Institut ”Jožef Stefan”, Ljubljana, Slovenija

Towards Coleman-like description of vacuum decay in the transverse field Ising model

Yutaro Shoji

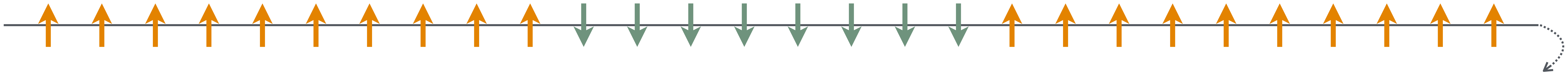
Work in progress with G. Lagnese

Belica workshop, 2-4.10 2024

Introduction

Transverse field Ising model

1D spin chain system



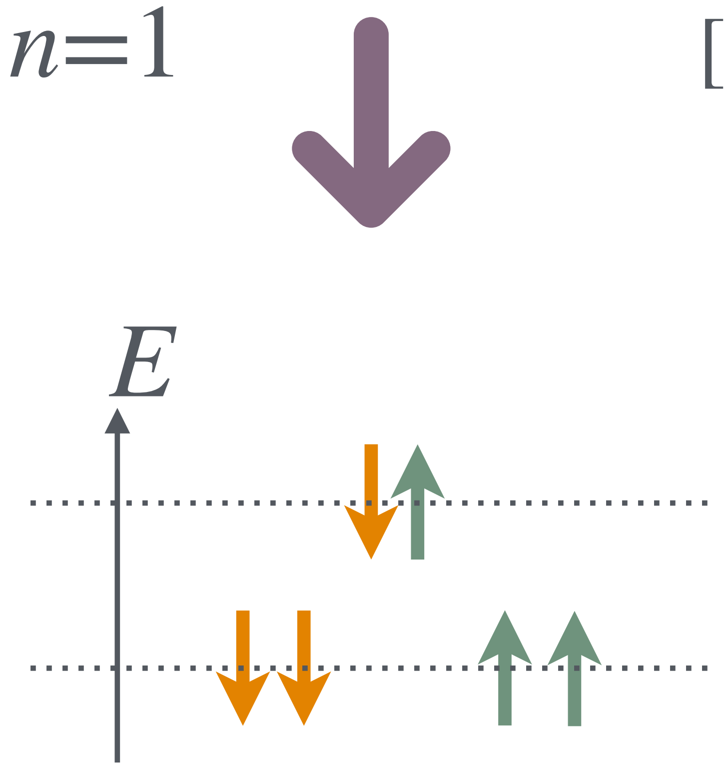
$$H = -\sum_{n=1}^{N}\left(\sigma_n^z\sigma_{n+1}^z + \underline{h_x\sigma_n^x} + h_z\sigma_n^z\right)$$

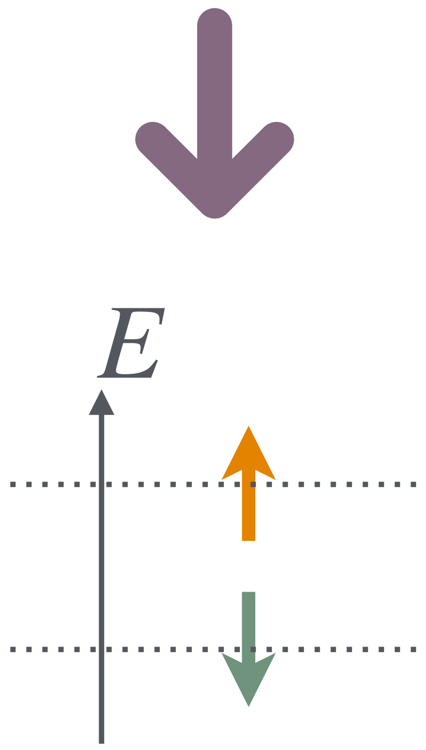
$0 < h_x < 1$

$h_z < 0$

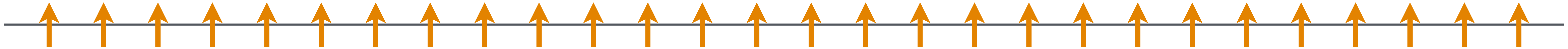
$[\sigma_n^x, \sigma_n^z] \neq 0$

Non-trivial evolution

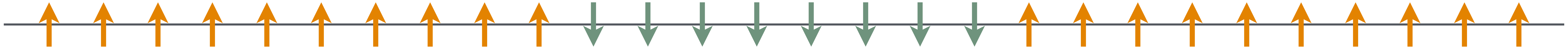




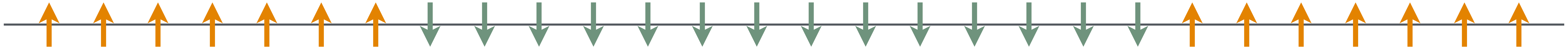
Vacuum decay in spin chain



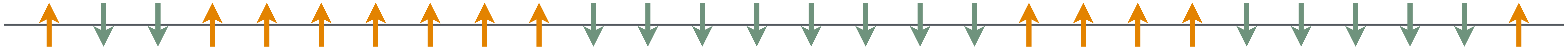
Quantum tunneling



Classical bubble expansion/Bloch oscillation



Thermalization



(It finally returns to the original state due to the quantum recurrence theorem but after a long long time)

Quantum simulations are available and it will be possibly realized in a real system

Vacuum decay rate in spin-chain vs in QFT

Spin-chain

$$E = E^{(0)} + E^{(1)} + E^{(2)} + \cdots$$

Non-local interaction

(Fermionic operators)

$$E^{(2)} = -\Delta E \sum_l \frac{|\langle 0 | V_{\text{int}} | \phi_l \rangle|^2}{E_l - i\gamma}$$

(False vacuum energy = 0)

$$\Gamma = -2\text{Im}E$$

$$\Delta E = 2|h_z|(1-h_x^2)^{1/8}$$

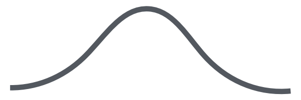
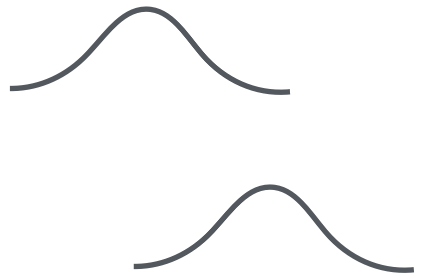
γ : Decay rate of $|\phi_l\rangle$

(Phenomenological parameter)

The decay rate is independent of γ if $\gamma \gg \Delta E$

QFT

$$\langle 0 | e^{-HT} | 0 \rangle \simeq e^{-\mathscr{E}VT}$$

$\simeq$ False vacuum +  +  $+ \cdots$

Bounce

Multi-bounce

$$\frac{\Gamma}{V} = -2\text{Im}\mathscr{E}$$

Imaginary part comes from negative mode

Bounce: O(4) symmetric saddle point of the action

$$\partial_\rho^2 \bar{\phi} + \frac{3}{\rho}\partial_\rho \bar{\phi} = \frac{dV}{d\phi}(\bar{\phi})$$

$$\bar{\phi}(\infty) = v_{\text{FV}}, \ \partial_\rho \bar{\phi}(0) = 0$$

What is the relation between them?

Can we "test" the vacuum decay in QFT using spin-chain?

Mismatch of prefactors?

[M. Lencsés, G. Mussardo, G. Takács, '22]

Contents

Vacuum decay in spin-chain

Why Fermions? Why non-local?

Fock state

$$a_n^\dagger |\downarrow\cdots\downarrow\rangle = |\underbrace{\downarrow\cdots\downarrow}_{n-1}\uparrow\downarrow\cdots\downarrow\rangle$$

$$\left.\begin{aligned}\{a_n, a_n^\dagger\} &= 1\\ \{a_n, a_n\} = \{a_n^\dagger, a_n^\dagger\} &= 0\end{aligned}\right\}$$

Fermionic

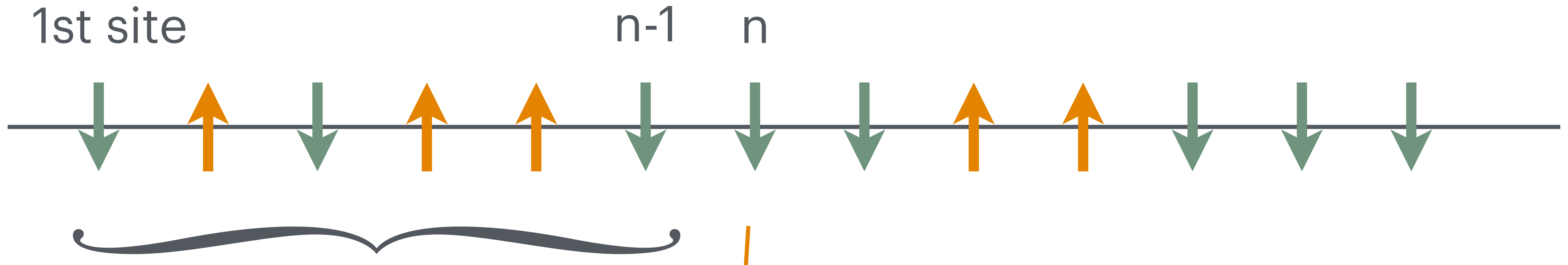
on the same site

$$\left.\begin{aligned}[a_n, a_m^\dagger] &= 0 \quad (n \neq m)\\ [a_n, a_m] = [a_n^\dagger, a_m^\dagger] &= 0\end{aligned}\right\}$$

Bosonic

on different sites

Jordan-Wigner transformation



The number of up spins

$$\psi_n^\dagger = \underbrace{\exp\left[i\pi\sum_{l=1}^{n-1} a_l^\dagger a_l\right]}_{\text{Non-local}} a_n^\dagger \qquad \psi_n = \exp\left[-i\pi\sum_{l=1}^{n-1} a_l^\dagger a_l\right] a_n$$

$$\left.\begin{aligned}\{\psi_n, \psi_m^\dagger\} &= \delta_{mn}\\ \{\psi_n, \psi_m\} = \{\psi_n^\dagger, \psi_m^\dagger\} &= 0\end{aligned}\right\}$$

Fermionic

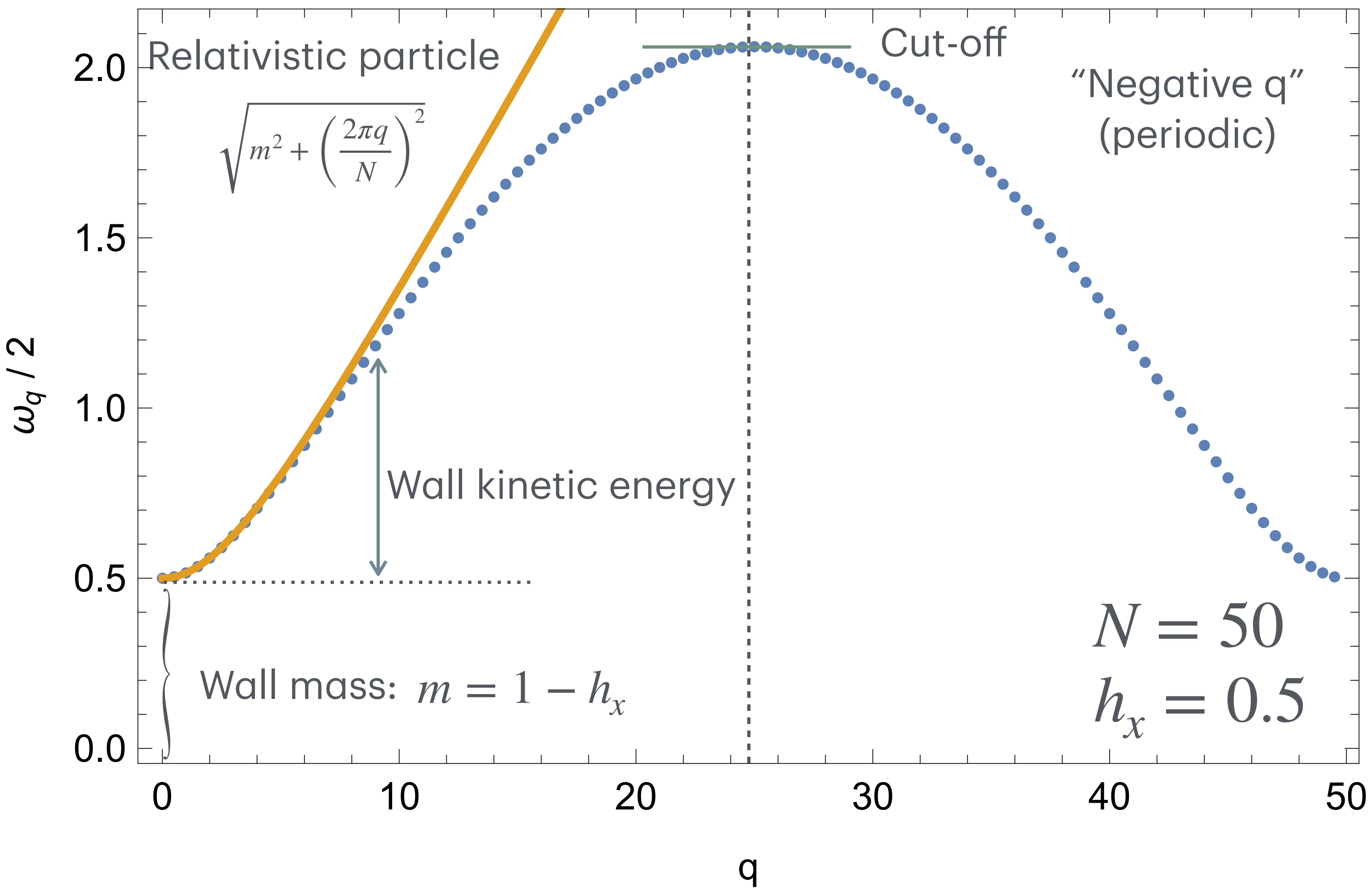
for all n, m

(Only for 1D spin-chain)

Pair creation of walls

$$\psi_n, \psi_n^\dagger \xrightarrow{\substack{\text{Fourier transformation} \\ \text{Bogoliubov transformation}}} \chi_q, \chi_q^\dagger$$

$$H = \frac{1}{N}\sum_q \omega_q \chi_q^\dagger \chi_q + V_{\text{int}} + V_0 \qquad \omega_q = 2\sqrt{\left(1-h_x\right)^2 + 4\sin^2\frac{\pi}{N}q}$$



Vacuum

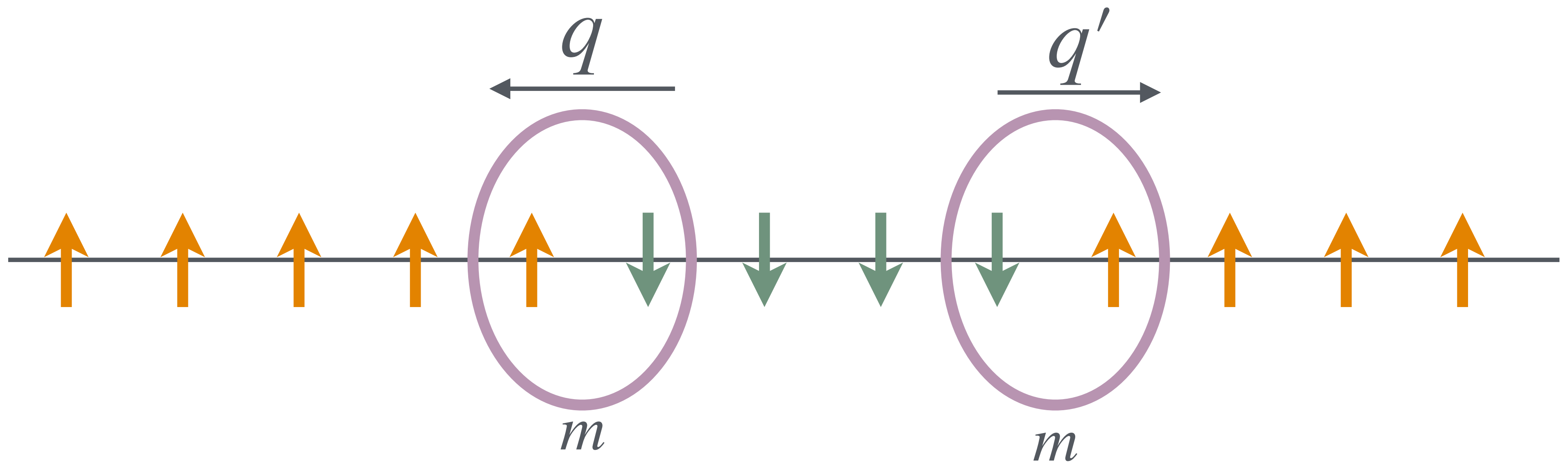
$$|0_{\text{even}}\rangle, \ |0_{\text{odd}}\rangle$$

roughly linear combinations of $|\uparrow\cdots\uparrow\rangle, |\downarrow\cdots\downarrow\rangle$

Excited states

(Always pair created)

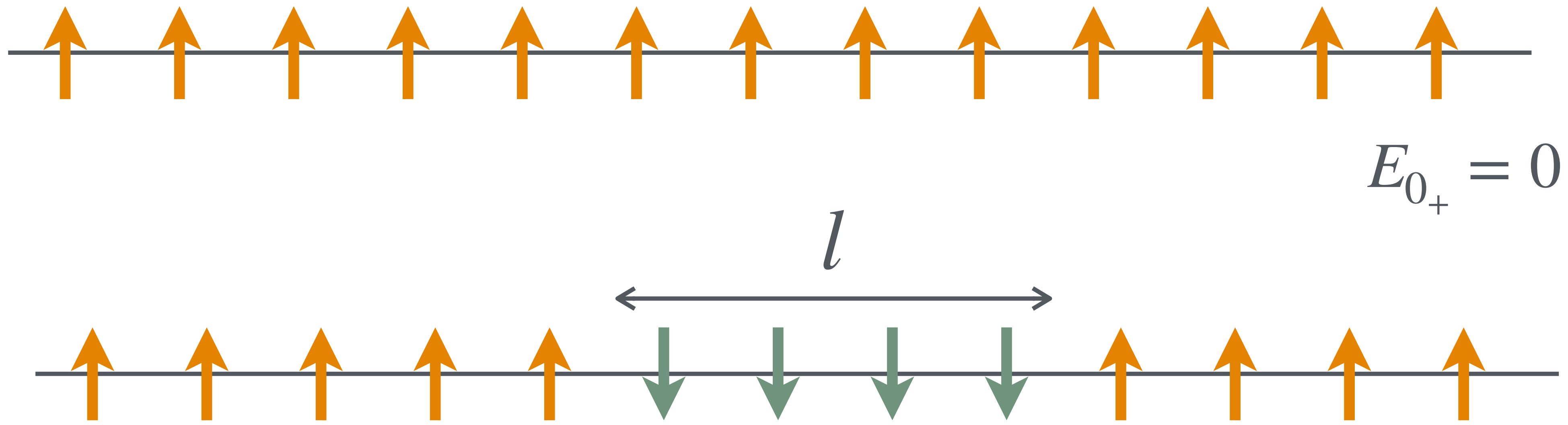
$$\chi_q^\dagger \chi_{q'}^\dagger |0_{\text{even}}\rangle, \ \chi_q^\dagger \chi_{q'}^\dagger |0_{\text{odd}}\rangle, \ \chi_q^\dagger \chi_{q'}^\dagger \chi_{q''}^\dagger \chi_{q'''}^\dagger |0_{\text{even}}\rangle, \cdots$$



Vacuum decay in spin-chain

$$H = \frac{1}{N}\sum_q \omega_q \chi_q^\dagger \chi_q + \underline{V_{\rm int}} + V_0$$

Creation/annihilation of walls
(Non-local)



$|0_+\rangle$

$E_{0_+} = 0$

$|\phi_l\rangle$

2nd order perturbation

$$E = E^{(0)} + E^{(1)} + E^{(2)} + \cdots$$

Fermionic, non-local

$$E^{(2)} = -\Delta E \sum_l \frac{|\langle 0 | V_{\rm int} | \phi_l \rangle|^2}{E_l - i\gamma}$$

Given outside of theory

$$\Gamma = -2{\rm Im}E$$

$$\Delta E = 2|h_z|(1-h_x^2)^{1/8}$$

γ : Decay rate of $|\phi_l\rangle$

Can we describe this decay in a field theoretical way?

Towards the path-integral formalism

The first step

$$\langle 0_+ | e^{-HT} | 0_+ \rangle \simeq e^{-E_0 T} \quad \text{at large T} \quad \Rightarrow \quad \int \mathscr{D}\phi e^{-S[\phi]} \ ?$$

Problem 1: States are discrete (either up or down)

★ Coherent state path integral

Integral over the complex plane

$$|z\rangle \equiv |z_1 \cdots z_N\rangle = e^{\sum_j z_j a_j^\dagger} |\downarrow \cdots \downarrow\rangle \qquad I = \left[\prod_n \int \frac{dz_n d\bar{z}_n}{\pi i} \frac{1}{(1+|z_n|^2)^2} \right] \frac{|z\rangle}{\sqrt{\langle z|z\rangle}} \frac{\langle z|}{\sqrt{\langle z|z\rangle}}$$

$$\langle f | e^{-HT} | i \rangle = \left[\prod_{n=0}^{M} \prod_{j=1}^{N} \int \frac{dz_{n,j} d\bar{z}_{n,j}}{\pi i (1+|z_{n,j}|^2)^2} \right] \frac{\langle f | z_N \rangle}{\sqrt{\langle z_N | z_N \rangle}} \frac{\langle z_0 | i \rangle}{\sqrt{\langle z_0 | z_0 \rangle}} \prod_{n=0}^{M-1} \underline{\frac{\langle z_{n+1} | e^{-H\delta\tau} | z_n \rangle}{\sqrt{\langle z_{n+1} | z_{n+1} \rangle} \sqrt{\langle z_n | z_n \rangle}}}$$

$$T = M\delta\tau$$

$$\simeq \frac{\langle z_{n+1} | z_n \rangle}{\sqrt{\langle z_{n+1} | z_{n+1} \rangle} \sqrt{\langle z_n | z_n \rangle}} \left(1 - \frac{\langle z_{n+1} | H | z_n \rangle}{\langle z_{n+1} | z_n \rangle} \delta\tau \right) \simeq \left[1 - \frac{\delta\tau}{1+|z_n|^2} \left(\frac{1}{2}(\bar{z}_n \partial_\tau z_n - z_n \partial_\tau \bar{z}_n) + H(z_n, \bar{z}_n) \right) \right]$$

The first step

$$\langle 0_+ | e^{-HT} | 0_+ \rangle \simeq e^{-E_0 T}$$

at large T

$$\Rightarrow \quad \int \mathscr{D}\phi \, e^{-S[\phi]} \; ?$$

Problem 1: States are discrete (either up or down)

★ Coherent state path integral

Integral over the complex plane

$$|z\rangle \equiv |z_1 \cdots z_N\rangle = e^{\sum_j z_j a_j^\dagger} | \downarrow \cdots \downarrow \rangle \qquad I = \left[\prod_n \int \frac{dz_n d\bar{z}_n}{\pi i} \frac{1}{(1+|z_n|^2)^2} \right] \frac{|z\rangle}{\sqrt{\langle z|z\rangle}} \frac{\langle z|}{\sqrt{\langle z|z\rangle}}$$

$$\langle f | e^{-HT} | i \rangle = \left[\prod_{n=0}^{M} \prod_{j=1}^{N} \int \frac{dz_{n,j} d\bar{z}_{n,j}}{\pi i (1+|z_{n,j}|^2)^2} \right] \frac{\langle f | z_N \rangle}{\sqrt{\langle z_N | z_N \rangle}} \frac{\langle z_0 | i \rangle}{\sqrt{\langle z_0 | z_0 \rangle}} \prod_{n=0}^{M-1} \underline{\frac{\langle z_{n+1} | e^{-H\delta\tau} | z_n \rangle}{\sqrt{\langle z_{n+1} | z_{n+1} \rangle} \sqrt{\langle z_n | z_n \rangle}}}$$

$$T = M\delta\tau$$

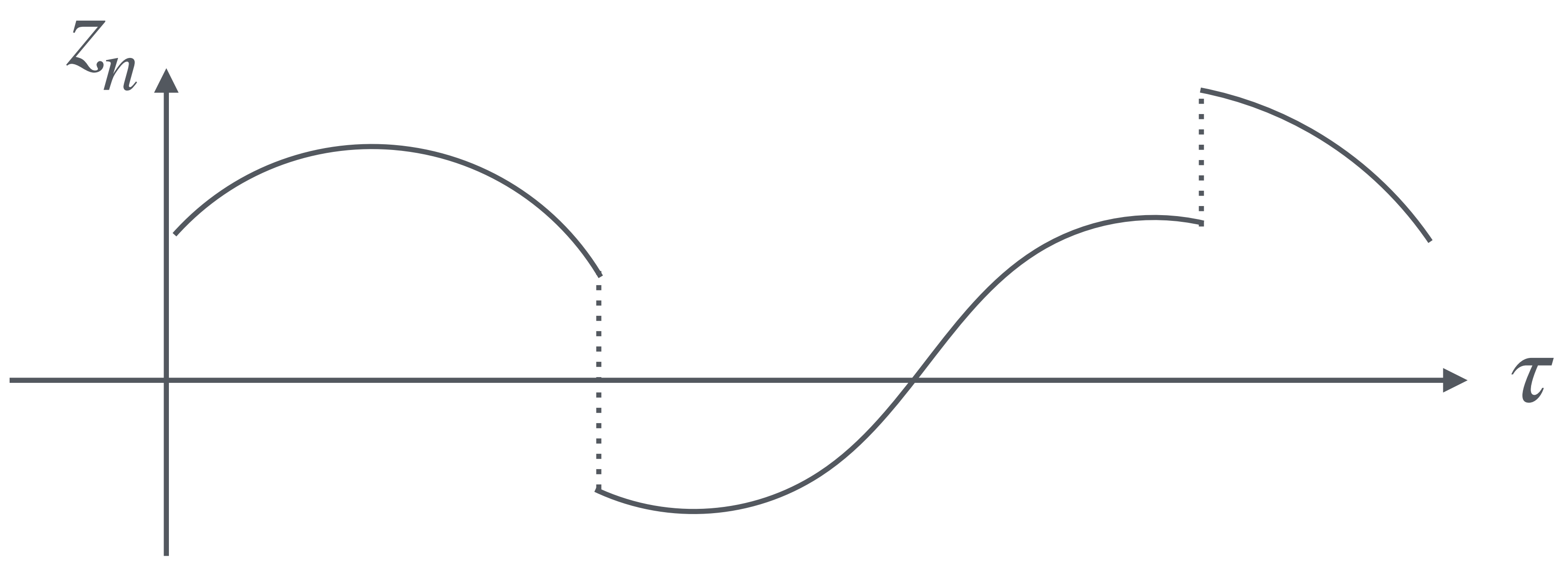
$$\simeq \frac{\langle z_{n+1} | z_n \rangle}{\sqrt{\langle z_{n+1} | z_{n+1} \rangle} \sqrt{\langle z_n | z_n \rangle}} \left(1 - \frac{\langle z_{n+1} | H | z_n \rangle}{\langle z_{n+1} | z_n \rangle} \delta\tau \right) \simeq \xcancel{\left[1 - \frac{\delta\tau}{1+|z_n|^2} \left(\frac{1}{2}(\bar{z}_n \partial_\tau z_n - z_n \partial_\tau \bar{z}_n) + H(z_n, \bar{z}_n) \right) \right]}$$

The second step

$$\frac{\langle z_{n+1}|z_n\rangle}{\sqrt{\langle z_{n+1}|z_{n+1}\rangle}\sqrt{\langle z_n|z_n\rangle}}\left(1-\frac{\langle z_{n+1}|H|z_n\rangle}{\langle z_{n+1}|z_n\rangle}\delta\tau\right)\not\simeq\left[1-\frac{\delta\tau}{1+|z_n|^2}\left(\frac{1}{2}(\bar{z}_n\partial_\tau z_n-z_n\partial_\tau\bar{z}_n)+H(z_n,\bar{z}_n)\right)\right]$$

because $z_{n+1}\not\to z_n$ as $\delta\tau\to 0$

Problem 2: Paths are not continuous



Not sure whether this really causes a problem

Problem 3: Off-diagonal elem. of Hamiltonian

$$\langle z_{n+1}|H|z_n\rangle$$

Then, it is complex and also it does not fit the path integral form

$$\langle 0_+|e^{-HT}|0_+\rangle=\int\mathscr{D}\bar{z}\mathscr{D}z e^{-\sum_n L(\bar{z}_n,z_n)\delta\tau}$$

★ We found a transformation such that

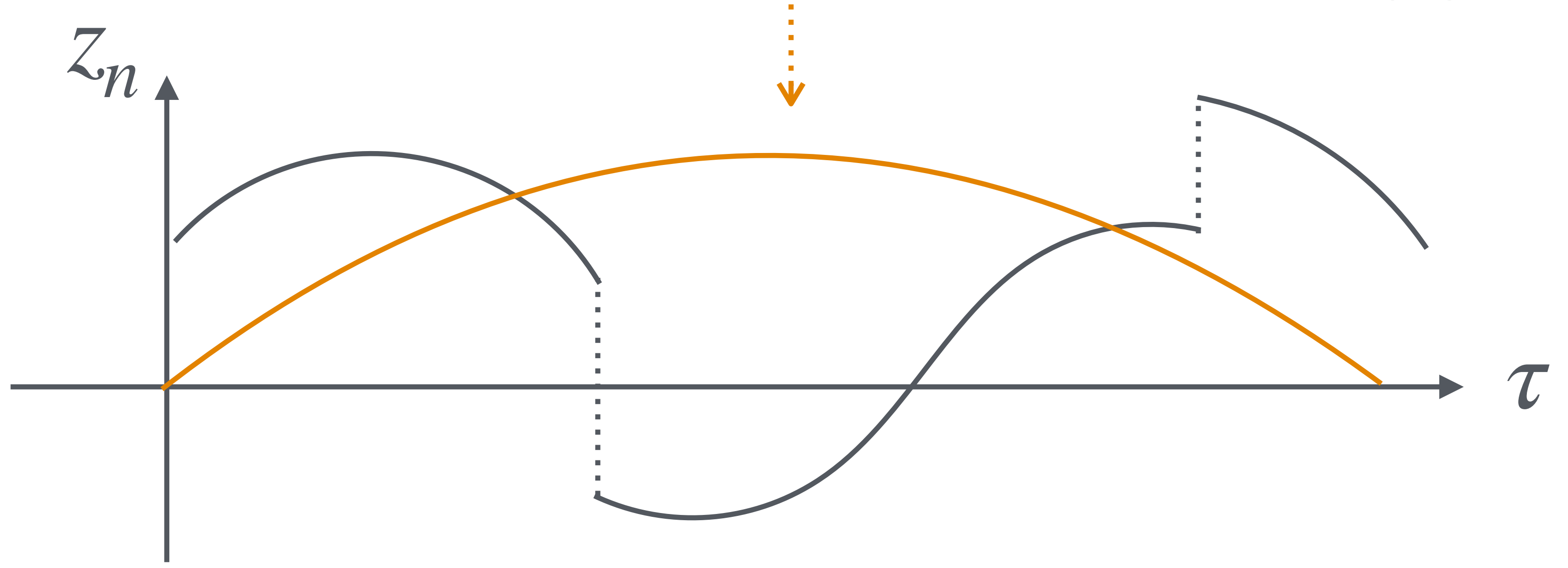
$$\tilde{H}(\tilde{z}_n,\bar{\tilde{z}}_n)=\langle\tilde{z}_n|H|\tilde{z}_n\rangle+(\text{Jacobian})$$

The third step

Question: Is there a saddle point of the action described by a bounce?

$$\langle 0_+ | e^{-HT} | 0_+ \rangle = \int \mathscr{D}\bar{\tilde{z}} \mathscr{D}\tilde{z} e^{-\sum_n K(\bar{\tilde{z}}_{n+1}, \tilde{z}_n) - \sum_n \tilde{H}(\bar{\tilde{z}}_n, \tilde{z}_n)\delta\tau} \simeq e^{-S_0}[1 + \cdots] + e^{-\underline{S_b}}[1 + \boxed{\cdots}] + \cdots$$

Imaginary part?



Bounce

$$K[\bar{z}_{n+1}, z_n] = -\sum_{j=1}^{N} \ln \frac{1 + \bar{z}_{j,n+1} z_{j,n}}{\sqrt{1 + |z_{j,n+1}|^2} \sqrt{1 + |z_{j,n}|^2}}$$

$$\left| \frac{1 + \bar{z}_{j,n+1} z_{j,n}}{\sqrt{1 + |z_{j,n+1}|^2} \sqrt{1 + |z_{j,n}|^2}} \right| \leq 1$$

~This is work in progress.~

Summary

- Vacuum decay in one-dimensional system can be simulated directly and also may be observed in real system (and there is already one for thermal decay). It may provide an explicit check of the theory of vacuum decay.

- The relation between vacuum decay in the 1D spin system and in the QFT is still not well understood. (There may be a mismatch of the prefactor.)

- We are trying to develop a bosonic path integral formalism for the spin-chain to look for a bounce-like saddle point of the action. Once it is found, we can also calculate the prefactor explicitly.