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Functional determinants: Gauge sector (4D)

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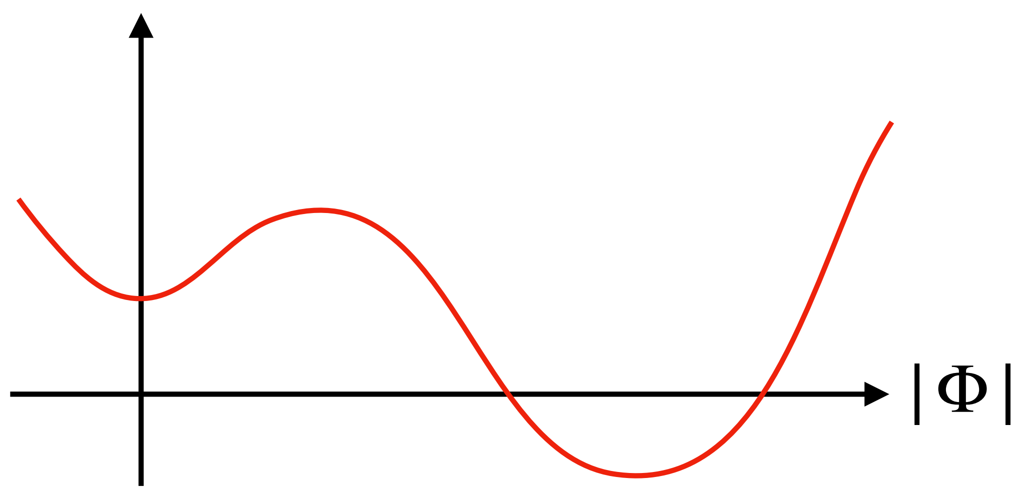
Setup

Gauge fixing

One complex scalar + U(1) gauge boson



$$\mathscr{L} = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + |D\Phi|^2 + V(|\Phi|) + \mathscr{L}_{\text{GF}} + \mathscr{L}_{\text{ghost}}$$

Background gauge

~R_ξ gauge in the broken phase~

$$\mathscr{L}_{\text{GF}} = \frac{1}{2\xi}\left[\partial_\mu A_\mu - 2\xi g(\Re\Phi)(\Im\Phi)\right]^2$$

$$\mathscr{L}_{\text{ghost}} = \bar{c}\left[-\partial^2 + 2\xi g^2|\Phi|^2\right]c$$

Fermi gauge

~R_ξ gauge in the symmetric phase~

$$\mathscr{L}_{\text{GF}} = \frac{1}{2\xi}\left[\partial_\mu A_\mu\right]^2$$

$$\mathscr{L}_{\text{ghost}} = \bar{c}\left[-\partial^2\right]c$$

Gauge-scalar mixing

Does the background gauge kill the mixing term? No!

$$\Phi = \frac{\bar{\phi} + h + ia}{\sqrt{2}}$$

$\bar{\phi}$**: Bounce**

Kinetic term:

$$|D\Phi|^2 \supset gA_\mu[a(\partial_\mu\bar{\phi}) - \bar{\phi}(\partial_\mu a)]$$

When the background is not constant, A_μ and a mix with each other

Gauge fixing term (BG):

$$\frac{1}{2\xi}\left[\partial_\mu A_\mu - 2\xi g(\Re\Phi)(\Im\Phi)\right]^2 \supset gA_\mu[a(\partial_\mu\bar{\phi}) + \bar{\phi}(\partial_\mu a)]$$

Partial wave expansions

Lorentz (pseudo-)scalar

$$h(x) = h_{l,m_A,m_B}(r) Y_{l,m_A,m_B}(\Omega)$$

$$a(x) = a_{l,m_A,m_B}(r) Y_{l,m_A,m_B}(\Omega)$$

$$c(x) = c_{l,m_A,m_B}(r) Y_{l,m_A,m_B}(\Omega)$$

Lorentz vector

$$A_\mu(x) = a^S_{l,m_A,m_B}(r)\frac{x_\mu}{r} Y_{l,m_A,m_B}(\Omega)$$
$$+ a^L_{l,m_A,m_B}(r)\frac{r}{L}\partial_\mu Y_{l,m_A,m_B}(\Omega)$$
$$+ a^{T1}_{l,m_A,m_B}(r) i\epsilon_{\mu\nu\rho\sigma} V^{(1)}_\nu L_{\rho\sigma} Y_{l,m_A,m_B}(\Omega)$$
$$+ a^{T2}_{l,m_A,m_B}(r) i\epsilon_{\mu\nu\rho\sigma} V^{(2)}_\nu L_{\rho\sigma} Y_{l,m_A,m_B}(\Omega)$$

They mix with each other

$$L_{\mu\nu} = \frac{i}{\sqrt{2}}(x_\mu \partial_\nu - x_\nu \partial_\mu)$$

$V^{(i)}_\mu$: independent vectors

Functional determinant

Background gauge

$$\Delta_l = \partial_r^2 + \frac{3}{r}\partial_r - \frac{L^2}{r^2} \qquad L = \sqrt{l(l+2)}$$

$[S, L, a]$

$l = 0$

BG vs Fermi

$$\mathscr{M}_0^{(Sa)} = \begin{pmatrix} \frac{1}{\xi}\left(-\Delta_0 + \frac{3}{r^2} + \xi g^2 \bar{\phi}^2\right) & 2g\bar{\phi}' \\ 2g\bar{\phi}' & -\Delta_0 + \frac{(\Delta_0 \bar{\phi})}{\bar{\phi}} + \xi g^2 \bar{\phi}^2 \end{pmatrix}$$

$l > 0$

Diagonalisable

Zero at $r = 0$ and $r = \infty$

$$\mathscr{M}_l^{(SLa)} = \begin{pmatrix} -\Delta_l + \frac{3}{r^2} + g^2\bar{\phi}^2 & -\frac{2L}{r^2} & 2g\bar{\phi}' \\ -\frac{2L}{r^2} & -\Delta_l - \frac{1}{r^2} + g^2\bar{\phi}^2 & 0 \\ 2g\bar{\phi}' & 0 & -\Delta_l + \frac{(\Delta_0\bar{\phi})}{\bar{\phi}} + \xi g^2\bar{\phi}^2 \end{pmatrix}$$

$$+ \left(1 - \frac{1}{\xi}\right) \begin{pmatrix} \partial_r^2 + \frac{3}{r}\partial_r - \frac{3}{r^2} & -L\left(\frac{1}{r}\partial_r - \frac{1}{r^2}\right) & 0 \\ L\left(\frac{1}{r}\partial_r + \frac{3}{r^2}\right) & -\frac{L^2}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$[c, \bar{c}]$

$l \geq 0$

$$\mathscr{M}_l^{(c\bar{c})} = -\Delta_l + \xi g^2 \bar{\phi}^2$$

$\times 2$

cancel accidentally when $\xi = 1$

$[T1, T2]$

$l > 0$

$$\mathscr{M}_l^{(T)} = -\Delta_l + g^2\bar{\phi}^2$$

$\times 2$

BG gauge with $\xi = 1$ is often used for numerical calculations

Functional determinant

Fermi gauge

$$\Delta_l = \partial_r^2 + \frac{3}{r}\partial_r - \frac{L^2}{r^2} \qquad L = \sqrt{l(l+2)}$$

$[S, L, a]$

$l = 0$

BG vs Fermi

$$\mathscr{M}_0^{(Sa)} = \begin{pmatrix} \frac{1}{\xi}\left(-\Delta_0 + \frac{3}{r^2} + \xi g^2\bar{\phi}^2\right) & g\bar{\phi}' - g\bar{\phi}\partial_r \\ 2g\bar{\phi}' + g\bar{\phi}\partial_r + \frac{3}{r}g\bar{\phi} & -\Delta_0 + \frac{(\Delta_0\bar{\phi})}{\bar{\phi}} \end{pmatrix}$$

$l > 0$

$$\mathscr{M}_l^{(SLa)} = \begin{pmatrix} -\Delta_l + \frac{3}{r^2} + g^2\bar{\phi}^2 & -\frac{2L}{r^2} & g\bar{\phi}' - g\bar{\phi}\partial_r \\ -\frac{2L}{r^2} & -\Delta_l - \frac{1}{r^2} + g^2\bar{\phi}^2 & -\frac{L}{r}g\bar{\phi} \\ 2g\bar{\phi}' + g\bar{\phi}\partial_r + \frac{3}{r}g\bar{\phi} & -\frac{L}{r}g\bar{\phi} & -\Delta_l + \frac{(\Delta_0\bar{\phi})}{\bar{\phi}} \end{pmatrix}$$

$$+\left(1 - \frac{1}{\xi}\right)\begin{pmatrix} \partial_r^2 + \frac{3}{r}\partial_r - \frac{3}{r^2} & -L\left(\frac{1}{r}\partial_r - \frac{1}{r^2}\right) & 0 \\ L\left(\frac{1}{r}\partial_r + \frac{3}{r^2}\right) & -\frac{L^2}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$[c, \bar{c}]$

$l \geq 0$

$$\mathscr{M}_l^{(c\bar{c})} = -\Delta_l$$

$\times 2$

$[T1, T2]$

$l > 0$

$$\mathscr{M}_l^{(T)} = -\Delta_l + g^2\bar{\phi}^2$$

$\times 2$

Prefactor

$$\mathscr{A} = J_T \left[\prod_{l=0}^{\infty} \left(\frac{\det' \mathscr{M}_l^{(h)}}{\det \widehat{\mathscr{M}}_l^{(h)}} \right)^{-(l+1)^2/2} \right]$$

$$\times \left[\prod_{l=1}^{\infty} \left(\frac{\det \mathscr{M}_l^{(T)}}{\det \widehat{\mathscr{M}}_l^{(T)}} \right)^{-(l+1)^2} \right]$$

Manifestly gauge independent

$$\times \left[\prod_{l=0}^{\infty} \left(\frac{\det \mathscr{M}_l^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_l^{(c\bar{c})}} \right)^{(l+1)^2} \right]$$

=1 for Fermi gauge

$$\times V_G J_G \left(\frac{\det' \mathscr{M}^{(Sa)_0}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} \right)^{-1/2} \left[\prod_{l=1}^{\infty} \left(\frac{\det \mathscr{M}_l^{(SLa)}}{\det \widehat{\mathscr{M}}_l^{(SLa)}} \right)^{-(l+1)^2/2} \right]$$

Gauge dependence should be canceled

Gauge independence

Gelfand-Yaglom for gauge sector

Proof is available in [M. Endo, T. Moroi, M. M. Nojiri, YS, '17]

$$\frac{\det \mathscr{M}_l^{(SLa)}}{\det \widehat{\mathscr{M}}_l^{(SLa)}} = \left[\lim_{r\to\infty} \frac{\det \left(\psi_1(r) \quad \psi_2(r) \quad \psi_3(r) \right)}{\det \left(\hat{\psi}_1(r) \quad \hat{\psi}_2(r) \quad \hat{\psi}_3(r) \right)} \right] \left[\lim_{r\to 0} \frac{\det \left(\psi_1(r) \quad \psi_2(r) \quad \psi_3(r) \right)}{\det \left(\hat{\psi}_1(r) \quad \hat{\psi}_2(r) \quad \hat{\psi}_3(r) \right)} \right]^{-1}$$

$$\mathscr{M}_l^{(SLa)} \psi_i(r) = 0 \qquad \widehat{\mathscr{M}}_l^{(SLa)} \hat{\psi}_i(r) = 0$$

Three independent solutions that are regular at r=0

(For the Fermi gauge, one cannot take the same initial conditions for the two determinants)

Semi-analytic decomposition

Case1: gauge symmetry broken at the false vacuum

BG gauge [M. Endo, T. Moroi, M. M. Nojiri, YS, ’17]

$$\frac{\det \mathcal{M}_l^{(c\bar{c})}}{\det \widehat{\mathcal{M}}_l^{(c\bar{c})}} = \frac{f_l^{(c\bar{c})}(\infty)}{\hat{f}_l^{(c\bar{c})}(\infty)}$$

$$\frac{\det \mathcal{M}_0^{(Sa)}}{\det \widehat{\mathcal{M}}_0^{(Sa)}} = \frac{\bar{\phi}(\infty)}{\bar{\phi}(0)}\left(\frac{f_0^{(c\bar{c})}(\infty)}{\hat{f}_0^{(c\bar{c})}(\infty)}\right)^2$$

$$\frac{\det \mathcal{M}_l^{(SLa)}}{\det \widehat{\mathcal{M}}_l^{(SLa)}} = \frac{\bar{\phi}(0)}{\bar{\phi}(\infty)}\frac{f_l^{(\eta)}(\infty)}{\hat{f}_l^{(\eta)}(\infty)}\left(\frac{f_l^{(c\bar{c})}(\infty)}{\hat{f}_l^{(c\bar{c})}(\infty)}\right)^2$$

Fermi gauge [M. Endo, T. Moroi, M. M. Nojiri, YS, ’17]

$$\frac{\det \mathcal{M}_l^{(c\bar{c})}}{\det \widehat{\mathcal{M}}_l^{(c\bar{c})}} = 1$$

$$\frac{\det \mathcal{M}_0^{(Sa)}}{\det \widehat{\mathcal{M}}_0^{(Sa)}} = \frac{\bar{\phi}(\infty)}{\bar{\phi}(0)}$$

$$\frac{\det \mathcal{M}_l^{(SLa)}}{\det \widehat{\mathcal{M}}_l^{(SLa)}} = \frac{\bar{\phi}(0)}{\bar{\phi}(\infty)}\frac{f_l^{(\eta)}(\infty)}{\hat{f}_l^{(\eta)}(\infty)}$$

FP modes

$$\left(\Delta_l - \xi g^2\bar{\phi}^2\right) f_l^{(c\bar{c})} = 0$$

Physical modes

$$\left(\Delta_l - g^2\bar{\phi}^2\right) f_l^{(\eta)} - \frac{2\bar{\phi}'}{r^2\bar{\phi}}\partial_r\left(r^2 f_l^{(\eta)}\right) = 0$$

Gauge zero mode

Gauge zero mode

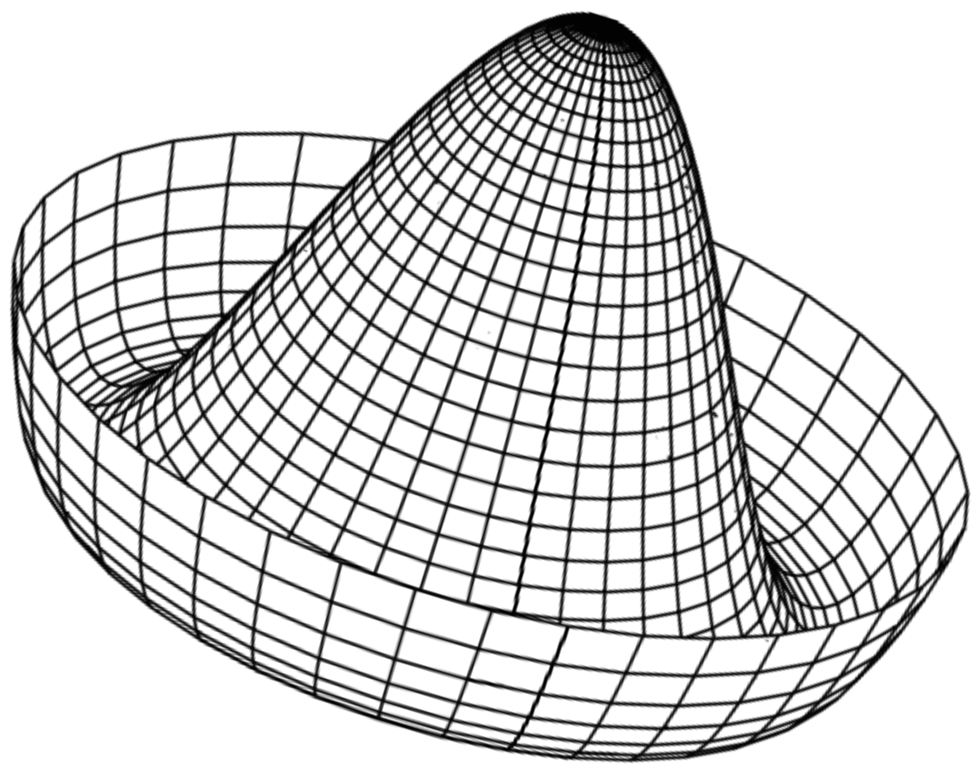
Case2: gauge symmetry restored at the false vacuum

[A. Kusenko, K. M. Lee, E. J. Weinberg, ’97]

BG gauge

$$\begin{pmatrix} \frac{1}{\xi}\left(-\Delta_0+\frac{3}{r^2}+\xi g^2\bar{\phi}^2\right) & 2g\bar{\phi}' \\ 2g\bar{\phi}' & -\Delta_0+\frac{(\Delta_0\bar{\phi})}{\bar{\phi}}+\xi g^2\bar{\phi}^2 \end{pmatrix}\begin{pmatrix} \partial_r f^{(c\bar{c})} \\ g\bar{\phi} f^{(c\bar{c})} \end{pmatrix}=0$$

Global symmetry: $\Phi \to ?,\ A_\mu \to ?$



Scalar potential is lifted

$J_G = ?$

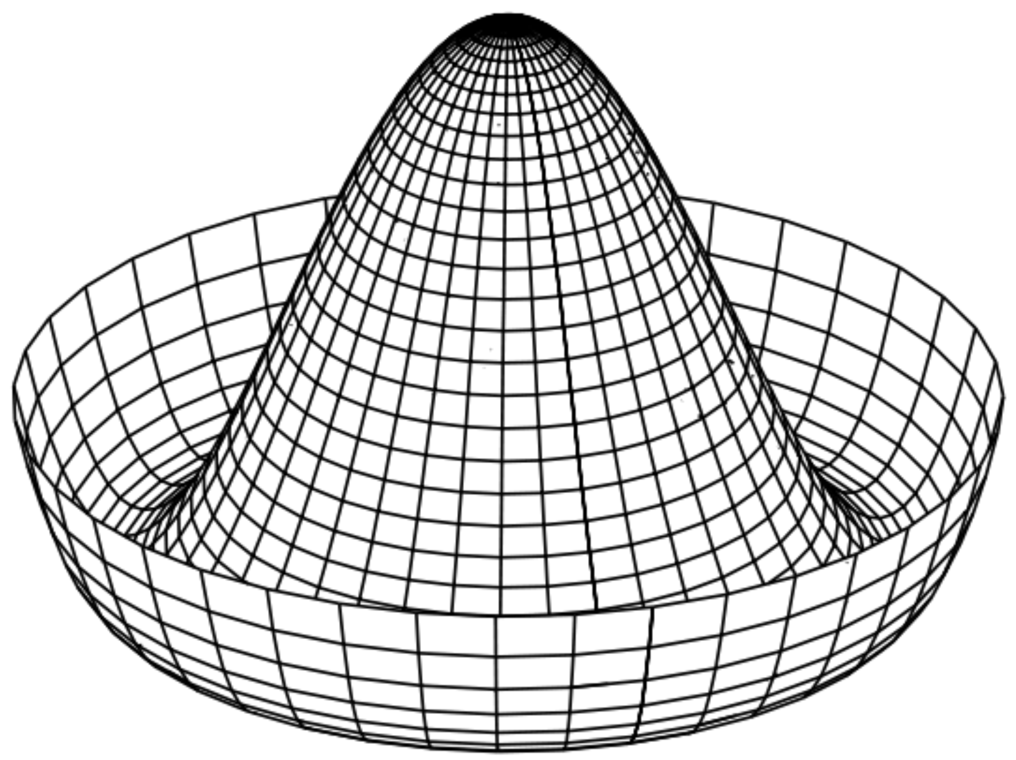
$V_G = ?$

Determine these to reproduce the Fermi gauge result

Fermi gauge

$$\begin{pmatrix} \frac{1}{\xi}\left(-\Delta_0+\frac{3}{r^2}+\xi g^2\bar{\phi}^2\right) & g\bar{\phi}'-g\bar{\phi}\partial_r \\ 2g\bar{\phi}'+g\bar{\phi}\partial_r+\frac{3}{r}g\bar{\phi} & -\Delta_0+\frac{(\Delta_0\bar{\phi})}{\bar{\phi}} \end{pmatrix}\begin{pmatrix} 0 \\ g\bar{\phi} \end{pmatrix}=0$$

Global symmetry: $\Phi \to e^{i\theta}\Phi,\ A_\mu \to A_\mu$



$$J_G = \sqrt{\pi\int \mathrm{d}r r^3\bar{\phi}^2}$$

$$V_G = 2\pi$$

Semi-analytic decomposition

Case2: gauge symmetry restored at the false vacuum

BG gauge [M. Endo, T. Moroi, M. M. Nojiri, YS, '17]

$$\frac{\det \mathcal{M}_l^{(c\bar{c})}}{\det \widehat{\mathcal{M}}_l^{(c\bar{c})}} = \frac{f_l^{(c\bar{c})}(\infty)}{\hat{f}_l^{(c\bar{c})}(\infty)}$$

$$\frac{\det' \mathcal{M}_0^{(Sa)}}{\det \widehat{\mathcal{M}}_0^{(Sa)}} = \frac{1}{2\pi m_h \bar{\phi}(0)} \lim_{r\to\infty} \frac{1}{r^3 \bar{\phi}(r) f_0^{(h)}(r)} \frac{\pi r^3 \partial_r f_0^{(c\bar{c})}(r)}{\xi g^2 f_0^{(c\bar{c})}(r)} \left(\frac{f_0^{(c\bar{c})}(\infty)}{\hat{f}_0^{(c\bar{c})}(\infty)} \right)^2$$

$$\frac{\det \mathcal{M}_l^{(SLa)}}{\det \widehat{\mathcal{M}}_l^{(SLa)}} = \frac{l}{2} \frac{\bar{\phi}(0)}{m_h} \lim_{r\to\infty} \frac{f_l^{(\eta)}(r)}{r \hat{f}_l^{(h)}(r) \bar{\phi}(r)} \left(\frac{f_l^{(c\bar{c})}(\infty)}{\hat{f}_l^{(c\bar{c})}(\infty)} \right)^2$$

Fermi gauge [M. Endo, T. Moroi, M. M. Nojiri, YS, '17]

$$\frac{\det \mathcal{M}_l^{(c\bar{c})}}{\det \widehat{\mathcal{M}}_l^{(c\bar{c})}} = 1$$

$$\frac{\det' \mathcal{M}_0^{(Sa)}}{\det \widehat{\mathcal{M}}_0^{(Sa)}} = \frac{1}{2\pi m_h \bar{\phi}(0)} \lim_{r\to\infty} \frac{1}{r^3 \bar{\phi}(r) f_0^{(h)}(r)} J_G^2$$

$$\frac{\det \mathcal{M}_l^{(SLa)}}{\det \widehat{\mathcal{M}}_l^{(SLa)}} = \frac{l}{2} \frac{\bar{\phi}(0)}{m_h} \lim_{r\to\infty} \frac{f_l^{(\eta)}(r)}{r \hat{f}_l^{(h)}(r) \bar{\phi}(r)}$$

FP modes

$$\left(\Delta_l - \xi g^2 \bar{\phi}^2\right) f_l^{(c\bar{c})} = 0$$

Physical modes

$$\left(\Delta_l - g^2 \bar{\phi}^2\right) f_l^{(\eta)} - \frac{2\bar{\phi}'}{r^2 \bar{\phi}} \partial_r \left(r^2 f_l^{(\eta)} \right) = 0 \qquad \left(\Delta_l - m_h^2\right) \hat{f}_l^{(h)} = 0$$

Jacobian for the BG gauge

$$\mathscr{A} = V_G J_G \left(\frac{\det' \mathscr{M}^{(Sa)_0}}{\det \widehat{\mathscr{M}}_0^{(Sa)}}\right)^{-1/2} \left(\frac{\det \mathscr{M}_0^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_0^{(c\bar{c})}}\right) \times \cdots$$

BG gauge

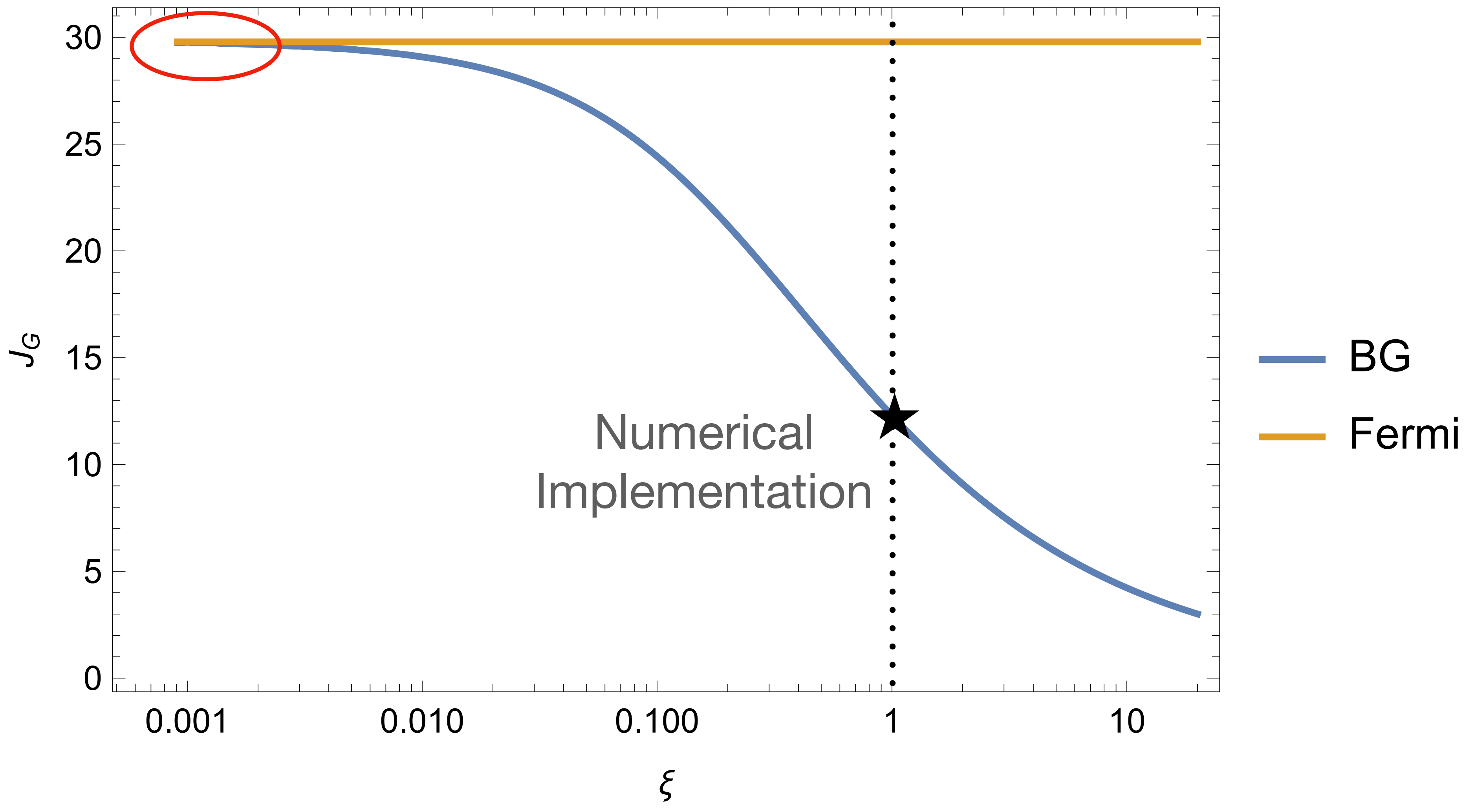
$$V_G J_G^{\mathrm{BG}} \left(\frac{\det' \mathscr{M}_0^{(Sa)}}{\det \widehat{\mathscr{M}}_0^{(Sa)}}\right)^{-1/2} \left(\frac{\det \mathscr{M}_0^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_0^{(c\bar{c})}}\right) = V_G \sqrt{2\pi m_h \bar{\phi}(0)} \lim_{r\to\infty} \sqrt{r^3 \bar{\phi}(r) f_0^{(h)}(r)}\, J_G^{\mathrm{BG}} \sqrt{\frac{\xi g^2 f_0^{(c\bar{c})}(r)}{\pi r^3 \partial_r f_0^{(c\bar{c})}(r)}}$$

Fermi gauge

$$V_G J_G \left(\frac{\det' \mathscr{M}_0^{(Sa)}}{\det \widehat{\mathscr{M}}_0^{(Sa)}}\right)^{-1/2} \left(\frac{\det \mathscr{M}_0^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_0^{(c\bar{c})}}\right) = V_G \sqrt{2\pi m_h \bar{\phi}(0)} \lim_{r\to\infty} \sqrt{r^3 \bar{\phi}(r) f_0^{(h)}(r)}$$

$$J_G^{\mathrm{BG}} = \sqrt{\frac{\pi r^3 \partial_r f_0^{(c\bar{c})}(r)}{\xi g^2 f_0^{(c\bar{c})}(r)}}$$

Massless would-be NG boson
(Same flat direction)



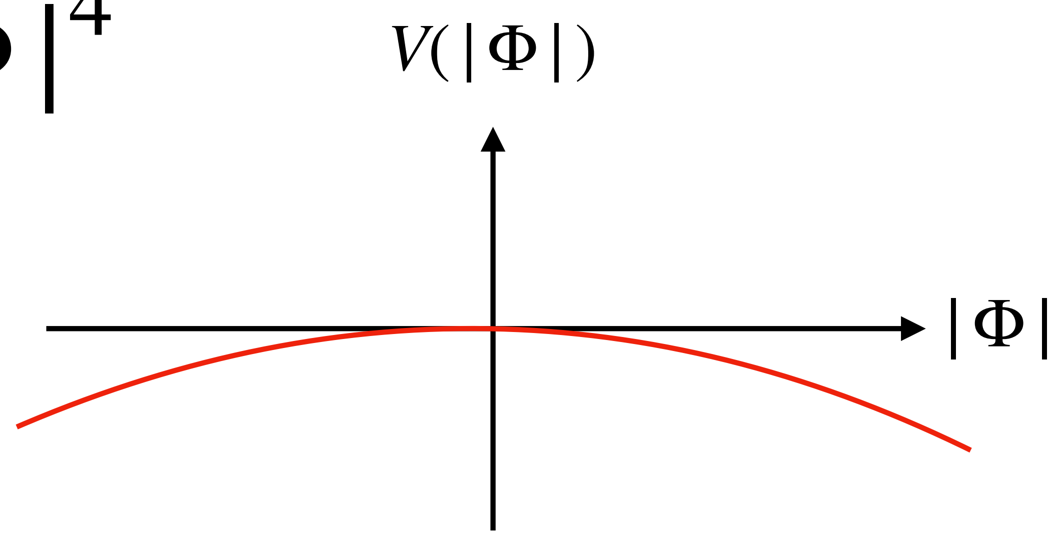
Fubini instanton

Fubini instanton and decay rate

Potential

$$V(|\Phi|) = \lambda|\Phi|^4$$

$$(\lambda < 0)$$



Bounce

$$\bar{\phi}(r) = \sqrt{\frac{8}{|\lambda|}}\frac{R}{R^2 + r^2}$$

R: Arbitrary constant

Decay rate

$$\gamma = \int \mathrm{d}R \mathscr{A} e^{-\mathscr{B}} \qquad \mathscr{B} = \frac{8\pi^2}{3|\lambda|}$$

$$\mathscr{A} = J_D J_T \left[\prod_{l=0}^{\infty}\left(\frac{\det' \mathscr{M}_l^{(h)}}{\det \widehat{\mathscr{M}}_l^{(h)}}\right)^{-(l+1)^2/2}\right]$$

Translational+dilatational zero modes

$$\times \left[\prod_{l=1}^{\infty}\left(\frac{\det \mathscr{M}_l^{(T)}}{\det \widehat{\mathscr{M}}_l^{(T)}}\right)^{-(l+1)^2}\right]$$

$$\times \left[\prod_{l=0}^{\infty}\left(\frac{\det \mathscr{M}_l^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_l^{(c\bar{c})}}\right)^{(l+1)^2}\right]$$

$$\times V_G J_G \left(\frac{\det' \mathscr{M}^{(Sa)_0}}{\det \widehat{\mathscr{M}}_0^{(Sa)}}\right)^{-1/2} \left[\prod_{l=1}^{\infty}\left(\frac{\det \mathscr{M}_l^{(SLa)}}{\det \widehat{\mathscr{M}}_l^{(SLa)}}\right)^{-(l+1)^2/2}\right]$$

Gauge zero mode

Gauge independence

Fermi gauge

[A. Andreassen, W. Frost, M. D. Schwartz, ’17; S. Chigusa, T. Moroi, YS, ’17 & ’18]

$$\frac{\det \mathscr{M}_l^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_l^{(c\bar{c})}} = 1$$

$$\frac{\det \mathscr{M}_l^{(SLa)}}{\det \widehat{\mathscr{M}}_l^{(SLa)}} = \frac{l}{l+2} \frac{\Gamma(l+1)\Gamma(l+2)}{\Gamma(l+1-z_g)\Gamma(l+2+z_g)}$$

Gauge independent

$$\frac{\det' \mathscr{M}_0^{(Sa)}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} = \frac{|\lambda|}{16\pi} J_G^2$$

$$\frac{\det \mathscr{M}_l^{(T)}}{\det \widehat{\mathscr{M}}_l^{(T)}} = \frac{\Gamma(l+1)\Gamma(l+2)}{\Gamma(l+1-z_g)\Gamma(l+2+z_g)}$$

$$J_G = \sqrt{\pi \int dr r^3 \bar{\phi}^2} = \lim_{r\to\infty} \sqrt{\frac{8\pi R^2}{|\lambda|} \ln r}$$

$$z_g = -\frac{1}{2}\left(1 - \sqrt{1 - \frac{8g^2}{|\lambda|}}\right)$$

Jacobian itself is divergent (gauge zero mode is not normalisable)

Multi-field bounce

Symmetries

Gauge charges of bounce fields

$$D_\mu \phi = (\partial_\mu + g_a T^a A_\mu^a)\phi$$

$$(T^a)^T = -T^a \qquad [T^a, T^b] = -f^{abc}T^c$$

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \vdots \end{pmatrix}$$: Vector of real scalars

$$a : \overbrace{1,\cdots,n_B}^{\text{Massive at false vacuum}}, \underbrace{n_B+1,\ldots,n_G}_{\text{Massless at false vacuum}} = n_B + n_U$$

$$A : 1,\cdots,n_U$$ is the index for these

$$M_{ia} = -\sum_k g_a T^a_{ik}\bar{\phi}_k$$

* $M^T M$ is the gauge boson mass matrix

* We ignore the gauge bosons that do not couple to the bounce

Symmetries broken by bounce

Symmetry of the action (Fermi gauge)

$$S_E[\bar{\phi}] = S_E[e^{\theta^A \tilde{T}^A + \theta^\alpha_G \tilde{T}^\alpha_G}\bar{\phi}]$$

$$\alpha : 1,\cdots,n_{NG}$$

Pure NG bosons

$$\tilde{T}^A = \sum_B \kappa_{AB} T^B$$

False vacuum is symmetric, bounce is not

$$\bar{\phi}(\infty) = e^{\theta^A \tilde{T}^A + \theta^\alpha_G \tilde{T}^\alpha_G}\bar{\phi}(\infty)$$

$$\bar{\phi}(r) \neq e^{\theta^A \tilde{T}^A + \theta^\alpha_G \tilde{T}^\alpha_G}\bar{\phi}(r)$$

Orthogonal (would-be) NG bosons

$$\int d^4x (\tilde{T}^A\bar{\phi})^T(\tilde{T}^B\bar{\phi}) = 0 \quad (A \neq B)$$

$$\int d^4x (\tilde{T}^\alpha_G\bar{\phi})^T(\tilde{T}^\beta_G\bar{\phi}) = 0 \quad (\alpha \neq \beta)$$

$$\int d^4x (\tilde{T}^A\bar{\phi})^T(\tilde{T}^\beta_G\bar{\phi}) = 0$$

(This is automatic)

Jacobian

$$J_G^{\mathrm{BG}} = \underbrace{\sqrt{\det \mathscr{K}^G}}_{\text{Global symmetry}} \underbrace{\sqrt{\det \mathscr{K}}\, \det \kappa \left(\prod_A g_A^2 \right)^{-1/2}}_{\text{Gauge symmetry}}$$

$$\mathscr{K}^G_{\alpha\beta} = \pi \int \mathrm{d}r r^3 \left(\tilde{T}^\alpha_G \bar{\phi} \right)^T \left(\tilde{T}^\beta_G \bar{\phi} \right) \qquad \mathscr{K}_{AB} = \lim_{r\to\infty} \frac{\pi r^3}{\xi} \left[\left(\partial_r \Psi_0^{(c\bar{c})}(r) \right) \left(\Psi_0^{(c\bar{c})}(r) \right)^{-1} \right]_{AB}$$

$$(\Delta_0 - \xi M^T M)\Psi_0^{(c\bar{c})} = 0$$

V_G : Group volume measured with $\tilde{T}^A, \tilde{T}^\alpha$

Fubini instanton

Multi-field quartic potential

$$V = \lambda_1 |\Phi_1|^4 + \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \cdots$$

Proof is available in [S. Oda, YS, D. Takahashi, ’19]

$$V = \lambda_{\rm eff} |\Phi|^4$$

(negatively largest direction)

Standard Model

ELVAS

https://github.com/YShoji-HEP/ELVAS

README MIT license

ELVAS

C++ Package for ELectroweak VAcuum Stability

Introduction

ELVAS is a C++ package for the calculation of the decay rate of a false vacuum at the one-loop level, based on the formulae developed in [1, 2]. ELVAS is applicable to models with the following features:

- Only one scalar boson is responsible for the vacuum decay.
- Classical scale invariance (approximately) holds. In particular, the potential of the scalar field responsible for the vacuum decay should be well approximated by the quartic form for the calculation of the bounce solution. (Thus, the bounce is nothing but the so-called Fubini instanton.)
- The instability of the scalar potential occurs due to RG effects; thus, the quartic coupling constant becomes negative at a high scale.

If you use ELVAS in scholarly work, please cite [1] and [2].

Download and Install

ELVAS requires a c++ compiler that supports C++14 and the *boost* library (http://www.boost.org/users/download/#live). The required version of *boost* is 1.59.0 or higher.

Unix

For UNIX systems, follow the instruction below.