Radiation from Global Axion Strings

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based on 2401.17253 with K. Saikawa, J. Redondo & A. Vaquero

and work in progress with J. Redondo, I. Y. Rybak & A. Drew

2nd Cosmic WISPers School
Ljubljana, June 12th 2024

Background: G. Pierobon

Departamento de Física Teórica
Universidad Zaragoza

Centro de Astropartículas y Física de Altas Energías
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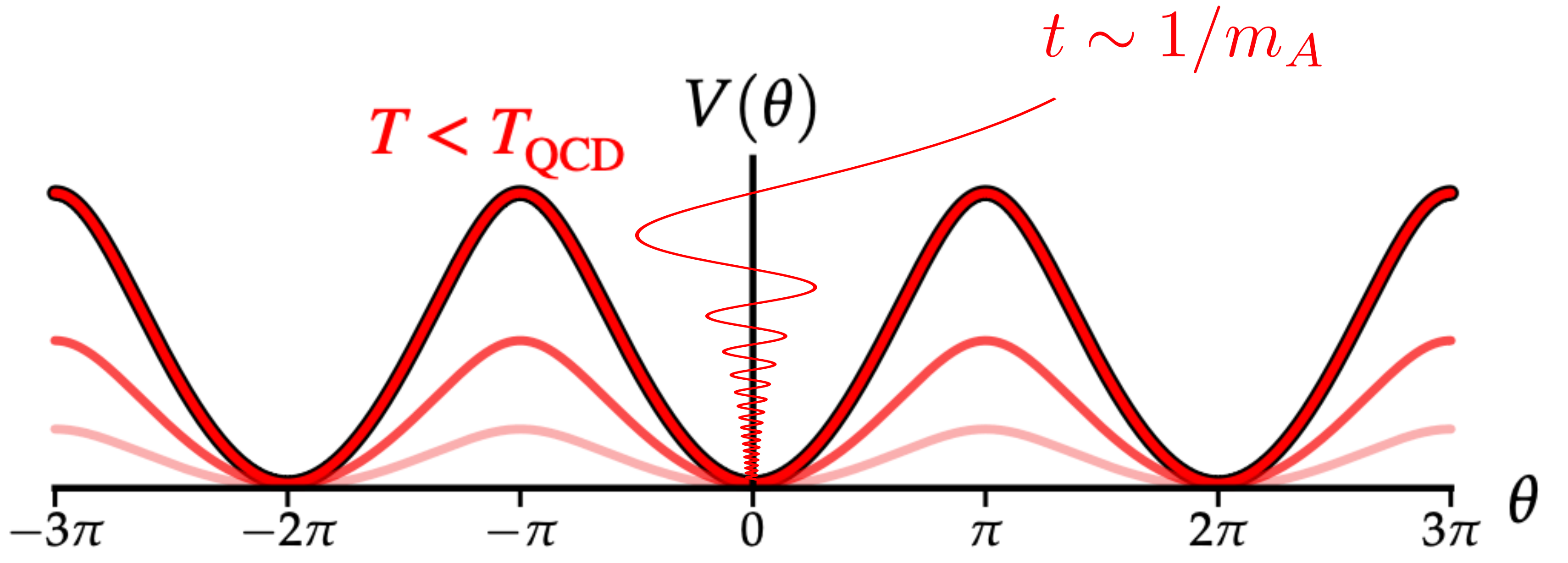
COSMIC WISPERS

COST EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

QCD Axion

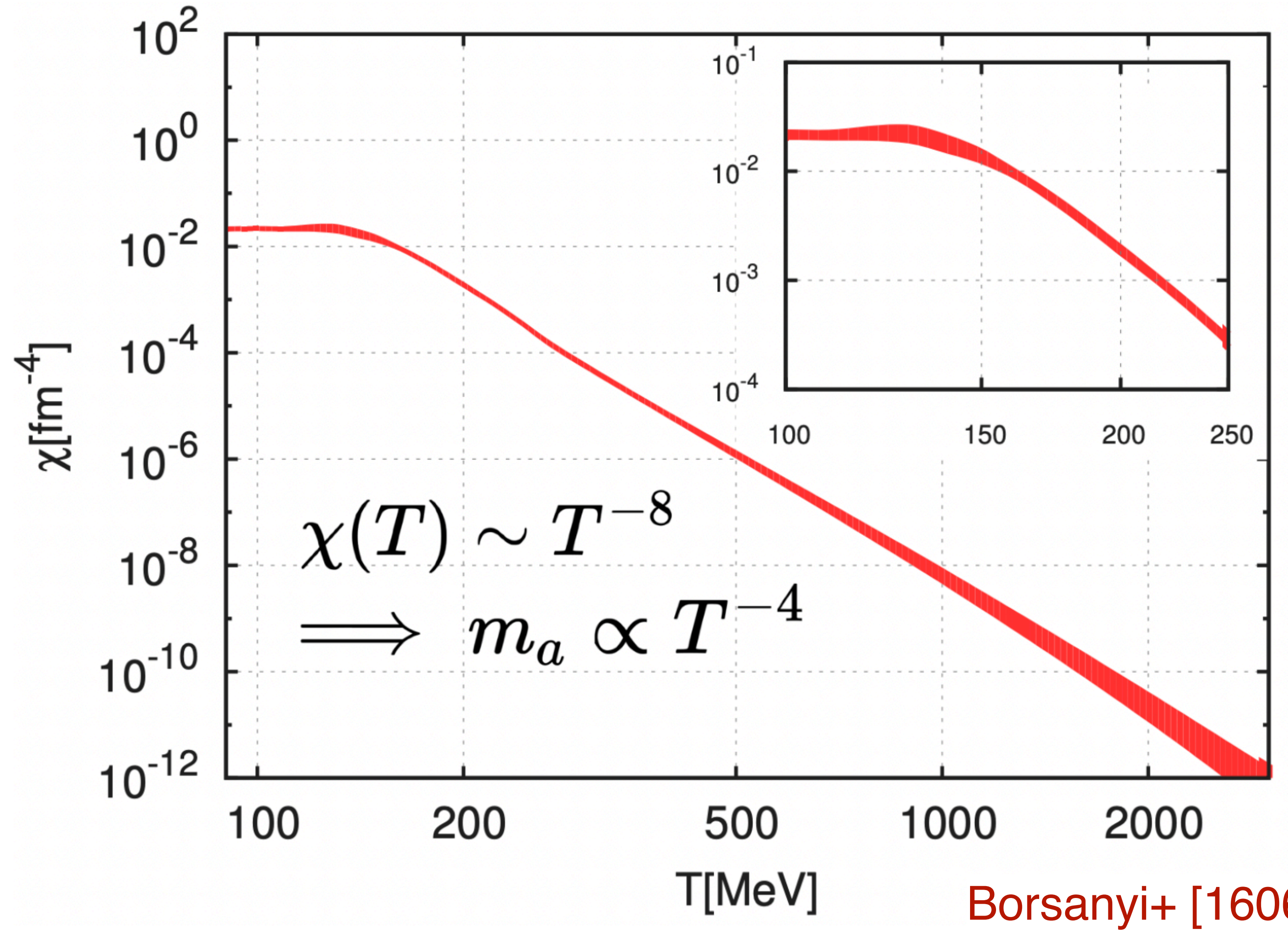
- The QCD Axion
  - Pseudo Nambu-Goldstone boson associated with the spontaneous breaking of the global Peccei-Quinn (PQ) *U(1)* symmetry at the scale f_a.
  - Dynamical solution to the strong-CP problem.
  - Suitable candidate for Cold Dark Matter.
- Acquires a mass below the QCD scale.

$$V(\theta) \approx \chi(T)(1-\cos\theta) = m_a^2(T) f_a^2 (1-\cos\theta)$$



O'Hare [2403.17697]

$$m_A|\theta|^2 R^3 = \frac{\rho_A}{m_A} R^3 \equiv N_A$$



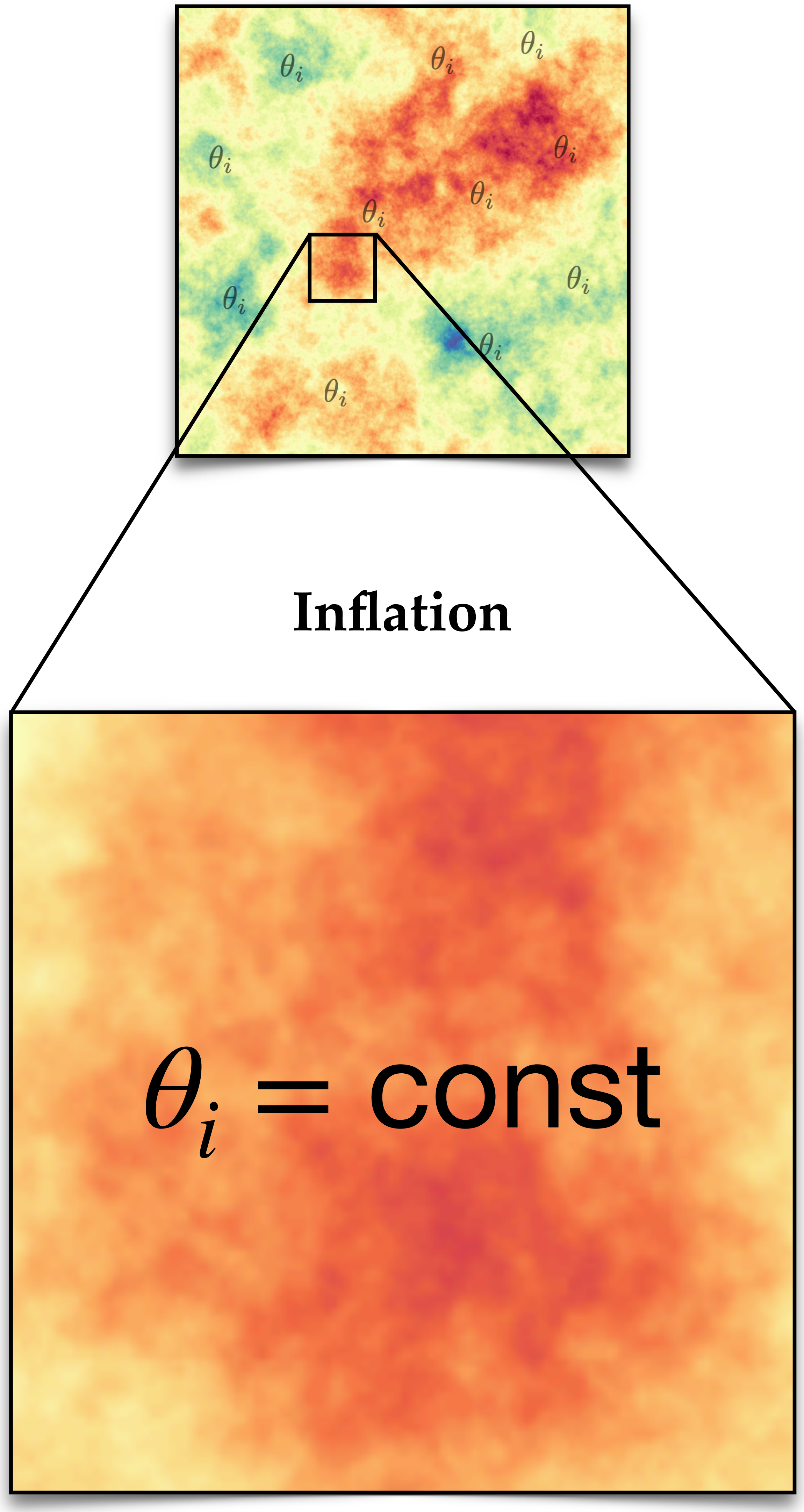
Borsanyi+ [1606.07494]

* Throughout this talk, when we refer to the axion, we implicitly mean the **QCD axion** (i.e. solves the strong CP problem)

Pre- vs. Post-Inflationary Scenario

Pre-Inflationary Scenario

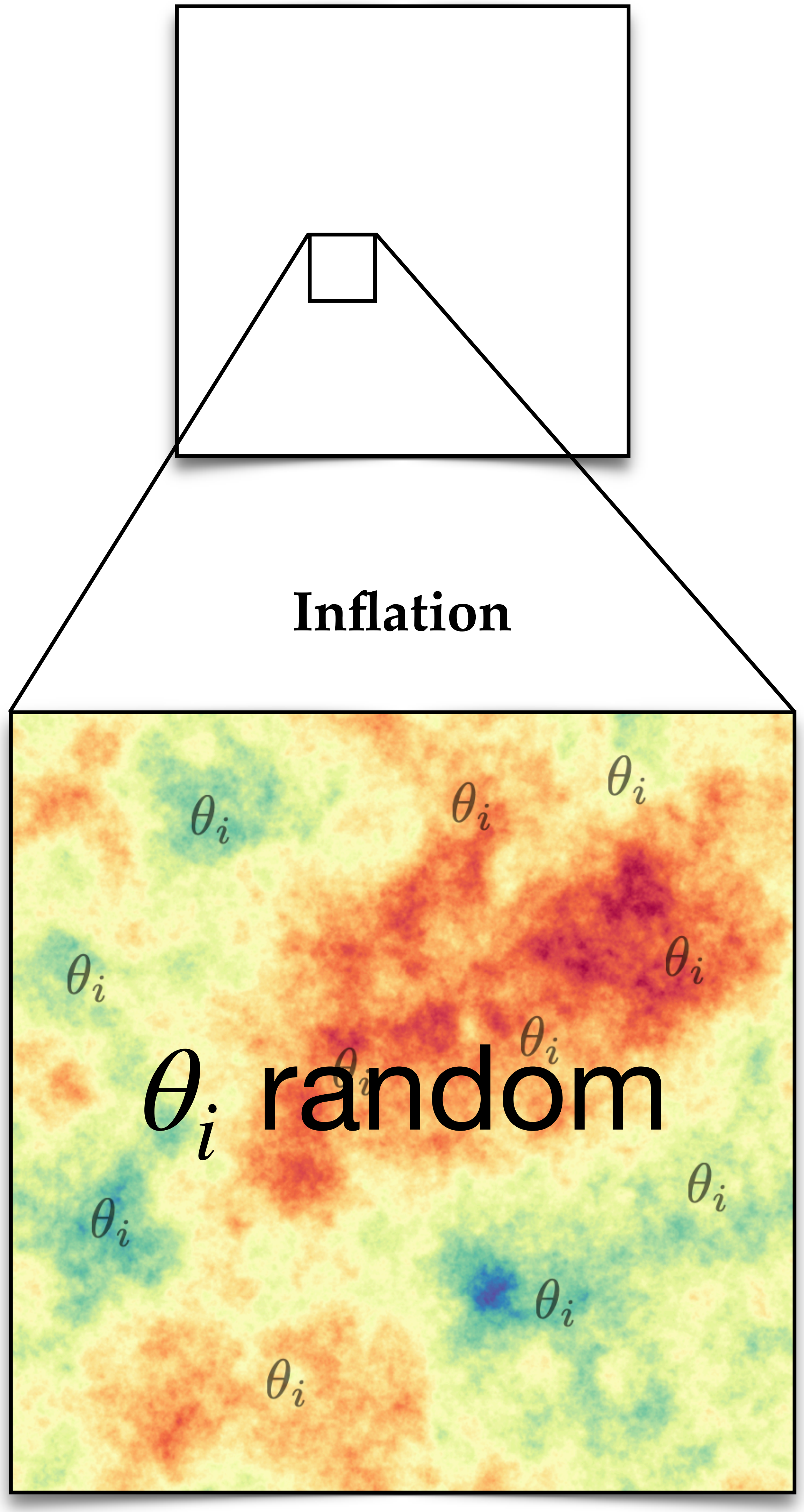
PQ broken **before** and **during** inflation



$$\sqrt{\langle \theta_i \rangle} = ???$$

Post-Inflationary Scenario

PQ broken **after** inflation

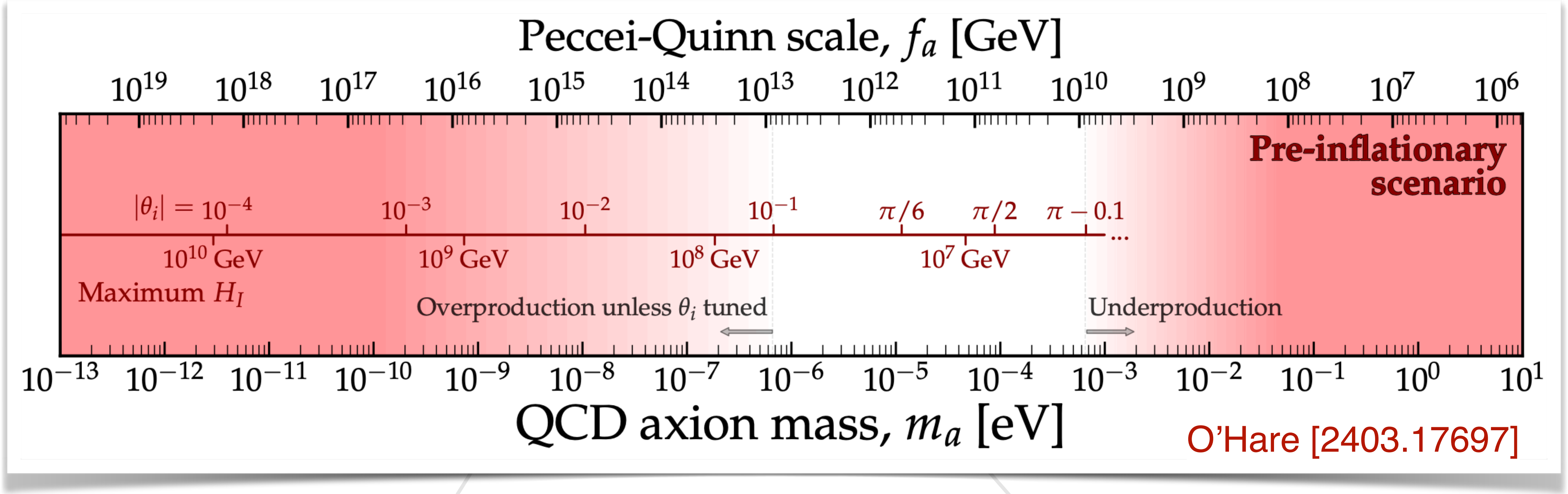


$$\sqrt{\langle \theta_i \rangle} \sim 2$$

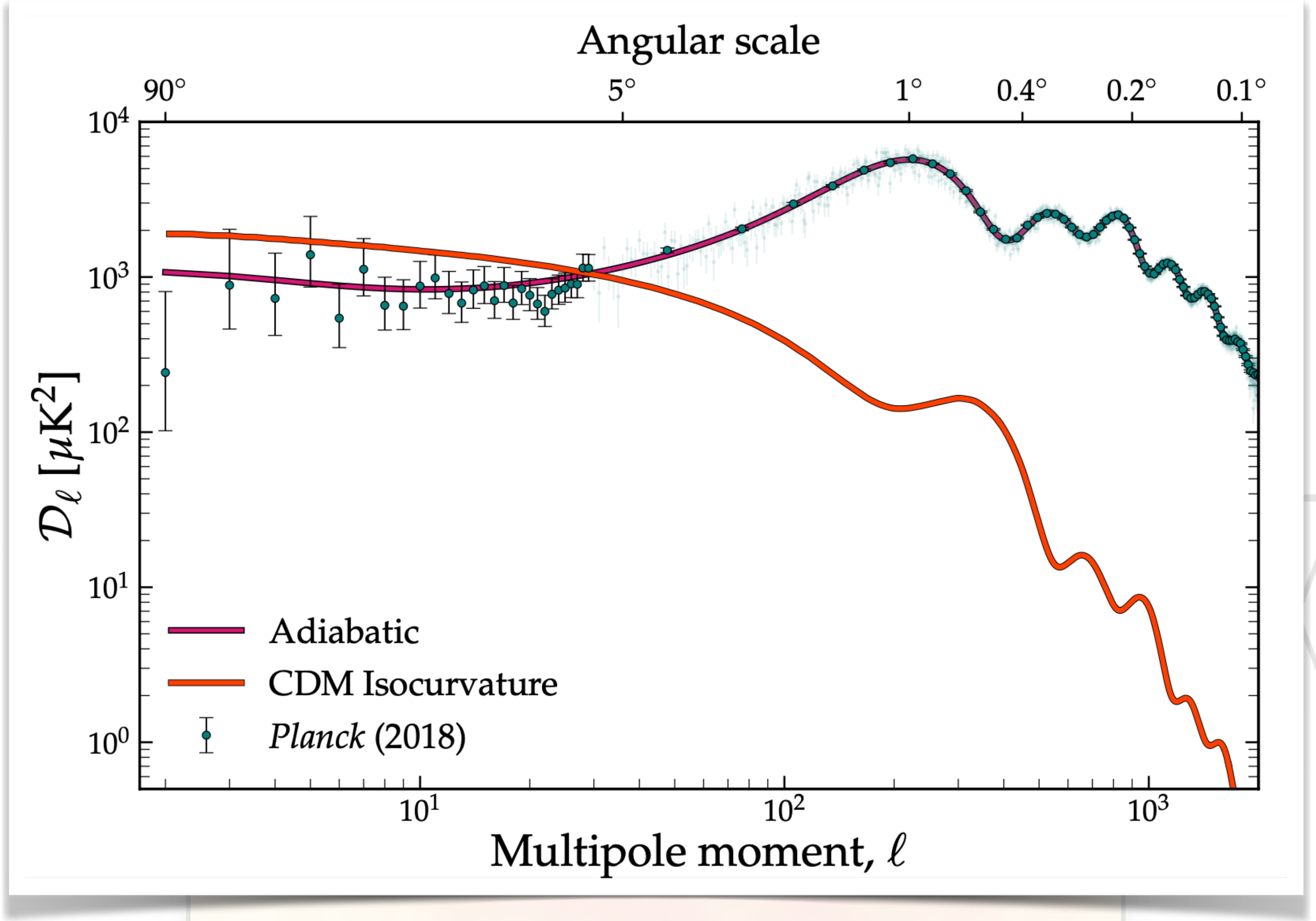
Pre- vs. Post-Inflationary Scenario

Pre-Inflationary Scenario

PQ broken **before** and **during** inflation



O'Hare [2403.17697]



PLANCK [1907.12875]

Post-Inflationary Scenario

PQ broken **after** inflation

$\sqrt{\langle\theta_i\rangle} \sim 2$

Pre- vs. Post-Inflationary Scenario

Pre-Inflationary Scenario

PQ broken **before** and **during** inflation

$\langle \theta_i \rangle = ???$

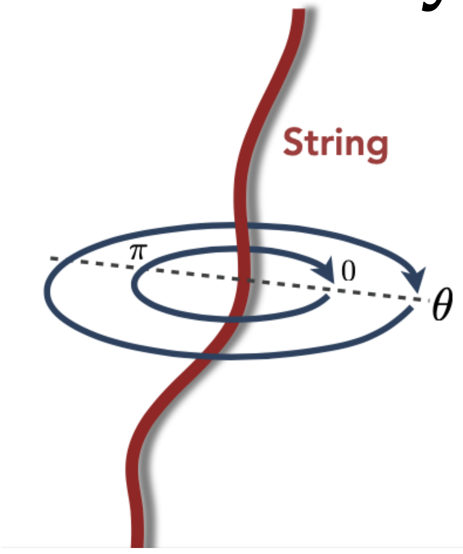
Post-Inflationary Scenario

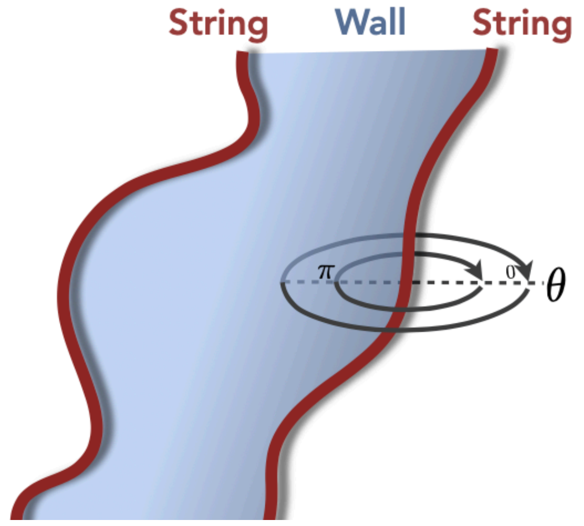
PQ broken **after** inflation

- Allows in principle for precise axion mass prediction

$$\Omega_a h^2 = 0.12 \quad \Rightarrow \quad m_a = ???\mu\text{eV}$$

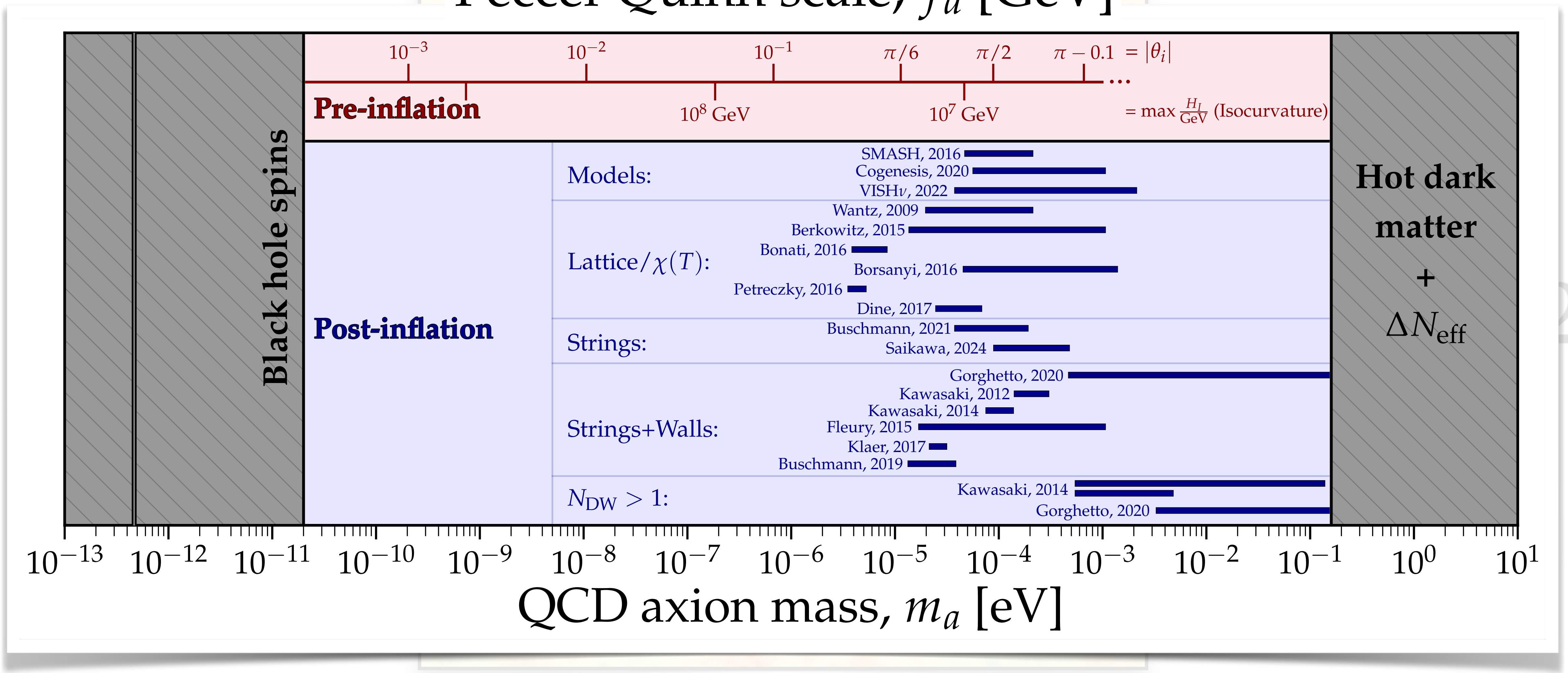
- **Subtlety:** Formation of Strings (+ Domain Walls)



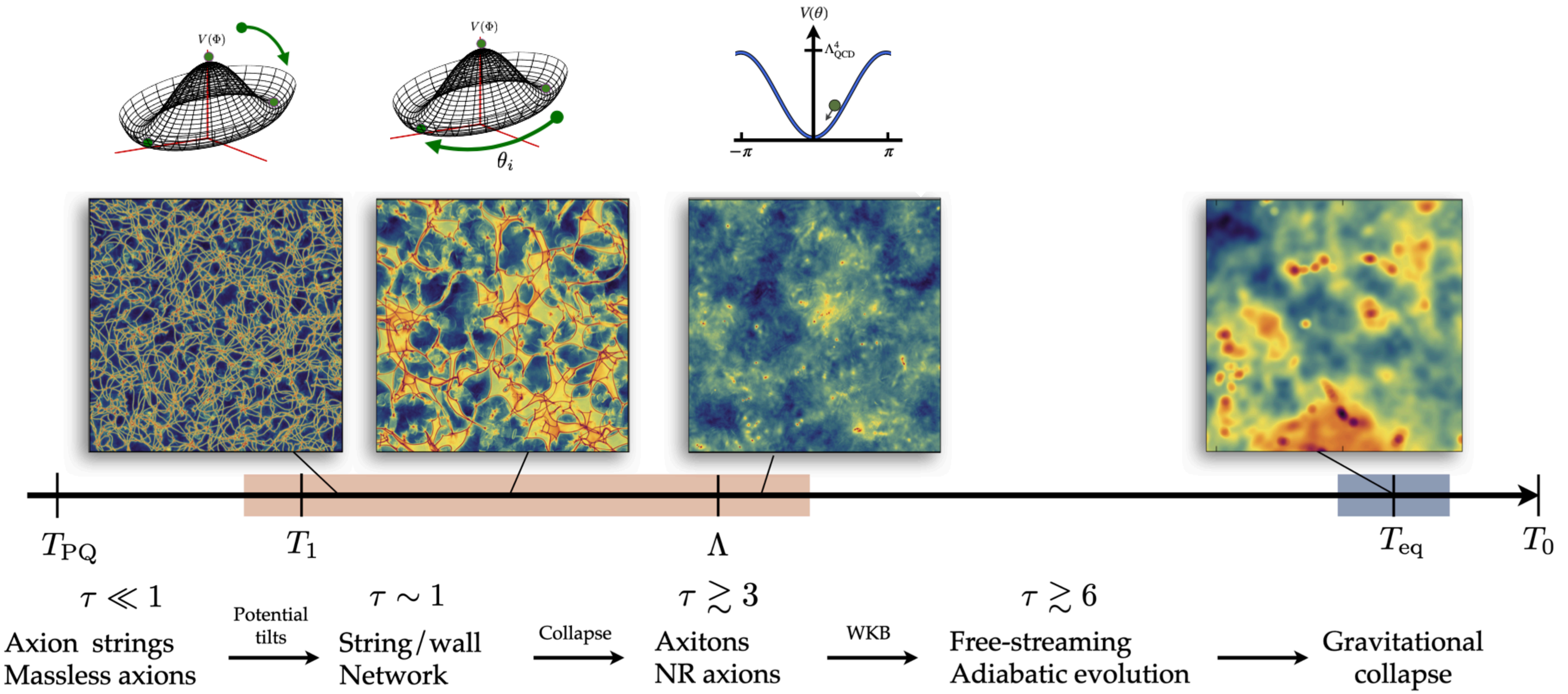


Inflation

O'Hare [2403.17697]



Cosmological Evolution in the Post-Inflationary Scenario



O'Hare+ [2110.11014]

How to simulate Axion Strings?

- Solve the classical EOM for a complex scalar field in comoving coordinates, discretised on a lattice:

$$\partial_\tau^2 \phi - \nabla^2 \phi + \lambda \phi \left(|\phi|^2 - \tau^2 \right) = 0$$

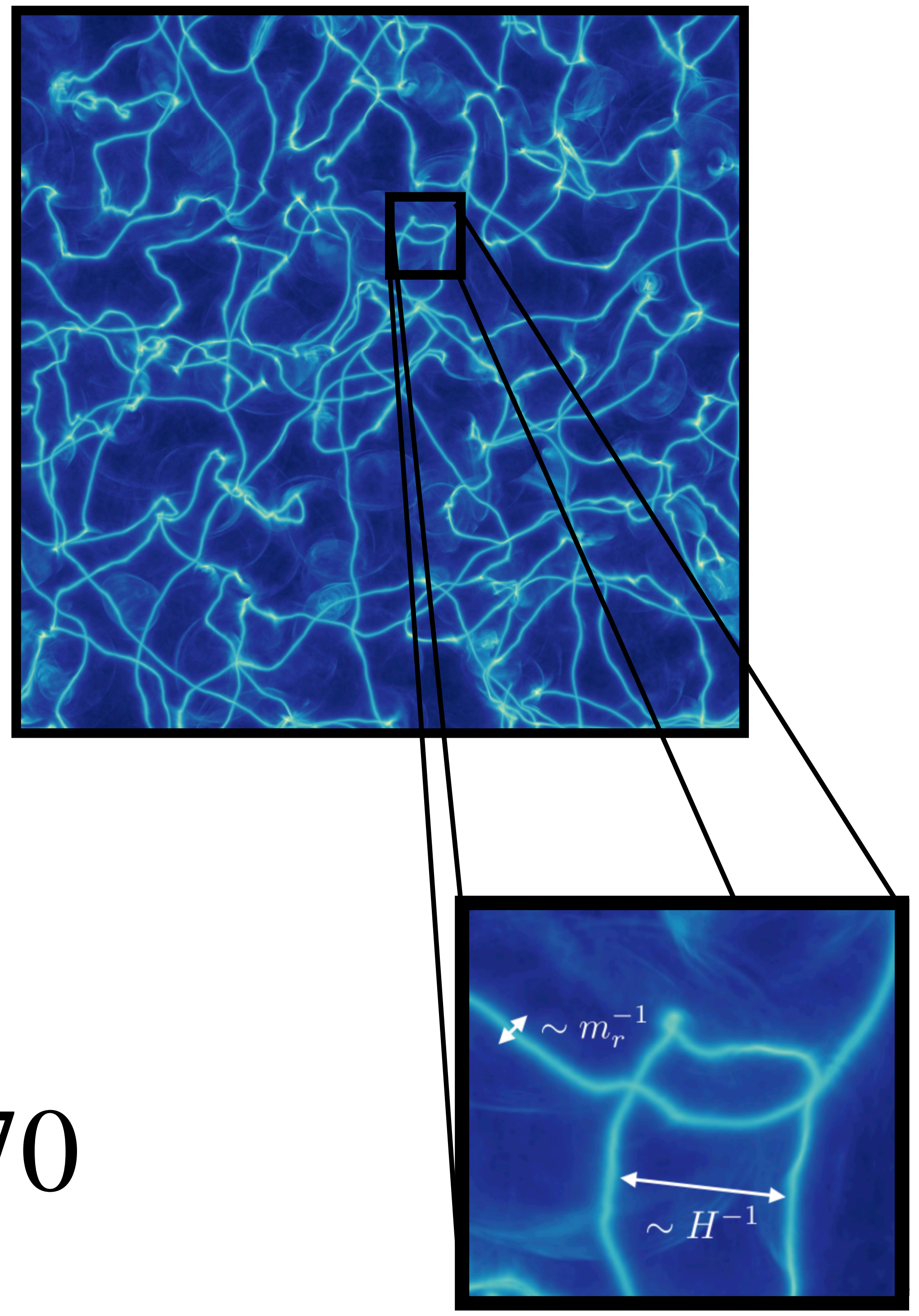
- **Tricky:** Simulations require proper resolution of two very different length scales.
    - String core radius

      $\sim m_r^{-1} \sim f_a^{-1}$ (m_r = radial mass)

    - Hubble radius

      $\sim H^{-1}$

    - Realistic value: $f_a / H_{\mathrm{QCD}} \sim 10^{30} \implies \log(m_r/H) \sim 70$



Jaxions Code

- State-of-the-art, highly parallelised C++ code to simulate the evolution of the axion dark matter field in the early Universe.
- Available on Github: https://github.com/veintemillas/jaxions.

jaxions v0

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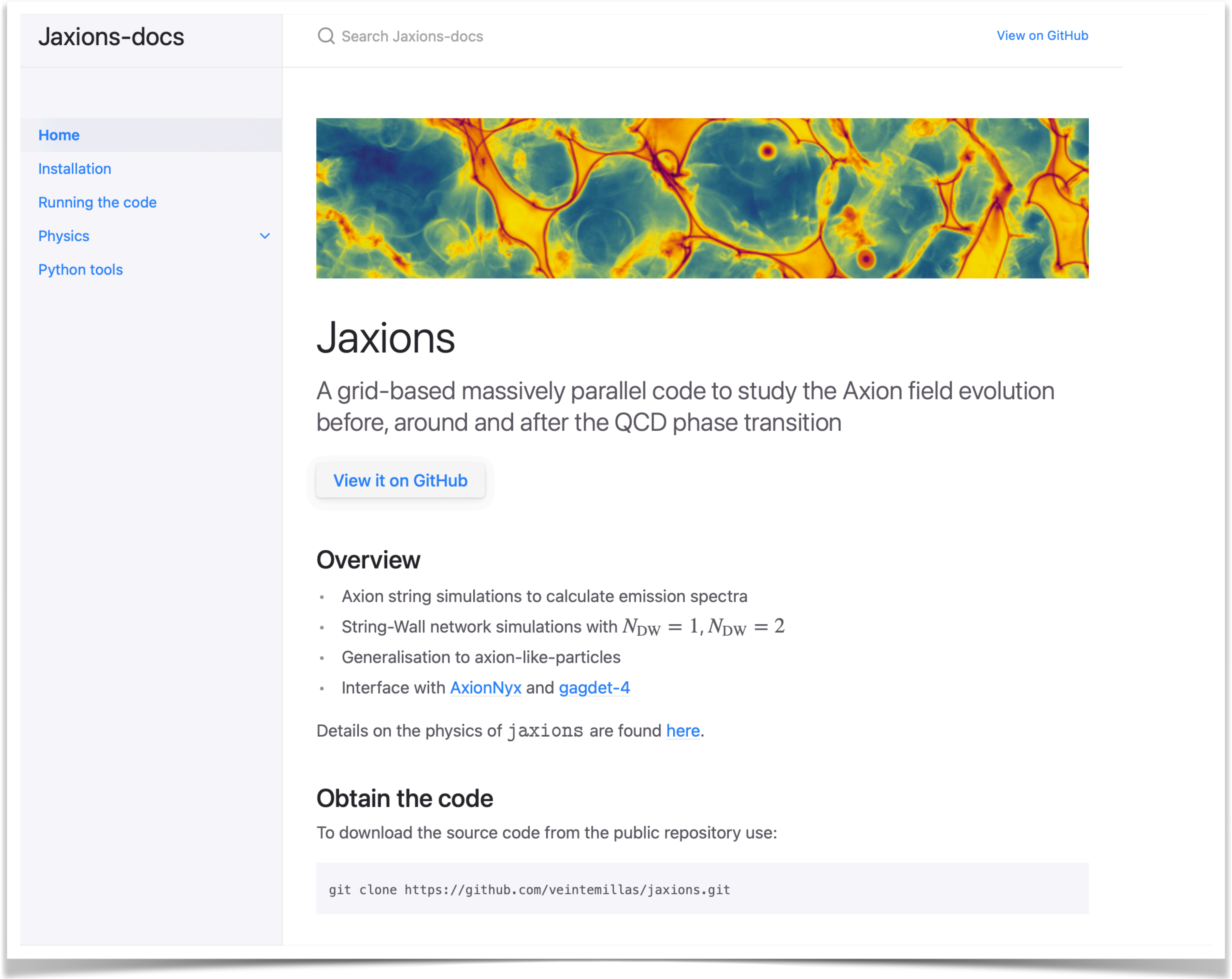
Abstract. We describe the `jaxions` numerical code to simulate the evolution and properties of the axion dark matter field.



The Issue of large $\log(m_r/H)$

- Evolution of the string density suggests that the energy density of the system is of order

$$\rho \sim 8\pi\xi \log(m_r/H) H^2 f_a^2$$

This leads to an enhancement by a factor of $\sim \xi \log(m_r/H)$ in comparison to the typical density $H^2 f_a^2$ at QCD temperatures.

- Does this imply an enhancement of the axion abundance (and therefore of the dark matter mass)?

- We need to know how this energy is partitioned into radiated axions (i.e. the axion spectrum).

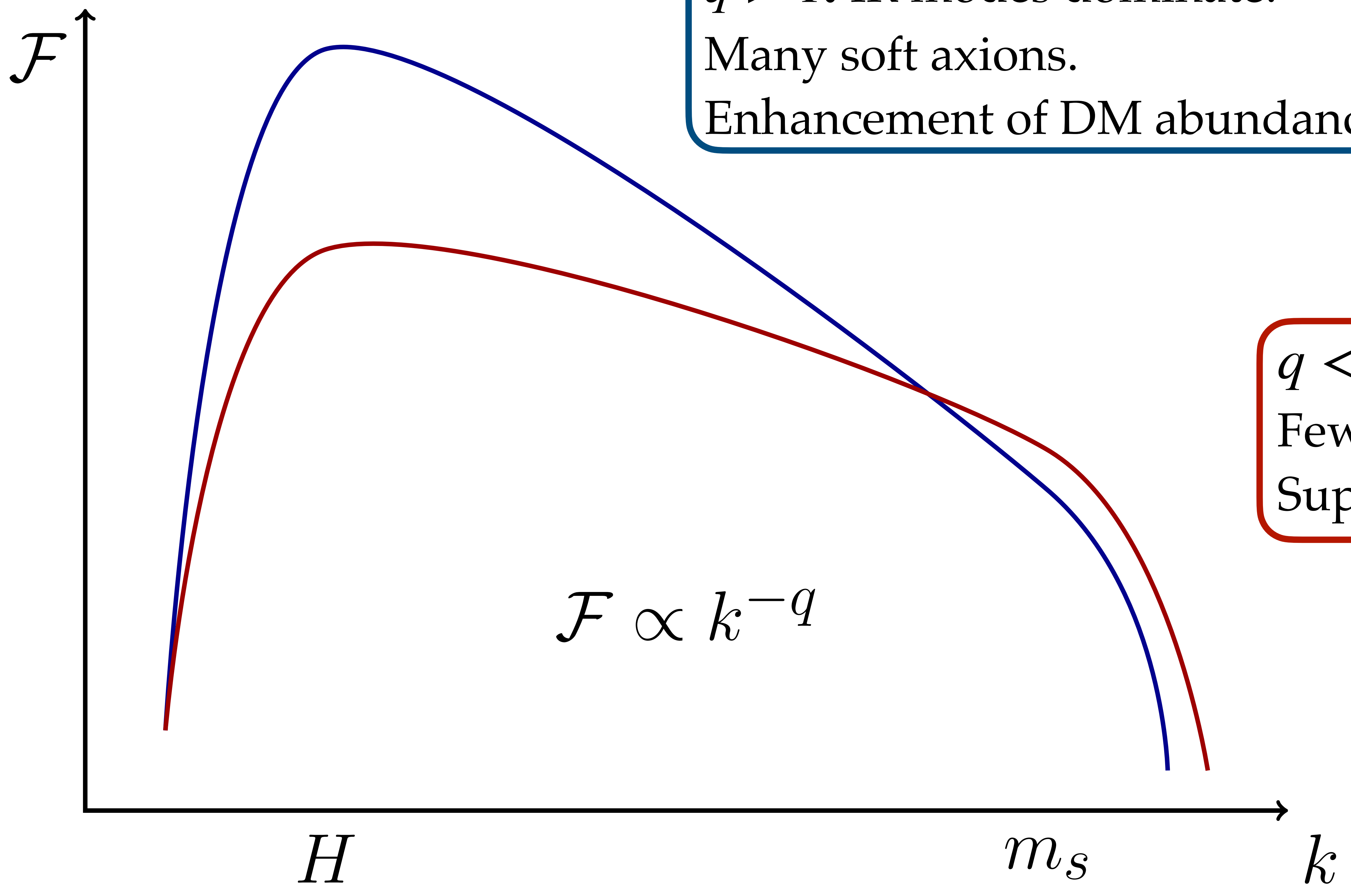
Axion Radiation from Strings

- Differential energy transfer rate:

$$\mathcal{F}\left(\frac{k}{RH}, \frac{m_r}{H}\right) \equiv \frac{1}{(f_a H)^2}\frac{1}{R^3}\frac{\partial}{\partial t}\left(R^4 \frac{\partial \rho_a}{\partial k}\right)$$

(R: scale factor)

- Slope is important! Gorghetto+ [1806.04677], Buschmann+ [2108.05368], Saikawa, MK+ [2401.17253]

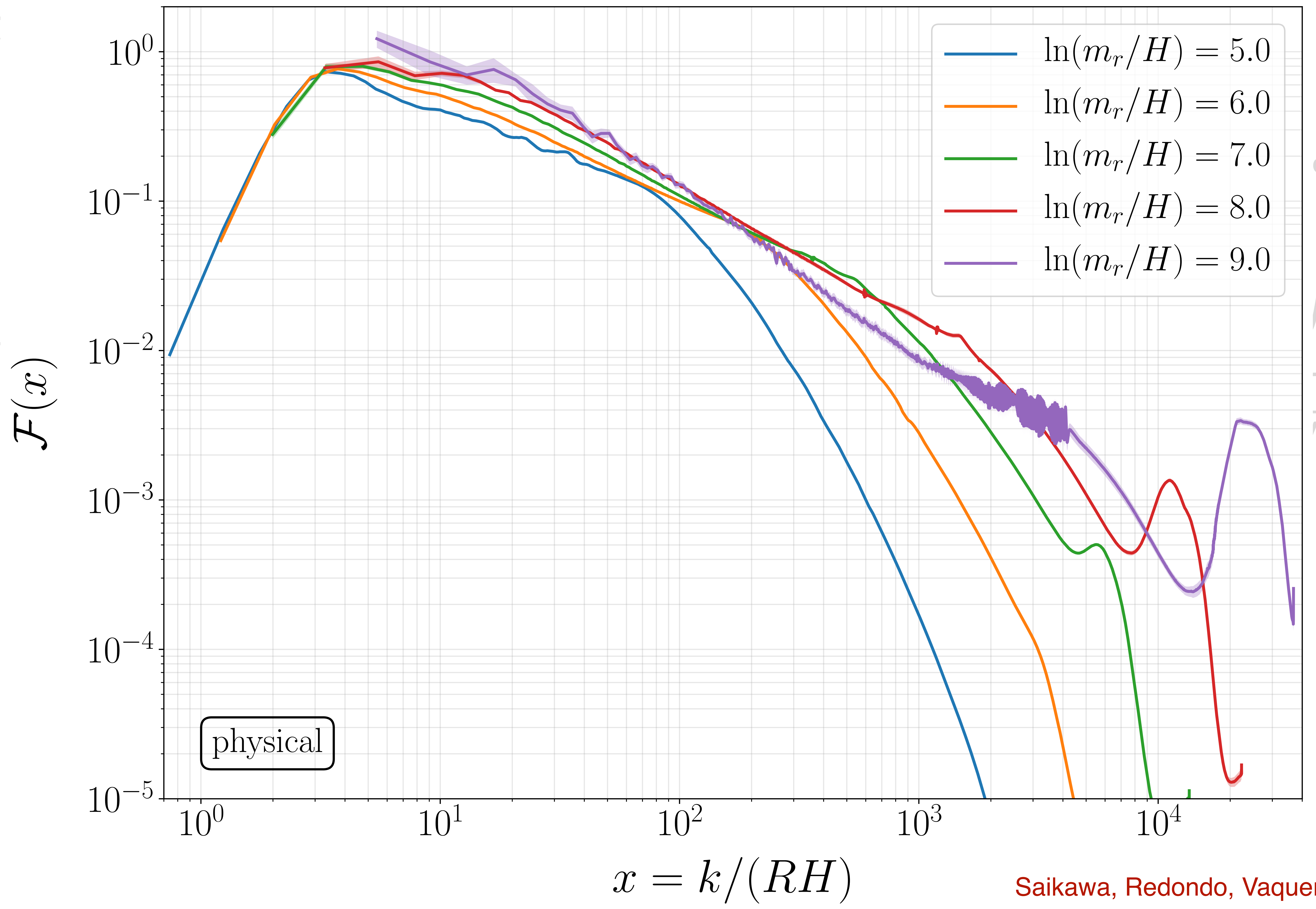


$q > 1$: IR modes dominate.
Many soft axions.
Enhancement of DM abundance.

$$\frac{n_a}{f_a^2 H} = \frac{1}{f_a^2 H}\int \mathrm{d}k \frac{1}{\omega}\frac{\partial \rho_a}{\partial k} = \int^{\tau} \frac{\mathrm{d}\tau'}{\tau'} \int \frac{\mathrm{d}x'}{x'} \mathcal{F}$$

$q < 1$: UV modes dominate.
Fewer hard axions.
Suppression of DM abundance.

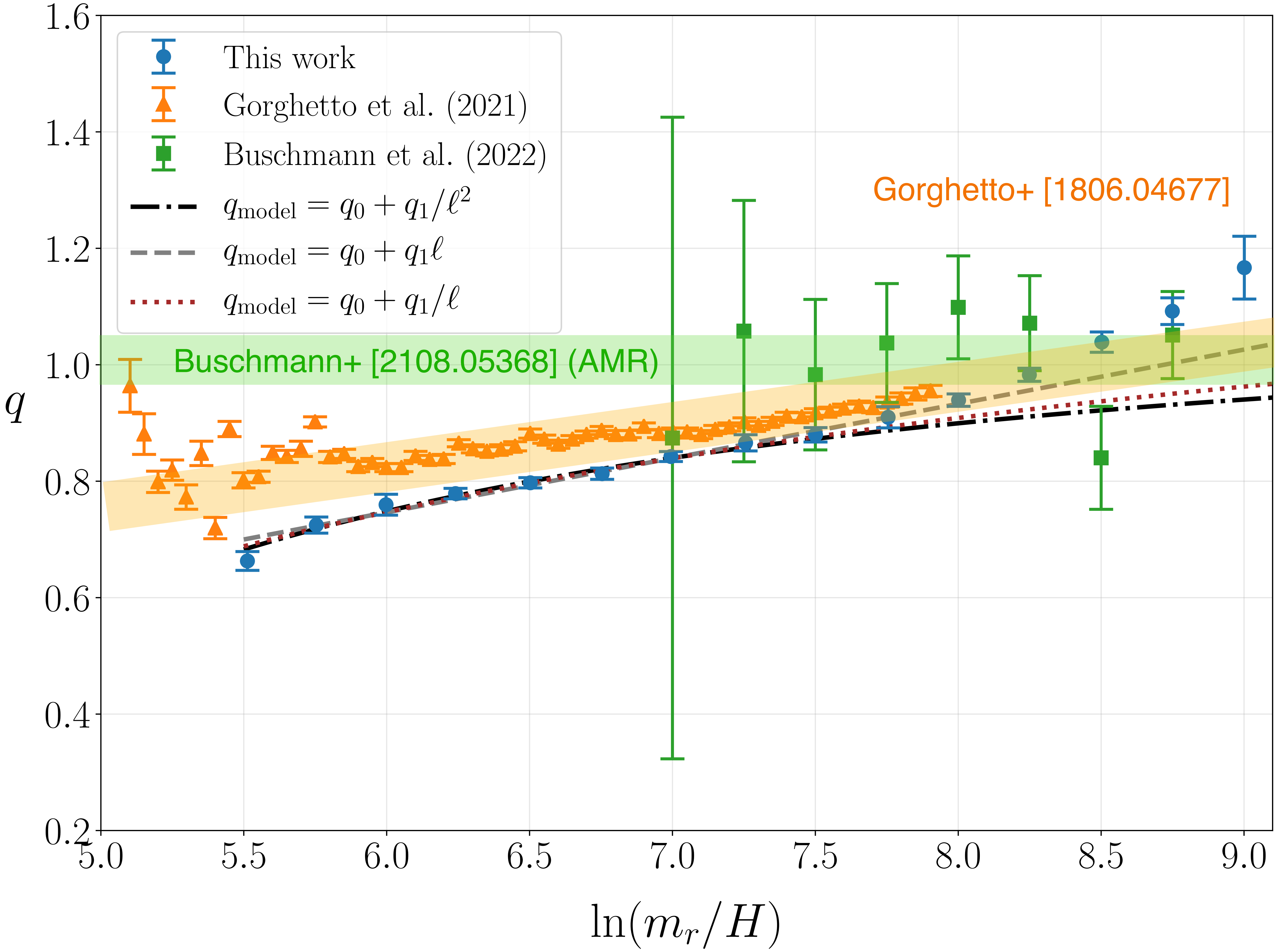
Axion Radiation from Strings



Saikawa, Redondo, Vaquero, MK [2401.17253]

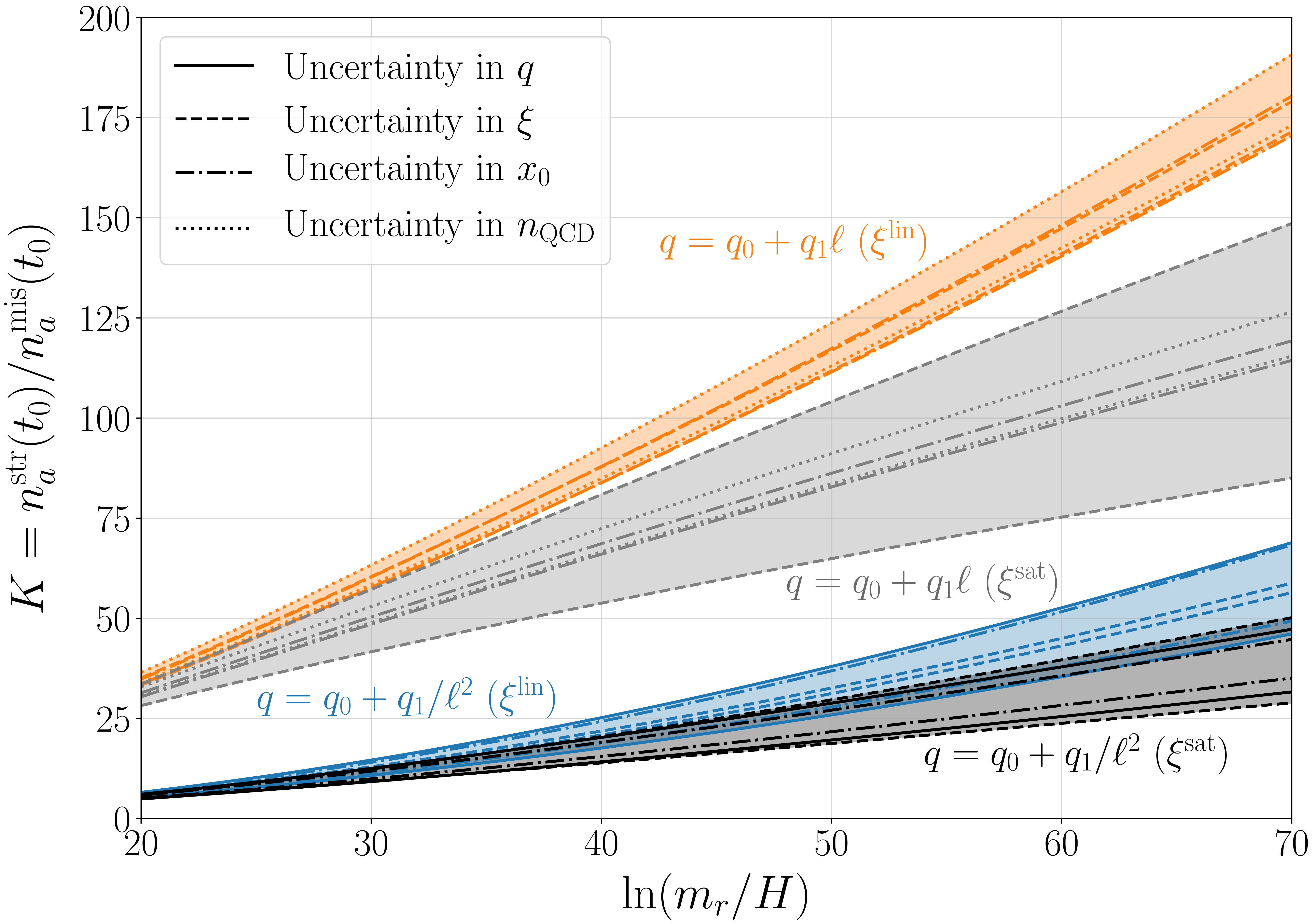
What can bias the Results?

- There are several systematic effects, that could explain discrepancies in the literature:
    - Initial conditions
    - Axion field oscillations
    - Discretisation effects



Saikawa, Redondo, Vaquero, MK [2401.17253]

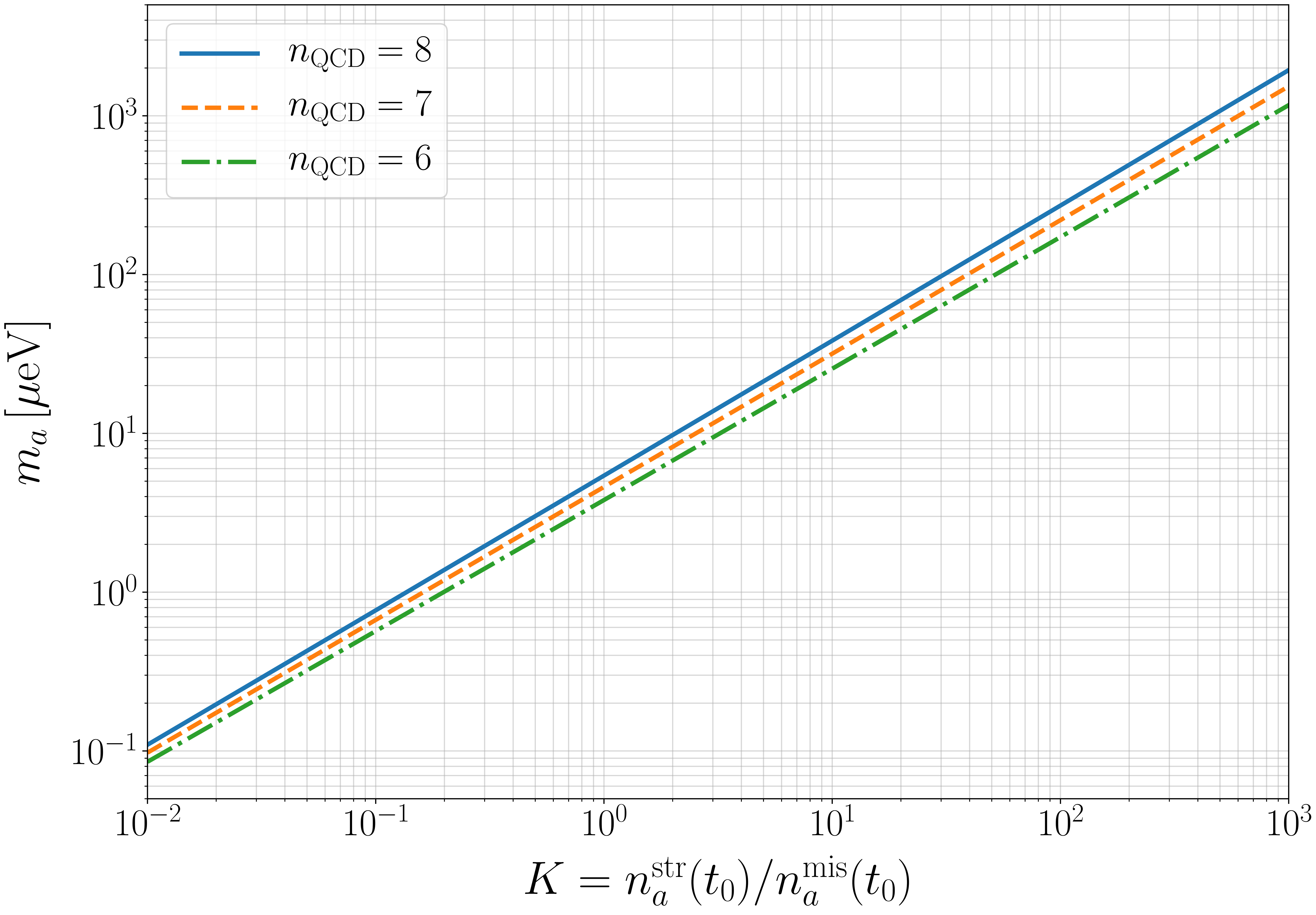
Axion Production: Strings vs. Misalignment



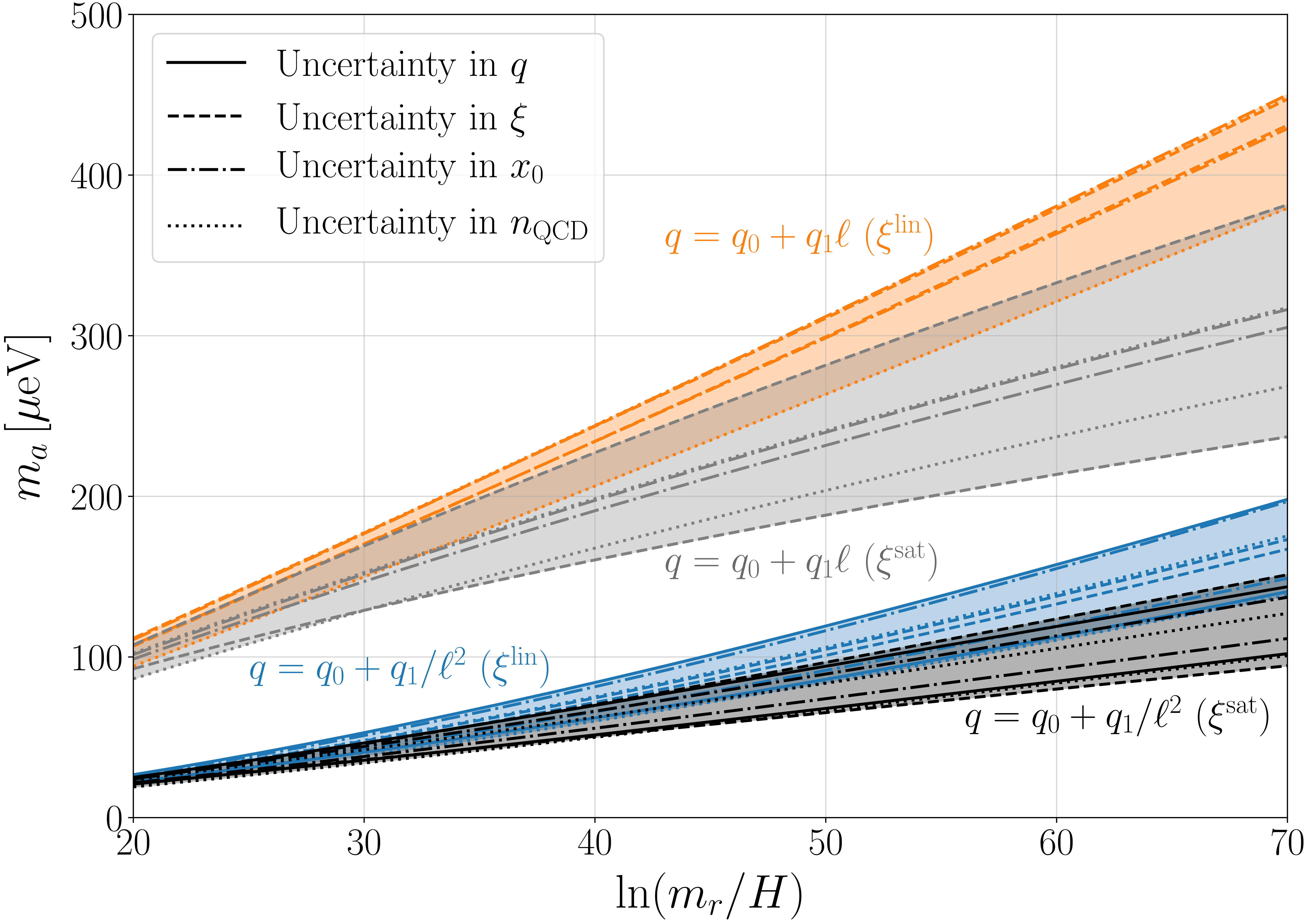
Known for "standard" angle-averaged misalignment:

$$\Omega_a h^2 = K \Omega_a^{\rm mis} h^2$$

Production Efficiency vs. Axion Mass



Axion Dark Matter Mass Prediction



Challenging ....

MADMAX, ALPHA, ORGAN

Summary Part I

- Understanding of the global string dynamics is very important for a precise prediction of the axion dark matter mass in the post-inflationary scenario.
- Our simulations predict $95\ \mu\text{eV} \lesssim m_a \lesssim 450\ \mu\text{eV}$.
- Fast developments in recent simulations allow us to have a better understanding, albeit serious discrepancies.
- There are several systematic effects that could bias the result, that could explain these discrepancies:
  - Initial conditions
  - Axion field oscillations
  - Discretisation effects
- Further improvement in the dynamical range would be helpful to be sure of the extrapolation.

If time allows …

Towards the Continuum Limit of Global Loop Decays

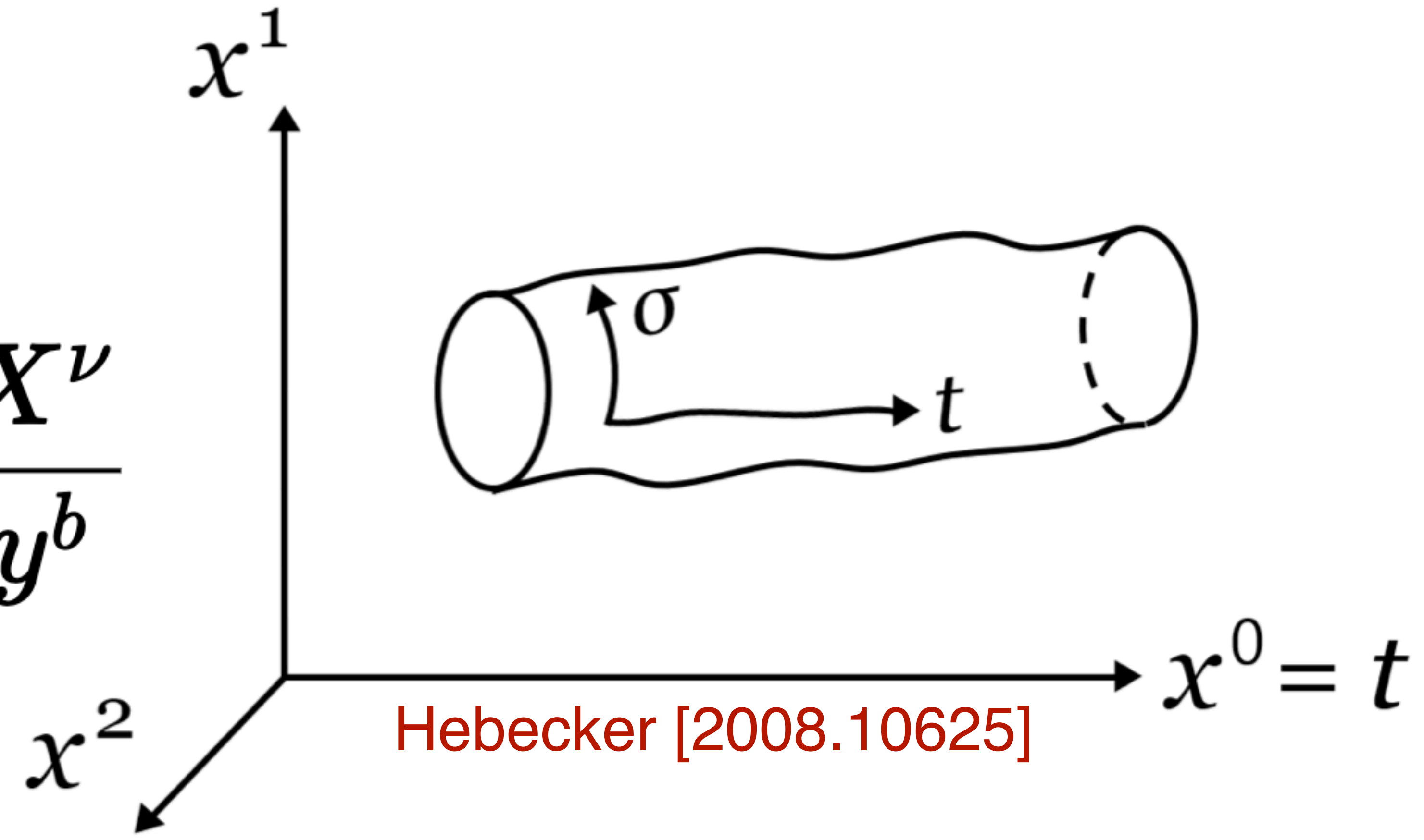
ongoing project(s) with J. Redondo, I. Y. Rybak and A. Drew

String Dynamics

- (Local) String dynamics governed by the **Nambu-Goto** action:

$$S_{\rm NG} = \mu_0 \int \mathrm{d}^2\sigma \sqrt{-h} \quad \text{with } h = \det\left(h_{ab}\right), \; h_{ab} = \eta_{\mu\nu} \frac{\partial X^\mu}{\partial y^a} \frac{\partial X^\nu}{\partial y^b}$$

Nambu (1970), Goto (1971)



Hebecker [2008.10625]

- For **Global Strings,** couple to antisymmetric **Kalb-Ramond** field:

$$S_{\rm KR} = S_{\rm NG} - \frac{1}{6} \int \mathrm{d}^4x F^{\mu\alpha\beta} F_{\mu\alpha\beta} + 2\pi f_a \int \mathrm{d}^2\sigma B_{\alpha\beta} \partial_a X^\alpha \partial_b X^\beta \epsilon^{ab}$$

Kalb & Ramond (1974), Dabholkhar & Quashnock (1990)

- Can be used to compute the axion field around strings (in close analogy to EM):

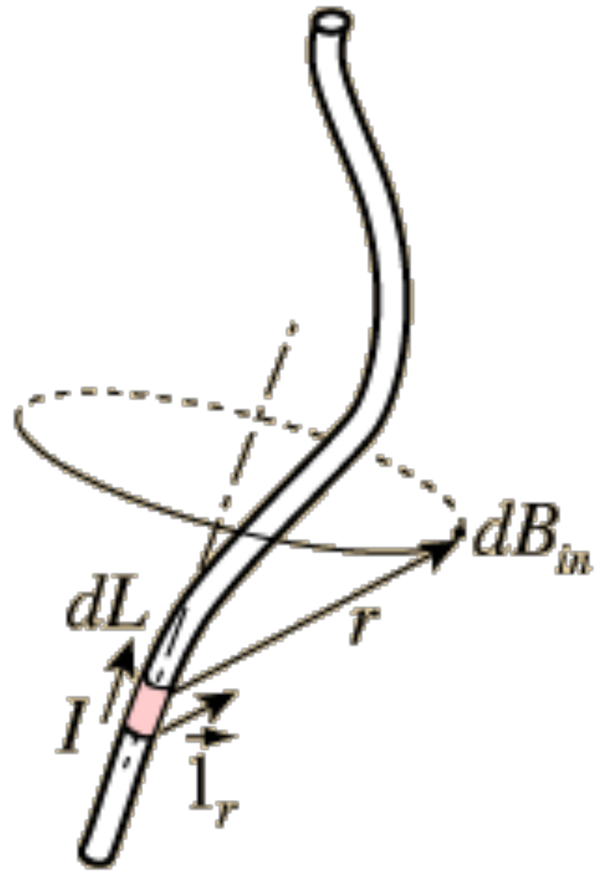
$$f_A \partial_\mu \theta = \frac{1}{6} \epsilon_{\mu\nu\alpha\beta} F^{\nu\alpha\beta}$$

e.g. Fleury & Moore [1509.00026]

Constructing the Axion Field around Strings

- Contribution of a short, straight section of the string to the "**axionic *B*-field**":

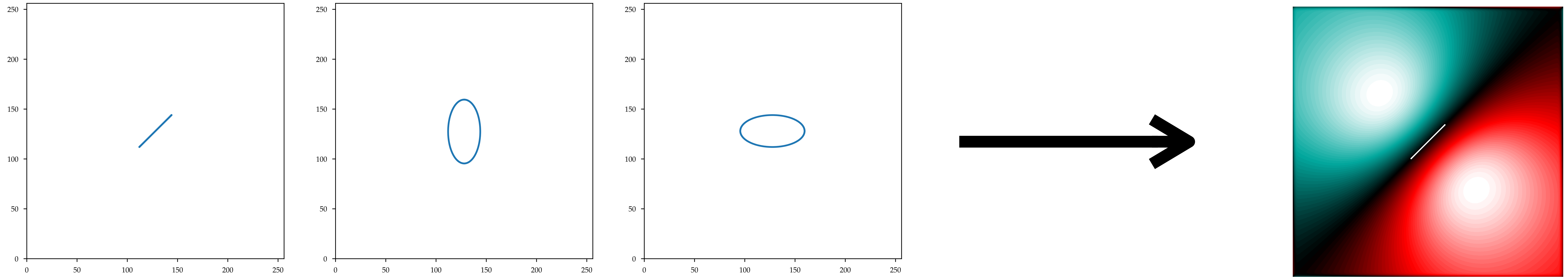
$$\nabla\theta = K \int d\sigma \frac{(\mathbf{x}-\mathbf{X}(\sigma))\times\mathbf{X}'}{|\mathbf{x}-\mathbf{X}(\sigma)|^3}$$



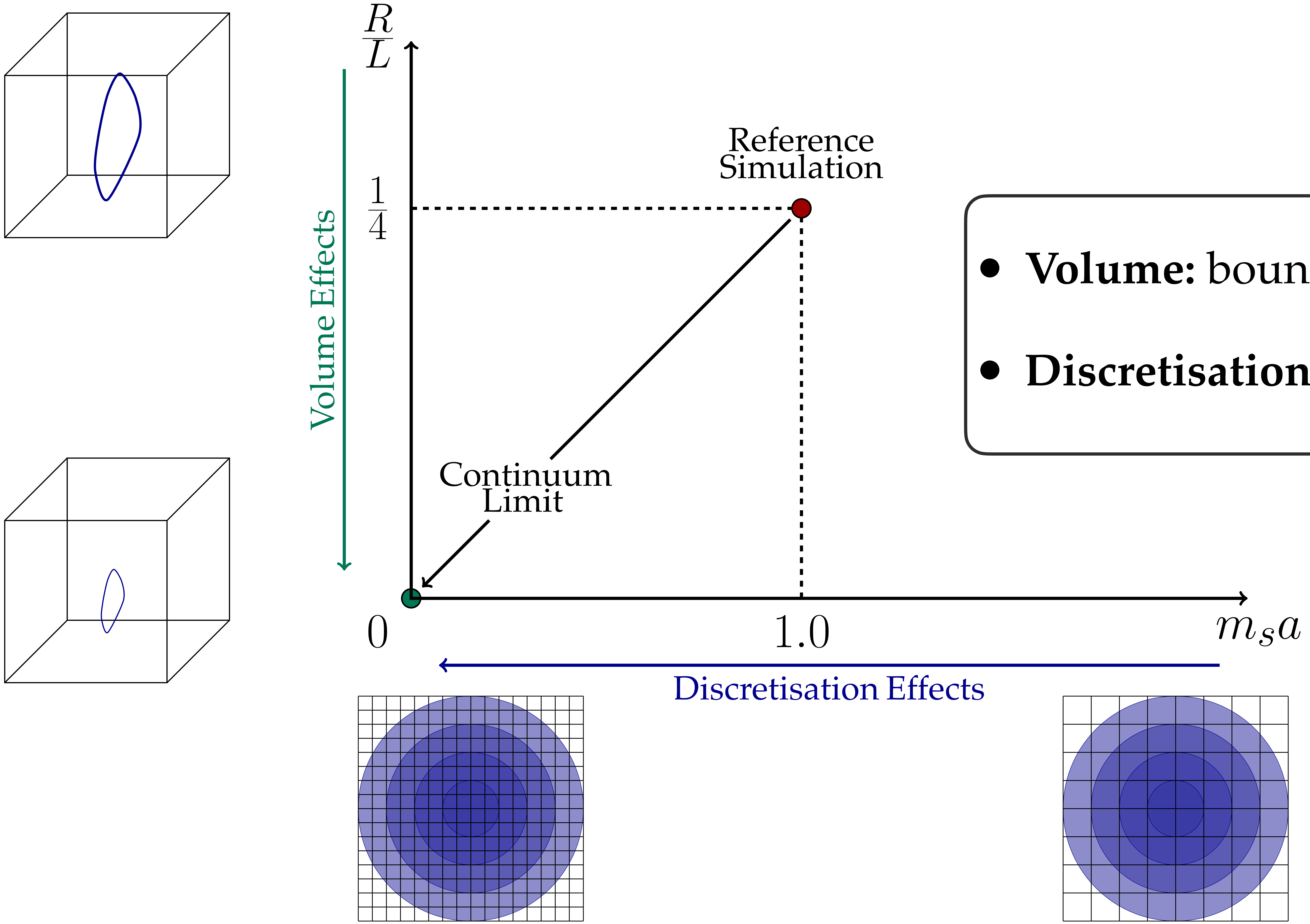
Biot-Savard law (Link)

- Calculate links to construct the axion field in the full plane:

$$\theta_{\mathbf{x}+\mathbf{d}x} - \theta_{\mathbf{x}} = \int_x^{x+dx} d^3\mathbf{x}\cdot\nabla\theta = -\frac{1}{2}\int_x^{x+dx} d^3\mathbf{x}\cdot\int d\sigma\frac{(\mathbf{x}-\mathbf{X}(\sigma))\times\mathbf{X}'}{|\mathbf{x}-\mathbf{X}(\sigma)|^3}$$

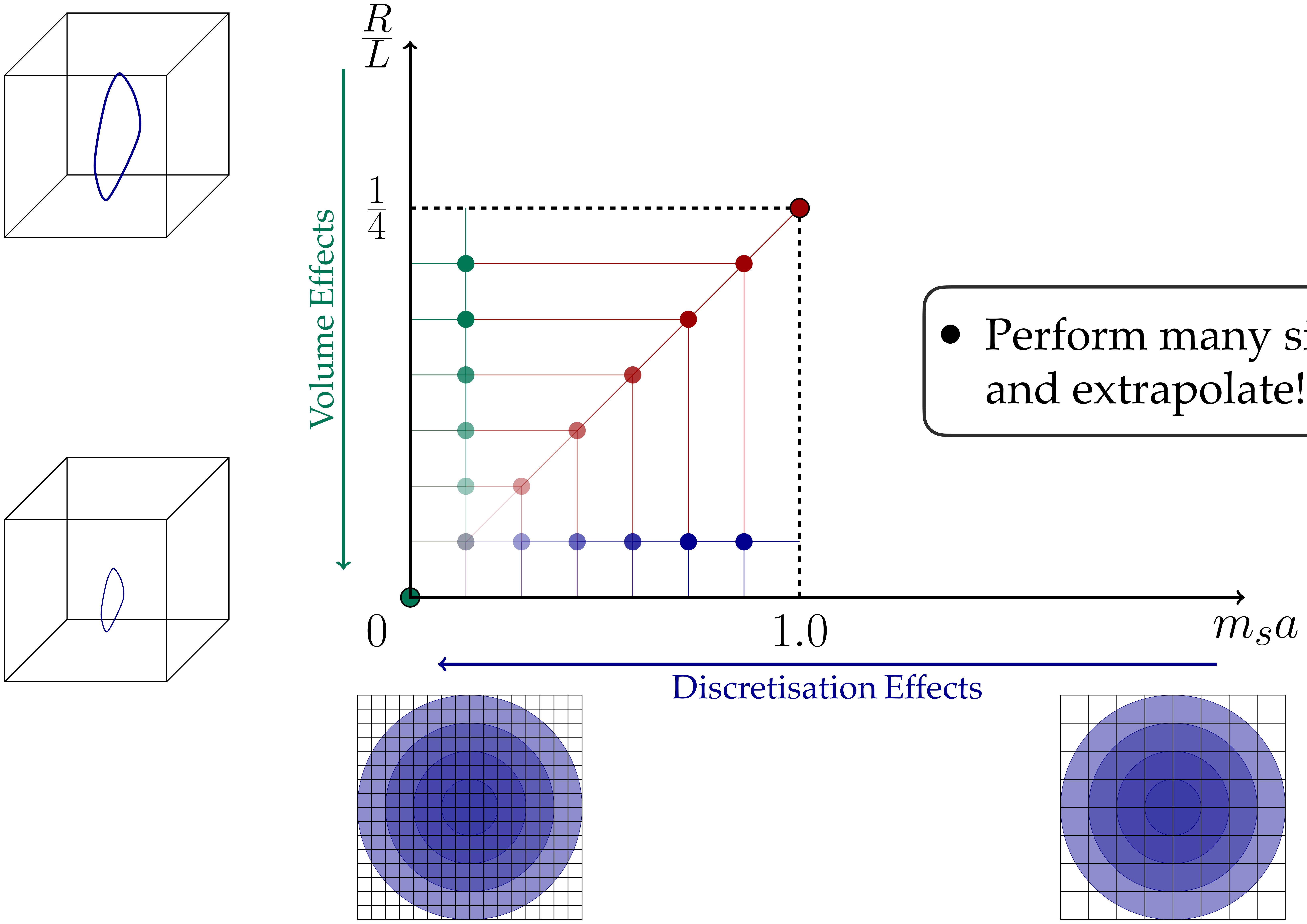


Towards the Continuum Limit



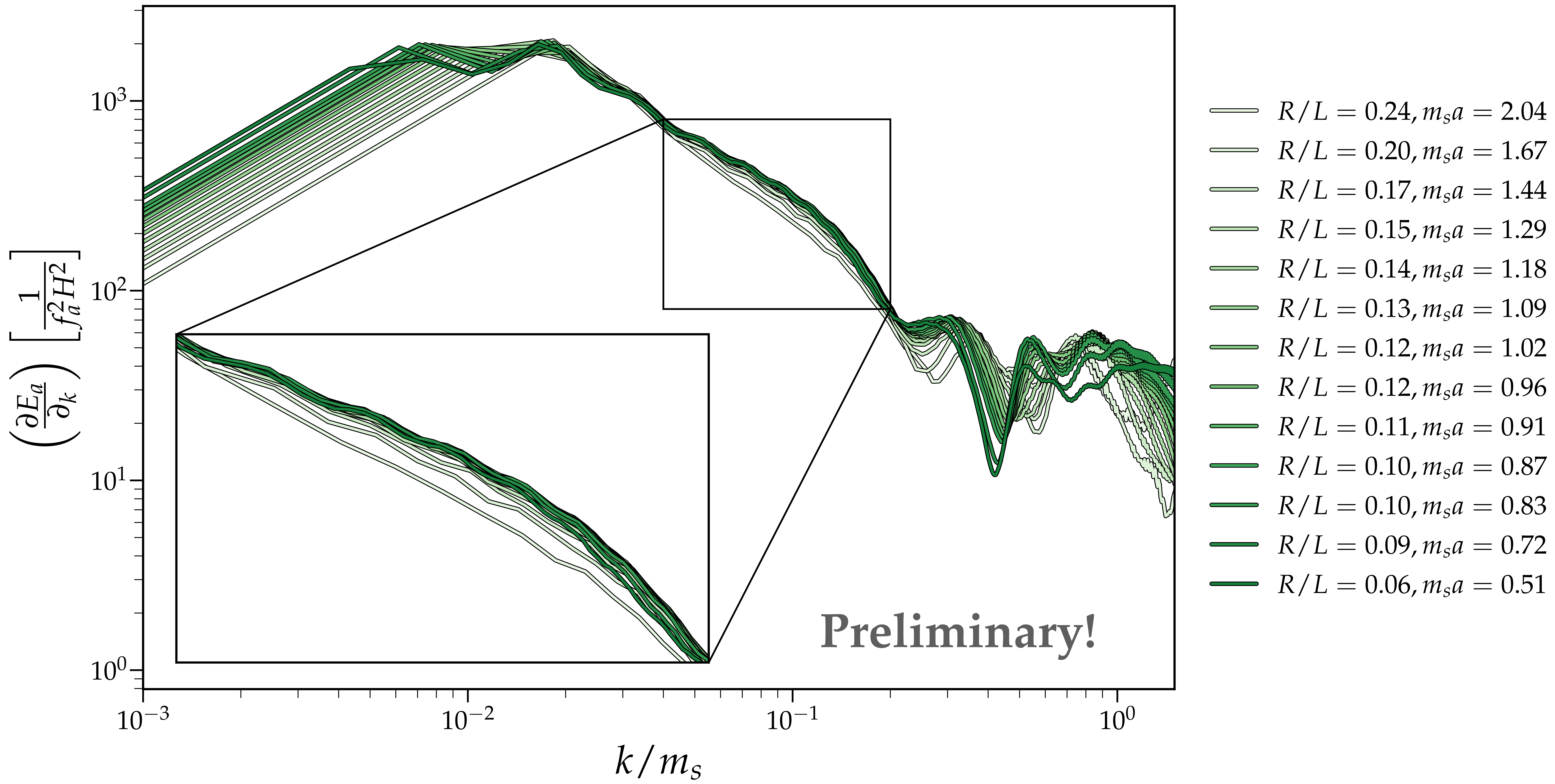
- **Volume:** boundary effects, periodic copies?
- **Discretisation:** resolution of the string cores

Towards the Continuum Limit



- Perform many simulations, test convergence and extrapolate!

Towards the Radiation Spectrum of Individual Strings

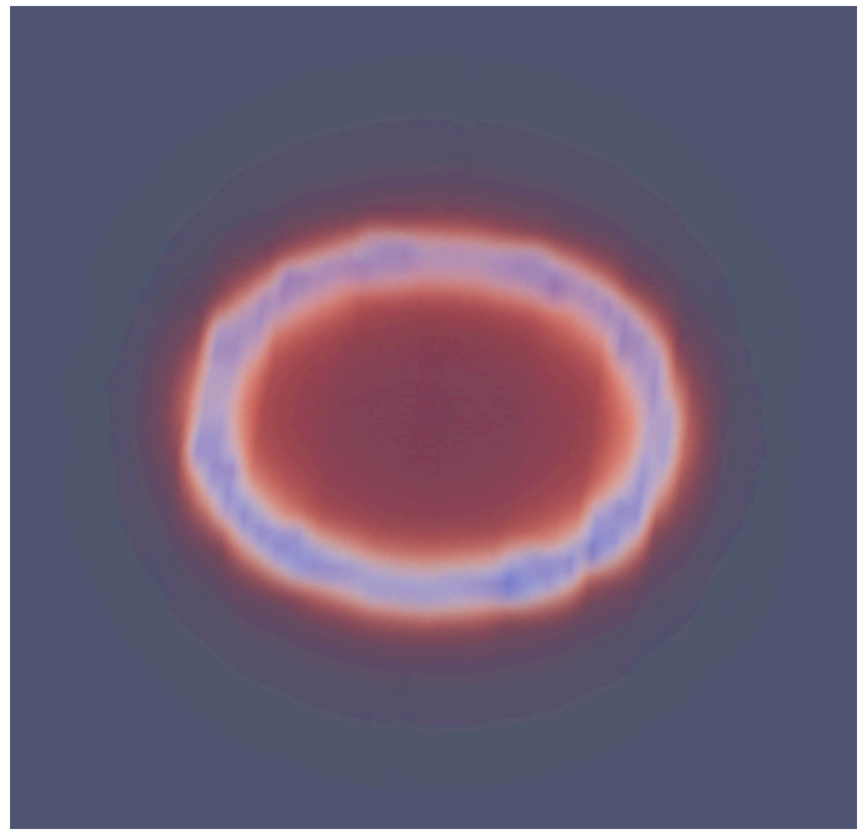


New Insights into the Network Spectrum?

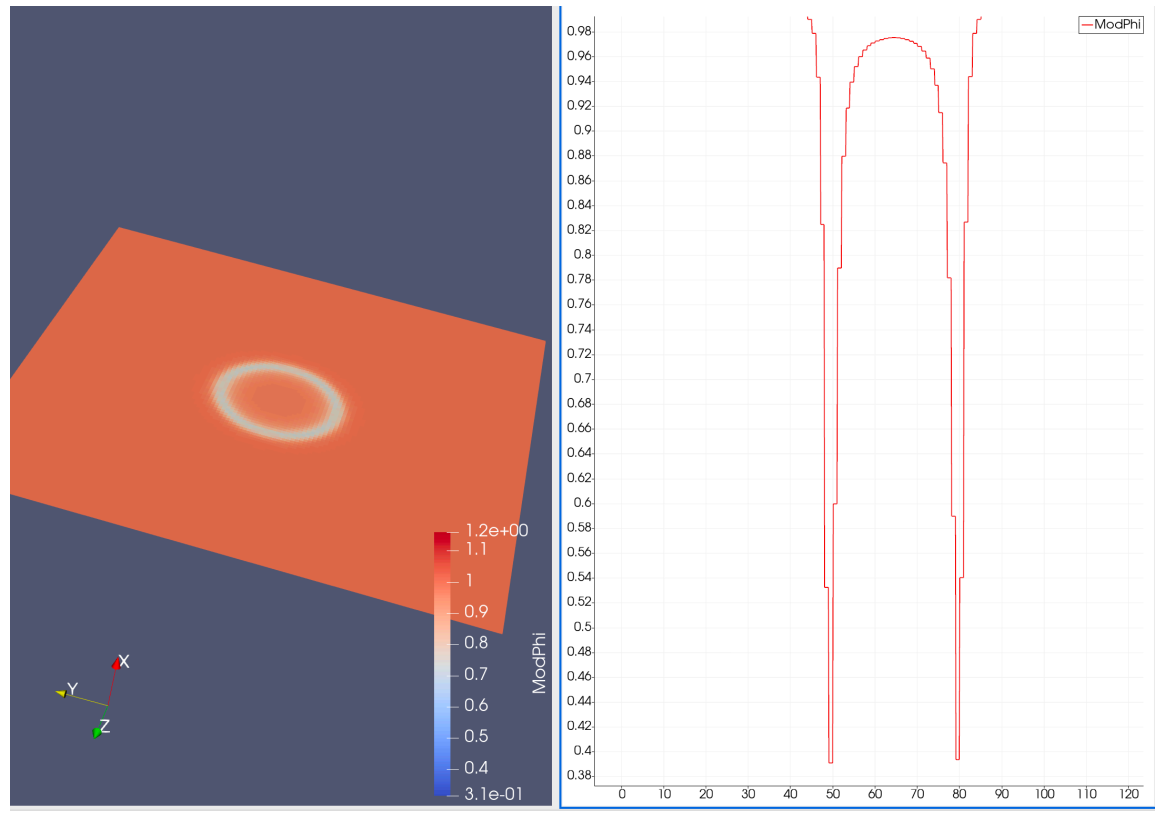
- We are running lots of simulations with different individual string configurations and try to obtain their radiation spectrum.

- We use both static-grid and **Adaptive Mesh Refinement** simulations and try to directly sync and compare results from different codes to better understand the discrepancies. (COSMIC WISPERS CA21106 STSM with Amelia Drew in Cambridge)

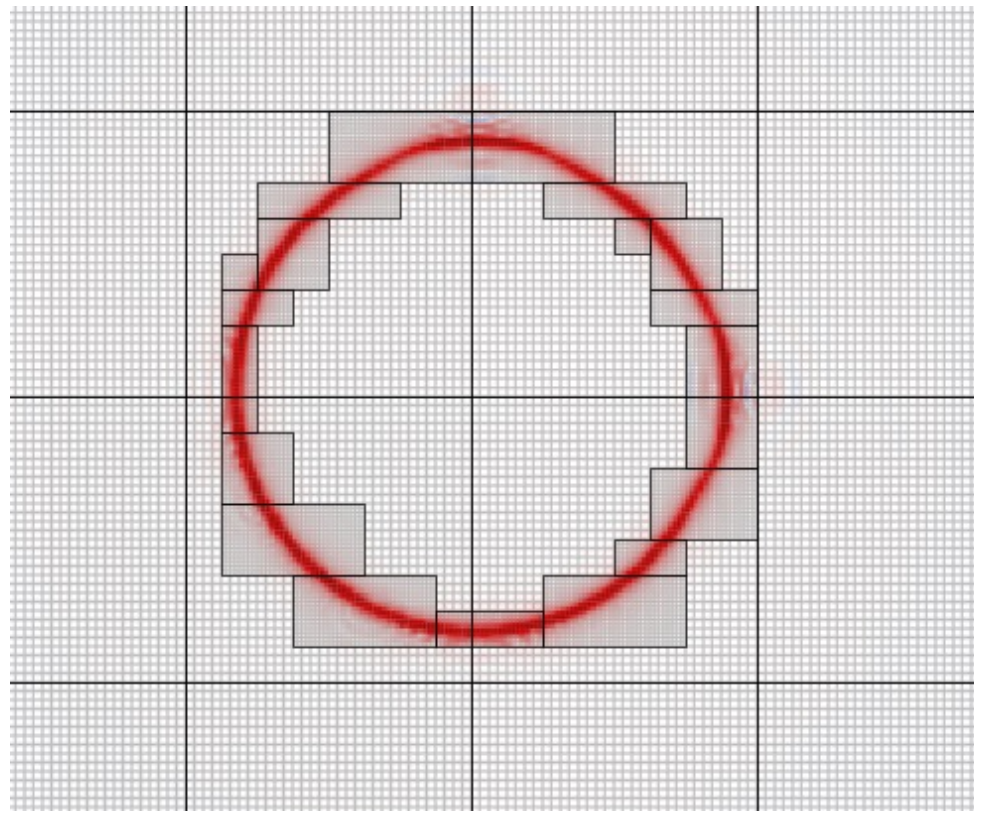
- The goal is to understand the contributions from the constituents of the string network in more detail.



Jaxions ICs in Chombo



Visualisation with ParaView



axioNyx AMR Code

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Thank you for your attention! Any questions?

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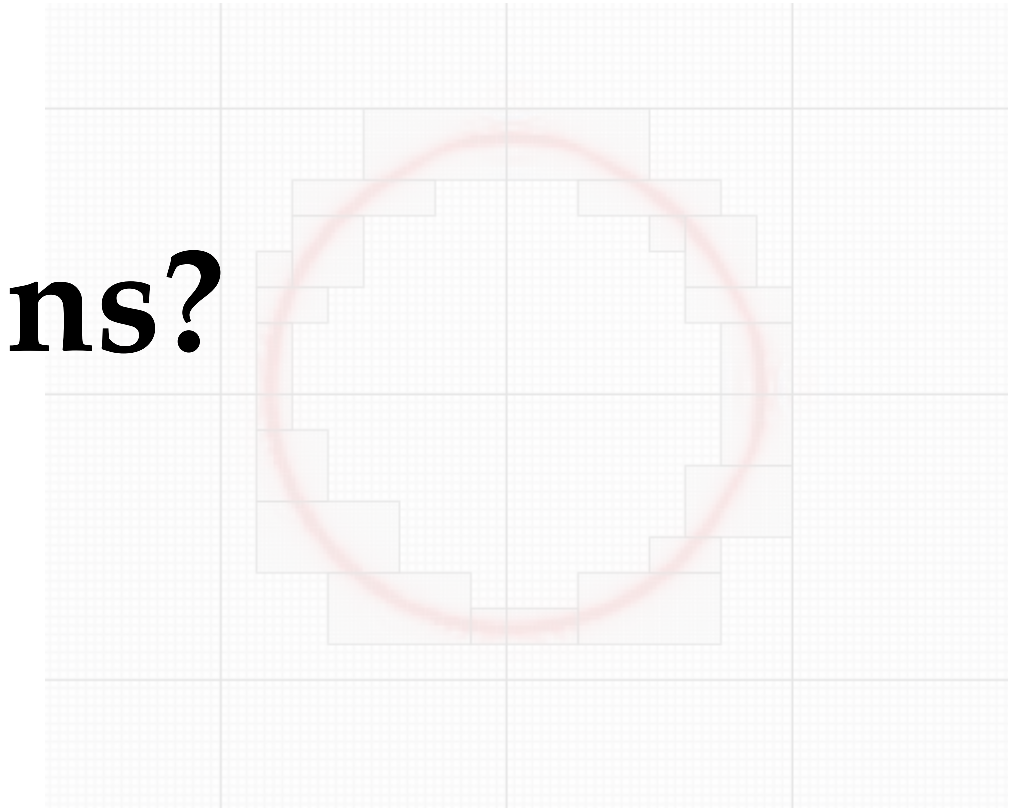
Jaxions ICs in Chombo

Visualisation with ParaView



axioNyx AMR Code

Backup Slides

General

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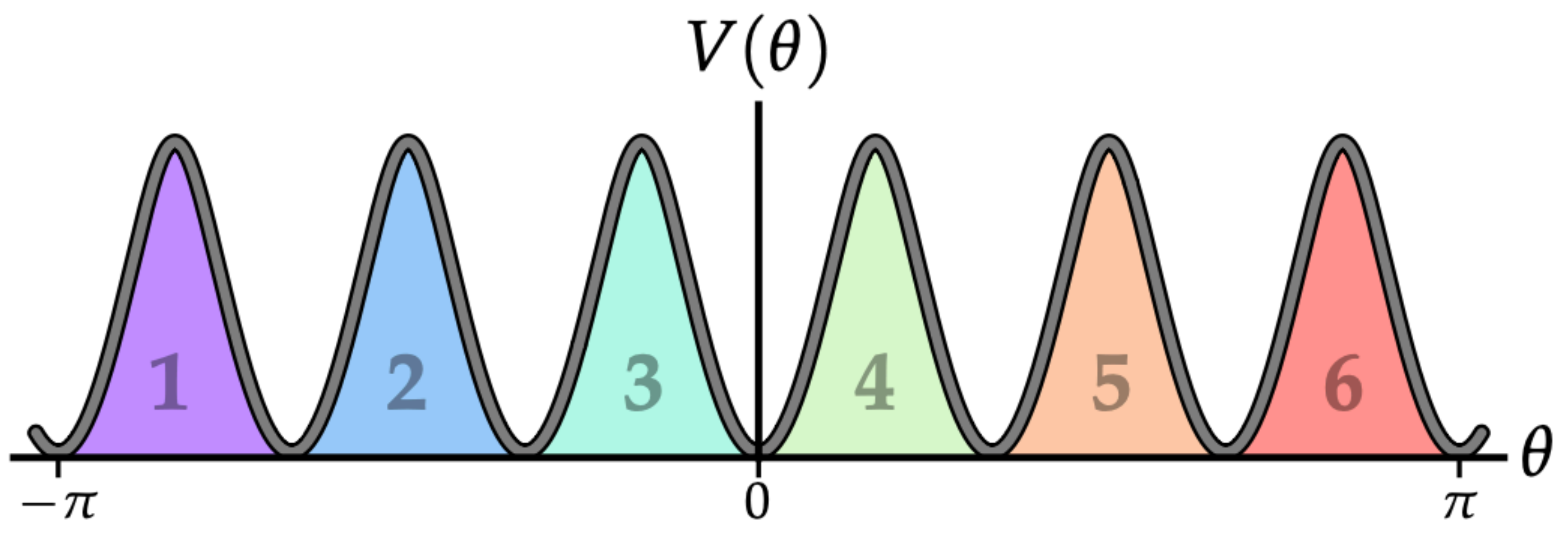
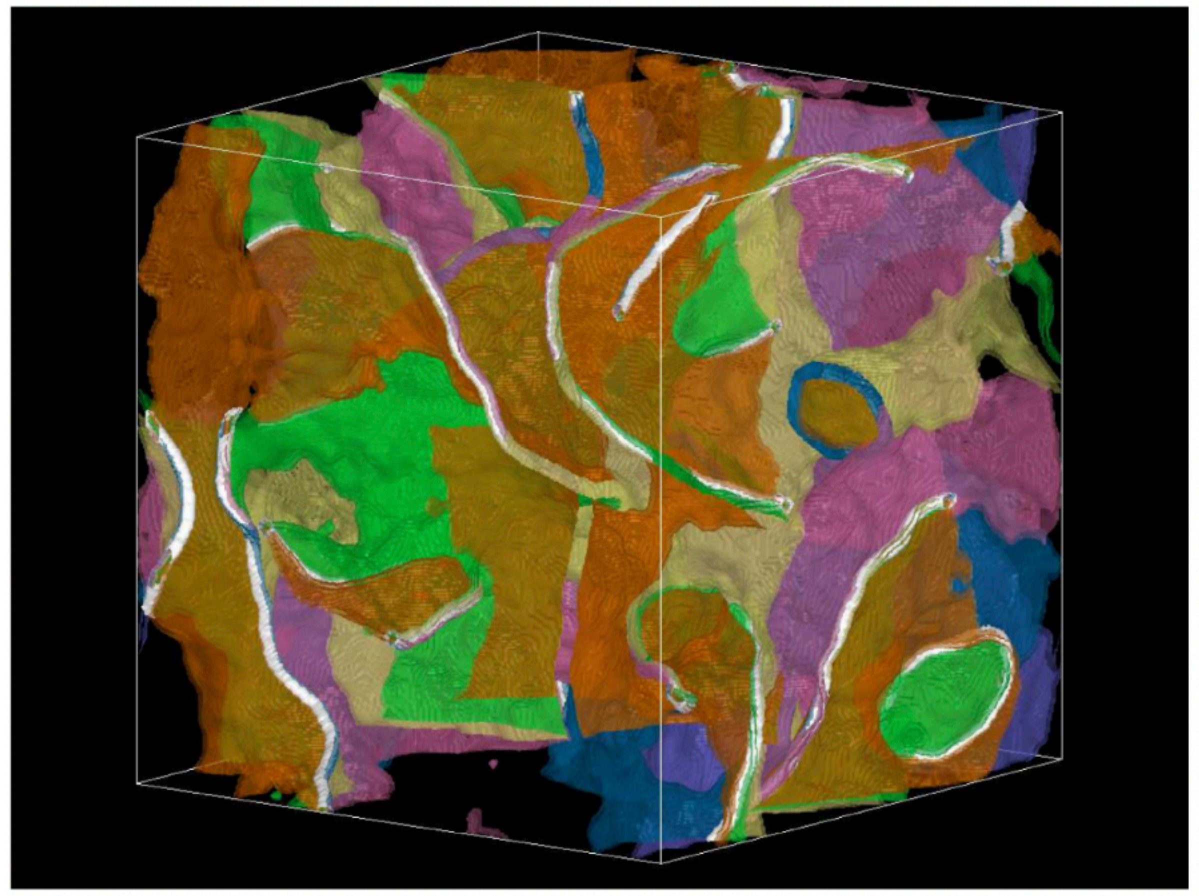
$N_{\rm DW} > 1$: Axion Domain Wall Problem

- Axion cycles around $N_{\rm DW}$ times between $(-\pi, \pi)$
- In general we get more axions from wall decay, so preferred m_a is higher.
- Phenomenologically difficult. Domain wall network gets stuck and overwhelms the cosmic energy density.
- Must have some preferred minimum!

$$V(\theta) \approx -\chi(T)\cos(N_{\rm DW}\theta)$$





(e) $N_{\rm DW} = 6$

Hiramatsu+ [1207.3166]

More details on the recent Paper

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Simulation Overview

- More than 1500 simulations performed at
  - RAVEN and COBRA supercomputers at Max Planck Computing and Data Facility (MPCDF)
  - SQUID supercomputer at Cybermedia Center, Osaka University
- Box sizes of up to 11.264^3 (256 CPU nodes)

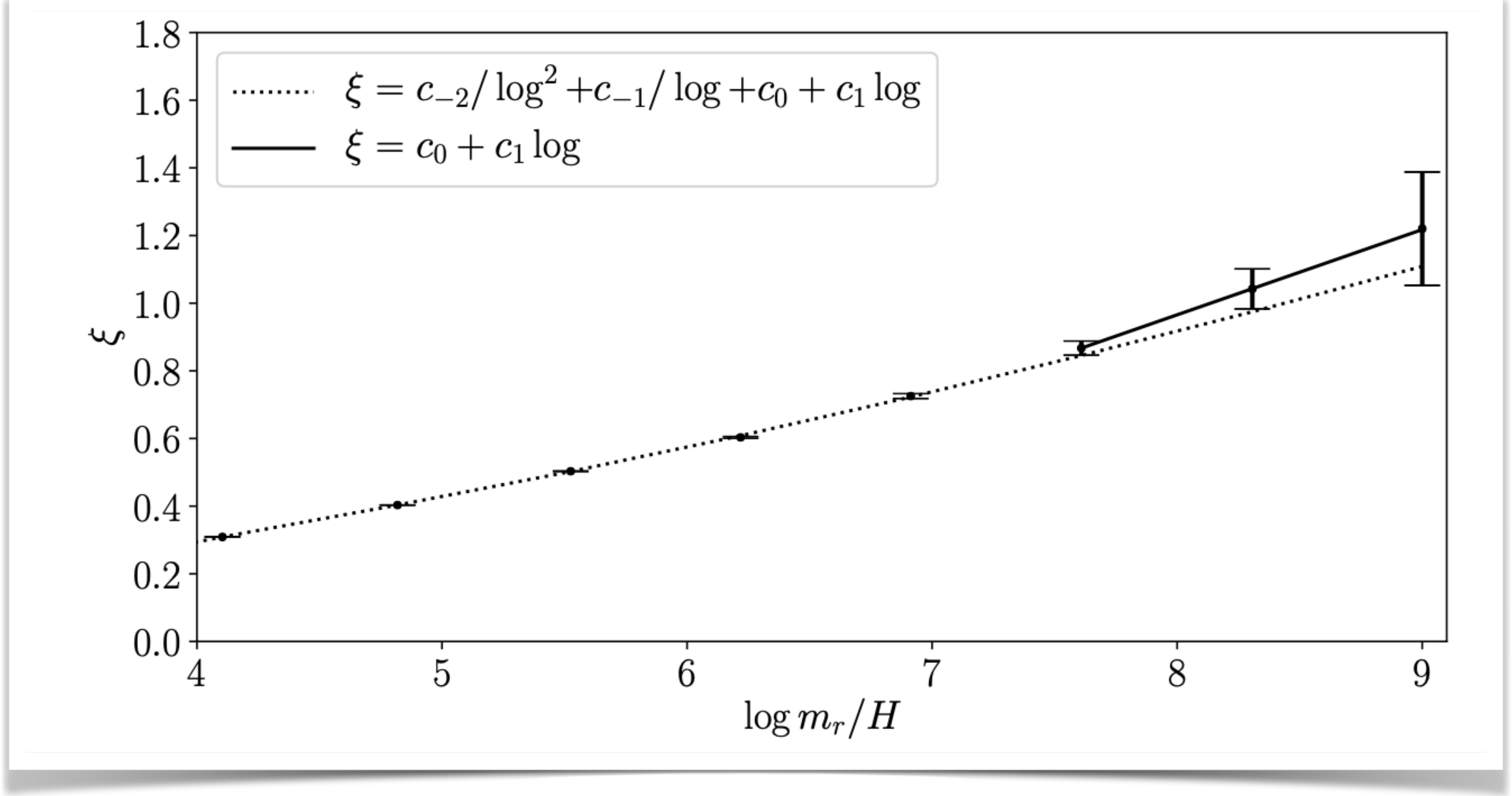
Type[a]	Grid size (N^3)	Laplacian	Final time (τ_f/L)	$\ln(m_r/H)$ at τ_f	Parameter	Number of simulations
Physical	11264^3	4-neighbours	0.625	9.08	$\bar{\lambda} = 195799$	20
Physical	4096^3	1-neighbour	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	4096^3	2-neighbours	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	4096^3	3-neighbours	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	4096^3	4-neighbours	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	3072^3	4-neighbours	0.5	7.34	$\bar{\lambda} = 14563.6$	30
Physical	3072^3	4-neighbours	0.5	7.74	$\bar{\lambda} = 32768$	30
Physical	3072^3	4-neighbours	0.5	8.08	$\bar{\lambda} = 64225.3$	30
Physical	3072^3	4-neighbours	0.5	8.37	$\bar{\lambda} = 114178$	30
Physical	2048^3	4-neighbours	0.55	7.12	$\bar{\lambda} = 6400$	30×30[b]
Physical	1024^3	4-neighbours	0.5	6.23	$\bar{\lambda} = 1600$	30×4[c]
Physical	3072^3	4-neighbours	0.458367	7.5	$\bar{\lambda} = 28571.2$	30
Physical	2560^3	4-neighbours	0.550042	7.5	$\bar{\lambda} = 13778.5$	30
Physical	2048^3	4-neighbours	0.687552	7.5	$\bar{\lambda} = 5643.68$	30
Physical	1536^3	4-neighbours	0.916735	7.5	$\bar{\lambda} = 1785.69$	30
Physical	1024^3	4-neighbours	1.3751	7.5	$\bar{\lambda} = 352.73$	30
PRS	8192^3	4-neighbours	0.55	6.80	$m_r a = 0.2$	20
PRS	8192^3	4-neighbours	0.55	7.21	$m_r a = 0.3$	20
PRS	8192^3	4-neighbours	0.55	7.72	$m_r a = 0.5$	20
PRS	8192^3	4-neighbours	0.55	8.06	$m_r a = 0.7$	20
PRS	8192^3	4-neighbours	0.55	8.41	$m_r a = 1.0$	20
PRS	8192^3	4-neighbours	0.55	8.82	$m_r a = 1.5$	20
PRS	4096^3	4-neighbours	0.55	7.72	$m_r a = 1.0$	30
PRS	2048^3	4-neighbours	0.55	7.03	$m_r a = 1.0$	30
PRS	1024^3	4-neighbours	0.55	6.33	$m_r a = 1.0$	30
PRS	2048^3	4-neighbours	0.5	6.93	$m_r a = 1.0$	1

Logarithmic Growth of String Density

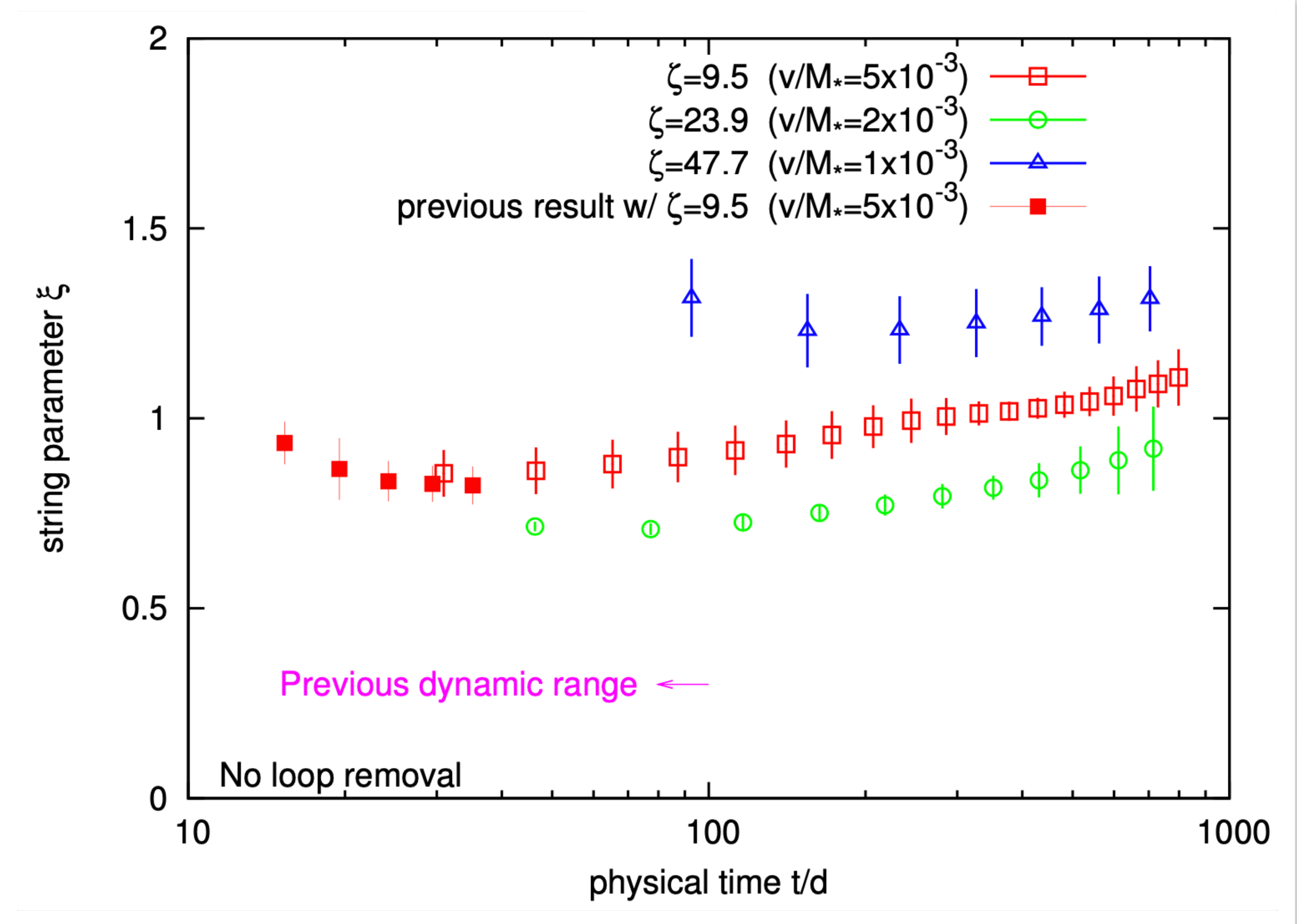
- String density parametrised as

$$\xi = \frac{l_{\rm string} t^2}{\mathcal{V}} = \frac{\rho_{\rm string} t^2}{\mu}$$

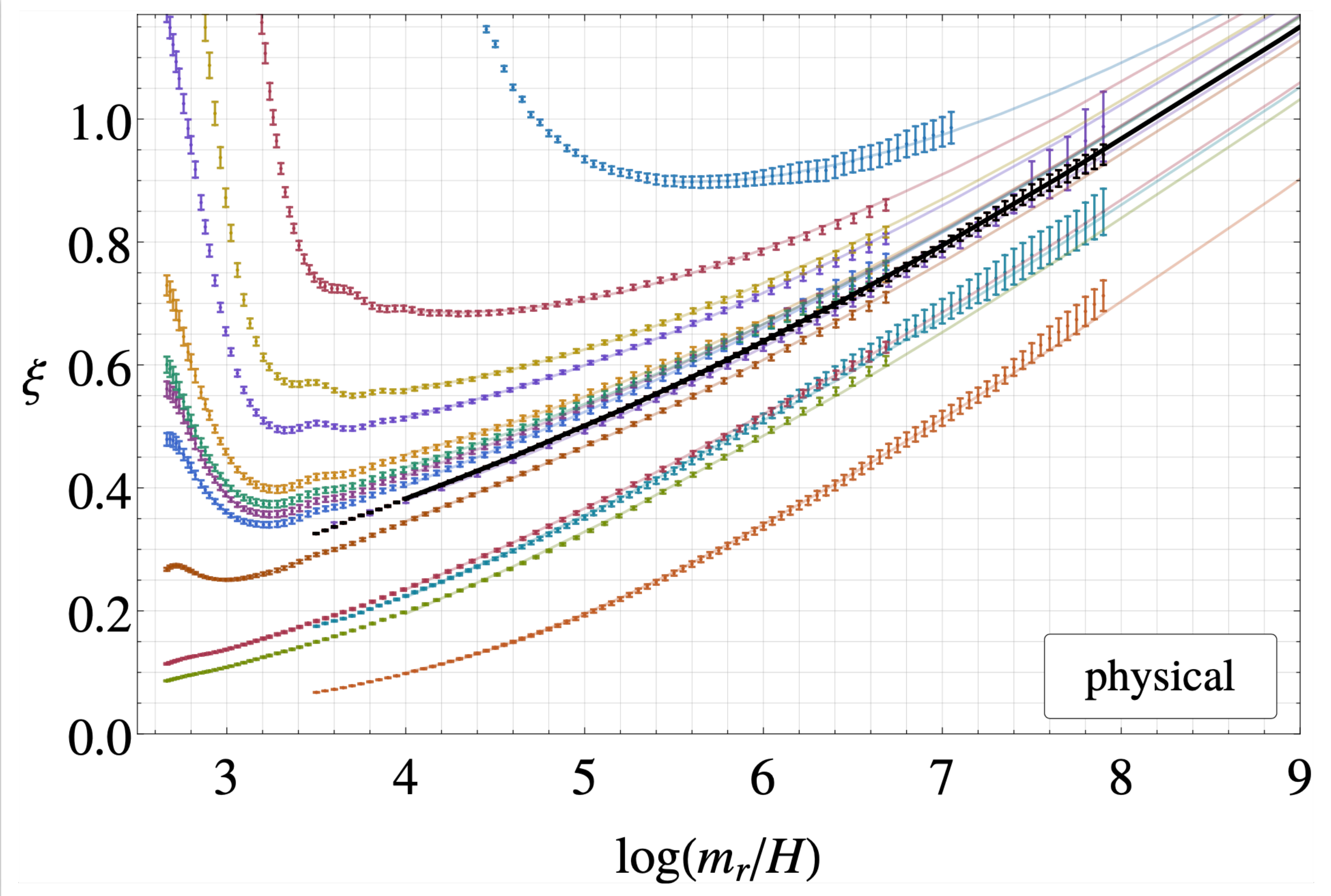
- Most recent simulations observe increase in ξ.



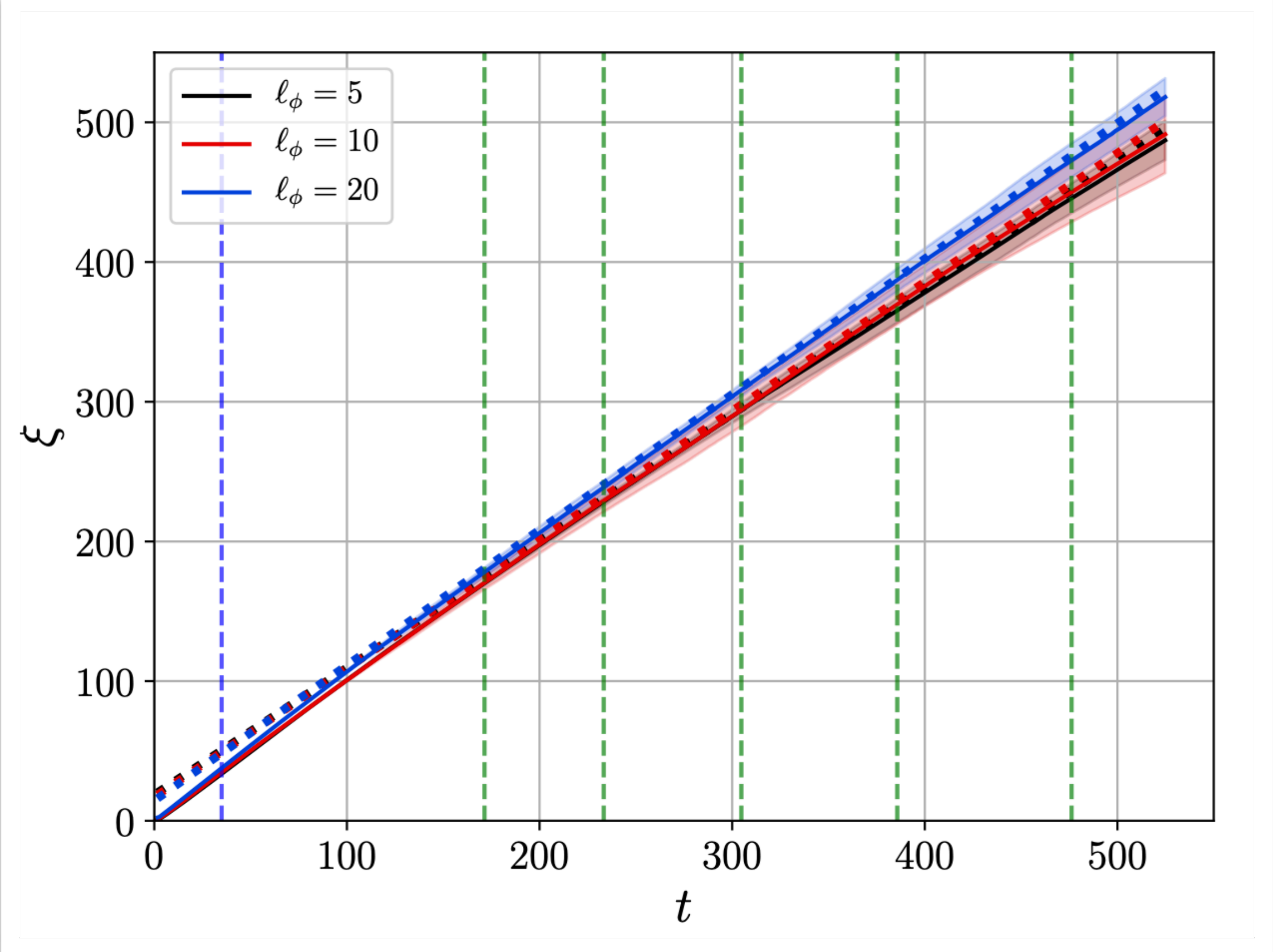
Buschmann+ [2108.05368]



Kawasaki+ [1806.05566]



Gorghetto+ [2007.04990]

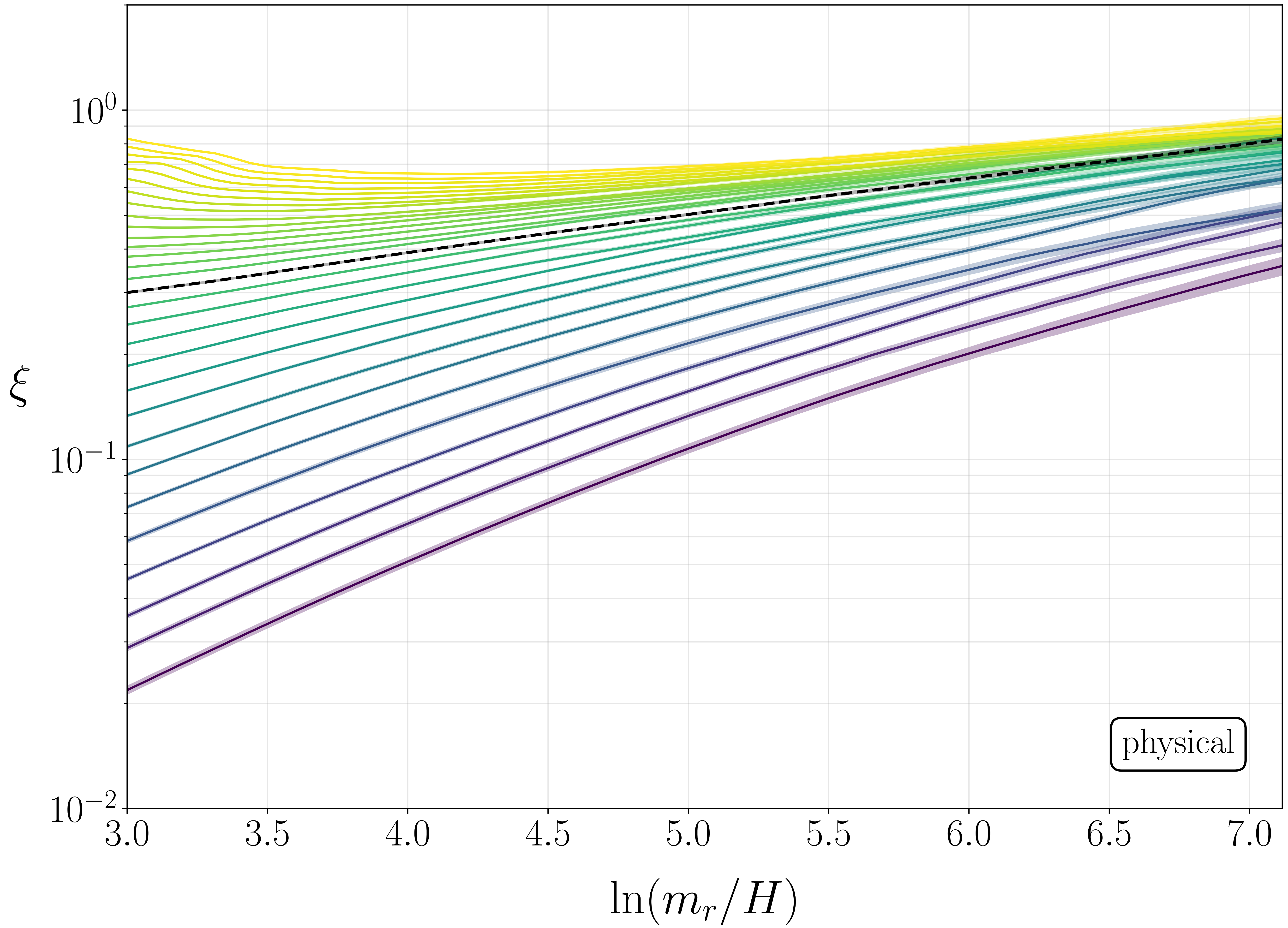


Hindmarsh+ [1908.03522]

Evolution of String Density

- Model evolution of string network density: $\frac{d\xi}{dt} = \frac{C(x)}{t}(\xi_c(\ell(t)) - \xi(t))$

Klaer & Moore [1912.08058]



Two reasonable fits to the data:

$$\xi^{\rm lin} = -0.19(3) + 0.205(7)\log(m_r/H)$$

$$\xi^{\rm sat} = \frac{2.5(1.4) + 0.23(6)\log(m_r/H)}{1 + 0.02(4)\log(m_r/H)}$$

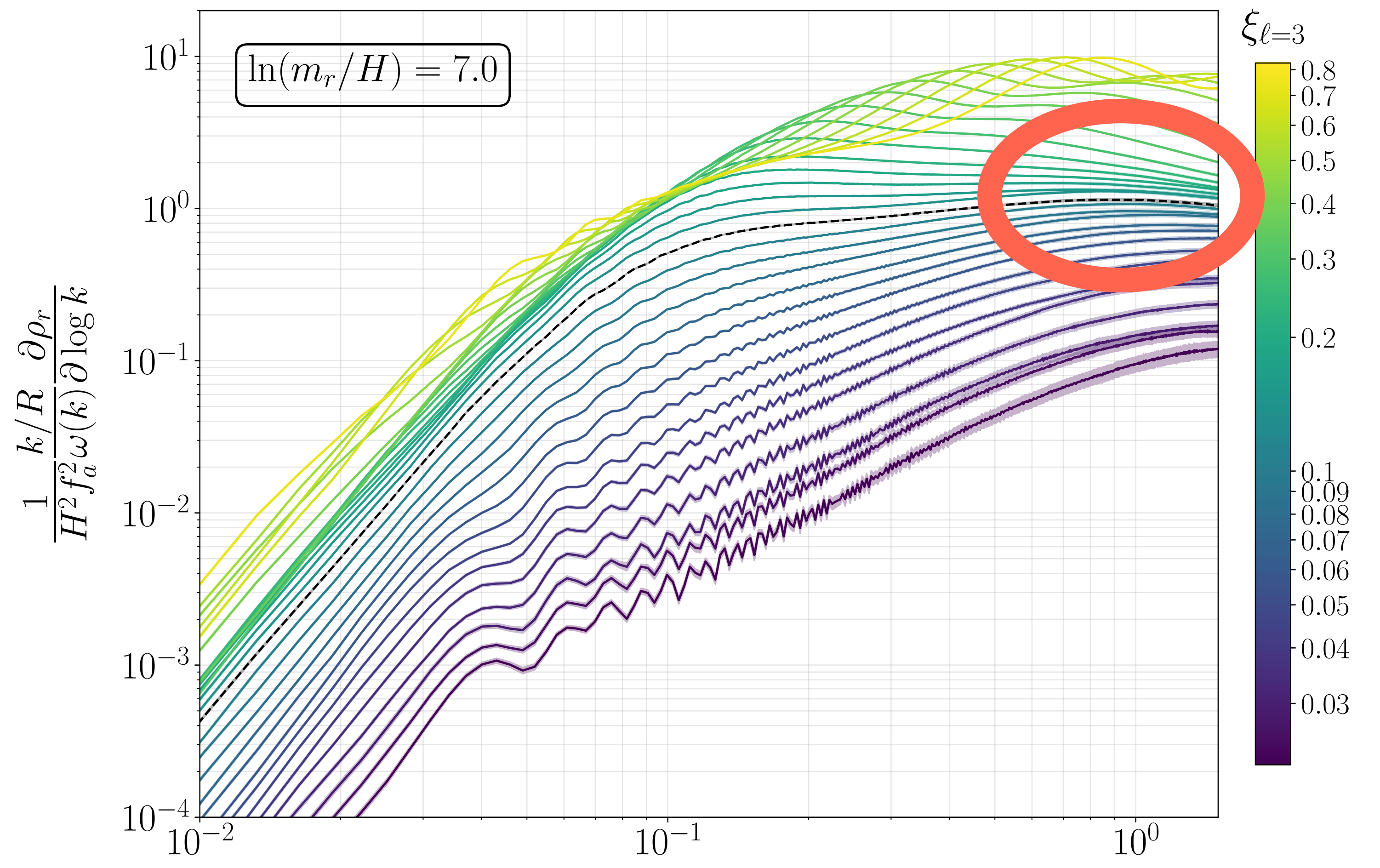
with

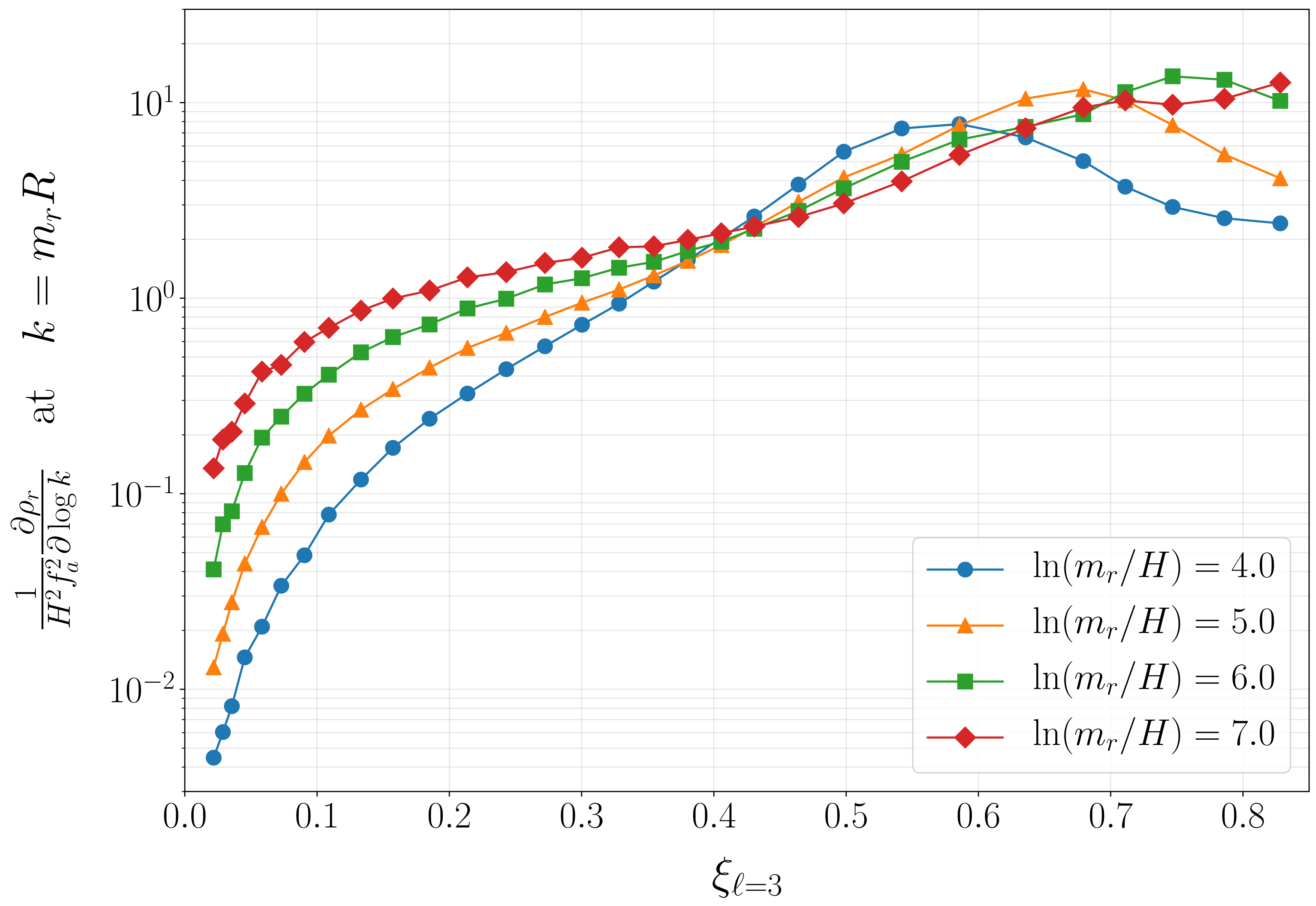
$$\xi^{\rm lin}(\log = 70) \sim 13.8(5)$$

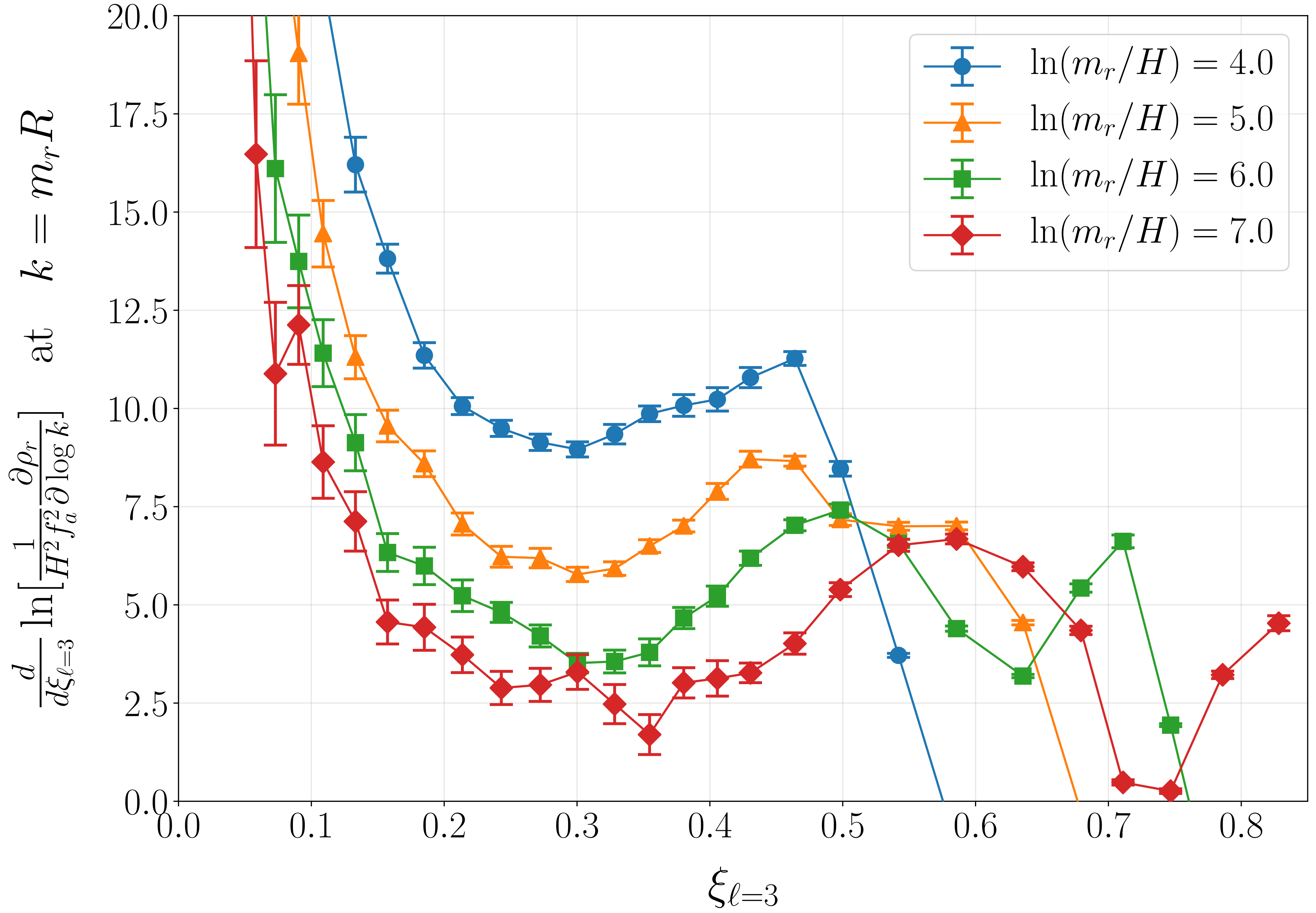
$$\xi^{\rm sat}(\log = 70) \sim 7(3)$$

Existence of an Attractor?

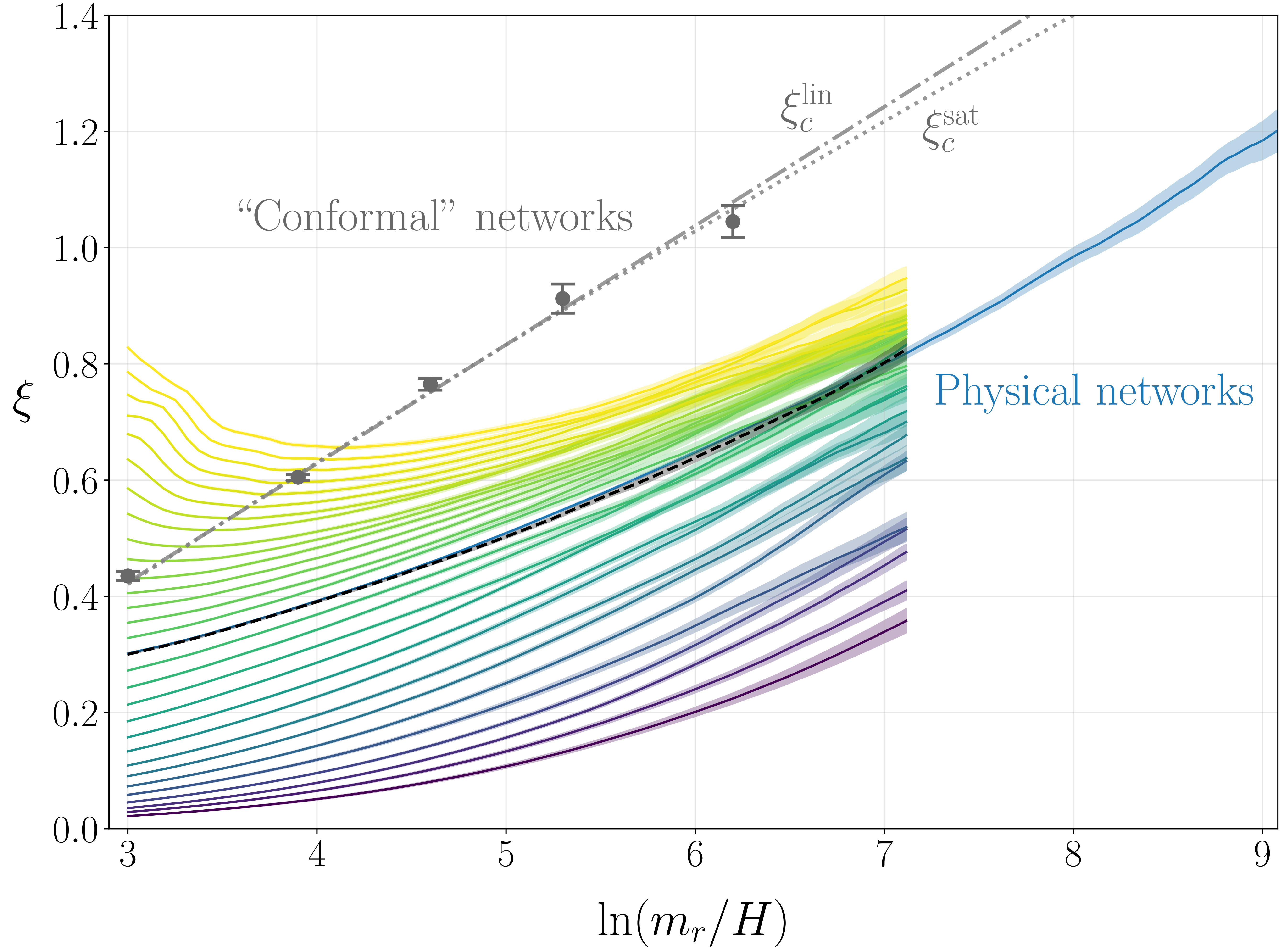
- Convergence behaviour is best observed in radial field spectra.
- Saxions are produced at $k/R \sim m_r$
- Look for point that is least sensitive to ICs.







Evolution of String Density



Fleury & Moore [1509.00026]
Gorghetto+ [1806.04677; 2007.04990]
Kawasaki+ [1806.05566]
Hindmarsh+ [1908.03522]
Buschmann+ [2108.05368]

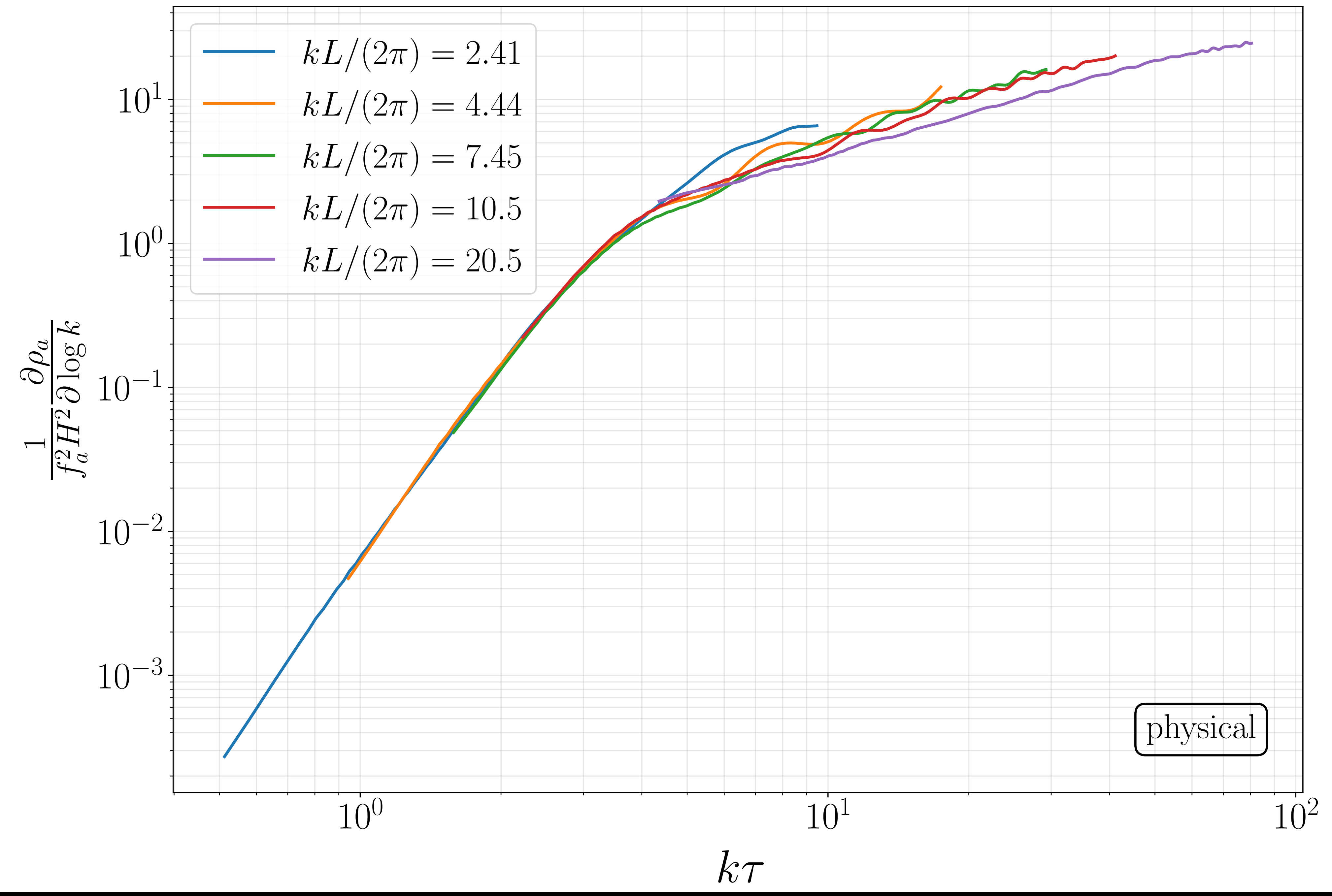
Logarithmic growth and "attractor" behaviour compatible with previous findings.

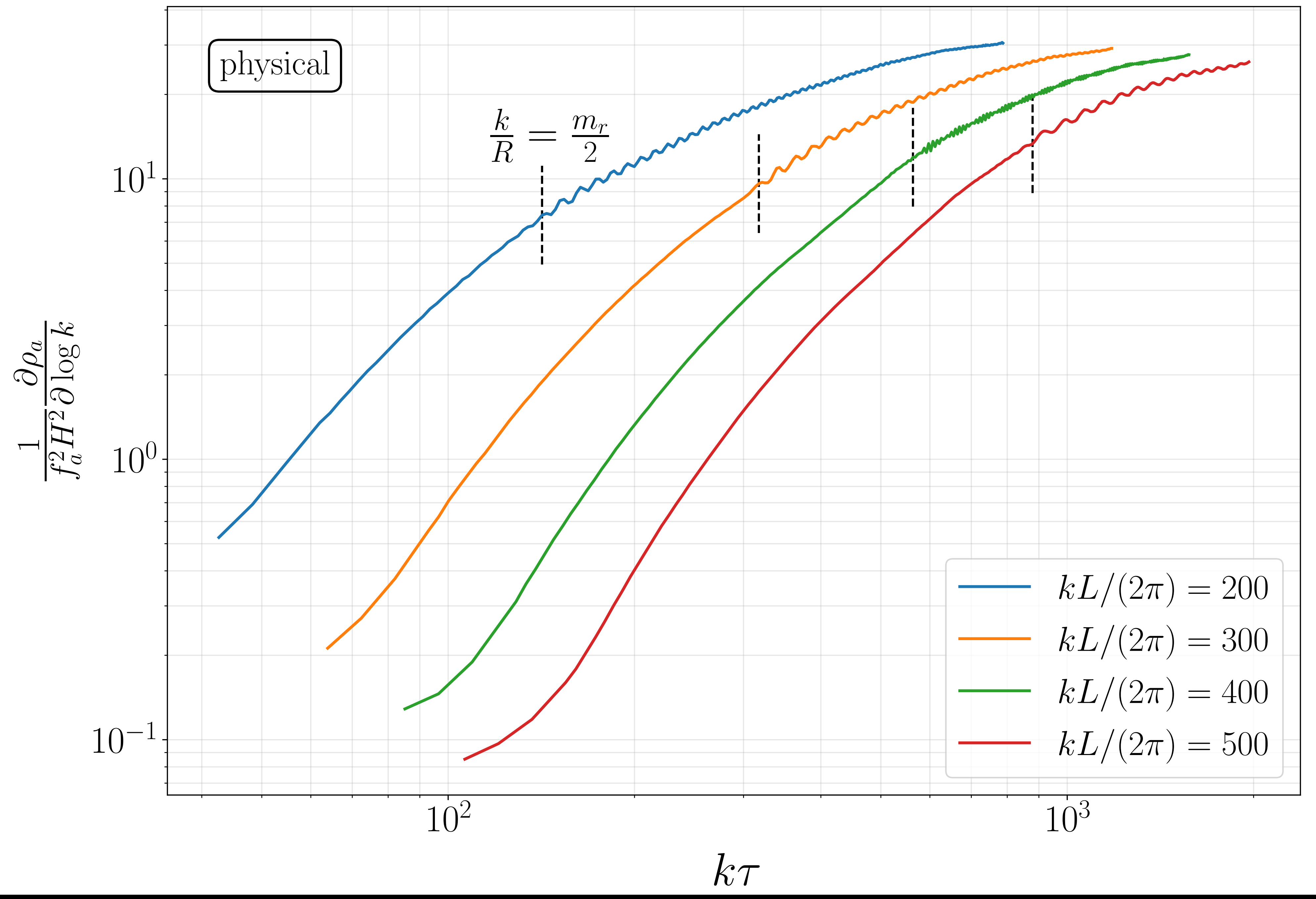
Axion Mode Evolution

- To calculate the differential spectrum, we need to know the time evolution of one mode:

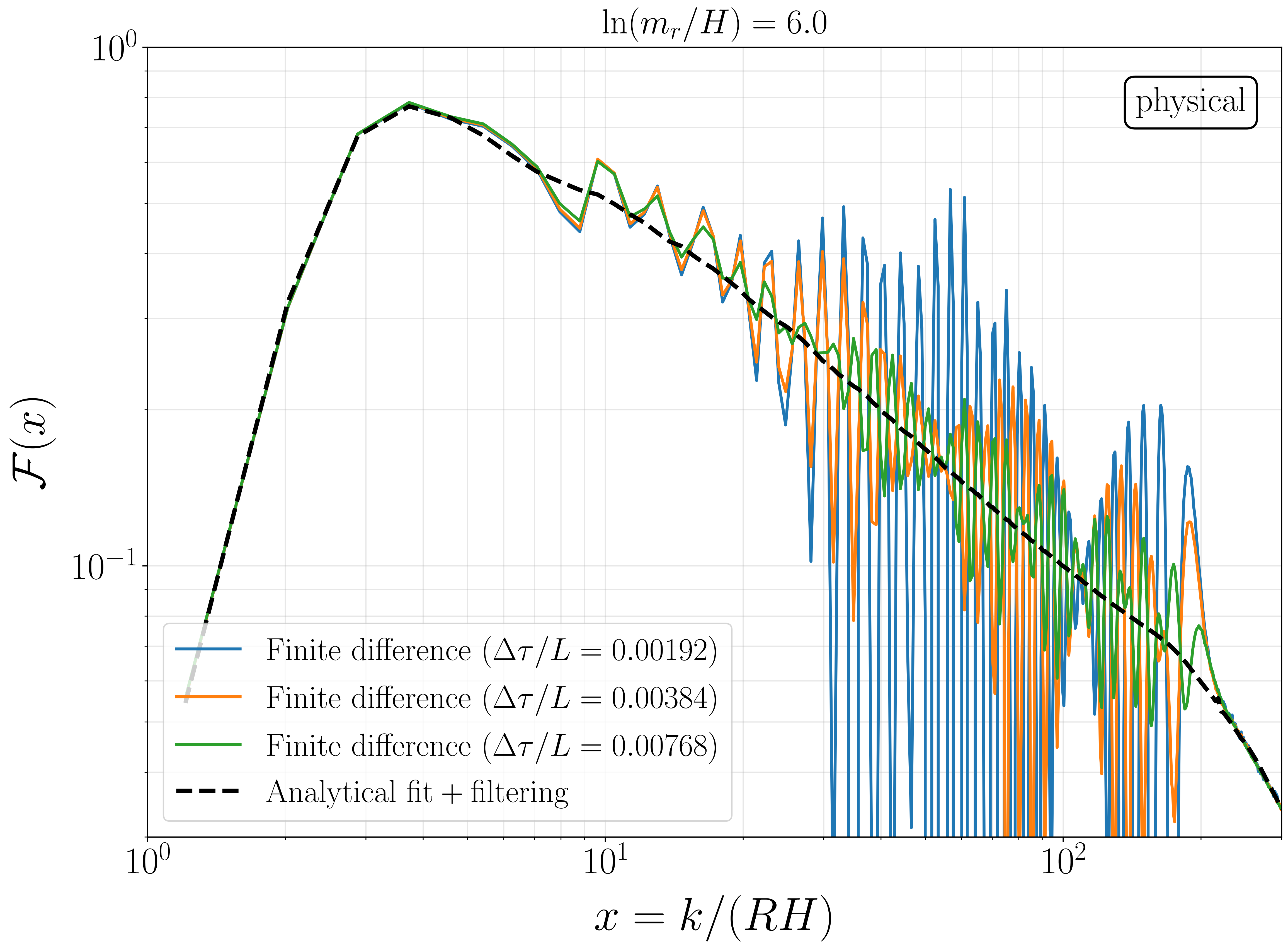
$$\mathcal{F} = \frac{1}{(f_a H)^2} \frac{1}{R^3} \frac{\partial}{\partial t} \left(R^4 \frac{\partial \rho_a}{\partial k} \right)$$

- Contains oscillating components with frequency $\sim 2k$, interpreted as axion field oscillations after the horizon entry or production from the radial field.





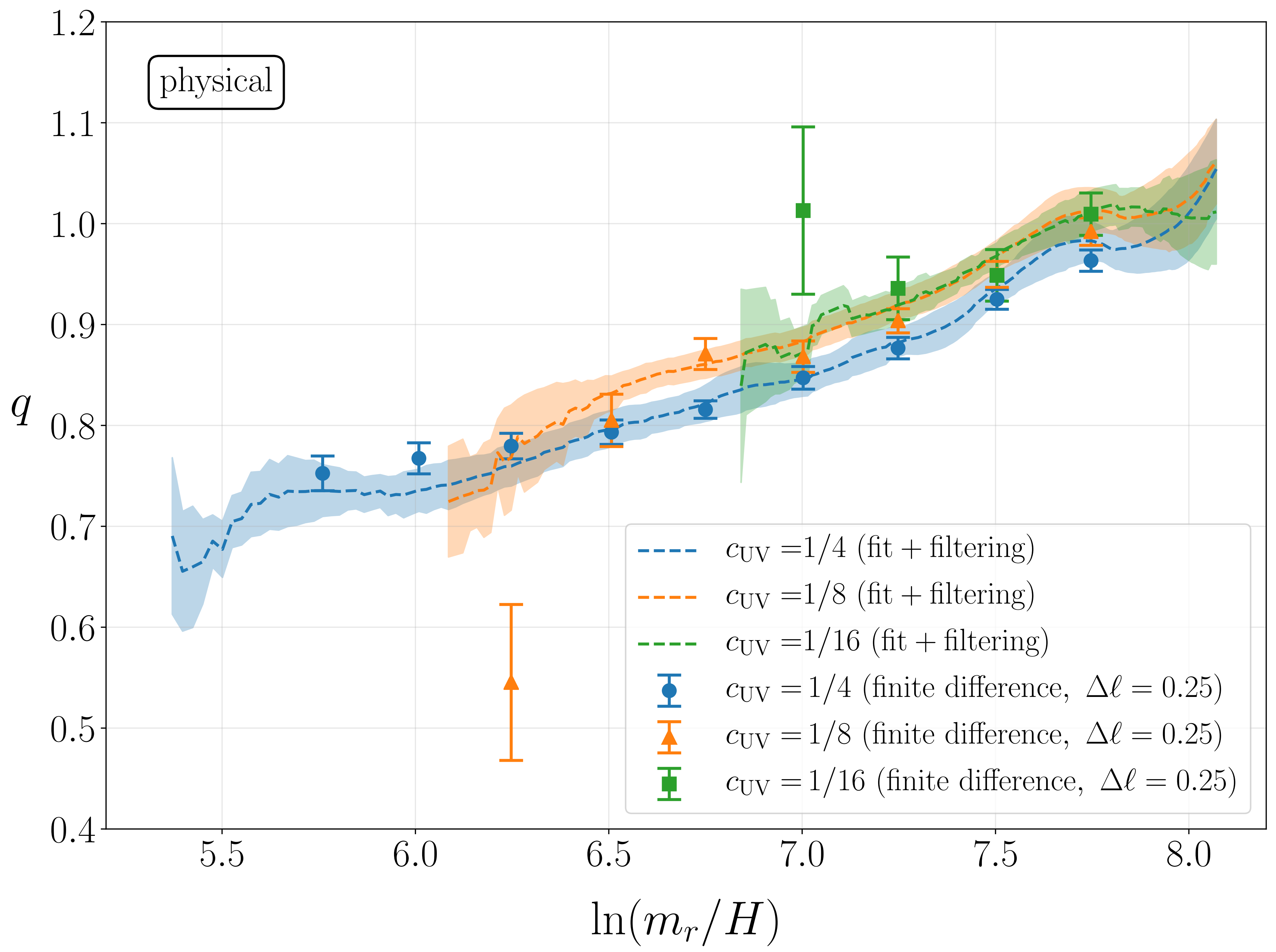
Calculation of the Instantaneous Spectrum



- Simple finite difference leads to a lot of contaminations from axion field oscillations.
- One can reduce them by applying a filter to remove high frequency components in the mode evolution data.

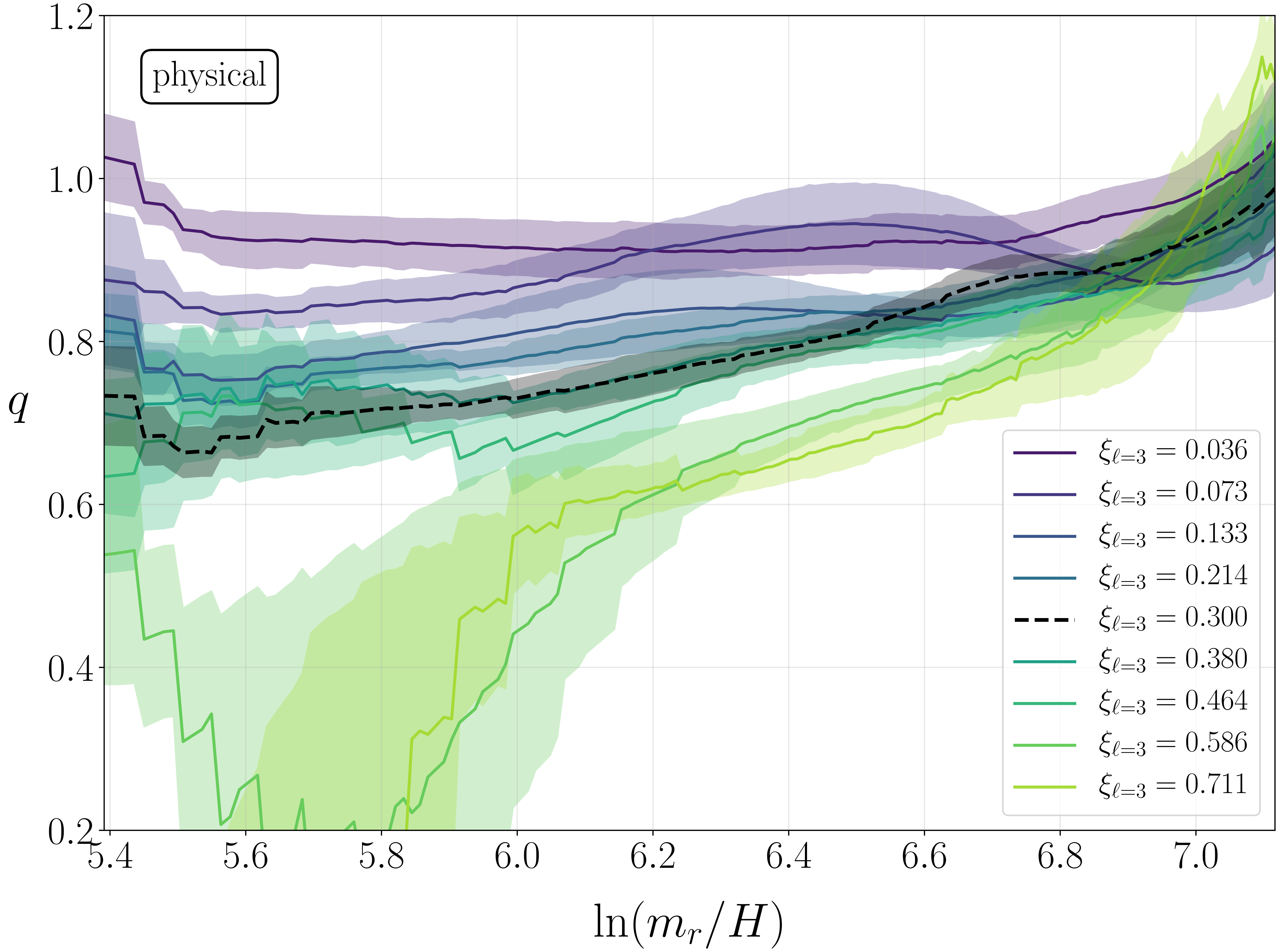
Axion Field Oscillations

- Fit power law $\mathcal{F} \sim k^{-q}$ to the data in the range $c_{\rm IR} H < k/R < c_{\rm UV} m_r$



- The oscillations in the IR modes have an impact on the measurement of q.
- The effect can be alleviated by taking a broader range for the fit.

Initial Conditions

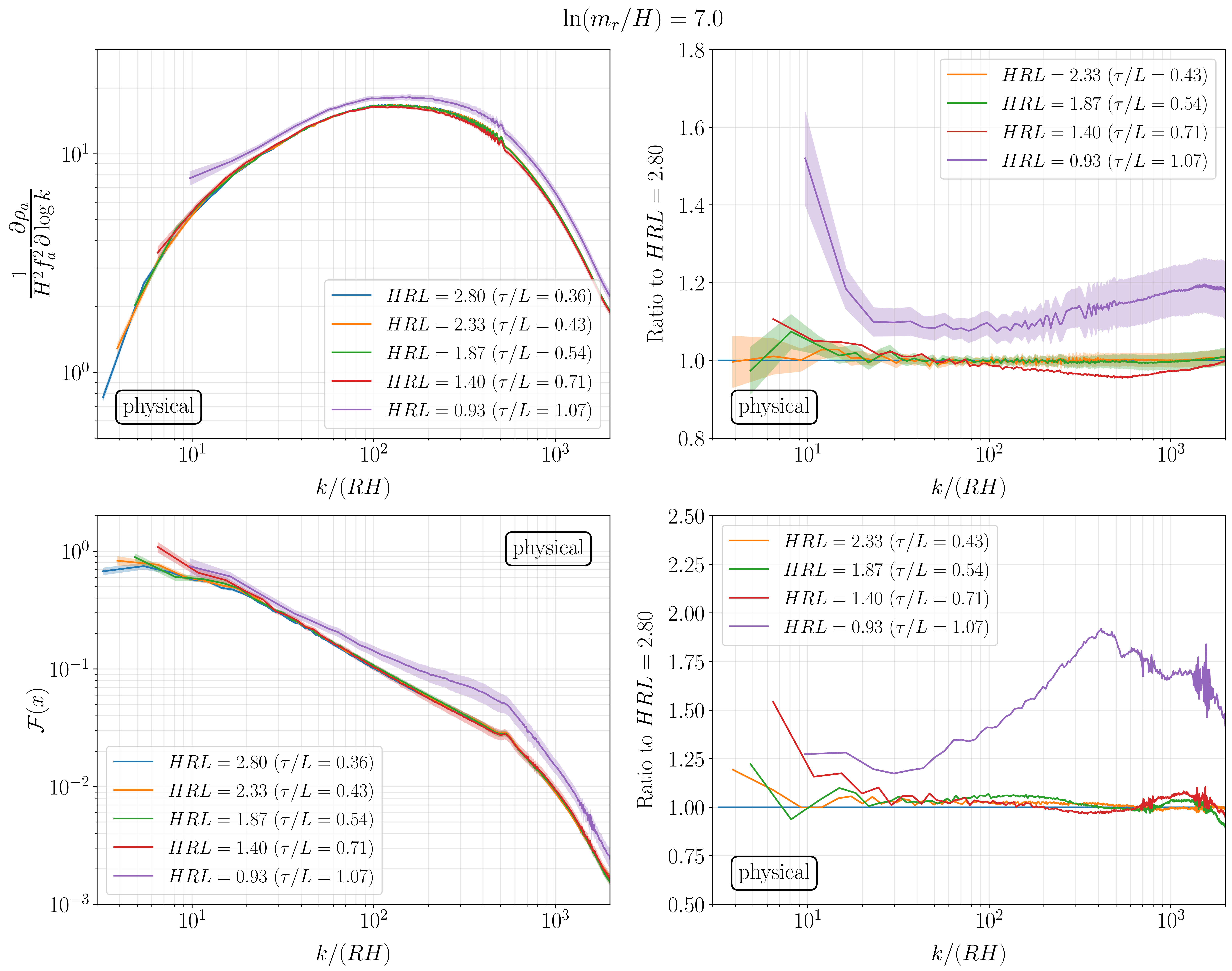


- Differences in the initial string density affect the slope of the radiation spectrum.
- Overdense (underdense) initial conditions could bias the estimation of q towards lower (higher) values.

Finite Volume Effects

- Fix $m_r a = 1.0$ and vary ratio of phys. box size RL to Hubble radius H^{-1} at $\ln(m_r/H) = 7$
- Results converge for $HRL \gtrsim 1.4$ (or $\tau/L \lesssim 0.7$)
- We terminate the simulations at $\tau/L \leq 0.625$

Should **not** be a problem!



Technicalities

Masking the Spectrum

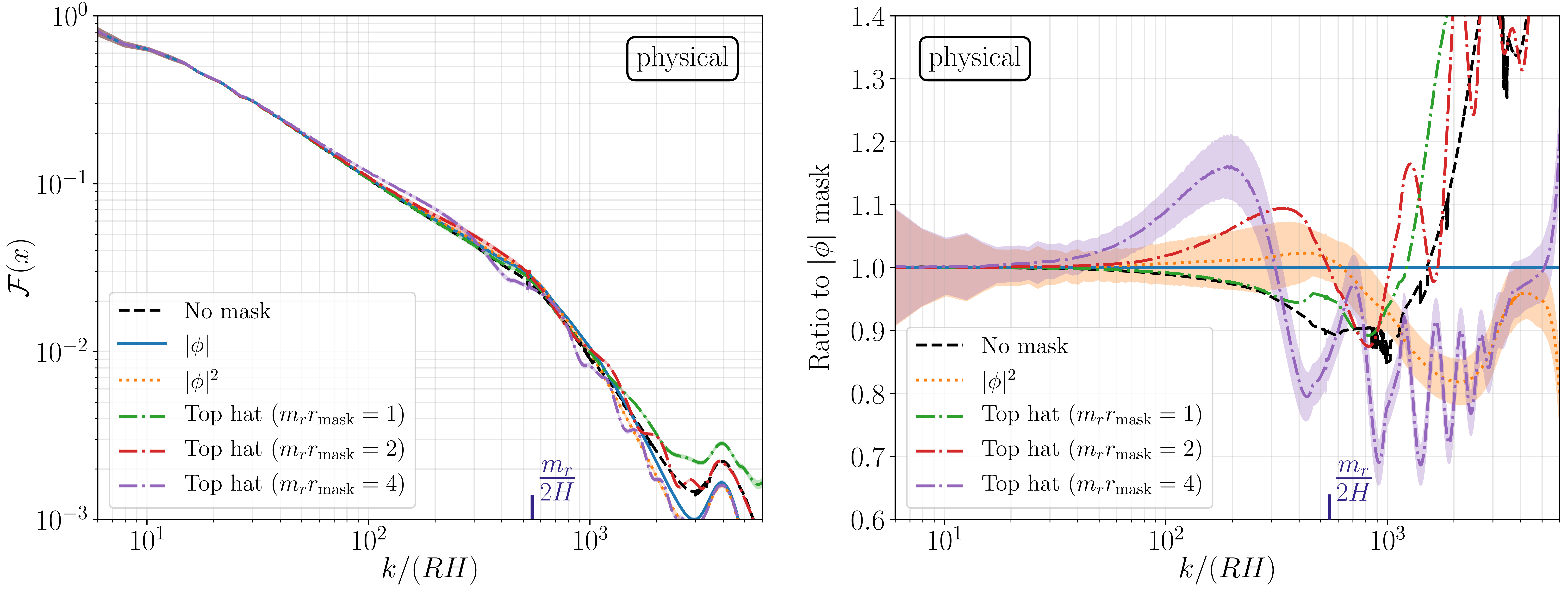
- Try to mitigate the contamination from the string core, we can introduce masks to to compute derivatives:

$$\dot{X}^{\text{mask}}(\boldsymbol{x}) = M(\boldsymbol{x})\dot{X}(\boldsymbol{x})$$

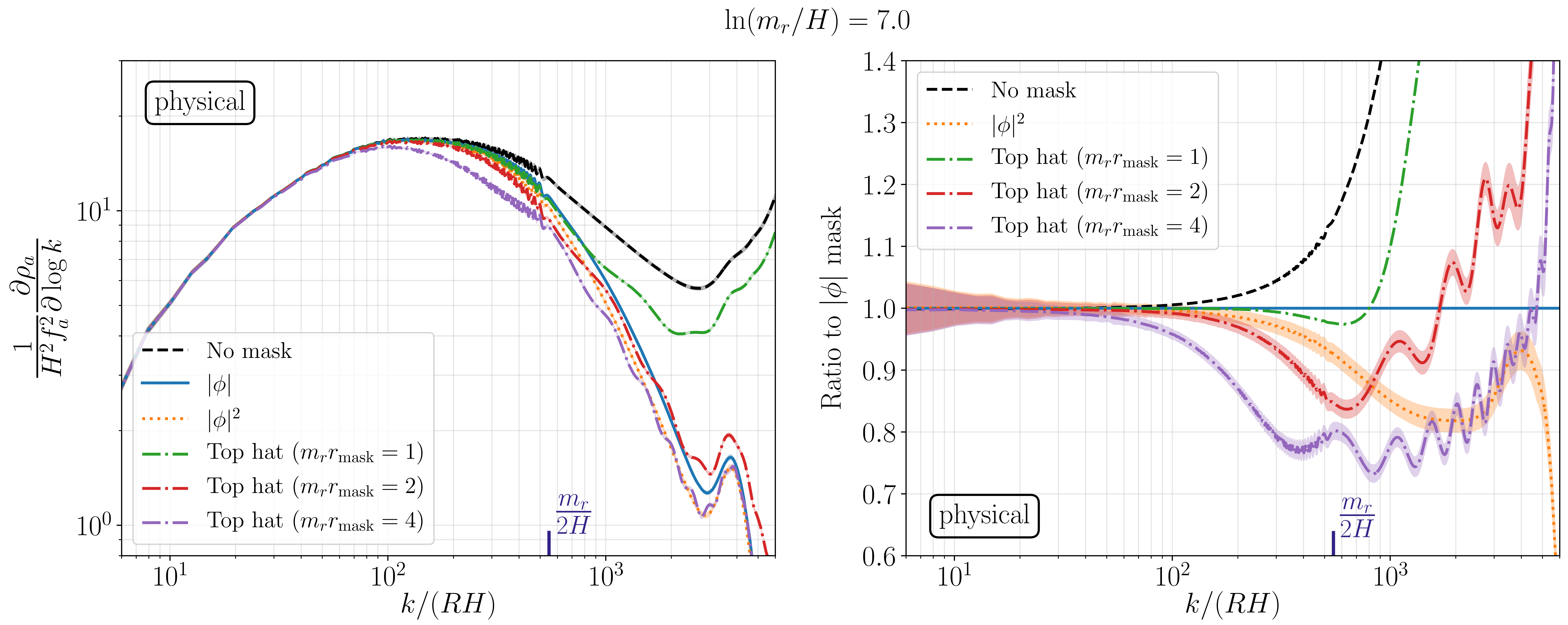
- Simple choice is to use the fact that the value of the radial field $|\phi|$ is zero inside the core.

$$M(x) = \left(\frac{|\phi(x)|}{f_a}\right)^k$$

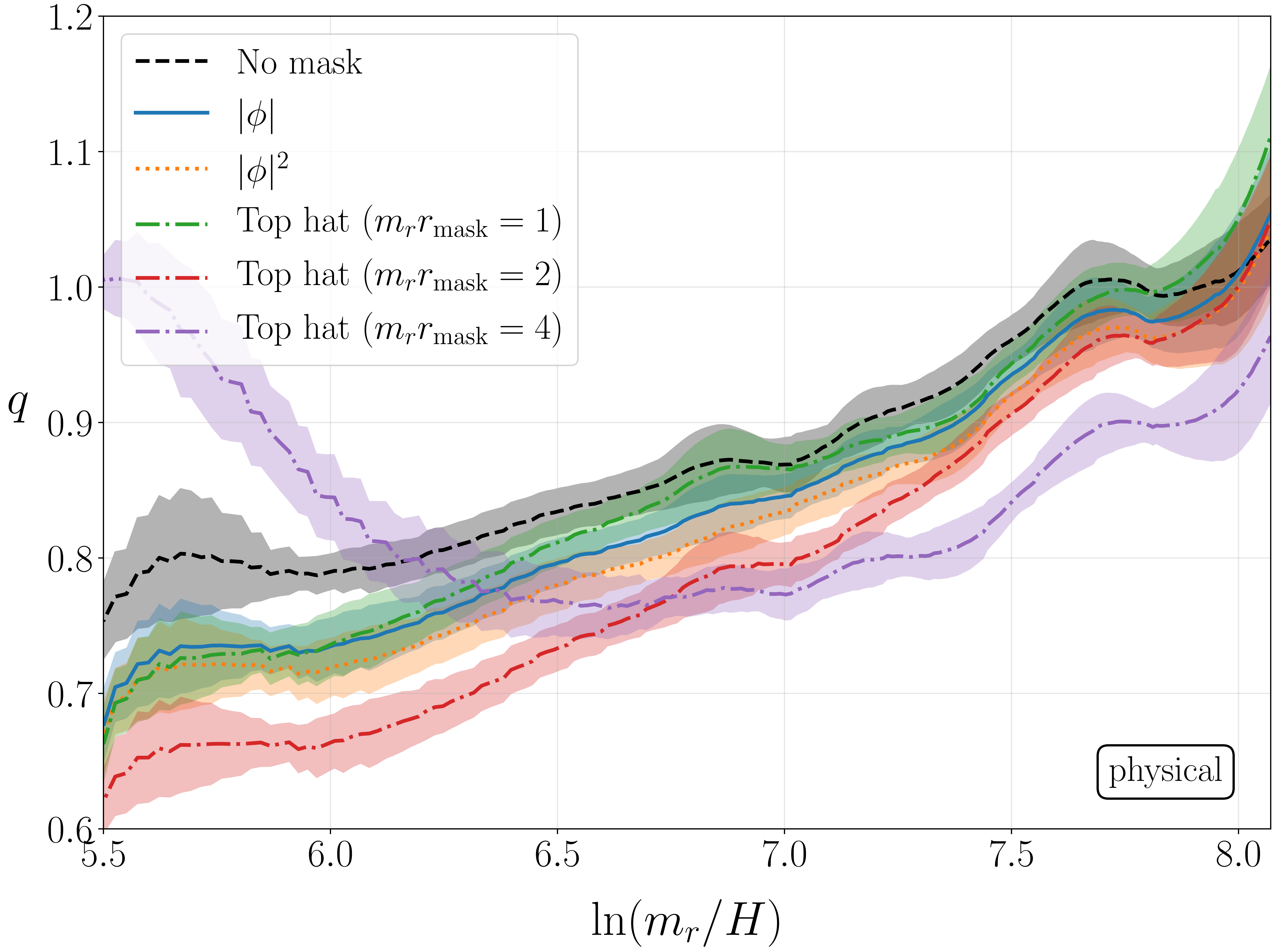
Masking the Spectrum



Masking the Spectrum



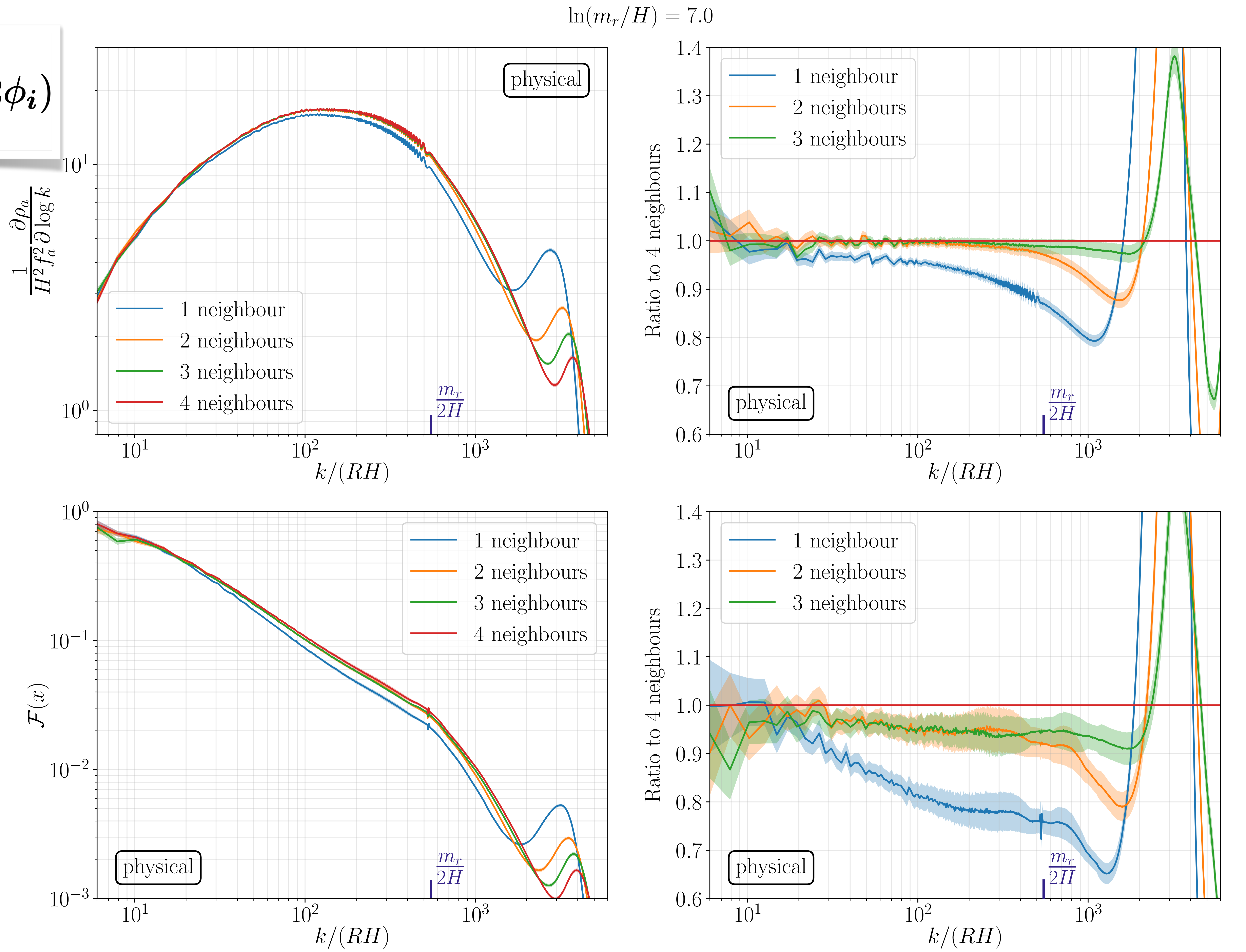
Masking the Spectrum



Discretisation of the Laplacian

$$(\nabla^2\phi)_{\boldsymbol{i}} = \frac{1}{\delta^2}\sum_{u=x,y,z}\sum_{n=1}^{N_g} C_n(\phi_{\boldsymbol{i}+n\boldsymbol{n}_u} + \phi_{\boldsymbol{i}-n\boldsymbol{n}_u} - 2\phi_{\boldsymbol{i}})$$

- Spectrum **underestimated** at intermediate momenta for smaller N_g
- Observation of peak-like structure in the UV, height related to N_g

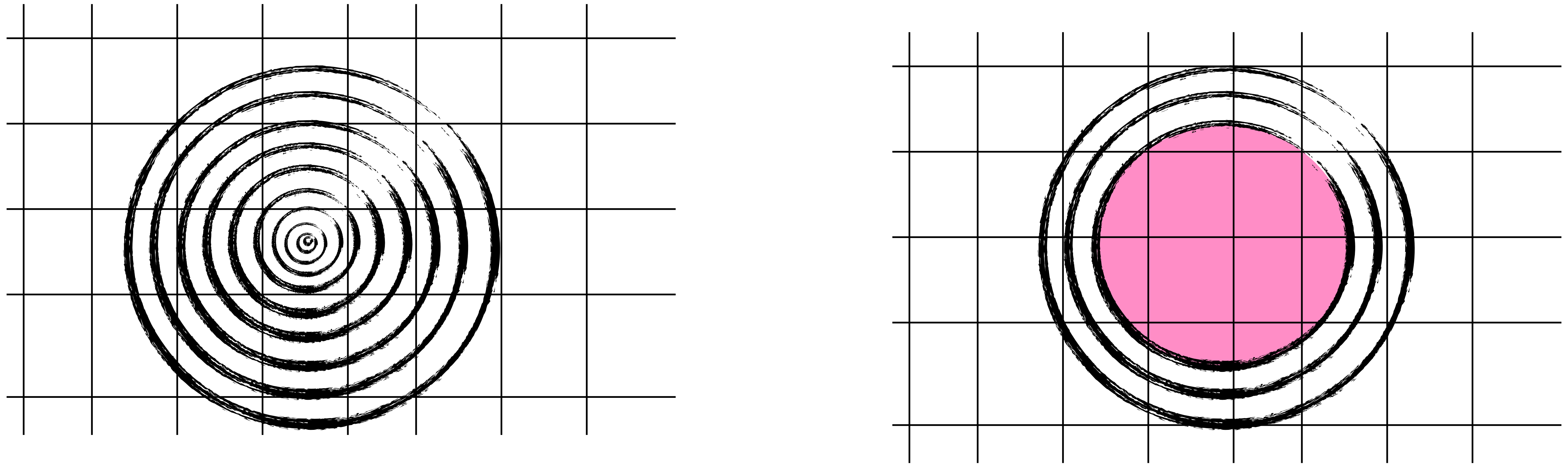


Dynamical Range (AMR)

How can we reach a larger dynamical range?

- **Brute Force**: Larger simulations on more powerful supercomputers
- **Better:** Use the given computational power more efficiently: **AMR!**
- **In addition:** Study effective models that allow us to study the network dynamics at high tension (**Moore strings**) with 2+3 extra degrees of freedom (two additional complex scalars + one vector field)

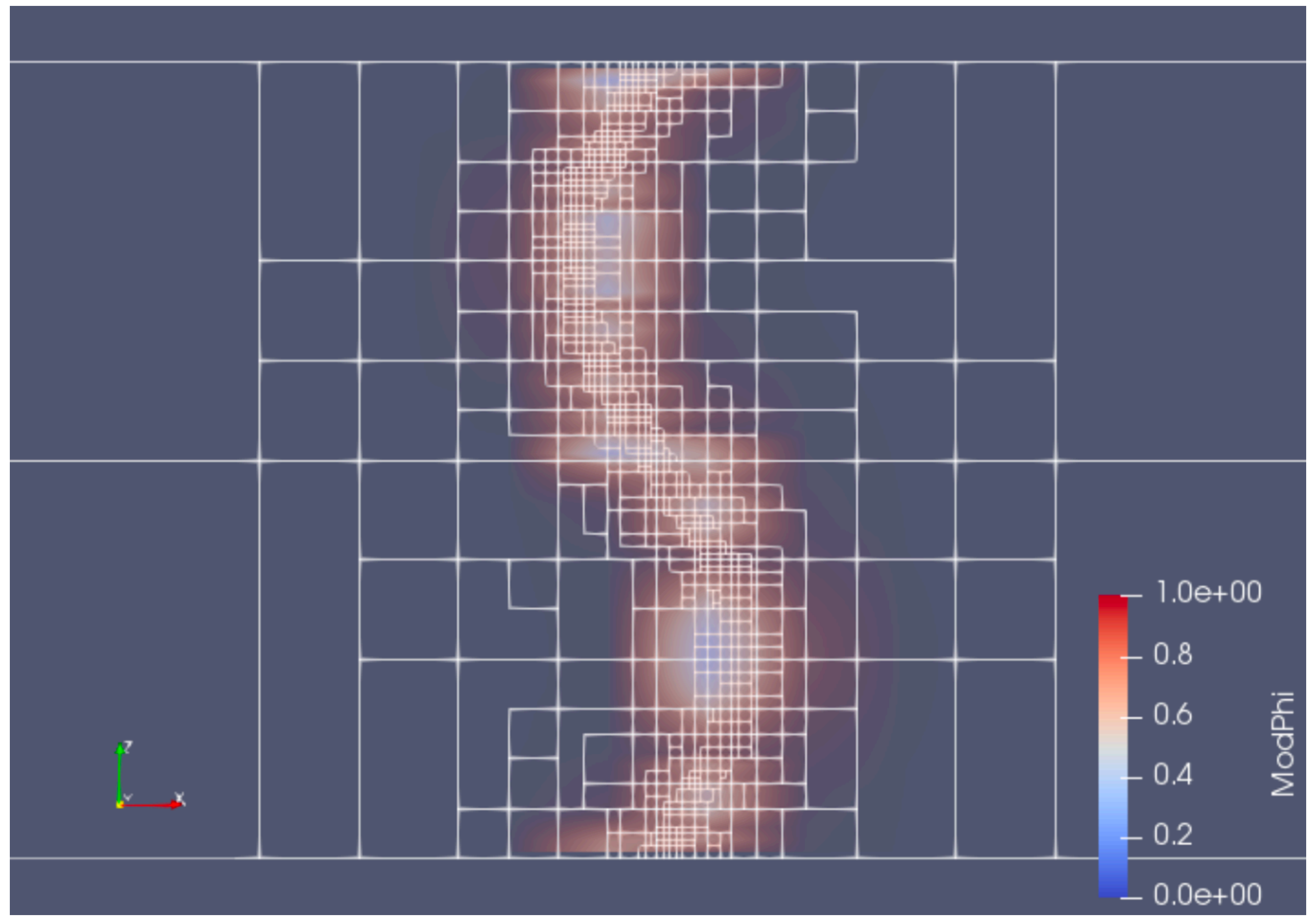
Klaer, Moore [1707.05566, 1708.07521, 1912.08058]



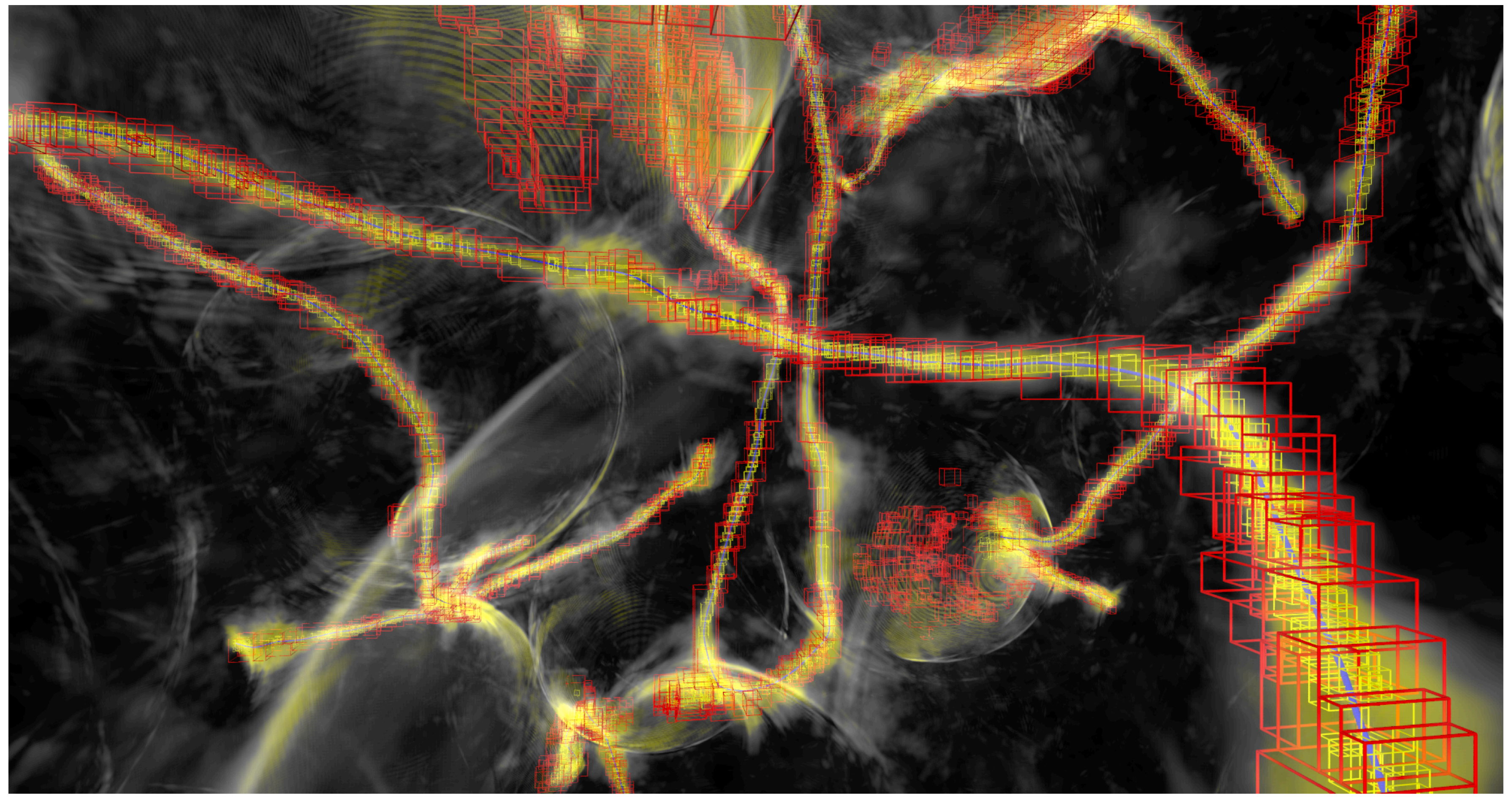
$$\log \sim 2(q_1^2 + q_2^2)$$

Adaptive Mesh Refinement (AMR)

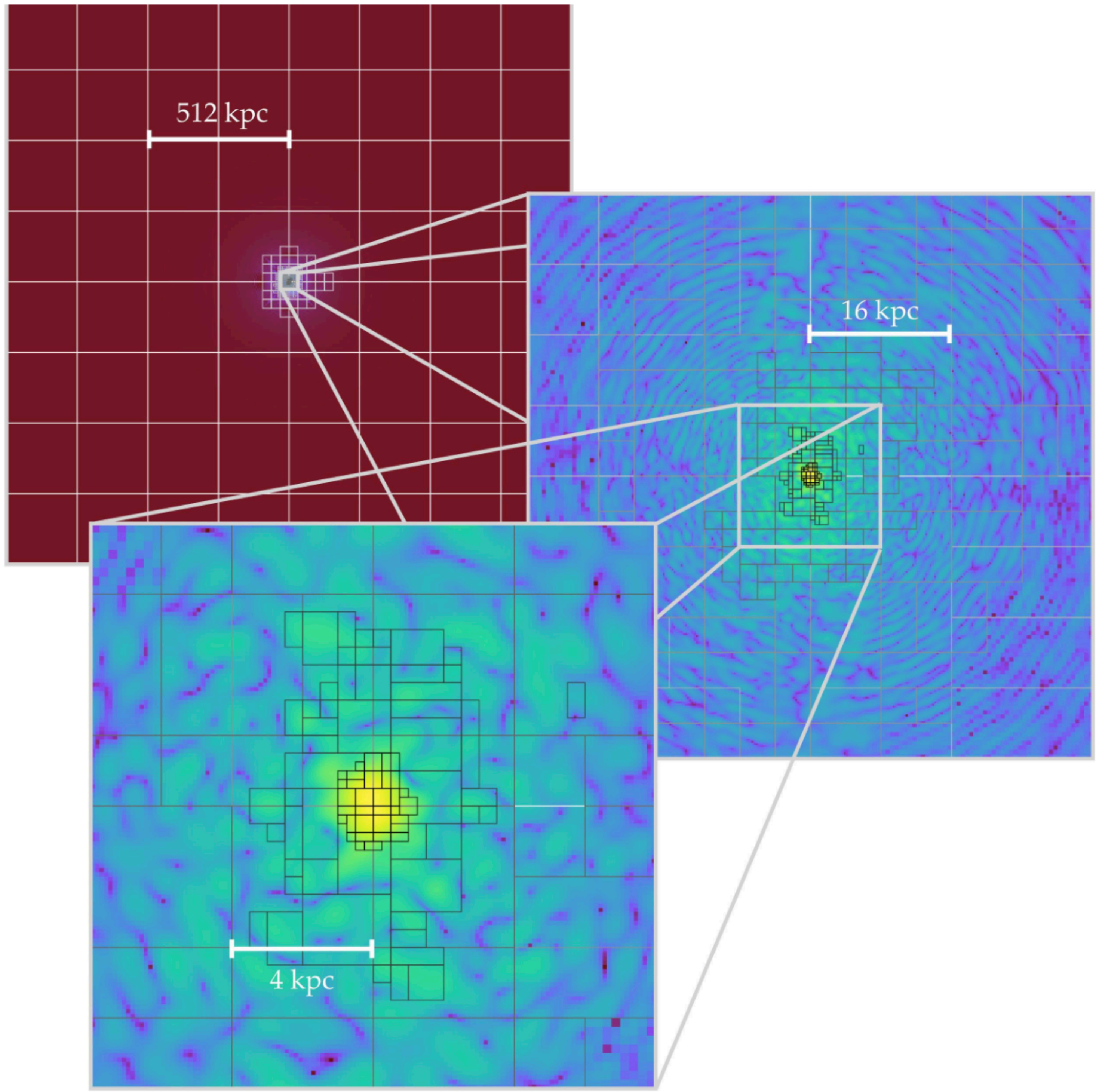
- **Idea:** Focus computational power on specific parts of the grid
- Nowadays widely used in cosmological simulation codes, numerical relativity **and** in axion string simulations
- Current codes mostly based on AMReX: https://amrex-codes.github.io/amrex/



Drew & Shellard [1910.01718]
"GRChombo"



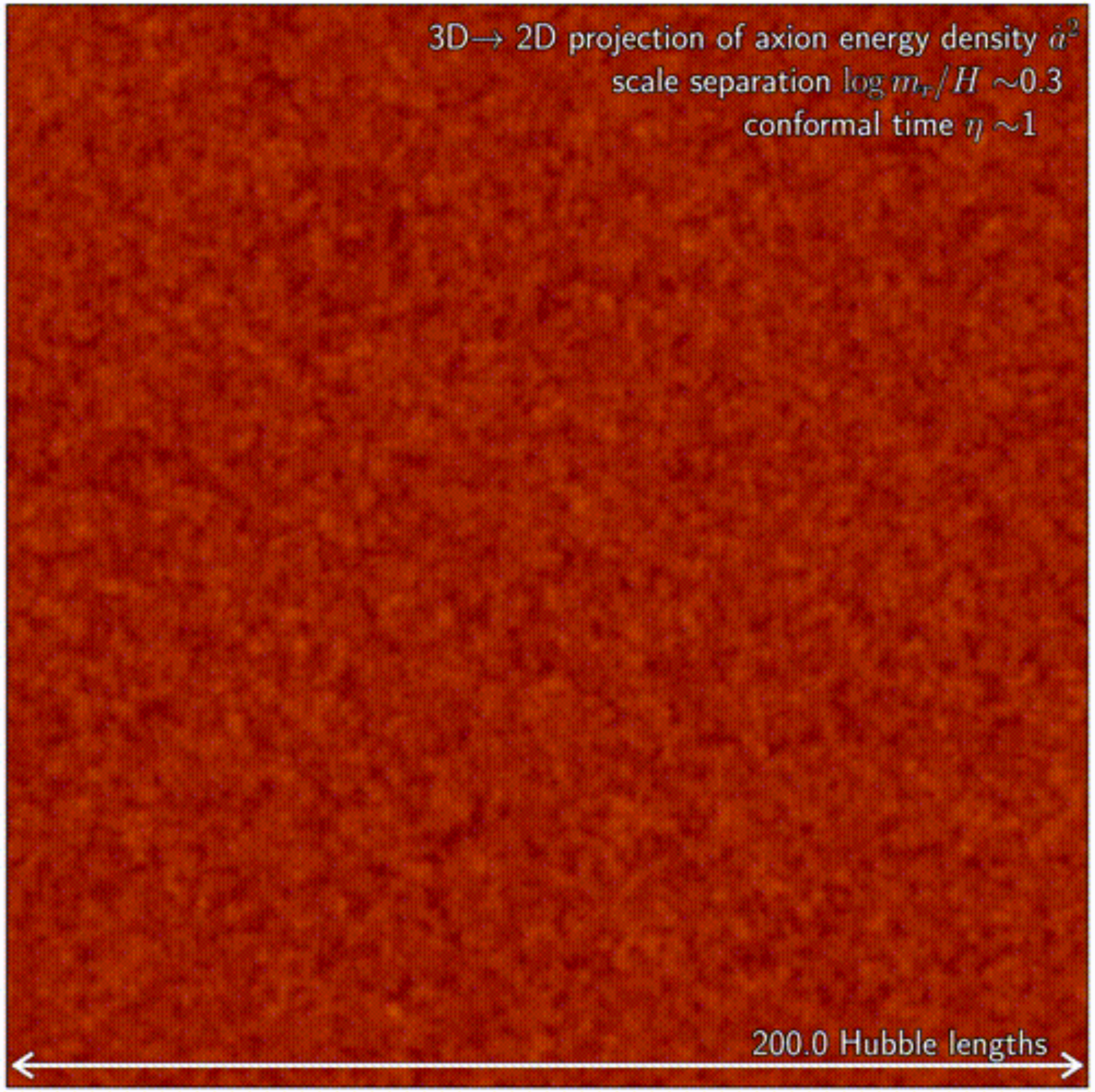
Benabou+ [2308.01334]
"sledgehamr"



Schwabe+ [2007.08256]
"axioNyx"

Adaptive Mesh Refinement (AMR)

- Movie not available in PDF format -



Buschmann [2404.02950]

- **Idea:** Focus co… the grid
- Nowadays wid… des, numerical relat…
- Current codes … https://amrex-…

Drew & Shellard [1910.01718]
"GRChombo"

"sledgehamr"

Schwabe+ [2007.08256]
"axioNyx"

Potential Improvement with AMR

- We can estimate the RAM needed to perform an AMR complex scalar simulation:

$$\mathrm{RAM} = 2 \times 2 \times 4\ \mathrm{bytes}\ \times \left(N_0^3 + \frac{\pi n_c n_r^2}{4} \frac{r^\ell - 1}{r - 1} N_p \right)$$

$N_p = \xi \times 6(L/(N_0\tau))^2 \times N_0^3$
Fleury & Moore [1509.00026]

- This takes into account, that we refine only around the strings and that we want to balance the RAM between the root and the refined grids

- Suggests time-dependent number of refinement levels:.

$$\ell + 7 \simeq \log_2\left(N_0^3/(\pi N_p)\right) = \log_2\left(N_0^2 \tau^2/(\pi 6 \xi L^2)\right)$$

- Results in log ~ 13,16,18 for base grids of $N_0 = 2048,\ 4096,\ 8192$ with $\ell = 9,\ 11,\ 13$. In practice not so trivial ..

Potential Improvement with AMR