Fermion determinants:

- Grassmann variables (statistics)
- Dirac representation of $O(4)$ (spin)
- pheno implications (top quark)
- zero modes (?)

spin-statistics theorem:

$\Rightarrow$ particles with half-integer spin have Fermi statics

Grassmann Variables:

Set of anticommuting objects $\{\vartheta_i\}_{i=1}^{N}$

$$\vartheta_i \vartheta_j + \vartheta_j \vartheta_i = 0 \quad \to \quad \vartheta_i^2 = 0 \quad \forall i$$

for finite N, there exist reps. of $\{\vartheta_i\}$ in terms of $2N \times 2N$ matrices example: $N = 1$ $\qquad \vartheta\vartheta + \vartheta\vartheta = 2\vartheta^2 = 0$

$$\vartheta = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \vartheta^2 = 0$$

$N = 2$ $\qquad 2N \times 2N = 4 \times 4$

$$\vartheta_1 = \left(\begin{array}{cc|cc} 0 & 1 & \multicolumn{2}{c}{0} \\ 0 & 0 & & \\ \hline \multicolumn{2}{c|}{0} & 0 & 1 \\ & & 0 & 0 \end{array}\right) \qquad \vartheta_2 = \left(\begin{array}{cc|cc} \multicolumn{2}{c|}{0} & 1 & 0 \\ & & 0 & -1 \\ \hline \multicolumn{2}{c|}{0} & \multicolumn{2}{c}{0} \end{array}\right)$$

$$\vartheta_1^2 = 0 \quad \vartheta_2^2 = 0 \quad \vartheta_1\vartheta_2 = -\vartheta_2\vartheta_1$$

$$\mathcal{Q} = \{ c_0 + c_1\vartheta_1 + \dots + c_N\vartheta_N + c_{12}\vartheta_1\vartheta_2 + c_{13}\vartheta_1\vartheta_3 + \dots$$
$$+ \dots + c_{12\dots N}\vartheta_1\vartheta_2\vartheta_3\dots\vartheta_N \} \equiv \text{Grassmann algebra}$$

$$\{x_i\}_{i=1}^{M} \qquad x_i x_j - x_j x_i = 0$$

$$\vartheta_1\vartheta_2\vartheta_1 = -\vartheta_1\vartheta_1\vartheta_2 = -\vartheta_1^2\vartheta_2 = 0$$

$$P(x_i)$$

Berezin Integral

$$\int_a^b f(x,y,\dots)\,dx = F(y,\dots)$$

$$\int a \underbrace{f}_{f(\vartheta)} + b \underbrace{g}_{g(\vartheta)} = a\int f + b\int g$$

$$\Downarrow$$

$$\int d\vartheta = 0 \qquad \int d\vartheta\,\vartheta = 1$$

$$\frac{\partial}{\partial\vartheta} 1 = 0 \qquad \frac{\partial}{\partial\vartheta}\vartheta = 1$$

Multiple integrals: $\quad d\vartheta_i\, d\vartheta_j = -d\vartheta_j\, d\vartheta_i$

$$\int d\vartheta_1 \dots d\vartheta_N\, \vartheta_N \dots \vartheta_1 = 1$$

$$\boxed{\int d\vartheta_1\, d\bar\vartheta_1 \dots d\vartheta_N\, d\bar\vartheta_N\; e^{\bar\vartheta_i M_{ij} \vartheta_j} = \det M}$$

$$\int dx_1 \dots dx_N\; e^{-\frac{1}{2} x_i A_{ij} x_j} = \sqrt{\frac{\pi}{\det A}}$$

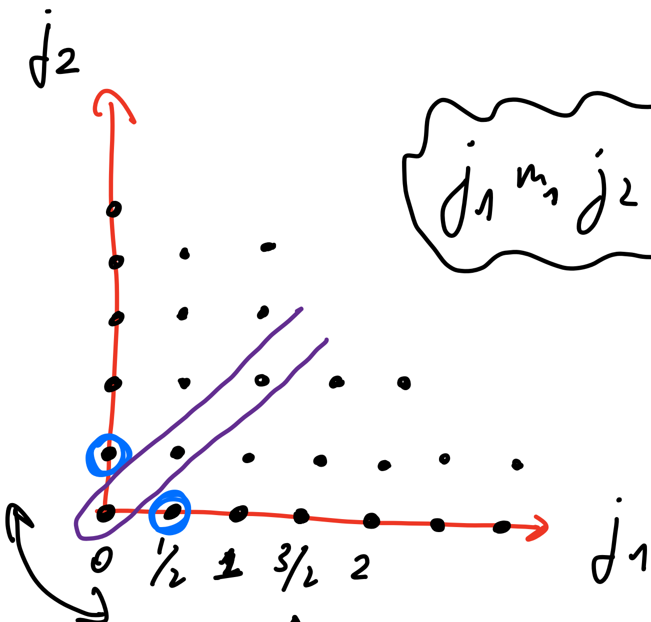
$$\int d\vartheta_1\, d\bar\vartheta_1\, d\vartheta_2\, d\bar\vartheta_2\; e^{M_{11}\bar\vartheta_1\vartheta_1 + M_{12}\bar\vartheta_1\vartheta_2 + \dots}$$

$$= \int d\vartheta_1 \dots d\bar\vartheta_2 \left[\bar\vartheta_2\vartheta_2\bar\vartheta_1\vartheta_1 \left(M_{11}M_{22} - M_{12}M_{21} \right) + \dots \right]$$

$$\Gamma\big|_\psi \sim \int [d\psi][d\bar\psi]\; e^{\bar\psi S''(\phi_0)\psi} = \det S''(\phi_0)$$

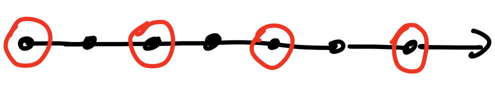
* Weyl

* $$\int d\vartheta_1 .. d\vartheta_N\; e^{\frac{1}{2}\vec\vartheta M \vec\vartheta} = Pf(M)$$



$j_1\, m_1\, j_2\, m_2$

$$SO(3,1) \to SO(4)$$
$$\downarrow$$
$$\cong SU(2) \times SU(2)$$
$$\sim SO(3)$$
$$\rho_{ij}$$



$$\psi \sim \left(\tfrac{1}{2}, 0\right) \oplus \left(0, \tfrac{1}{2}\right)$$

$$L_{ij} = x_i \partial_j - x_j \partial_i$$

$$S''(\phi_{cl})$$

$$=$$

$$\not\partial + \frac{y_t}{\sqrt{2}} \phi_{cl}(x)$$

$$= \gamma^{\mu}_{\alpha\beta} \partial_\mu + \delta_{\alpha\beta} \frac{y_t}{\sqrt{2}} \phi_{cl}(x)$$

$$\psi_\alpha \qquad \Lambda \phi(x) = \phi(\Lambda x)$$

$$(\Box + \phi_{cl}^2) \phi(x)$$

$$\Phi(x) = \sum_{l, m_1, m_2} R_l(\rho)\, Y_{l m_1 m_2}(\vartheta\, \varphi_1\, \varphi_2)$$

$$[O, L_{\mu\nu}] = 0$$

$$(L_{\mu\nu})_{\alpha\beta} = (x_i \partial_j - x_j \partial_i)\, \delta_{\alpha\beta} + [\gamma_i\, \gamma_j]_{\alpha\beta}$$

$$\psi_\alpha(x) \longrightarrow \rho(\Lambda)_{\alpha\beta}\, \psi_\beta(\Lambda x)$$

Infinitesimally:

$$\psi \to \psi + \alpha_{ij} L_{ij} \psi$$

generator of rotations

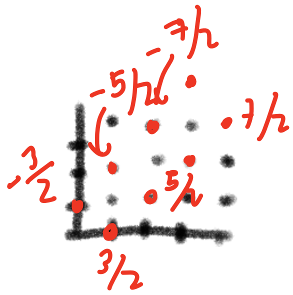
$$L_{ij} = x_i \partial_j - x_j \partial_i + [\gamma_i \gamma_j]$$

$\Lambda \cdot \psi$ is a solution iff ψ is

$$\Rightarrow [\mathcal{O}, L_{ij}] = 0$$

$$\Phi(\vec{x}) = \sum_{\ell,m} R_{\ell,m}(r)\, Y_{\ell,m}(\vartheta,\varphi)$$

$$\Phi(\vartheta,\phi) = \sum_{\ell,m} c_{\ell,m}\, Y_{\ell,m}(\vartheta,\varphi)$$



Avan, de Vega:

$$\psi^\alpha(x^\mu) = \sum_{k,m_L,m_R} \left(\alpha_+^k(r)\, \mathcal{Y}^\alpha_{k,m_L;\,k+\frac{1}{2}\, m_R}(\Theta) \right)$$

$$+\ \beta_+(r)\ \bar{\mathcal{Y}}^{\alpha}_{k,m_L;\,k+\frac{1}{2}\,m_R}(\Theta)$$

$(2j+1)$

$\deg[(k,\ k+\tfrac{1}{2})]$

$(2k+1)\left(2(k+\tfrac{1}{2})+1\right)$

$= (2k+1)(2k+2)$

$$+\ \alpha_-(r)\ \mathcal{Y}^{\alpha}_{k,m_L;\,k-\frac{1}{2}\,m_R}(\Theta) + \beta_-(r)\ \bar{\mathcal{Y}}^{\alpha}_{k,m_L;\,k-\frac{1}{2}\,m_R}(\theta)\Big)$$

$$\begin{pmatrix} -\frac{d^2}{dr^2}+\frac{J(J-1)}{r^2}+\frac{y_t^2}{2}h^2(r) & \frac{y_t}{\sqrt{2}}h'(r) \\ \frac{y_t}{\sqrt{2}}h'(r) & \cdots\ \frac{J(J+1)}{r^2}\ \cdots \end{pmatrix}\begin{pmatrix}\alpha^{\pm}_J(r)\\ \beta^{\pm}_J(r)\end{pmatrix} = 0$$

$$\alpha \sim r^J \qquad \beta \sim r^{J+1}$$

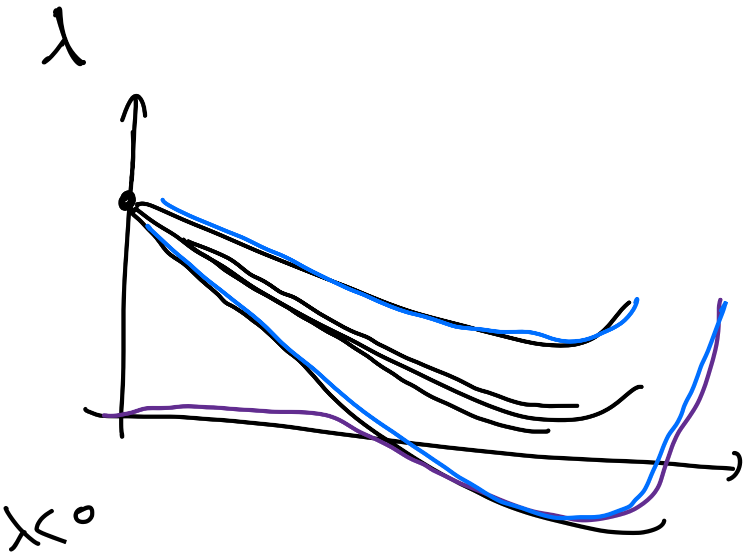
$$\rho_\alpha = \frac{\alpha}{\alpha_0} \qquad \rho_\beta = \frac{\beta}{\beta_0}$$

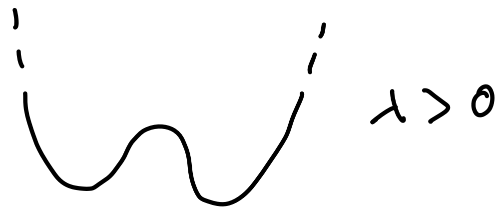
for each J

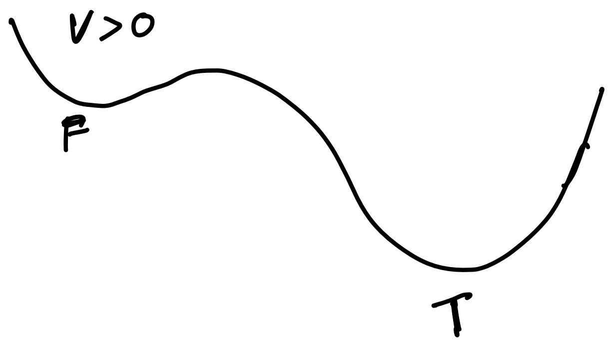
$$\lim_{r\to\infty} \det\begin{pmatrix}\rho^1_\alpha(r) & \rho^2_\alpha(r)\\ \rho^1_\beta(r) & \rho^2_\beta(r)\end{pmatrix}$$

y_t $\qquad$ $y_t \bar{t} \phi t$

y_t^2







$V \geq 0$

$V = 0$

$$\det D \sim \frac{1}{2} \det D^2$$