What's next in Left-Right

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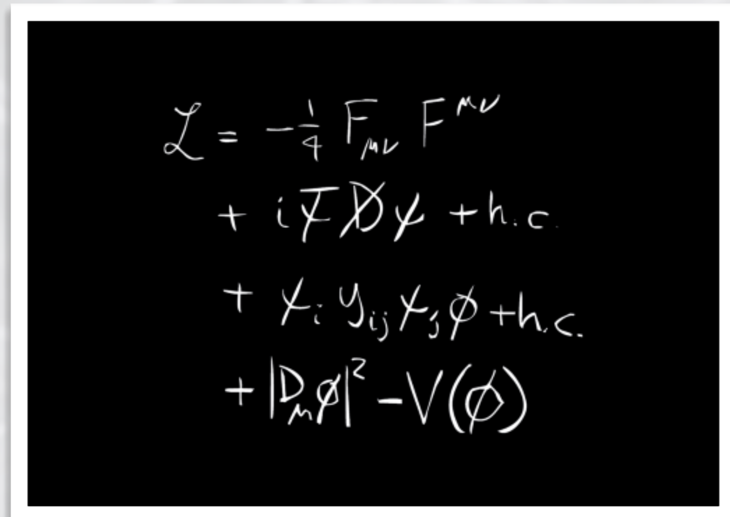
Bled 2024

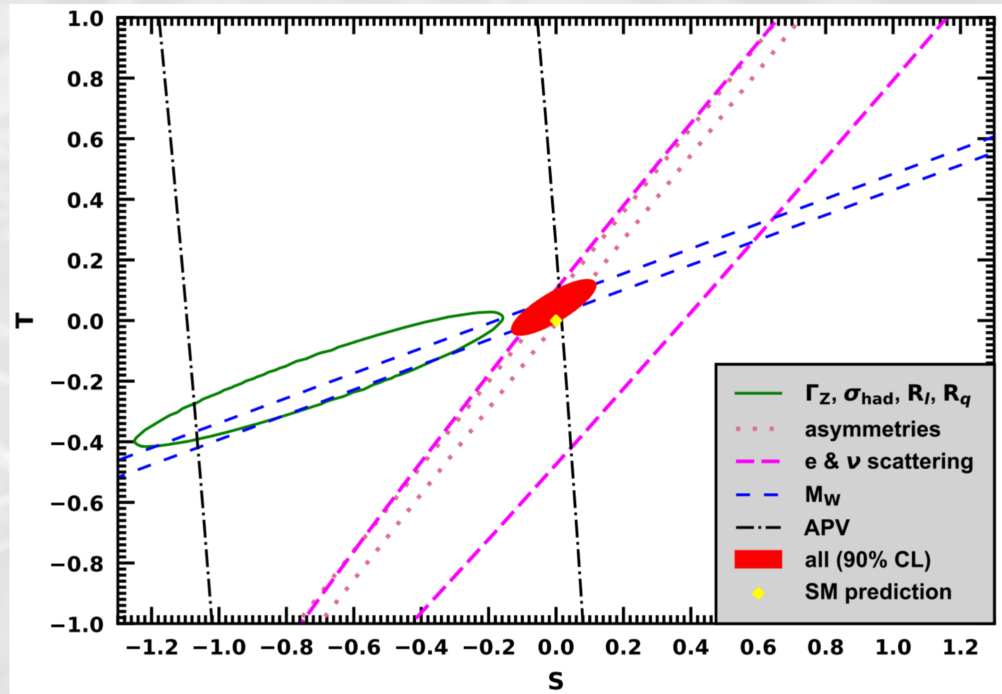
With Miha Nemevšek and Fabrizio Nesti: 2403.07756

The Standard Model — A success story

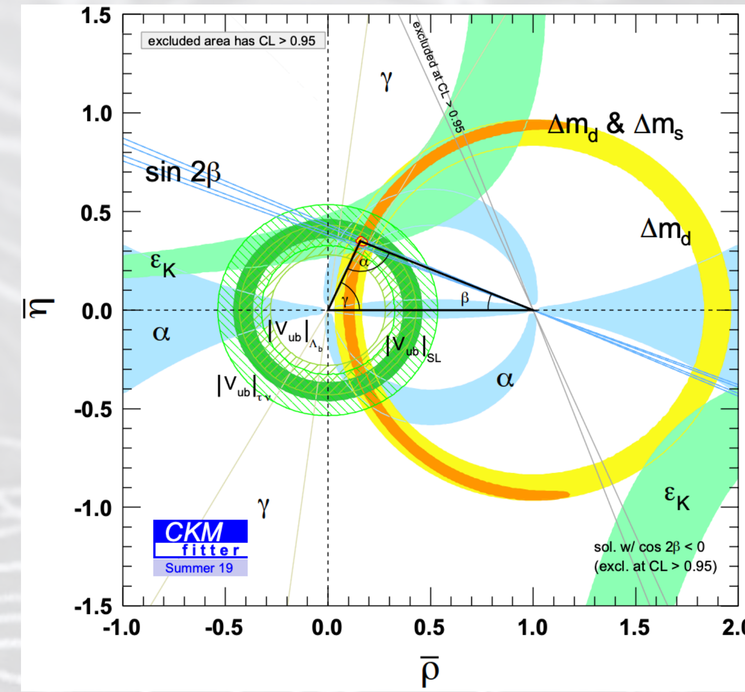


$$\mathscr{G}_{\rm SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

+ some ***accidental*** **symmetries**



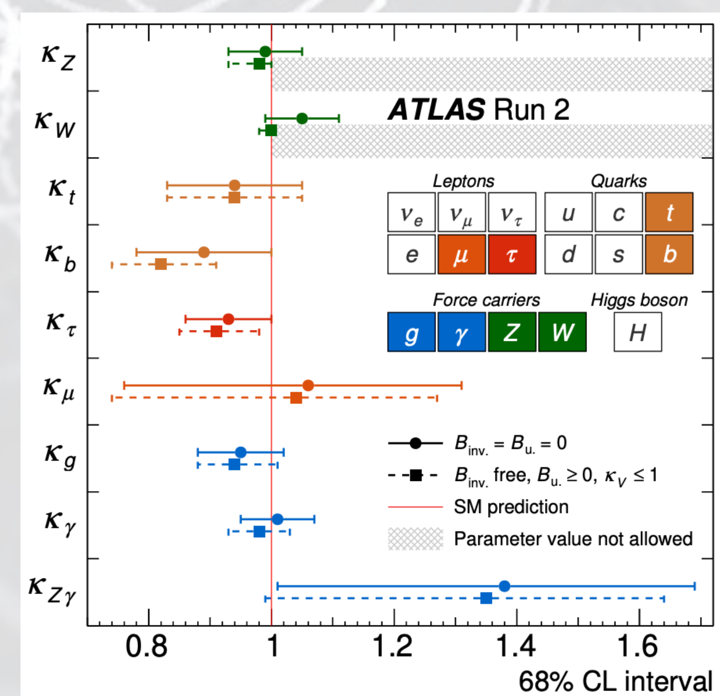
Electroweak fit



CKM paradigm of flavour mixing

Hundreds of experimental measurements overwhelmingly confirm the SM!

⇒ **Just need to completely understand the Higgs sector now?**



Higgs couplings

Strong arguments in f(l)avour of New Physics!

Observations unaccounted for in SM: ν-oscillations, Dark matter,

baryon asymmetry of the Universe (also some theoretical caveats…)

How to unveil the NP model at work?

⇒Test SM symmetries with flavour observables:

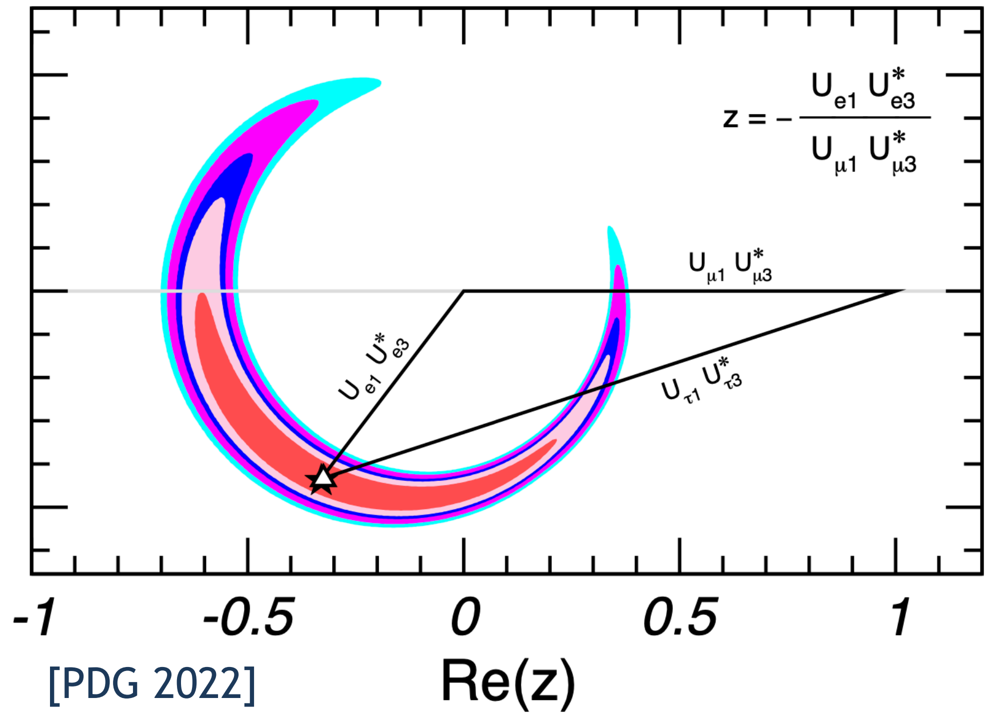
(c)LFV, lepton flavour universality violation, …

ν-oscillations 1st laboratory *evidence* of New Physics!

- New mechanism of mass generation? **Majorana fields?**
- New sources of CP violation?

Several experimental puzzles remain:

- Absolute mass scale?
- Mass ordering? (NO vs IO)
- CP violation maximal?



[PDG 2022]

Making neutrino masses

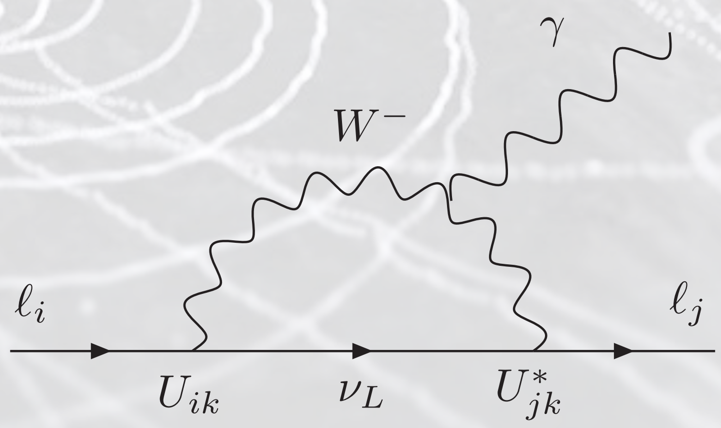
Neutrinos oscillate ⇒ **neutral lepton flavour violated, neutrinos are massive,**
new sources of **CPV?**

Extend SM to accommodate $\nu_\alpha \rightsquigarrow \nu_\beta$: ad-hoc 3 ν_R ⇒ Dirac masses, "**SM**$_{m_\nu}$", $U_{\mathbf{PMNS}}$

In **SM**$_{m_\nu}$: **flavour-universal** lepton couplings, lepton number conserved

cLFV possible … but not observable! $\mathrm{BR}(\mu \to e\gamma) \propto |\sum U^*_{\mu i} U_{ei} m^2_{\nu_i}/m^2_W| \simeq 10^{-54}$

EDMs still tiny… (2-loop from δ_{CP}, $|d_\ell| \sim 10^{-35} e\mathrm{cm}$)



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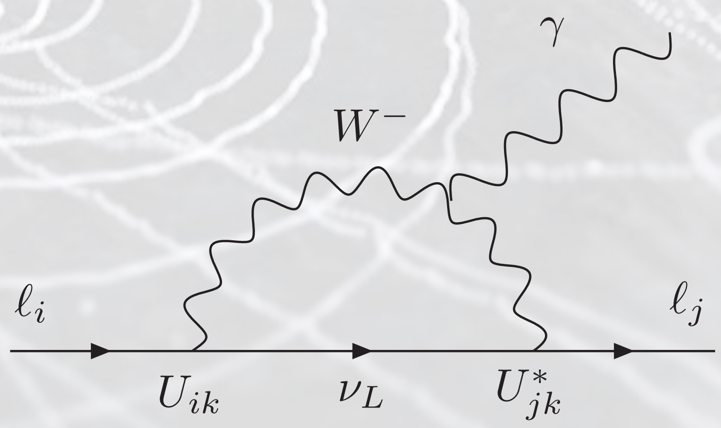


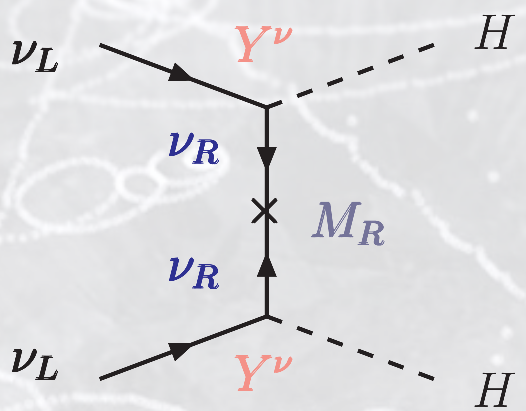
Nothing forbids an additional mass term of the form $\mathscr{L} \supseteq \frac{m_{RR}}{2} \bar{\nu}_R \nu_R^C$!

⇒ Neutrinos become **Majorana** particles — also SM-like neutrinos: $\mathscr{L}_{\text{eff}} \sim \frac{m_{LL}}{2} \bar{\nu}_L \nu_L^C$

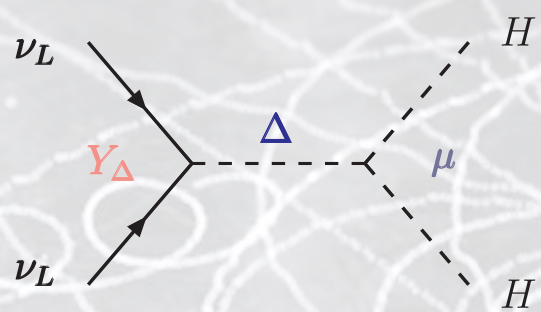
Making neutrino masses

Effective mass term $\mathscr{L}_{\text{eff}} \sim \frac{m_{LL}}{2}\,\bar{\nu}_L\,\nu_L^C$ from Weinberg operator: $\mathscr{L}^{d=5} \sim \frac{h_{ij}}{2\Lambda}(H\,L_i\,H\,L_j)$

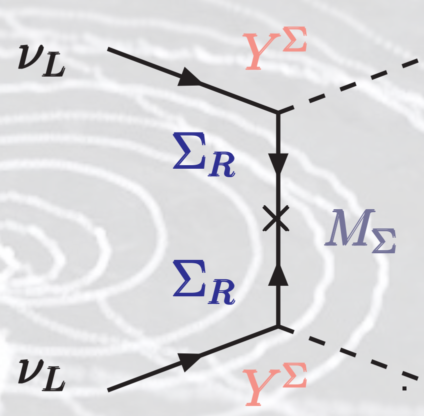
Different realisations: $\mathscr{O}^5_{\text{typeI}} \sim (L_i^T\,H)(L_j^T H)$, $O^5_{\text{typeII}} \sim (L_i^T \sigma_a L_j)(H^T \sigma_a H)$, $\mathscr{O}^5_{\text{typeIII}} \sim (L_i^T \sigma_a H)(L_j^T \sigma_a H)$



Type I (fermion singlet)



Type II (scalar triplet)



Type III (fermion triplet)

Mass terms: $m_\nu^I \sim -\,v^2 Y_\nu^T \frac{1}{M_R} Y_\nu$, $m_\nu^{II} \sim -\,v^2 Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} \sim -\,Y_\Delta v_\Delta$, $m_\nu^{III} \sim -\,Y_\Sigma^T \frac{v^2}{2M_\Sigma} Y_\Sigma$

Countless more possibilities with higher odd-dimensional operators or loop-level realisations…

(Actually they are countable, see e.g. [John Gargalionis and Ray Volkas: 2009.13537]

Making neutrino masses

Mechanisms of m_ν generation: account for oscillation data

and ideally address SM issues — **BAU** (leptogenesis), **DM** candidates, strong **CP**, **hierarchy**,...

Many well motivated possibilities, featuring distinct **NP states** (singlets, triplets)

Realised at very different scales $\Lambda_{\text{EW}} \rightsquigarrow \Lambda_{\text{GUT}}$

$\Rightarrow$ Expect *very* different phenomenological impact

Compare "vanilla" **type I seesaw** vs. low-scale seesaw:

High scale: $\mathcal{O}(10^{10-15}$ GeV)	Low scale: $\mathcal{O}$(MeV - TeV)
Theoretically **"natural"** $Y^\nu \sim 1$	Finetuning of Y^ν (or approximate LN conservation)
"Vanilla" leptogenesis	Leptogenesis possible (resonant, ...)
Decoupled new states	New states **within experimental reach!**
	Collider, high-intensities ("leptonic observables")

$\Rightarrow$ low-scale seesaws (and variants): **non-decoupled states**, modified lepton currents!

$\Rightarrow$ rich phenomenology at **colliders, high intensities** and low energies

testability!!

(Also expect tight constraints)

Introducing Left-Right: Motivation

Features: Mohapatra, Senjanović ‘75

- Combination of **type I** & **type II** seesaw mechanism, new states $\sim \mathcal{O}(\mathrm{TeV})$
- Can address the **strong CP problem** (see e.g. [2107.10852])
- Lightest **right-handed neutrino** can be a **Dark Matter candidate** [2312.00129]
- Low(ish)-scale **leptogenesis** can be implemented [C. Hati et al. '18]
- Left-right symmetry $\mathcal{G}_{\mathrm{LR}} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

  appears in the breaking of **GUTs**, e.g.:

$$SO(10) \to SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \to \mathcal{G}_{LR} \to \mathcal{G}_{\mathrm{SM}}$$

Introducing Left-Right: Motivation

Feature

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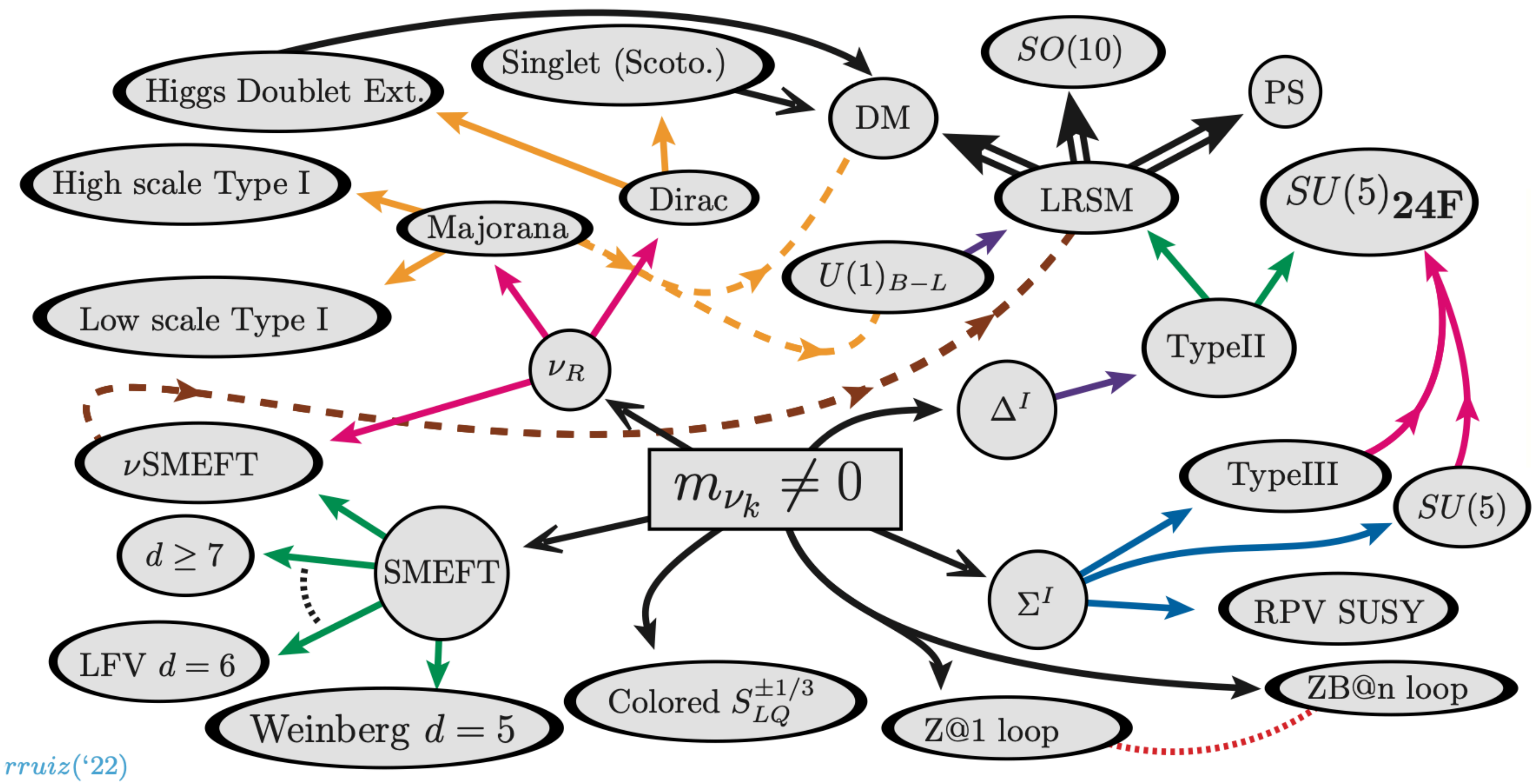
These core ideas can be realized in ***many*** ways!

Minkowski ('77); Yanagida ('79); Glashow & Levy ('80); Gell-Mann et al., ('80); Mohapatra & Senjanović ('82); + *many* others



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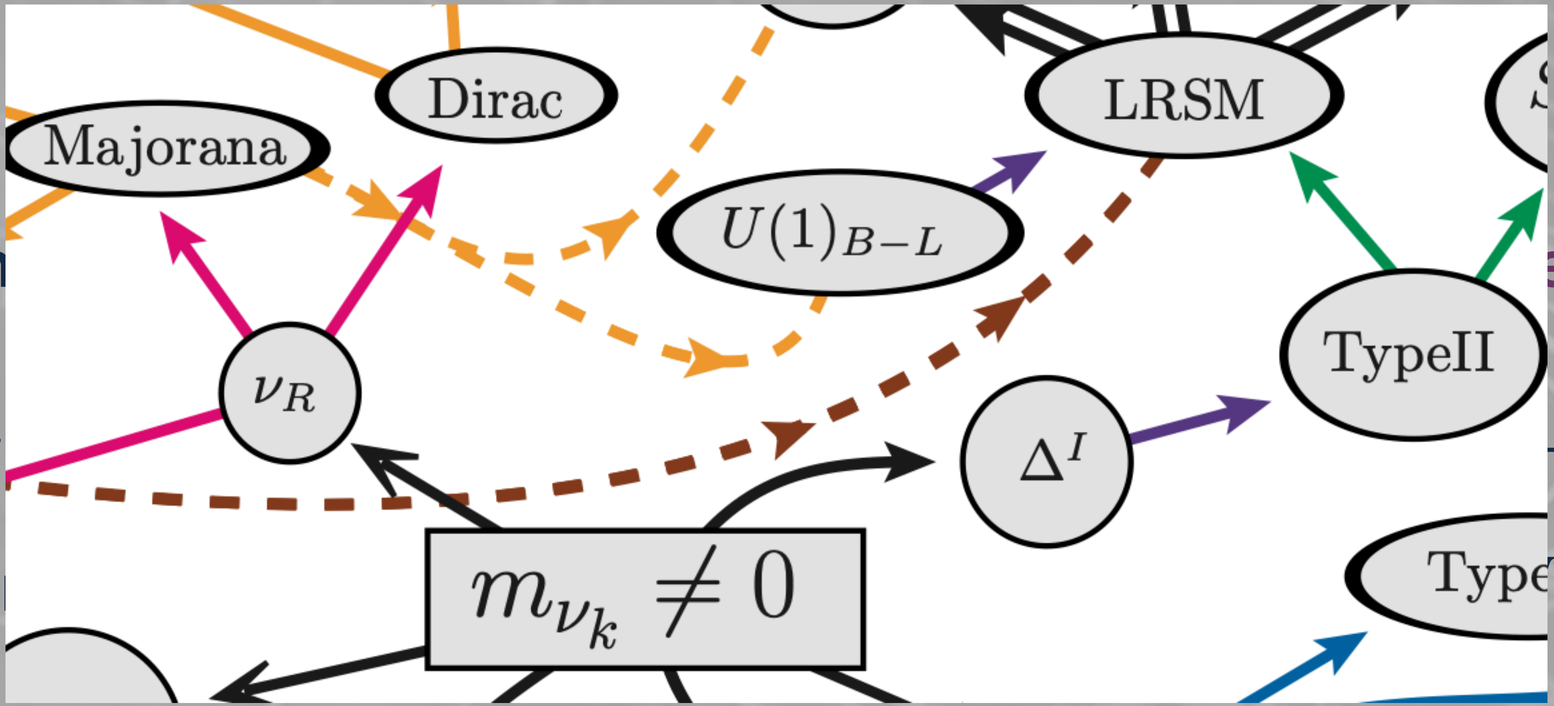

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- Or can appear spontaneously by **gauging parity**: [A. Maeizza and F. Nesti 2111.11076]

Introducing Left-Right: Motivation

Goals:

- Go back to the diagonalisation of the Lagrangian **beyond simplifying assumptions**
- Cast all Lagrangian parameters in **physical and measurable parameters**
- Include **NLO QCD Corrections**
- Implementation in **Feynrules** for more precise signal modelling in Collider physics

With Miha Nemevšek and Fabrizio Nesti: 2403.07756

See also: https://sites.google.com/site/leftrighthep

Introducing Left-Right: Model overview

SM Gauge group is extended: $\mathcal{G}_{\mathrm{LR}} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

Right-handed SM fermion singlets are promoted to $SU(2)_R$**-doublets**

$\Rightarrow$ **Add RH neutrinos**, $U(1)_{B-L}$-anomalies automatically cancelled

Scalar sector: different (minimal) possibilities, bi-doublet + 2 doublets
or (like here) bi-doublet + 2 triplets

Physical spectrum: **SM +** $N_R\,,W_R^{\pm}\,,Z_R\,,\Delta_{R,L}^{\pm\pm}\,,\Delta_L^{+}\,,\Delta_L^{0}\,,\chi_L^{0}\,,\Delta_R^{0}\,,A^0\,,H^0\,,H^{\pm}$

Field content: $\mathcal{G}_{\rm LR} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

Fermions: $Q_{L,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}$, $L_{L,R} \begin{pmatrix} \nu \\ \ell \end{pmatrix}$ **(3 generations)**

Gauge Fields: $SU(2)_{L,R}$-gauge fields, $A_{L,R} = A^a_{L,R} \frac{\sigma^a}{2}$, $A^{\pm}_{L,R} = \frac{A^1_{L,R} \mp i A^2_{L,R}}{\sqrt{2}}$

$U(1)_{B-L}$-gauge field B + **QCD** $SU(3)_c$

(Complex)
Scalar Fields: $SU(2)_{L,R}$-gauge triplets, $\Delta_{L,R} = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$, $(1,3,1,2)_L$ & $(1,1,3,2)_R$

$SU(2)_{L,R}$-gauge bi-doublet, $\phi = \begin{pmatrix} \phi_1^{0*} & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}$, **(1, 2, 2, 0)**

Electrical charge: $Q = T^3_L + T^3_R + \frac{B-L}{2}$

Diagonalising the Lagrangian: Gauge sector

Higgs Mechanism: Scalar multiplets acquire vevs at

$$\langle\phi\rangle = \begin{pmatrix} v_1 & 0 \\ 0 & -e^{i\alpha}v_2 \end{pmatrix}, \langle\Delta_{L,R}\rangle = \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix} \text{ with } v = \sqrt{v_1^2 + v_2^2} = 246\,\text{GeV}, \tan\beta = \frac{v_2}{v_1}$$

Leading to masses for gauge bosons from the scalar-kinetic terms:

$$\mathscr{L}_{\text{kin}} = |D\phi|^2 + |D\Delta_L|^2 + |D\Delta_R|^2$$

$$D^\mu\phi = \partial^\mu - ig(A_L^\mu\phi - \phi A_R^\mu)$$

$$D^\mu\Delta_{L,R} = \partial\Delta_{L,R} - ig[A_{L,R}^\mu, \Delta_{L,R}] - ig'B^\mu\Delta_{L,R}$$

(Order of fields matters due to matrix representation)

Manifest left-right model: $g_L = g_R \equiv g$ leads to conservation of $\mathscr{C}$ or $\mathscr{P}$

(more on this later)

Diagonalising the Lagrangian: Gauge sector

Charged gauge fields mass matrix:

$$(A_L^-, A_R^-)\, M^2_{W_{L,R}} \begin{pmatrix} A_L^+ \\ A_R^+ \end{pmatrix} \Rightarrow M^2_{W_{L,R}} = \frac{g^2}{2} v_R^2 \begin{pmatrix} \epsilon^2 & e^{-i\alpha}\epsilon^2 \sin 2\beta \\ e^{i\alpha}\epsilon^2 \sin 2\beta & 2+\epsilon^2 \end{pmatrix} \quad \epsilon = \frac{v}{v_R}$$

$M_{W_{L,R}}$ is diagonalised with a **unitary** rotation: $\begin{pmatrix} A_L^+ \\ A_R^+ \end{pmatrix} = U_W \begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix}$

with $U_W = \begin{pmatrix} c_\xi & s_\xi e^{-i\alpha} \\ -s_\xi e^{i\alpha} & c_\xi \end{pmatrix}$ and $s_\xi \simeq \frac{\epsilon^2}{2} s_{2\beta} \simeq \frac{M^2_{W_L}}{M^2_{W_R}} s_{2\beta}$ (Up to $\mathcal{O}(\epsilon^2)$)

Mixing controlled by $\beta = \arctan(v_2/v_1)$

The mass eigenvalues then become:

$$M_{W_L} \simeq \frac{gv}{\sqrt{2}}, \quad M_{W_R} = g v_R \left(1 + \frac{\epsilon^2}{4}\right)$$

Input parameters: $M_{W_L}, M_{W_R}, \tan\beta, \alpha, g$

Diagonalising the Lagrangian: Gauge sector

$$(A_{3L}, A_{3R}, B)\, M^2_{Z_{L,R}} \begin{pmatrix} A_{3L} \\ A_{3R} \\ B \end{pmatrix} \Rightarrow M^2_{Z_{L,R}} = \frac{g^2}{2} v_R^2 \begin{pmatrix} \epsilon^2 & -\epsilon^2 & 0 \\ -\epsilon^2 & 4+\epsilon^2 & -4r \\ 0 & -4r & 4r^2 \end{pmatrix} \qquad r = \frac{g'}{g}$$

$M_{Z_{L,R}}$ is diagonalised with an **orthogonal** rotation: $\begin{pmatrix} A_{3L} \\ A_{3R} \\ B \end{pmatrix} = O_Z \begin{pmatrix} A \\ Z_L \\ Z_R \end{pmatrix}$

The mass eigenvalues then become:

$$M_A = 0\,,\ M_{Z_L} = \frac{gv}{\sqrt{1 + \frac{1}{1+2r^2}}}\,, \quad \text{(Up to } \mathcal{O}(\epsilon^2))$$

$$M_{Z_R} \simeq g v_R \sqrt{2(1+r^2)} \left(1 + \frac{\epsilon^2}{8(1+r^2)^2} \right)$$

Mixing is fixed:

$$O_Z = \begin{pmatrix} s_w & -c_w & 0 \\ s_w & s_w t_w & -\frac{\sqrt{c_{2w}}}{c_w} \\ \sqrt{c_{2w}} & \sqrt{c_{2w}} t_w & t_w \end{pmatrix} + \frac{\epsilon^2}{4} \begin{pmatrix} 0 & 0 & \frac{c_{2w}^{3/2}}{c_w^3} \\ 0 & -\frac{c_{2w}^2}{c_w^5} & -\frac{c_{2w}^{3/2} t_w^2}{c_w^3} \\ 0 & \frac{c_{2w}^{3/2} t_w}{c_w^4} & -\frac{c_{2w}^2 t_w}{c_w^4} \end{pmatrix}.$$

Diagonalising the Lagrangian: Gauge sector

From the mass eigenvalues:

$$M_A = 0\,,\; M_{Z_L} \simeq \frac{gv}{\sqrt{1+\frac{1}{1+2r^2}}}\,,$$

$$M_{Z_R} \simeq g v_R \sqrt{2(1+r^2)}\left(1+\frac{\epsilon^2}{8(1+r^2)^2}\right)$$

We can fix c_w and therefore r in the on-shell scheme: $c_w = \frac{M_{W_L}}{M_{Z_L}} \;\Rightarrow r = \frac{s_w}{\sqrt{c_{2w}}}$

Using M_{W_L}, M_{Z_L} and g as input parameters:

$$r \simeq \sqrt{\frac{M_{W_L}^2}{2M_{W_L}^2 - M_{Z_L}^2} - 1} \simeq 0.63$$

$$\Rightarrow M_{Z_R} = \frac{\epsilon^2 M_{W_R}}{4\sqrt{2}} \frac{M_{W_L}^2}{(2M_{W_L}^2 - M_{Z_L}^2)^{3/2}} + \sqrt{2}\frac{M_{W_R}}{2M_{W_L}^2 - M_{Z_L}^2} \simeq 1.67 M_{W_R}$$

Diagonalising the Lagrangian: Gauge sector

Input parameters: $M_{Z_L}\,,M_{W_L}\,,M_{W_R}\,,\ \tan\beta\,,\alpha\,,g$

M_{W_L}, M_{Z_L} and g take their measured values and define $s_w\,,\alpha_e$, etc…

M_{W_R} is limited by direct searches to be $M_{W_R} \gtrsim 6\,\text{TeV}$

Measurements of the neutron EDM d_n limit $\sin\alpha\tan 2\beta \lesssim 5.8\times 10^{-12}$ [2107.10852]

$$d_n \propto \bar{\theta} \simeq \frac{m_t}{2m_b}\sin\alpha\tan 2\beta$$

Dominant decay channels: $\text{BR}(W_R^\pm \to q_u q_d) \simeq 75\,\%\,,\ \text{BR}(W_R^\pm \to \ell^\pm N) \simeq 24\,\%$

$$\text{BR}(Z_R \to q\bar{q}) \simeq 55\,\%\,,\ \text{BR}(Z_R \to NN) \simeq 17\,\%\,,\ \text{BR}(Z_R \to W_L^+ W_L^- h) \simeq 12\,\%$$

(mg5_aMC with default parameters)

Diagonalising the Lagrangian: Scalar sector

Scalars are complex

$$\Rightarrow \Delta_{L,R} = \begin{pmatrix} \frac{\Delta^+_{L,R}}{\sqrt{2}} & \Delta^{++}_{L,R} \\ v_{L,R} + \mathrm{Re}\Delta^0_{L,R} + i\mathrm{Im}\Delta^0_{L,R} & -\frac{\Delta^+_{L,R}}{\sqrt{2}} \end{pmatrix}, \quad \phi = \begin{pmatrix} v_1 + \mathrm{Re}\phi^0_1 - i\mathrm{Im}\phi^0_1 & \phi^+_2 \\ \phi^-_1 & v_2 + \mathrm{Re}\phi^0_2 + i\mathrm{Im}\phi^0_2 \end{pmatrix}$$

$\Rightarrow$ some of the **pseudo-scalar** excitations are eaten by the (massive) gauge bosons

The most general $\mathscr{P}$- (and $\mathscr{C}$-) symmetric potential is given by:

$$\mathscr{P} : \phi \to \phi^\dagger\,,\ \Delta_L \leftrightarrow \Delta_R \qquad \mathscr{C} : \phi \to \phi^T\,,\ \Delta_L \leftrightarrow \Delta_R^*$$

$$\begin{aligned}
\mathcal{V} = &-\mu_1^2\left[\phi^\dagger\phi\right] - \mu_2^2\left(\left[\tilde\phi\phi^\dagger\right] + \left[\tilde\phi^\dagger\phi\right]\right) - \mu_3^2\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right) \\
&+ \lambda_1\left[\phi^\dagger\phi\right]^2 + \lambda_2\left(\left[\tilde\phi\phi^\dagger\right]^2 + \left[\tilde\phi^\dagger\phi\right]^2\right) + \lambda_3\left[\tilde\phi\phi^\dagger\right]\left[\tilde\phi^\dagger\phi\right] + \lambda_4\left[\phi^\dagger\phi\right]\left(\left[\tilde\phi\phi^\dagger\right] + \left[\tilde\phi^\dagger\phi\right]\right) \\
&+ \rho_1\left(\left[\Delta_L\Delta_L^\dagger\right]^2 + \left[\Delta_R\Delta_R^\dagger\right]^2\right) + \rho_2\left(\left[\Delta_L\Delta_L\right]\left[\Delta_L^\dagger\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right]\right) + \rho_3\left[\Delta_L\Delta_L^\dagger\right]\left[\Delta_R\Delta_R^\dagger\right] \\
&+ \rho_4\left(\left[\Delta_L\Delta_L\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right] + \left[\Delta_L^\dagger\Delta_L^\dagger\right]\left[\Delta_R\Delta_R\right]\right) + \alpha_1\left[\phi^\dagger\phi\right]\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right) \\
&+ \left(\alpha_2\left(\left[\tilde\phi\phi^\dagger\right]\left[\Delta_L\Delta_L^\dagger\right] + \left[\tilde\phi^\dagger\phi\right]\left[\Delta_R\Delta_R^\dagger\right]\right) + \text{h.c.}\right) + \alpha_3\left(\left[\phi\phi^\dagger\Delta_L\Delta_L^\dagger\right] + \left[\phi^\dagger\phi\Delta_R\Delta_R^\dagger\right]\right) \\
&+ \beta_1\left(\left[\phi\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_2\left(\left[\tilde\phi\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\tilde\phi^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_3\left(\left[\phi\Delta_R\tilde\phi^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\tilde\phi\Delta_R^\dagger\right]\right)
\end{aligned}$$

In the case of $\mathscr{C}$, additional phases appear:

$\Rightarrow$ the parameters $\mu_2\,, \lambda_2\,, \lambda_4\,, \rho_4$ and β_i can now be complex, in $\mathscr{P}$ only α_2 carries the phase δ_2

Diagonalising the Lagrangian: Scalar sector

The most general $\mathscr{P}$- (and $\mathscr{C}$-) symmetric potential is given by:

$$\begin{aligned}
\mathcal{V} = &-\mu_1^2 \left[\phi^\dagger\phi\right] - \mu_2^2\left(\left[\tilde{\phi}\phi^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\right) - \mu_3^2\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right) \\
&+ \lambda_1\left[\phi^\dagger\phi\right]^2 + \lambda_2\left(\left[\tilde{\phi}\phi^\dagger\right]^2 + \left[\tilde{\phi}^\dagger\phi\right]^2\right) + \lambda_3\left[\tilde{\phi}\phi^\dagger\right]\left[\tilde{\phi}^\dagger\phi\right] + \lambda_4\left[\phi^\dagger\phi\right]\left(\left[\tilde{\phi}\phi^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\right) \\
&+ \rho_1\left(\left[\Delta_L\Delta_L^\dagger\right]^2 + \left[\Delta_R\Delta_R^\dagger\right]^2\right) + \rho_2\left(\left[\Delta_L\Delta_L\right]\left[\Delta_L^\dagger\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right]\right) + \rho_3\left[\Delta_L\Delta_L^\dagger\right]\left[\Delta_R\Delta_R^\dagger\right] \\
&+ \rho_4\left(\left[\Delta_L\Delta_L\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right] + \left[\Delta_L^\dagger\Delta_L^\dagger\right]\left[\Delta_R\Delta_R\right]\right) + \alpha_1\left[\phi^\dagger\phi\right]\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right) \\
&+ \left(\alpha_2\left(\left[\tilde{\phi}\phi^\dagger\right]\left[\Delta_L\Delta_L^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\left[\Delta_R\Delta_R^\dagger\right]\right) + \text{h.c.}\right) + \alpha_3\left(\left[\phi\phi^\dagger\Delta_L\Delta_L^\dagger\right] + \left[\phi^\dagger\phi\Delta_R\Delta_R^\dagger\right]\right) \\
&+ \beta_1\left(\left[\phi\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_2\left(\left[\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_3\left(\left[\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger\right]\right)
\end{aligned}$$

The minimisation conditions $\frac{\partial\mathscr{V}}{\partial S_i} = 0$ and $\frac{\partial^2\mathscr{V}}{\partial S_i \partial S_j} > 0$ lead us to:

$$\begin{aligned}
\mu_1^2 &= 2\left(\lambda_1 + s_{2\beta}c_\alpha\lambda_4\right)v^2 + \left(\alpha_1 - \alpha_3\frac{s_\beta^2}{c_{2\beta}}\right)v_R^2\,, \\
\mu_2^2 &= \left(s_{2\beta}\left(2c_{2\alpha}\lambda_2 + \lambda_3\right) + \lambda_4\right)v^2 \\
&\quad + \frac{1}{2c_\alpha}\left(2c_{\alpha+\delta_2}\alpha_2 + \alpha_3\frac{t_{2\beta}}{2c_\alpha}\right)v_R^2\,, \\
\mu_3^2 &= \left(\alpha_1 + \left(2c_{\alpha+\delta_2}\alpha_2 s_{2\beta} + \alpha_3 s_\beta^2\right)\right)v^2 + 2\rho_1 v_R^2 \\
\alpha_2 s_{\delta_2} &= \frac{s_\alpha}{4}\left(\alpha_3 t_{2\beta} + 4\left(\lambda_3 - 2\lambda_2\right)s_{2\beta}\epsilon^2\right).
\end{aligned}$$

$$\begin{aligned}
v_L = &\frac{\epsilon^2 v_R}{\left(1 + t_\beta^2\right)\left(2\rho_1 - \rho_3\right)}\Big(-\beta_1 t_\beta\cos(\alpha - \theta_L) \\
&+ \beta_2\cos(\theta_L) + \beta_3 t_\beta^2\cos(2\alpha - \theta_L)\Big).
\end{aligned}$$

For exact solvability we assume $\beta_i = v_L = 0$ and keep only the phase δ_2 (no impact on collider pheno)

In any case: $v_L \ll v \ll v_R \simeq \mathscr{O}(\text{TeV})$

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

Let's start with the "easy" ones that don't mix (in units of v_R) :

$$m^2_{\Delta_R^{++}} = 4\rho_2 + \frac{c_{2\beta}}{c_\beta^4}\alpha_3\epsilon^2\,,$$

$v_L = 0 \Rightarrow$ no mixing of $\Delta_L\,,\Delta_R^{++}$

$$m^2_{\Delta_L^{++}} = (\rho_3 - 2\rho_1) - \frac{t_\beta^4 - 2c_{2\alpha}t_\beta^2 + 1}{t_\beta^4 - 1}\alpha_3\epsilon^2\,,$$

$$m^2_{\Delta_L^{+}} = (\rho_3 - 2\rho_1) - \frac{\left(t_\beta^2 + 1\right)^2 - 4t_\beta^2 c_{2\alpha}}{2\left(t_\beta^4 - 1\right)}\alpha_3\epsilon^2\,,$$

$$m^2_{\Delta_L^{0}} = m^2_{\chi_L^0} = (\rho_3 - 2\rho_1) + s_{2\beta}t_{2\beta}s_\alpha^2\alpha_3\epsilon^2\,,$$

Take as input parameters: $m_{\Delta_R^{++}}$, $m_{\Delta_L^0}$, (and $\tan\beta$ and α), solve for $\rho_{2,3}$

ρ_1 and α_3 are fixed by other masses

$\Rightarrow$ Mass spectrum of Δ_L follows a sum rule:

$$m^2_{\Delta_L^{++}} - m^2_{\Delta_L^{+}} = m^2_{\Delta_L^{+}} - m^2_{\Delta_L^{0}} = v^2\alpha_3\frac{c_{2\beta}}{2}$$

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

Now the singly charged scalars:

$$(\phi_1^-, \phi_2^-, \Delta_R^-)\, M_+^2 \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \Delta_R^+ \end{pmatrix} \quad \textbf{with} \quad M_+^2 = \alpha_3 v_R^2 \begin{pmatrix} \frac{s_\beta^2}{c_{2\beta}} & -e^{-i\alpha}\frac{t_{2\beta}}{2} & -\epsilon e^{-i\alpha}\frac{s_\beta}{\sqrt{2}} \\ -e^{i\alpha}\frac{t_{2\beta}}{2} & \frac{c_\beta^2}{c_{2\beta}} & \epsilon\frac{c_\beta}{\sqrt{2}} \\ -\epsilon e^{i\alpha}\frac{s_\beta}{\sqrt{2}} & \epsilon\frac{c_\beta}{\sqrt{2}} & \epsilon^2\frac{c_{2\beta}}{2} \end{pmatrix}$$

M_+ is diagonalised with a unitary rotation (up to $\mathscr{O}(\epsilon^2)$:

$$\begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \Delta_R^+ \end{pmatrix} = U_+ \begin{pmatrix} \varphi_L^+ \\ H^+ \\ \varphi_R^+ \end{pmatrix}$$

$$\begin{aligned} U_+ = &\begin{pmatrix} c_\beta & -e^{-i\alpha}s_\beta & 0 \\ e^{i\alpha}s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1+\frac{1}{4}\epsilon^2 c_{2\beta}^2 & \frac{\epsilon c_{2\beta}}{\sqrt{2}} \\ 0 & -\frac{\epsilon c_{2\beta}}{\sqrt{2}} & -1+\frac{1}{4}\epsilon^2 c_{2\beta}^2 \end{pmatrix} \begin{pmatrix} 1-\frac{1}{4}\epsilon^2 s_{2\beta}^2 & 0 & \frac{\epsilon s_{2\beta}}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{\epsilon s_{2\beta}}{\sqrt{2}} & 0 & 1-\frac{1}{4}\epsilon^2 s_{2\beta}^2 \end{pmatrix} \\ \simeq &\begin{pmatrix} c_\beta & e^{-i\alpha}s_\beta & 0 \\ e^{i\alpha}s_\beta & -c_\beta & 0 \\ 0 & 0 & -1 \end{pmatrix} + \frac{\epsilon}{\sqrt{2}} \begin{pmatrix} 0 & 0 & e^{-i\alpha}s_\beta \\ 0 & 0 & c_\beta \\ e^{i\alpha}s_{2\beta} & -c_{2\beta} & 0 \end{pmatrix} + \frac{\epsilon^2}{2} \begin{pmatrix} -4c_\beta s_\beta^4 & -e^{-i\alpha}s_\beta c_{2\beta}^2 & 0 \\ -4e^{i\alpha}s_\beta c_\beta^4 & c_\beta c_{2\beta}^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

$\Rightarrow$ $\varphi_{L,R}^\pm$ are the goldstones of $W_{L,R}^\pm$ and remain massless

The remaining (fixed) mass is

$$m_{H^+} \simeq \sqrt{\frac{\alpha_3}{c_{2\beta}}}\, v_R \left(1 + \epsilon^2 \frac{c_{2\beta}^{3/2}}{4}\right)$$

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

Goldstones are decoupled in an independent rotation

We are left with 4 neutral states in the basis $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})$

$$m_0^2 = \begin{pmatrix} 4\epsilon^2\left(\lambda_1 + \frac{4tc_\alpha\left(\lambda_4\left(t^2+1\right)+4\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2}\right) & 2\epsilon\left(\alpha_1 - \frac{t^2X\left(t^2-s_{2\alpha+\delta_2}/s_{\delta_2}\right)}{\left(t^2+1\right)^2}\right) & \frac{4\epsilon^2\left(t^2c_{2\alpha}-1\right)\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} & \frac{4t^2\epsilon^2 s_{2\alpha}\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} \\ 2\epsilon\left(\alpha_1 - \frac{t^2X\left(t^2-s_{2\alpha+\delta_2}/s_{\delta_2}\right)}{\left(t^2+1\right)^2}\right) & Y & \frac{2tX\epsilon\left(t^2c_{2\alpha}-1\right)s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} & \frac{2t^3X\epsilon s_{2\alpha}s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} \\ \frac{4\epsilon^2\left(t^2c_{2\alpha}-1\right)\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} & \frac{2tX\epsilon\left(t^2c_{2\alpha}-1\right)s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} & X + \frac{16\lambda_2\epsilon^2\left(t^2c_{2\alpha}-1\right)^2}{\left(t^2+1\right)^2} & \frac{16\lambda_2 t^2\epsilon^2 s_{2\alpha}\left(t^2c_{2\alpha}-1\right)}{\left(t^2+1\right)^2} \\ \frac{4t^2\epsilon^2 s_{2\alpha}\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} & \frac{2t^3X\epsilon s_\alpha s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} & \frac{16\lambda_2 t^2\epsilon^2 s_{2\alpha}\left(t^2c_{2\alpha}-1\right)}{\left(t^2+1\right)^2} & X + \frac{16\lambda_2 t^4\epsilon^2 s_{2\alpha}^2}{\left(t^2+1\right)^2} \end{pmatrix}$$

First we decouple the **SM**-like Higgs h from the rest via a 2-1 rotation around θ:

$$m_h^2 = v^2\left(4\lambda_1 + \frac{64\lambda_2 t^2 c_\alpha^2}{(t^2+1)^2} + \frac{16\lambda_4 tc_\alpha}{t^2+1} - Y\tilde{\theta}^2\right)$$

$$\tilde{\theta} = \frac{\theta}{\epsilon} = \left(\frac{2\alpha_1}{Y} + \frac{X}{Y}\frac{t^2(1-t^2)}{1+t^2}\frac{\sin(2\alpha+\delta_2)}{\sin(\delta_2)}\right)$$

m_h and θ will be taken as input to solve for λ_1 and α_1

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

We are left with 4 neutral states in the basis $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})$

Remarkably, setting $\lambda_3 = 2\lambda_2$ allows to determine the remaining rotations *exactly*:

We rotate $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})^T = O_N(h, \Delta_R, H, A)^T$

$$\theta \equiv \epsilon\,\tilde{\theta} \equiv \theta_{21} = \epsilon\left[\frac{2\alpha_1}{Y} - \frac{2X\left(t^4 - t^2\, s_{2\alpha+\delta_2}/s_{\delta_2}\right)}{Y\left(t^2+1\right)^2}\right],$$

$$\phi \equiv \epsilon^2\tilde{\phi} \equiv \theta_{31} = \epsilon^2\frac{\left(t^2 c_{2\alpha} - 1\right)}{(1+t^2)^2}\left[\frac{32tc_\alpha\lambda_2 + 4\lambda_4(1+t^2)}{X} - 2\,t\,\tilde{\theta}\, s_{\alpha+\delta_2}/s_{\delta_2}\right],$$

$$\theta_{41} = \epsilon^2\frac{t^2 s_{2\alpha}}{(1+t^2)^2}\left[\frac{32tc_\alpha\lambda_2 + 4\lambda_4(t^2+1)}{X} - 2\,t\,\tilde{\theta}\, s_{\alpha+\delta_2}/s_{\delta_2}\right],$$

$$\theta_{34} = \cot^{-1}\left[\cot(2\alpha) - \frac{\csc(2\alpha)}{t^2}\right],$$

$$\eta \equiv \theta_{23} = -\frac{1}{2}\tan^{-1}\left[\frac{4tX\epsilon\sqrt{t^4 - 2c_{2\alpha}t^2 + 1}\, s_{\alpha+\delta_2}/s_{\delta_2}}{(t^2+1)^2\left(Y\tilde{\theta}^2\epsilon^2 - \frac{16(t^4-2c_{2\alpha}t^2+1)\lambda_2\epsilon^2}{(t^2+1)^2} - X + Y\right)}\right]$$

$$h \text{ part of } \Re\Delta_R: \quad \theta \equiv \theta_{21} \simeq -(O_N)_{2,1},$$

$$H \text{ part of } \Re\Delta_R: \quad \eta \equiv \theta_{23} = \arcsin[(O_N)_{2,3}/c_\theta],$$

$$h \text{ part of } \Re\phi_{20}: \quad \phi \equiv \theta_{31} \simeq -(O_N)_{3,1},$$

θ, ϕ, η can be taken as **input** parameters!

Mixing angles also control scalar couplings to SM-gauge bosons & quarks!

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

We rotate $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})^T = O_N(h, \Delta_R, H, A)^T$ and get the mass eigenvalues:

$$
\begin{aligned}
m_h^2 &= \epsilon^2 \left(4\lambda_1 + \frac{64\lambda_2 t^2 c_\alpha^2}{(t^2+1)^2} + \frac{16\lambda_4 t c_\alpha}{t^2+1} - Y\tilde{\theta}^2 \right), \\
m_\Delta^2 &= Y + \sec(2\eta) \left[(Y-X)s_\eta^2 + \epsilon^2 \left(Y\tilde{\theta}^2 c_\eta^2 - \frac{16\lambda_2 \left(t^4 - 2c_{2\alpha}t^2 + 1\right)}{(t^2+1)^2} s_\eta^2 \right) \right] \\
m_H^2 &= X - \sec(2\eta) \left[(Y-X)s_\eta^2 + \epsilon^2 \left(Y\tilde{\theta}^2 s_\eta^2 - \frac{16\lambda_2 \left(t^4 - 2c_{2\alpha}t^2 + 1\right)}{(t^2+1)^2} c_\eta^2 \right) \right] \\
m_A^2 &= X \,,
\end{aligned}
$$

The masses $m_h\,, m_\Delta\,, m_H, m_A$ are taken as input parameters to determine the potential

And we get another sum rule: $$m_{H^+}^2 - m_A^2 = v^2 \alpha_3 \frac{c_{2\beta}}{2} \sim O(150\,\mathrm{GeV})^2$$

Mass splitting $|m_H^2 - m_A^2|$ must be small to ensure perturbativity of λ_2 : $|m_H^2 - m_A^2| \lesssim 16v^2$

Diagonalising the Lagrangian: Fermions

The various vevs in the model induce tree-level mass terms for all fermions:

Only the bi-doublet couples to quarks, giving masses to up- and down-type quarks

$$\mathcal{L}_Y^q = \bar{Q}'_L \left(Y_q \, \phi + \tilde{Y}_q \, \tilde{\phi} \right) Q'_R + \text{H.c.} \,,$$
$$\mathcal{L}_Y^\ell = \bar{L}'_L \left(Y_\ell \, \phi + \tilde{Y}_\ell \, \tilde{\phi} \right) L'_R + $$
$$+ \bar{L}'^c_L i\sigma_2 \Delta_L Y^M_L L'_L + \bar{L}'^c_R i\sigma_2 \Delta_R Y^M_R L'_R + \text{H.c.} \,.$$

$$M_u = Y_q \, v_1 + \tilde{Y}_q \, e^{-i\alpha} v_2$$
$$M_d = -Y_q \, e^{i\alpha} v_2 - \tilde{Y}_q \, v_1$$

Which are diagonalised as:

$$M_u = U_{uL} \, m_u \, U^\dagger_{uR} \,, \qquad M_d = U_{dL} \, m_d \, U^\dagger_{dR}$$

From these mixings we can define the **CKM** and its **right-handed** (measurable) analogue:

$V_L^{\text{CKM}} \equiv U^\dagger_{uL} U_{dL}$, $V_R^{\text{CKM}} \equiv U^\dagger_{uR} U_{dR}$ (V_R can has additional phases in the case of $\mathscr{C}$)

The quark Yukawas are then fully determined from measurable inputs: quark masses and mixings

$$Y_q = \frac{1}{v_1^2 - v_2^2} \left(M_u \, v_1 + e^{-i\alpha} M_d \, v_2 \right)$$
$$\tilde{Y}_q = -\frac{1}{v_1^2 - v_2^2} \left(M_d \, v_1 + e^{i\alpha} M_u \, v_2 \right)$$

Diagonalising the Lagrangian: Fermions

Both triplets and the bi-doublet induce mass-terms for the leptons:

$$M_\ell = -Y_\ell v_2 e^{i\alpha} + \tilde{Y}_\ell v_1\,, \quad M_D = Y_\ell v_1 - \tilde{Y}_\ell v_2 e^{-i\alpha}\,, \quad M_L = v_L Y_L^M\,, \quad M_R = v_R Y_R^M$$

In which M_D is a mass-term between **LH** and RH neutrinos, M_L and M_R are Majorana

The charged lepton mass M_ℓ is easily diagonalised: $M_\ell = U_{\ell L}\, m_\ell\, U_{\ell R}^\dagger$

And the Yukawas of the bi-doublet are given by:

$$Y_\ell = \frac{1}{v_1^2 - v_2^2}\left(M_D\, v_1 + M_\ell\, e^{-i\alpha} v_2\right)$$

$$\tilde{Y}_\ell = -\frac{1}{v_1^2 - v_2^2}\left(M_\ell\, v_1 + M_D\, e^{i\alpha} v_2\right)$$

Due to M_D, **LH** and **RH** neutrinos mix with each other, we need to diagonalise the mass matrix:

$$\bar{n}_L' M_n n_L^{\prime c} = \begin{pmatrix}\bar{\nu}_L' & \bar{\nu}_R^{\prime c}\end{pmatrix} \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^{\prime c} \\ \nu_R' \end{pmatrix}$$

Diagonalising the Lagrangian: Fermions

Due to M_D, **LH** and **RH** neutrinos mix with each other, we need to diagonalise the mass matrix:

$$\bar{n}'_L M_n n'^c_L = (\bar{\nu}'_L \ \bar{\nu}'^c_R) \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu'^c_L \\ \nu'_R \end{pmatrix}$$

Majorana mass matrix is complex symmetric: (block-) diagonalised via Autonne-Takagi factorisation:

$$\tilde{W}^T M_n \tilde{W} = \begin{pmatrix} m_{\text{light}} & \mathbb{0} \\ \mathbb{0} & m_{\text{heavy}} \end{pmatrix}$$

Perturbative diagonalisation (expand in M_R^{-1}) gives us:

$$M_\nu \simeq M_L - M_D M_R^{-1} M_D^T \,, \qquad M_N \simeq M_R$$

In which the blocks are diagonalised via the unitary matrices V_ν and V_N:

$$V_\nu^T M_\nu V_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \,, \quad V_N^\dagger M_N V_N^* = \text{diag}(m_{N_1}, m_{N_2}, m_{N_3})$$

Diagonalising the Lagrangian: Fermions

Due to M_D, LH and RH neutrinos mix with each other, we need to diagonalise the mass matrix:

$$\bar{n}'_L M_n n'^c_L = (\bar{\nu}'_L \; \bar{\nu}'^c_R) \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu'^c_L \\ \nu'_R \end{pmatrix}$$

Majorana mass matrix is complex symmetric: (block-) diagonalised via Autonne-Takagi factorisation:

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Type II mass term

Type I mass term

Perturbative diagonalisation (expand in M_R^{-1}) gives us:

$$M_\nu \simeq M_L - M_D M_R^{-1} M_D^T \,, \qquad M_N \simeq M_R$$

In which the blocks are diagonalised via the unitary matrices V_ν and V_N:

$$V_\nu^T M_\nu V_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \,, \quad V_N^\dagger M_N V_N^* = \text{diag}(m_{N_1}, m_{N_2}, m_{N_3})$$

Diagonalising the Lagrangian: Fermions

The full rotation matrix is approximately given by (up to M_R^{-1}):

$$W = \begin{pmatrix} \sqrt{\mathbb{1} - BB^\dagger} V_\nu & BV_N^* \\ -B^\dagger V_\nu & \sqrt{\mathbb{1} - B^\dagger B} V_N^* \end{pmatrix} \simeq \begin{pmatrix} V_\nu & B_1 V_N^* \\ -B_1^\dagger V_\nu & V_N^* \end{pmatrix} .$$

With $B_1 = M_D^\dagger M_R^{-1\dagger}$

Charged lepton currents can be cast as:

$$\mathcal{L}_{cc}^\ell = \frac{g_L}{\sqrt{2}} \bar{\ell}_L \gamma^\mu \mathcal{U}_L n_L W_L^\mu + \frac{g_R}{\sqrt{2}} \bar{\ell}_R \gamma^\mu \mathcal{U}_R n_R W_R^\mu$$

With the 3×6 mixing matrices given by:

$$(\mathcal{U}_L)_{\alpha i} = \sum_{k=1}^{3} (V_{\ell L}^\dagger)_{\alpha k} W_{ki} \, ,$$

$$(\mathcal{U}_R)_{\alpha i} = \sum_{k=1}^{3} (V_{\ell L}^\dagger)_{\alpha k} W_{(k+3)i} \, .$$

The first 3×3 block of $\mathcal{U}_L$ can be identified as the **LH** would-be PMNS, the second 3×3 block of $\mathcal{U}_R$ as its **RH** analogue

$\mathcal{U}_R$ could be measured in $W_R^\pm \to \ell N$ decays

Diagonalising the Lagrangian: Fermions

In the case of $\mathscr{C}$, the neutrino sector is further restricted, the symmetry properties under

$$\mathscr{C}: \phi \leftrightarrow \phi^T, \Delta_L \leftrightarrow \Delta_R^*$$

restrict the Yukawa couplings and therefore the mass terms to

$$Y_\ell = Y_\ell^T, \ \tilde{Y}_\ell = \tilde{Y}_\ell^T, \ Y_L^M = Y_R^M, \ M_D = M_D^T, \ M_L = \frac{v_L}{v_R} M_R$$

From the light and heavy masses

$$M_\nu \simeq M_L - M_D M_R^{-1} M_D^T, \qquad M_N \simeq M_R$$

The Dirac mass matrix (and therefore all Yukawas) is **fully determined** by measurable inputs $(m_{\nu_i}, m_{N_i}, \mathscr{U}_L, \mathscr{U}_R)$

$$M_D = M_N \sqrt{\frac{v_L}{v_R}\mathbb{1} - M_N^{-1} M_\nu}$$

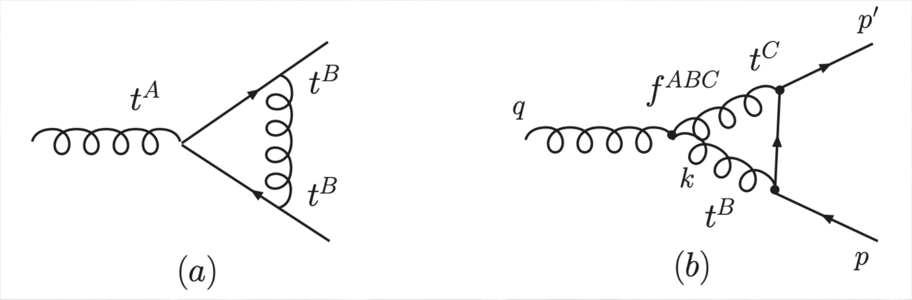
With the help of the Cayley-Hamilton theorem the square root is analytically calculable and of the form

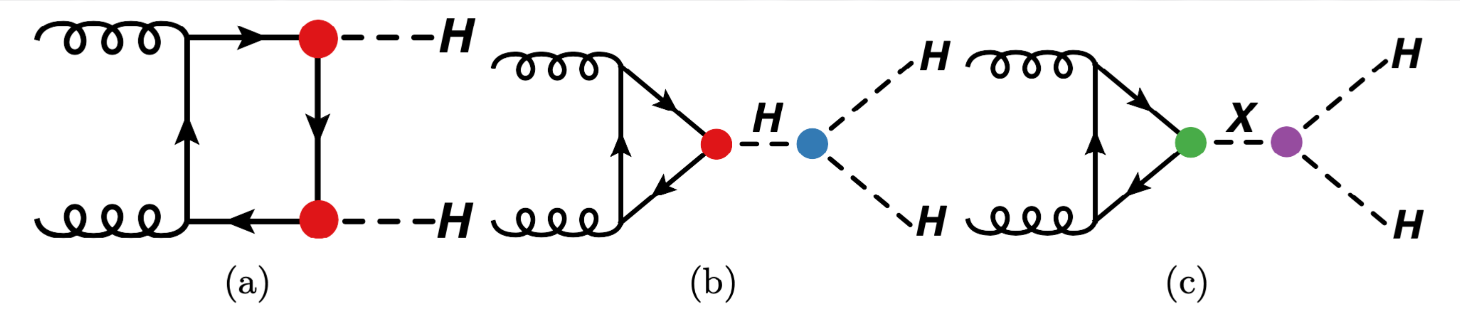
$$\sqrt{A} = c_0\,\mathbb{1} + c_1\,A + c_2\,A.A$$

c_i are functions of invariants of A
(ask Miha about it 🤓)

What’s next: QCD renormalisation

Going beyond tree-level: $\mathcal{O}(\alpha_s)$ corrections to quark vertices + new topology: ggF





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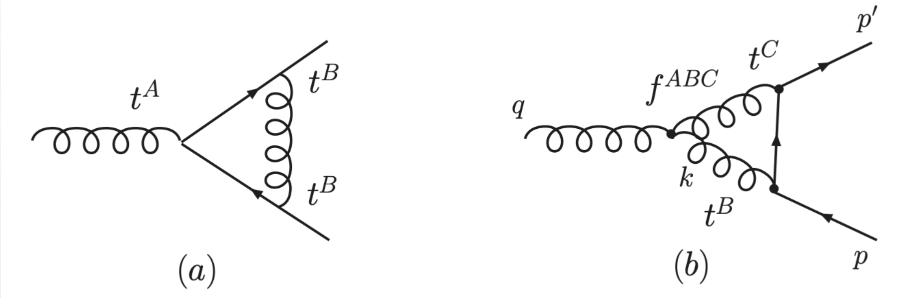
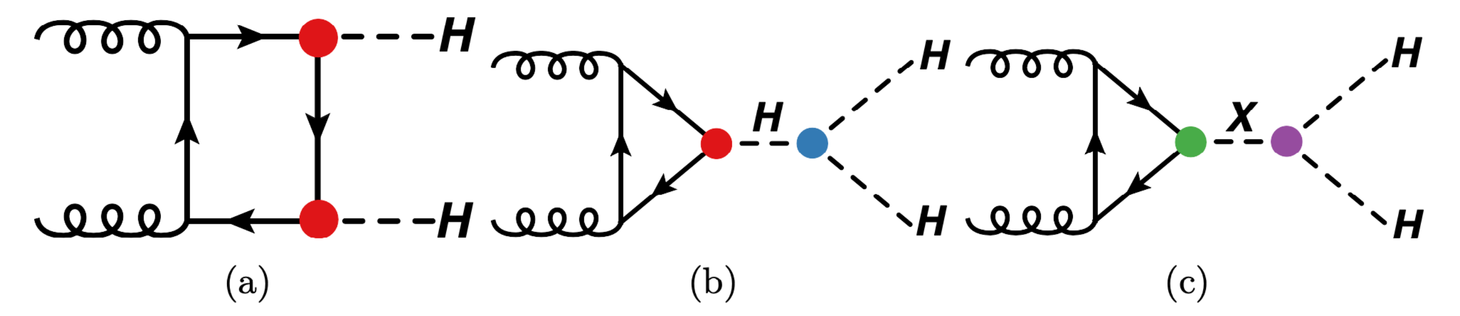
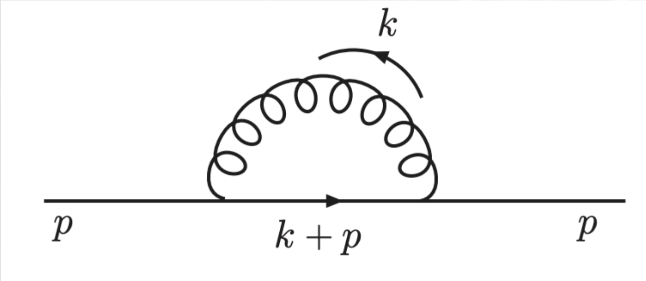
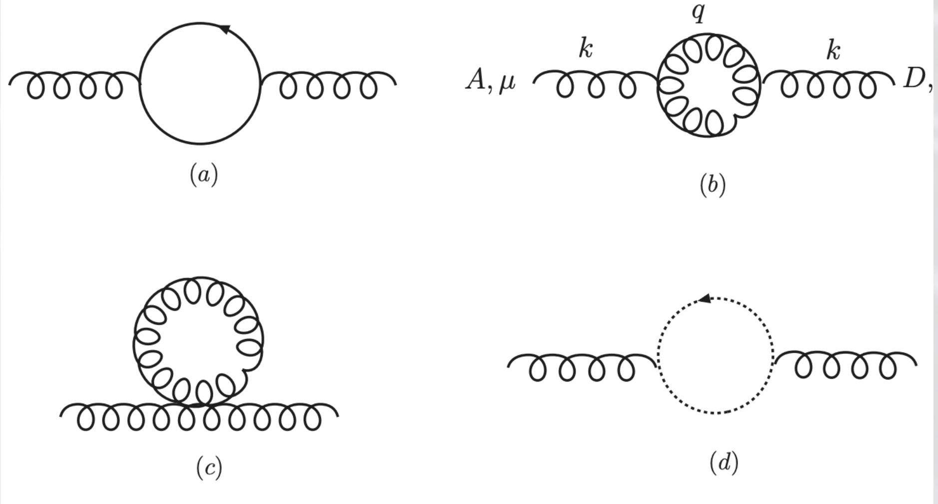




To absorb UV divergences: compute counter terms from self-energies



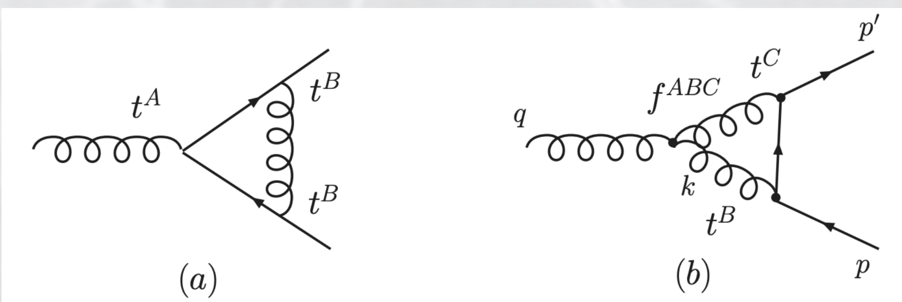


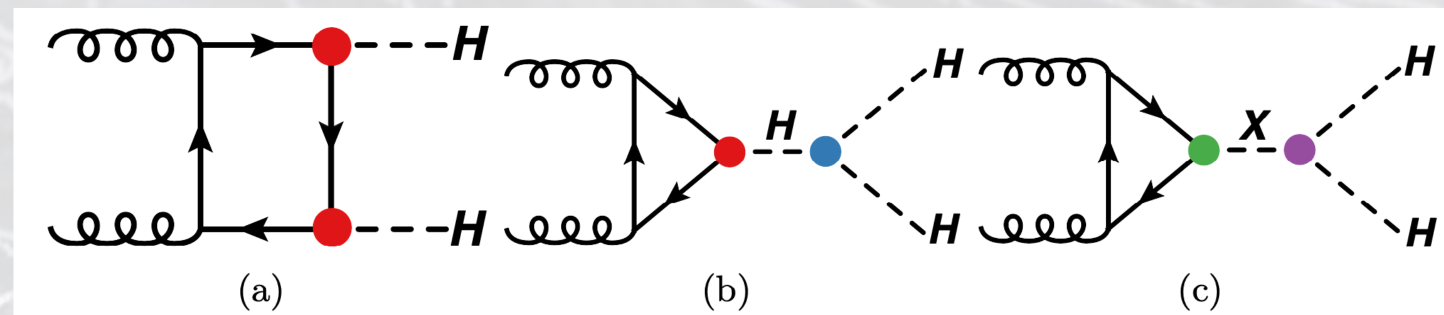
+ ISR/FSR diagrams to absorb IR divergences

Let's go through it in detail….

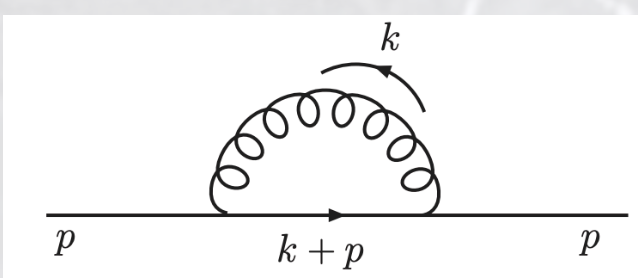
What’s next: QCD renormalisation

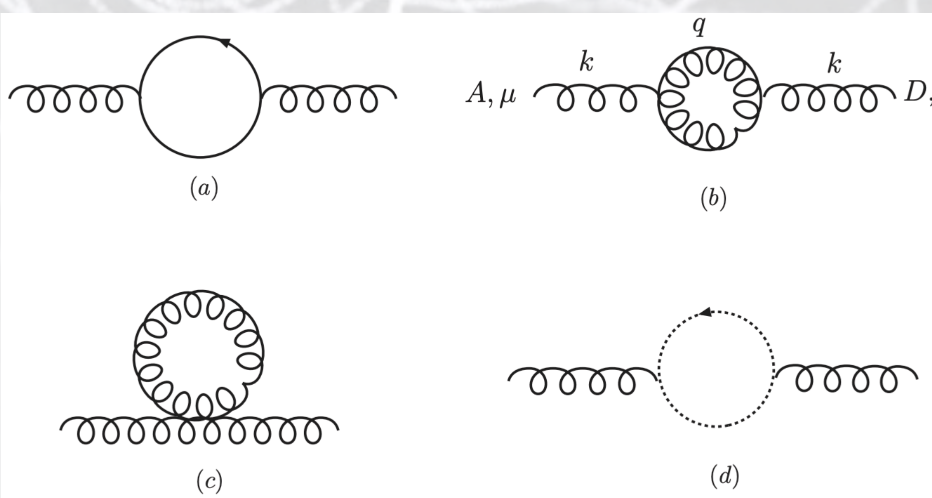
Going beyond tree-level: $\mathcal{O}(\alpha_s)$ corrections to quark vertices + new topology: ggF





To absorb UV divergences: compute counter terms from self-energies





+ ISR/FSR diagrams to absorb IR divergences

~~Let’s go through it in detail…~~ procedure has been automised in MoGRe [1907.04898] 😎

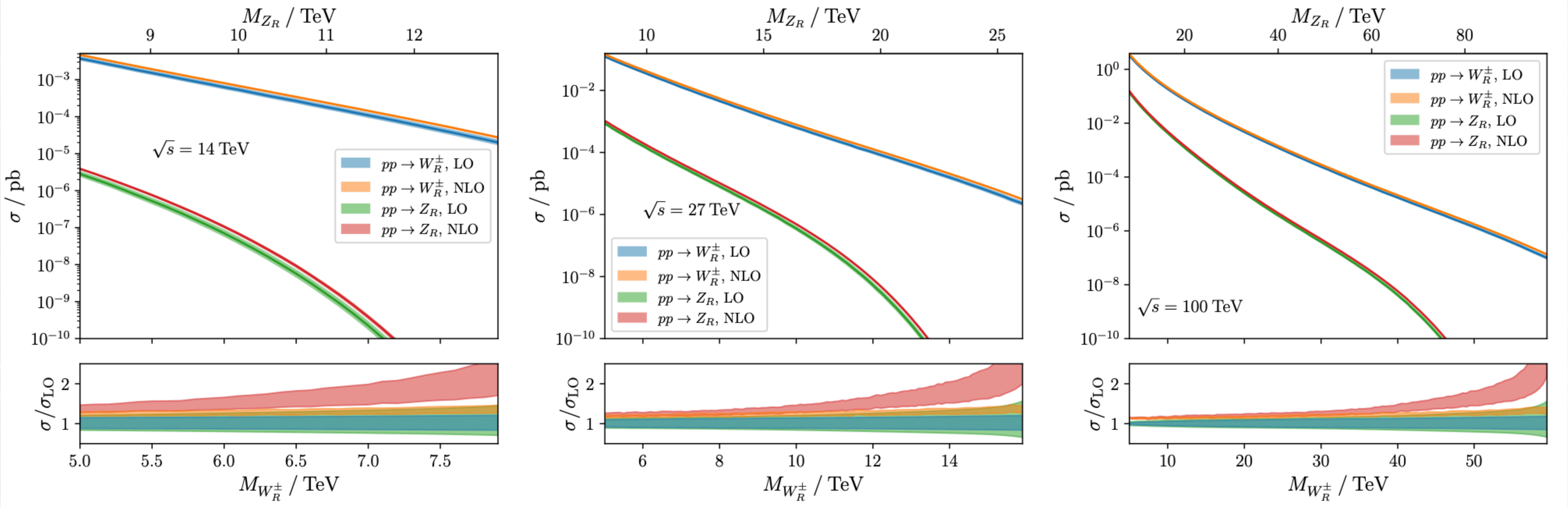
Some numerical results

Using MG5_aMC_@NLO 3.5.3:

- Set light neutrino/charged lepton/light quark masses/Yukawas to 0
- MG5 default run card with default cuts
- No shower, no detector simulation, just fixed-order NLO/LO cross-sections
- All parameters fixed to default values (see previous tables)
- PDF-sets: NNPDF40, dynamical scale variation

Some numerical results

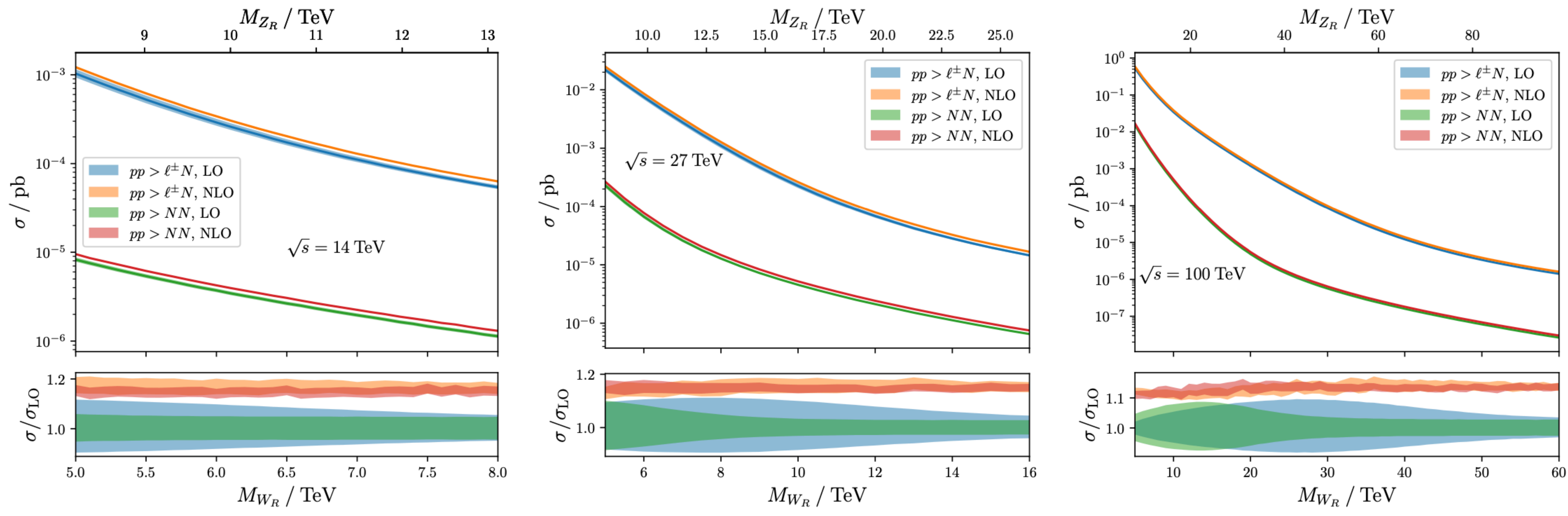
Drell-Yan production of the heavy gauge bosons:



Remember: $M_{Z_R} \simeq 1.67 M_{W_R}$, becomes kinematically inaccessible at LHC

Some numerical results

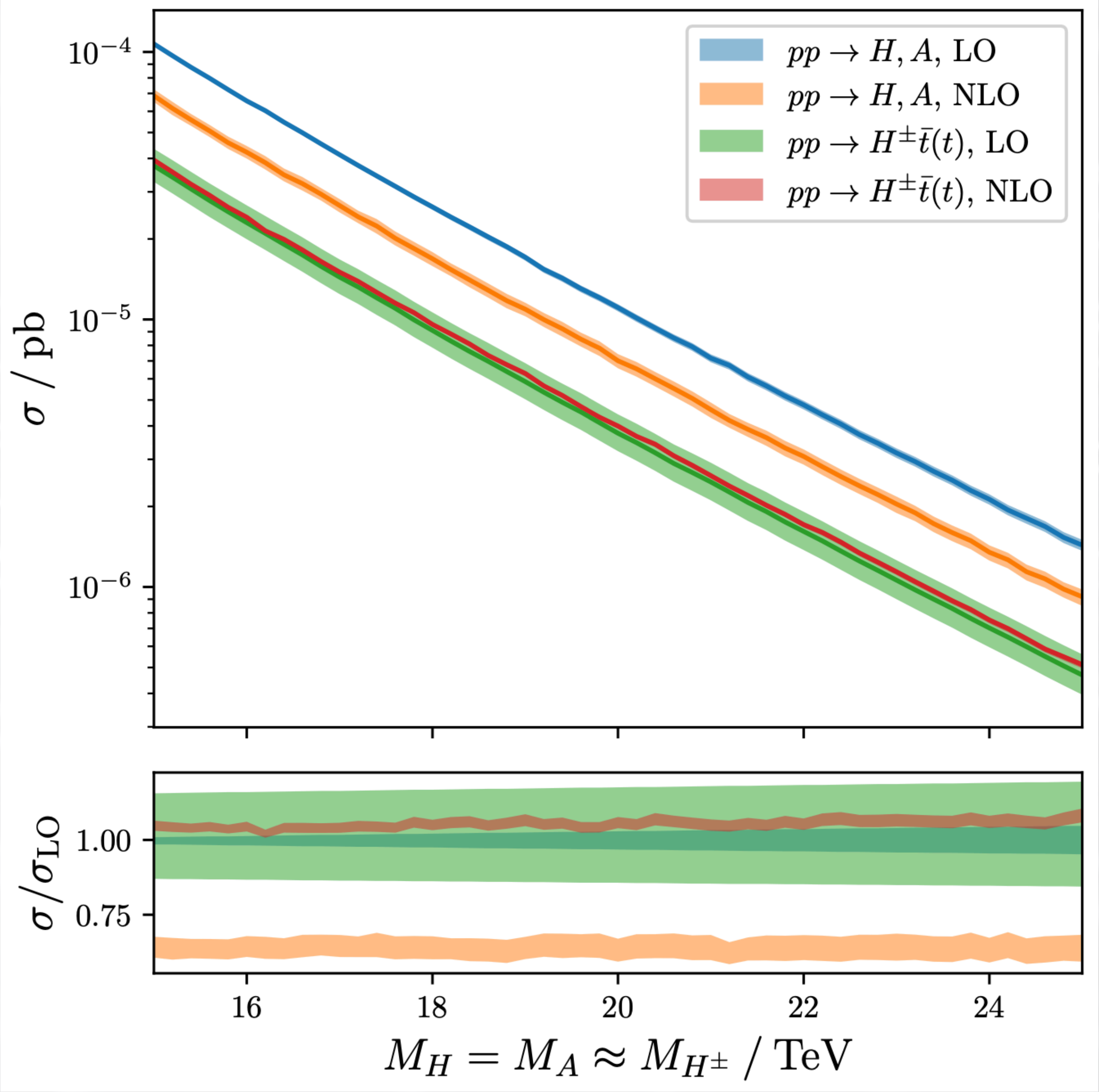
s-channel Drell-Yan production of heavy N



(Mostly) mediated by $pp \to W_R^{\pm} \to \ell^{\pm}N, pp \to Z_R \to NN$

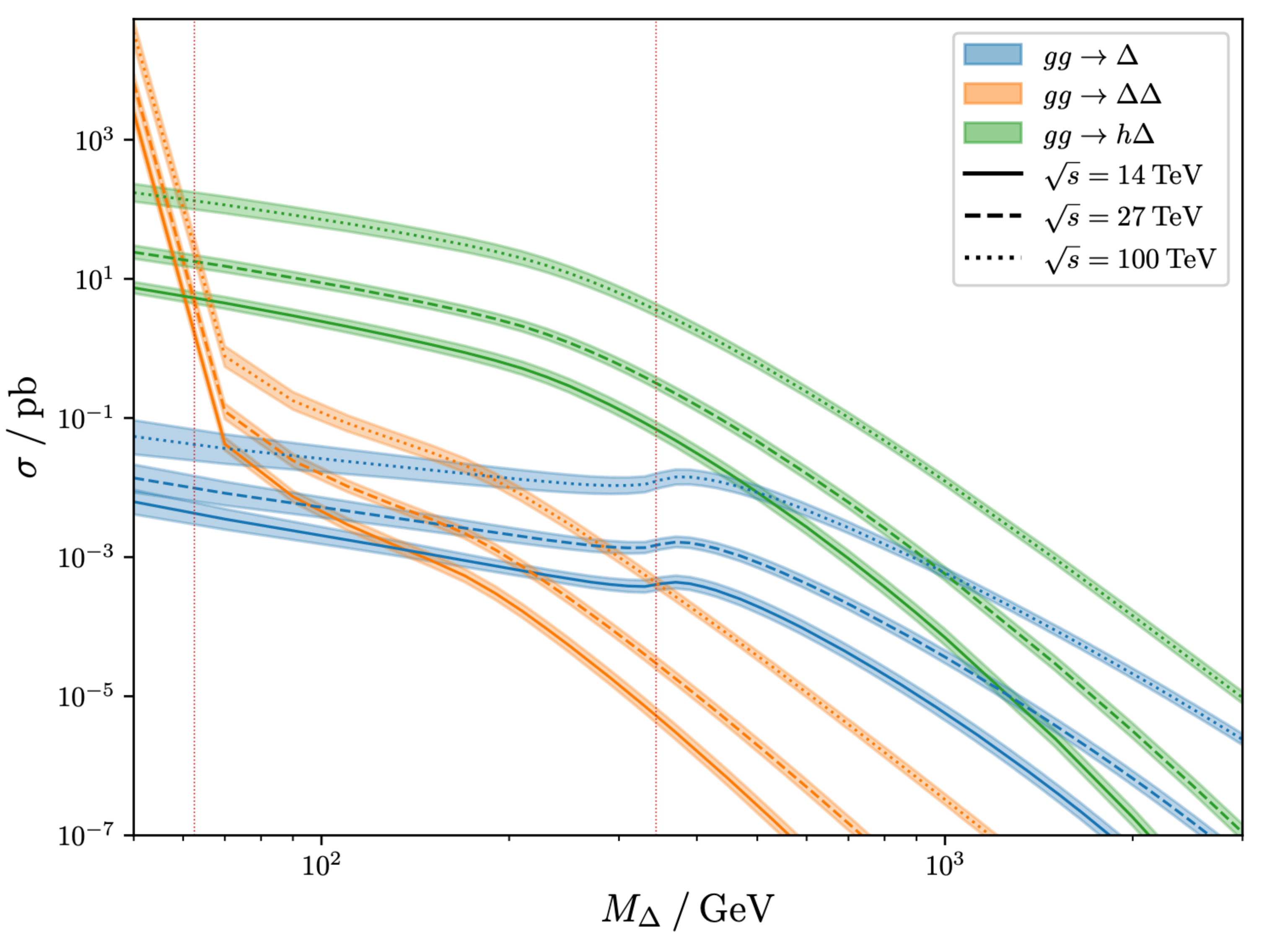
Production of heavy scalars:

- ggF process highly suppressed at large x
- Dominant coupling to b-quarks via top Yukawa
- QCD corrections negative due to interplay of top-mass/Yukawa renormalisation



Production of light scalars:

- ggF depending on scalar mixing angles
- $gg \to h \to \Delta\Delta$ can be resonant
- "Δ-strahlung" dominates, scales with m_{W_R} and $\tan(\beta)$



Full and automatic computation of genuine loop-induced QCD processes!

Conclusion

Left-Right symmetric model (**LRSM**) well motivated theory Framework

- Gives origin to **neutrino masses**
- Can be used to address many others of the **SM issues**
- Features numerous new states around the TeV **scale**

New model file:

- **All mixings** are calculated
- New parameter inversion: cast all parameters in **physical (measurable) parameters**
- Includes full **QCD NLO corrections** for the first time
- Also a parity violating version of the model file where $g_L \neq g_R$