From vacuum decay to gravitational waves

MARCO MATTEINI

marco.matteini@ijs.si

Jožef Stefan Institute

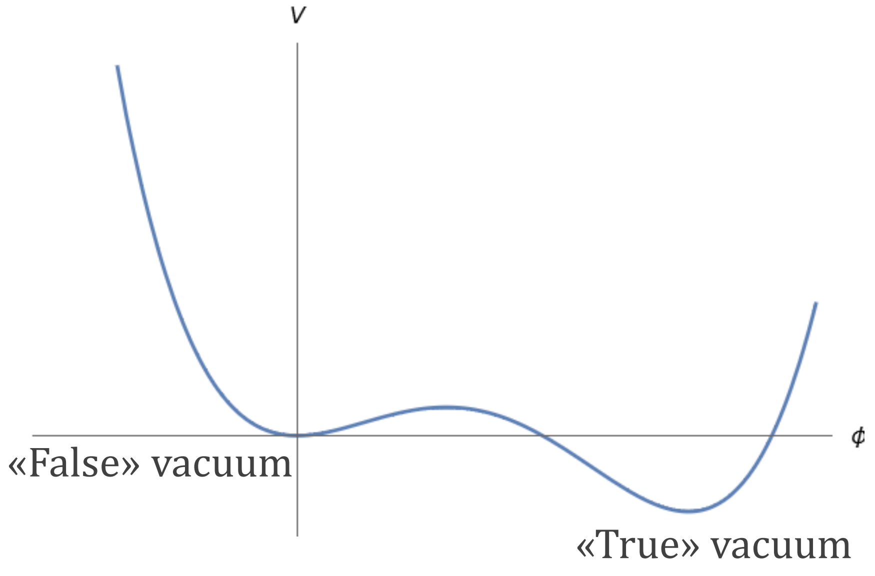
University of Ljubljana Faculty of Mathematics and Physics

Based on:
A. Ivanov, MM, M. Nemevšek, L. Ubaldi
10.1007/JHEP03(2022)209
MM, M. Nemevšek, Y. Shoji, L. Ubaldi
2404.17632
work in progress with V. Brdar, M.Finetti, A. Morais, M. Nemevšek

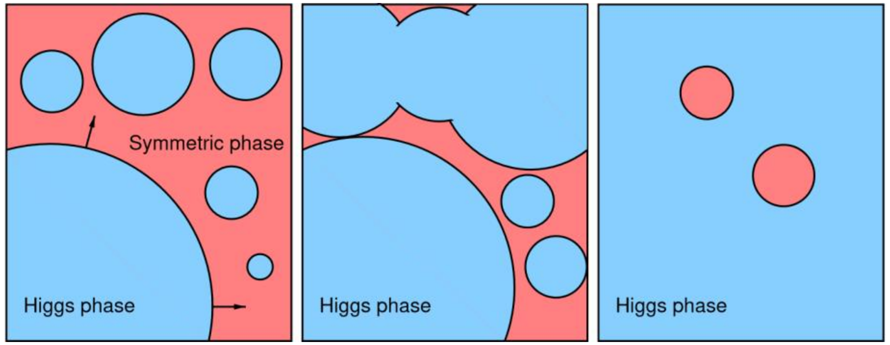
BASICS OF VACUUM DECAY

- Simple example: single scalar

- Metastability of the false vacuum
- Decay to the true vacuum (tunneling under the barrier)
- 1° order phase transition: bubble nucleation



- Bubble expansion: conversion of false vacuum to true vacuum



[*Hindmarsh, Lüben, Lumma, Pauly,* 2008.09136]

- 1-loop decay rate (per unit volume) for Euclidean dimension D

$$\frac{\Gamma}{\mathcal{V}} = \left(\frac{S_R}{2\pi\hbar}\right)^{\frac{D}{2}} \left|\frac{\det' \mathcal{O}}{\det \mathcal{O}_{\rm FV}}\right|^{-\frac{1}{2}} e^{-\frac{S_R}{\hbar} - S_{\rm ct}} \left(1 + \mathcal{O}(\hbar)\right)$$

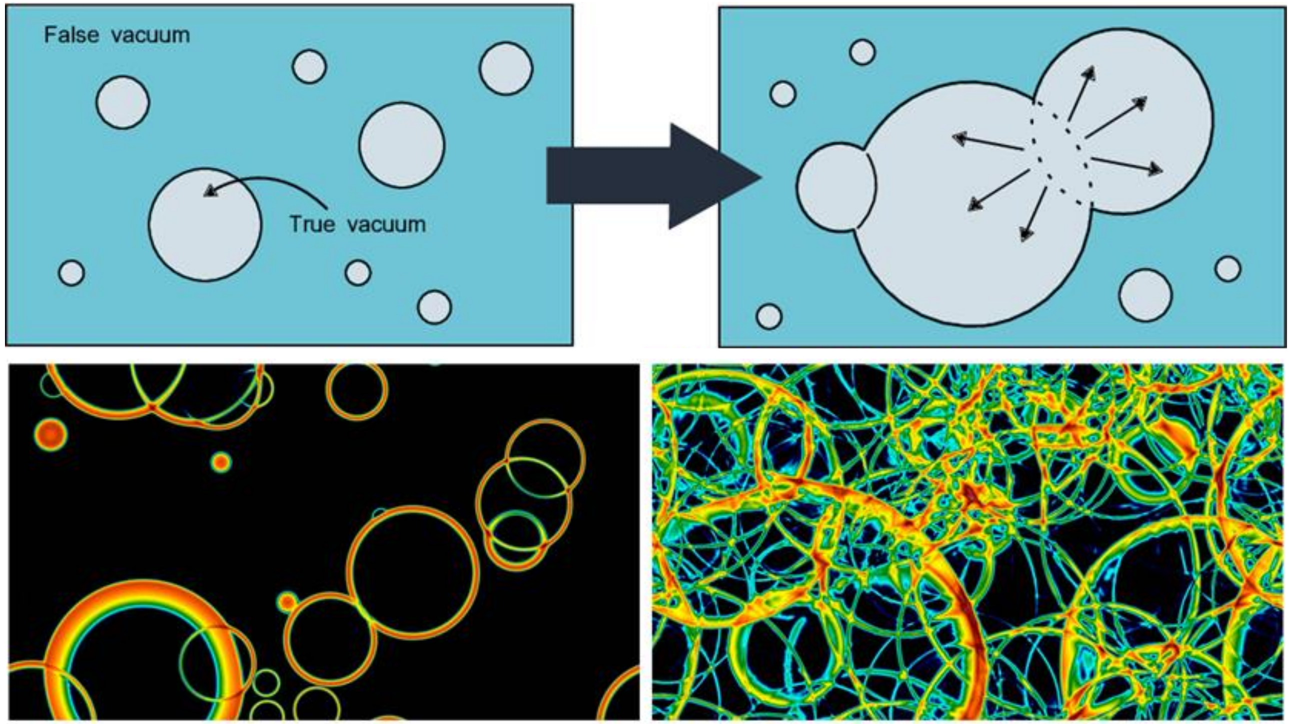
fluctuations — classical solution («bounce»)

GWs FROM PHASE TRANSITIONS

- Early universe: 1° order cosmological PT is a sign of BSM physics!
- Signature: Gravitational Waves. Stochastic background: many uncorrelated, unresolved sources

GWs FROM PHASE TRANSITIONS

- Early universe: 1° order cosmological PT is a sign of BSM physics!
- Signature: Gravitational Waves. Stochastic background: many uncorrelated, unresolved sources
- Sources of GWs from PTs:
  - Bubble collisions
  - Sound waves in the plasma
  - Turbulence in the plasma

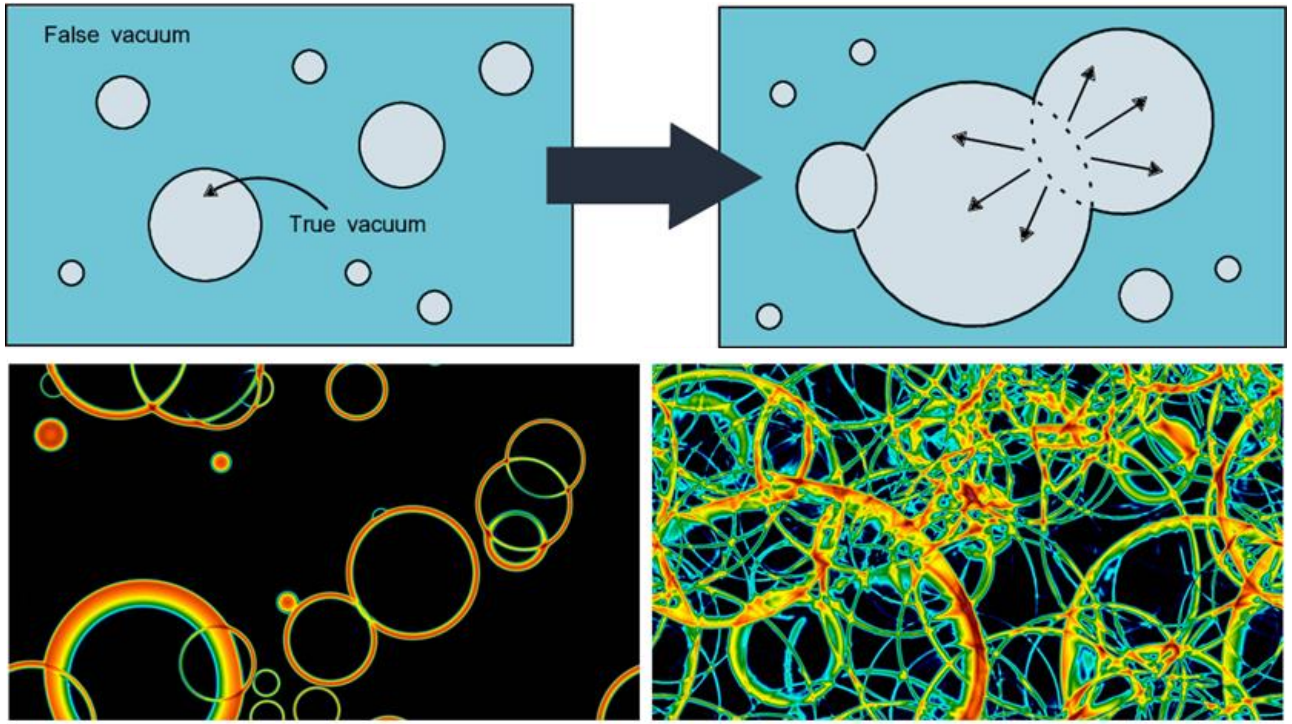


[*Weir*, 1705.01783]

GWs FROM PHASE TRANSITIONS

- Early universe: 1° order cosmological PT is a sign of BSM physics!
- Signature: Gravitational Waves. Stochastic background: many uncorrelated, unresolved sources

- Sources of GWs from PTs:
  - Bubble collisions
  - Sound waves in the plasma
  - Turbulence in the plasma
- Relevant temperatures:
  - Nucleation: 1 bubble per Hubble volume
  - Percolation: connected region of TV phase
- Phase transition parameters:
  - Strength α: energy released by the vacuum transition normalized to the radiation energy
  - Inverse duration β: time derivative of Γ at percolation
  - Wall velocity ξ_w



[*Weir*, 1705.01783]

NUCLEATION & PERCOLATION

- Nucleation temperature T_n

$$\int_{t_c}^{t_n} \mathrm{d}t \, \frac{\Gamma(t)}{H(t)^3} = \int_{T_n}^{T_c} \frac{\mathrm{d}T}{T} \frac{\Gamma(T)}{H(T)^4} = 1$$

Approximate criterion

$$\frac{\Gamma}{H^4} \approx 1 \longrightarrow \frac{S_3}{T_n} \approx 4 \log \left(\frac{T_n}{H} \right)$$

- Percolation temperature T_p (at least 34% of the comoving volume has been converted to the TV)

$$I(t) = \frac{4\pi}{3} \int_{t_c}^{t} \mathrm{d}t' \, \Gamma(t') \, a(t')^3 \, r(t,t')^3 \longrightarrow I(T) = \frac{4\pi v_w}{3} \int_{T}^{T_c} \mathrm{d}T' \, \frac{\Gamma(T')}{H(T') \, T'^4} \left(\int_{T}^{T'} \frac{\mathrm{d}T''}{H(T'')} \right)^3$$

Stronger requirement for completion: decreasing FV volume $V_{false} = a^3(t) \, e^{-I(t)}$

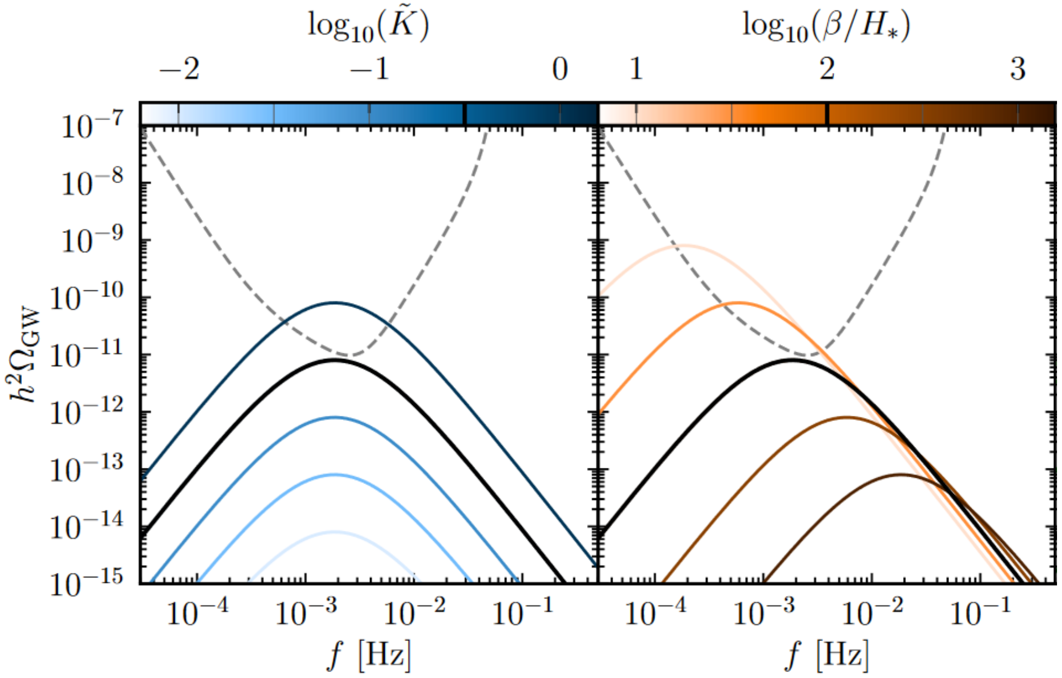
$$\frac{1}{V_{\mathrm{false}}} \frac{\mathrm{d}V_{\mathrm{false}}}{\mathrm{d}t} = 3H(t) - \frac{\mathrm{d}I(t)}{\mathrm{d}t} = H(T) \left(3 + T \frac{\mathrm{d}I(T)}{\mathrm{d}T} \right) < 0$$

GW POWER SPECTRUM

Particle physics model → PT params: $H_*, \alpha, \beta, \xi_w$ → Power spectrum: $h^2\Omega_{GW}(f; H_*, \alpha, \beta, \xi_w)$

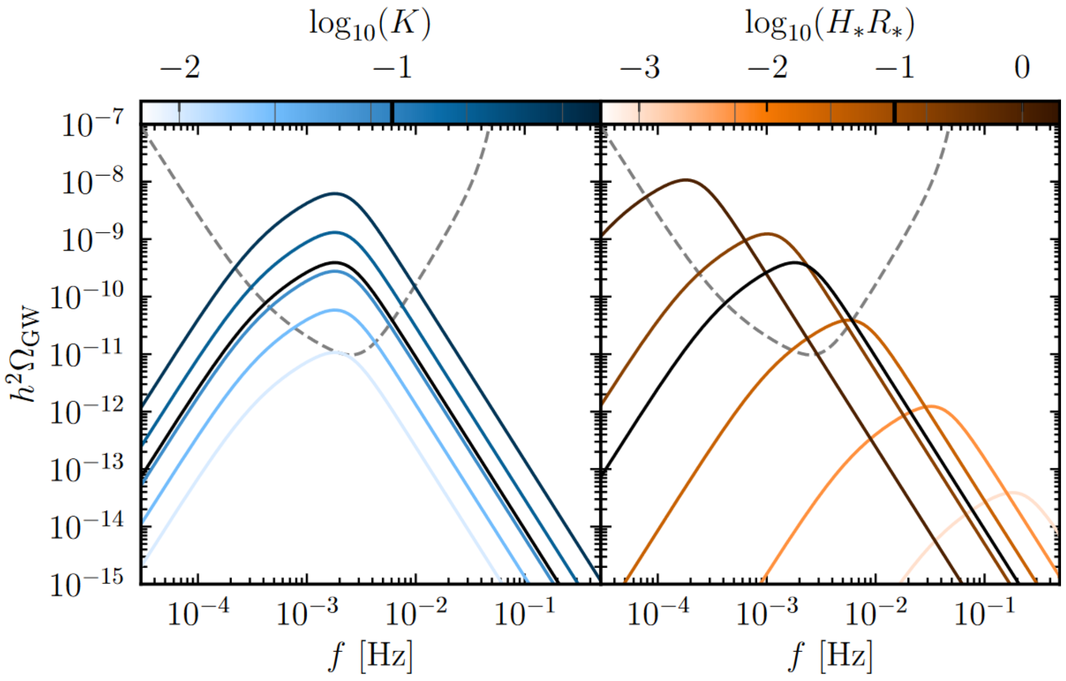
- Templates from [*LISA Cosmology Working Group*, 2403.03723]

-Broken power law for bubble collisions



$\tilde{K} \equiv \alpha/(1+\alpha)$

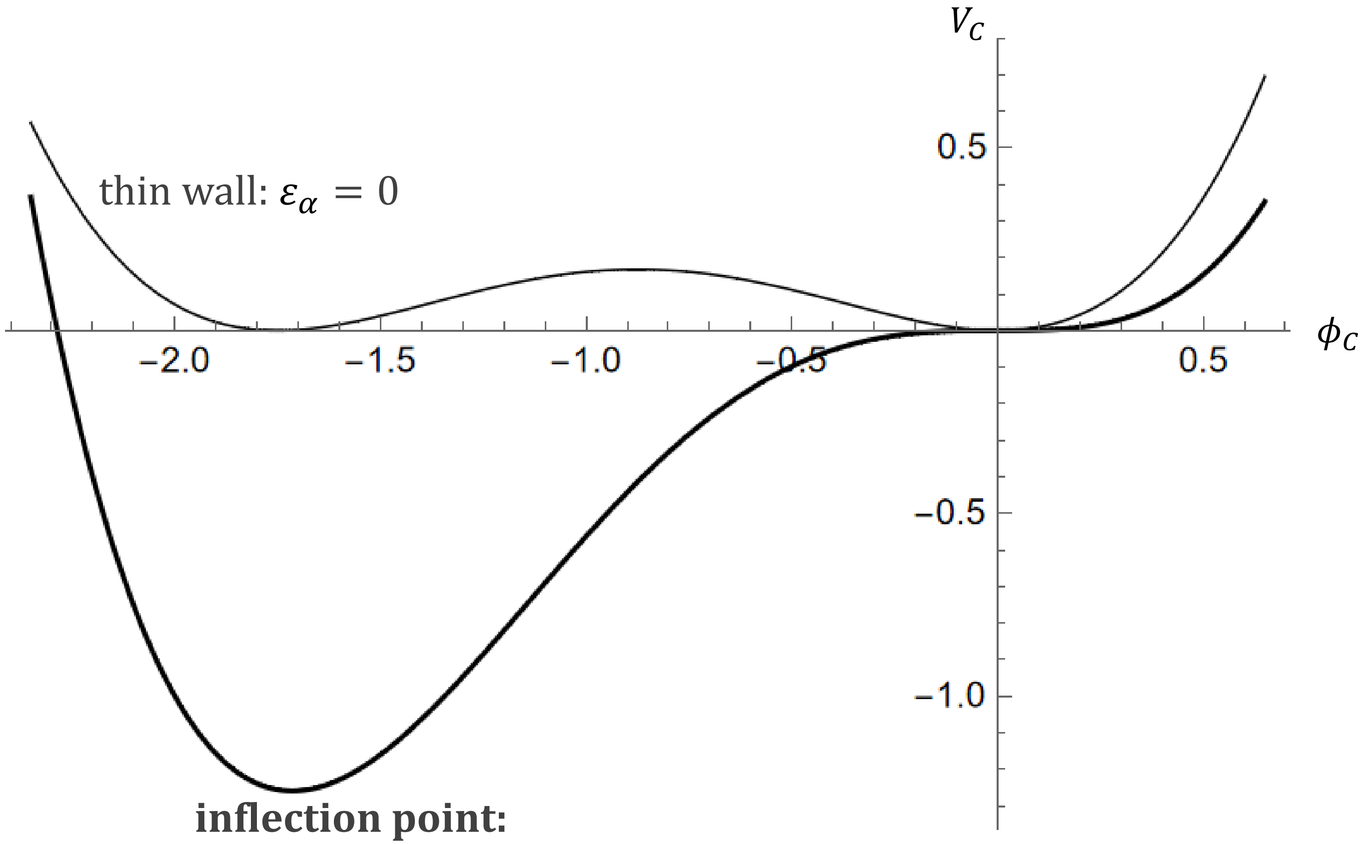
-Double broken power law for sound waves and turbulence



$K \simeq 0.6\,\kappa\,\alpha/(1+\alpha)$ $\quad H_* R_* = (8\pi)^{1/3} \max(\xi_w, c_s)\,\dfrac{H_*}{\beta}$

THIN & THICK WALL

- Single real scalar with potential: $V_C(\phi_C) = \frac{1}{2}m^2\phi_C^2 + \eta\phi_C^3 + \frac{1}{8}\lambda_C\phi_C^4$



- Dimensionless quantities

$$\varphi_C \equiv \frac{2\eta}{m^2}\phi_C$$

$$S = \frac{\Omega\ m^{6-D}}{4\eta^2} S_C(\epsilon_\alpha)$$

$$\varepsilon_\alpha \equiv 1 - \lambda_C \frac{m^2}{4\eta^2}$$

thin-wall expansion parameter

$$\varphi_C(z) = \sum_{n=0} \varepsilon_\alpha^n\ \varphi_{Cn}(z)$$

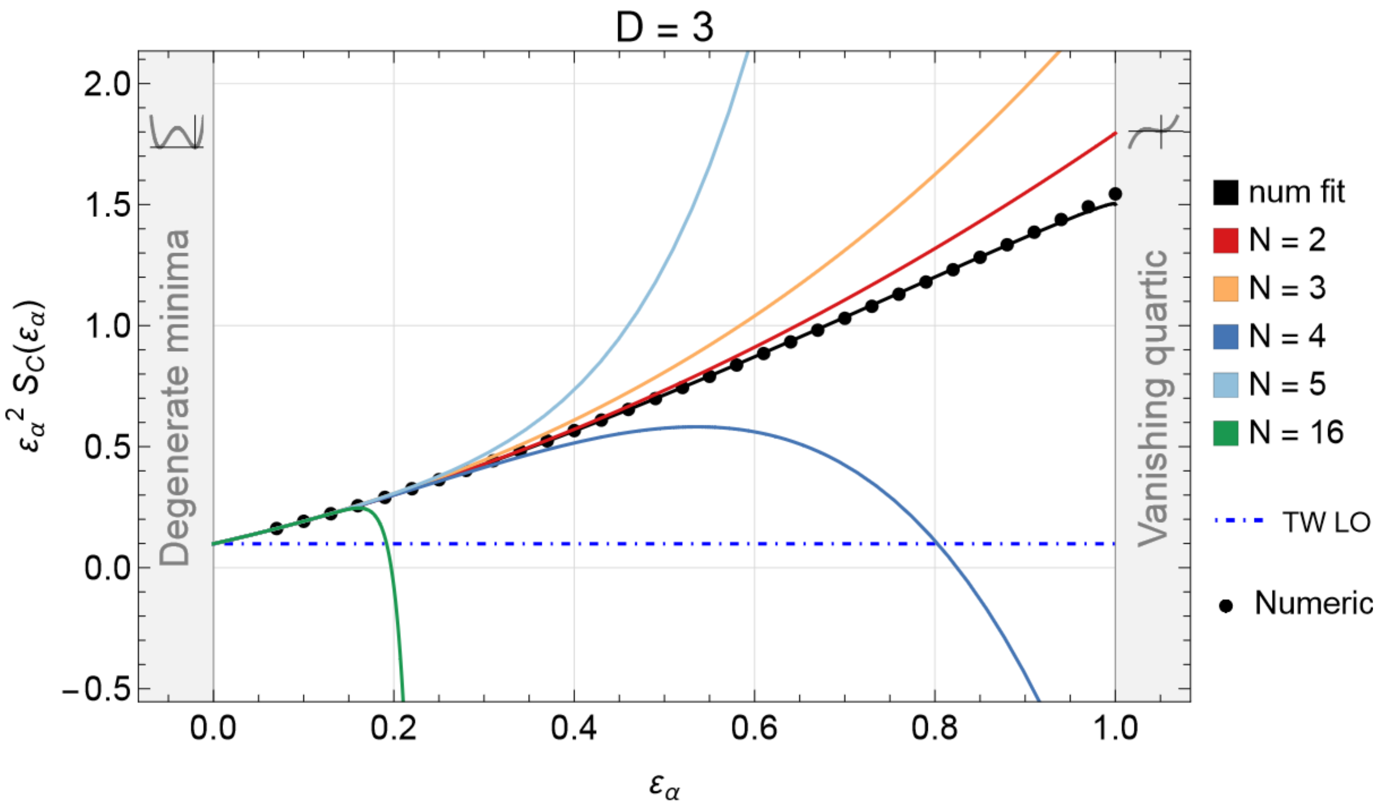
$$S_C(\epsilon_\alpha) = \sum_{n=0}^{N} \epsilon_\alpha^n\, S_C^{(n)}(\epsilon_\alpha)$$

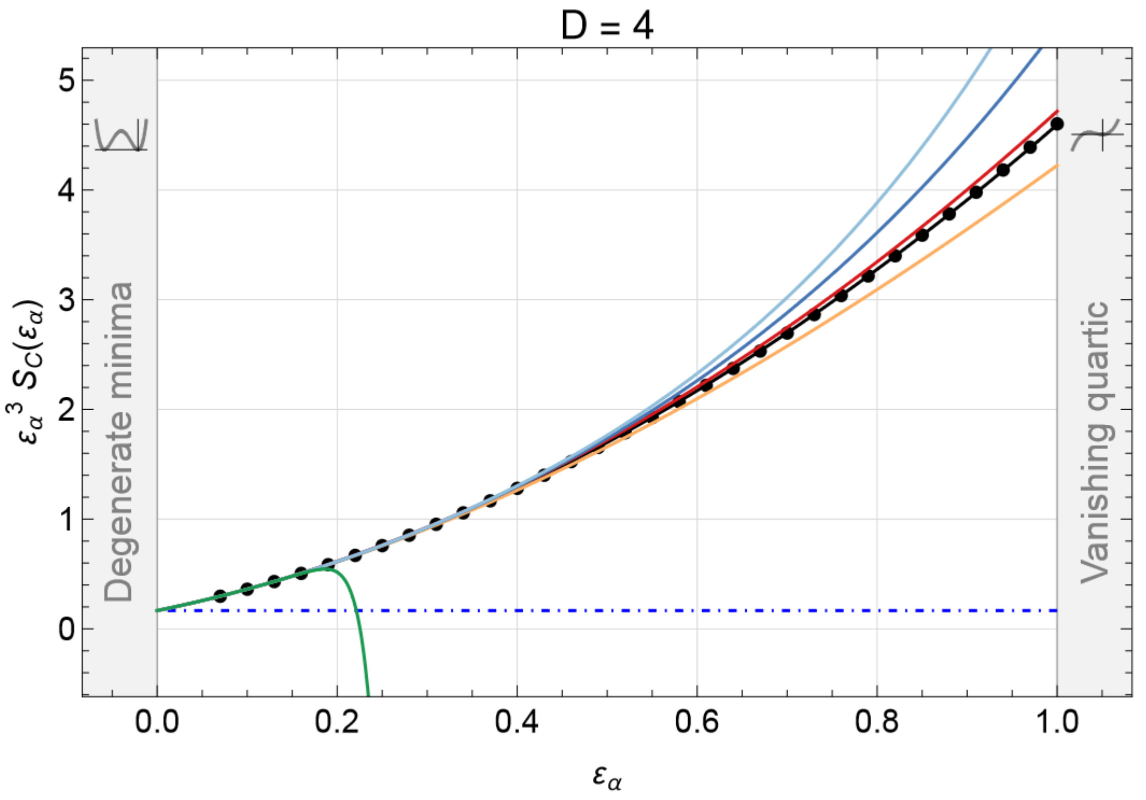
THIN & THICK WALL

$$S_C(\epsilon_\alpha) = \sum_{n=0}^{N} \epsilon_\alpha^n \, S_C^{(n)}(\epsilon_\alpha) \longleftarrow S_C^{(2)}(\epsilon_\alpha) = \boxed{\frac{1}{\epsilon_\alpha^{D-1}}} \left(\frac{D-1}{3}\right)^{D-1} \frac{2}{3D} \left(1 + \epsilon_\alpha \frac{3D+8}{2} + \epsilon_\alpha^2 \frac{9D^3 - 11D^2 + 138D - 12D\pi^2 - 64}{8(D-1)}\right)$$

THIN & THICK WALL

$$S_C(\epsilon_\alpha) = \sum_{n=0}^{N} \epsilon_\alpha^n \, S_C^{(n)}(\epsilon_\alpha) \longleftarrow S_C^{(2)}(\epsilon_\alpha) = \boxed{\frac{1}{\epsilon_\alpha^{D-1}}} \left(\frac{D-1}{3}\right)^{D-1} \frac{2}{3D} \left(1 + \epsilon_\alpha \frac{3D+8}{2} + \epsilon_\alpha^2 \frac{9D^3 - 11D^2 + 138D - 12D\pi^2 - 64}{8(D-1)}\right)$$





The truncated expansion works well away from thin wall!

APPLICATION TO THERMAL POTENTIAL

- Temperature-dependent coefficients in $V_C(\phi_C) = \frac{1}{2}m^2\phi_C^2 + \eta\phi_C^3 + \frac{1}{8}\lambda_C\phi_C^4$ $\quad \varepsilon_\alpha \equiv 1 - \lambda_C \frac{m^2}{4\eta^2}$

- Can we calculate e.g. the nucleation temperature analytically?
- Start from the approximate criterion (at the electroweak scale)

$$\frac{S_3}{T} \simeq 140 \qquad \text{with} \qquad S_3 = \frac{32\pi}{81}\frac{m^3}{4\eta^2}\frac{1}{\varepsilon_\alpha^2}\left[1 + \frac{17}{2}\varepsilon_\alpha + \left(\frac{247}{8} - \frac{9\pi^2}{4}\right)\varepsilon_\alpha^2\right]$$

APPLICATION TO THERMAL POTENTIAL

- Temperature-dependent coefficients in $V_C(\phi_C) = \frac{1}{2}m^2\phi_C^2 + \eta\phi_C^3 + \frac{1}{8}\lambda_C\phi_C^4$ $\quad \varepsilon_\alpha \equiv 1 - \lambda_C \frac{m^2}{4\eta^2}$

- Can we calculate e.g. the nucleation temperature analytically?
- Start from the approximate criterion (at the electroweak scale)

$$\frac{S_3}{T} \simeq 140 \qquad \text{with} \qquad S_3 = \frac{32\pi}{81}\frac{m^3}{4\eta^2}\frac{1}{\varepsilon_\alpha^2}\left[1 + \frac{17}{2}\varepsilon_\alpha + \left(\frac{247}{8} - \frac{9\pi^2}{4}\right)\varepsilon_\alpha^2\right]$$

- Expand in the two regimes:

- Degeneracy ($m^2 = 4\eta^2/\lambda_C$): expand S_3 around T_C and check for self-consistency at T_n
- Inflection ($m^2 \to 0$ i.e. $\epsilon_\alpha \simeq 1$): expand S_3 around T_{inf} and check for self-consistency at T_n

FLUID – FIELD MODEL

- Widely used model for numerical simulations: cosmic fluid - order parameter field

$$V(\phi, T) = \frac{1}{2}\gamma(T^2 - T_0^2)\phi^2 - \frac{1}{3}AT\phi^3 + \frac{1}{4}\lambda\phi^4$$

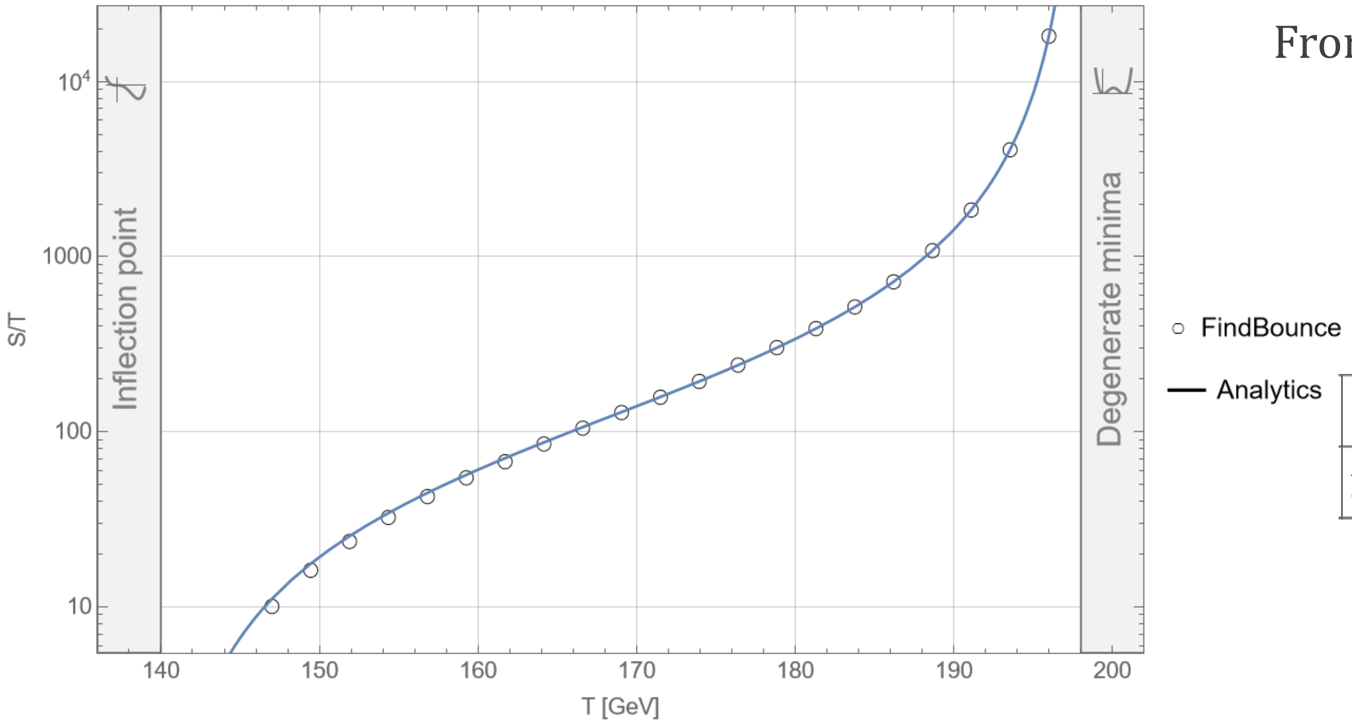
- Phase structure: Degenerate minima at $T_C = \frac{\sqrt{9\gamma\lambda}}{\sqrt{9\gamma\lambda - 2A^2}}T_0$, inflection point at T_0

FLUID – FIELD MODEL

- Widely used model for numerical simulations: cosmic fluid - order parameter field

$$V(\phi,T)=\frac{1}{2}\gamma(T^2-T_0^2)\phi^2-\frac{1}{3}AT\phi^3+\frac{1}{4}\lambda\phi^4$$

- Phase structure: Degenerate minima at $T_C=\frac{\sqrt{9\gamma\lambda}}{\sqrt{9\gamma\lambda-2A^2}}T_0$, inflection point at T_0



From [*Hindmarsh, Huber, Rummukainen, Weir*, 1504.03291]:

γ	A	λ	T_0 [GeV]	T_c [GeV]	T_N [GeV]	α_{T_N}
1/18	$\sqrt{10}/72$	10/648	140	$\sqrt{2}\,T_0=197.99$	$0.86\,T_C=170.27$	$\alpha_N=0.01$

From our analytical action:

γ	A	λ	T_0 [GeV]	T_c [GeV]	$T_N^{(S/T)}$ [GeV]	$T_N^{(\Gamma/H)}$ [GeV]	α_{T_N}
1/18	$\sqrt{10}/72$	10/648	140	197.99	170.04	170.22	0.0104

NUCLEATION IN FLUID – FIELD MODEL

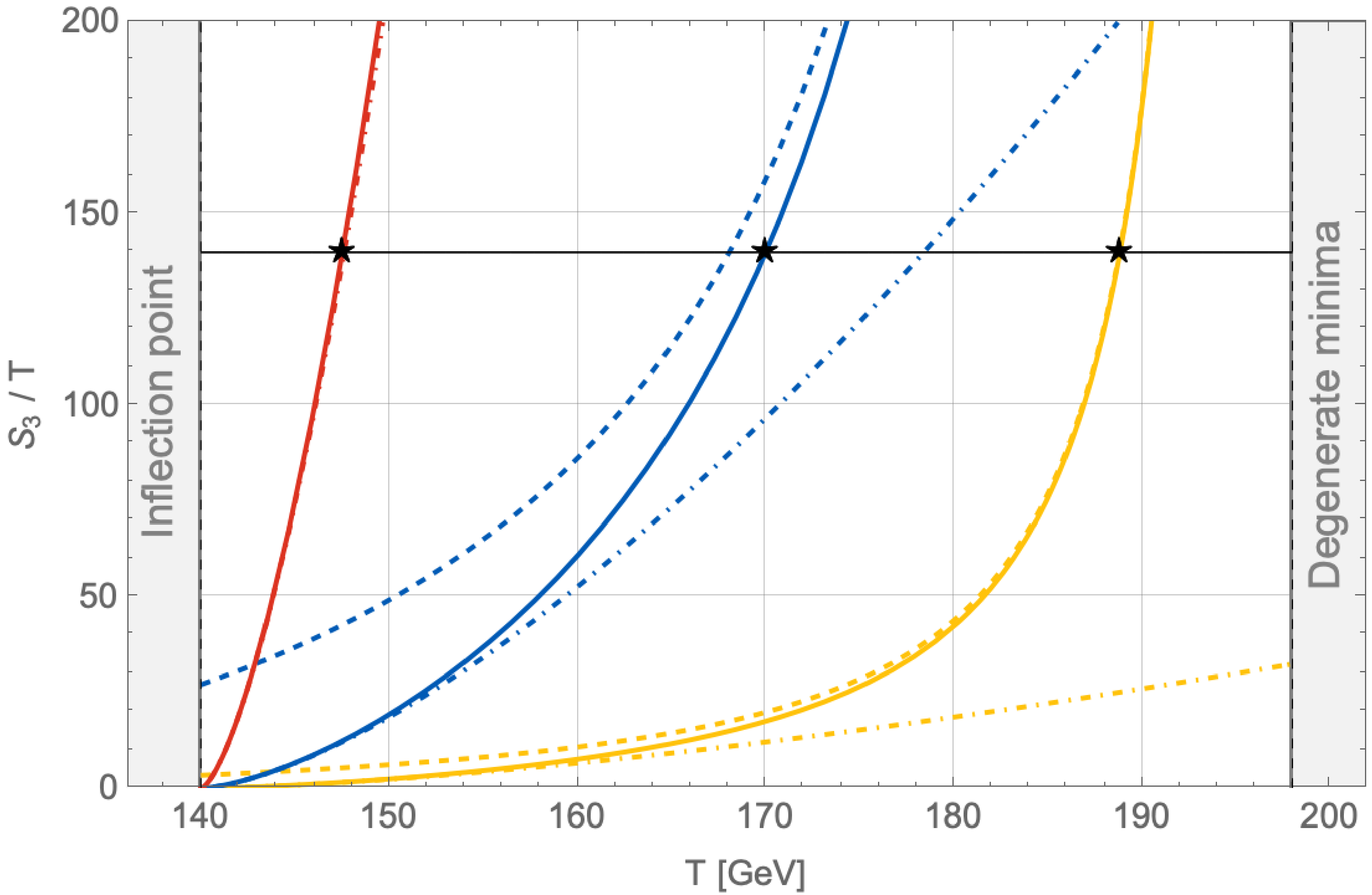
Bench.	1	2	3
γ	$1/18$	$2/9$	$1/9$
A	$\sqrt{10}/72$	$\sqrt{10}/9$	$\sqrt{10}/144$
λ	$10/648$	$160/648$	$1.25/648$
T_0	140	140	140
T_c	198	198	198
★ $T_N^{(S/T)}$	170.04	188.82	147.34
$T_N^{(T_c)}$	168.08	188.80	129.93
$T_N^{(T_0)}$	178.41	293.65	147.62

Temperatures in GeV

— Analytical action

- - - Expansion around T_C

·-·· Expansion around T_0



MATHEMATICA PACKAGE

- In development: `Mathematica` package that calculates
  Relevant temperatures, PT parameters and GW spectrum for a given (single field) particle physics model

- Input the bubble wall velocity & the potential: - direct user implementation, - `DRalgo` [*Ekstedt, Schicho, Tenkanen*, 2205.08815]

- Phase tracing: either analytical or numerical, checks for overlap of pairs of phases

  If overlap found, checks for existence of critical temperaure

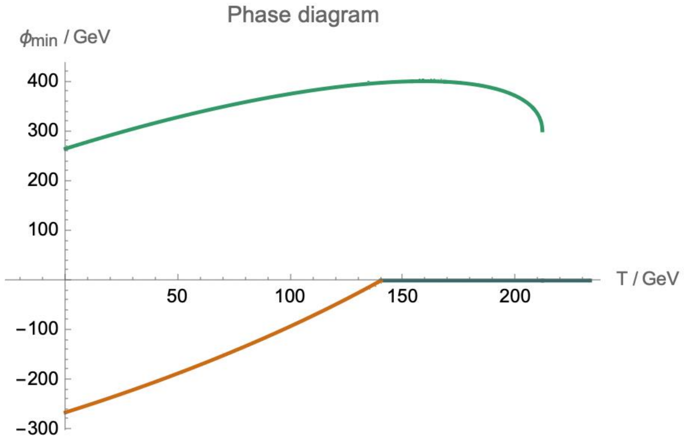
- Bounce action: either analytics or `FindBounce` [*Guada, Nemevšek, Pintar*, 2002.00881] + fit

- Calculation of nucleation and percolation + percolation condition

- Calculation of PT parameters

- GW spectrum calculation via templates from [*LISA Cosmology Working Group*, 2403.03723]

In[7]:=
```
trs = TBounce[V, vw, "TracingMethod" → NSolve,
    "PlotAction" → True, "PlotGWSpectrum" → True] // Quiet // EchoTiming
```



Looping over pairs of phases

Found transition at critical temperature

» T_c → 197.99

Computing nucleation temperature via $\Gamma/H^4 \approx 1$ criterion and bisection method...

» $T_n^{estimate}$ → 170.887 S_3/T = 150.903 Γ/H^4 = 0.996576

Fitting action...

» Action function → ActionFunction[Type: PWLaurent Domain: {149., 198.}]

Computing nucleation temperature via $\int dT/T\ \Gamma/H^4 \approx 1$ criterion and action fit method...

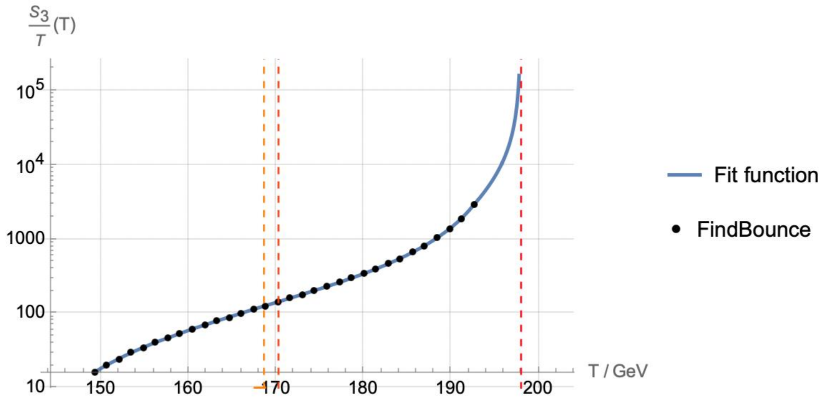
» T_n → 170.259 S_3/T = 143.223 Γ/H^4 = 2025. $\int_{T_n}^{T_c} \frac{dT}{T} \frac{\Gamma}{H^4}$ = 0.999938

Computing phase transition parameters...

Solving $I_{\mathcal{F}}(T_p)$ = 0.34 for Tp

Searching for Tp with FindRoot...

» T_p → 168.6

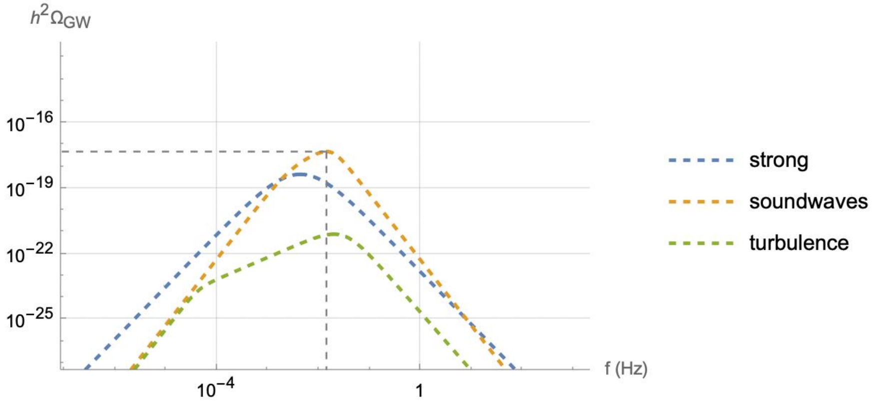


» α → 0.0104057

» β/H → 1702.73

» Percolation condition: satisfied $(-1.34517 \times 10^{-11})$

Computing GW spectrum...



» Transition → Transition[α: 0.0104 β/H: 1.70×10^3]

32.4182

Out[7]= {Transition[α: 0.0104 β/H: 1.70×10^3]}

SUMMARY AND OUTLOOK

- Detection of gravitational waves opens up a new window to study the very early universe
- Cosmological phase transitions are a source of GWs and a clear sign of BSM physics

- The thin wall approximation works in a wider range of parameter space than previously thought
- Analytical results can be used for phenomenologically relevant scenarios

- Mathematica package for fully automated analysis of a cosmological phase transition is on the way
- Improvements: more robust phase tracing, bubble wall velocity, multi-field scenarios

SUMMARY AND OUTLOOK

- Detection of gravitational waves opens up a new window to study the very early universe
- Cosmological phase transitions are a source of GWs and a clear sign of BSM physics

- The thin wall approximation works in a wider range of parameter space than previously thought
- Analytical results can be used for phenomenologically relevant scenarios

- Mathematica package for fully automated analysis of a cosmological phase transition is on the way
- Improvements: more robust phase tracing, bubble wall velocity, multi-field scenarios

THANK YOU!

EUCLIDEAN ACTION EXPANSION

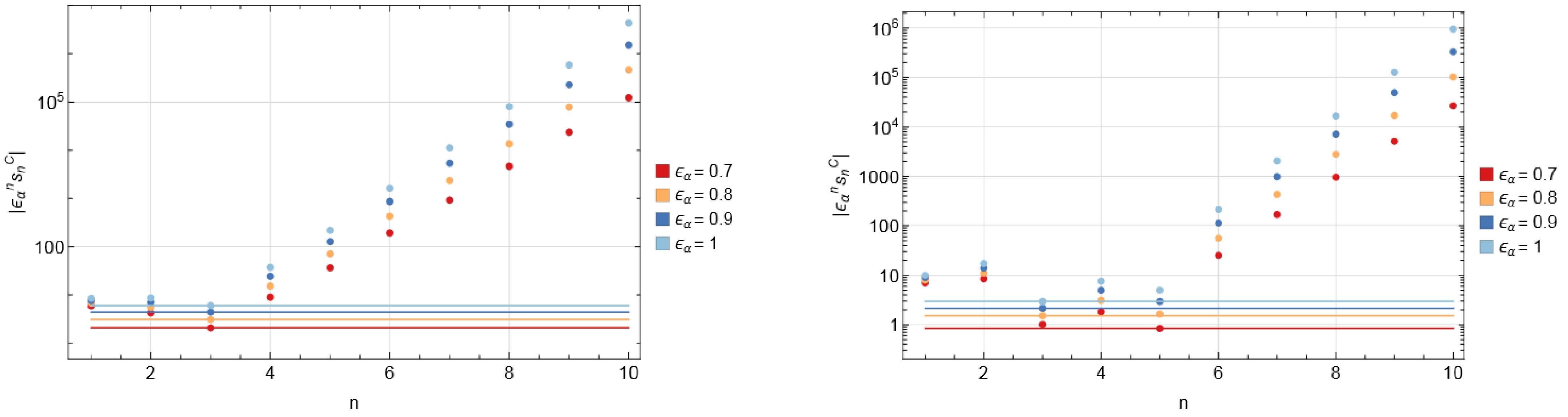


FIG. 2. Absolute values of each term of $S_C^{(10)}(\varepsilon_\alpha)$ in (22) for $\varepsilon_\alpha = 0.7, 0.8, 0.9, 1$. The left panel is for $D = 3$, the right for $D = 4$. The horizontal lines indicate the minimum values.

ANALYTICS FOR NUCLEATION

- Potential $V(\phi,T) = \frac{c_2(T^2-m^2)}{2}\phi^2 - c_3 T\phi^3 + \frac{c_4}{8}\phi^4$
- From expansion around T_C: $T_n = N/D$

$$N = \frac{48c_3^6T_C^3\left(945c_4T_C + \pi\left(12\pi^2-119\right)\sqrt{c_2}\sqrt{\frac{c_3^2T_C^2}{c_2c_4}}\right)}{c_4} - 72c_2c_3^4T_C^3\left(315c_4T_C + 4\pi\left(\pi^2-9\right)\sqrt{c_2}\sqrt{\frac{c_3^2T_C^2}{c_2c_4}}\right)$$
$$+ c_2^2c_4c_3^2T_C^3\left(2835c_4T_C + 4\pi\left(9\pi^2-74\right)\sqrt{c_2}\sqrt{\frac{c_3^2T_C^2}{c_2c_4}}\right)$$
$$- \frac{8\sqrt{\pi}c_3^4T_C^{7/2}}{c_4^{3/2}}\left[96\pi\left(6\pi^2-17\right)c_3^6T_C + 16c_3^4\left(2835\sqrt{c_2}c_4^2\sqrt{\frac{c_3^2T_C^2}{c_2c_4}} + 2\pi\left(25-9\pi^2\right)c_2c_4T_C\right)\right.$$
$$\left.+4c_2^{3/2}c_4^2c_3^2\left(\pi\left(9\pi^2-25\right)\sqrt{c_2}T_C - 5670c_4\sqrt{\frac{c_3^2T_C^2}{c_2c_4}}\right) + 2835c_2^{5/2}c_4^4\sqrt{\frac{c_3^2T_C^2}{c_2c_4}}\right]^{1/2}$$

- From expansion around inflection:

$$T_n = m + \frac{9\sqrt[3]{2}\,105^{2/3}\sqrt[3]{c_2^6c_3^4m^4}}{\left(323\pi - 18\pi^3\right)^{2/3}c_2^3\sqrt[3]{m}}$$

$$D = \frac{c_3^2T_C^2}{c_4}\left(16c_3^4\left(2835c_4T_C + \pi\left(36\pi^2-391\right)\sqrt{c_2}\sqrt{\frac{c_3^2T_C^2}{c_2c_4}}\right)\right.$$
$$- 8c_2c_4c_3^2\left(2835c_4T_C + 2\pi\left(18\pi^2-169\right)\sqrt{c_2}\sqrt{\frac{c_3^2T_C^2}{c_2c_4}}\right)$$
$$\left.+c_2^2c_4^2\left(2835c_4T_C + 4\pi\left(9\pi^2-74\right)\sqrt{c_2}\sqrt{\frac{c_3^2T_C^2}{c_2c_4}}\right)\right)$$

PT STRENGTH AND DURATION

- Different possible definitions for the strength

Given $\epsilon(\phi, T) = 3aT^4 + V(\phi, T) - T\frac{\partial V}{\partial T}, \quad p = aT^4 - V(\phi, T), \quad \theta = \frac{\epsilon - 3p}{4}, \quad w = \epsilon + p$

$$\alpha_\theta = \left.\frac{\theta(\phi_s, T) - \theta(\phi_b, T)}{3aT^4}\right|_{T_N} \qquad \alpha_N = \left.\frac{w(\phi_s, T) - w(\phi_b, T)}{3aT^4}\right|_{T_N}$$

Latent heat density

$$\left[V(\phi_{\mathrm{FV}}, T) - V(\phi_{\mathrm{TV}}, T) - \frac{T}{4}\left(\frac{\partial V}{\partial T}(\phi_{\mathrm{FV}}, T) - \frac{\partial V}{\partial T}(\phi_{\mathrm{TV}}, T)\right)\right]$$

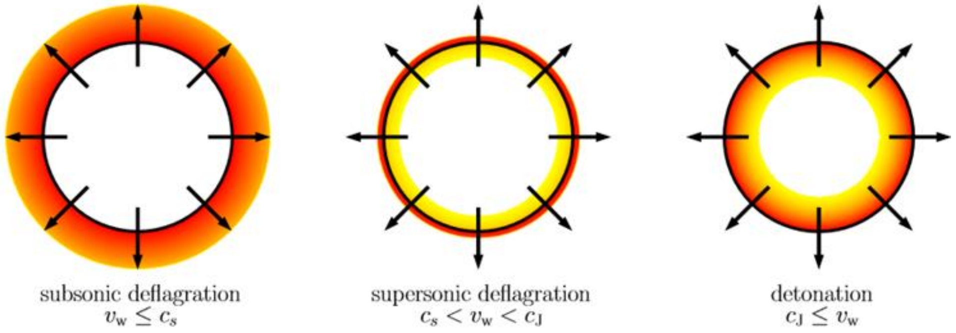
- Inverse duration $\beta = \frac{\mathrm{d}}{\mathrm{d}T}\left[\log \Gamma(T)\right]_{T=T_p}$

WALL VELOCITY

- Outward pressure vs Friction (plasma particles reflect and gain mass)
- Wall velocity from $|P_{out}| = |P_{in}|$

- Coupled system of equations for the scalar and the particles in the plasma

$$\Box\phi + V_T'(\phi) + \sum \frac{dm^2}{d\phi} \int \frac{d^3p}{(2\pi)^3 2E}\,\delta f(p,x) = 0 \qquad \partial_t f_i + \dot{\vec{x}}\,\partial_{\vec{x}} f_i + \dot{\vec{p}}\,\partial_{\vec{p}} f_i = -C_i[f_i]$$



GW POWER SPECTRUM

- Energy momentum tensor of GWs: $T^{\mathrm{gw}}_{\mu\nu} = \frac{1}{32\pi G}\langle \partial_\mu h_{ij} \partial_\nu h_{ij} \rangle$

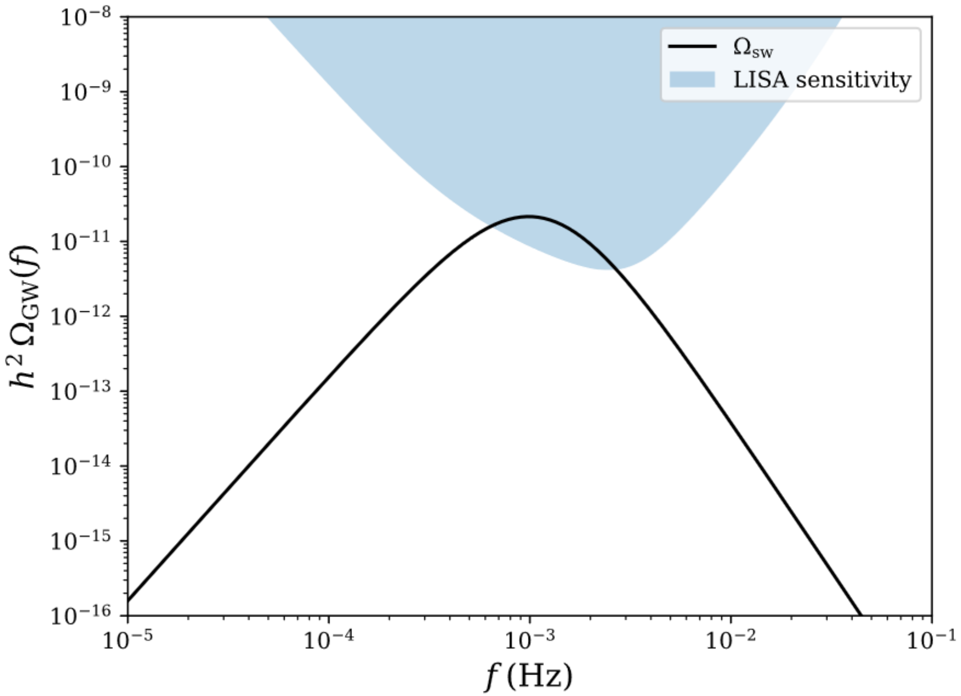
- Energy density: $\rho_{\mathrm{gw}} = \frac{1}{32\pi G}\langle \dot{h}^2_{ij} \rangle$

$$\Omega_{\mathrm{gw}} = \frac{\rho_{\mathrm{gw}}}{\rho_{\mathrm{tot}}}$$

$$H^2 = \frac{8\pi G \rho_{\mathrm{tot}}}{3}$$

+ frequency space

Power spectrum



[LISA Cosmology Working Group, 1910.13125]

COUPLED FIELD – FLUID MODEL

- Potential & eq. of state: $V(\phi,T) = \frac{1}{2}\left(T^2 - T_0^2\right)\gamma\phi^2 - \frac{1}{3}AT\phi^3 + \frac{1}{4}\lambda\phi^4$, $\epsilon(T,\phi) = 3aT^4 + V(\phi,T) - T\frac{\partial V}{\partial T}$
- Energy-momentum tensor: $p(T,\phi) = aT^4 - V(\phi,T)$

$$T^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - \tfrac{1}{2}g^{\mu\nu}(\partial\phi)^2 + [\epsilon + p]\,U^\mu U^\nu + g^{\mu\nu}p$$

$$\begin{cases} [\partial_\mu T^{\mu\nu}]_{\text{field}} = (\partial_\mu\partial^\mu\phi)\partial^\nu\phi - \dfrac{\partial V}{\partial\phi}\partial^\nu\phi = \delta^\nu \\ [\partial_\mu T^{\mu\nu}]_{\text{fluid}} = \partial_\mu[(\epsilon+p)U^\mu U^\nu] - \partial^\nu p + \dfrac{\partial V}{\partial\phi}\partial^\nu\phi = -\delta^\nu \end{cases}$$

← $\delta^\nu = \eta U^\mu\partial_\mu\phi\partial^\nu\phi$

- Numerical simulations: $U^i = WV^i$, $E = W\epsilon$, $Z_i = W(\epsilon+p)U_i$

$$\begin{cases} -\ddot{\phi} + \nabla^2\phi - \dfrac{\partial V}{\partial\phi} = \eta W(\dot{\phi} + V^i\partial_i\phi) \\ \dot{E} + \partial_i(EV^i) + p[\dot{W} + \partial_i(WV^i)] - \dfrac{\partial V}{\partial\phi}W(\dot{\phi} + V^i\partial_i\phi) \\ \qquad = \eta W^2(\dot{\phi} + V^i\partial_i\phi)^2 \\ \dot{Z}_i + \partial_j(Z_iV^j) + \partial_i p + \dfrac{\partial V}{\partial\phi}\partial_i\phi = -\eta W(\dot{\phi} + V^j\partial_j\phi)\partial_i\phi \end{cases}$$

➡ GWs:

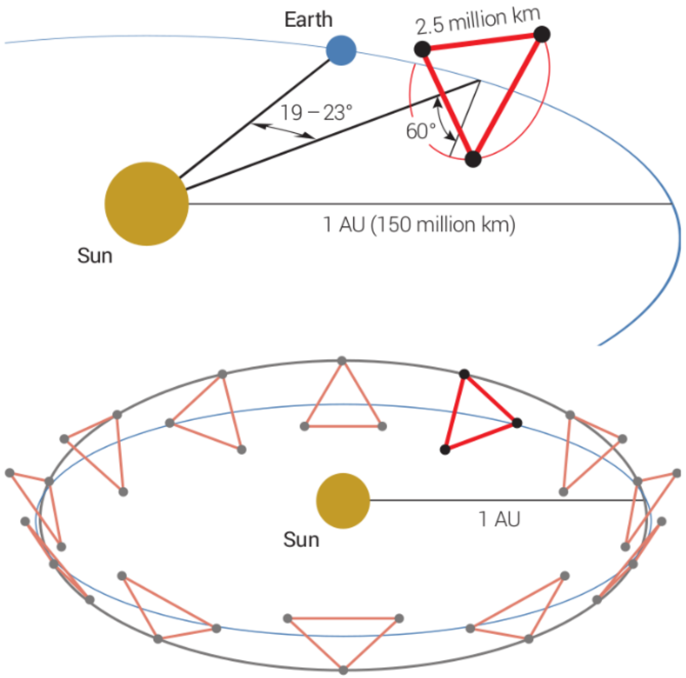
$$\tau^{\phi}_{ij} = \partial_i\phi\partial_j\phi, \quad \tau^{\text{f}}_{ij} = W^2(\epsilon+p)V_iV_j$$

$$h_{ij}(\mathbf{k},t) = (16\pi G)\lambda_{ij,kl}(\mathbf{k})\int_0^t dt'\,\frac{\sin[k(t-t')]}{k}\tau_{kl}(\mathbf{k},t')$$

LISA MISSION

- Laser Interferometer Space Antenna
- ESA – expected to launch in 2030s
- 3 satellites orbiting Earth, arms of 2.5M km
- Lasers and photodetectors which detect small changes in separation through time delays of signals

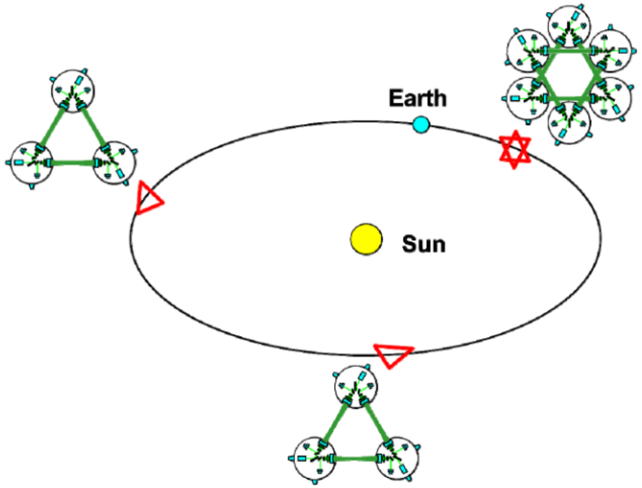
- Most sensitive in the range $10^{-3} - 10^{-2}\ Hz$

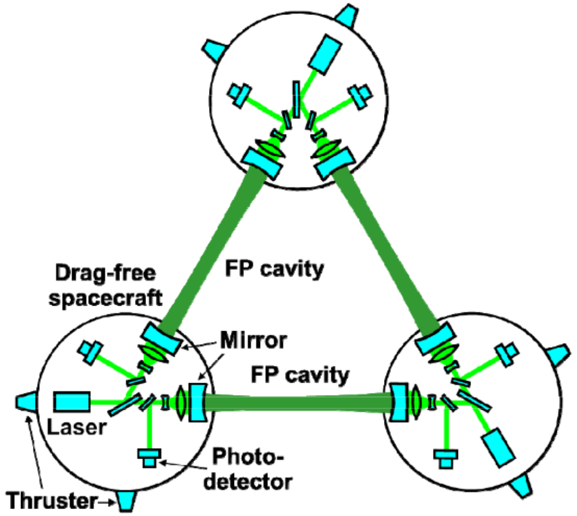


[Amaro-Seoane et al., 1702.00786]

DECIGO MISSION

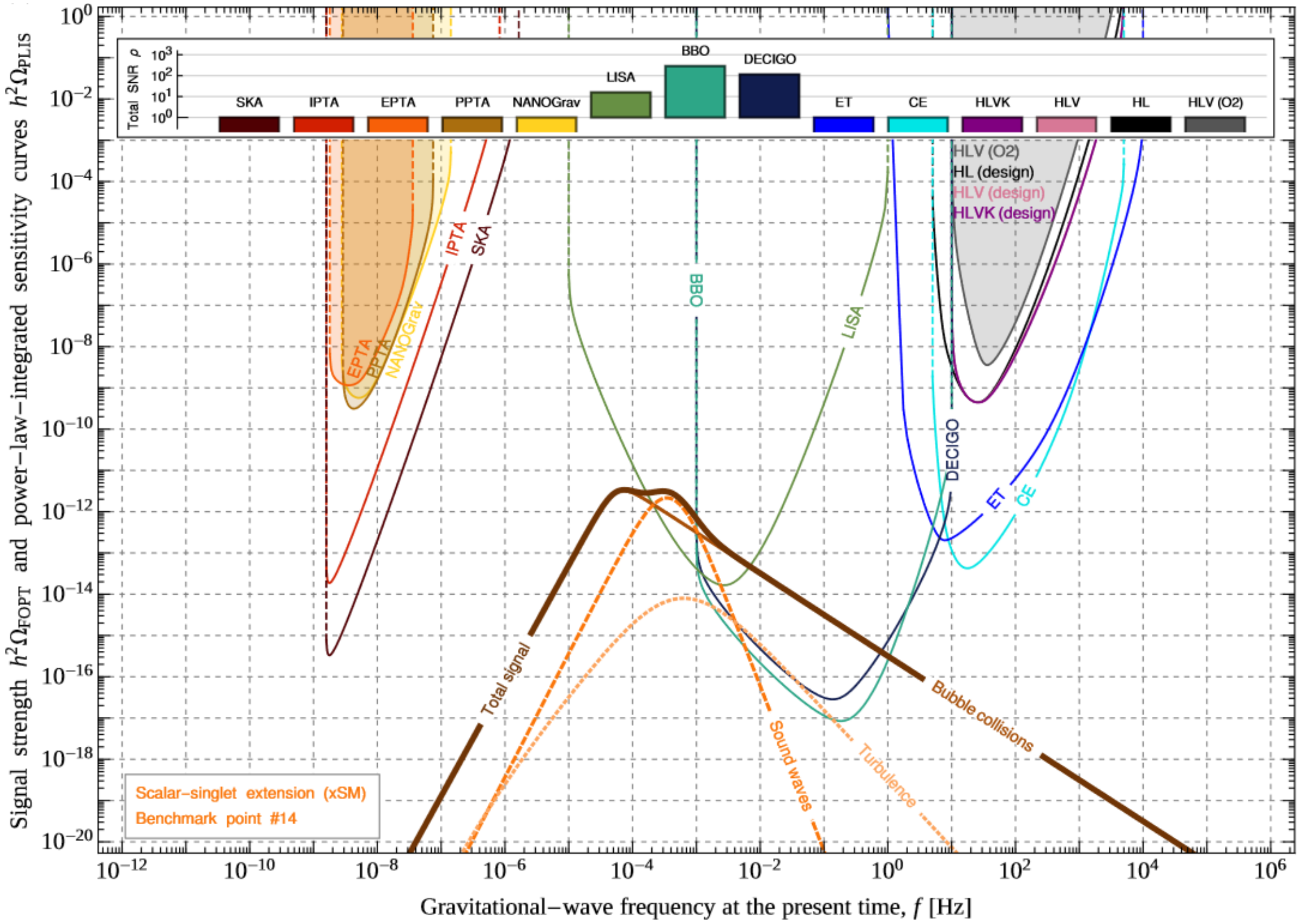
- Deci-hertz Interferometer Gravitational Wave Observatory
- Japanese project – expected to launch in 2030s
- Four clusters of observatories placed in the heliocentric orbit.
- Each cluster: three spacecraft, which form three Fabry-Perot Michelson interferometers with an arm length of 1,000 km
- Most sensitive in the range $0.1 - 10\ Hz$





[Kawamura, Ando, Seto, Sato, Musha et al., 2006.13545]

AN EXAMPLE OF SIGNAL



[Schmitz, 2002.04615]