf{p}^2\rangle \mathcal{F}(m_{N_i})}{\langle \mathbf{p}^2\rangle + m_{N_i}^2}$$

$$
\begin{aligned}
|m_{\beta\beta}^{\text{eff}}| &= \left| m_{\beta\beta}^{\nu} + m_N \Theta_{e1}^2 \frac{\langle \mathbf{p}^2 \rangle}{\langle \mathbf{p}^2 \rangle + m_N^2} + m_N (1 + r_\Delta) \Theta_{e2}^2 \frac{\langle \mathbf{p}^2 \rangle}{\langle \mathbf{p}^2 \rangle + m_N^2 (1 + r_\Delta)^2} \right| \\
&= \left| \alpha m_{\beta\beta}^{\nu} + \beta m_N \Theta_{e1}^2 \right|
\end{aligned}
$$

where

$$
\alpha \equiv 1 - \frac{\langle \mathbf{p}^2 \rangle}{\langle \mathbf{p}^2 \rangle + m_N^2 (1 + r_\Delta)^2}, \quad \beta \equiv \frac{\langle \mathbf{p}^2 \rangle}{\langle \mathbf{p}^2 \rangle + m_N^2} - \frac{\langle \mathbf{p}^2 \rangle}{\langle \mathbf{p}^2 \rangle + m_N^2 (1 + r_\Delta)^2}
$$

$|m_{\beta\beta}^{\text{eff}}| = \alpha |m_{\beta\beta}^{\nu}|$



$|m_{\beta\beta}^{\text{eff}}| = \beta m_N |\Theta_{e1}|^2$

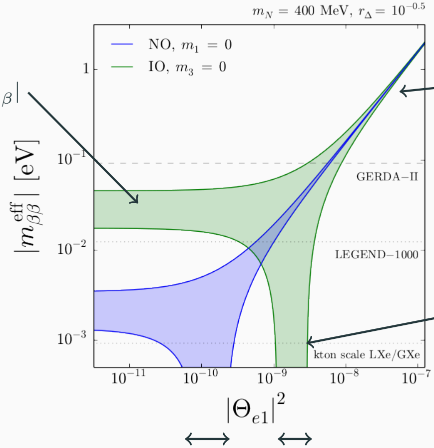
$|\Theta_{e1}|^2 = \dfrac{\alpha |m_{\beta\beta}^{\nu}|}{\beta m_N}$

Light Neutrino + HNL Contribution (3+2)

We are interested in the limit $r_\Delta \ll 1$ and $m_N^2 \gg \langle \mathbf{p}^2 \rangle$

$$\alpha \approx 1, \quad \beta = \frac{2 r_\Delta \langle \mathbf{p}^2 \rangle}{m_N^2}$$

$|m_{\beta\beta}^{\text{eff}}| = |m_{\beta\beta}^{\nu}|$

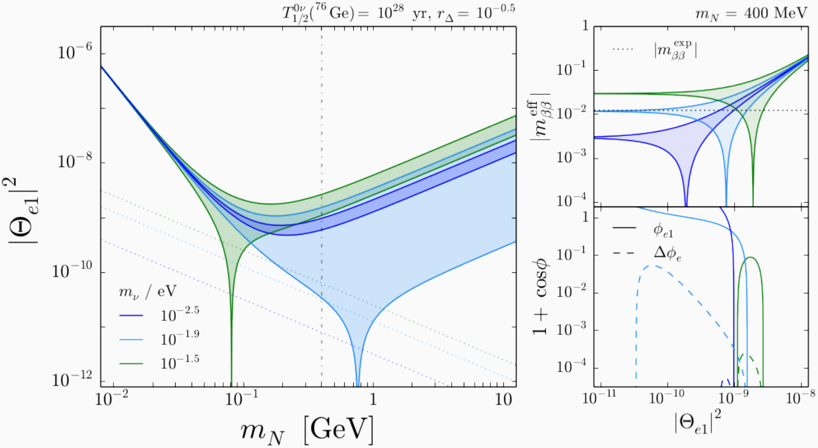


$$|m_{\beta\beta}^{\text{eff}}| = \frac{2 r_\Delta \langle \mathbf{p}^2 \rangle |\Theta_{e1}|^2}{m_N}$$

$$|\Theta_{e1}|^2 = \frac{m_N |m_{\beta\beta}^{\nu}|}{2 r_\Delta \langle \mathbf{p}^2 \rangle}$$

Light Neutrino + HNL Contribution (1+2)

$$|m_{\beta\beta}^{\text{eff}}| = \left|\alpha m_\nu + \beta m_N \Theta_{e1}^2\right| \quad \Rightarrow \quad \cos\phi_{e1} = \frac{\left|m_{\beta\beta}^{\text{exp}}\right|^2 - \alpha^2 m_\nu^2 - \beta^2 m_N^2 |\Theta_{e1}|^4}{2\alpha\beta m_\nu m_N |\Theta_{e1}|^2}$$



Direct Searches

HNL Production at Fixed-Target Experiments

Decays of pseudoscalar mesons $P = \{\pi, K, D, D_s\}$ produced in proton beam target

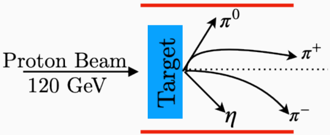
1) Two-body leptonic decays:

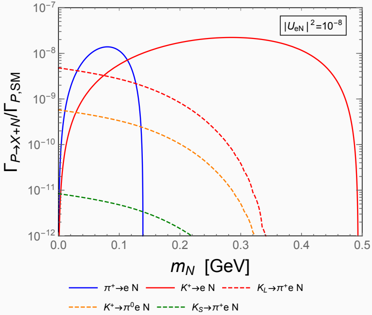
$$\mathrm{Br}(P^+ \to \ell_\alpha^+ N) \propto G_F^2 |U_{\alpha N}|^2 f_2\left(\frac{m_{\ell_\alpha}^2}{m_P^2}, \frac{m_N^2}{m_P^2}\right)$$

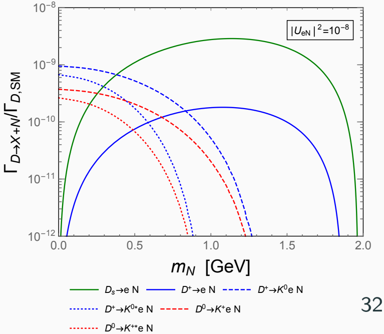
$$f_2\left(\frac{m_{\ell_\alpha}^2}{m_P^2}, \frac{m_N^2}{m_P^2}\right) \propto \frac{m_N^2}{m_P^2}\left(1 - \frac{m_N^2}{m_P^2}\right)^2$$

2) Three-body semi-leptonic decays:

$$\mathrm{Br}(P^+ \to P'^0 \ell_\alpha^+ N) \propto G_F^2 |U_{\alpha N}|^2 \underbrace{f_3\left(\frac{m_{\ell_\alpha}^2}{m_P^2}, \frac{m_N^2}{m_P^2}\right)}_{f_+(q^2), f_-(q^2)}$$





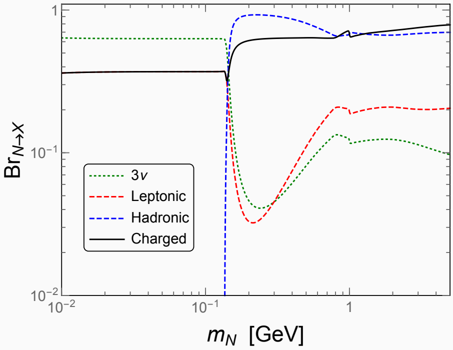


[Berryman, de Gouvêa, Fox, Kayser, Kelly, Raaf, JHEP 02 (2020) 174]

HNL Decays

Long-lived HNLs can decay inside the fiducial volume, e.g. Argon-based detector

- Invisible ($N \to 3\nu$) and neutral semi-leptonic ($N \to \nu\pi^0$) decays not detected



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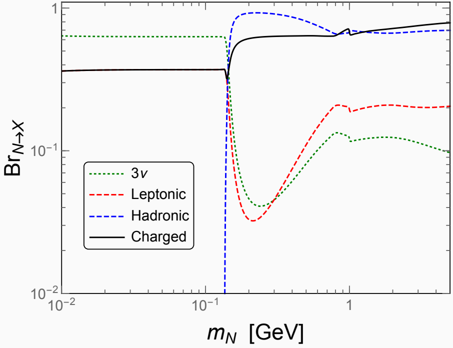

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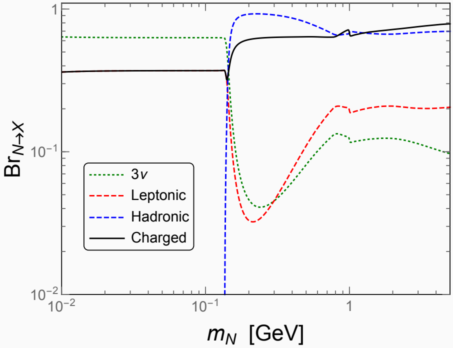
  - ⇒ For two-body decays, invariant mass reconstruction can suppress SM background



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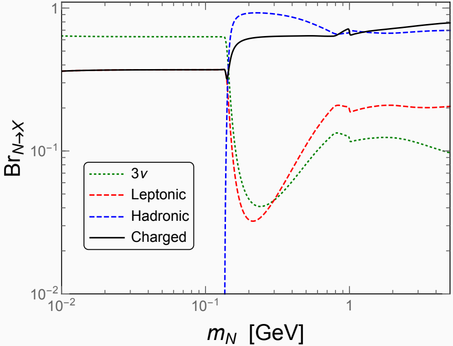
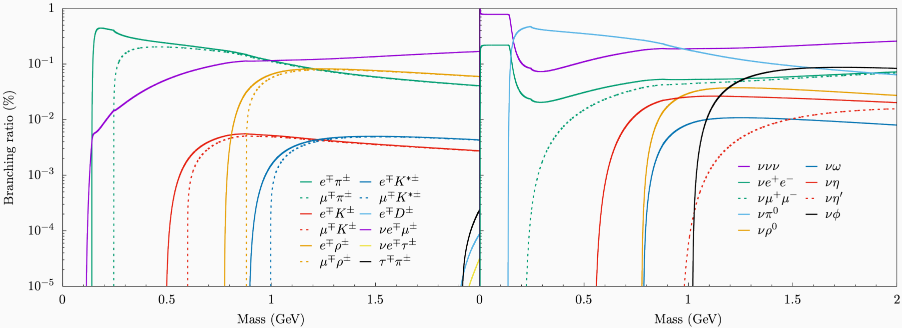
  - $\Rightarrow$ For two-body decays, invariant mass reconstruction can suppress SM background
  - $\Rightarrow$ For $N \to \nu\ell_\alpha^+\ell_\beta^-$ $(\alpha, \beta = e, \mu)$, backgrounds are low (mis-ID of $\pi^\pm$)



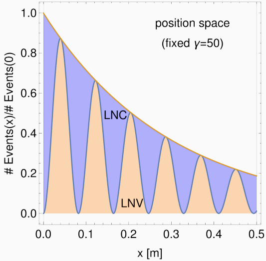
HNL Decays



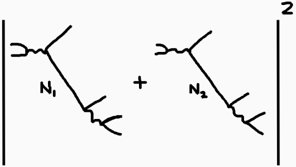
[Ballett, Boschi, Pascoli, JHEP **03** (2020) 111]

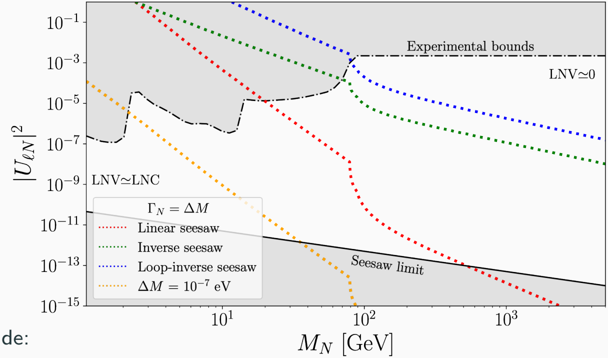
[Coloma, Fernández-Martínez, González-López, Hernández-García, Eur. Phys. J. C 81 (2021)]

HNL Oscillations



[Antusch, Cazzato and Fischer, Mod. Phys. Lett. A 34 (2019)]





For all r_Δ of interest must include:

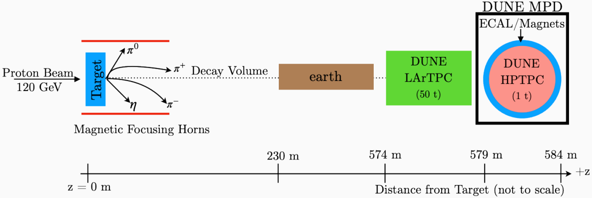
e.g. $N \to e^\mp \pi^\pm$

[Fernández-Martínez, Marcano, Naredo-Tuero, JHEP **03** (2023) 057]

[S. Antusch, J. Hajer, and J. Rosskopp (2022)]

Using PYTHIAv8 (considering DUNE with 6.6×10^{21} POT at 120 GeV):

- Simulated meson production (prod. fractions N_P and momentum profiles)



[Berryman, de Gouvêa, Fox, Kayser, Kelly, Raaf, JHEP 02 (2020) 174]

DUNE: Detector Modelling and Simulation

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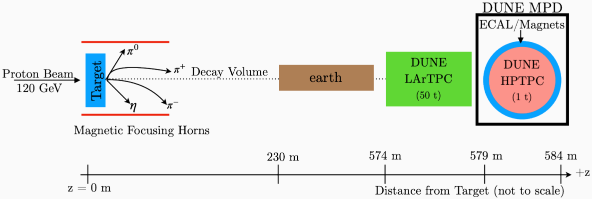

[Berryman, de Gouvêa, Fox, Kayser, Kelly, Raaf, JHEP 02 (2020) 174]

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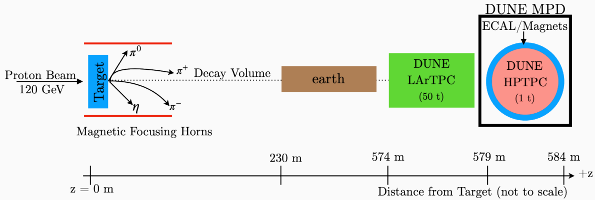
- Simulated meson production (prod. fractions N_P and momentum profiles)
- Rest-frame decays of mesons to HNLs, boosted to lab frame
- HNLs required to decay to charged tracks inside the DUNE ND (for simplicity, assuming a conical cross section)

$$\epsilon_{\text{geo}} = \frac{1}{N_{\text{tot}}} \sum_{\text{cut}} e^{-\frac{m_N \Gamma_N}{p_{N_z}} L} \left(1 - e^{-\frac{m_N \Gamma_N}{p_{N_z}} \Delta\ell_{\text{det}}}\right), \quad L = 574 \text{ m}, \ \Delta\ell_{\text{det}} = 5 \text{ m}$$

$$\frac{p_{N_T}}{p_{N_z}} < \theta_{\text{det}} \sim 7 \times 10^{-3}$$



[Berryman, de Gouvêa, Fox, Kayser, Kelly, Raaf, JHEP 02 (2020) 174]

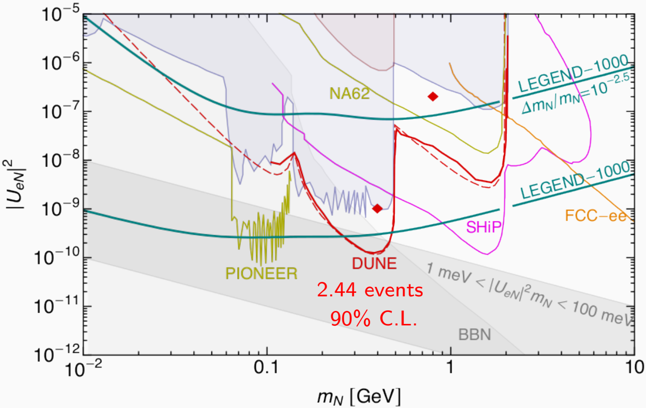
DUNE Sensitivity

Putting this all together obtain the DUNE sensitivity

$$\mathcal{N}_{\text{sig}}^{\text{DUNE}} = \sum_{P,\,\text{charged}} N_P \cdot \text{Br}(P \to N) \cdot \text{Br}(N \to \text{charged}) \cdot \epsilon_{\text{geo}}$$

In the phenomenological model:

$$N_{\text{sig}}^{\text{DUNE}} = \mathcal{N}_{\text{sig}}^{\text{DUNE}}(m_N, |\Theta_{e1}|^2) + \mathcal{N}_{\text{sig}}^{\text{DUNE}}\left(m_N(1+r_\Delta), |\Theta_{e2}|^2\right)$$

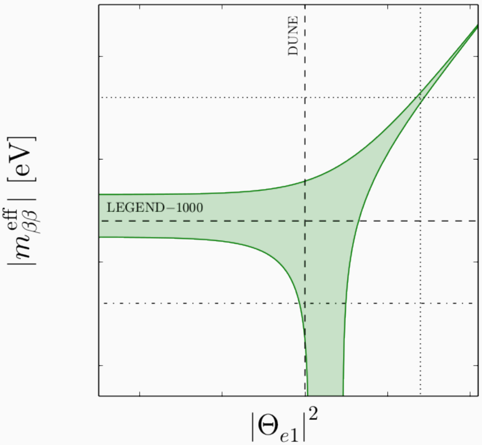


$0\nu\beta\beta$ Decay vs. Direct Searches

Analytical Comparison

For simplicity, we consider the 1+2 setup

- ⇒ Captures the relevant limits of the 3+2 model
- ⇒ Only $|U_{eN}|^2$, i.e. electron channels (e.g. $N \to e^{\pm}\pi^{\mp}$) in DUNE



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We can get an analytical estimate of r_Δ probed by LEGEND-1000 and DUNE

$\Rightarrow$ For $m_N < m_K$ ($m_N > m_K$), $K^+ \to e^+ N$ ($D_s^+ \to e^+ N$) dominates

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LEGEND-1000 and DUNE signals in the $|\Theta_{e1}|^2 \gg m_\nu / m_N$ limit

$$|m_{\beta\beta}| \approx \beta m_N |\Theta_{e1}|^2 \quad \Rightarrow \quad |\Theta_{e1}|^2 \propto \frac{m_N}{r_\Delta (T^{0\nu}_{1/2})^{1/2}}$$

$$N^{\text{DUNE}}_{\text{sig}} \propto \mathcal{A}(m_N)|\Theta_{e1}|^4 + \mathcal{A}\big(m_N(1+r_\Delta)\big)\frac{|\Theta_{e1}|^4}{(1+r_\Delta)^2} \quad \underset{r_\Delta \ll 1}{\Rightarrow} \quad |\Theta_{e1}|^2 \propto \sqrt{\frac{N^{\text{DUNE}}_{\text{sig}}}{\mathcal{A}(m_N)}}$$

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Giving,

$$r_\Delta \sim 1.5 \times 10^{-3} \left(\frac{m_N}{800 \text{ MeV}}\right) \left(\frac{300}{N_{\text{sig}}^{\text{DUNE}}}\right)^{1/2} \left(\frac{10^{28} \text{ yr}}{T^{0\nu}_{1/2}}\right)^{1/2}$$

Statistical Analysis – Likelihoods

Assume that LEGEND-1000 and DUNE are simple counting experiments following

$$\text{Pois}\left(n_{\text{obs}} \mid \lambda_{\text{sig}} + \lambda_{\text{bkg}}\right) \propto \frac{\left(\lambda_{\text{sig}} + \lambda_{\text{bkg}}\right)^{n_{\text{obs}}} e^{-\left(\lambda_{\text{sig}} + \lambda_{\text{bkg}}\right)}}{\Gamma\left(n_{\text{obs}} + 1\right)}$$

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Given the model hypothesis $\boldsymbol{\theta} = \{m_\nu, m_N, r_\Delta, |\Theta_{e1}|^2, \phi_{e1}\}$, *expected* number of events, with $\mathcal{E} = 6632$ kg · yr, $\mathcal{B} = 6.4 \times 10^{-5}/(\text{kg} \cdot \text{yr})$:

$$\lambda_{\text{sig}}^{\text{LEGEND}} = \frac{\ln 2 \cdot N_A \cdot \mathcal{E}}{m_A \cdot T_{1/2}^{0\nu}(\boldsymbol{\theta})}, \quad \lambda_{\text{bkg}}^{\text{LEGEND}} = \mathcal{E} \cdot \mathcal{B} = 0.4 \text{ cts}$$

$$\lambda_{\text{sig}}^{\text{DUNE}} = N_{\text{sig}}^{\text{DUNE}}(\boldsymbol{\theta}), \qquad \lambda_{\text{bkg}}^{\text{DUNE}} \approx 0$$

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Given data $\mathbf{D}$ and model parameters $\boldsymbol{\theta}$, global likelihood is

$$\mathcal{L}(\mathbf{D}|\boldsymbol{\theta}) = \text{Pois}\left(n_{\text{obs}}^{\text{LEGEND}} \mid \lambda_{\text{sig}}^{\text{LEGEND}} + \lambda_{\text{bkg}}^{\text{LEGEND}}\right) \cdot \text{Pois}\left(n_{\text{obs}}^{\text{DUNE}} \mid \lambda_{\text{sig}}^{\text{DUNE}}\right)$$

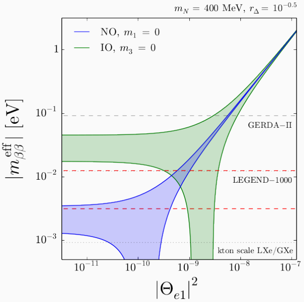
MCMC Likelihood Scan with Benchmark Points

We would like to estimate the posterior probability of the HNL hypothesis $\boldsymbol{\theta}$ given data $\mathbf{D}$

$$p(\boldsymbol{\theta}|\mathbf{D}) \propto \mathcal{L}(\mathbf{D}|\boldsymbol{\theta}) \cdot \pi(\boldsymbol{\theta})$$

Perform MCMC scan (Metropolis-Hastings) of parameter space with flat priors:

- Consider 4 benchmark scenarios b: $\boldsymbol{\theta}_0 = \boldsymbol{\theta}_b$



Scenario b	m_ν [eV]	m_N [MeV]	$\|\Theta_{e1}\|^2$	r_Δ	$\lambda_{\rm sig}^{\rm DUNE}$	$\lambda_{\rm sig}^{\rm LEGEND}$	$T_{1/2}^{0\nu}$ [yr]
1	$10^{-1.9}$	400	$10^{-9.0}$	$10^{-0.5}$	76.7	5.94	$10^{27.8}$
2	$10^{-2.5}$	400	$10^{-9.0}$	$10^{-0.5}$	76.7	2.73	$10^{28.1}$
3	$10^{-1.9}$	800	$10^{-6.7}$	$10^{-2.5}$	325	15.5	$10^{27.4}$
4	$10^{-2.5}$	800	$10^{-6.7}$	$10^{-2.5}$	325	12.3	$10^{27.5}$

MCMC Likelihood Scan with Benchmark Points

We would like to estimate the posterior probability of the HNL hypothesis $\boldsymbol{\theta}$ given data $\mathbf{D}$

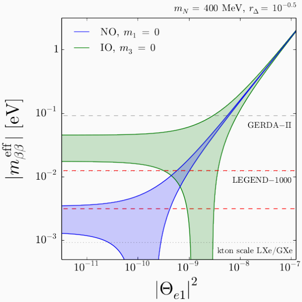
$$p(\boldsymbol{\theta}|\mathbf{D}) \propto \mathcal{L}(\mathbf{D}|\boldsymbol{\theta}) \cdot \pi(\boldsymbol{\theta})$$

Perform MCMC scan (Metropolis-Hastings) of parameter space with flat priors:

- Consider 4 benchmark scenarios b: $\boldsymbol{\theta}_0 = \boldsymbol{\theta}_b$
- $\mathbf{D}$: Signals at LEGEND-1000 (✓/✗) and DUNE (✓/✗)

$$✓ : n_{\text{obs}} = \lambda_{\text{sig}}(\boldsymbol{\theta}_b) + \lambda_{\text{bkg}}$$

$$✗ : n_{\text{obs}} = \lambda_{\text{bkg}}$$



Scenario b	m_ν [eV]	m_N [MeV]	$\lvert\Theta_{e1}\rvert^2$	r_Δ	$\lambda_{\text{sig}}^{\text{DUNE}}$	$\lambda_{\text{sig}}^{\text{LEGEND}}$	$T_{1/2}^{0\nu}$ [yr]
1	$10^{-1.9}$	400	$10^{-9.0}$	$10^{-0.5}$	76.7	5.94	$10^{27.8}$
2	$10^{-2.5}$	400	$10^{-9.0}$	$10^{-0.5}$	76.7	2.73	$10^{28.1}$
3	$10^{-1.9}$	800	$10^{-6.7}$	$10^{-2.5}$	325	15.5	$10^{27.4}$
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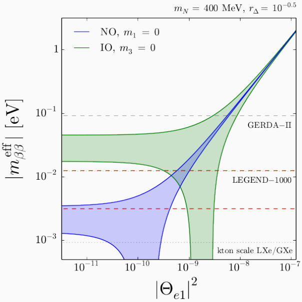
$$✗ : n_{\text{obs}} = \lambda_{\text{bkg}}$$

- Markov chain $[\boldsymbol{\theta}_0, \boldsymbol{\theta}_1, \ldots]$ to approximate $p(\boldsymbol{\theta}|\mathbf{D})$



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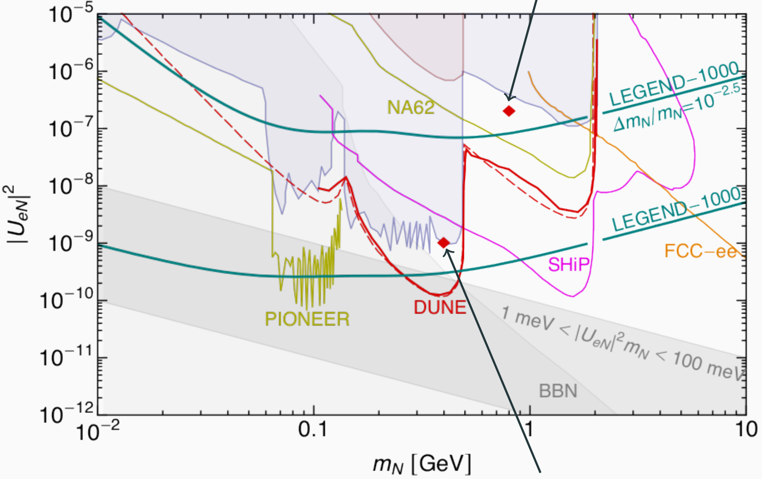
Benchmarks 3 and 4



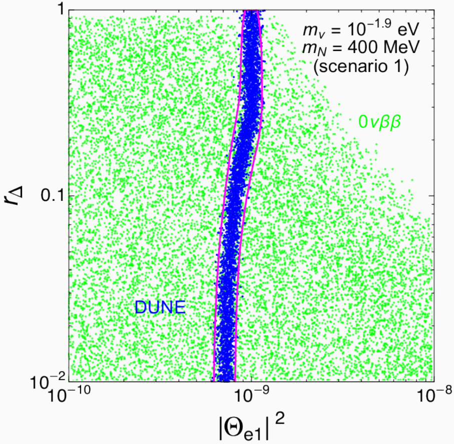
Benchmarks 1 and 2

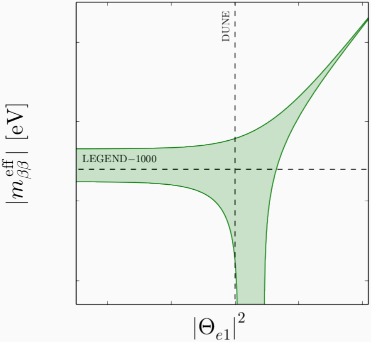
LEGEND-1000 (✓) and DUNE (✓)

Benchmark 1:

- $|\Theta_{e1}|^2 \approx 10^{-9}$ from DUNE
- m_ν saturates $T^{0\nu}_{1/2}$ half-life

$\Rightarrow$ r_Δ upper limit



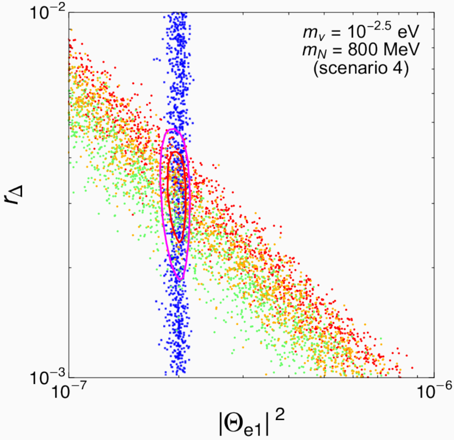


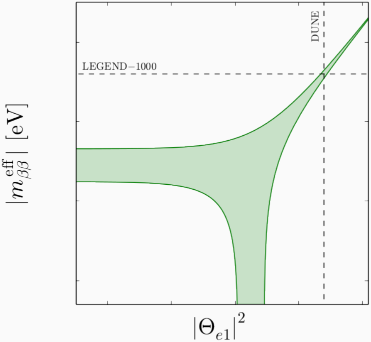
LEGEND-1000 (✓) and DUNE (✓)

Benchmark 4:

- $|\Theta_{e1}|^2 \approx 2 \times 10^{-7}$ from DUNE
- HNL pair dominates $T^{0\nu}_{1/2}$ half-life

$\Rightarrow$ $2 \times 10^{-3} \lesssim r_\Delta \lesssim 5 \times 10^{-3}$



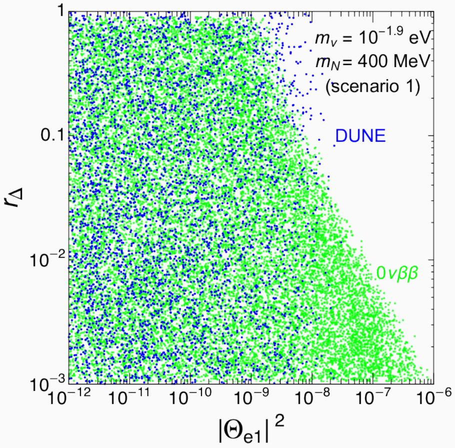


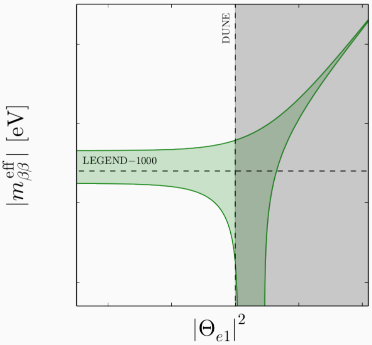
LEGEND-1000 (✓) and DUNE (✗)

Benchmark 1:

- $|\Theta_{e1}|^2 \lesssim 10^{-8}$ from DUNE
- m_ν saturates $T^{0\nu}_{1/2}$ half-life

$\Rightarrow |\Theta_{e1}|^2$ and r_Δ upper limits





Benchmark 4:

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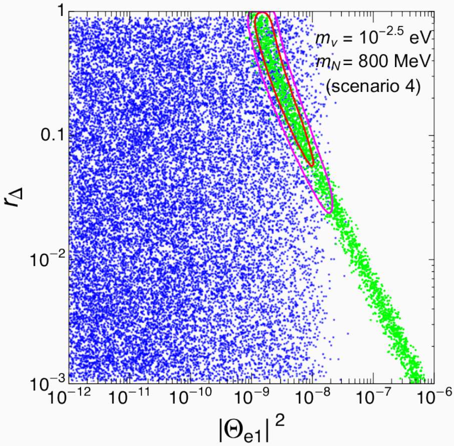
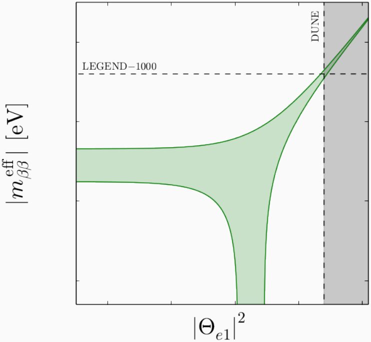
$\Rightarrow r_\Delta \gtrsim 2 \times 10^{-2}$





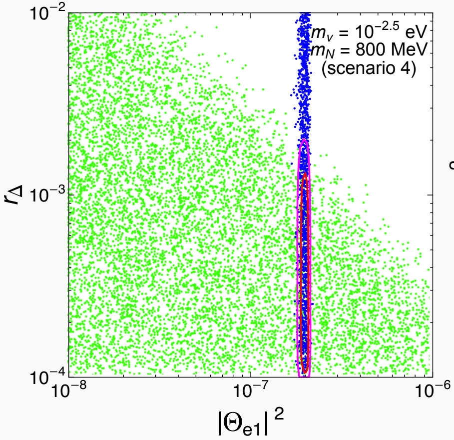
LEGEND-1000 (✗) and DUNE (✓)

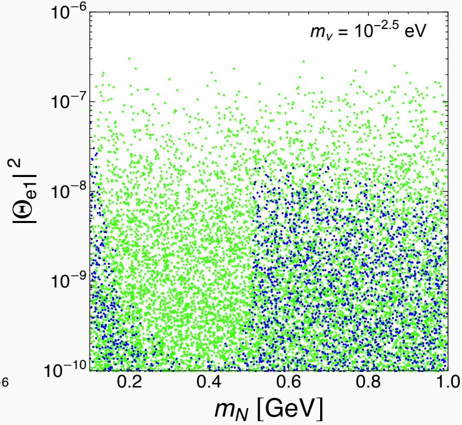
- $|\Theta_{e1}|^2$ from DUNE

$\Rightarrow$ r_Δ upper limit for both benchmarks

LEGEND-1000 (✗) and DUNE (✗)

- $|\Theta_{e1}|^2$ upper limit





Summary

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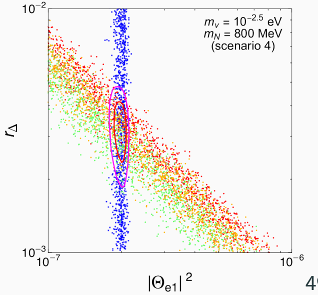

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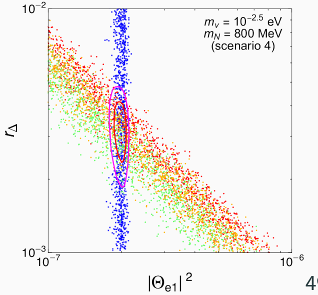

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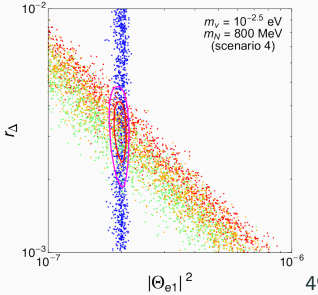

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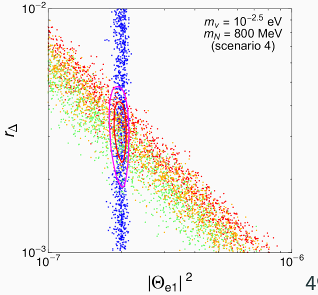

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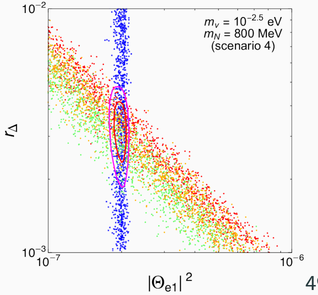

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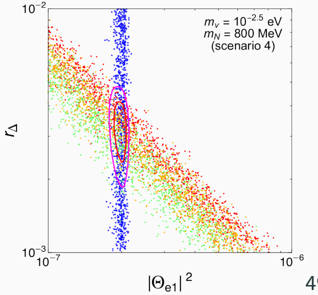

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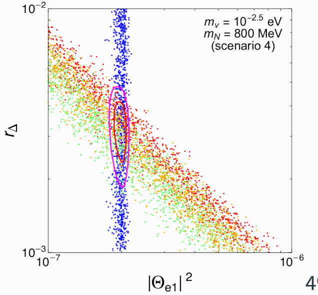

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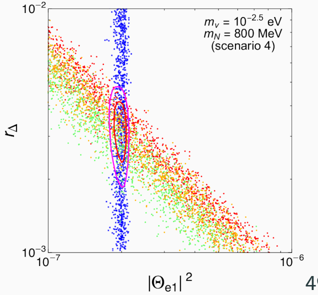
⇒ Same as above with regards to DUNE signal



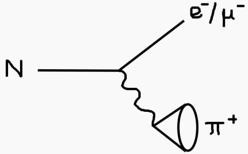
3+2 Model Scan

In the 3+2 model, muon channels are also relevant at DUNE

- DUNE signal including, e.g. $N \to \mu^{\pm}\pi^{\mp}$

$$N_{\text{sig}}^{\text{DUNE}}(\boldsymbol{\theta}) = \mathcal{N}_{\text{sig}}^{\text{DUNE}}(m_N, |\Theta_{e1}|^2, |\Theta_{\mu 1}|^2) + \mathcal{N}_{\text{sig}}^{\text{DUNE}}\left(m_N(1+r_\Delta), |\Theta_{e2}|^2, |\Theta_{\mu 2}|^2\right)$$

$$\Rightarrow \boldsymbol{\theta} = \{\alpha_{21}, m_N, r_\Delta, |\Theta_{e1}|^2, \phi_{e1}\}_{\text{NO/IO}}$$



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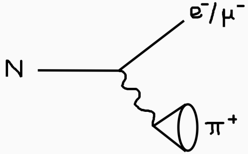

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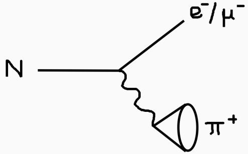

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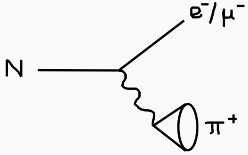
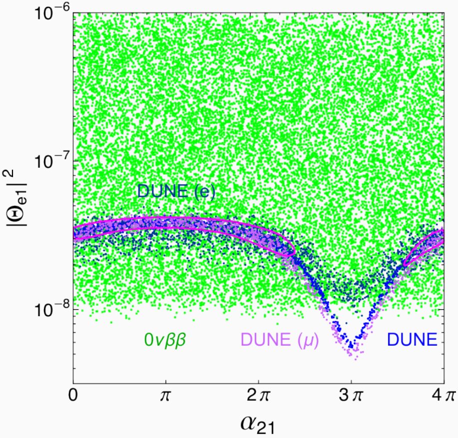
- Repeat MCMC scan with $\mathcal{L}'$

$\Rightarrow$ 3+2 model parameter space further constrained

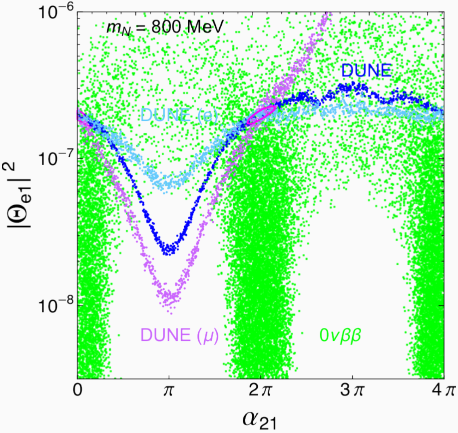


3+2 Model Scan

Normal Ordering



Inverted Ordering



Preliminary

Conclusions

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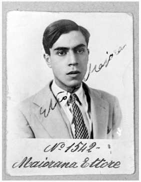
DUNE and $0\nu\beta\beta$ decay can probe the nature of HNLs with $m_N \in [100 \text{ MeV}, 2 \text{ GeV}]$

- Phenomenological parametrisation describing HNL pair contribution to m_ν



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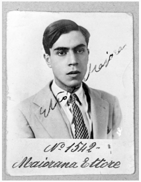

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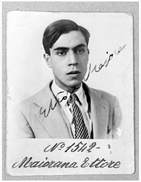

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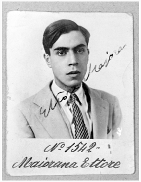

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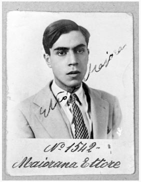
- Probes:



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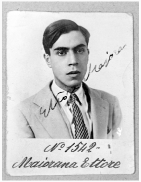
  - $\Rightarrow$ Constructive/destructive interference of HNL pair with ν in $0\nu\beta\beta$ decay



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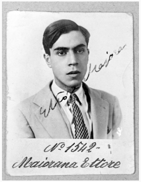

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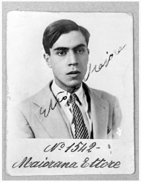

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Conclusions

DUNE and $0\nu\beta\beta$ decay can probe the nature of HNLs with $m_N \in [100\ \text{MeV}, 2\ \text{GeV}]$

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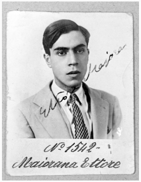
  - $\Rightarrow$ $\boldsymbol{\theta} = (m_\nu, m_N, r_\Delta, |\Theta_{e1}|^2, \phi_{e1})$
  - $\Rightarrow$ Majorana and quasi-Dirac limits of HNL pair



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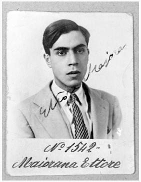

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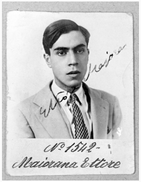

Thank you for listening!

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