From vacuum decay to gravitational waves

Based on:
A. Ivanov, MM, M. Nemevšek, L. Ubaldi: 10.1007/JHEP03(2022)209
MM, M. Nemevšek, Y. Shoji, L. Ubaldi: 2404.17632
work in progress with V. Brdar, M.Finetti, A. Morais, M. Nemevšek

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2ND TRAINING SCHOOL

COST ACTION COSMIC WISPERS (CA21106)

LJUBLJANA, 10-14 JUNE 2024

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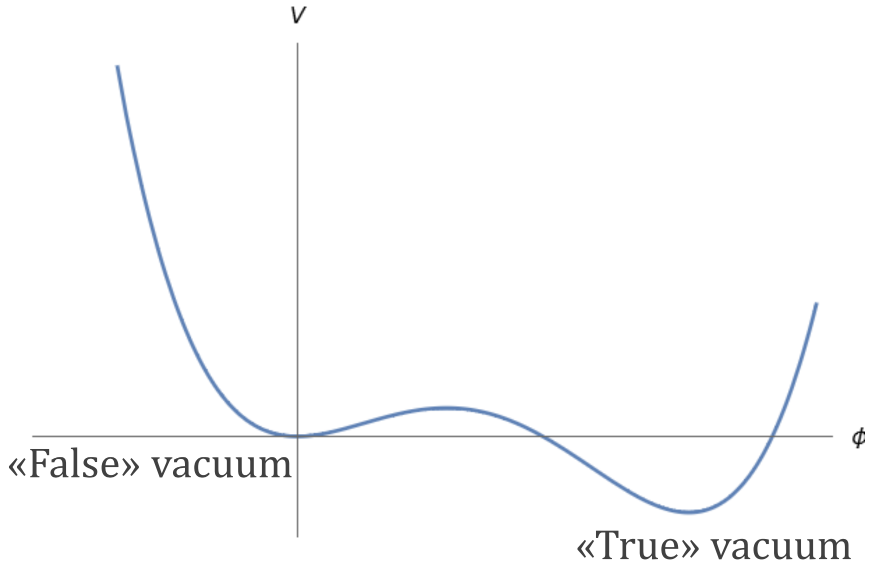
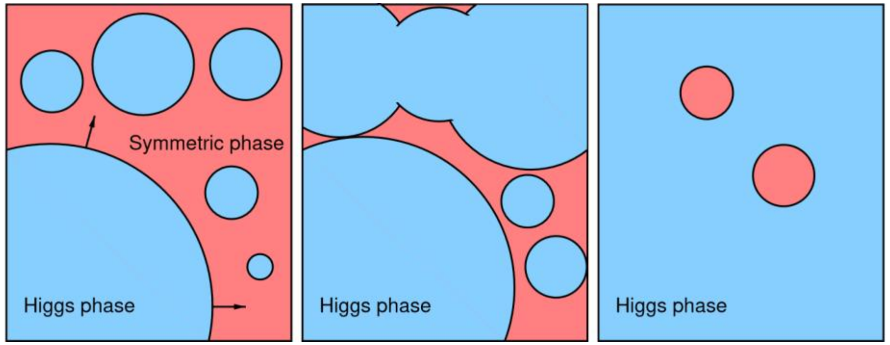
BASICS OF VACUUM DECAY

- Easiest example in field theory: single scalar ϕ

- Metastability of the false vacuum
- Decay to the true vacuum (tunneling under the barrier)
- 1° order phase transition: Bubble nucleation

- Bubble expansion: conversion of false vacuum to true vacuum





[*Hindmarsh, Lüben, Lumma, Pauly*, 2008.09136]

- 1-loop decay rate (per unit volume) for Euclidean dimension D

$$\frac{\Gamma}{\mathcal{V}} = \underbrace{\boxed{\left(\frac{S_R}{2\pi\hbar}\right)^{\frac{D}{2}} \left|\frac{\det' \mathcal{O}}{\det \mathcal{O}_{\rm FV}}\right|^{-\frac{1}{2}}}}_{\text{fluctuations}} \underbrace{\boxed{e^{-\frac{S_R}{\hbar}-S_{\rm ct}}}}_{\text{classical solution («bounce»)}} (1+\mathcal{O}(\hbar))$$

GWs FROM PHASE TRANSITIONS

- Early universe: 1° order cosmological PT is a sign of BSM physics!
- Signature: Gravitational Waves. Stochastic background: many uncorrelated, unresolved sources

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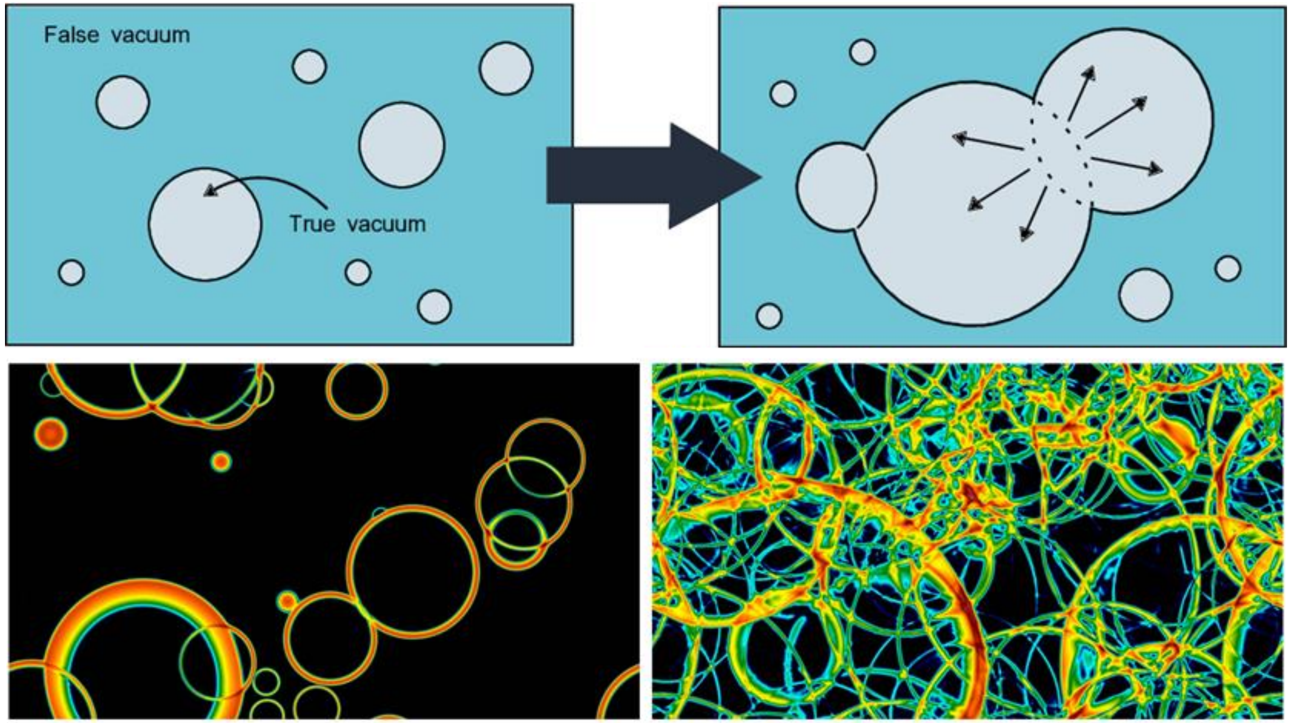

[*Weir*, 1705.01783]

- Sources of GWs from PTs:
  - Bubble collisions
  - Sound waves in the plasma
  - Turbulence in the plasma

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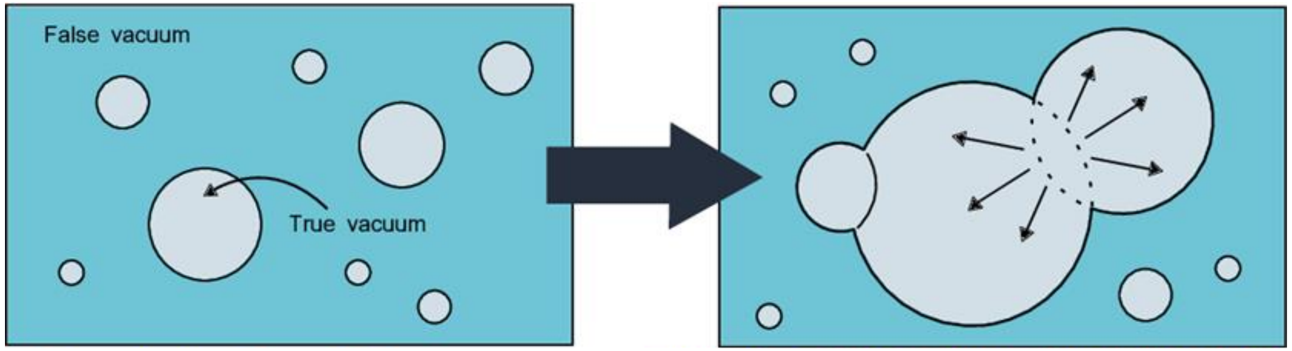

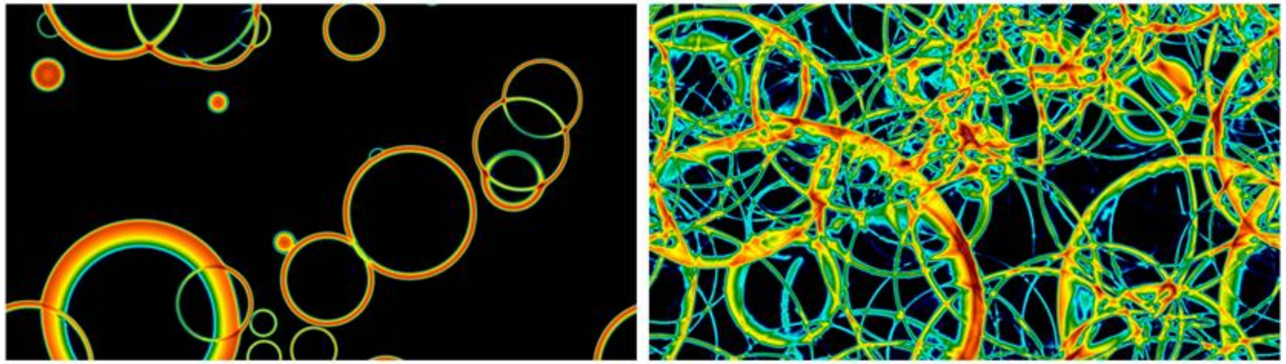

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- Sources of GWs from PTs:
  - Bubble collisions
  - Sound waves in the plasma
  - Turbulence in the plasma
- Relevant temperatures:
  - Nucleation: 1 bubble per Hubble volume
  - Percolation: connected region of TV phase
- Phase transition parameters:
  - Strength: energy released by the vacuum transition normalized to the radiation energy density
  - Duration: time derivative of Γ at percolation

- Very early universe opaque to light, but transparent to GWs!

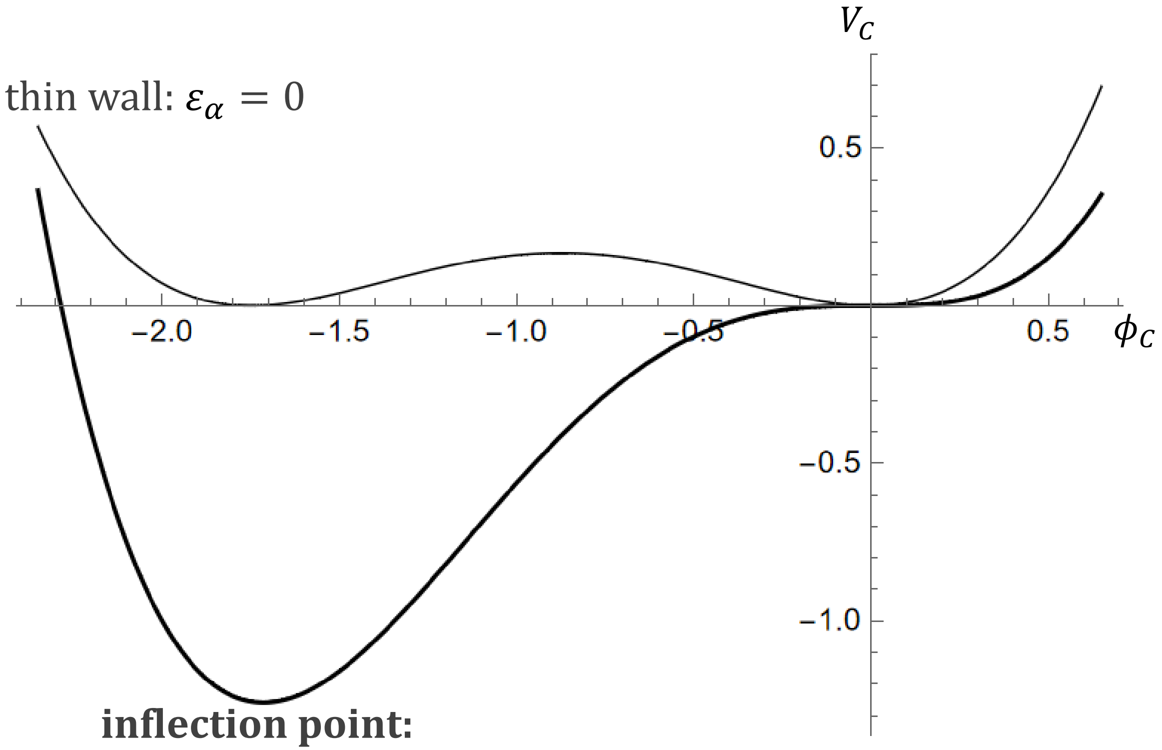
THIN & THICK WALL

- Single real scalar with potential:

$$V_C(\phi_C) = \frac{1}{2}m^2\phi_C^2 + \eta\phi_C^3 + \frac{1}{8}\lambda_C\phi_C^4$$

- Dimensionless quantities $\varphi_C \equiv \frac{2\eta}{m^2}\phi_C$, $\boxed{\varepsilon_\alpha \equiv 1 - \lambda_C\frac{m^2}{4\eta^2}}$

thin wall: $\varepsilon_\alpha = 0$



inflection point:
$\boldsymbol{m^2 \to 0}$

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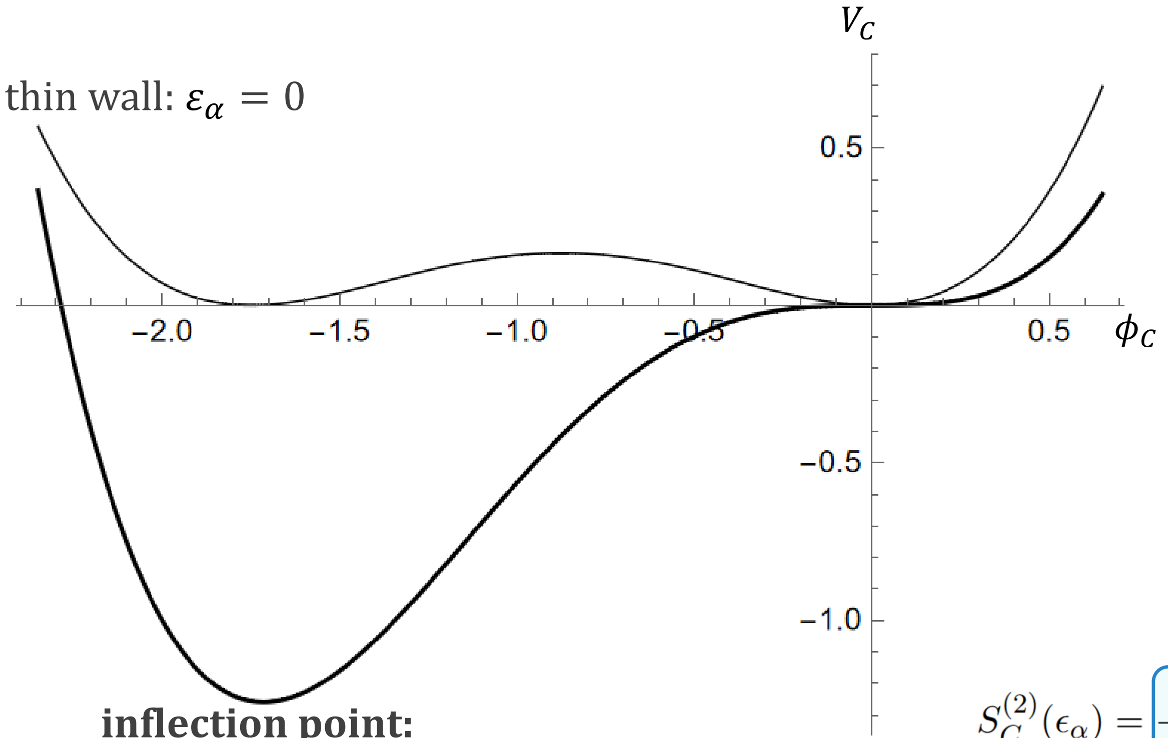

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- Dimensionless quantities $\varphi_C \equiv \frac{2\eta}{m^2}\phi_C$, $\boxed{\varepsilon_\alpha \equiv 1 - \lambda_C \frac{m^2}{4\eta^2}}$
- ε_α serves as expansion parameter: thin wall expansion

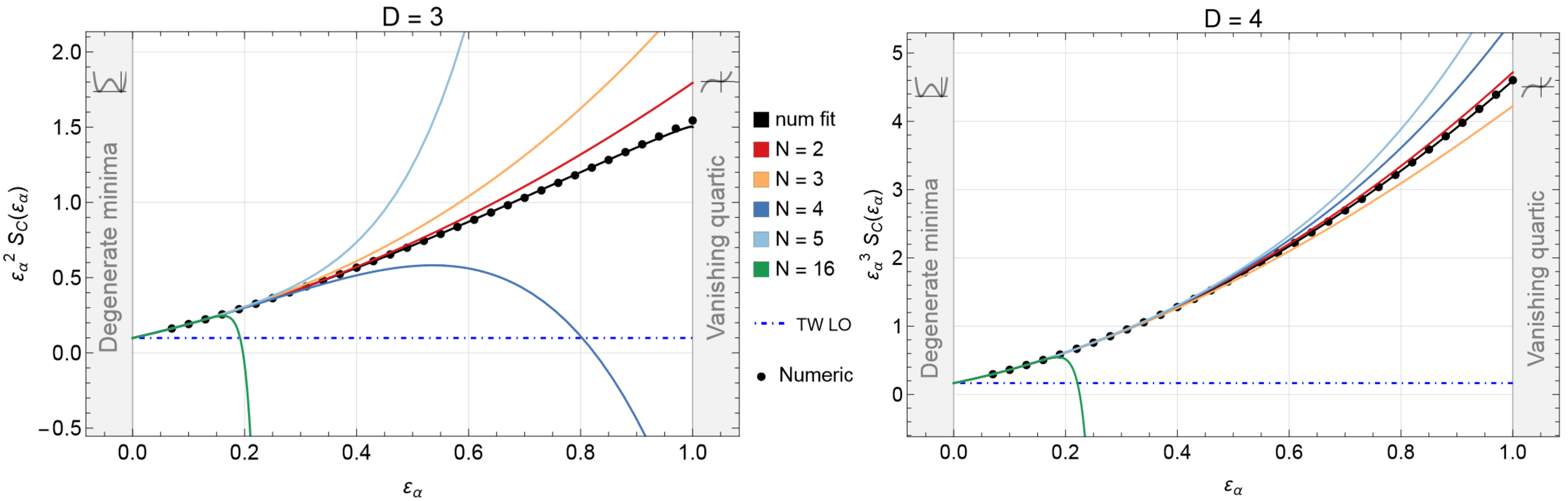
$$\varphi_C(z) = \sum_{n=0} \varepsilon_\alpha^n \, \varphi_{Cn}(z)$$

- EOM: $\ddot{\phi} + \frac{D-1}{\rho}\dot{\phi} = \frac{\mathrm{d}V}{\mathrm{d}\phi}$ solve order by order in ε_α
- Action:

$$S = \Omega \int_0^\infty \mathrm{d}\rho \; \rho^{D-1}\left(\frac{1}{2}\dot{\phi}^2 + V - V_{\mathrm{FV}}\right) = \frac{\Omega \; m^{6-D}}{4\eta^2} S_C(\epsilon_\alpha)$$

$$S_C^{(2)}(\epsilon_\alpha) = \boxed{\frac{1}{\epsilon_\alpha^{D-1}}}\left(\frac{D-1}{3}\right)^{D-1}\frac{2}{3D}\left(1 + \epsilon_\alpha\frac{3D+8}{2} + \epsilon_\alpha^2\frac{9D^3 - 11D^2 + 138D - 12D\pi^2 - 64}{8(D-1)}\right)$$

EUCLIDEAN ACTION



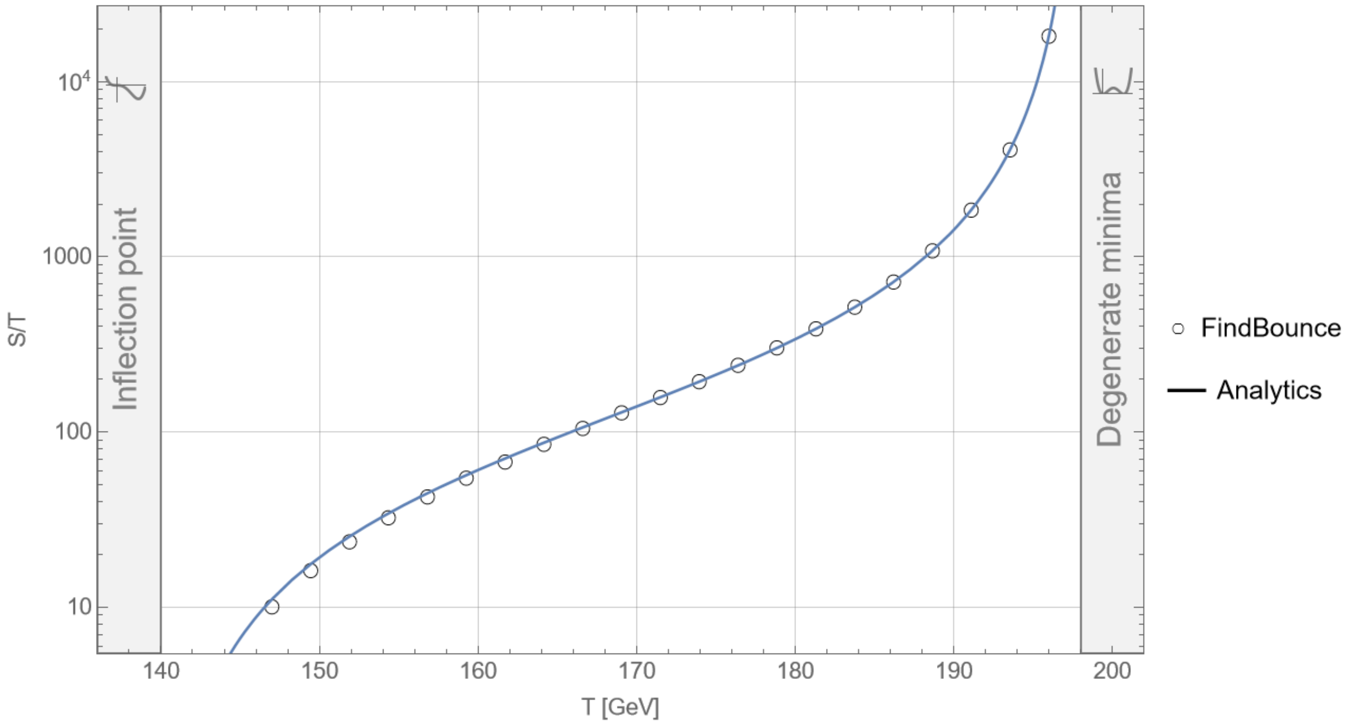
Works well away from thin wall!

EARLY UNIVERSE APPLICATION

- Example: cosmic fluid - order parameter field model (often adopted for numerical simulations)

$$V(\phi, T) = \frac{1}{2}\gamma(T^2 - T_0^2)\phi^2 - \frac{1}{3}AT\phi^3 + \frac{1}{4}\lambda\phi^4$$

- Phase structure: Degenerate minima at $T_C = \frac{\sqrt{9\gamma\lambda}}{\sqrt{9\gamma\lambda - 2A^2}} T_0$, inflection point at T_0



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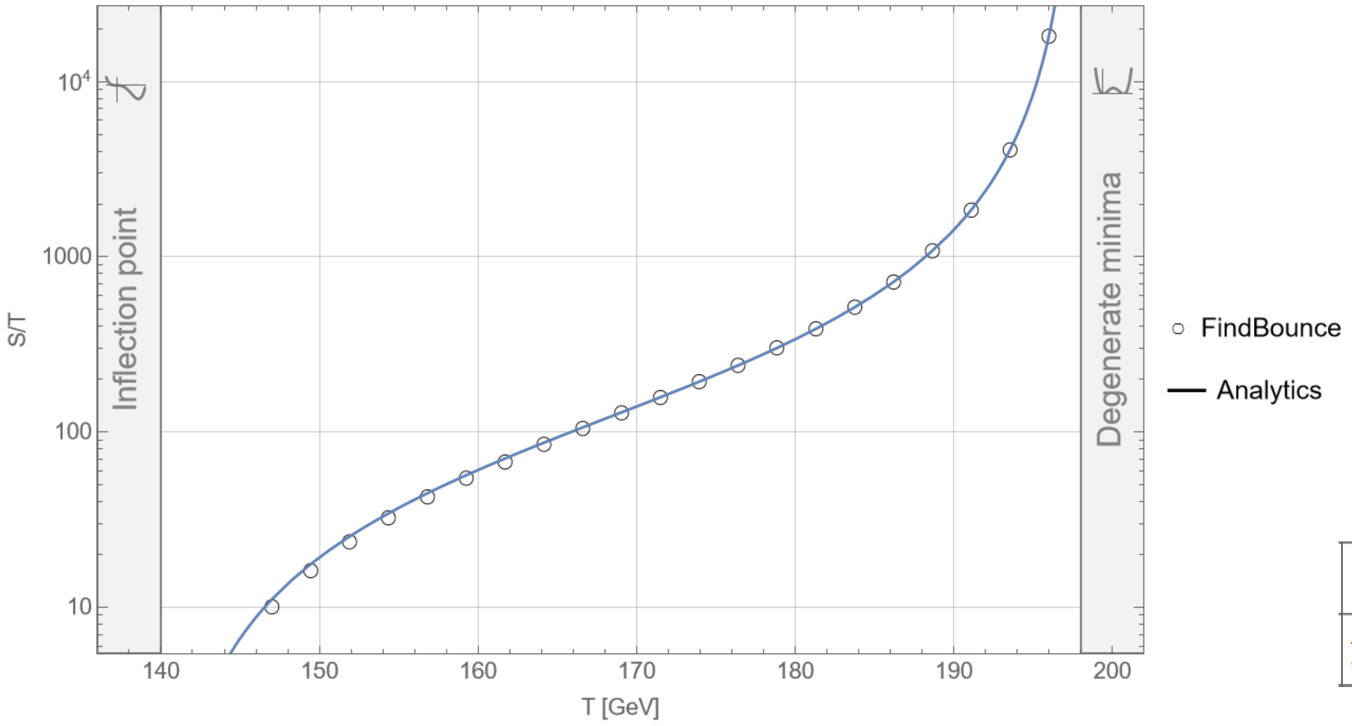

- Obtain nucleation & percolation temperatures
- Obtain PT parameters

From 1504.03291 :

γ	A	λ	T_0 [GeV]	T_c [GeV]	T_N [GeV]	α_{T_N}
1/18	$\sqrt{10}/72$	10/648	140	$\sqrt{2}\,T_0 = 197.99$	$0.86\,T_C = 170.27$	$\alpha_N = 0.01$

From our analytical action:

γ	A	λ	T_0 [GeV]	T_c [GeV]	$T_N^{(S/T)}$ [GeV]	$T_N^{(\Gamma/H)}$ [GeV]	α_{T_N}
1/18	$\sqrt{10}/72$	10/648	140	197.99	170.04	170.22	0.0104

SUMMARY AND OUTLOOK

- Detection of gravitational waves opens up a new window to study the very early universe
- Cosmological phase transitions are a source of GWs and a clear sign of BSM physics

- The thin wall approximation works in a wider range of parameter space than previously thought
- Analytical results can be used for phenomenologically relevant scenarios

- Apply our results to phase transitions e.g. in dark sectors

Thank you!

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DIFFERENT ORDERS OF THE EUCLIDEAN ACTION

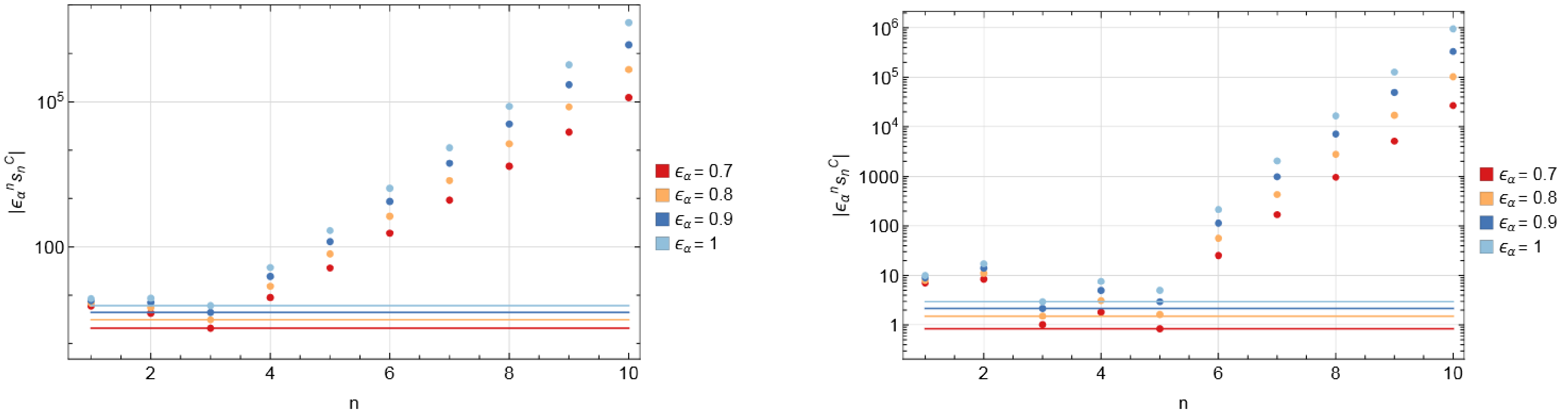


FIG. 2. Absolute values of each term of $S_C^{(10)}(\varepsilon_\alpha)$ in (22) for $\varepsilon_\alpha = 0.7, 0.8, 0.9, 1$. The left panel is for $D = 3$, the right for $D = 4$. The horizontal lines indicate the minimum values.

NUCLEATION AND PERCOLATION

- Nulceation temperature T_n

$$\int_{t_c}^{t_n} \mathrm{d}t \, \frac{\Gamma(t)}{H(t)^3} = \int_{T_n}^{T_c} \frac{\mathrm{d}T}{T} \frac{\Gamma(T)}{H(T)^4} = 1$$

Approximate criterion for fast transitions

$$\frac{S_3}{T_n} \approx 4 \log \left(\frac{T_n}{H} \right)$$

- Percolation temperature T_p (at least 34% of the comoving volume has been converted to the TV)

$$I(t) = \frac{4\pi}{3} \int_{t_c}^{t} \mathrm{d}t' \, \Gamma(t') \, a(t')^3 \, r(t,t')^3 \longrightarrow I(T) = \frac{4\pi v_w}{3} \int_{T}^{T_c} \mathrm{d}T' \, \frac{\Gamma(T')}{H(T') \, T'^4} \left(\int_{T}^{T'} \frac{\mathrm{d}T''}{H(T'')} \right)^3$$

Stronger requirement: decreasing FV volume

$$\frac{1}{V_{\rm false}} \frac{\mathrm{d}V_{\rm false}}{\mathrm{d}t} = 3H(t) - \frac{\mathrm{d}I(t)}{\mathrm{d}t} = H(T) \left(3 + T \, \frac{\mathrm{d}I(T)}{\mathrm{d}T} \right) < 0$$

PT STRENGTH AND DURATION

- Different possible definitions for the strength

Given $\epsilon(\phi, T) = 3aT^4 + V(\phi, T) - T\frac{\partial V}{\partial T}, \quad p = aT^4 - V(\phi, T), \quad \theta = \frac{\epsilon - 3p}{4}, \quad w = \epsilon + p$

$$\alpha_\theta = \left.\frac{\theta(\phi_s, T) - \theta(\phi_b, T)}{3aT^4}\right|_{T_N}$$

$$\alpha_N = \left.\frac{w(\phi_s, T) - w(\phi_b, T)}{3aT^4}\right|_{T_N}$$

Latent heat density

$$\left[V(\phi_{\rm FV}, T) - V(\phi_{\rm TV}, T) - \frac{T}{4}\left(\frac{\partial V}{\partial T}(\phi_{\rm FV}, T) - \frac{\partial V}{\partial T}(\phi_{\rm TV}, T)\right)\right]$$

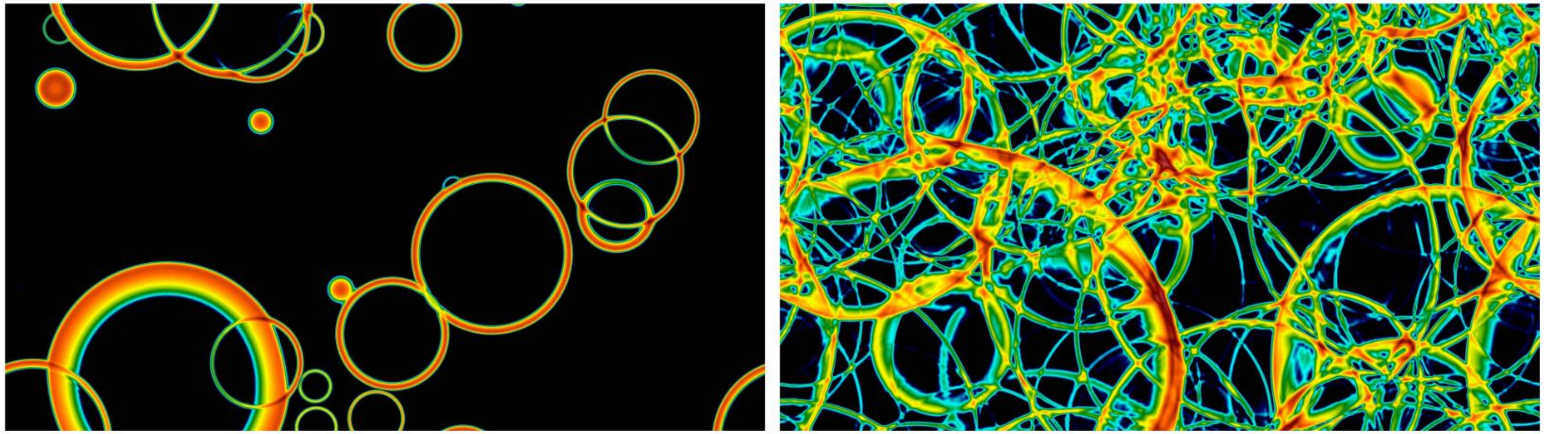
- Inverse duration $\beta = \frac{\mathrm{d}}{\mathrm{d}T}\left[\log \Gamma(T)\right]_{T=T_p}$

GW PRODUCTION FROM PTs

- Simulations for bubble expansion and collision. 3 stages of GW production:
- Bubbles collision and merger: short duration (usually subdominant, unless there is supercooling);
- Acoustic stage: shells of fluid kinetic energy continue to expand into the plasma as sound waves, overlap and source gravitational waves (believed to be dominant);
- Turbulent phase: non-linearity in the fluid equations becomes important, the previous phases might produce turbulence (not well-understood).

- Example of simulation:

[Weir, 1705.01783]



GRAVITATIONAL WAVES POWER SPECTRUM

- Energy momentum tensor of GWs: $T^{\rm gw}_{\mu\nu} = \dfrac{1}{32\pi G}\langle \partial_\mu h_{ij} \partial_\nu h_{ij} \rangle$

- Energy density: $\rho_{\rm gw} = \dfrac{1}{32\pi G}\langle \dot{h}^2_{ij} \rangle$

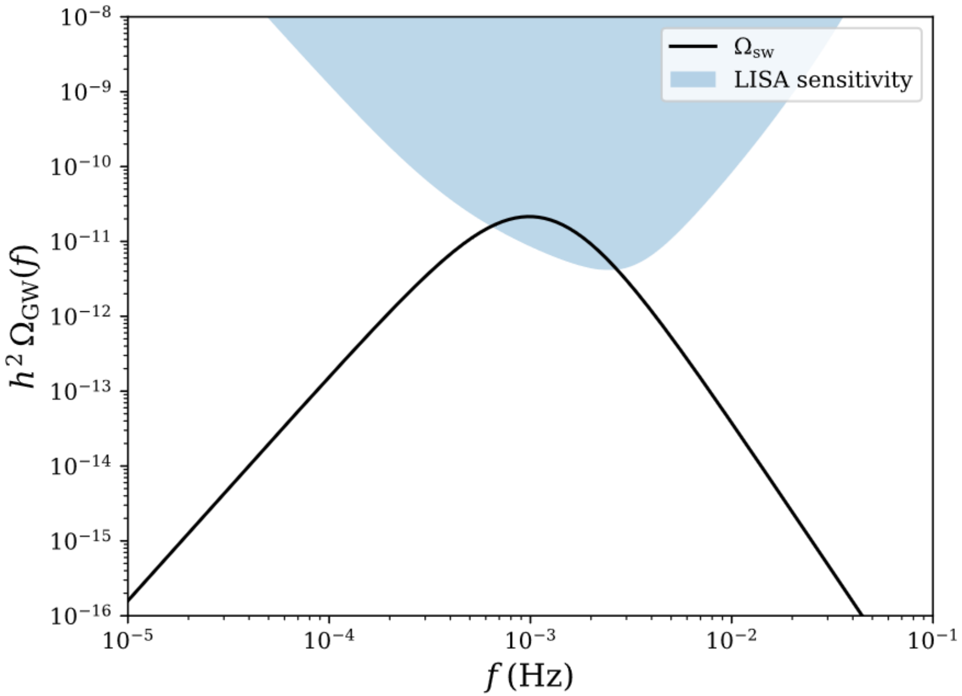
$$\Omega_{\rm gw} = \frac{\rho_{\rm gw}}{\rho_{\rm tot}}$$

$$H^2 = \frac{8\pi G \rho_{\rm tot}}{3}$$

+ frequency space

$\downarrow$

Power spectrum



[LISA Cosmology Working Group, 1910.13125]

COUPLED FIELD – FLUID MODEL

- Potential & eq. of state: $V(\phi,T) = \frac{1}{2}\left(T^2 - T_0^2\right)\gamma\phi^2 - \frac{1}{3}AT\phi^3 + \frac{1}{4}\lambda\phi^4$, $\epsilon(T,\phi) = 3aT^4 + V(\phi,T) - T\frac{\partial V}{\partial T}$

  $p(T,\phi) = aT^4 - V(\phi,T)$
- Energy-momentum tensor:

$$T^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - \tfrac{1}{2}g^{\mu\nu}(\partial\phi)^2 + [\epsilon + p]\,U^\mu U^\nu + g^{\mu\nu}p$$

$$\begin{cases} [\partial_\mu T^{\mu\nu}]_{\text{field}} = (\partial_\mu\partial^\mu\phi)\partial^\nu\phi - \dfrac{\partial V}{\partial\phi}\partial^\nu\phi = \delta^\nu \\ [\partial_\mu T^{\mu\nu}]_{\text{fluid}} = \partial_\mu[(\epsilon+p)U^\mu U^\nu] - \partial^\nu p + \dfrac{\partial V}{\partial\phi}\partial^\nu\phi = -\delta^\nu \end{cases} \longleftarrow \quad \delta^\nu = \eta U^\mu\partial_\mu\phi\partial^\nu\phi$$

- Numerical simulations: $U^i = WV^i$, $E = W\epsilon$, $Z_i = W(\epsilon+p)U_i$

$$\begin{cases} -\ddot{\phi} + \nabla^2\phi - \dfrac{\partial V}{\partial\phi} = \eta W(\dot{\phi} + V^i\partial_i\phi) \\ \dot{E} + \partial_i(EV^i) + p[\dot{W} + \partial_i(WV^i)] - \dfrac{\partial V}{\partial\phi}W(\dot{\phi} + V^i\partial_i\phi) \\ \qquad = \eta W^2(\dot{\phi} + V^i\partial_i\phi)^2 \\ \dot{Z}_i + \partial_j(Z_iV^j) + \partial_i p + \dfrac{\partial V}{\partial\phi}\partial_i\phi = -\eta W(\dot{\phi} + V^j\partial_j\phi)\partial_i\phi \end{cases} \Longrightarrow$$

GWs:

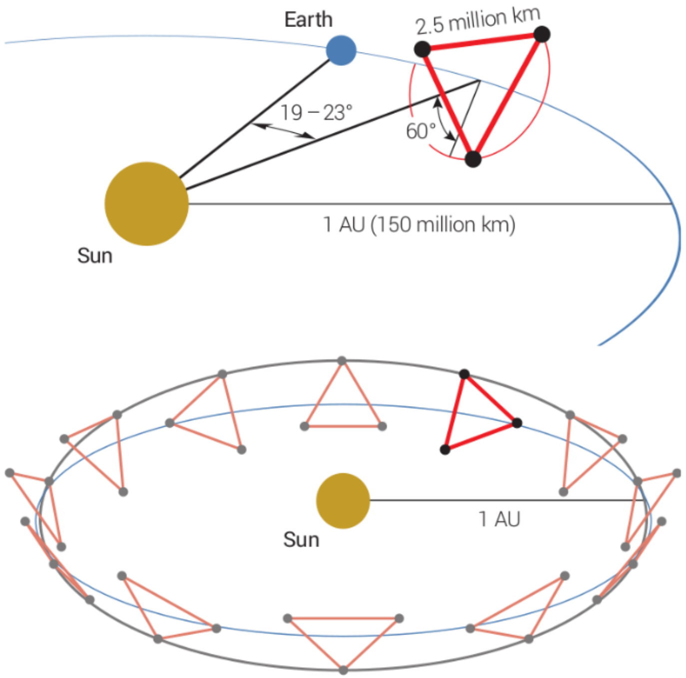
$$\tau^{\phi}_{ij} = \partial_i\phi\partial_j\phi, \quad \tau^{\text{f}}_{ij} = W^2(\epsilon+p)V_iV_j$$

$$h_{ij}(\mathbf{k},t) = (16\pi G)\lambda_{ij,kl}(\mathbf{k})\int_0^t dt'\,\frac{\sin[k(t-t')]}{k}\tau_{kl}(\mathbf{k},t')$$

LISA MISSION

- Laser Interferometer Space Antenna
- ESA – expected to launch in 2030s
- 3 satellites orbiting Earth, arms of 2.5M km
- Lasers and photodetectors which detect small changes in separation through time delays of signals

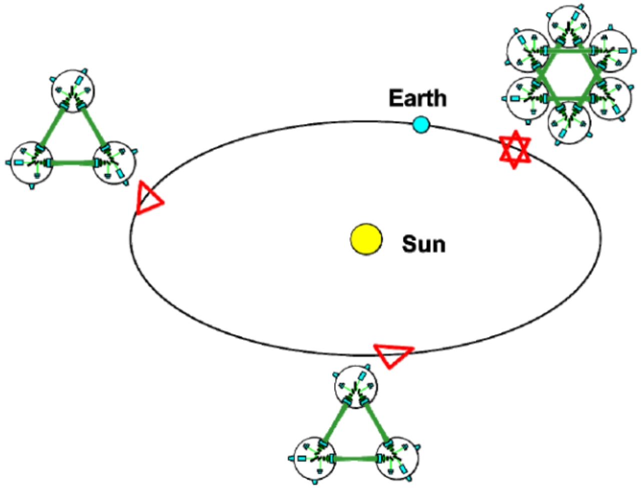
- Most sensitive in the range $10^{-3} - 10^{-2}\ Hz$

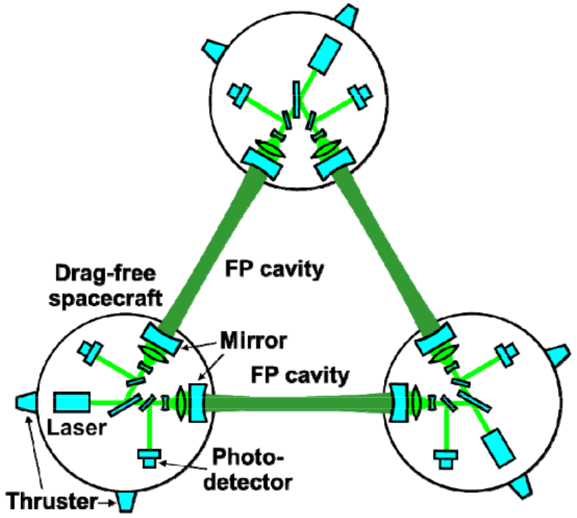


[Amaro-Seoane et al., 1702.00786]

DECIGO MISSION

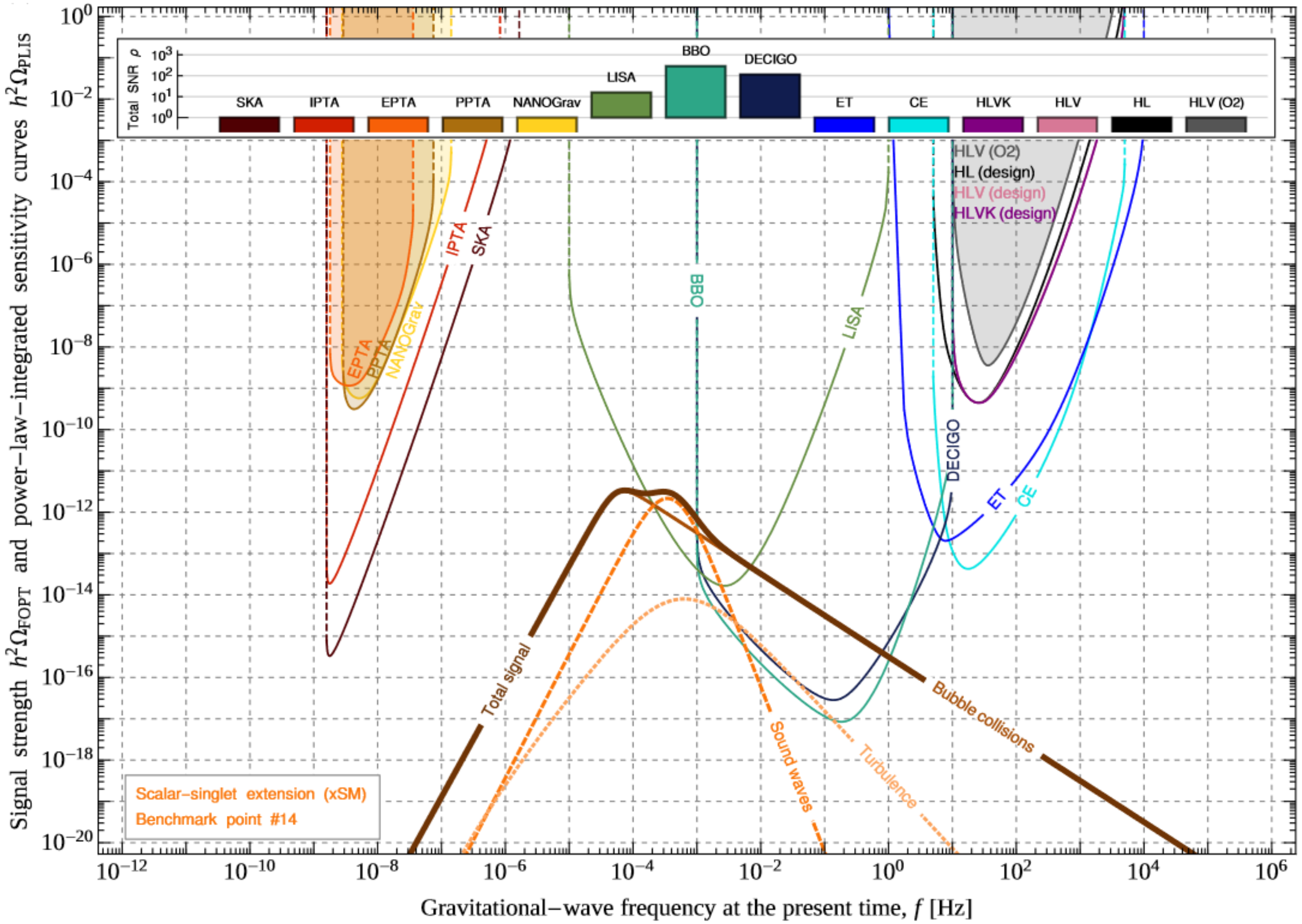
- Deci-hertz Interferometer Gravitational Wave Observatory
- Japanese project – expected to launch in 2030s
- Four clusters of observatories placed in the heliocentric orbit.
- Each cluster: three spacecraft, which form three Fabry-Perot Michelson interferometers with an arm length of 1,000 km
- Most sensitive in the range $0.1 - 10\ Hz$





[Kawamura, Ando, Seto, Sato, Musha et al., 2006.13545]

AN EXAMPLE OF SIGNAL



[Schmitz, 2002.04615]