A path towards $D \to \rho \ell \nu$ in lattice QCD

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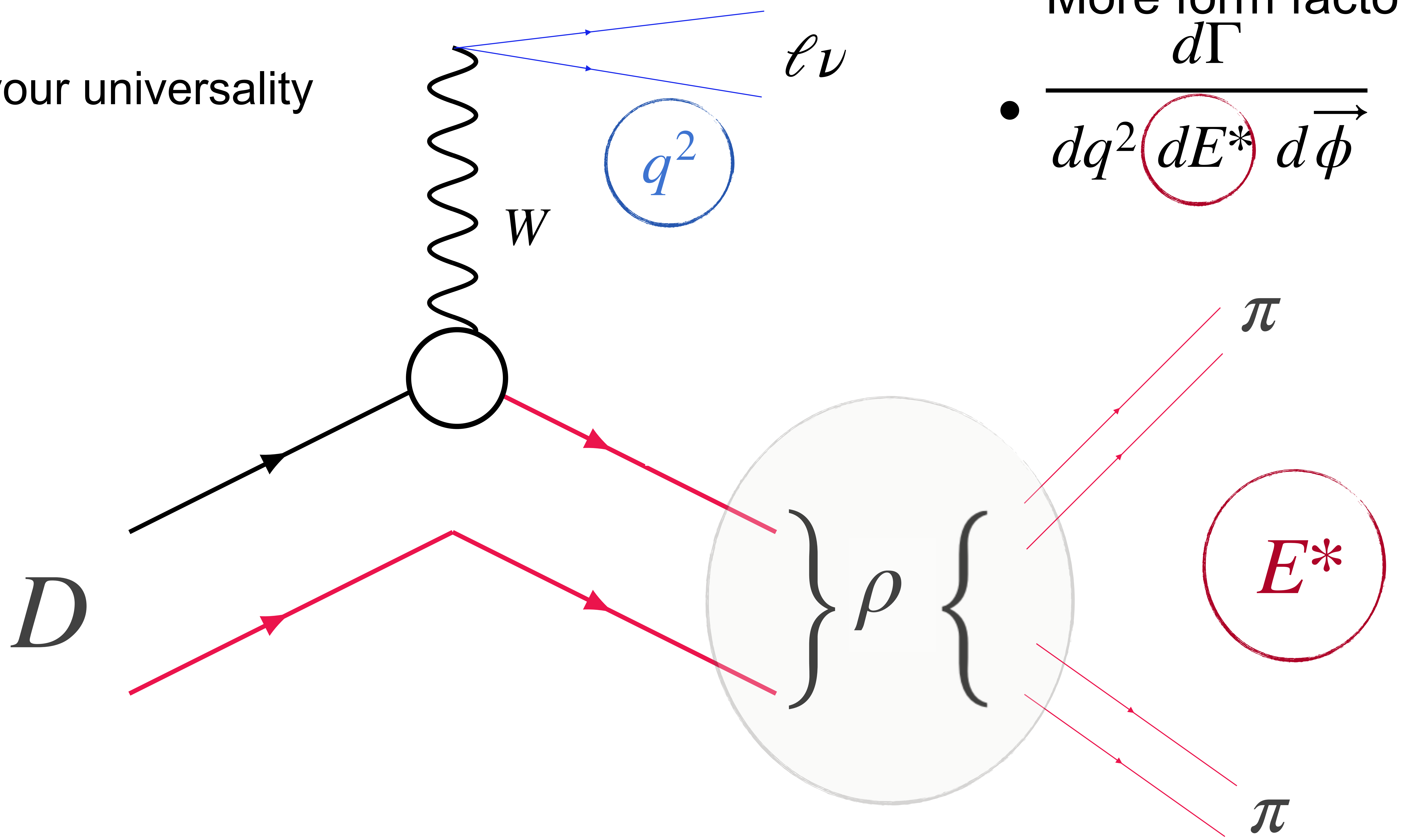
$$D \to \rho \ell \nu \to \pi\pi\ell\nu$$

- V_{cd}
- Lepton flavour universality
- NP search

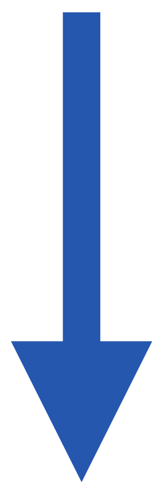
Advantage:

- More form factors (4)
- $\dfrac{d\Gamma}{dq^2\, dE^*\, d\vec{\phi}}$

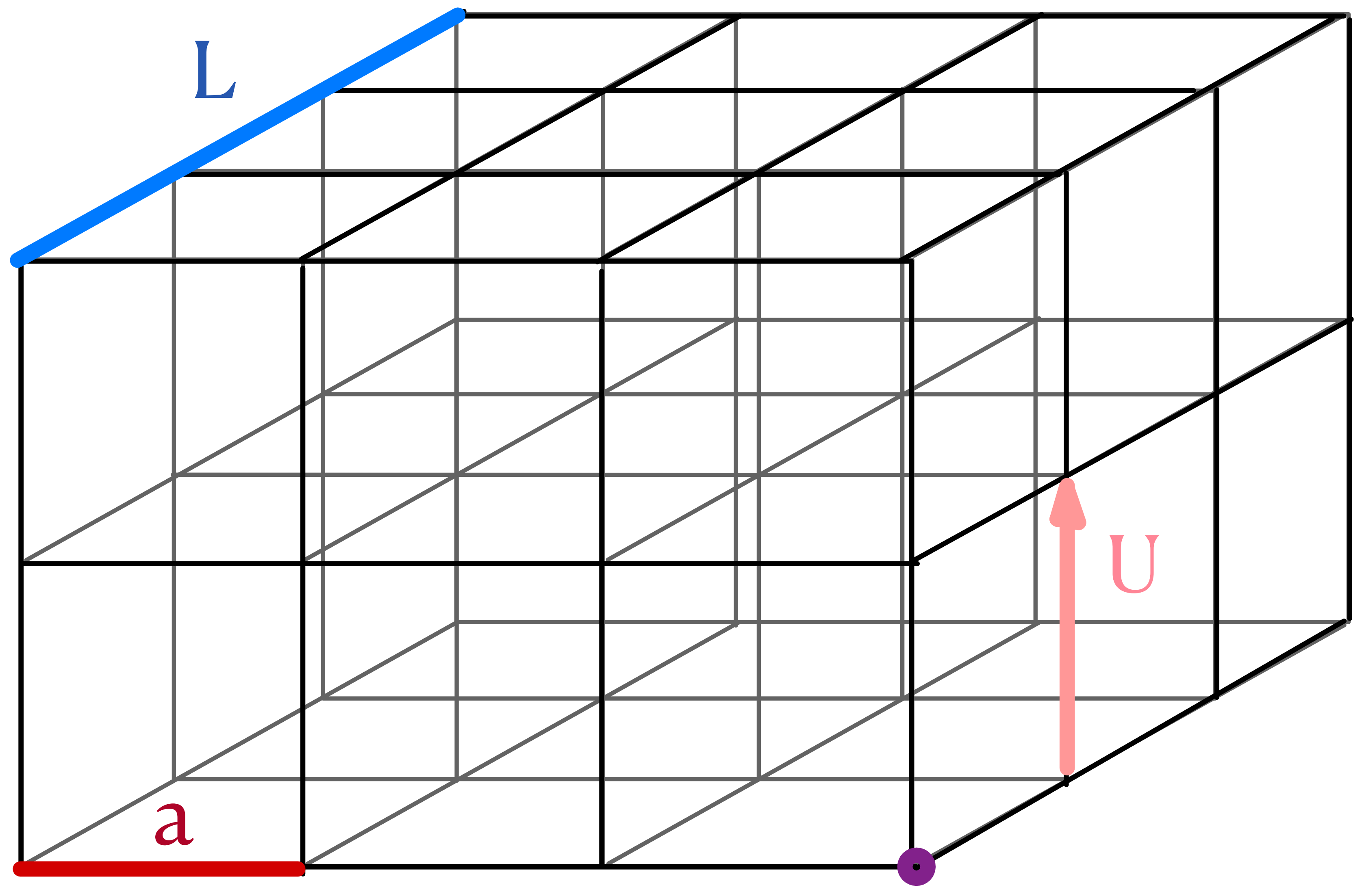


Introduction to Lattice QCD

- Wick rotation: $t \to i\tau$
- Path integral: $\langle O \rangle \propto \int \mathscr{D}\bar{\psi}\mathscr{D}\psi\mathscr{D}U \; O \; e^{-S_f} e^{-S_g}$
- $\langle O \rangle \propto \int \mathscr{D}U \; O \det[\not{D}] e^{-S_g}$



- $\langle O \rangle \propto \sum_i O(\{U\}_i)$



ρ on 4 Ensembles

ρ: $\mathrm{I}=1,\ \mathrm{J}=1$

- $\rho \to \pi\pi, \quad Br \approx 100\,\%$
- $\rho \to \pi\gamma, \quad Br = (4.5 \pm 0.5) \times 10^{-4}$
- $\rho^0 \to \pi\pi\gamma,\ Br = (9.9 \pm 1.6) \times 10^{-3}$

C13

- $a = 0.114\ \mathrm{fm}$
- $N_s^3 \times N_t = 32^3 \times 96$
- $m_\pi = 317\ \mathrm{MeV}$

D5

- $a = 0.088\ \mathrm{fm}$
- $N_s^3 \times N_t = 48^3 \times 64$
- $m_\pi = 280\ \mathrm{MeV}$

D6

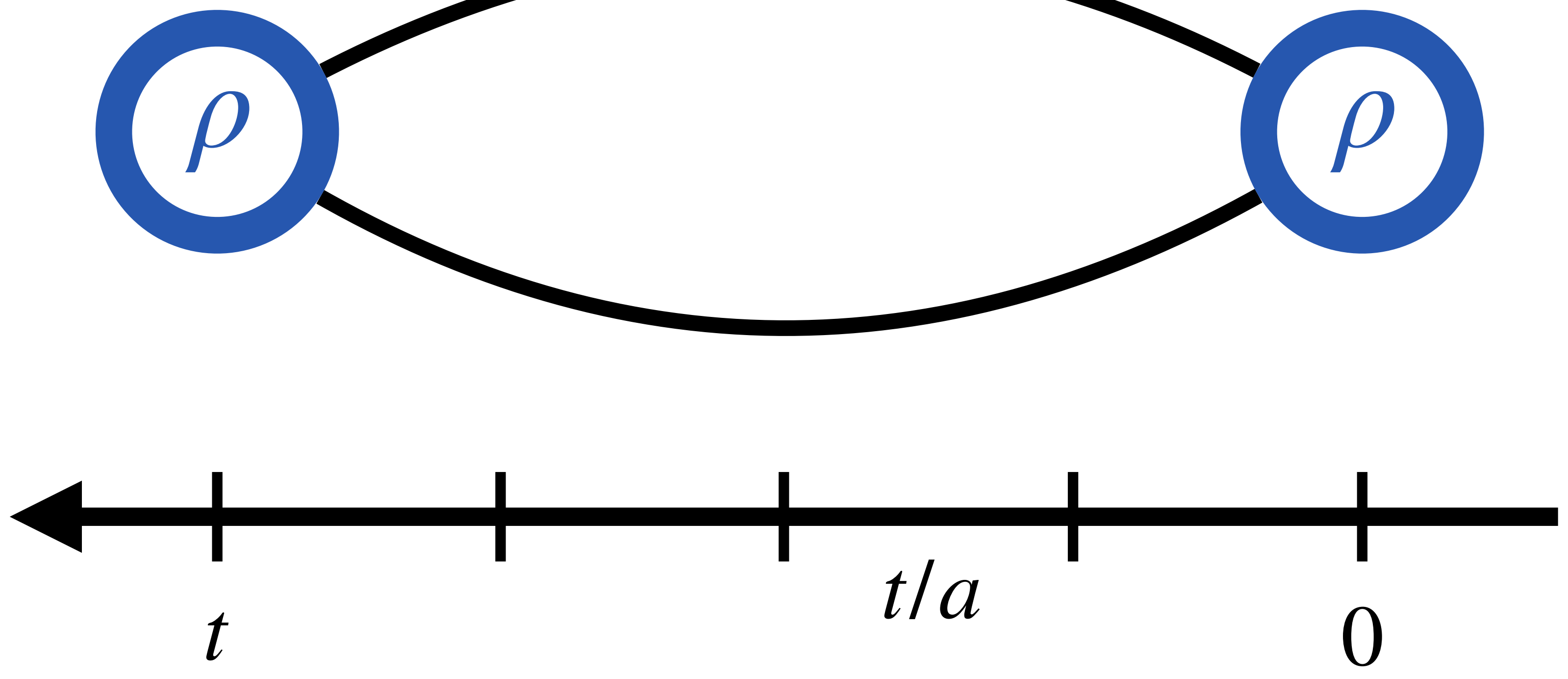
- $a = 0.088\ \mathrm{fm}$
- $N_s^3 \times N_t = 48^3 \times 96$
- $m_\pi = 176\ \mathrm{MeV}$

E5

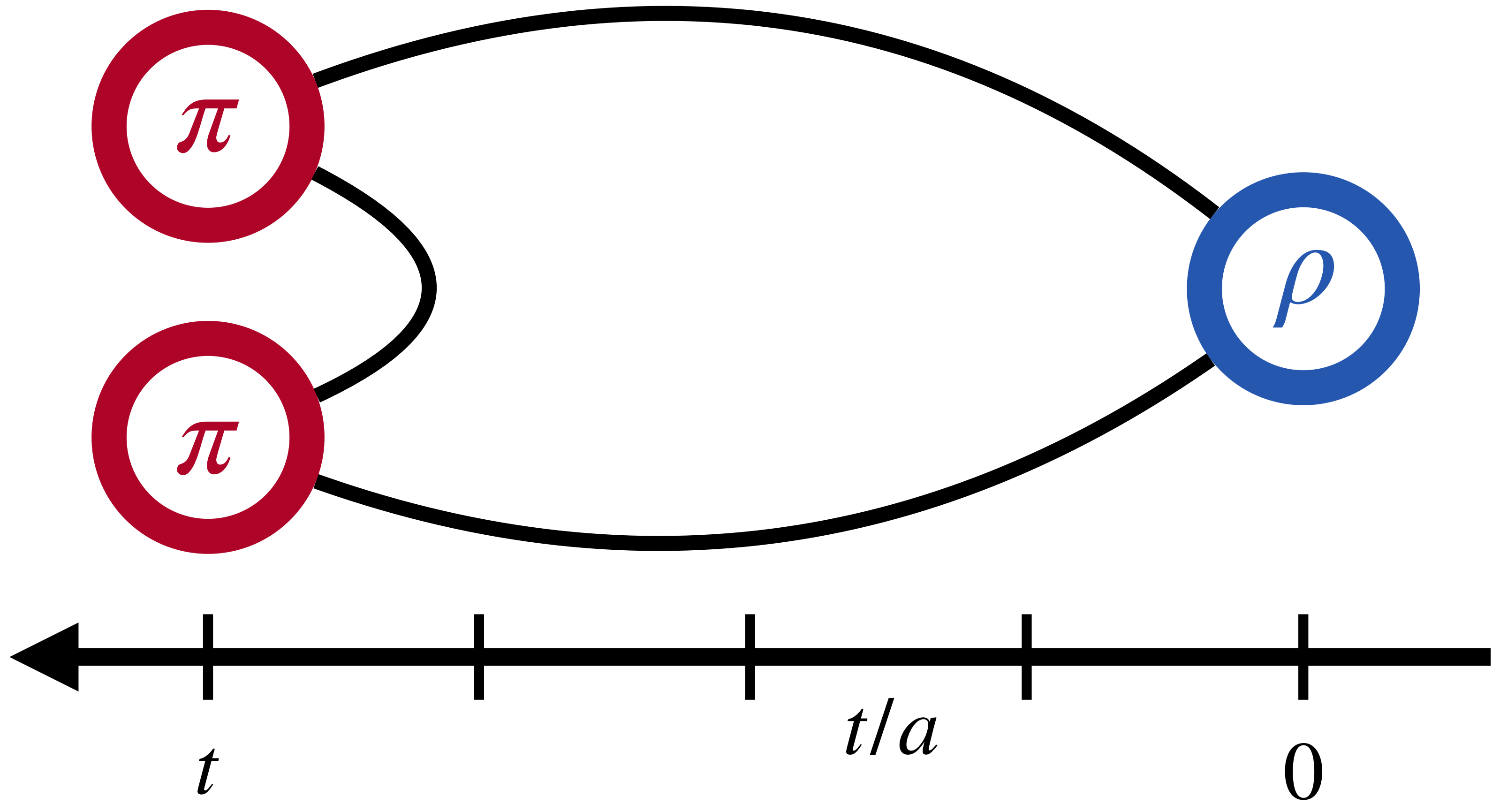
- $a = 0.068\ \mathrm{fm}$
- $N_s^3 \times N_t = 48^3 \times 128$
- $m_\pi = 287\ \mathrm{MeV}$

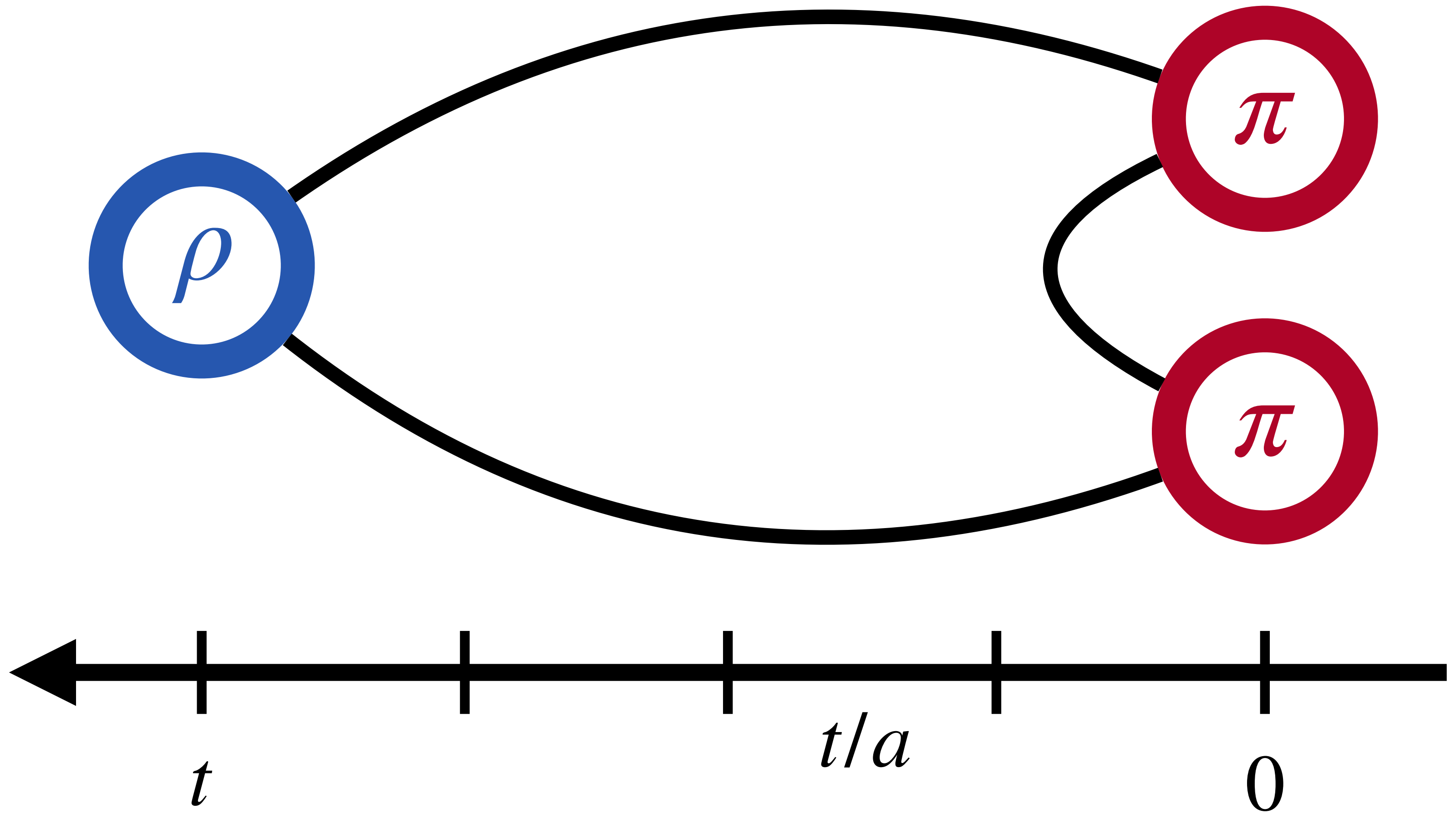
Source: PDGLive

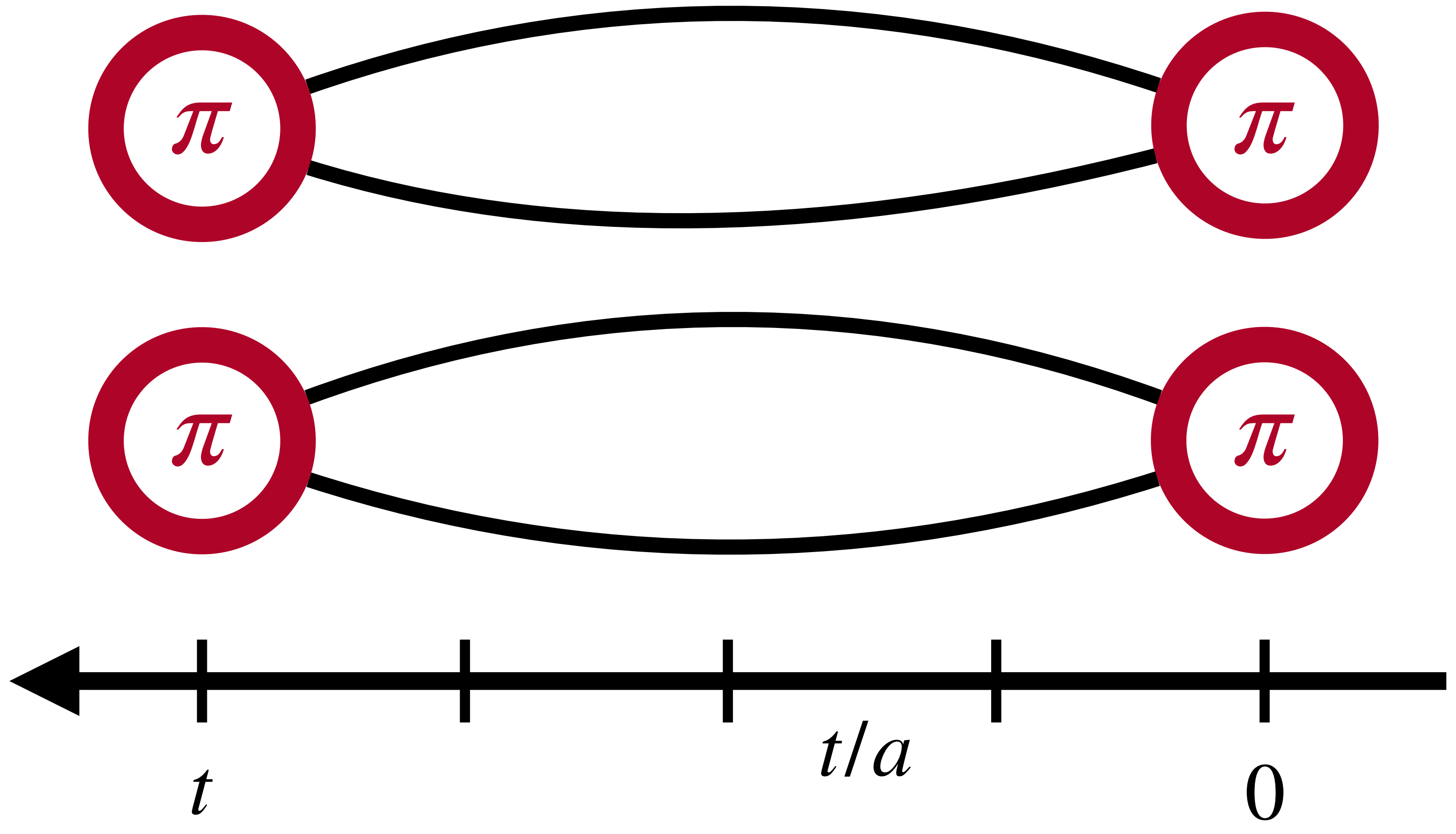
2-point correlation functions

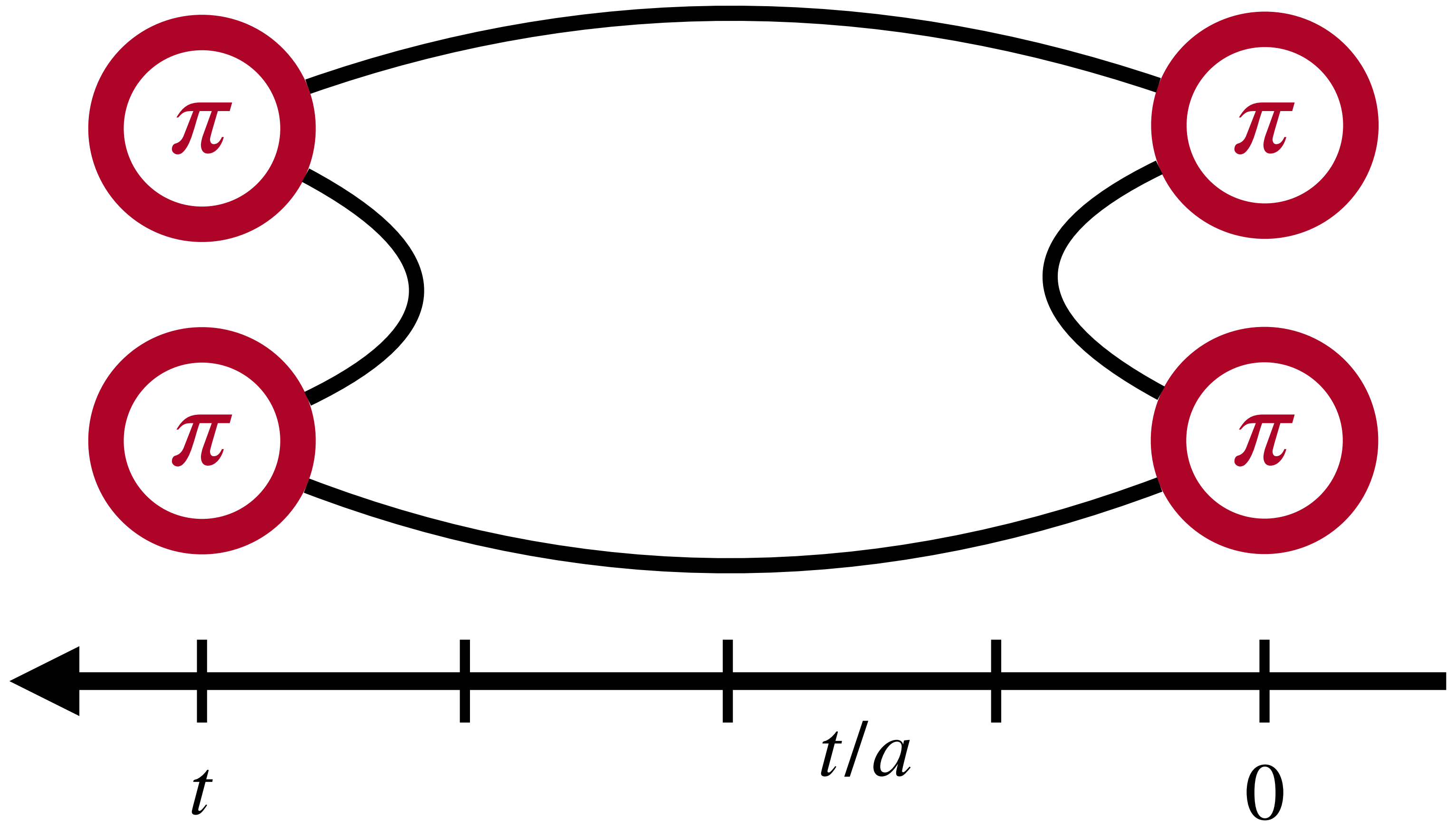


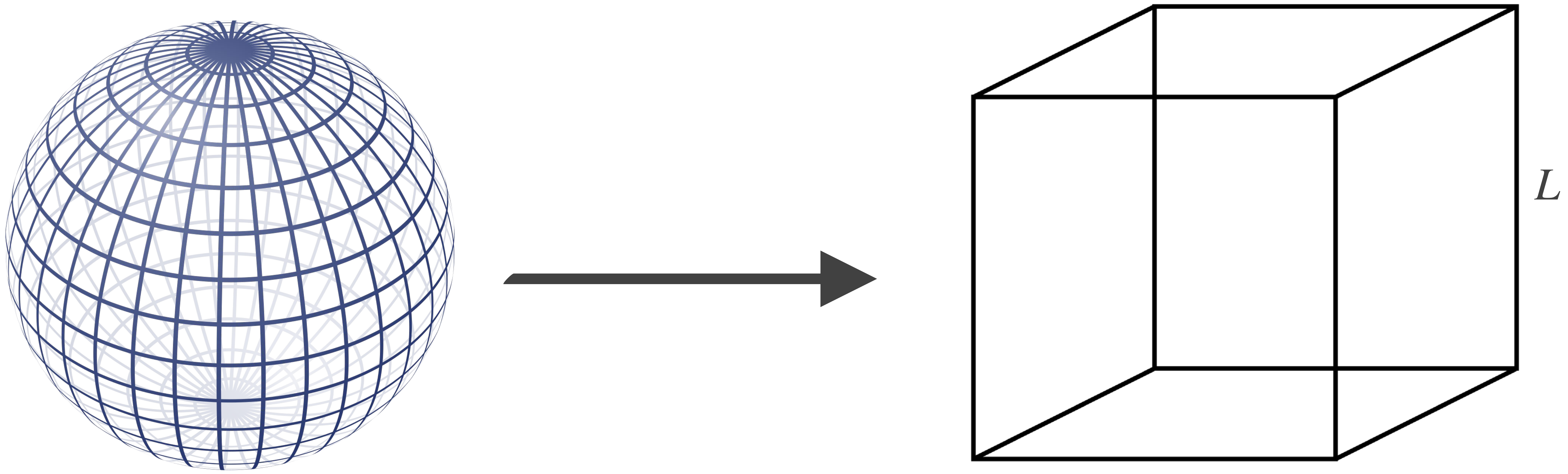
$$C_{ij}(t) = \langle O_i^\dagger(t) O_j(0) \rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2E_n} e^{-E_n t}$$











infinite volume:

- $O(3)$ symmetry
- infinite irreps (J^P)

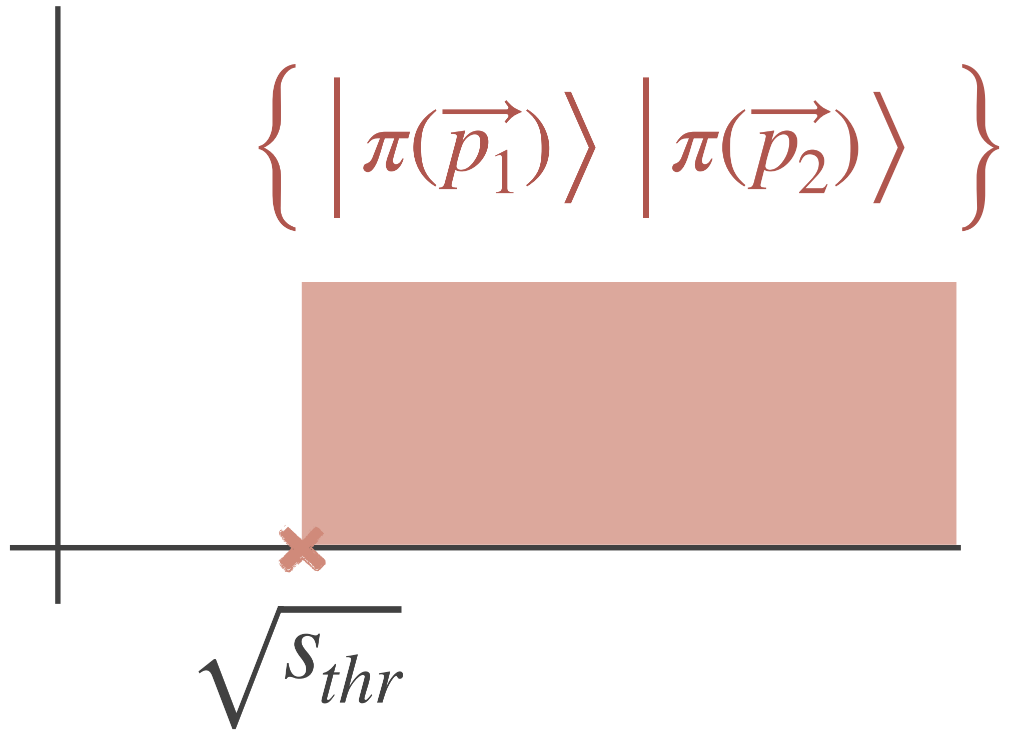
$$J, \overrightarrow{P} = \vec{p}_1 + \vec{p}_2$$

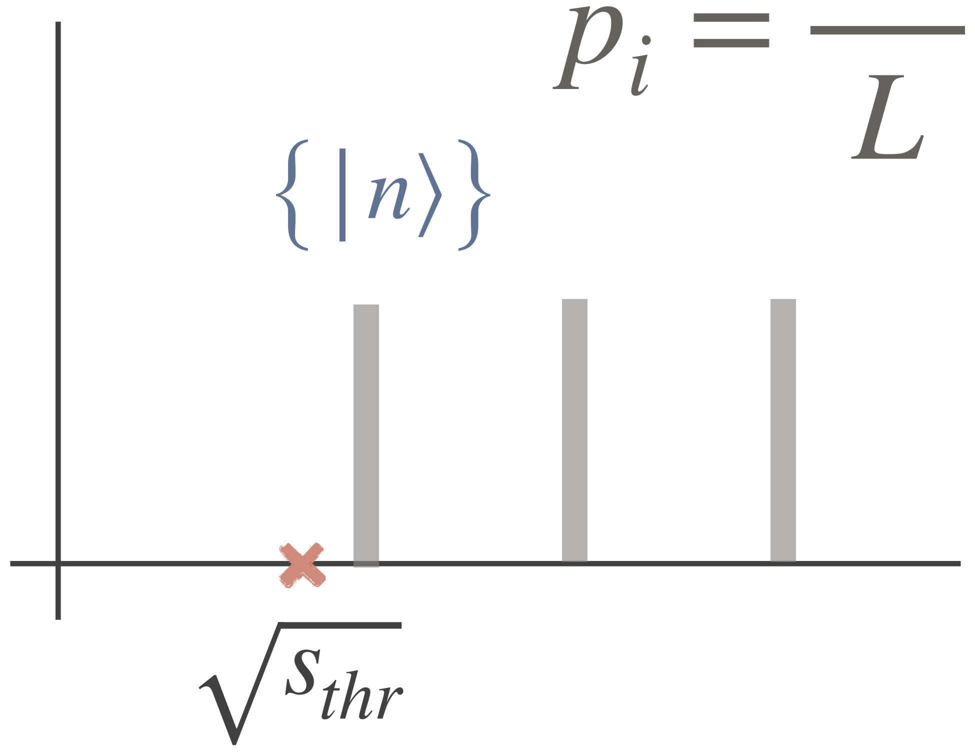
finite volume:

- discrete symmetries, Λ

$$L, \Lambda, \overrightarrow{P} = \vec{p}_1 + \vec{p}_2$$

$$\vec{p}_i = \frac{2\pi}{L}\overrightarrow{m} \quad \overrightarrow{m} \in Z^3$$





Projection operator

$$P_r^{(\Lambda)} = \frac{s_\Lambda}{g} \sum_{a=1}^{g} B_{rr}^{(\Lambda)}(R_a) B(R_a)$$

Projector — Irrep — Reducible rep

Project:

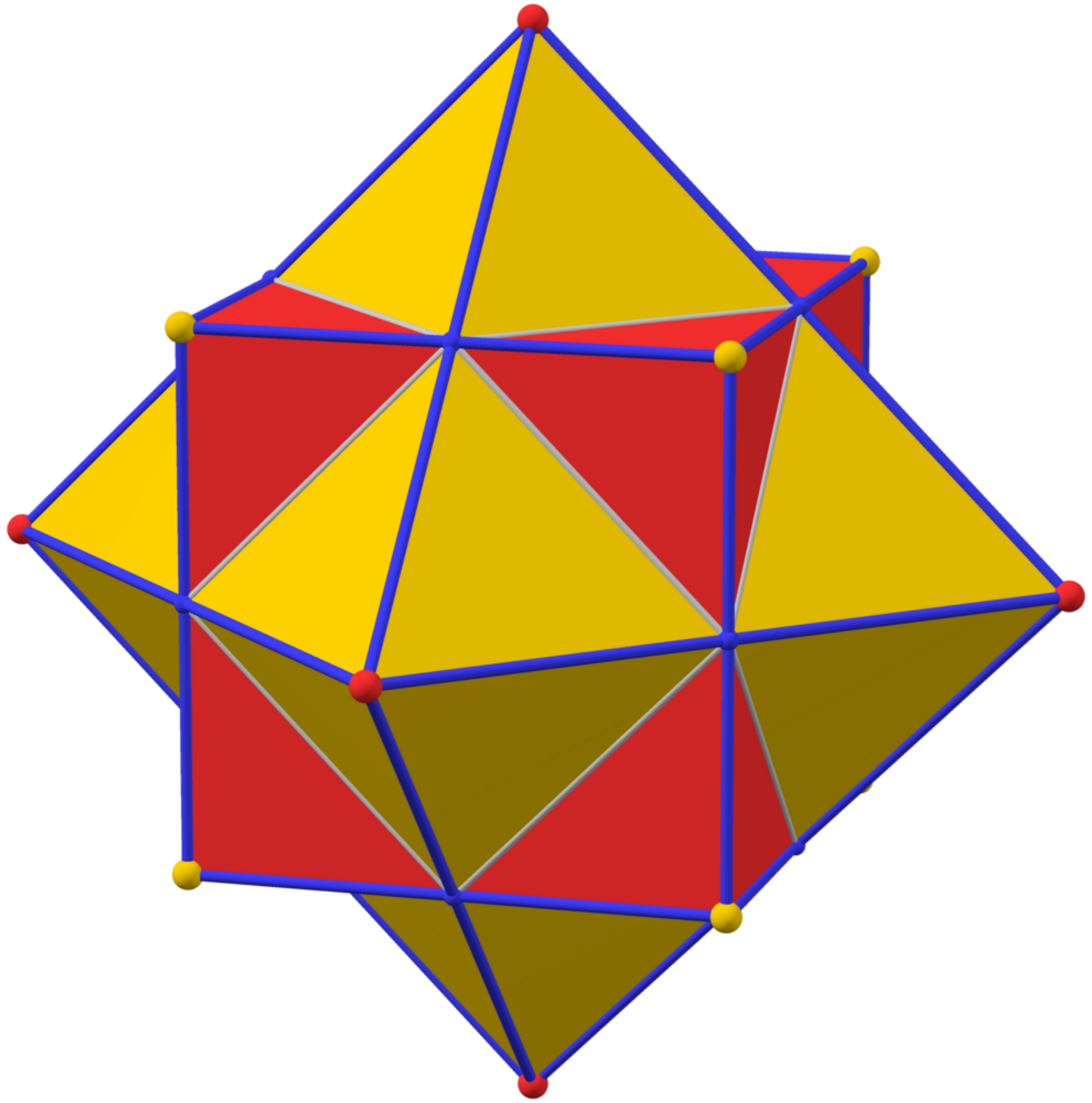
$\pi(p_1)\pi(p_2) \rightarrow \Lambda$

ρ (vector) $\rightarrow \Lambda$

$Y_{l,m} \rightarrow \Lambda$

$$C^{J,P} \longrightarrow C^{L,\Lambda,P}$$

Lattice at rest: O_h symmetry



$$C_{ij}^{\Lambda} = \langle O_i^{\dagger} O_j \rangle$$

ρ operators:

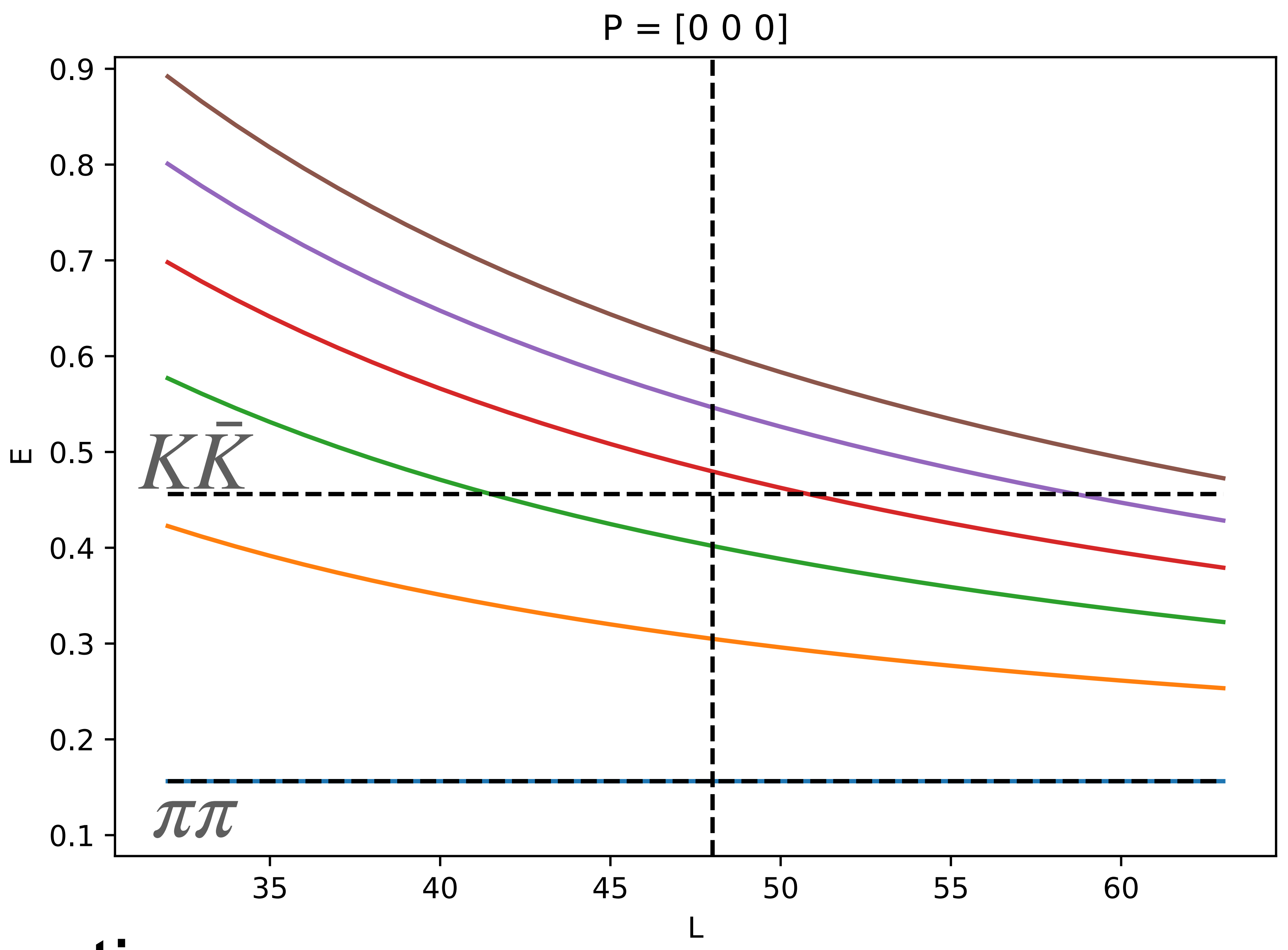
$$O_1(t,P) = \sum_x \bar{d}(t,x)\ \gamma_i\ u(t,x) e^{P \cdot x}$$

$$O_2(t,P) = \sum_x \bar{d}(t,x)\ \gamma_0 \gamma_i\ u(t,x) e^{P \cdot x}$$

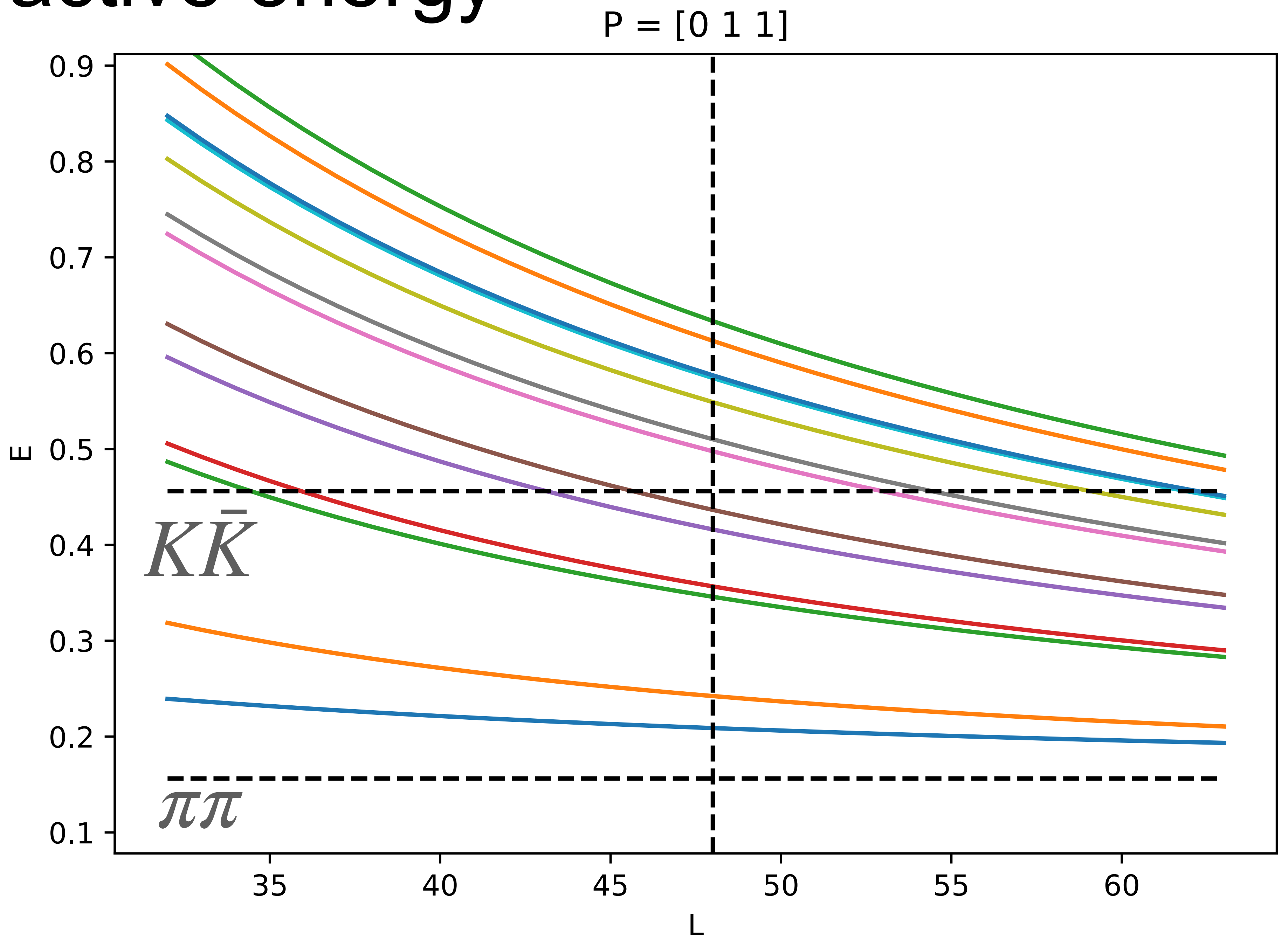
$\pi\pi$ operators:

$$O_{3+a}(t,P) = O_{\pi\pi}(t,P,p_a)$$

$$O_{\pi\pi}(t,P,p) = \frac{1}{\sqrt{2}} \left(\pi^+(t,\frac{P}{2}+p)\pi^0(t,\frac{P}{2}-p) - \pi^0(t,\frac{P}{2}+p)\pi^+(t,\frac{P}{2}-p) \right)$$



$\pi\pi$ non-interactive energy



$$C_{ij}^{\Lambda}(t) = \sum_n \frac{Z_i^{\Lambda n *} Z_j^{\Lambda n}}{2E_n^{\Lambda}} e^{-E_n^{\Lambda} t}$$

GEVP

$$C_{ij}^{\Lambda}(t) v_j^{\Lambda n}(t) = \lambda^{\Lambda n}(t, t_0) C_{ij}^{\Lambda}(t_0) v_j^{\Lambda n}(t)$$

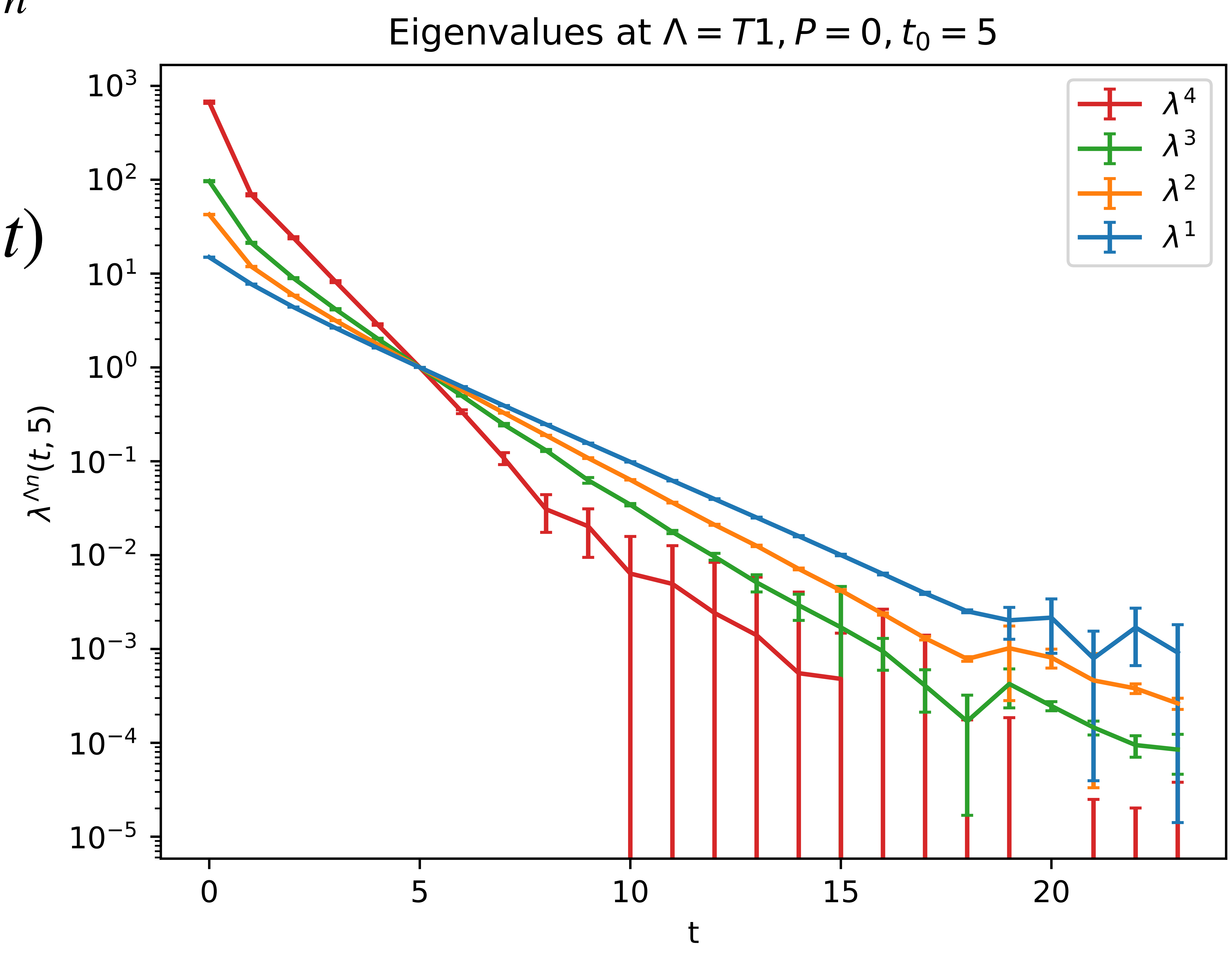
Simple model

$$\lambda^{\Lambda n}(t, t_0) \overset{t \to \infty}{=} e^{-E_n^{\Lambda}(t-t_0)}$$

Excited state pollution model:

$$\lambda^{\Lambda n}(t, t_0) = (1-A) e^{-E_n^{\Lambda}(t-t_0)} + A e^{-E_m^{\Lambda}(t-t_0)}$$

C13 ensemble:



Jay, Neil arXiv:2008.01069

AIC

Fit multiple models:

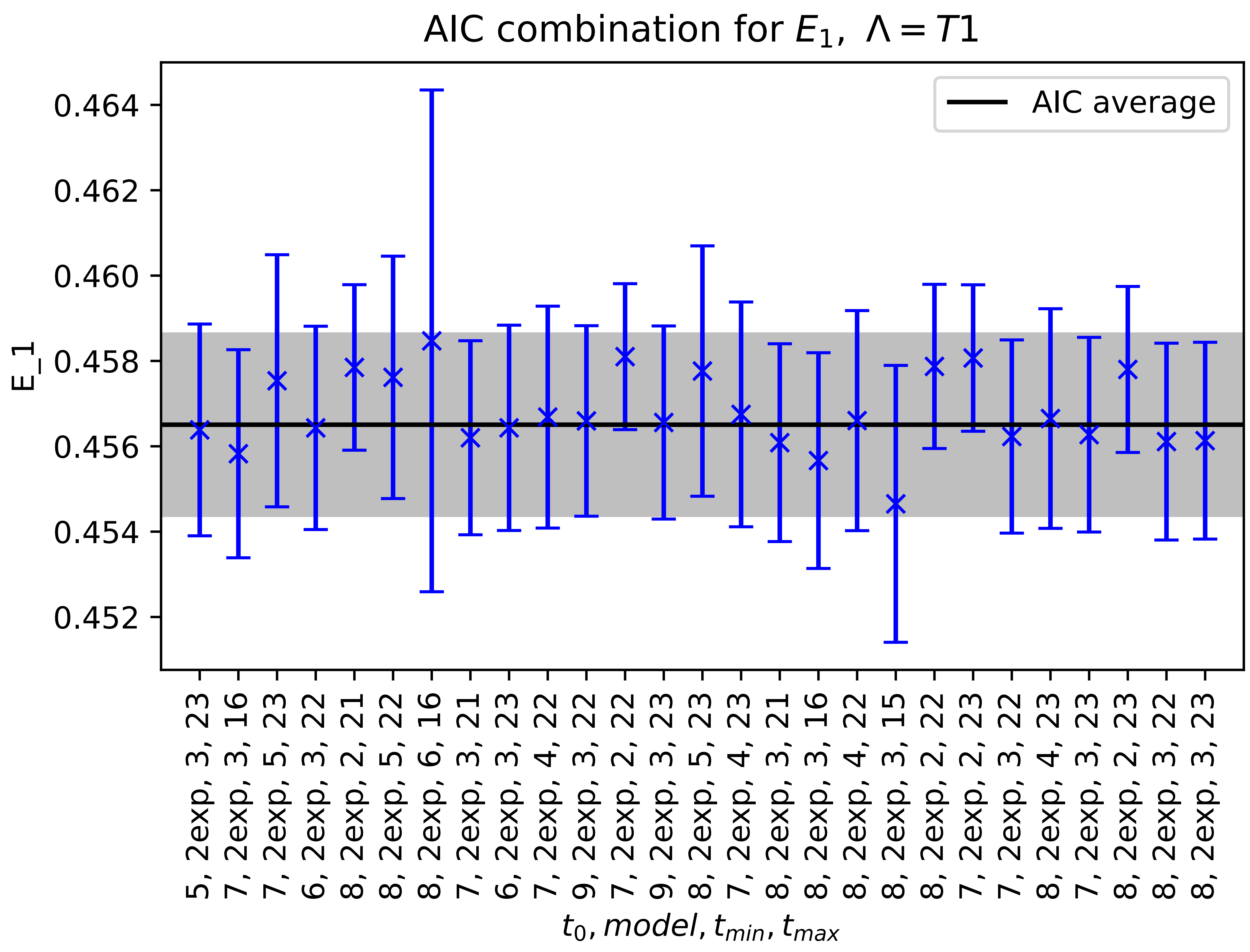
- Range: $[t_{min}, t_{max}]$
- t_0
- 1exp, 2exp

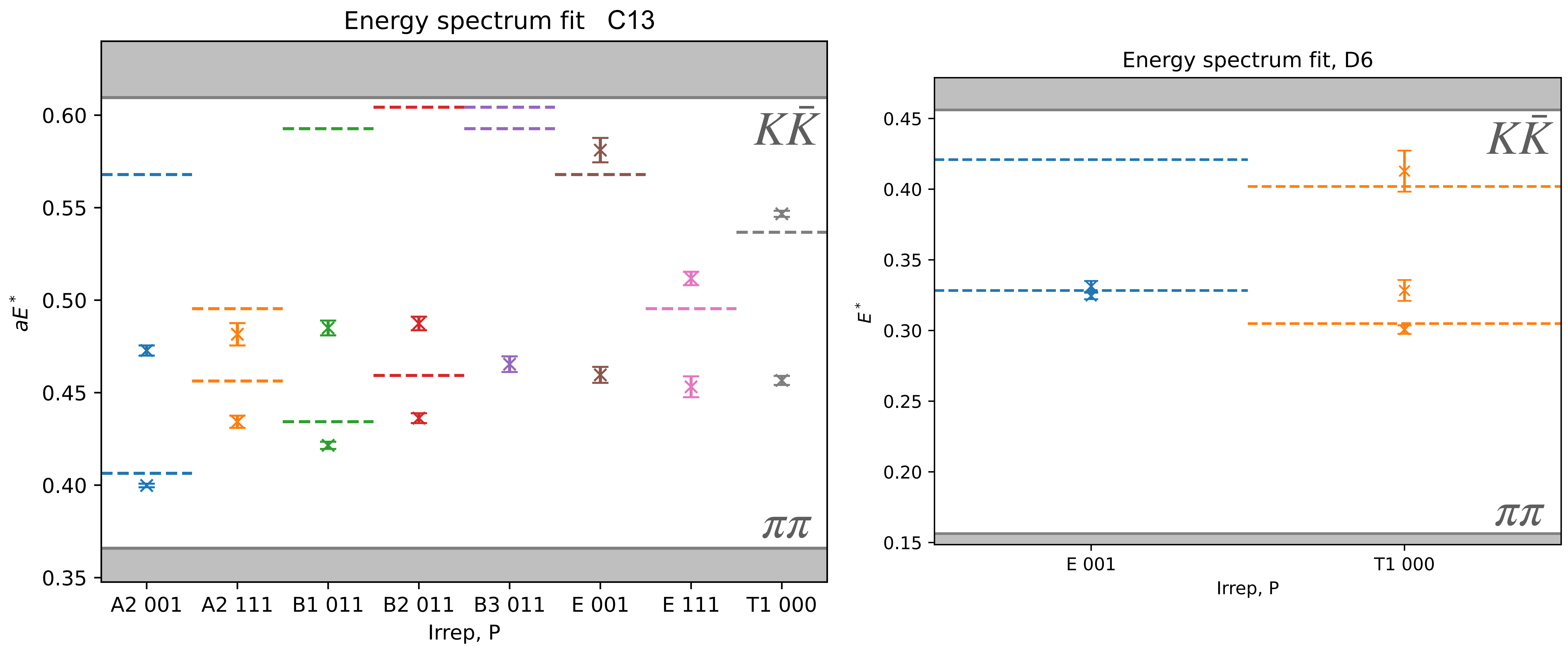
Average over models:

$$AIC = \frac{1}{N_D} \log \frac{\chi^2}{N_D} + 2N_{par} + 2N_{ex}$$

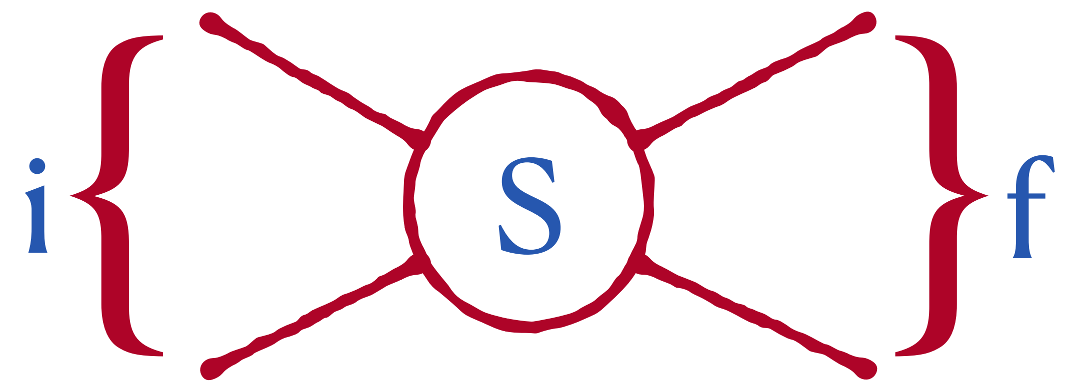
$$\omega_m = \frac{\exp[-0.5\Delta AIC_m]}{\sum_k \exp[-0.5\Delta AIC_k]}$$

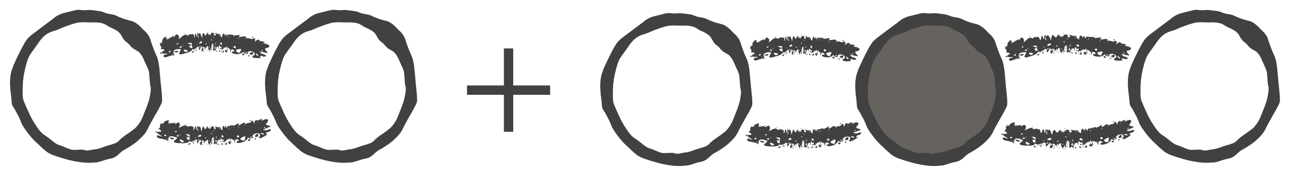
$$\{E^\Lambda\} = \sum_m \omega_m \{E_m^\Lambda\}$$

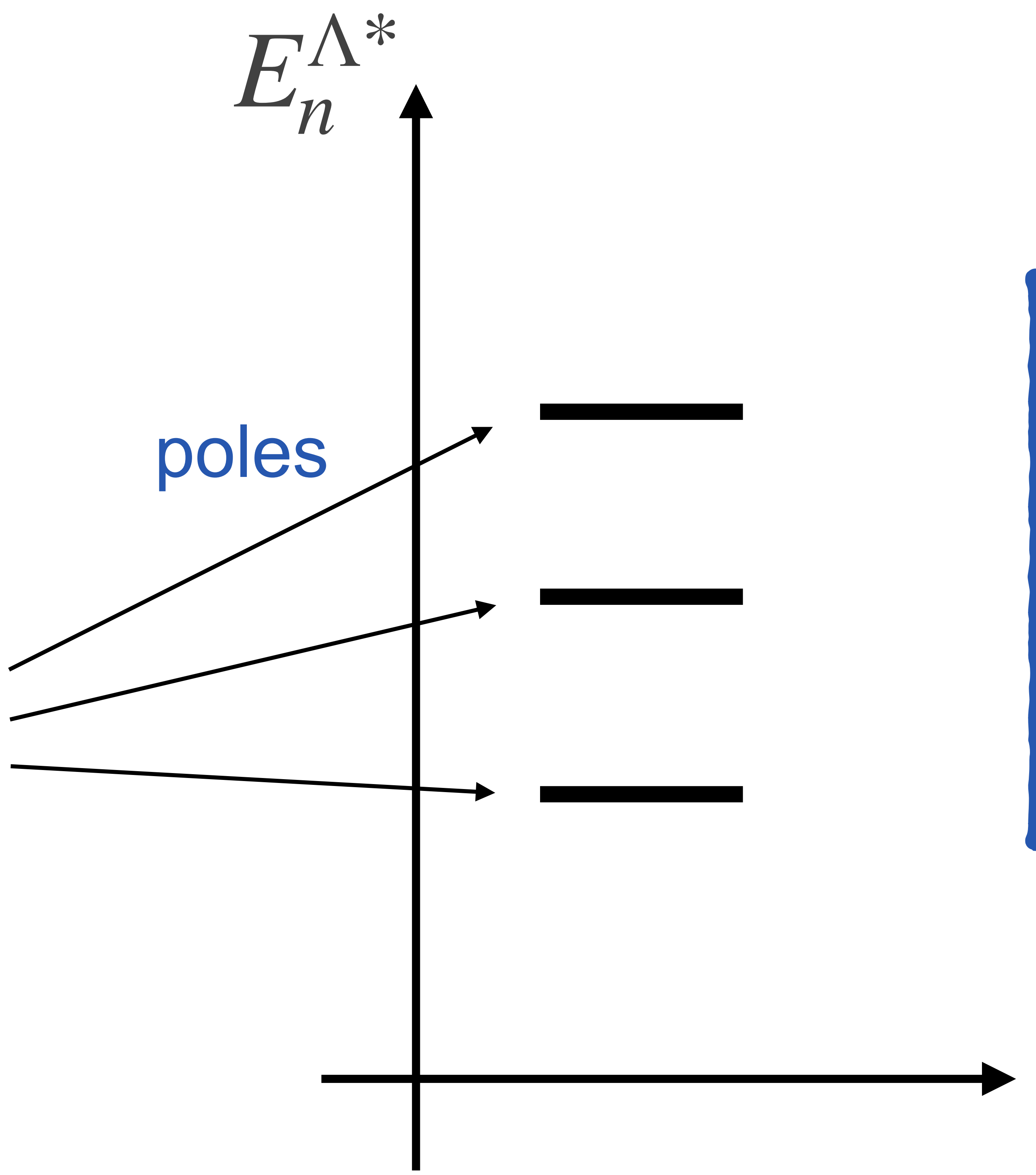


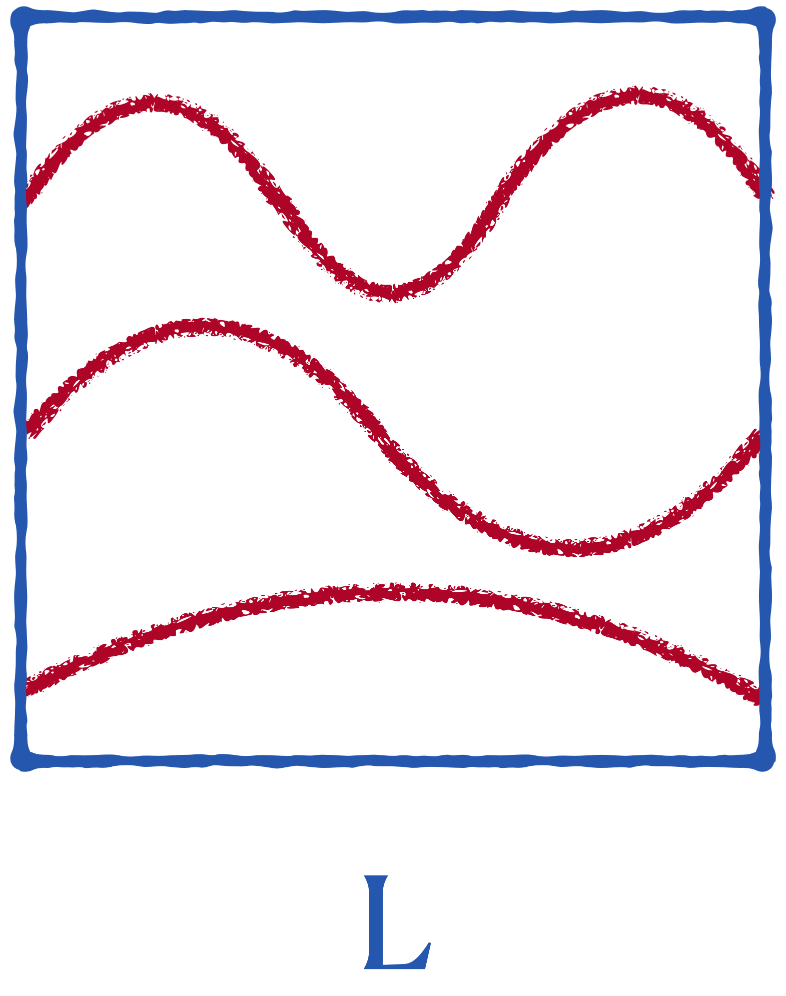


Energy spectrum fit C13
0.60
0.55
0.50
0.45
0.40
0.35
aE^*
$K\bar{K}$
$\pi\pi$
A2 001
A2 111
B1 011
B2 011
B3 011
E 001
E 111
T1 000
Irrep, P
Energy spectrum fit, D6
0.45
0.40
0.35
0.30
0.25
0.20
0.15
E^*
$K\bar{K}$
$\pi\pi$
E 001
T1 000
Irrep, P



$C_L^{(2)} =$  $+ \ldots$





$$C_L^{(2)} = C_\infty^{(2)} - A' \frac{1}{F_\Lambda^{-1}(E^*) + T_\Lambda(E^*)} A$$

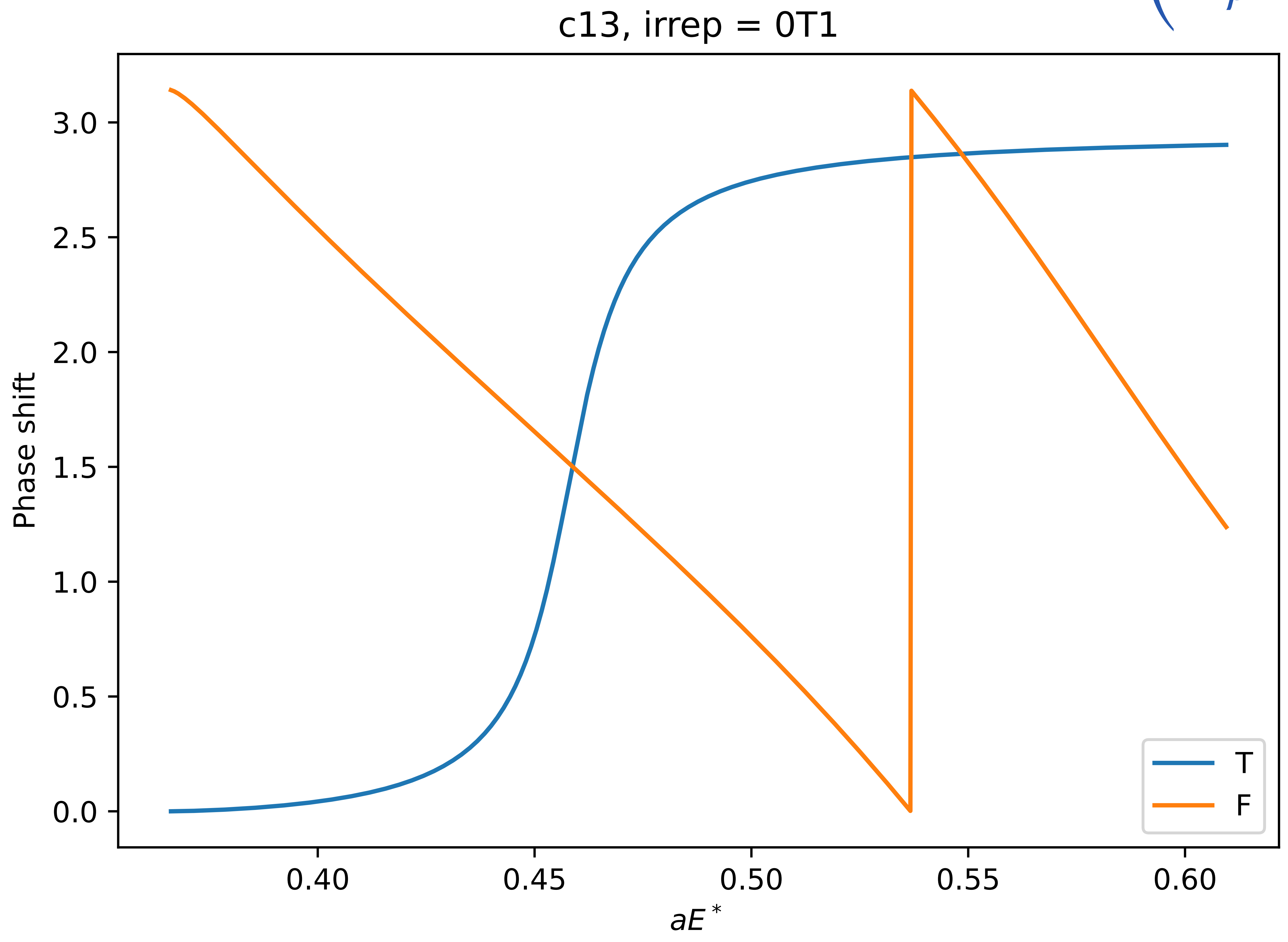
discrete spectrum where:

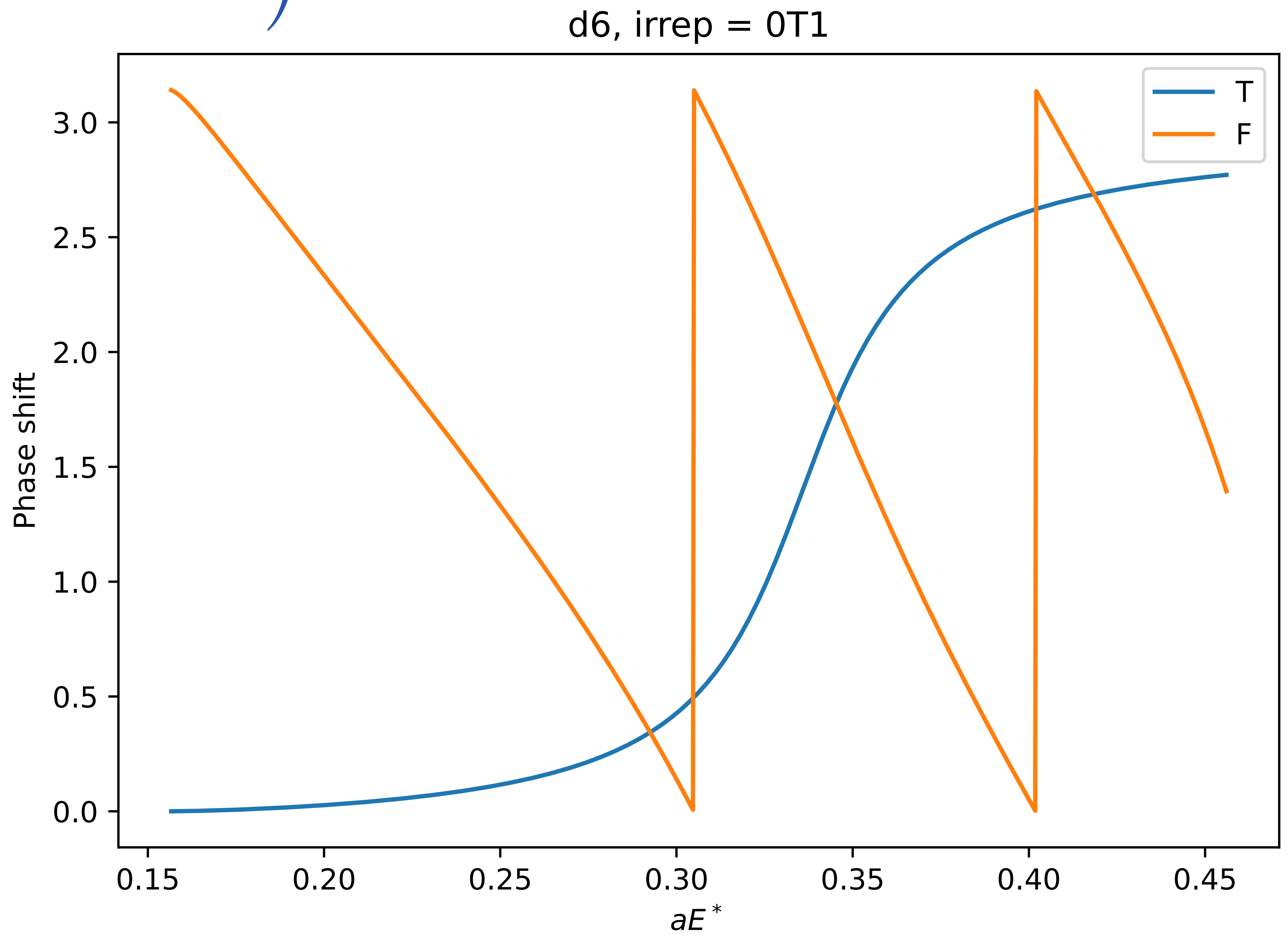
$$\det\left[F_\Lambda^{-1}(E^*) + T_\Lambda(E^*)\right] = 0$$

Luscher NPB354
Rummukainen, Gottlieb hep-lat/9503028
Kim, Sharpe, Sachrajda hep-lat/0507006
Briceno 1401.3312
Woss, Wilson, Dudek 2001.08474
Briceno, Dudek, Young 1706.06223

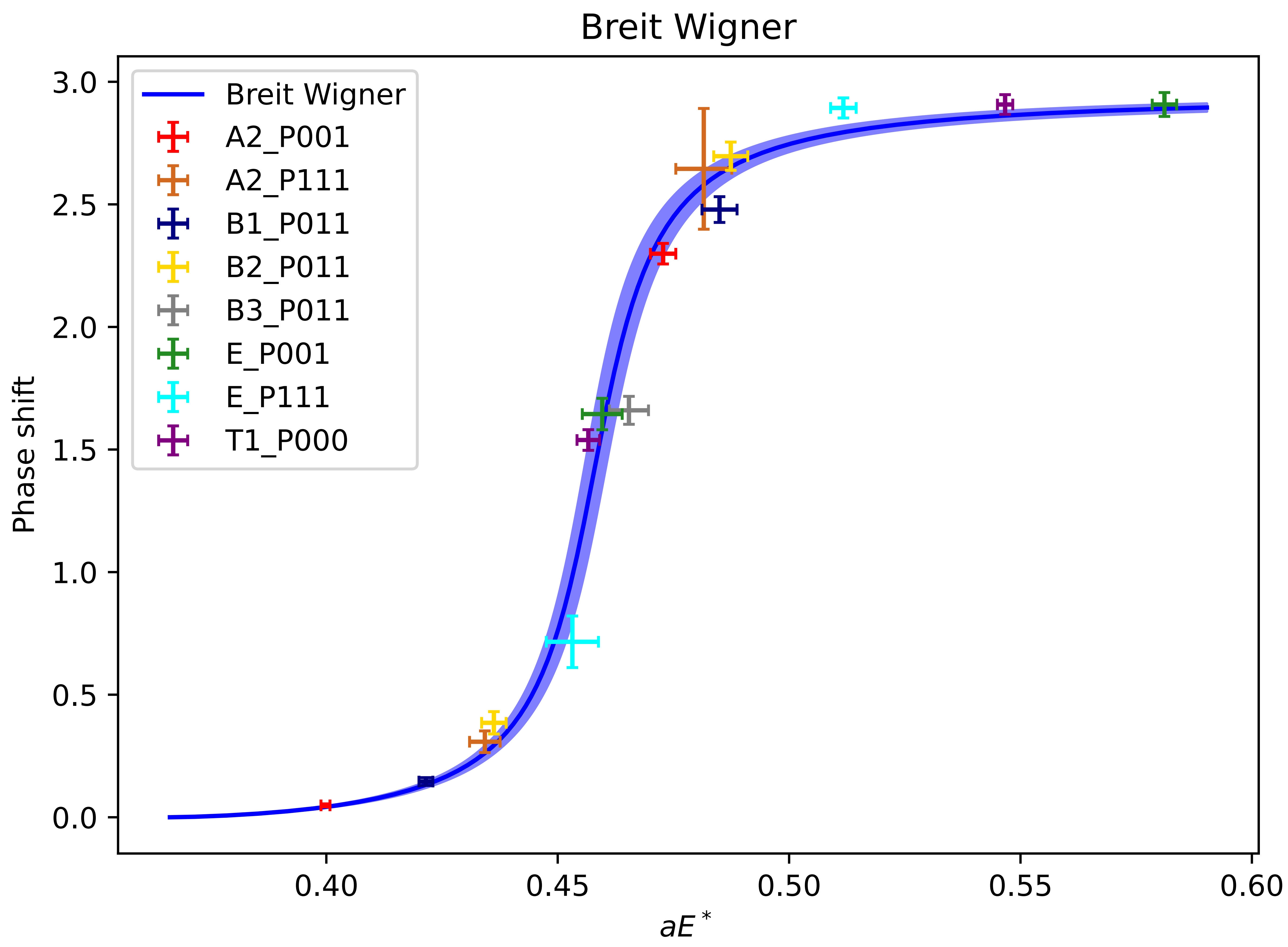
$$F_{lm,l'm'} = \frac{q^*}{8\pi E^*}\left(\delta_{mm'}\delta_{ll'} - i\frac{4\pi}{q^*}\sum_{l_i,m_i}\frac{\sqrt{4\pi}}{q^{*l}}c_{l,m}(E^*)\int Y^*_{lm}Y_{l_im_i}Y_{l'm'}d\Omega\right)$$

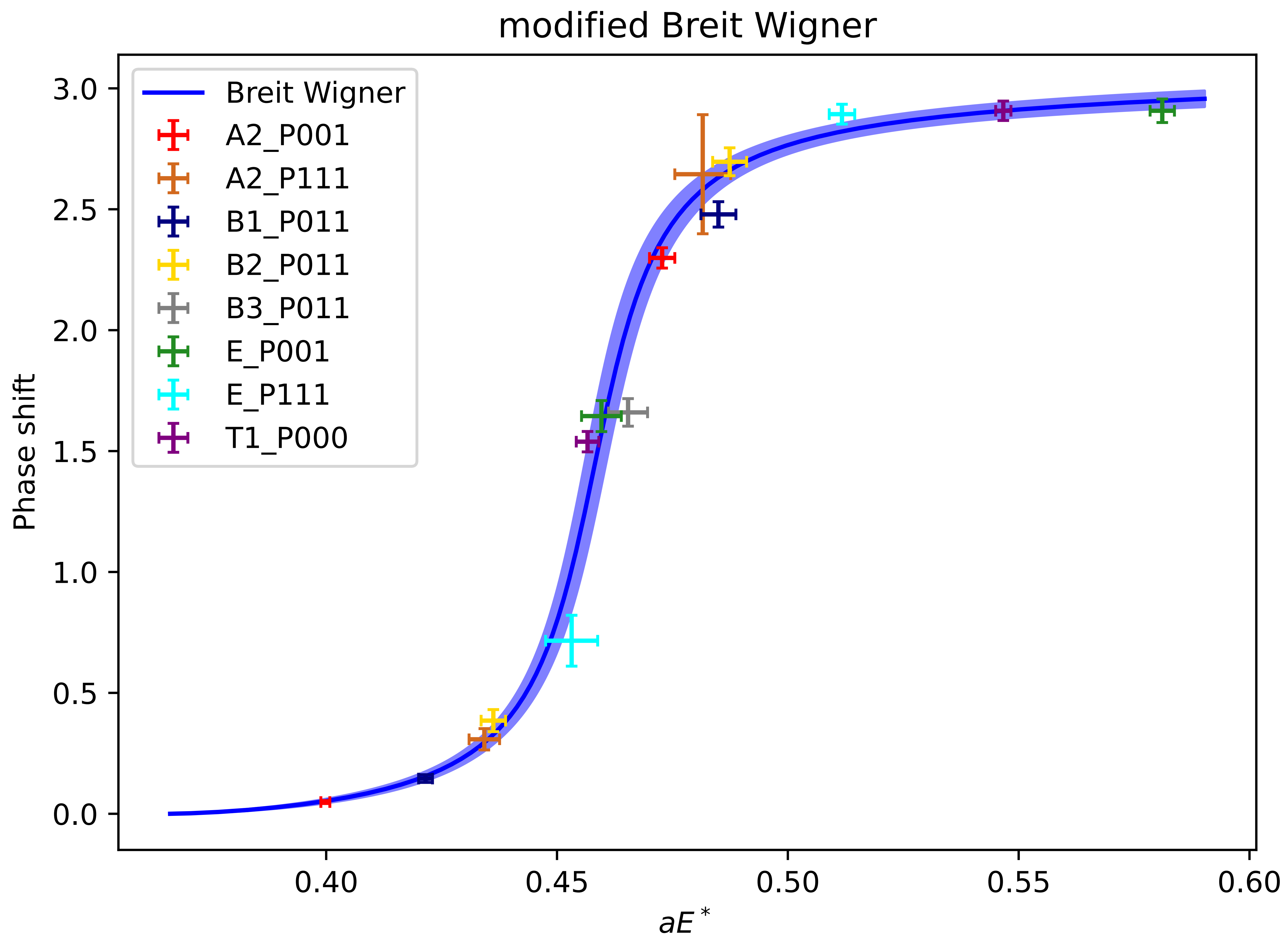
$$T = \frac{16\pi E^*}{k}\frac{E^*\Gamma}{\left(m_\rho^2 - E^{*2} - iE^*\Gamma\right)},\ \Gamma = g^2_{\rho\pi\pi}\frac{k^3}{E^{*2}}$$





C13 QUANTIZATION CONDITION FIT





$g = 5.6 \pm 0.3$
$m_\rho = 794 \pm 5$ MeV

$\chi^2/\mathrm{Dof} = 0.50$

$g = 5.7 \pm 0.3$
$m_\rho = 794 \pm 5$ MeV
$r = 1.8 \pm 1.8$ fm

$\chi^2/\mathrm{Dof} = 0.39$

Chiral extrapolation

Spectroscopy

- Analyse ensembles: C13, D5, D6, E5

Done

VEGA

- Chiral and continuum extrapolation to the physical limit

EG: breit-wigner

$$T = \frac{16\pi E^*}{k} \frac{E^*\Gamma}{\left(m_\rho^2 - E^{*2} - iE^*\Gamma\right)}, \ \Gamma = g_{\rho\pi\pi}^2 \frac{k^3}{E^{*2}}$$

$$m_\rho = m_{\rho,0} + c_1 m_\pi^2 + d_1 a^2$$

$$g_{\rho\pi\pi} = g_{\rho\pi\pi,0} + c_2 m_\pi^2 + d_2 a^2$$

Conclusion

- ρ spectroscopy on C13 ensemble ✓

TODO:

- ρ spectroscopy on E5, D6, D5
- Chiral and continuum extrapolation to the physical limit
- Analyze $D \to \rho\ell\nu$ matrix elements