Thermodynamics from the S-matrix reloaded

based on arXiv: 2408.06729 w Emanuele Gendy & Joan Elias Miró

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Thermal Field Theory

why?

- Systems that are at the same time in (1) particle physics regime (i.e. relativity + QM) and (2) statistical physics regime, i.e. with temperature T or chemical potential μ turned on, where

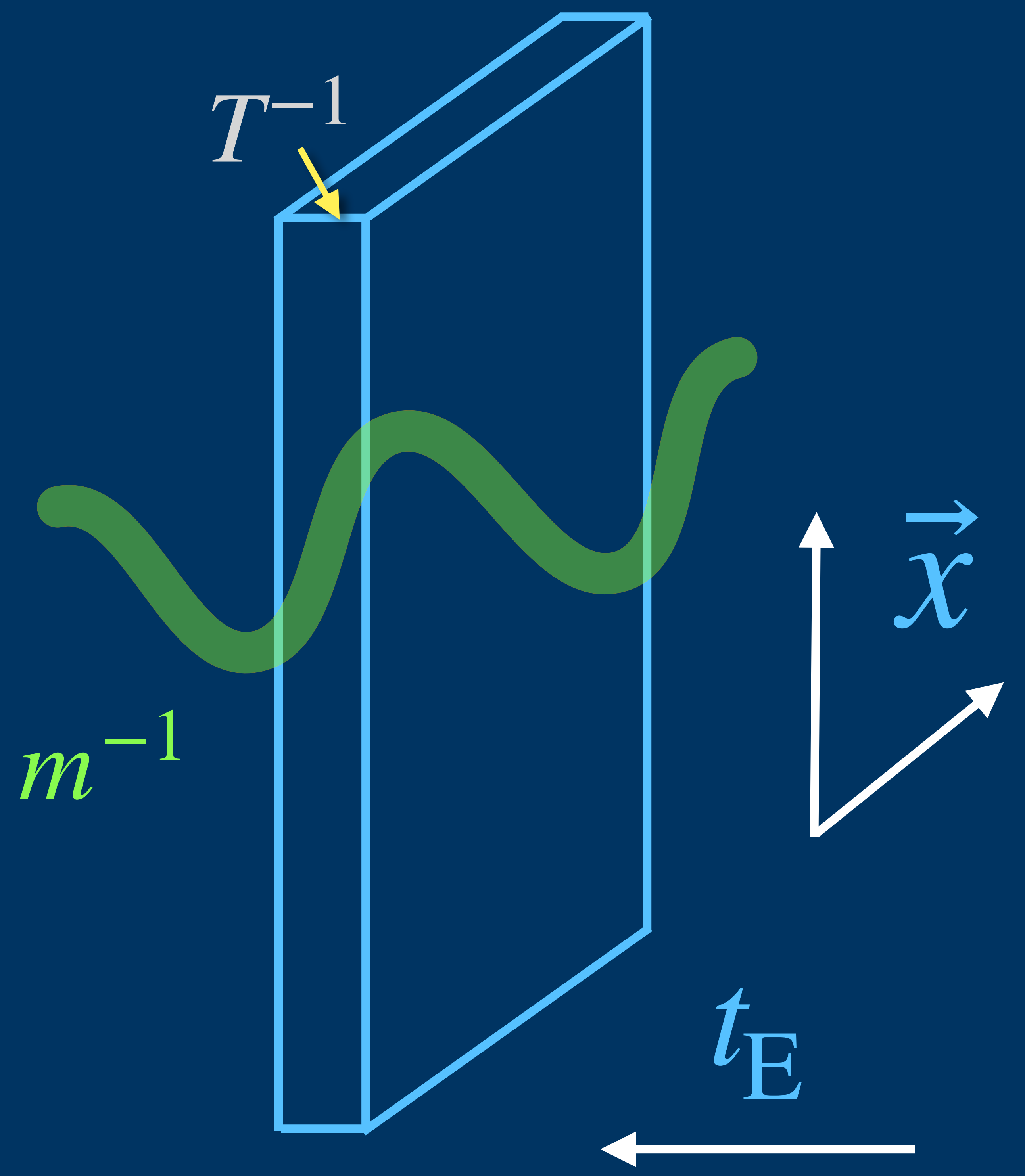
$$T \text{ or } \mu \gg m$$

- Early Universe physics
- Heavy ion collisions
- Extremely dense stellar objects or configurations (e.g. supernovae)

Thermal Field Theory

Overview

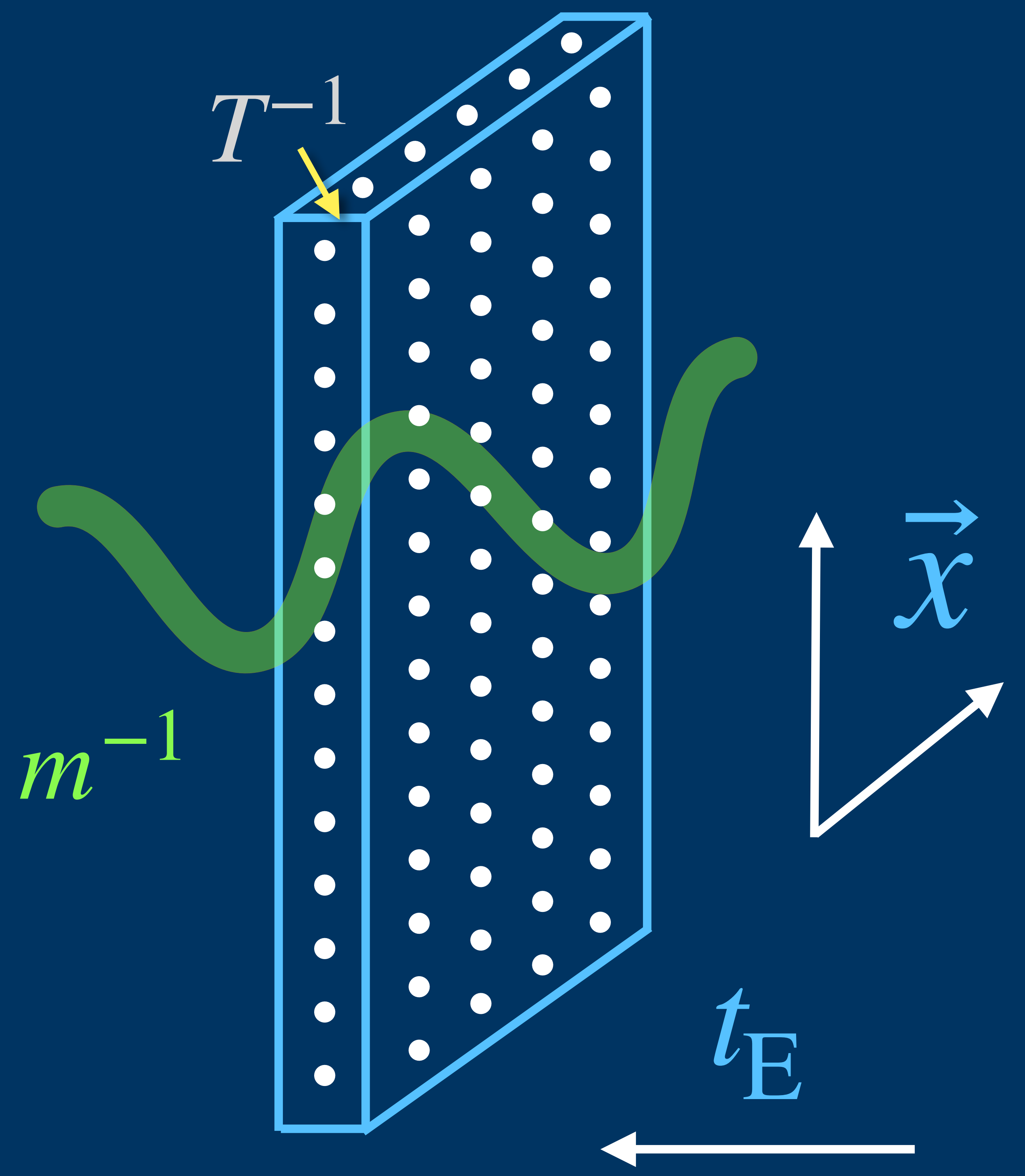
- Two standard methods to treat this regime, both based on a representation of the partition function in terms of a Euclidean path integral



- Time is an interval $[0, T^{-1}]$ and fields have periodic (bosons) or anti-periodic (fermions) b.c.
- <u>Perturbative</u> expansion with Feynman diagrams having modified propagator to account for the geometry of spacetime
- $T \gg m$ is suggestive of a dimensional reduction procedure

Thermal Field Theory
Overview

- Two standard methods to treat this regime, both based on a representation of the partition function in terms of a Euclidean path integral



- Spacetime is replaced by a lattice, with the Euclidean time direction having fewer points to mimic finite temperature
- The path integral is then performed numerically in a non-perturbative fashion

Thermal Field Theory

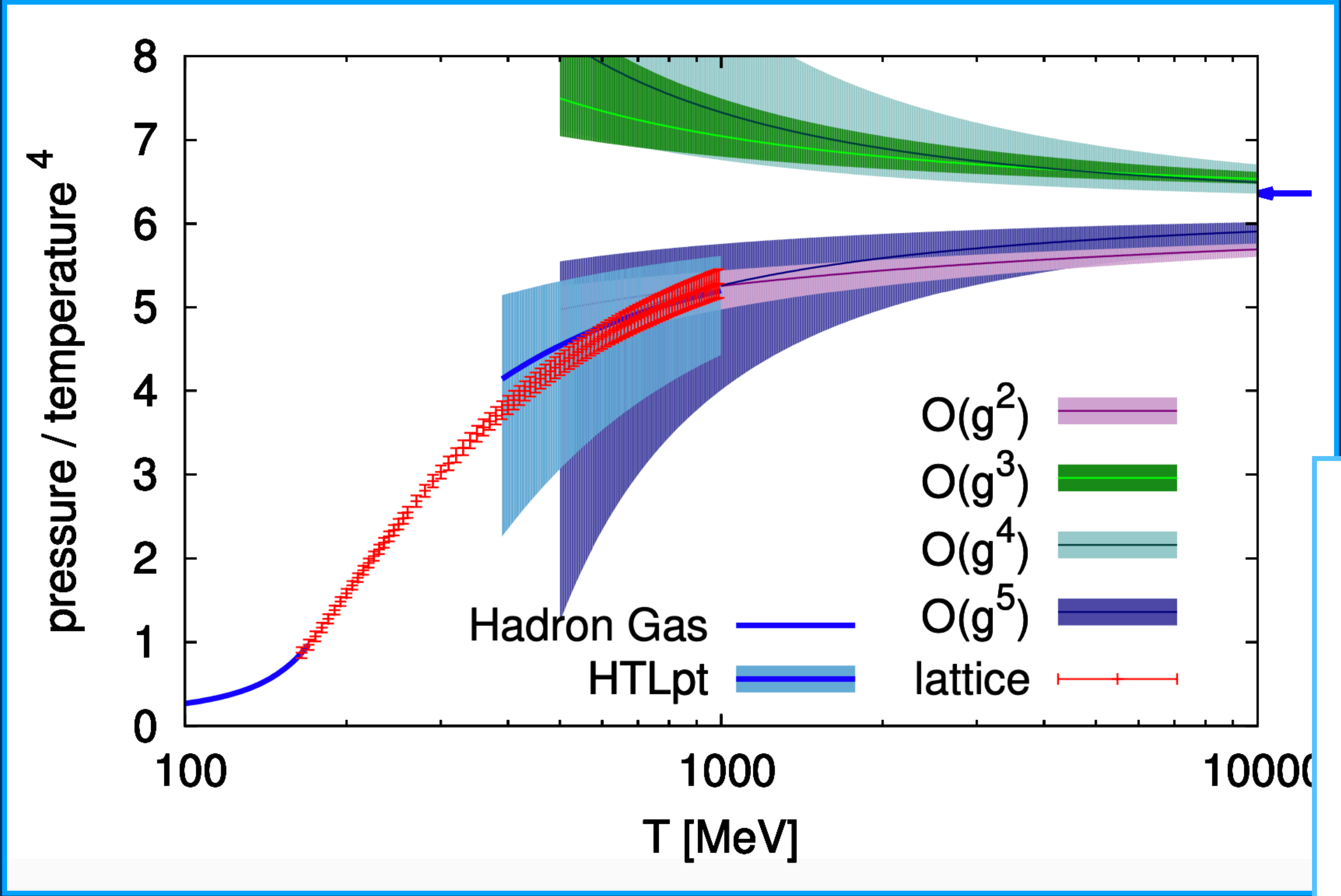
Open questions

Perturbative

- In realistic models like QCD the perturbative series is found to be badly behaved
- Several attempts to get a better convergence have been tried with some success
- **<u>Linde problem</u>**: starting from 4 loops strongly coupled cromo-magnetic modes enter and render the perturbative expansion meaningless

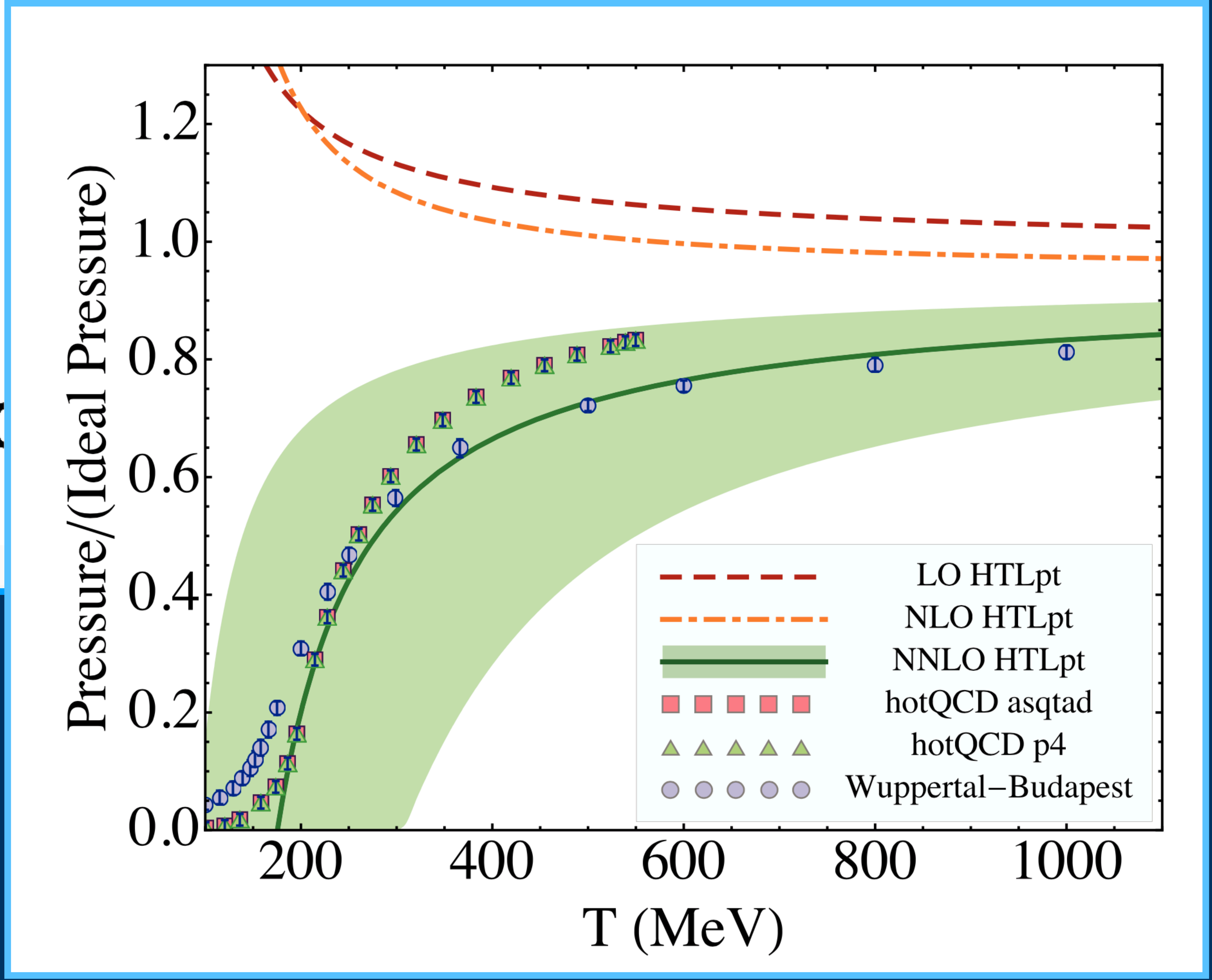
Lattice

- **<u>Sign problem</u>**: main limitation of the lattice approach, as it renders the inclusion of a chemical potential numerically untreatable
- Measure of the path integral not positive definite (oscillatory behaviour of the integrand)



Borsanyi's slides 2018

Andersen et al 2011



Thermal Field Theory

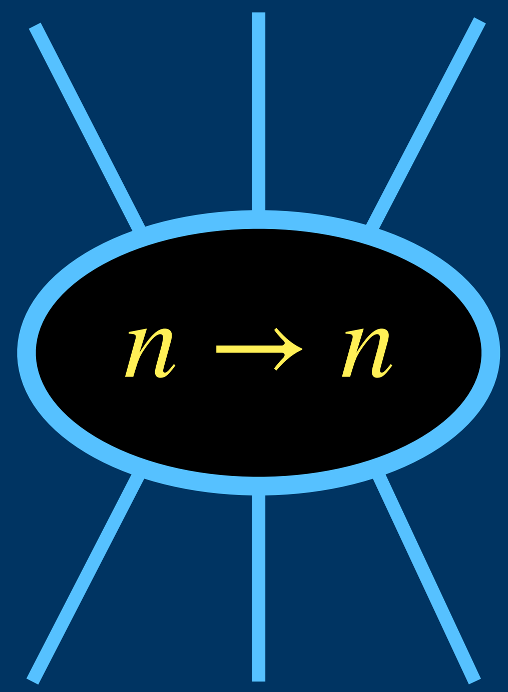
A third way?

- In both approaches one encounters a problem with a name
- Due to the limitations of lattice methods in treating the $\mu \neq 0$ regime, it would be interesting to have a better control on perturbative methods
- We follow a third path
- Instead of starting from a Lagrangian, we consider a formulation of statistical mechanics where the dynamical information is encoded in **scattering amplitudes among the constituents of the system** (Dashen, Ma and Bernstein)

Thermal Field Theory from S-matrix

A third way

- Similar to the perturbative method, since one expands in $M_{n\to n}$
- It can be shown for simple scalar theories that



$\propto g^{2n-2}$

Vacuum bubbles with thermal propagators	$\equiv$	Forward amplitudes averaged over Bose or Fermi distributions

[arXiv:2408.00729]

- For gauge theories the amplitude approach is powerful in that M is gauge invariant (no ghosts, only physical polarisations)

Thermal Field Theory from S-matrix

A third way

- Moreover, the DMB approach neatly disentangles zero temperature dynamical information and the effect of temperature or chemical potential
- This makes it possible to recycle all the knowledge about QCD amplitudes (known to high loop orders and particle multiplicities)
- Different perspective on the same problem might suggest a different resummation method of the perturbative series
- In 2408.06729 we (1) show the power of the method at LO in QCD and (2) push the formalism to NLO and NNLO in a (1+1)-dimensional model of “flux tube long strings”

S-matrix approach

DMB master formula

The trace implies a sum over a complete set of states

$$F - F_0 = -\frac{1}{2\pi i}\int_0^\infty \mathrm{d}E\; e^{-\beta E}\; \mathrm{Tr}_c \ln S(E)$$

Boltzmann suppression

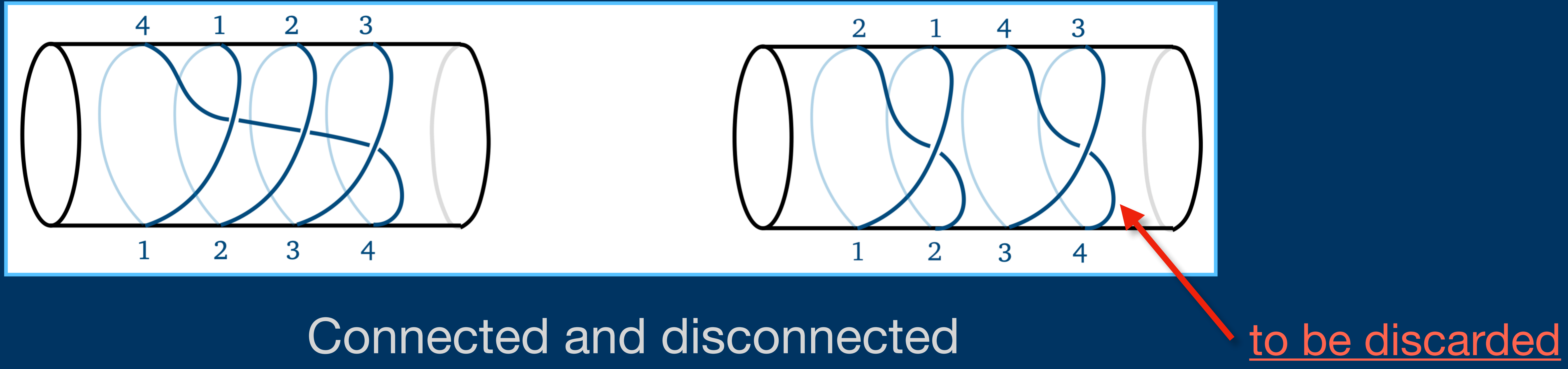
Not exactly the S-matrix:

$$\langle \beta \,|\, S(E = E_\alpha) \,|\, \alpha \rangle \equiv S_{\beta\alpha}$$

S-matrix approach

LO effects in QCD

- The formula can be unpacked by evaluating the trace on asymptotic states with increasing number of particles
- Keep only connected contributions to the trace to extract F



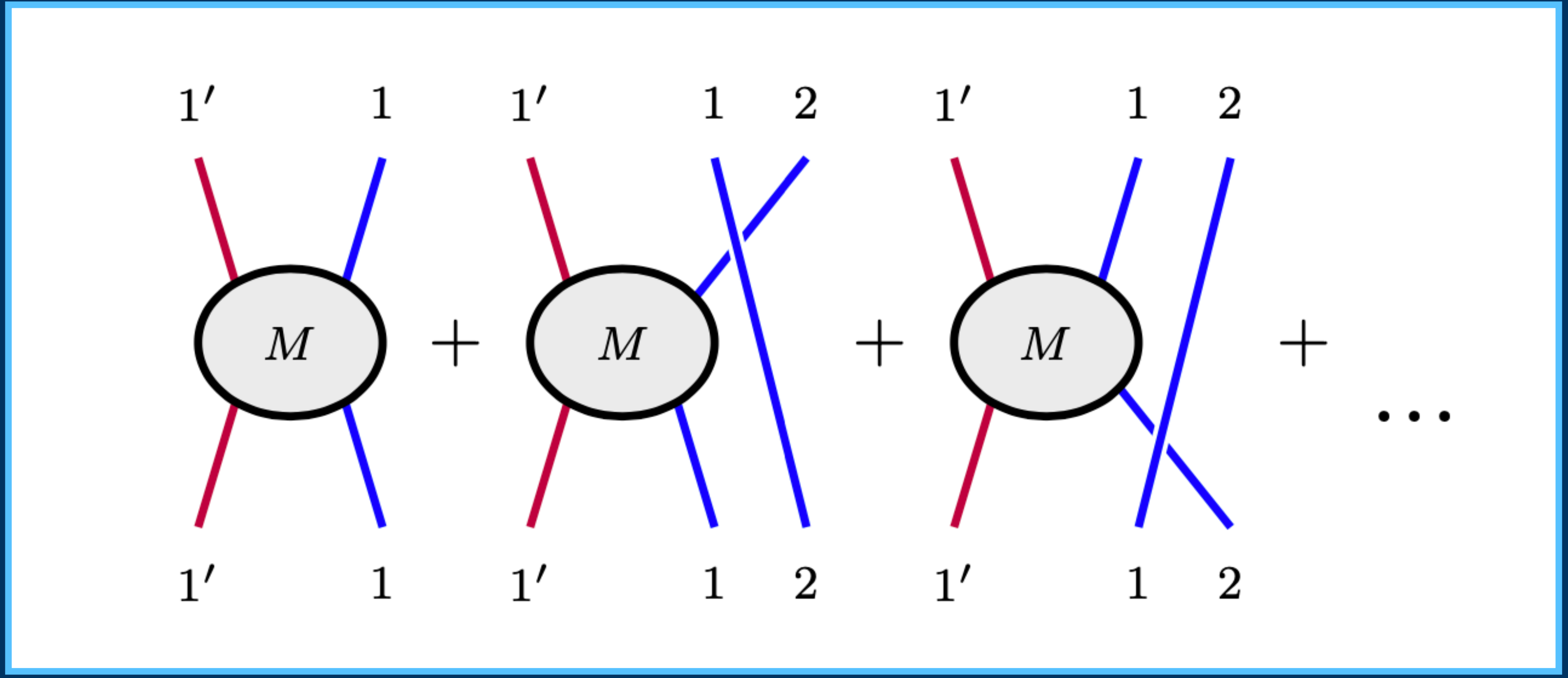
Connected and disconnected contributions to the free theory

S-matrix approach

LO effects in QCD

- Similarly, with interactions, also scattering histories with free propagating particles can be connected under the trace and must be included

- Summing over all such histories turns the Boltzmann weight into either BE or FD densities



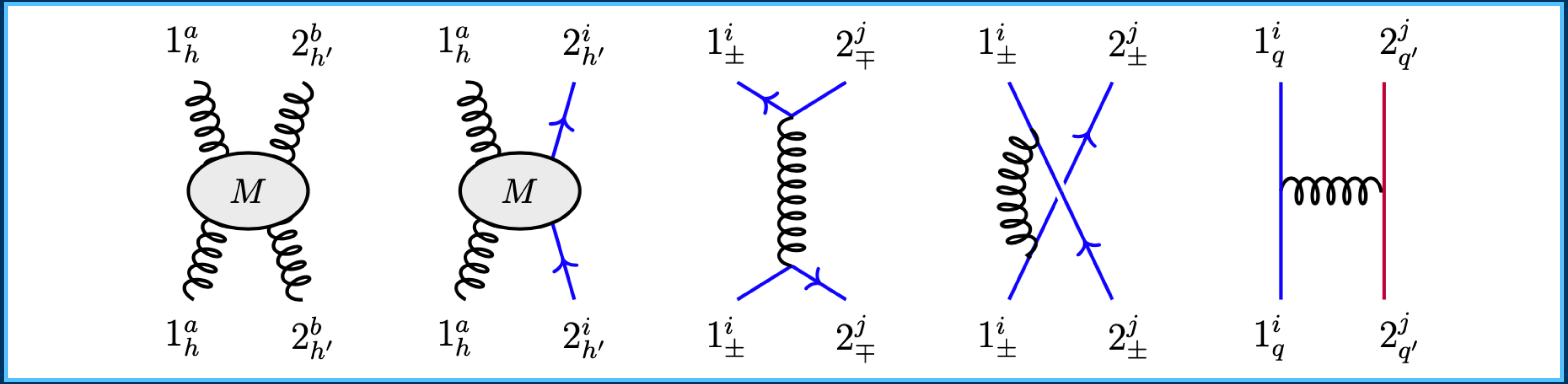
- We get the practical, well-known and easily derivable with other methods LO formula

$$f - f_0 = -\frac{1}{2}\left(\prod_{i=1}^{2}\int\frac{\mathrm{d}^d k_i}{(2\pi)^d 2E_i}\right) n(E_1)\, n(E_2)\, M_{1,2\to 1,2}$$

LO effects in QCD

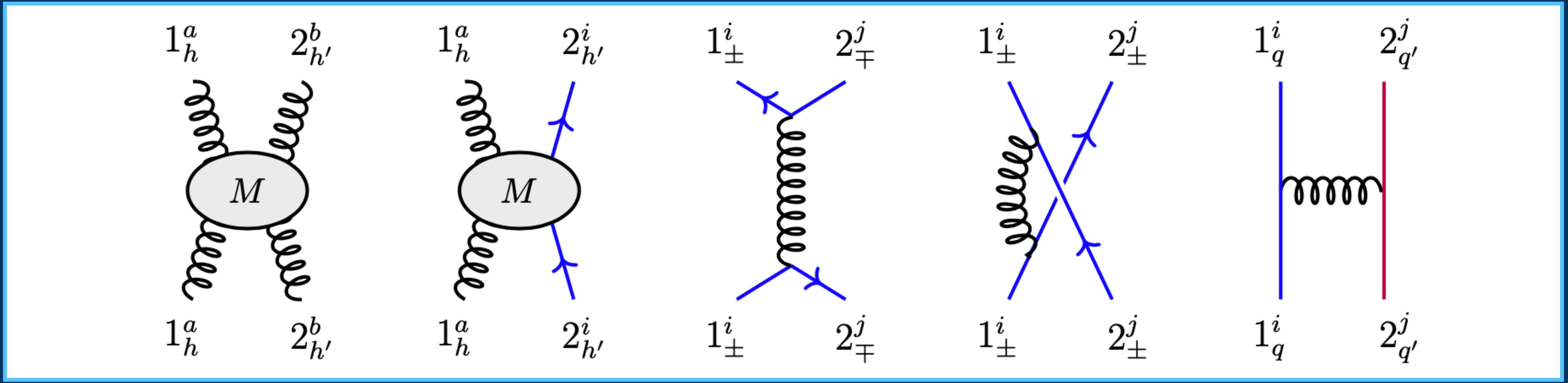
Parke-Taylor formula

$$M(1^a_-, 2^b_-, 3^c_+, 4^d_+) = -2g_s^2\langle 12\rangle^4\left[\frac{f^{abe}f^{cde}}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} + \frac{f^{ace}f^{bde}}{\langle 13\rangle\langle 32\rangle\langle 24\rangle\langle 41\rangle}\right]$$



- All amplitudes are encoded in compact expressions, like the celebrated Parke-Taylor formula for gluon amplitudes
- They admit a simple forward limit independent on the momenta
- Potential problems coming from $\lim_{t\to 0} s/t$ are avoided by first summing over colours

LO effects in QCD

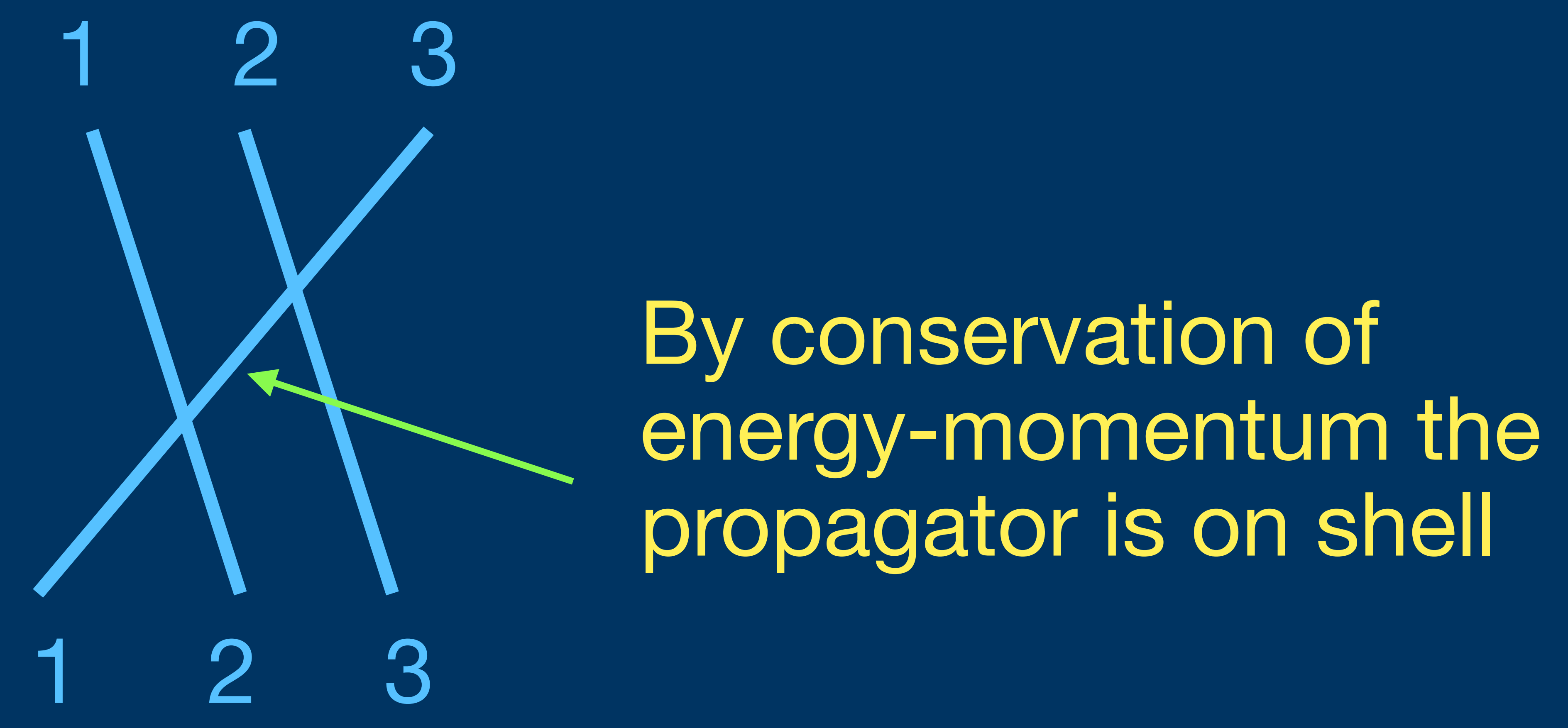


- Having computed the forward limits, one is left with the evaluation of integrals like $\int \mathrm{d}^d k \, E^{-1} \, n(E)$
- We reproduce the textbook result $f_{\mathrm{QCD}} = \alpha_s (N_c^2 - 1) \dfrac{\pi T^4}{36} \left(N_c + \dfrac{5}{4} N_f \right)$ without having to do complicated sum-integrals or invoke ghost fields to deal with a gauge theory
- However possible <u>complications from the strict forward limit</u> appear already

NLO effects

Forward divergences

- At NLO one encounters quite generically $3 \to 3$ diagrams that are singular in the forward limit (it's enough to have nonzero $2 \to 2$)



- Omnipresent problem, even for theories with no IR divergences

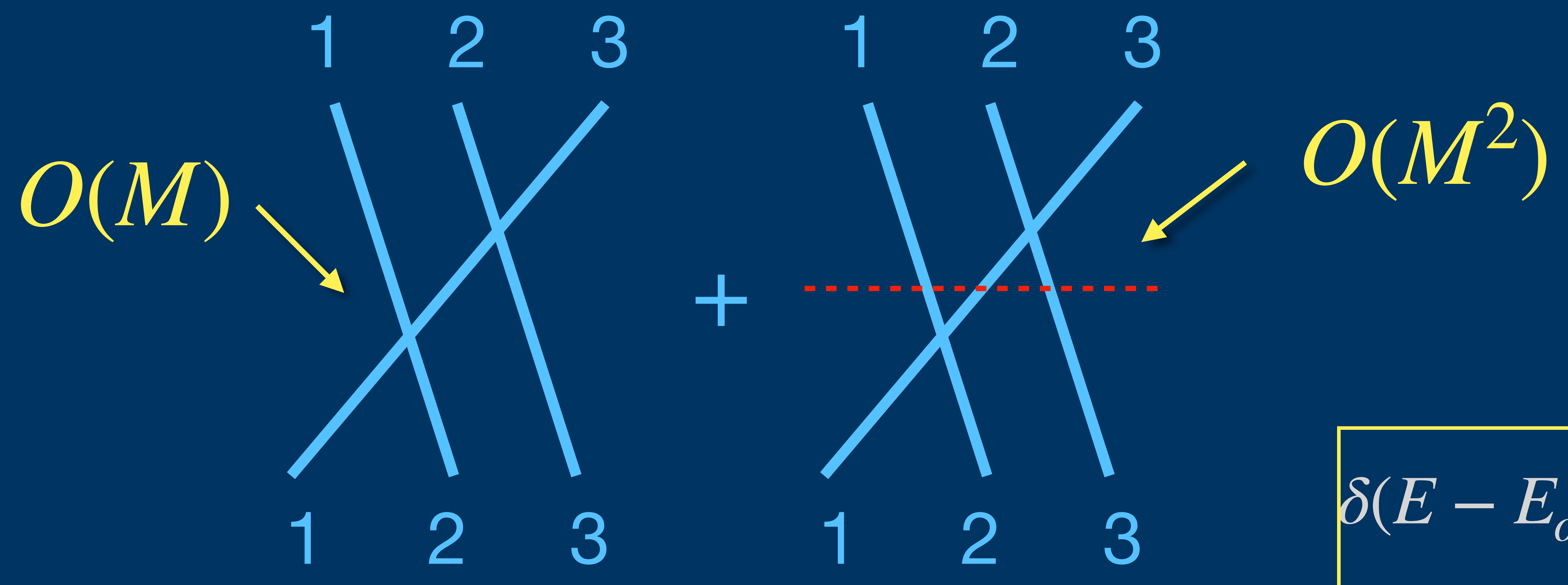
NLO effects

Flux Tube theory

- To disentangle spurious from intrinsic IR divergences we consider a (1+1)-dimensional model, derivatively coupled
- It is the theory of NG bosons coming from the spontaneous breaking of Poincaré symmetry by a long string (a world-sheet in spacetime)
- Pheno: it can describe any string below the string scale, e.g. flux tubes of QCD. It enjoys universality up to high orders in the derivative expansion (thanks to non-linearly realised Poincaré)
- It has been well studied, especially its “integrable version”, so we can compare DMB with other methods like Thermodynamic Bethe Ansatz (TBA)

Flux Tube theory

Resolution of spurious divergences

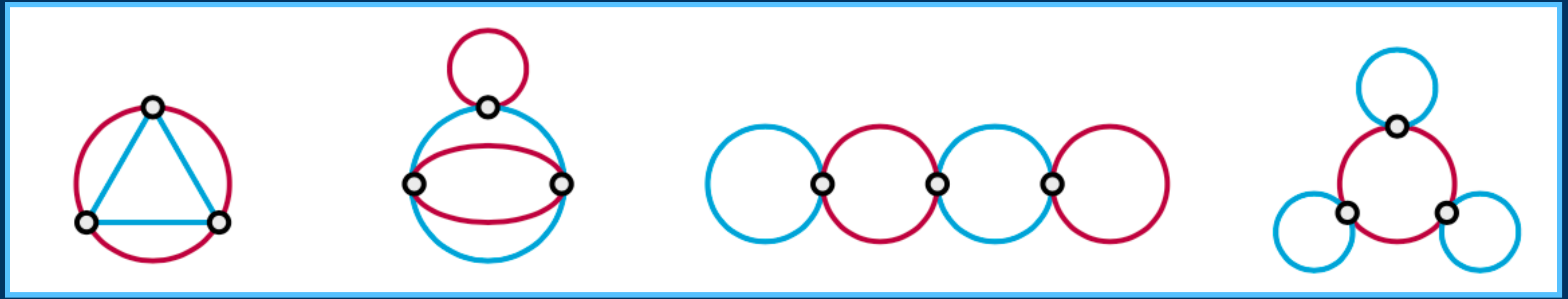


$$\delta(E - E_\alpha)\ \Re \frac{1}{E - E_\alpha + i\epsilon} = -\frac{1}{2}\,\partial_E\,\delta(E - E_\alpha)$$

- The logarithm $\ln S = \ln(1 + M)$ produces terms with more amplitude insertions but same global topology
- A propagator minus a cut propagator gives a well-defined distribution that can be safely integrated in $\mathrm{d}E\, e^{-\beta E}$

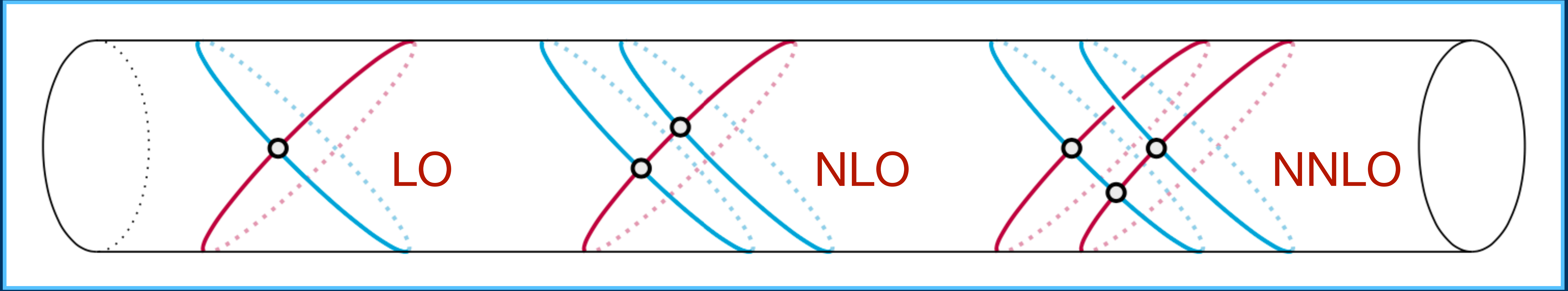
Flux Tube theory

Towards NNLO



NNLO vacuum topologies

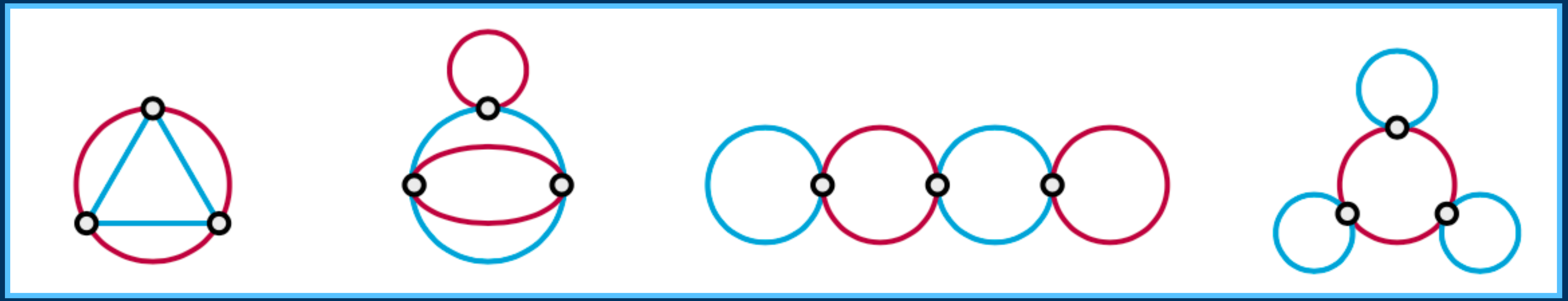
- By (1) properly grouping terms coming from more insertions of M and (2) using the known expressions for $M_{n \to n}$ coming from previous works on the integrable flux tube, we were able to compute the free energy up to NNLO



- For the first time using the DMB method up to this order

Flux Tube theory

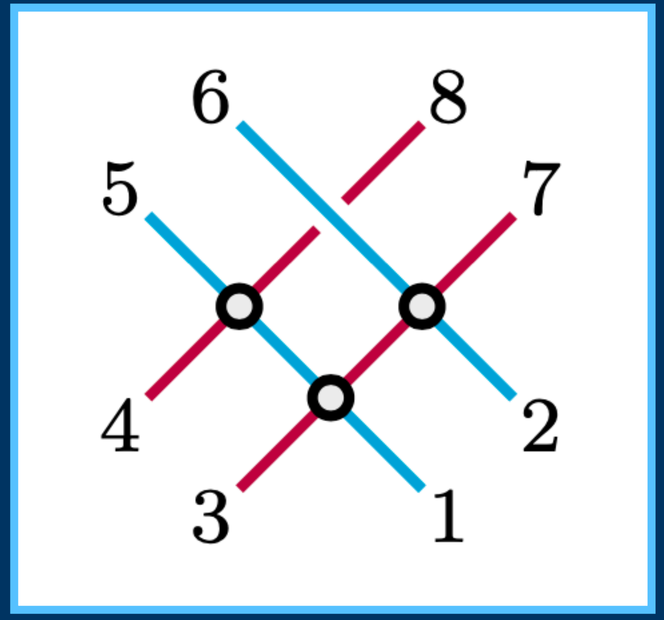
NNLO



NNLO vacuum topologies

- Each vertex in our diagrammatic expansion is a 4-pt vertex due to the underlying integrability of the theory
- Starting from a Lagrangian would imply the use of up to 8-pt vertices to go to NNLO
- UV divergences are already absorbed into the amplitude, i.e. renormalization is done already
- Diagrammatically, we find that only the (1-loop)^k topologies contribute at k loops

Overview and Future directions



*265 Feynman diagrams

- The Flux Tube theory can be studied at higher orders including integrability breaking effects (TBA uses integrability), which is an open area of research
- We can conclude that S-matrix based methods provide a consistent framework for computing thermodynamics in a perturbative expansion
- Example with no intrinsic IR divergences fully under control
- Building blocks are already structured, being a clever sum of Feynman diagrams* (when these are available)
- The method could provide new indications on how to cure the ill behaviour of thermal QCD perturbative series