Schwinger pair production in de Sitter
Regularizing negative conductivities

Long life to the Standard Model - Domačija Belica

António Torres Manso

Ongoing work with
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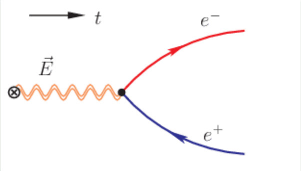
Soon on 2410.XXXXX

Spontaneous **particle creation** by **time-varying backgrounds**

- Schwinger Effect *W. Heisenberg, H. Euler (1936); J. Schwinger (1951)*
  - Production of charged particles from vacuum under **strong electric fields**
  - Time dependent vector potential



- Curved backgrounds *L. Parker (1966); S. W. Hawking (1975)*
  - Particle production from vacuum under **time dependent gravitational field**

Spontaneous **particle creation** by **time-varying backgrounds**

Combining the two examples: **Schwinger effect in de Sitter**

Inflationary Magnetogenesis

- Generate the **observed magnetic fields** present in voids our universe

Generation of Dark Sectors

- Candidates for non-thermal **dark matter**

During inflation (Φ), in practice this could be realized with

$$S = -\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + V(\Phi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{4f}\Phi F_{\mu\nu}\tilde{F}^{\mu\nu} + \mathcal{L}_{ch}(\phi, A_\nu)\right],$$

M. Bastero-Gil,P. Ferraz, L. Ubaldi, R. Vega-Morales 2023

Scalar QED in de Sitter

$$S = \int d^4x\sqrt{-g}\left\{-g^{\mu\nu}\left(\partial_\mu - ieA_\mu\right)\phi^*\left(\partial_\nu + ieA_\nu\right)\phi - (m_\phi^2 + \xi R)\phi^*\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right\},$$

- Set a constant a electric field

$$A_\mu = \frac{E}{H^2\tau}\delta_\mu^z, \qquad F_{\mu\nu}F^{\mu\nu} = -2E^2$$

- After canonically normalizing the scalar field ϕ e.o.m. for $q \equiv a\phi$

$$q_k'' + \omega_{\boldsymbol{k}}^2 q_{\boldsymbol{k}} = 0,$$

- Analytical solution with Whittaker functions

$$q_k = \frac{\mathrm{e}^{-\pi\lambda r/2}}{\sqrt{2k}} W_{i\lambda r,\mu}(2ik\tau)$$

Scalar QED in de Sitter

- A_ν e.o.m.

$$\nabla^\nu F_{\mu\nu} = J_\mu^\phi \quad \text{with} \quad J_\mu^\phi = \frac{ie}{2}\left\{\phi^\dagger\left(\partial_\mu + ieA_\mu\right)\phi - \phi\left(\partial_\mu - ieA_\mu\right)\phi^\dagger\right\} + \text{ h.c. }.$$

- With an electric field in the z-direction,

$$\left\langle 0 \left| J_z^\phi \right| 0 \right\rangle = \frac{2e}{a^2}\int \frac{d^3k}{(2\pi)^3}\left(k_z + eA_z\right)\left|q_k\right|^2 .$$

- **Divergent expectation value**. With a cut off momentum ζ T. Kobayashi, N. Afshordi 2014

$$\left\langle J_z^\phi \right\rangle = aH\frac{e^2E}{4\pi^2}\lim_{\zeta\to\infty}\left[\frac{2}{3}\left(\frac{\zeta}{aH}\right)^2 + \frac{1}{3}\ln\frac{2\zeta}{aH} - \frac{25}{36} + \frac{\mu^2}{3} + \frac{\lambda^2}{15} + F_\phi(\lambda,\mu,r)\right].$$

$$\lambda = \frac{eE}{H^2}, \qquad r = \frac{k_z}{k}, \qquad \mu^2 = \frac{9}{4} - \frac{m^2}{H^2} - \lambda^2 \quad \text{and} \quad m_\xi^2 = m_\phi^2 + 12\,\xi H^2.$$

- For fermions we have a similar expression, **same UV divergences**

State of the art on renormalization

Scalars

- 3 different regularization/renormalization procedures
  - Adiabatic Subtraction (AS) — *T. Kobayashi, N. Afshordi 2014*
  - Point Splitting (PS) — *T. Hayashinaka, J. Yokoyama 2016*
  - Pauli Villars (PV) — *M. Banyeres, G. Domenèch, J. Garriga 2018*
- All **agree** for $m > H$
- When $m < H$:
  - AS and PS the result leads to **negative conductivities** coming from $\log(m/H)$ term
  - In PV authors argue $\log(m/H)$ should be **reabsorbed in the running of electric charge**

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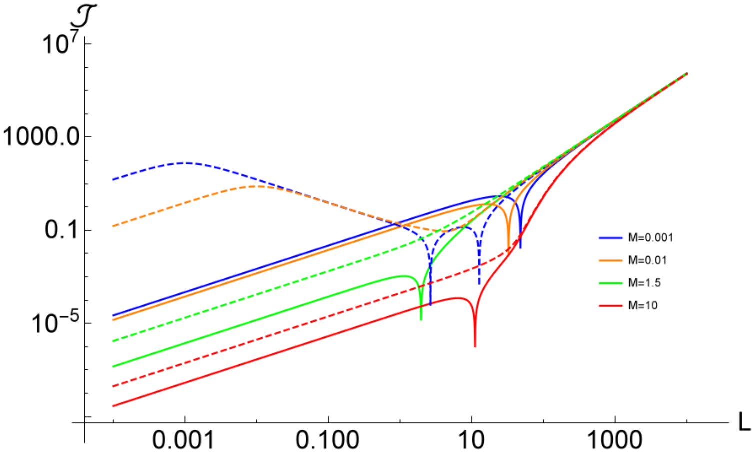
Fermions

- Only AS — *T. Hayashinaka, T. Fujita, J. Yokoyama 2016*
- Again, when $m < H$ **negative conductivities** coming from $\log(m/H)$ term
- But also more generally when $\lambda < 1$



T. Hayashinaka, T. Fujita, J. Yokoyama 2016

Renormalized currents with PV

- An arbitrary number of additional **auxiliary fields are introduced to cancel divergences**

The **regularized** current $\langle J_z\rangle_{\mathrm{reg}} = \lim_{\Lambda\to\infty}\sum_{i=0}^{3}(-1)^i \langle J_z\rangle_i = aH\frac{e^2E}{4\pi^2}\lim_{\Lambda\to\infty}\left[\frac{1}{6}\ln\frac{\Lambda^2}{H^2} - \frac{2\lambda^2}{15} + F_\phi(\lambda,\mu,r)\right].$

- $\ln \Lambda/H$ **divergence** to be reabsorbed with **renormalization** of the charge

$$\langle J_\mu\rangle_{reg} = (\delta_3 + 1)\,\nabla^\nu F_{\mu\nu}$$

$$\nabla^\nu F_{\mu\nu} = \langle J_\mu\rangle_{ren} = \langle J_\mu\rangle_{reg} - (-2aHE\delta_\nu^{\ z})\delta_3$$

Renormalized currents with PV

- In *Banyeres et al* $\delta_3 = -\frac{e^2}{48\pi^2} \ln \frac{\Lambda^2}{m^2}$ as $\mathbf{p^2 = 0}$ in the **vacuum polarization** diagram

$$\left\langle J_z^{\phi} \right\rangle_{\mathrm{ren}} = aH \frac{e^2 E}{4\pi^2} \left[\frac{1}{6} \ln \frac{m^2}{H^2} - \frac{2\lambda^2}{15} + F_{\phi}(\lambda, \mu, r) \right]$$

- It is argued that when problematic $\ln m/H$ is reabsorbed in **running** of **e** to scale H

Renormalized currents with PV

- In *Banyeres et al* $\delta_3 = -\frac{e^2}{48\pi^2} \ln \frac{\Lambda^2}{m^2}$ as $\mathbf{p^2} = \mathbf{0}$ in the **vacuum polarization** diagram

$$\left\langle J_z^{\phi} \right\rangle_{\rm ren} = aH \frac{e^2 E}{4\pi^2} \left[\frac{1}{6} \ln \frac{m^2}{H^2} - \frac{2\lambda^2}{15} + F_{\phi}(\lambda, \mu, r) \right]$$

- It is argued that when problematic $\ln m/H$ is reabsorbed in **running** of **e** to scale H

However, studying the e.o.m. we see that:

- To sustain a **constant background electric field** in de Sitter

  Need an *effective* tachyonic mass for the gauge field $\mathbf{m_A^2} = -\mathbf{2\,H^2}$

Renormalized currents with PV

Taking $p^2 = -2\,H^2$

$$\delta_3 = \left(\frac{e}{12\pi}\right)^2 \left(3\ln\left(\frac{m^2}{\Lambda^2}\right) - 12\left(\frac{m}{H}\right)^2 + 6\left(2\left(\frac{m}{H}\right)^2 + 1\right)^{3/2} \coth^{-1}\left(\sqrt{2\left(\frac{m}{H}\right)^2 + 1}\right) - 8\right)$$

- We find the **renormalized** current to be

$$\left\langle J_z^{\phi} \right\rangle_{\text{ren}}^{PV} = aH\frac{e^2E}{4\pi^2}\left[\frac{1}{3}\ln\frac{m}{H} - \frac{4}{9} - \frac{2}{3}\left(\frac{m}{H}\right)^2 - \frac{2\lambda^2}{15} + \frac{\left(1+2\left(\frac{m}{H}\right)^2\right)^{3/2}}{3}\coth^{-1}\left(\sqrt{2\left(\frac{m}{H}\right)^2+1}\right) + F_{\phi}\right]$$

- As we will see, $\log m/H$ will drop when problematic and J^{ϕ} is always positive

Renormalizing currents with AS

- The subtraction is done **mode by mode** removing the expectation evaluated in the adiabatic approx

WKB expansion $$q_{\boldsymbol{k}}(\tau) = \frac{1}{\sqrt{2W_{\boldsymbol{k}}(\tau)}} \exp\left\{-i\int^{\tau} d\tilde{\tau}\, W_{\boldsymbol{k}}(\tilde{\tau})\right\}$$

- **Running / Physical Scale AS** with an **arbitrary adiabatic expansion scale** $\bar{m}$

A. Ferreiro, S. Monin, J. Navarro Salas, F. Torrenti 2018, 2022, 2023

- Value of $\bar{m}$ has to be set to obtain the **appropriate adiabatic vacuum** evolution

$$\left\langle J_z^{\phi}\right\rangle_{\rm reg}^{AS} = \left\langle J_z^{\phi}\right\rangle - \left\langle J_z^{\phi}\right\rangle^{(2)} = aH\frac{e^2 E}{4\pi^2}\left[\frac{1}{3}\ln\frac{\bar{m}}{H} - \frac{2\lambda^2}{15} + F_{\phi}(\lambda,\mu,r)\right]$$

- If $m > H \quad \bar{m} = m;$
- If $m < H \quad \bar{m} = H$

Renormalizing currents with DR

- Applying DR, in the Whitaker function we have a **scaleless argument** and integral gives zero

- **Expanding** the argument for a **large energy-like quantity**,

$$e_k = \sqrt{k^2 + a^2 x^2}$$

  Isolates the divergent pieces and **introduce an artificial IR regulator**.

  A. V. Lysenko, O. O. Sobol, E. V. Gorbar, A. I. Momot, and S. I. Vilchinskii 2020, 2023

- We just **regularize the asymptotic piece** $\left\langle J_z^{\phi} \right\rangle^{e_k}$ with DR

Renormalizing currents with DR

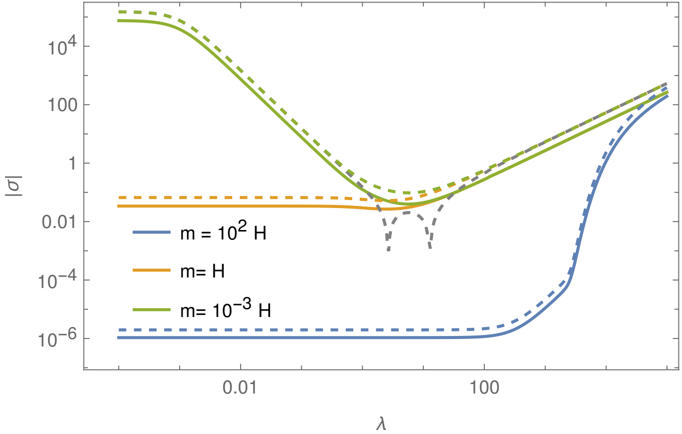
- We obtain a different regularization (minimal ?)

$$\left\langle J_z^\phi \right\rangle_{\mathrm{reg}}^{DR} = \left\langle J_z^\phi \right\rangle - \left\langle J_z^\phi \right\rangle^{e_k} + \left\langle J_z^\phi \right\rangle_{reg}^{e_k}$$

$$\left\langle J_z^\phi \right\rangle_{\mathrm{ren}}^{DR} = \left\langle J_z^\phi \right\rangle_{\mathrm{reg}}^{DR} - (-2aHE\delta_\nu^{\ z})\delta_3^{DR}$$
$$= aH\frac{e^2E}{4\pi^2}\left[\frac{1}{3}\ln\frac{2m}{H} - \frac{7}{18} - \left(\frac{m}{H}\right)^2 - \frac{4\lambda^2}{15} + \frac{\left(1+2\left(\frac{m}{H}\right)^2\right)^{3/2}}{3}\coth^{-1}\left(\sqrt{2\left(\frac{m}{H}\right)^2+1}\right) + F_\phi\right]$$

Renormalized Conductivities PV vs AS

- Successfully **removed the infrared divergences** ($\ln m/H$) that lead to negative conductivities



$$\sigma_z \equiv \frac{1}{aH} \frac{\langle J_z \rangle}{e^2 H}$$

Solid: PV
Dashed: AS
Grey: Old results

Renormalized Conductivities PV vs DR

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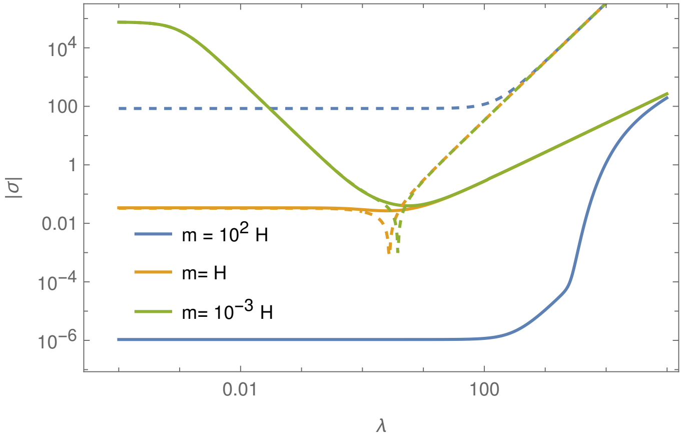


$$\sigma_z \equiv \frac{1}{aH}\frac{\langle J_z\rangle}{e^2 H}$$

DR **disagrees** when:

- $\lambda \gg 1$
- $m \gg H$

Non physical results

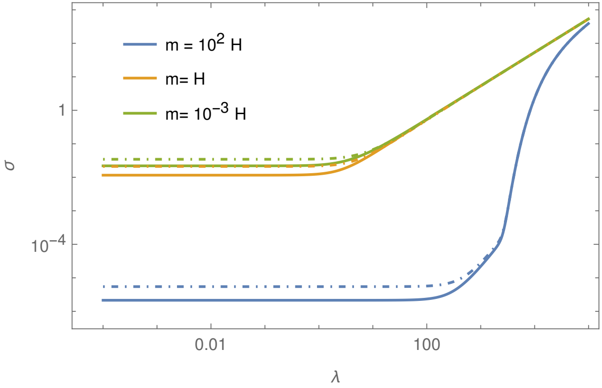
Solid: PV
Dashed: DR

Renormalized Conductivities (PV) Fermions vs Scalars

Successfully

- **Removed the infrared divergences** ($\ln m/H$) that lead to negative conductivities

- **Corrected fermion conductivities**



$$\sigma_z \equiv \frac{1}{aH}\frac{\langle J_z\rangle}{e^2 H}$$

Solid: Scalars with $\xi = 1/6$

DotDashed: Fermions

Hidden Discussion: Approximation with vacuum polarization

- In QFT **dispersion relation** obtained from the Klein-Gordon eq, for both scalars and fermions

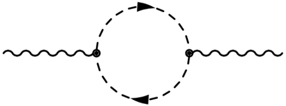
$$(\Box + m^2)\phi_s = 0 \implies \omega_p^2 = |p|^2 + m^2,$$

- In de Sitter ($R = 12H^2$), and for a scalar field

$$(\Box + m^2 + \xi R)\phi = (\Box + m_\xi^2)\phi = 0,$$

$\Box$ will include expansion effects on ϕ

- When interactions are more efficient than expansion effects (or at high internal momenta p)



$\Box$ is taken to flat space limit and

$$\langle 0|T\{\phi(0)\bar{\phi}(x)\}|0\rangle = \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m_\xi^2 + i\varepsilon} e^{ipx}.$$

Hidden Discussion: What about fermions?

$$\left(\Box + m_f^2 + \frac{1}{4}R\right)\psi = \left(\Box + \bar{m}_f^2\right)\psi = 0$$

- In this regime where the kinematics inside the loop may be treated as in Minkowski $\Box \simeq \partial^2$,

$$\omega_p^2 = |p|^2 + \bar{m}_f^2$$

  but for a propagator still need Dirac equation

Hidden Discussion: What about fermions?

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- With a modified action in Minkowski background

  $$S' = \int d^4x \{\overline{\psi'}[i\gamma^\mu \partial_\mu - \bar{m}_f]\psi'\}$$

  we have a wave equation that gives the same dispersion relation

  $$\left(\partial^2 + \bar{m}_f^2\right)\psi' = 0\,,$$

- Take this ”effective” action for the fermions in the 1-loop corrections to the photon propagator

  $$\langle 0|T\{\psi(0)\bar{\psi}(x)\}|0\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{i(\not{p} + \bar{m}_f)}{p^2 - \bar{m}_f^2 + i\varepsilon}e^{ipx}.$$

Conclusion & Outlook

- We have revised PV, AS and DR renormalization in the literature
- We were able to address and clarify literature's negative conductivities in $H > m$ case
  - **Unphysical** result comes from **wrong physical** conditions
- With both PV and AS we have always recovered physically sensible results
  - Currents show **small deviations**
  - In PV we seem to have a **better knowledge on the physical system**.
  - With the the physical scale AS criteria to determine the scale $\bar{m}$ seems more unsatisfactory.

Conclusion & Outlook

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Next steps

- Submit paper
- Apply this into generation of Dark Sectors during inflation
- Check Gravitational wave spectrum in Dark matter compatible scenarios

Backup

- An arbitrary number of additional **auxiliary fields are introduced to cancel divergences**
- The mass of these extra fields will then be sent to infinity, making them **non-dynamical**

Introduce **3** fields $$\sum_{i=0}^{3}(-1)^i = 0 \quad \text{and} \quad \sum_{i=0}^{3}(-1)^i m_i^2 = 0,$$

$$m_0 = m\,, \quad m_2^2 = 4\Lambda^2 - m^2 \quad \text{and} \quad m_1^2 = m_3^2 = 2\Lambda^2, \quad \Lambda \to \infty$$

The **regularized** current $$\langle J_z \rangle_{\mathrm{reg}} = \lim_{\Lambda\to\infty} \sum_{i=0}^{3}(-1)^i \langle J_z \rangle_i \,.$$

$$\left\langle J_z^{\phi} \right\rangle_{\mathrm{reg}} = aH\frac{e^2 E}{4\pi^2} \lim_{\Lambda\to\infty}\left[\frac{1}{6}\ln\frac{\Lambda^2}{H^2} - \frac{2\lambda^2}{15} + F_{\phi}(\lambda,\mu,r)\right]$$

- $\ln \Lambda/H$ **divergence** to be reabsorbed with **renormalization** of the charge

$$(\delta_3 + 1)\,\nabla^{\nu} F_{\mu\nu} = \langle J_\mu \rangle_{reg}$$
$$\langle J_\mu \rangle_{ren} = \nabla^{\nu} F_{\mu\nu} = \langle J_\mu \rangle_{reg} - (-2aHE\delta_\nu^{\ z})\delta_3$$

Constant eletric field in de Sitter

$$S = -\int d^4x \sqrt{-g}\,\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\,.$$

From Euler Lagrange equations we would expect

$$g^{\alpha\nu}g^{\beta\sigma}\partial_\alpha F_{\nu\sigma} = 0\,.$$

However, including the details on the physical setting of our problem, a de Sitter metric and $A_\mu = \frac{E}{H^2\tau}\delta^z_\mu$, we find

$$\begin{aligned} g^{\alpha\nu}g^{\beta\sigma}\partial_\alpha F_{\nu\sigma} &= g^{00}g^{ij}\partial_0 F_{0j} + g^{ij}g^{00}\partial_j F_{j0} \\ &= \left(-a^{-2}\right)a^{-2}\partial_\tau\left(-\frac{E}{\tau^2H^2}\delta^z_i\right) \\ &= -2a^{-4}\frac{E}{\tau^3H^2}\delta^z_i \neq 0\,. \end{aligned}$$

We see that an abelian gauge theory with only a kinetic term is not consistent with a constant electric field in a de Sitter background.

Constant eletric field in de Sitter

A possible solution is the inclusion of an effective mass term in the action

$$S = -\int d^4x \sqrt{-g} \left(\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_A^2 A_\mu A^\mu \right) .$$

Then, the Euler Lagrange equations give

$$\begin{aligned} g^{\alpha\nu} g^{\beta\sigma} \partial_\alpha F_{\nu\sigma} - m_A^2 g^{\beta\mu} A_\mu &= 0 \\ -a^{-4} 2 \frac{E}{\tau^3 H^2} \delta_i^z - m_A^2 a^{-2} \frac{E}{\tau H^2} \delta_i^z &= 0 \, , \end{aligned}$$

and the system becomes consistent for $m_A^2 = -2H^2$.

In order to have a consistent electric field in de Sitter, the gauge boson must have something like an effective tachyonic mass, or a source term, breaking the conformal invariance.

Revising AS

- In a **time-dependent background** the **vacuum** of the theory is generally **evolving** making the concept of "vacuum contribution" ambiguous

- The subtraction is done **mode by mode** removing the expectation evaluated in the adiabatic approx

WKB expansion $$q_{\boldsymbol{k}}(\tau) = \frac{1}{\sqrt{2W_{\boldsymbol{k}}(\tau)}} \exp\left\{-i\int^{\tau} d\tilde{\tau}\, W_{\boldsymbol{k}}(\tilde{\tau})\right\}$$

$$\left\langle J_z^{\phi}\right\rangle = -\frac{2e}{(2\pi)^3 a^2}\int d^3k\,(k_z + eA_z)\,\frac{1}{2W_{\boldsymbol{k}}}$$

Inserting the mode function q in the e.o.m.

$$W_{\boldsymbol{k}}^2 = \omega_{\boldsymbol{k}}^2 + \frac{3}{4}\left(\frac{W_{\boldsymbol{k}}'}{W_{\boldsymbol{k}}}\right)^2 - \frac{1}{2}\frac{W_{\boldsymbol{k}}''}{W_{\boldsymbol{k}}}$$

Expanded at the n^{th} order

$$W_{\boldsymbol{k}} = W_{\boldsymbol{k}}^{(0)} + W_{\boldsymbol{k}}^{(1)} + W_{\boldsymbol{k}}^{(2)} + \ldots$$

Running / Physical Scale AS

- Take $\Omega_{\boldsymbol{k}}^{\bar{m}}$ with **arbitrary adiabatic expansion scale** $\bar{m}$ (opposed to automatically set $\bar{m} = m$)
  A. Ferreiro, S. Monin, J. Navarro Salas, F. Torrenti 2018, 2022, 2023

$$\Omega_{\boldsymbol{k}}^{\bar{m}2} = (k_z + eA_z)^2 + k_x^2 + k_y^2 + a^2\bar{m}^2 = \omega_{\boldsymbol{k}}^2 + a^2(\bar{m}^2 - m^2) + \frac{a''}{a}$$

And set $W_{\boldsymbol{k}}^{2\,(0)} = \Omega_{\boldsymbol{k}}^{\bar{m}2}$

Find second order $W_{\boldsymbol{k}}^2$ with e.o.m. $W_{\boldsymbol{k}}^{2\,(2)} = \Omega_{\boldsymbol{k}}^{\bar{m}2} - a^2(\bar{m}^2 - m^2) - \frac{a''}{a} + \frac{3}{4}\left(\frac{\Omega_{\boldsymbol{k}}^{\bar{m}\prime}}{\Omega_{\boldsymbol{k}}^{\bar{m}}}\right)^2 - \frac{1}{2}\frac{\Omega_{\boldsymbol{k}}^{\bar{m}\prime\prime}}{\Omega_{\boldsymbol{k}}^{\bar{m}}}$

$$\left\langle J_z^{\phi} \right\rangle^{(2)} = \lim_{\zeta\to\infty} \frac{eaH^3}{(2\pi)^2}\left[\frac{2\lambda}{3}\left(\frac{\zeta}{aH}\right)^2 - \frac{2\lambda^3}{15} - \frac{\lambda}{3}\left(\frac{m}{H}\right)^2 + \frac{\lambda}{3}\ln\left(\frac{2\zeta}{a\bar{m}}\right) + \frac{\lambda}{18}\right]$$

- And the **renormalized** current is given by

$$\left\langle J_z^{\phi} \right\rangle_{\mathrm{ren}}^{AS} = \left\langle J_z^{\phi} \right\rangle - \left\langle J_z^{\phi} \right\rangle^{(2)} = aH\frac{e^2E}{4\pi^2}\left[\frac{1}{3}\ln\frac{\bar{m}}{H} - \frac{2\lambda^2}{15} + F_{\phi}(\lambda, \mu, r)\right]$$

(Similar to Banyeres et al)

- Value of $\bar{m}$ has to be set to obtain the **appropriate adiabatic vacuum** evolution