Axion electrodynamics in plasmas

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Presentation Overview

Theoretical basis

Genesis of Axion theory

1. Axial U(1) problem
2. QCD vacuum angle
3. Strong CP problem
4. PQ symmetry

The axion is represented by a spin-zero (boson) scalar field φ,

$$\mathcal{L}_{eff} = \underbrace{\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m_0^2\varphi^2}_{\text{free axion}} + \underbrace{\mathcal{L}_{\varphi\gamma}}_{\text{axion-photon}} + \underbrace{\cancel{\mathcal{L}_{\varphi\psi}}}_{\text{axion-fermion}} + \cdots \tag{1}$$

Axion-photon coupling

The axion couples to the electromagnetic field via:

$$\mathcal{L}_{\varphi\gamma} = \frac{\kappa}{4\mu_0}\varphi F_{\mu\nu}\tilde{F}^{\mu\nu} \tag{2}$$

$\kappa = \eta m_0$ is the coupling constant. η [GeV^{-2}] model dependent.

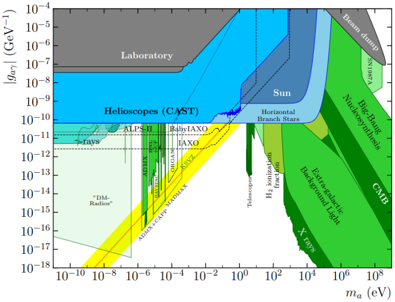

Figure: [Irastorza 2022] Sensitivity of the axion-detection experiments and cosmological bounds (green) in the coupling-mass parameter space. The QCD axion is shown in yellow.

Axion electromagnetism

The axion-photon coupling $\mathcal{L}_{\varphi\gamma}$ effectively rotates the fields $(\mathbf{E}, \mathbf{B})$ by an angle $\arctan(-\kappa\varphi)$ in the Maxwell equations [Visinelli 13]

$$\nabla \times (\mathbf{E} - c\kappa\varphi\mathbf{B}) = -\frac{\partial}{\partial t}(c\mathbf{B} + \kappa\varphi\mathbf{E}) \qquad \nabla \cdot (\mathbf{E} - \kappa\varphi\mathbf{B}) = \frac{\rho}{\varepsilon_0} \tag{3}$$

$$\nabla \times \left(\mathbf{B} + \frac{\kappa\varphi}{c}\mathbf{E}\right) = -\frac{1}{c^2}\frac{\partial}{\partial t}(\mathbf{E} - c\kappa\varphi\mathbf{B}) + \mu_0\mathbf{J} \qquad \nabla \cdot (\mathbf{B} - \frac{\kappa\varphi}{c}\mathbf{E}) = 0 \tag{4}$$

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The propagation for the axion field is given by a KG eq°

$$\left(\Box + m_\varphi^2 c^2\right)\varphi = \frac{2g}{\mu_0} \underbrace{\mathbf{E} \cdot \mathbf{B}}_{\propto F_{\mu\nu}\tilde{F}^{\mu\nu}} \tag{5}$$

Photon wave equation

Axion-Maxwell equations with dual symmetry:

$$\partial_\mu (R_\varphi \mathcal{F}^{\mu\nu}) = \mu_0 \mathcal{J}^\nu$$

$\mathcal{F}^{\mu\nu} = (F^{\mu\nu}, F^{*\mu\nu})^T$ dual vector; $F^{\mu\nu} \equiv \partial^\mu \mathcal{A}^\nu - \partial^\nu \mathcal{A}^\mu$;
$R_\varphi = \begin{pmatrix} 1 & -\kappa\varphi \\ \kappa\varphi & 1 \end{pmatrix}$ axion rotation matrix in dual space.

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Axion-photon dual wave equation:

$$\partial^\mu \partial_\mu \tilde{A}^\nu - \partial^\nu \partial_\mu \tilde{A}^\mu = \mu_0 J^\nu$$

$\tilde{A}^i = A^i + X^i$, with gauge offset $\partial^i \times X^i = \kappa\varphi E^i$, $\partial_0 X^i + \partial^i X_0 = \kappa\varphi B^i$

Axion-Polariton gauge

Lorenz gauge for axion-polariton in vacuum

$$\partial_\mu \tilde{A}^\mu = 0$$

Photon mass and axion-photon mass can not be simultaneously eliminated by a change of gauge.

This choice ensures usual current conservation $\partial_\mu j^\mu = 0$ and vanishing longitudinal axion-polariton via Ward-Takahashi

$$k_\mu \tilde{\epsilon}^\mu = 0$$

Note: fixing instead $\partial_\mu A^\mu = 0$, conserved current is $j^\nu - j^\nu_{ax}$,

$$j^\nu_{ax} = -\partial_\mu(\kappa\varphi)F^{*\mu,\nu}$$

Photon angular momentum

Effect on photon angular momentum:

$$\mathbf{S} = \frac{1}{c} \int d^3 x \pi_{\mathbf{A}} \times \mathbf{A} \tag{6}$$

$\pi^{\alpha} = \frac{\partial L}{\partial(\partial_0 A_{\alpha})} = -\mathbf{E} + \kappa\varphi\mathbf{B}. \to \mathbf{S} = \mathbf{S_0} + \mathbf{S}_{\chi}$

$$\mathbf{S}_{\chi} = \frac{1}{c} \int d^3 x \ \kappa\varphi(\nabla \times \mathbf{A}) \times \mathbf{A} = 0 \tag{7}$$

Axion coupling contribution to photon spin is vanishing $\Rightarrow$ no axion-polariton in vacuum (states are degenerate)

($J_z^{ax} = 0 \leftrightarrow J_z^{\gamma} = \pm 1$) impossible from longitudinal field alone.

Some more context

[Raffelt & Stodolsky]: mixing possible with QED vacuum polarization.

also [Raffelt & Stodolsky]: don't forget to add *neutral gas* contribution to birefringence effect for realistic astrophysical applications.

Plasma (the realistic astrophysical application):

*To take with a grain of salt

Plasmas $\rightarrow$ Langmuir Waves (plasmons)

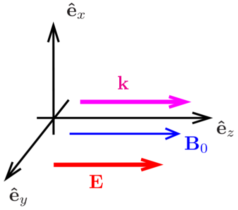

Figure: In plasmas, resistive effects typically induce $\mathbf{E} \cdot \mathbf{B} \neq 0$. A small displacement $\delta z = \xi$ of an electron from its equilibrium position induces oscillations:

$$\frac{\mathrm{d}^2\xi}{\mathrm{d}t^2} + \omega_{pe}^2 \xi = 0, \quad \omega_{pe} = \frac{4\pi n_e e^2}{m_e}$$

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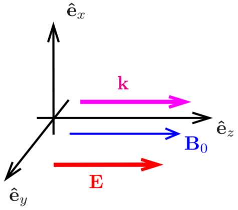

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At finite temperatures, $k \neq 0$,

$$\omega_{pl}^2 = \omega_{pe}^2 + S_e^2 k^2, \text{ plasmon bare disp.} \tag{8}$$

$$\omega_{\varphi}^2 = m_{\varphi}^2 c^4 + c^2 k^2, \text{ axion bare disp (KG)} \tag{9}$$

(8): $S_e = \sqrt{\frac{3k_b T_e}{m_e}}$ thermal speed

(9): $m_{\varphi} = \sqrt{m_0^2 + 2g^2 B_0^2/(c\mu_0)}$ axion **effective** mass

Axion-plasmon Waves

Equations (4) and (5) must be closed with continuity equations within a fluid treatment:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}) = 0 \tag{10}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right) u = \frac{\nabla P}{m_e n_e} - \frac{n_e}{m_e}(\mathbf{E} + \mathbf{u} \times \mathbf{B}). \tag{11}$$

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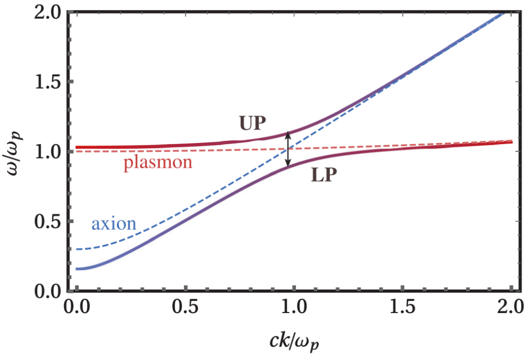
Yields a coupled dispersion relation on $(\tilde{n}, \tilde{\varphi})$:

$$(\omega^2 - \omega_{pl}^2)\tilde{n} - i\frac{gec}{m_e} B_0 k \tilde{\varphi} \tag{12}$$

$$(-\omega^2 + \omega_{\varphi}^2)\tilde{\varphi} + i\frac{\kappa e c^2}{\epsilon_0 k} B_0 \tilde{n} = 0 \tag{13}$$

Axion-plasmon polaritons [Terças+18]



$$\omega_{U,L}{}^2 = \frac{1}{2}\left(\omega_\varphi^2 + \omega_{pl}^2 \pm \sqrt{(\omega_{pl}^2 - \omega_\varphi^2)^2 + 4\Omega^4}\right) \quad (14)$$

$\Omega = (2\kappa^2 e^2 B_0^2 (n_0/m_e) c^5/\hbar)^{1/4}$ Rabi frequency

Rotating wave approximation

The system can be simplified (approximated) near the resonance $\omega_{pl} \simeq \omega_p$, with $\omega \to -i\frac{\partial}{\partial t}$:

$$\left(i\frac{\partial}{\partial t} - \omega_{pl}\right)\tilde{n} - \frac{igec^4 B_0}{\hbar k \omega_p}\tilde{\varphi} = 0$$

$$\left(i\frac{\partial}{\partial t} - \omega_{\varphi}\right)\tilde{\varphi} + \frac{igec^4 B_0}{\hbar k \omega_p}\tilde{n} = 0$$

Axion-plasmon quantization:

$$\tilde{n}(x) = \sum_k \mathcal{A}_k\left(\hat{a}_k e^{ikx} + h.c\right)\tilde{\varphi}(x) = \sum_k \mathcal{B}_k\left(\hat{b}_k e^{ikx} + h.c\right) \tag{15}$$

Axion-plasmon quantization

Heisenberg picture: $\frac{\mathrm{d}\hat{c}}{\mathrm{d}t} = \frac{i}{\hbar}[\hat{H}, \hat{c}]$,

$$\hat{H}_{\gamma\varphi} = \sum_k \omega_{pl} \hat{a}_k^\dagger \hat{a}_k + \sum_k \omega_\varphi \hat{b}_k^\dagger \hat{b}_k + \Omega \sum_k \hat{a}_k^\dagger \hat{b}_k + h.c \tag{16}$$

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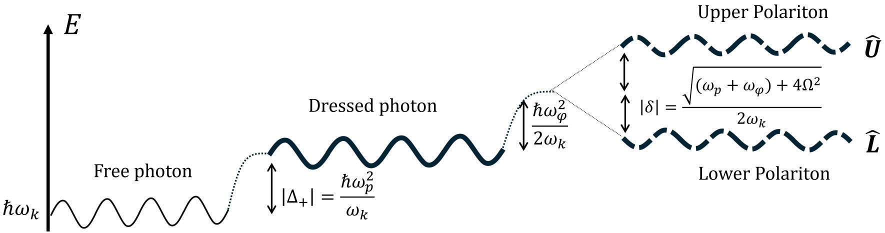
Mixing terms $\propto \Omega$ can be absorbed in diagonalization with polariton operators $\hat{L}_k = u_k \hat{a}_k - v_k \hat{b}_k$ and $\hat{U}_k = u_k \hat{b}_k + v_k \hat{a}_k$

$$\hat{H}'_{\gamma\varphi} = \tilde{\omega}_L \sum_k \hat{L}_k^\dagger \hat{L}_k + \tilde{\omega}_U \sum_k \hat{U}_k^\dagger \hat{U}_k \tag{17}$$

$$\boxed{\tilde{\omega}_{U,L} = \frac{1}{2}\left(\omega_\varphi + \omega_{pl} \pm \sqrt{(\omega_{pl} - \omega_\varphi)^2 + 4\Omega^2}\right)} \tag{18}$$

Conclusion

- $\cancel{U(1)_{PQ}} + \mathbf{B}_0 \xrightarrow[\text{mixing}]{\gamma_{\parallel} \leftrightarrow \varphi} \cancel{SO(2)_{EM}}$ axion-polariton level splitting



- Opens up much phenomenology: decay rates $\Gamma_{\varphi\to\gamma\gamma}$ [Terças+18], Boson oscillation [Capolupo 19], Casimir effect [Brevik 22], etc.

Discussion

- couplings neglected:
  1. $\mathcal{L}_{\psi\gamma}$: invitation to full quantization of AQED.
  2. $\mathcal{L}_{EH}$: can be added, but is it wise?