$T_{cc}^{+}(3875)$ via plane wave approach and including diquark-antidiquark operators

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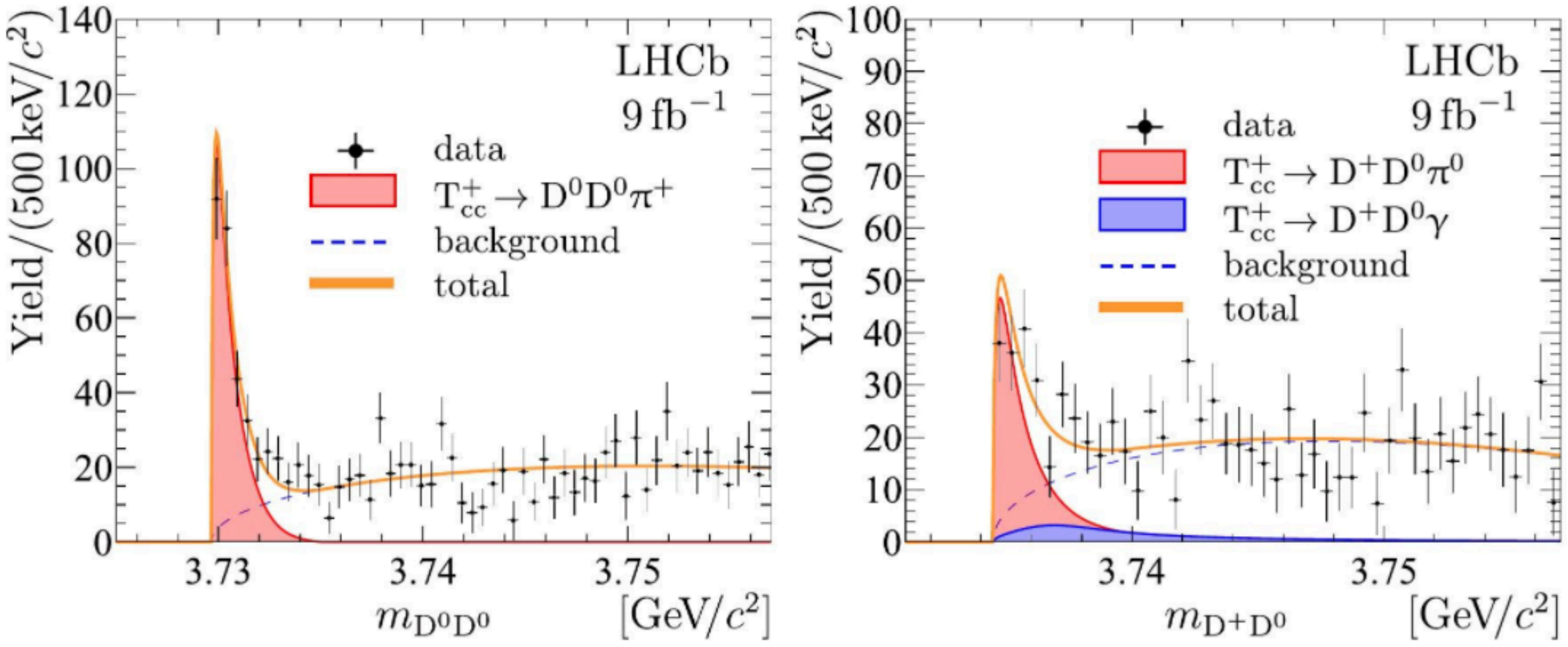
with **S. Prelovsek**, **E. O. Pacheco, L. Leskovec**, **M. Padmanath** and **S. Collins**.

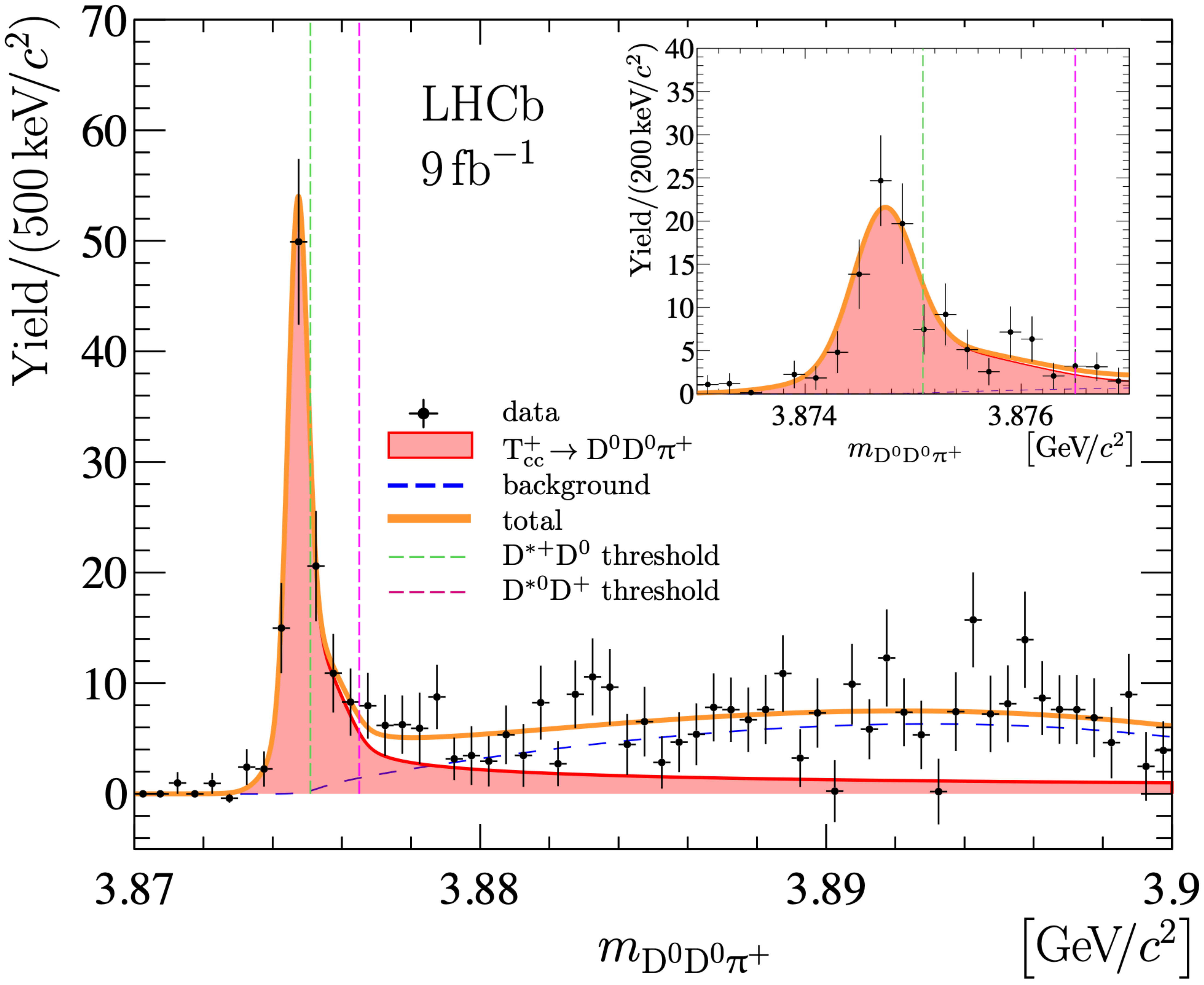


BRDA 2024, Belica, 4. 10. 2024

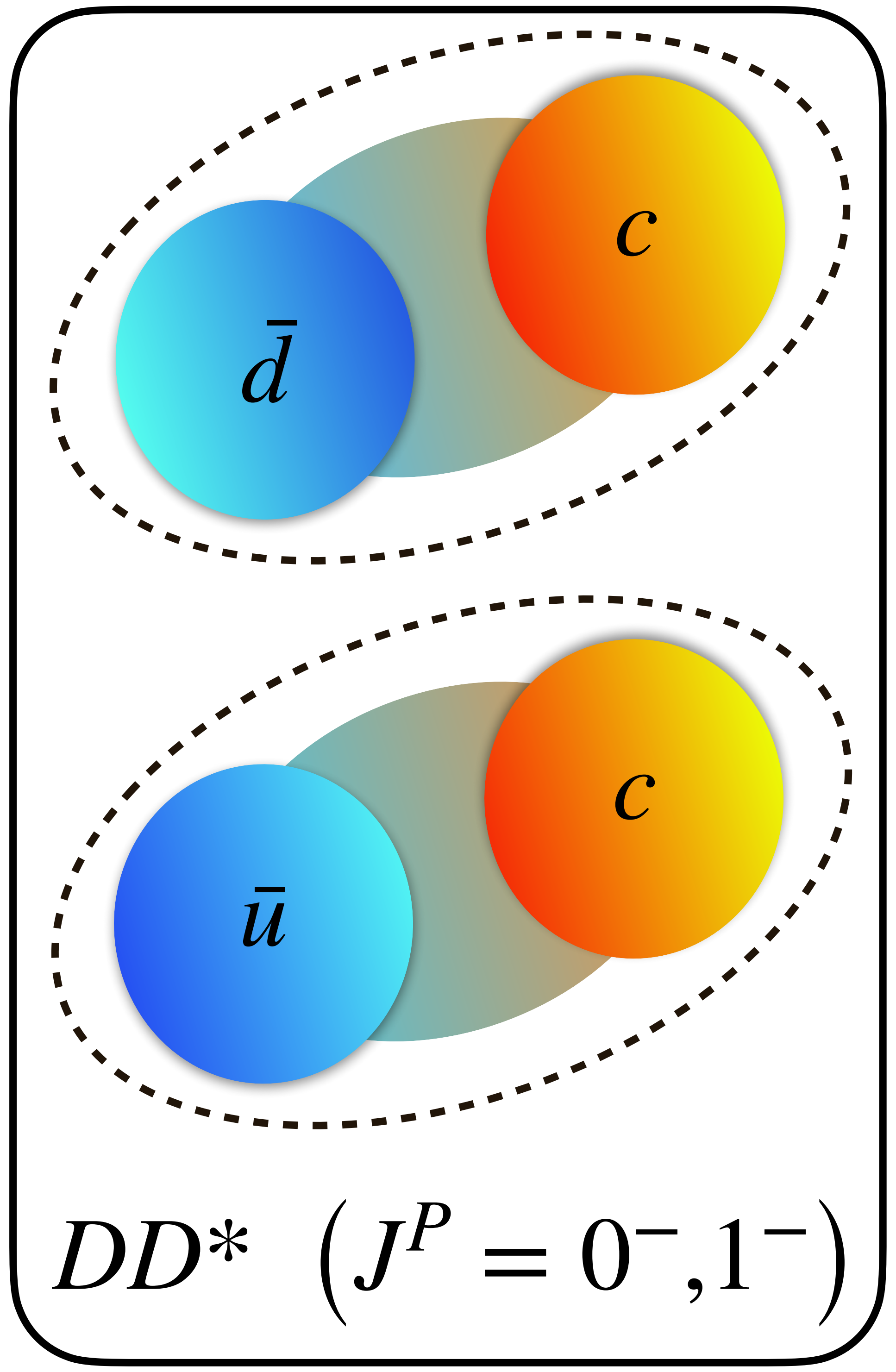
$$\delta m_{\text{pole}} = m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0}),$$

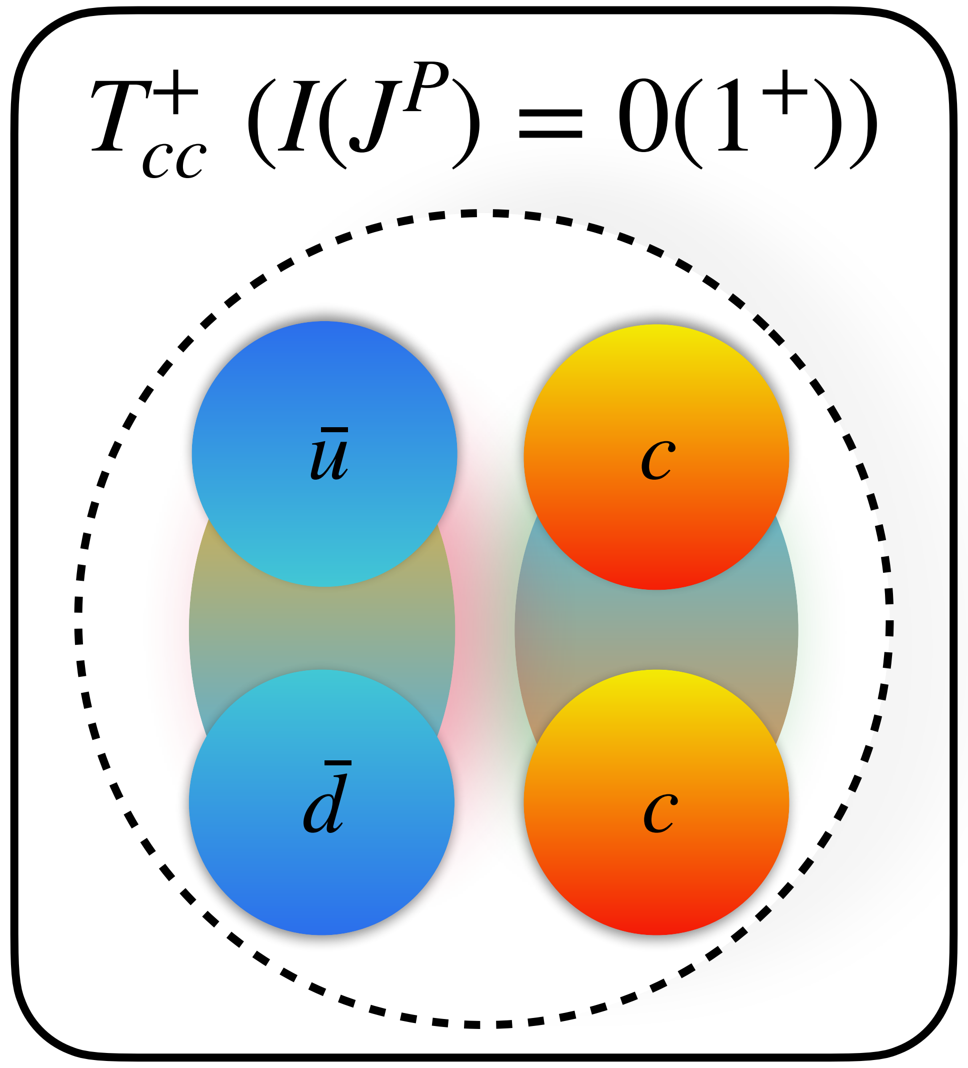
$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0}\ \text{keV}/c^2,$$
$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14}\ \text{keV}$$

A short intro on T_{cc}



- $cc\bar{u}\bar{d}$
- September 2021





Methods & basic observables of lattice QCD

$$\mathscr{L}_{QCD}(x) = \sum_f \bar{q}^{(f)}(x)\left(i\gamma^\mu\partial_\mu - m^{(f)} - g_S\gamma^\mu G^a_\mu t_a\right) q^{(f)}(x) - \frac{1}{4}G^{a\mu\nu}G_{a\mu\nu}$$

$$S_{QCD}\left[q,\bar{q},G\right] = \int \mathrm{d}^4x\,\mathscr{L}_{QCD}(x)$$

$$\mathscr{Z} = \int \mathscr{D}\bar{q}\mathscr{D}q\mathscr{D}U\; e^{-S_{QCD}[q,\bar{q},U]}$$

$$t_E = it_M$$

$$\langle\mathscr{O}\rangle = \langle\Omega\,|\,\mathscr{O}\,|\,\Omega\rangle = \frac{1}{\mathscr{Z}}\int \mathscr{D}\bar{q}\mathscr{D}q\mathscr{D}U\;\mathscr{O}\left[q,\bar{q},U\right] e^{-S_{QCD}}$$

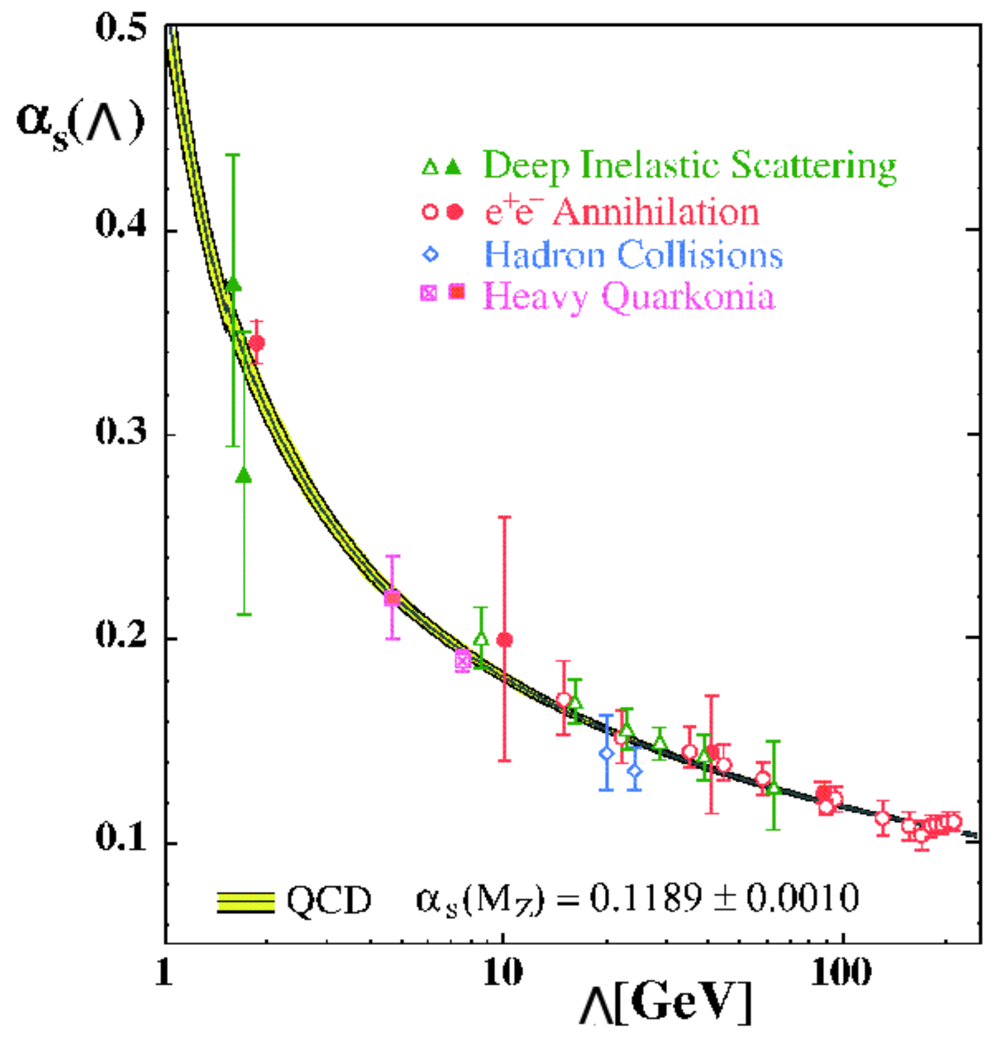
$$C_2^{ij}(t) = \langle\mathscr{O}^i(t)\mathscr{O}^{j\dagger}(t_0)\rangle =$$

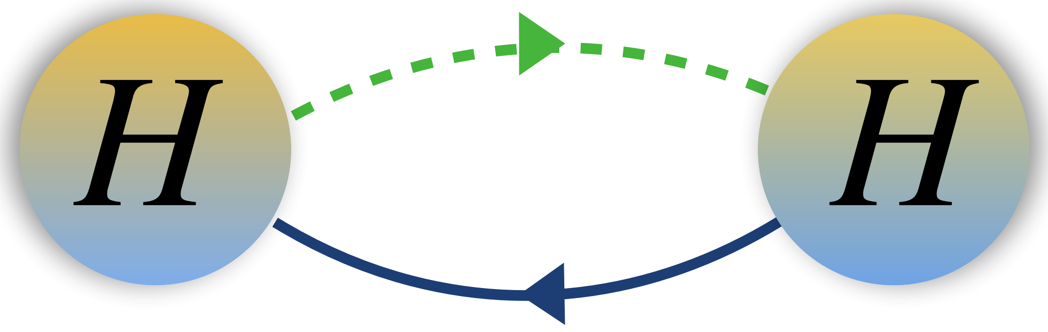
$$= \sum_n \langle\Omega\,|\,\mathscr{O}^i(t_0)\,|\,n\rangle\langle n\,|\,\mathscr{O}^{j\dagger}(t_0)\,|\,\Omega\rangle e^{-E_n(t-t_0)} =$$

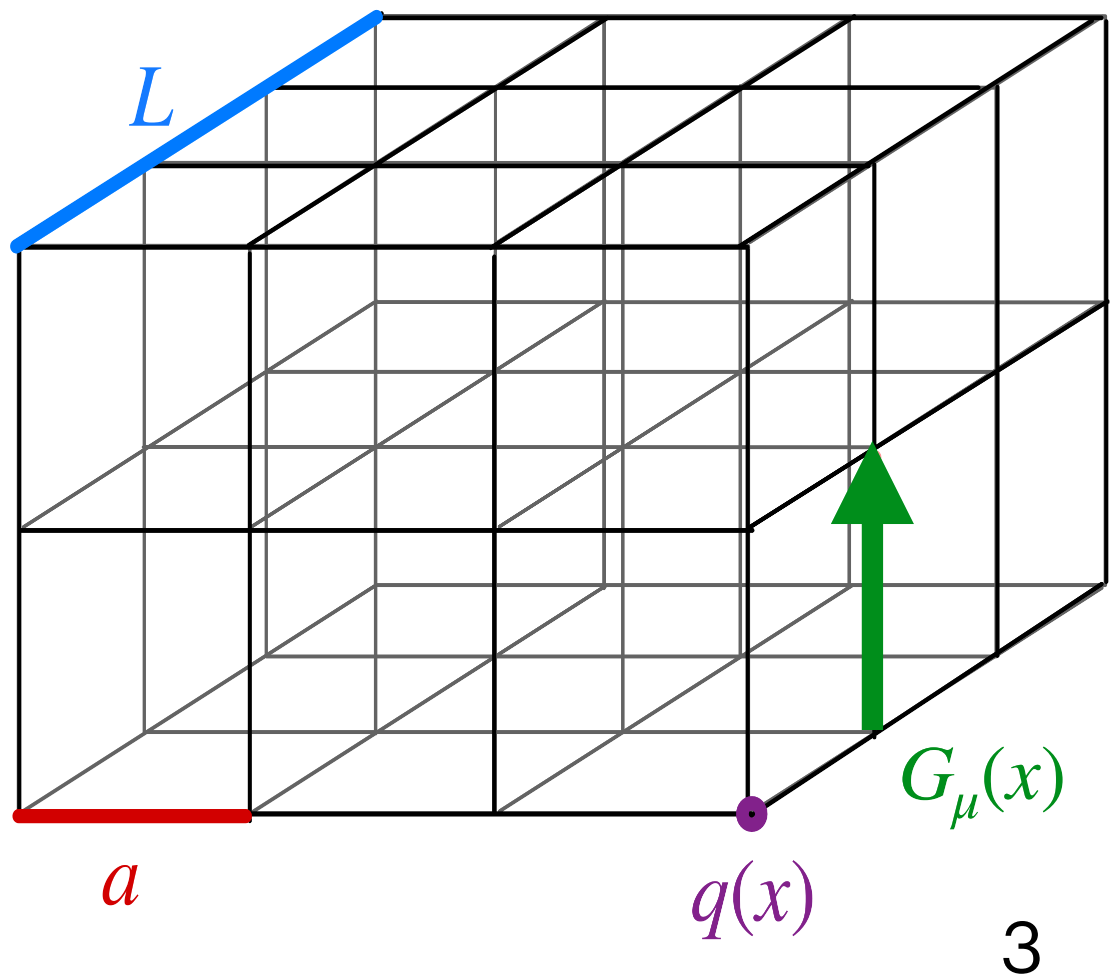
$$= \sum_n Z_n^{i*}Z_n^j e^{-E_n(t-t_0)}$$

E_n → **energies of the ground & excited states**

$Z_n^{i*}Z_n^j$ → **overlap factors for each operator and state**







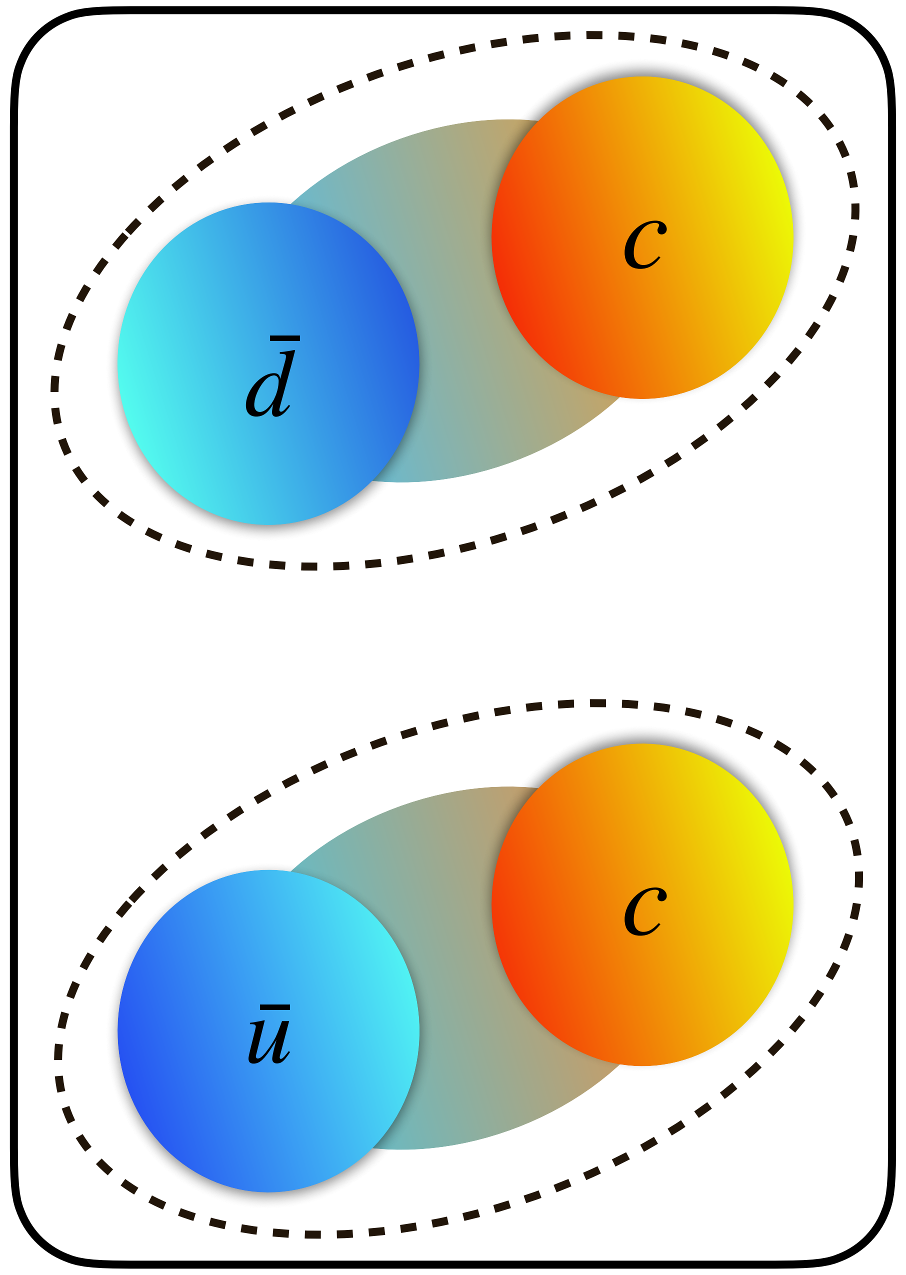
Addition of diquark-antidiquark operators

- basic scattering operators: M. Padmanath and S. Prelovsek, 2202.10110 (2022)

$$\mathcal{O}^{DD^*}\left(\vec{p}_1,\vec{p}_2\right) = \sum_{\vec{x}_1,\vec{x}_2} e^{i\vec{p}_1\cdot\vec{x}_1} e^{i\vec{p}_2\cdot\vec{x}_2}\cdot\left[\bar{u}(x_1)\Gamma_V c(x_1)\right]\left[\bar{d}(x_2)\Gamma_P c(x_2)\right] - \{\bar{u}\leftrightarrow\bar{d}\}$$

- implementation of $[cc]\left[\bar{u}\bar{d}\right]_{I=0}$ operators: E. O. Pacheco et al., 2312.13441 (2023)

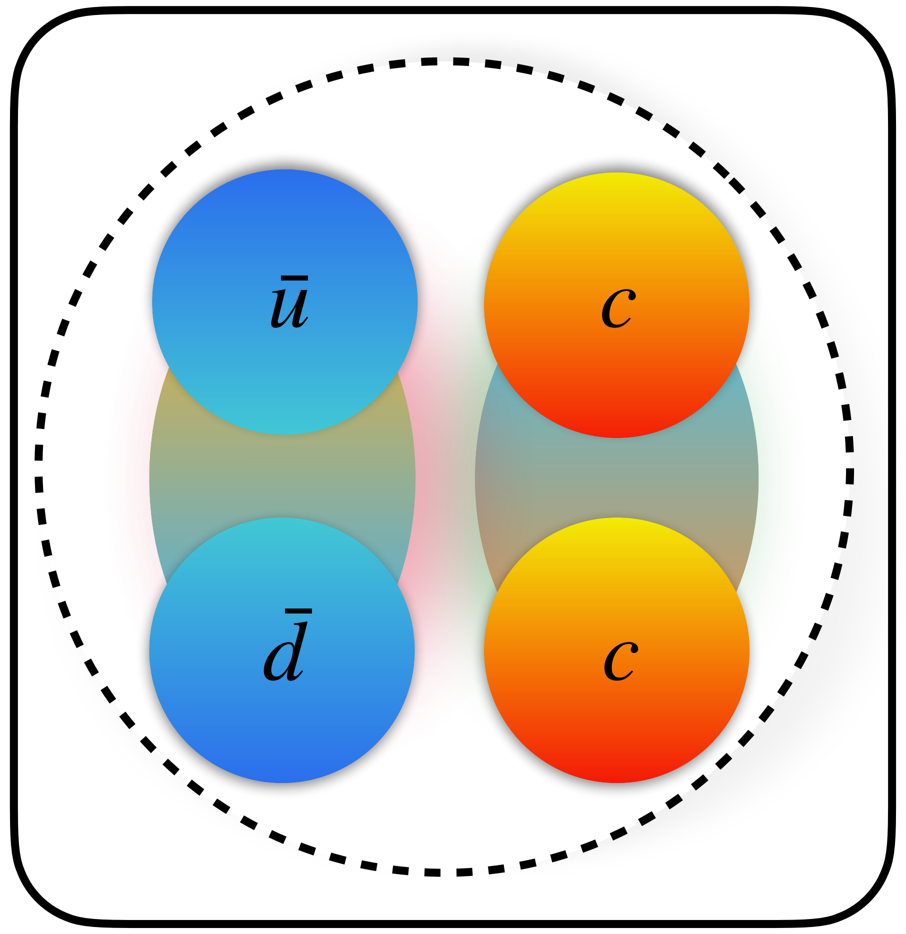
$$\mathcal{O}^{[cc][\bar{u}\bar{d}]}\left(\vec{p}\right) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\cdot\epsilon_{abc}\left[c_b^{\alpha}(x)(C\gamma_i)^{\alpha\beta}c_c^{\beta}(x)\right]\epsilon_{ade}\left[\bar{u}_d^{\gamma}(x)(C\gamma_5)^{\gamma\delta}\bar{d}_e^{\delta}(x)\right]$$



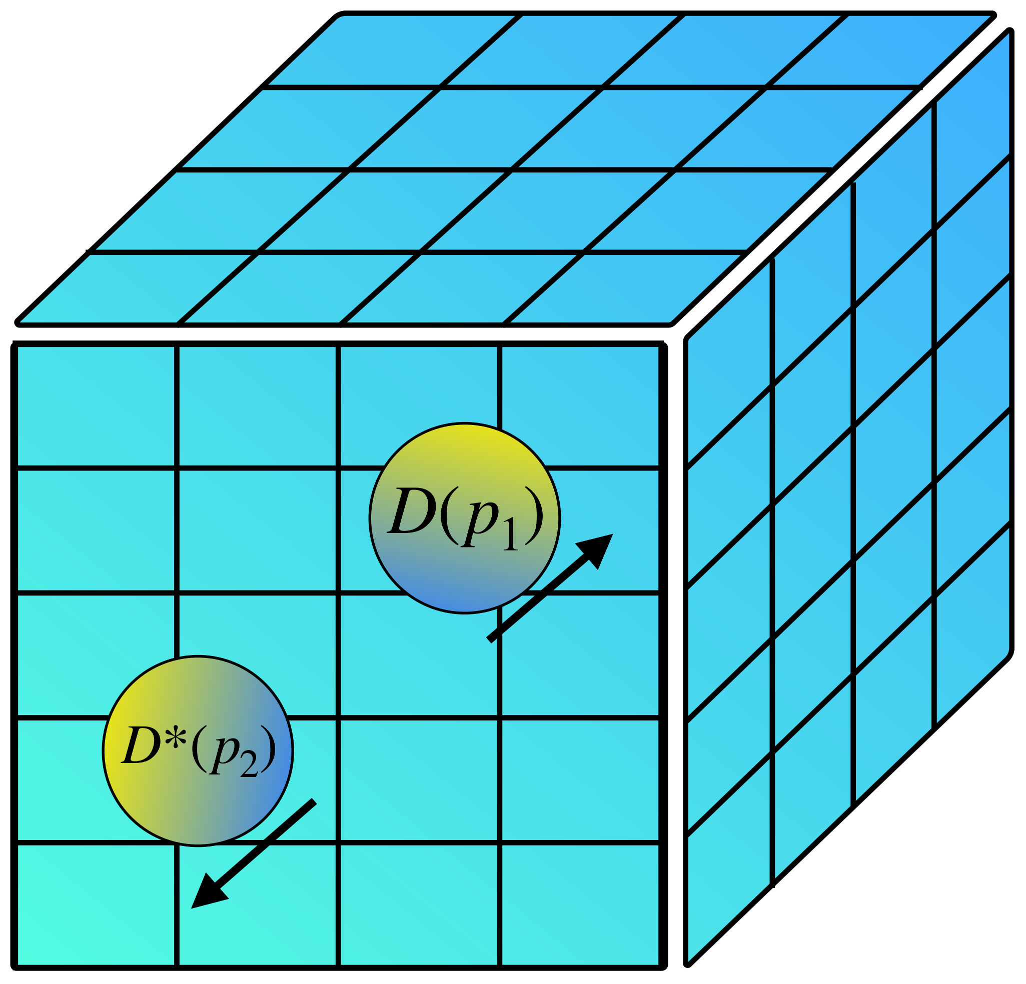
- **its effects are generally dependent on the heavy quark mass**
  - simulations were done for two heavy quark masses:

$$m_c^{lat} \sim m_c^{phys}, \quad m_b^{lat} \sim m_b^{phys}$$

in this talk I focus on this case



Finite-volume energy spectrum



$m_c^{lat} \sim m_c^{phys}$

T_1^+ operator basis
$\mathcal{O}_1, \mathcal{O}_2 \sim D(0)D^*(0)$
$\mathcal{O}_3 = [D(1)D^*(-1)]_{l=0}$
$\mathcal{O}_4 = [D(1)D^*(-1)]_{l=2}$
$\mathcal{O}_5 = D^{*0}(0)D^{*+}(0)$
$\mathcal{O}_6 = [cc]\,[\bar{u}\bar{d}]_{I=0}$

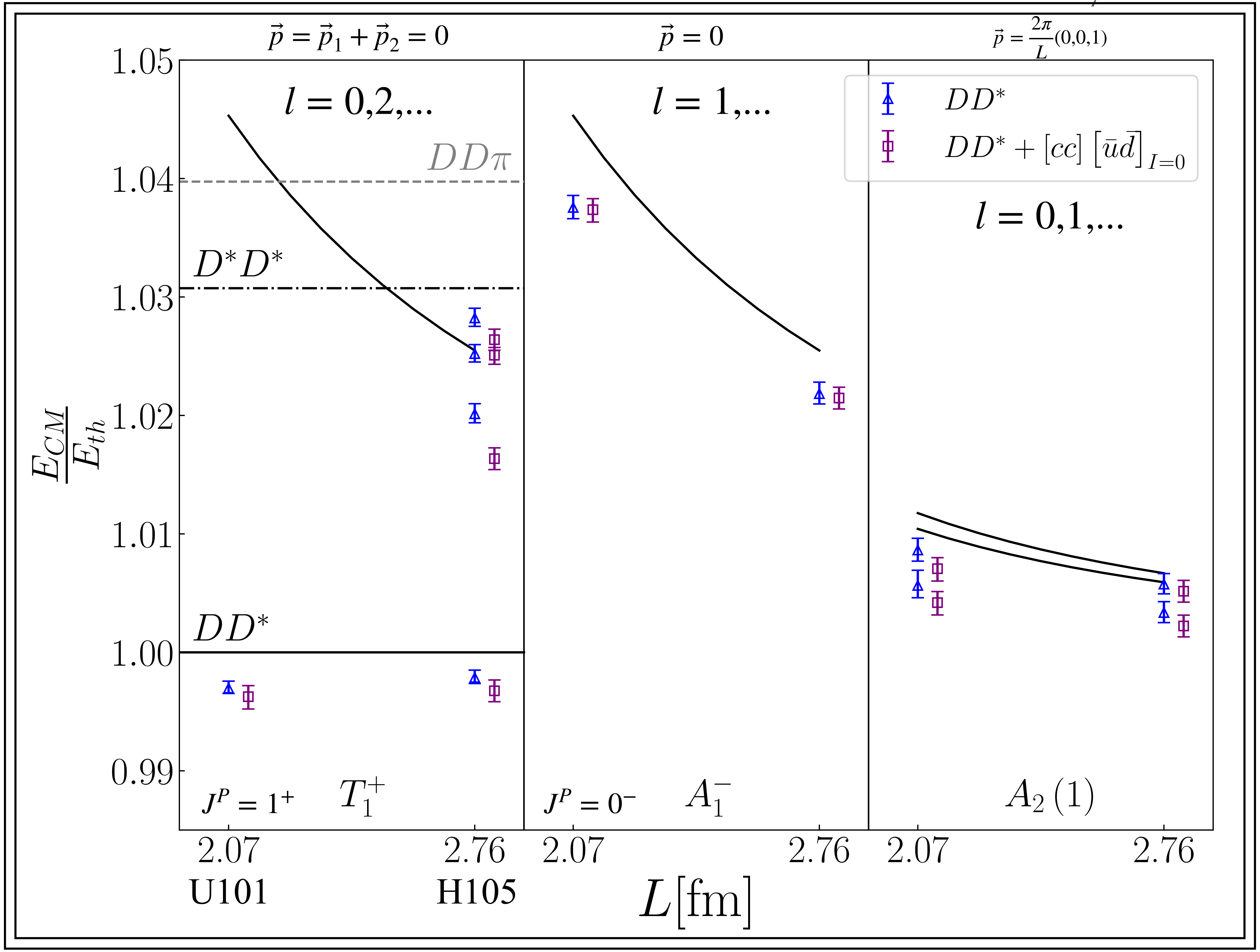
- two ensembles used, U101 and H105:

$N_L = 24,\ 32$

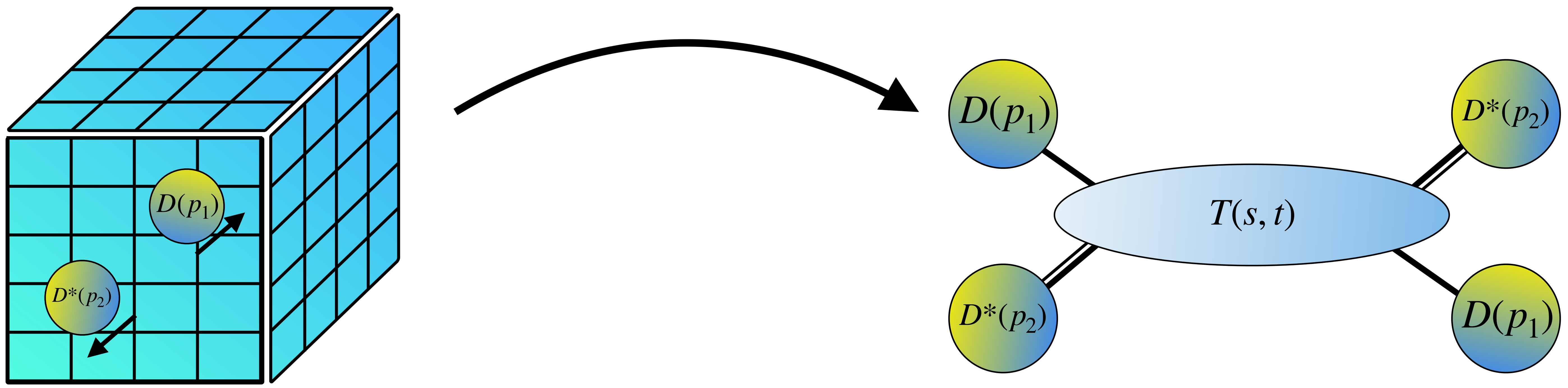
$m_\pi = 280(3)$ MeV

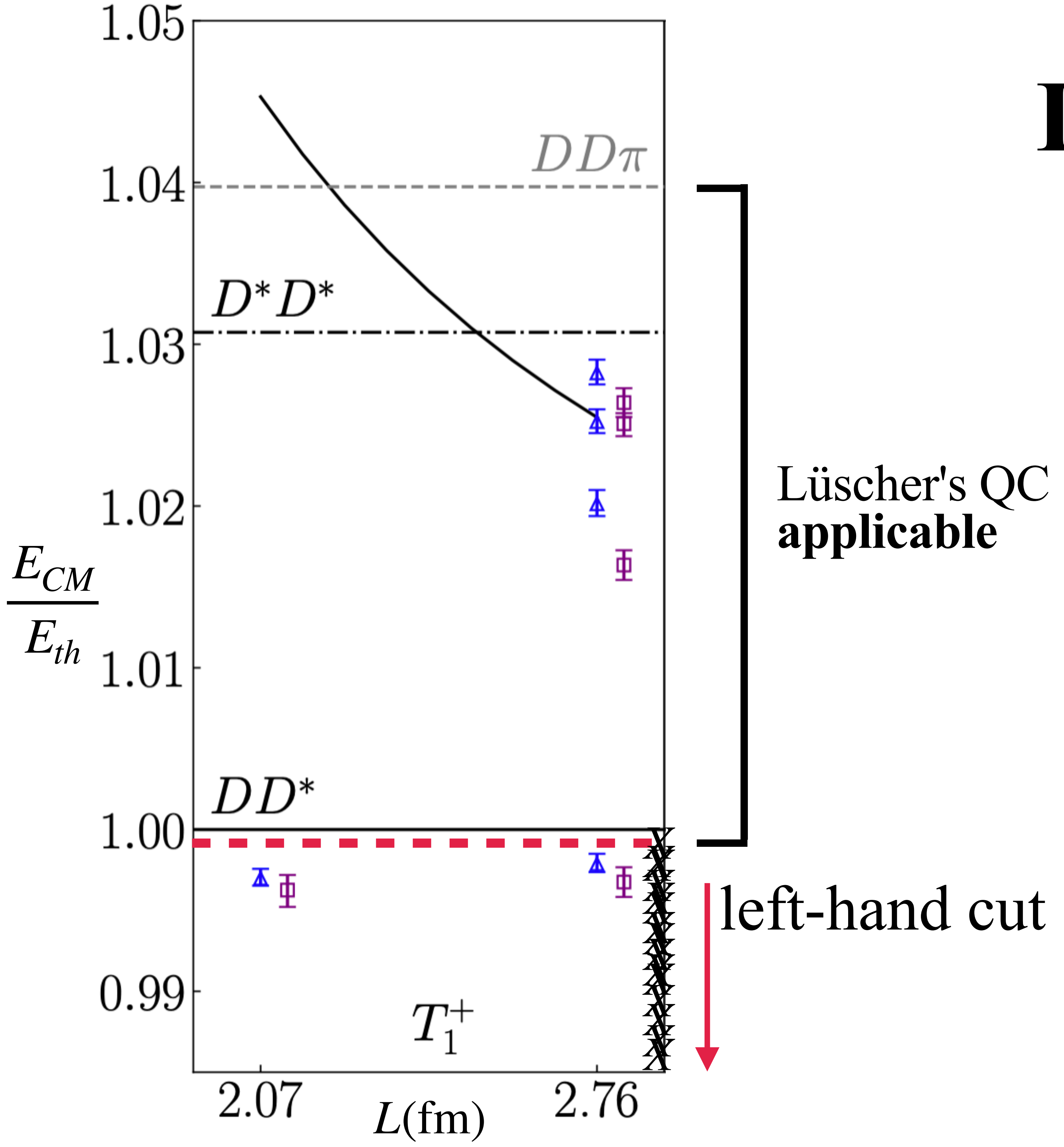
$a = 0.08636(98)(40)$ fm

Preliminary results



Search for the pole in DD^* scattering amplitude





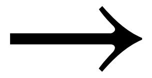
Lüscher's quantization condition

$$\det_{l,m}\left[F^{-1}(L,E) + T(E)\right] = 0$$

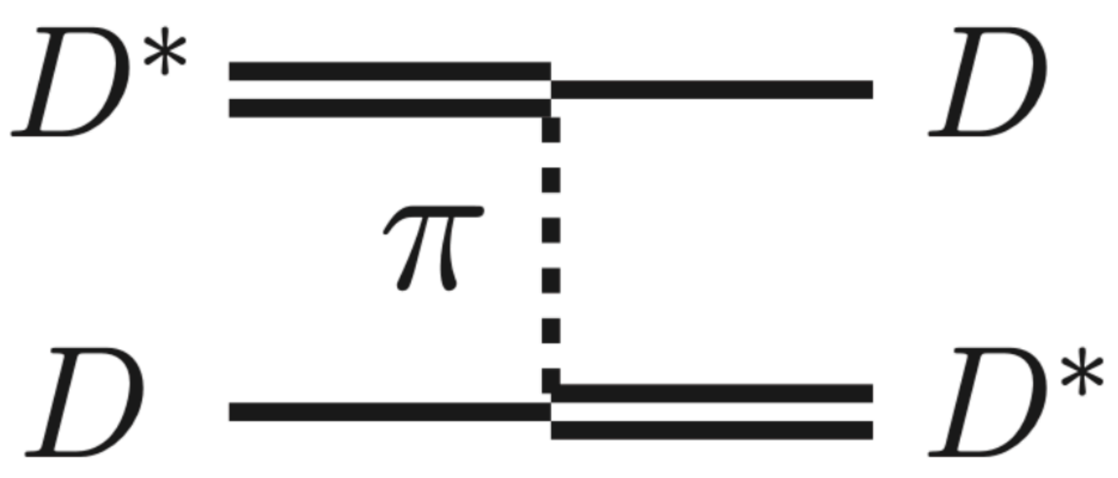
$F(L,E)$ - known kinematical function

- evaluated using **finite-volume energies** from LQCD

$T(E)$ - **continuum** scattering amplitude

 **The problem?**

- ❖ derivation assumes presence of no left/right hand cuts
- ❖ **one-pion exchange** in the u-channel
- ❖ appearance of left-hand cut very close to DD^* threshold



Search for the pole in DD^* scattering amplitude

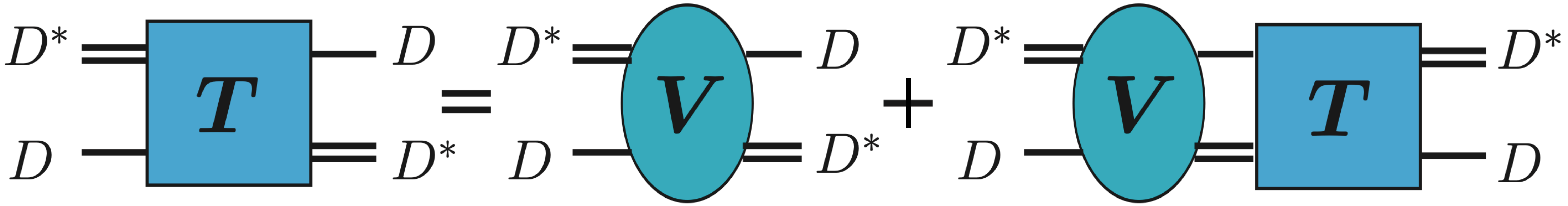
$\hat{T}$ - scattering matrix

$\hat{V}$ - potential

$\hat{G}$ - propagator

Lippmann-Schwinger equation

$$\hat{T} = \hat{V} + \hat{V}\hat{G}\hat{T}$$



in nonrelativistic regime:

$\hat{T} = \hat{G}^{-1}\left(\hat{G}^{-1} - \hat{V}\right)^{-1}\hat{V}$ ⟶ $\hat{T}$ **matrix poles**: $\det\left(\hat{G}^{-1} - \hat{V}\right) = 0$ ⟶ $\det\left(\hat{H} - E\hat{I}\right) = 0$

$$\hat{H} = \frac{\hat{p}^2}{2m_r} + \hat{V}$$

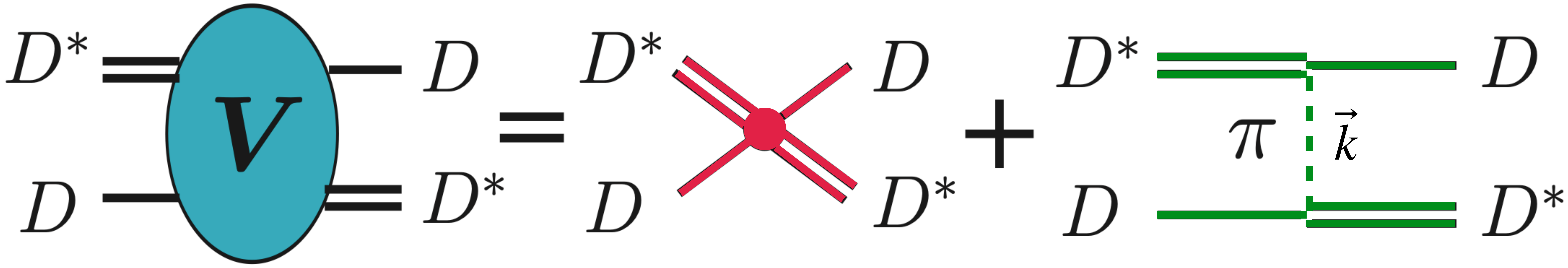
EFT: DD^* elastic scattering

- effective potential V derived from chiral EFT, up to $\mathcal{O}(p^2)$:

$$V\left(\vec{p},\vec{p}'\right) = \left(2c_{S0} + 2\left(\vec{p}^2 + \vec{p}'^2\right)c_{S2}\right)\left(\vec{\epsilon}\cdot\vec{\epsilon}'^*\right) + 2\left(\vec{p}'\cdot\vec{\epsilon}'^*\right)\left(\vec{p}\cdot\vec{\epsilon}\right)c_{P2} - \frac{3g^2}{4f_\pi^2}\frac{(\vec{k}\cdot\vec{\epsilon})(\vec{k}\cdot\vec{\epsilon}'^*)}{\vec{k}^2+\mu_\pi^2}$$

$\mu_\pi^2 = m_\pi^2 - (m_{D*} - m_D)^2 \longrightarrow$ effective pion mass

$\vec{k} = \vec{p} + \vec{p}'$



- low energy constants c_{S0}, c_{S2}, c_{P2} are treated as fit parameters

Fitting low energy constants: c_{S0}, c_{S2}, c_{P2}

in nonrelativistic regime: **projected to various lattice irreps** $\Lambda = T_1^+, A_1^-, A_2(1)$**:**

$$\det\left(\hat{H} - E\hat{I}\right) = 0 \longrightarrow \det\left(\hat{H}_\Lambda - E_\Lambda \hat{I}\right) = 0$$

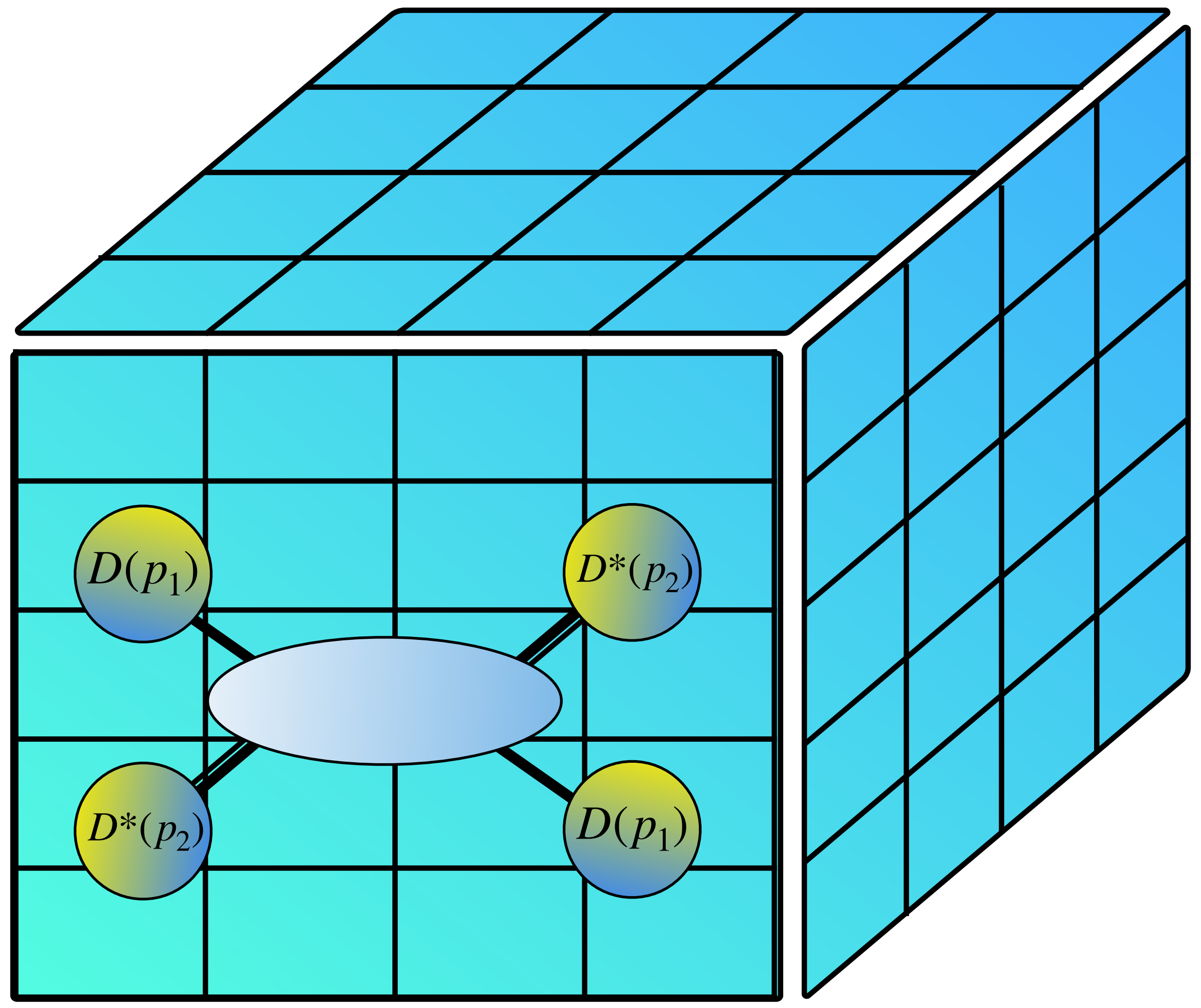
$$\hat{H}_\Lambda - E_\Lambda \hat{I} = \hat{U}_\Lambda \left[\hat{H} - E\hat{I}\right] \hat{U}_\Lambda^\dagger$$

$\hat{H} = \dfrac{\hat{p}^2}{2m_r} + \hat{V} \rightarrow$ well defined in **plane wave basis:**

$$|\vec{p}_1\rangle \otimes |\vec{p}_2, i\rangle$$

discretization:

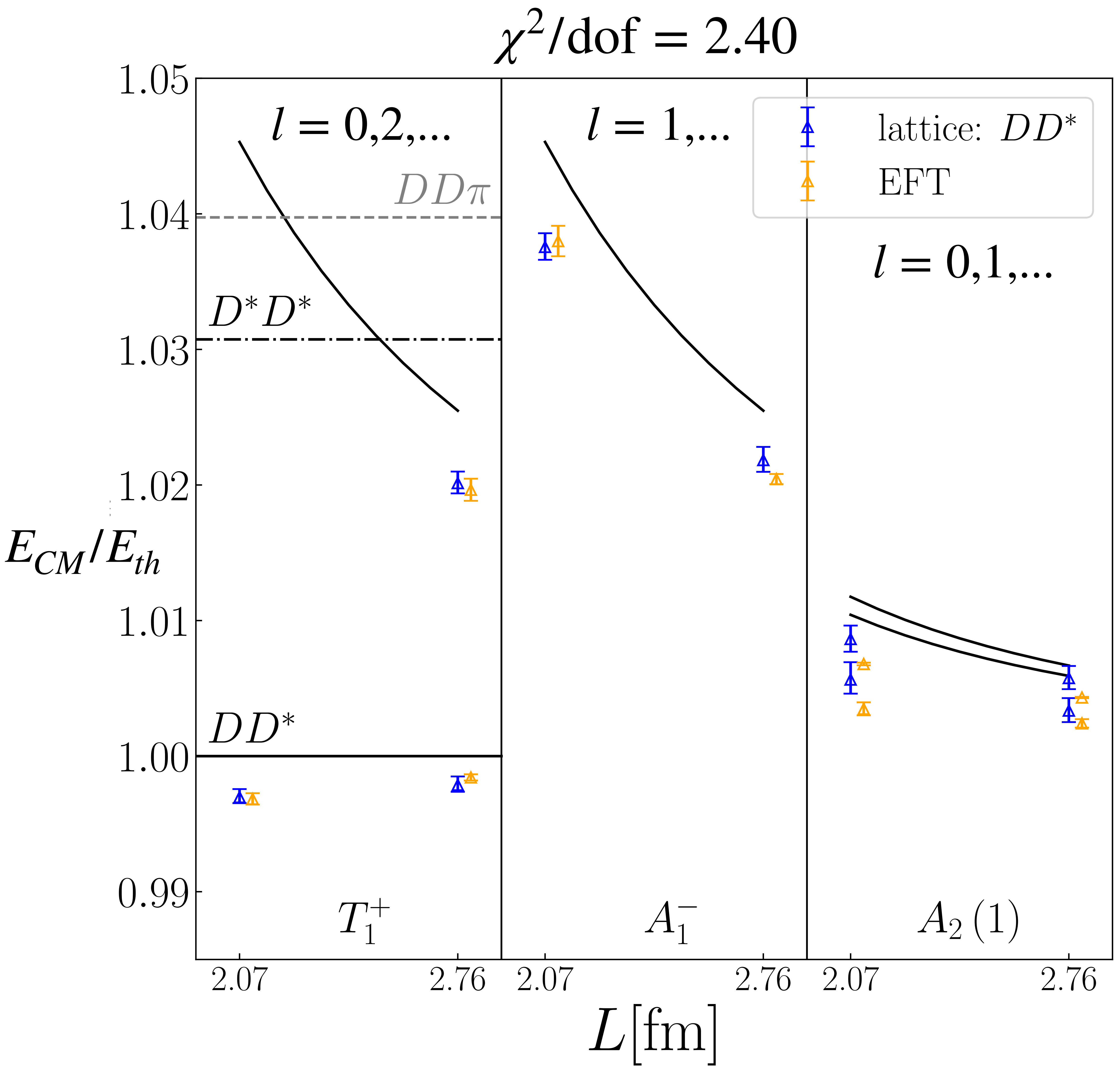
$$\vec{p} = \frac{2\pi}{L}\vec{n},\ \vec{n} \in Z^3$$
$$i = x, y, z$$

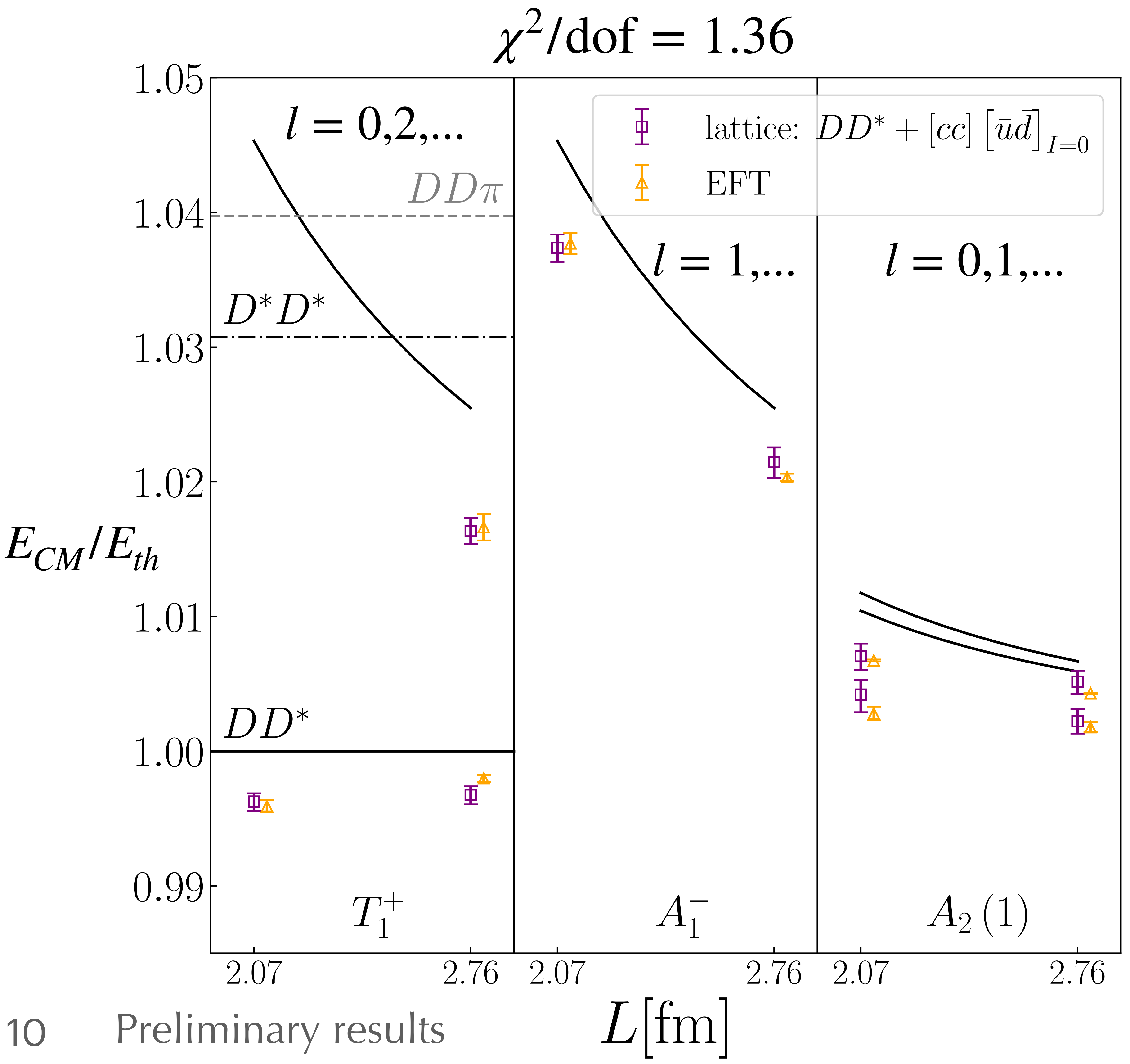


with additional constraints: $\vec{p}_1 + \vec{p}_2 = \vec{0}, \dfrac{2\pi}{L}(0,0,1)$

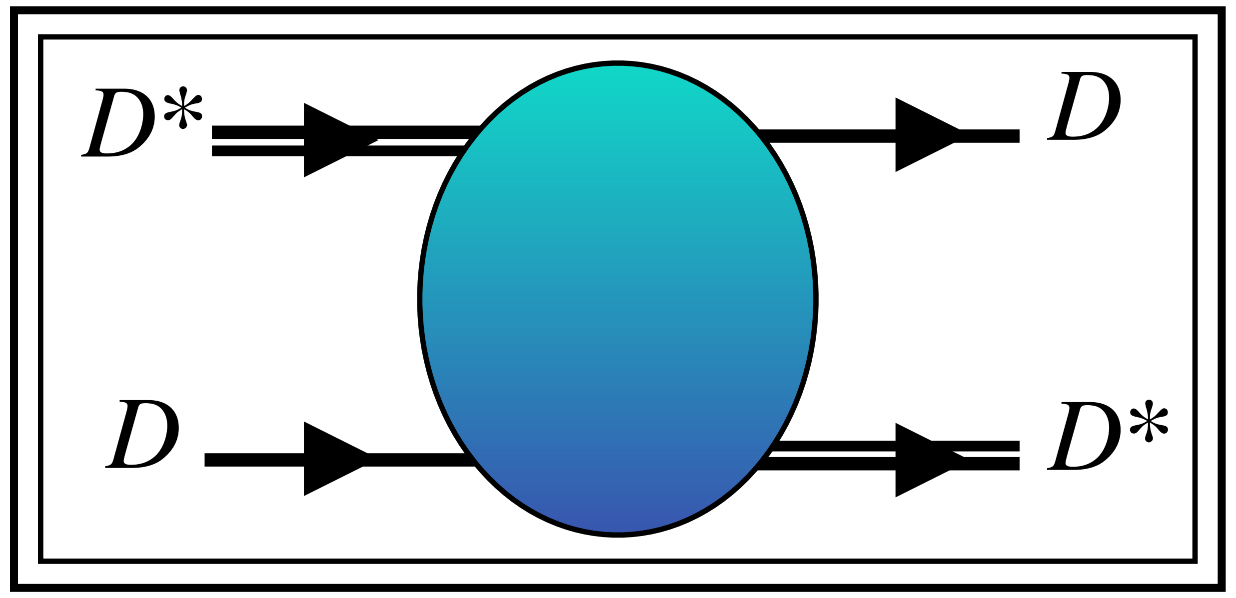
T_1^+, A_1^- $\quad$ $A_2(1)$

Fitting low energy constants: c_{S0}, c_{S2}, c_{P2}

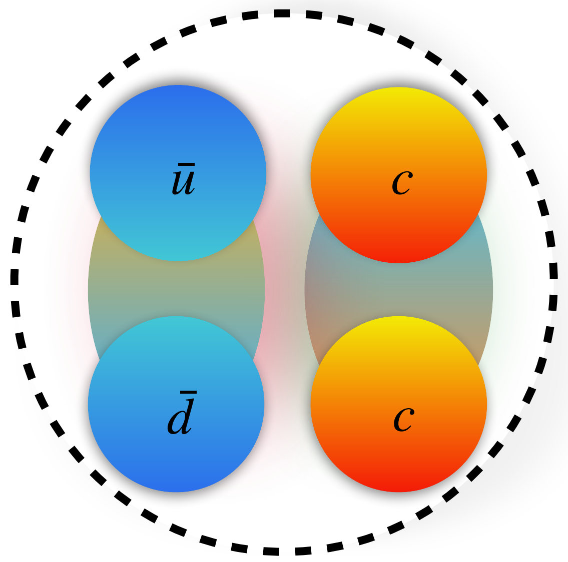




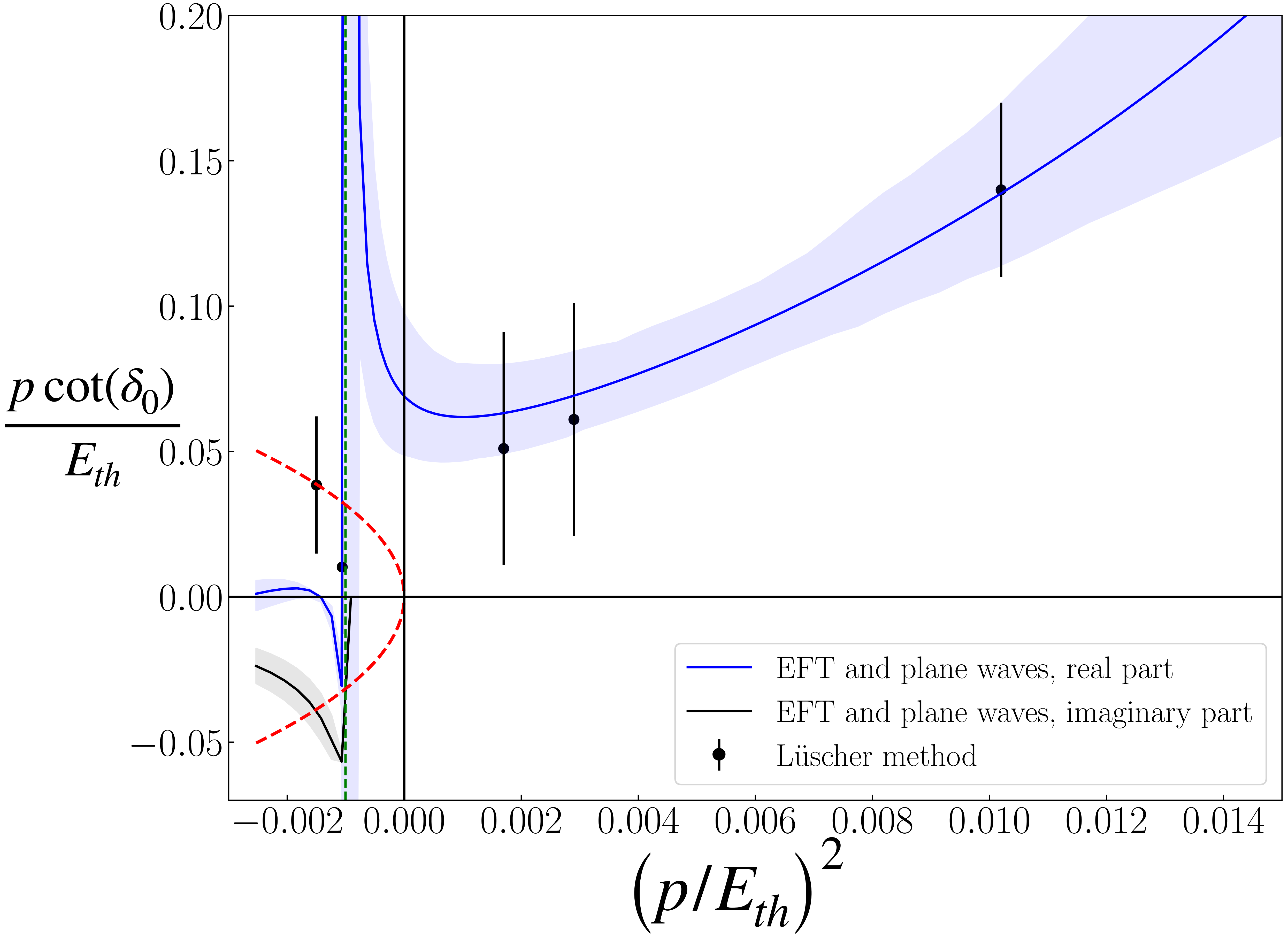
Preliminary results



Plane waves and Lüscher:
a comparison



without $[cc]\,[\bar{u}\bar{d}]_{I=0}$



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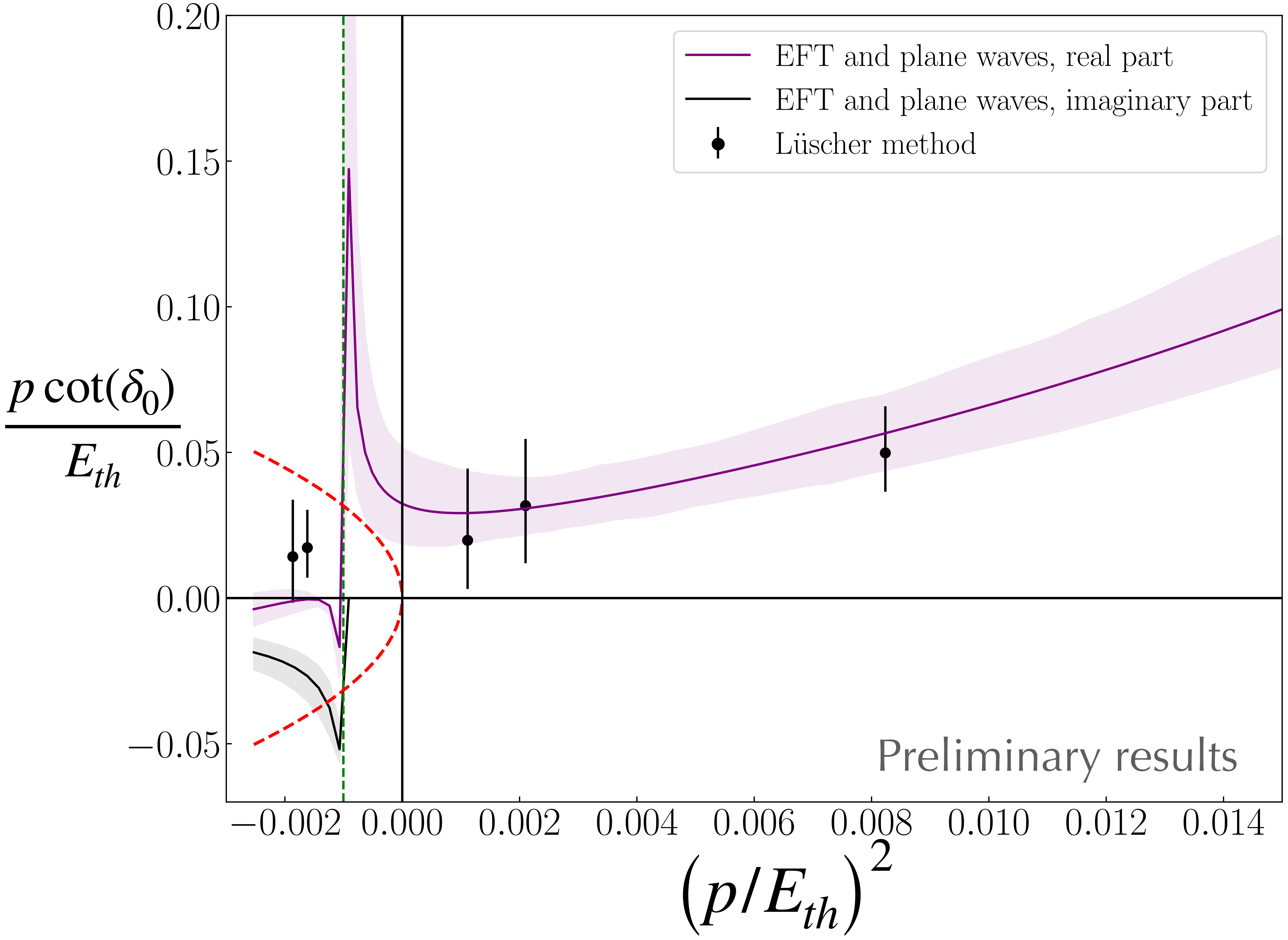
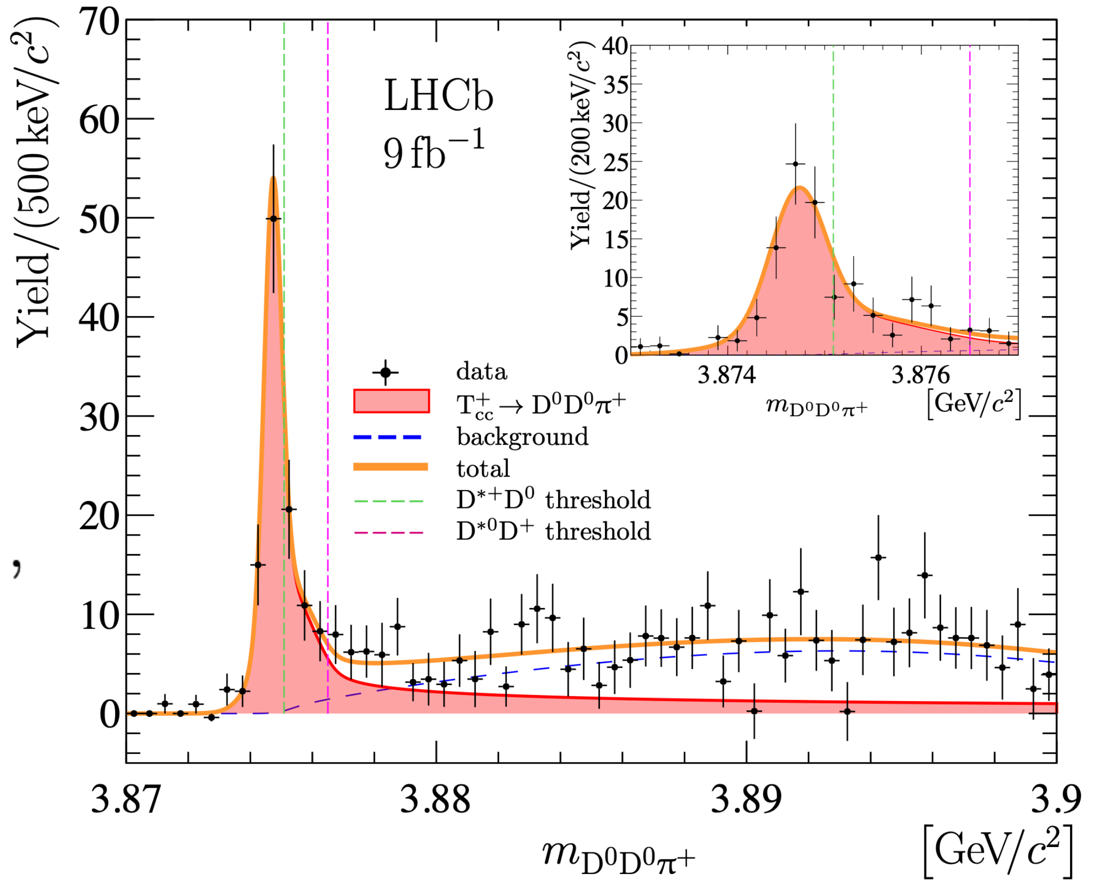


T_{cc} pole

LHCb:

$$\delta m_{\text{pole}} = m_{T_{cc}^+} - \left(m_{D^{*+}} + m_{D^0}\right),$$

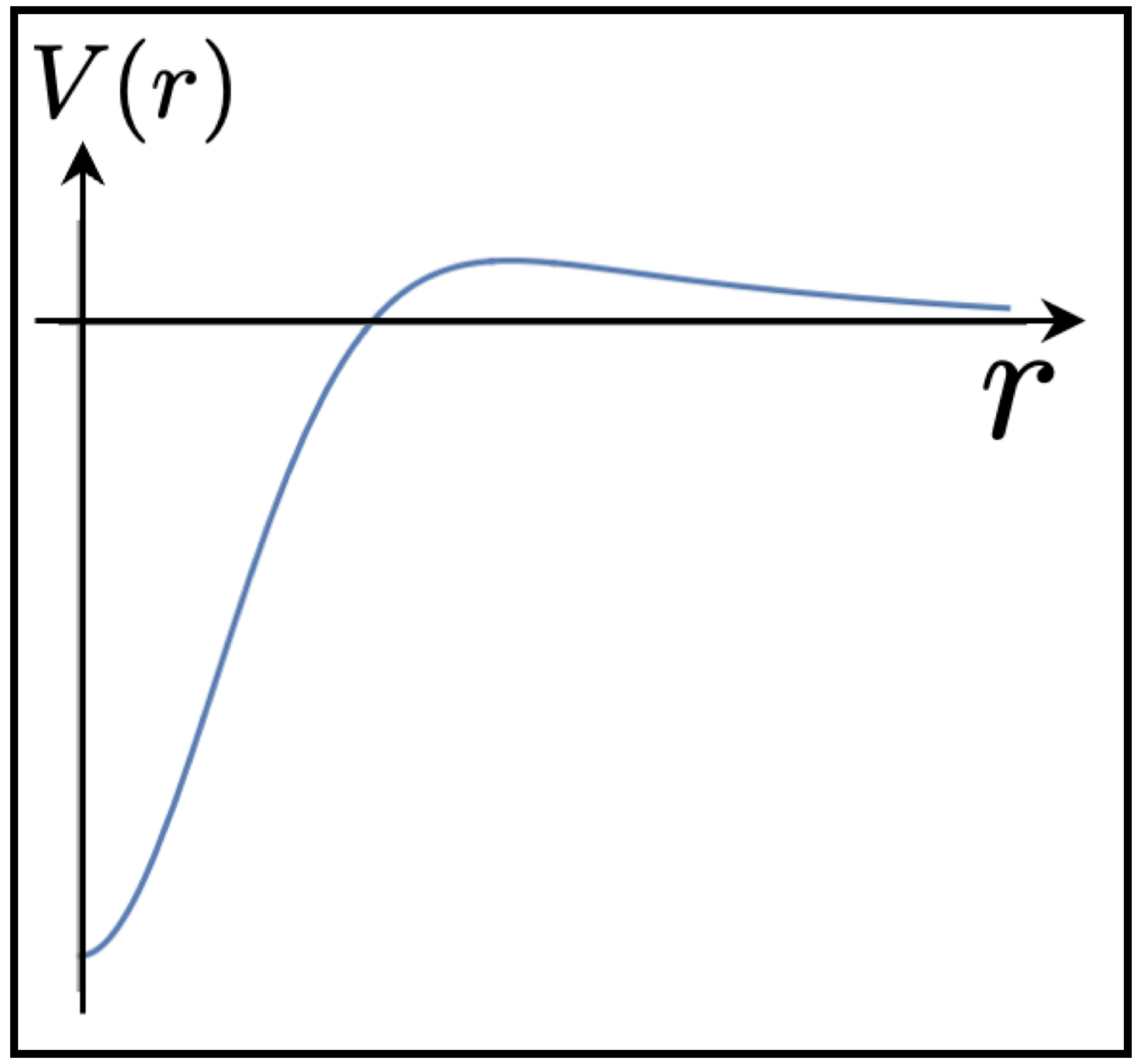
$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0} \text{ keV}/c^2,$$
$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14} \text{ keV}$$



- **subthreshold resonance** in both cases

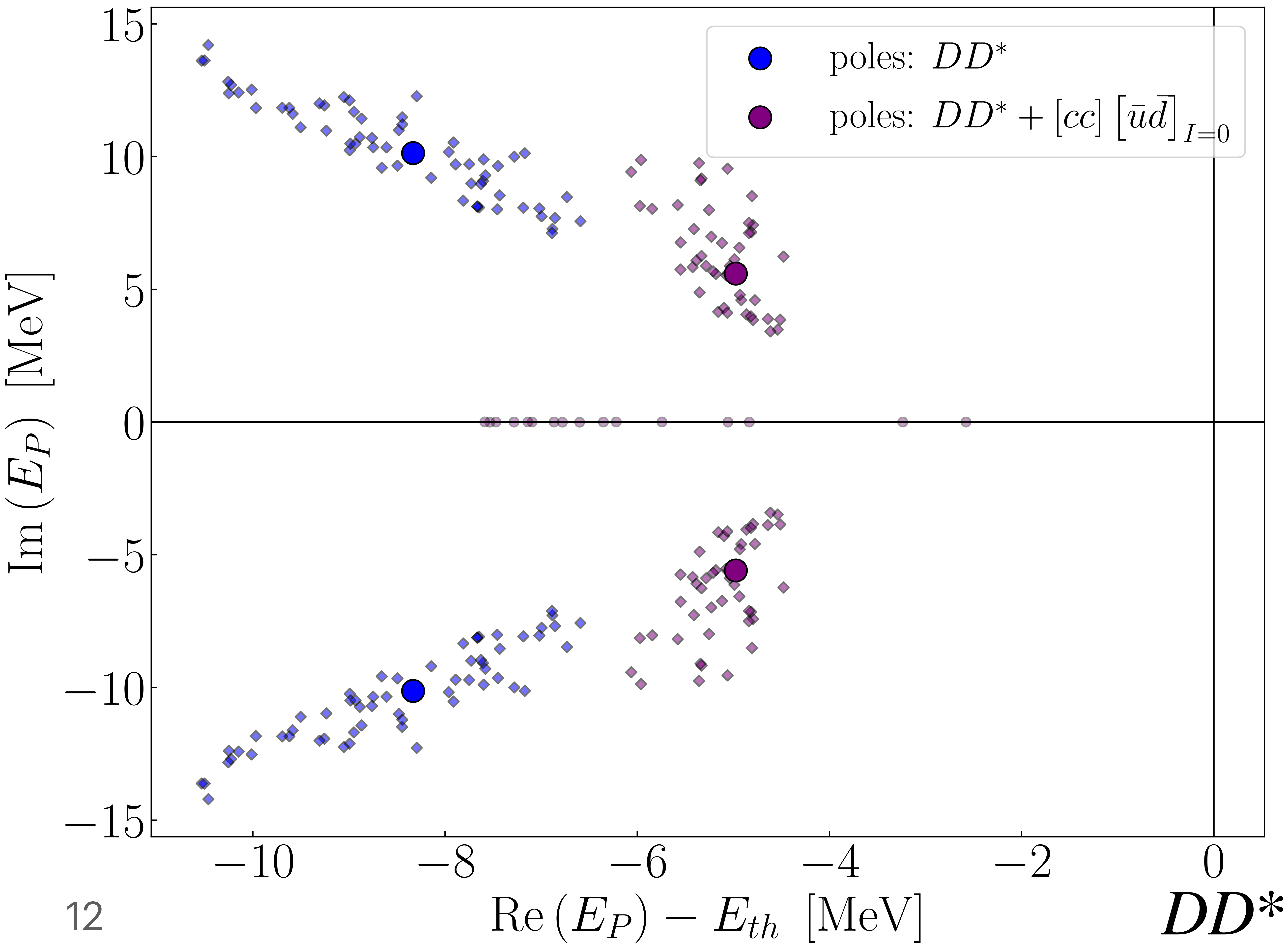
$$\text{Re}\left(E_P\right) - E_{th} = -8.33^{+1.79}_{-2.20} \text{ MeV}$$
$$\text{Im}\left(E_P\right) = -10.13^{+3.03}_{-4.07} \text{ MeV}$$

$$\text{Re}\left(E_P\right) - E_{th} = -4.99^{+0.56}_{-0.95} \text{ MeV}$$
$$\text{Im}\left(E_P\right) = -5.60^{+1.97}_{-3.55} \text{ MeV}$$



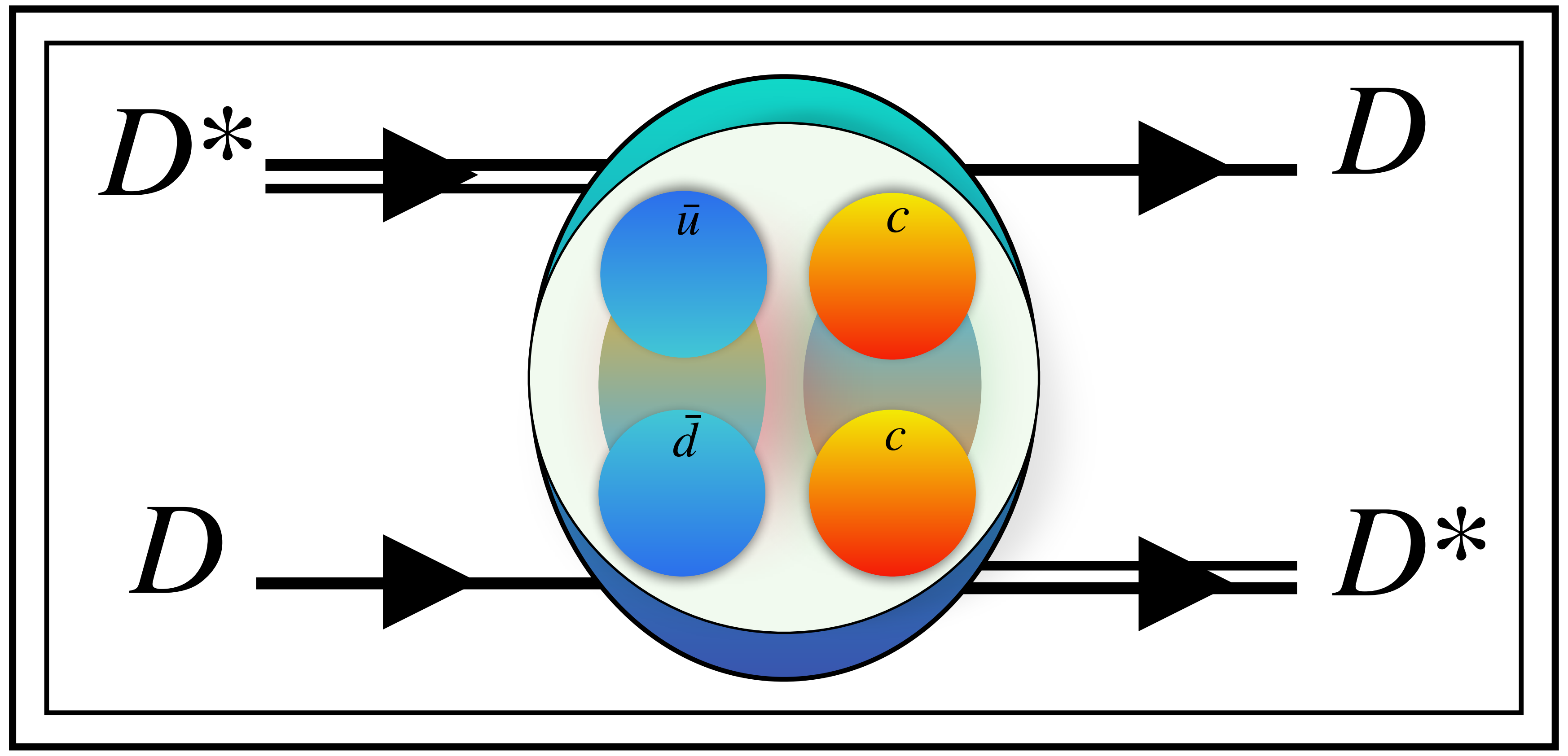
- Contact terms → short range attraction
- One-pion exchange → long range repulsion

$$m_\pi = 280 \text{ MeV}$$



DD*

Summary



- study of the T_{cc} pole:
  - addition of local diquark-antidiquark interpolators: **energy shifts, overlap factors**
  - explicit accounting for the left-hand cut: **effective potential** featuring OPE
  - in both cases: **subthreshold resonance**
    - shift towards the real line with the addition of $[cc]\left[\bar{u}\bar{d}\right]_{I=0}$, indicating stronger binding

backup slides
2pt correlator for large times

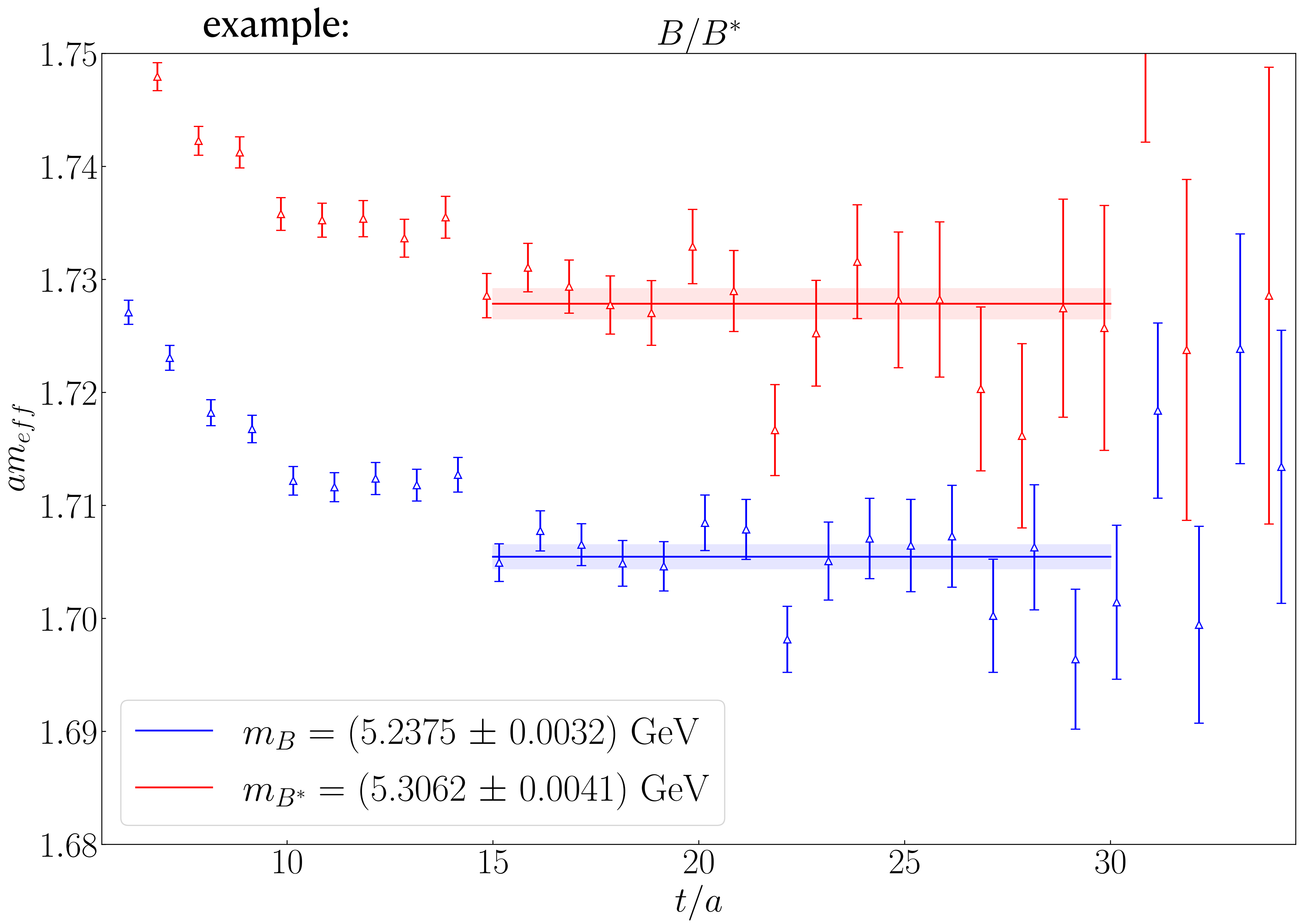
in the limit $t \to \infty$:

$$C_2(t) = \sum_n Z_n^* Z_n e^{-E_n(t-t_0)} \to |Z_0|^2 e^{-E_0(t-t_0)}$$

$$\downarrow$$

$$am_{eff}(t) = \ln\left(\frac{C_2(t)}{C_2(t+1)}\right)$$

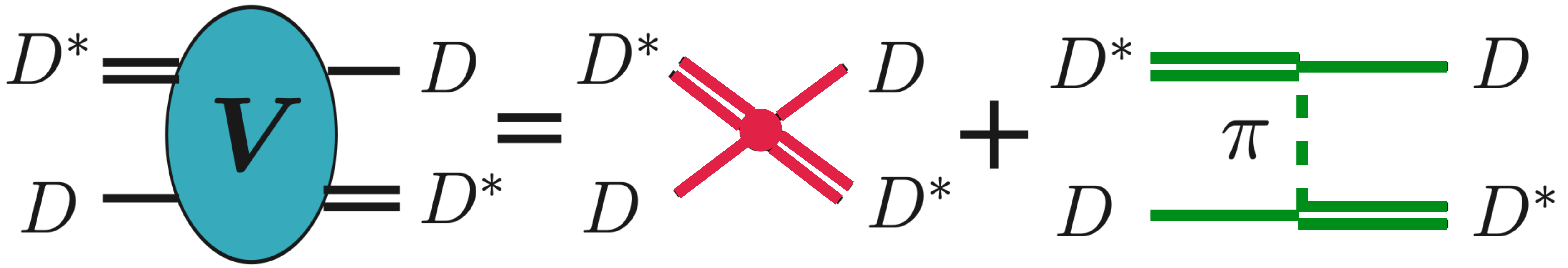
$$\downarrow$$



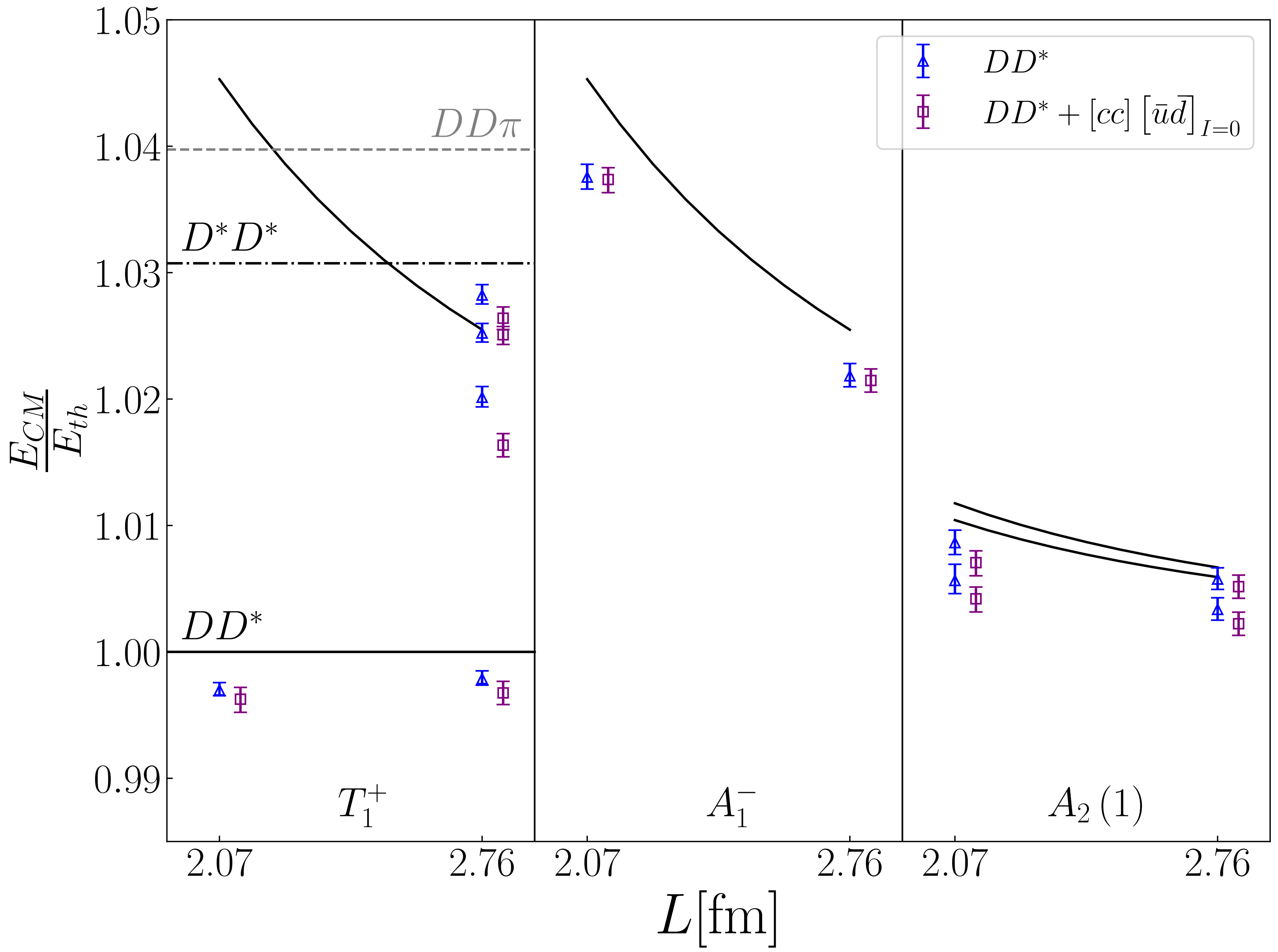
measure of excited state contamination → perform fits where statistical noise is small + am_{eff} sinks to its plateau

Cutoff function in $\hat{V}$

$$V\left(\vec{p},\vec{p}'\right) = \left(2c_{S0} + 2\left(\vec{p}^2 + \vec{p}'^2\right)c_{S2}\right)\left(\vec{\epsilon}\cdot\vec{\epsilon}'^{\,*}\right) + 2\left(\vec{p}'\cdot\vec{\epsilon}'^{\,*}\right)\left(\vec{p}\cdot\vec{\epsilon}\right)c_{P2} - \frac{3g^2}{4f_\pi^2}\frac{(\vec{k}\cdot\vec{\epsilon})(\vec{k}\cdot\vec{\epsilon}'^*)}{\vec{k}^2+\mu_\pi^2}$$



- two limitations on the validity of EFT potential:
  1. relativistic effects
  2. presence of higher thresholds (D^*D^*, $DD\pi$, …)



- we employ cutoff: $\tilde{V}(\vec{p},\vec{p}') = V(\vec{p},\vec{p}')\cdot\exp\left(-\frac{p^n + p'^n}{\Lambda^n}\right)$
  - sharp cutoff: $\Lambda = 0.65$ GeV, $n = 40$
    - → $(\Lambda + E_{th})/E_{th} \approx 1.07$

H105, T_1^+

$$m_c^{lat} \sim m_c^{phys}$$

$\mathcal{O}_1, \mathcal{O}_2 \sim D(0)D^*(0)$
$\mathcal{O}_3 = [D(1)D^*(-1)]_{I=0}$
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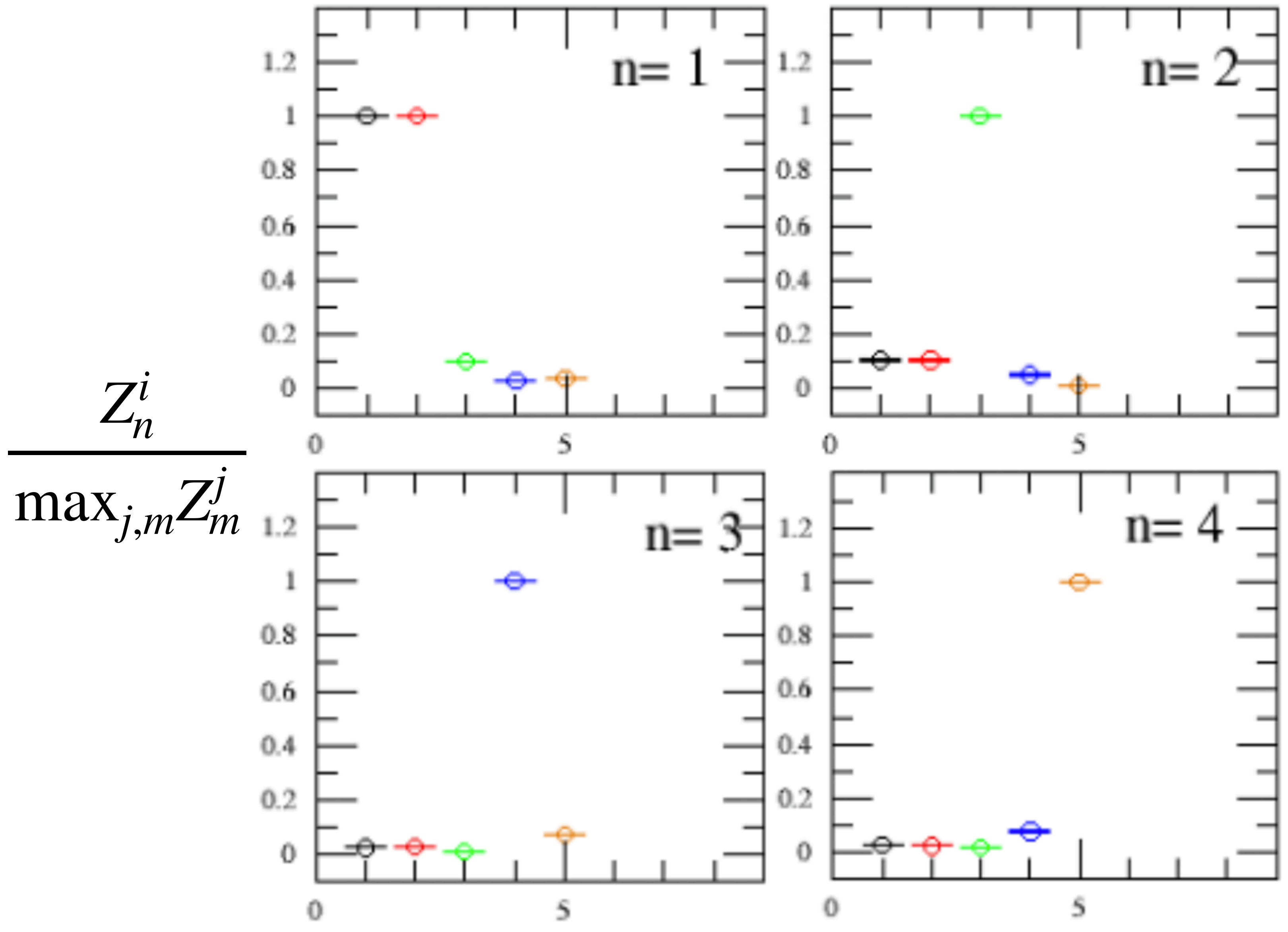
$\mathcal{O}_6 = [cc]\,[\bar{u}\bar{d}]_{I=0}$

without $[cc]\,[\bar{u}\bar{d}]_{I=0}$



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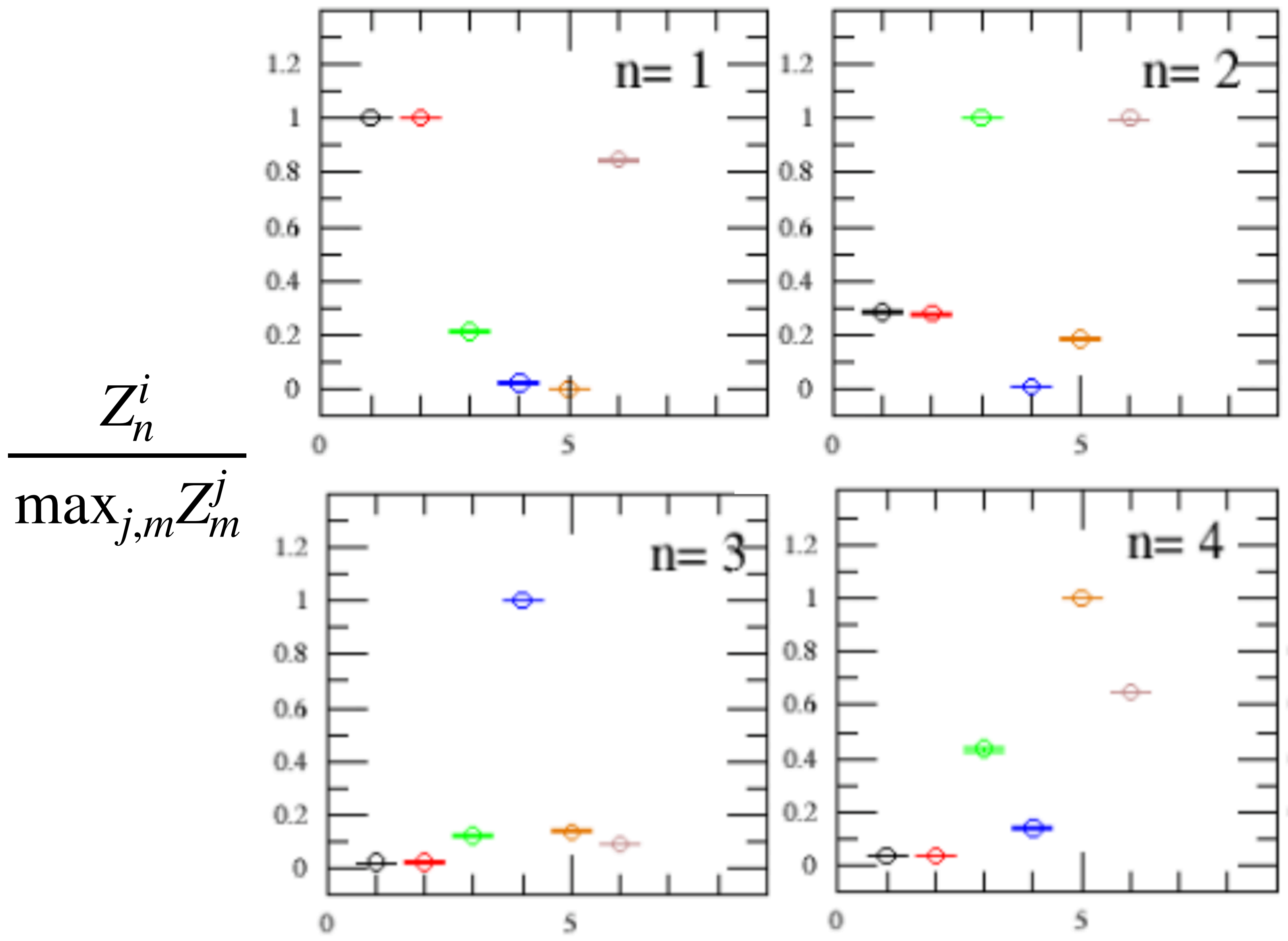

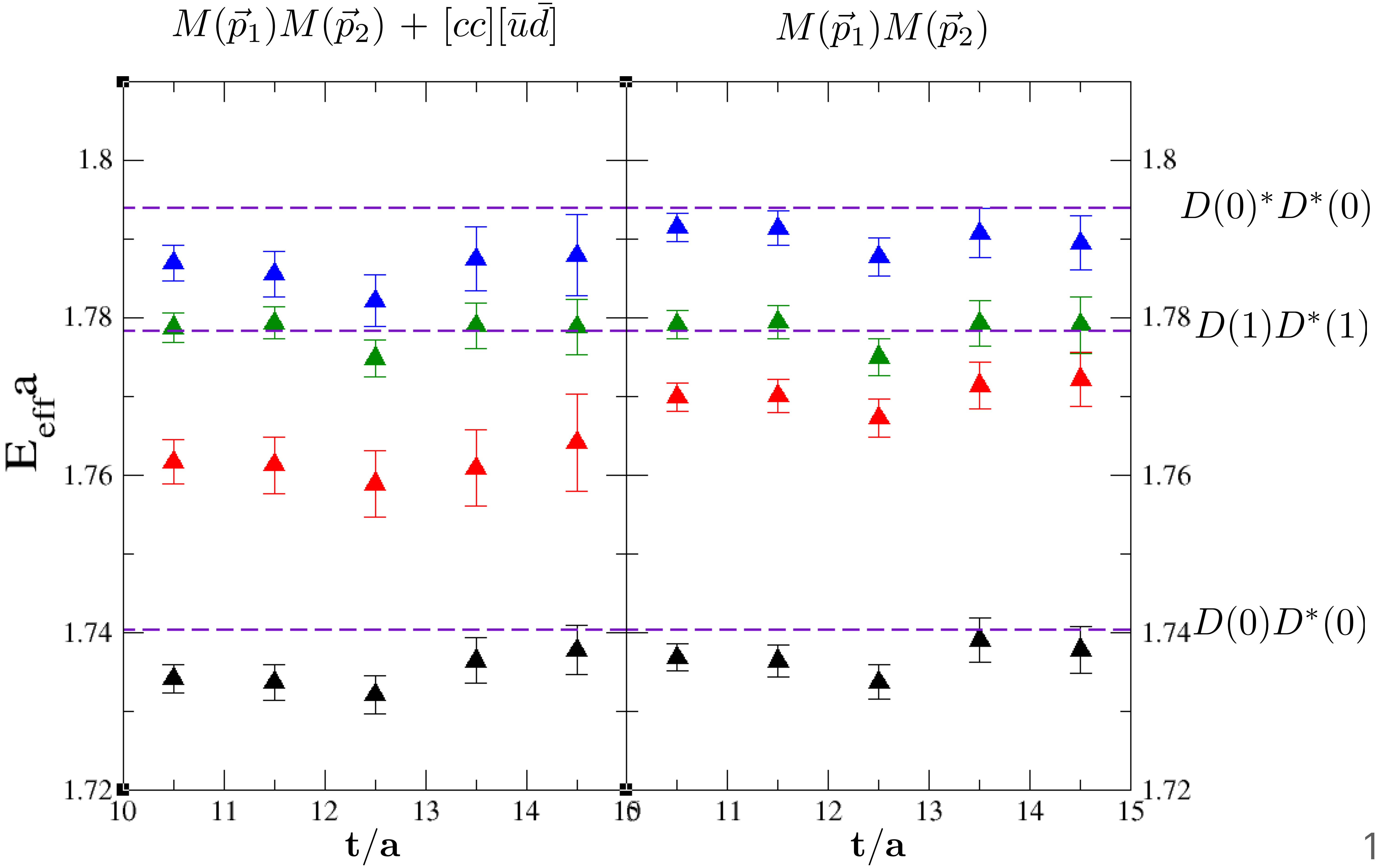

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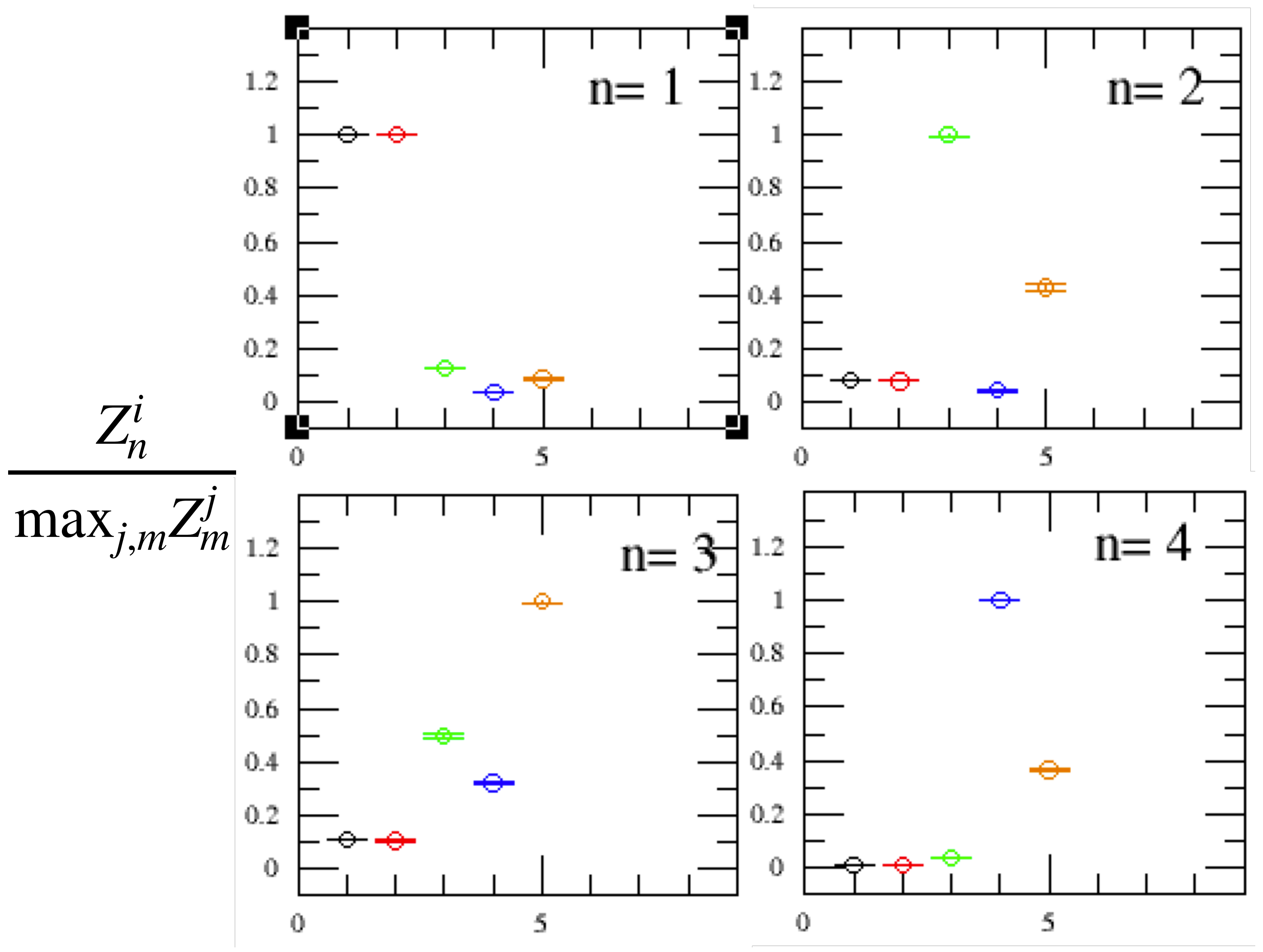

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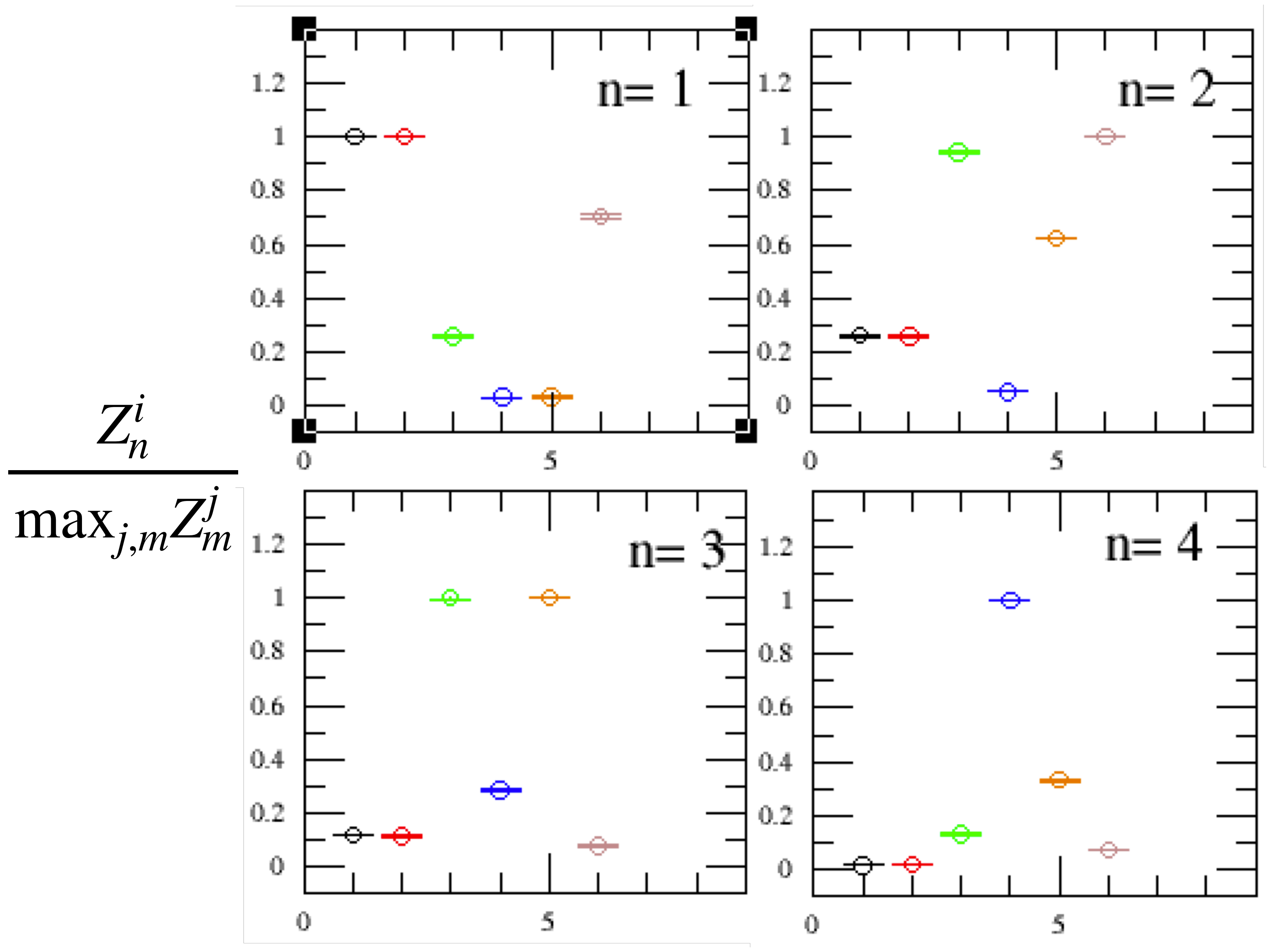

U101, T_1^+

$$m_c^{lat} \sim m_c^{phys}$$

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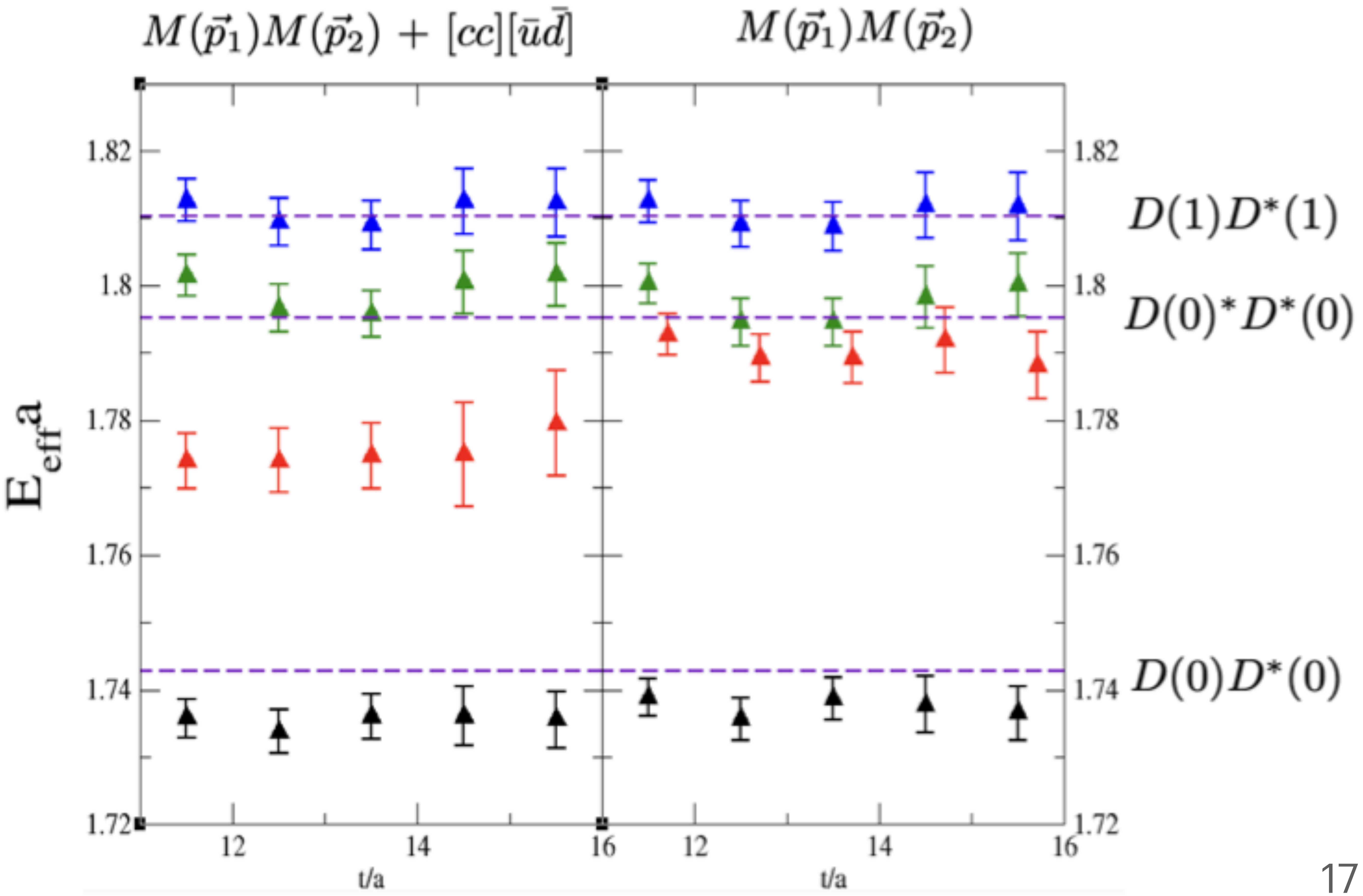

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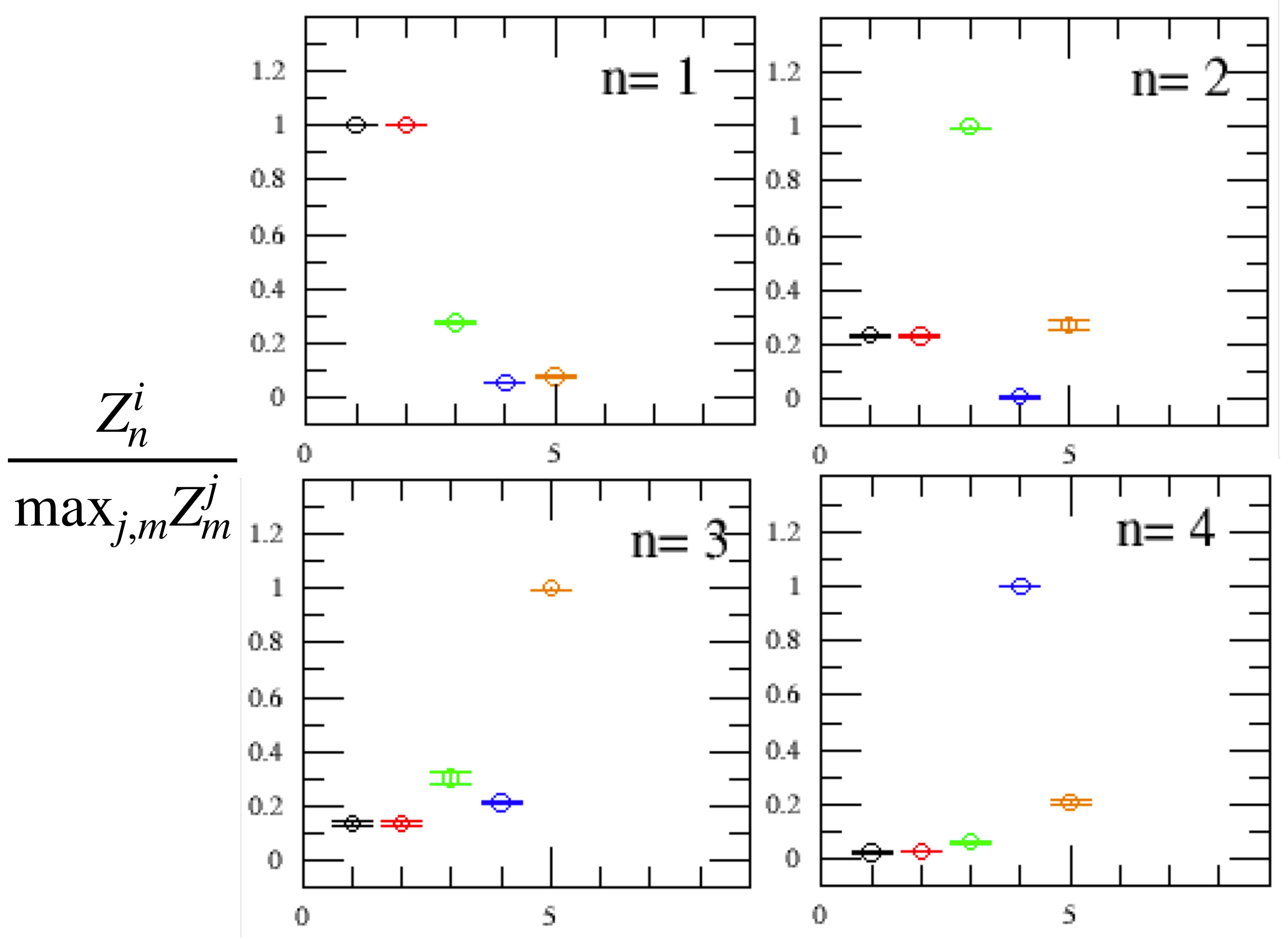

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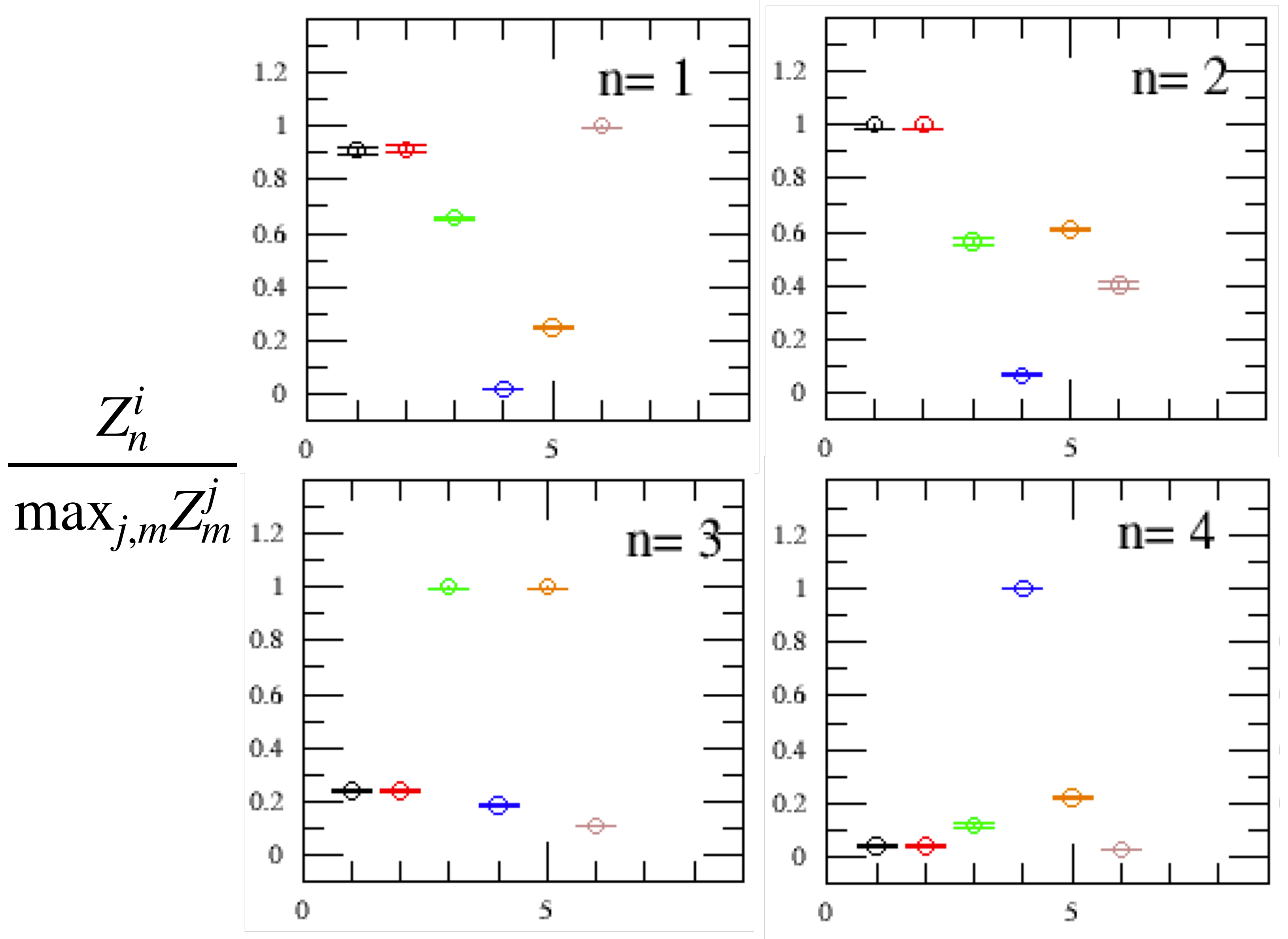

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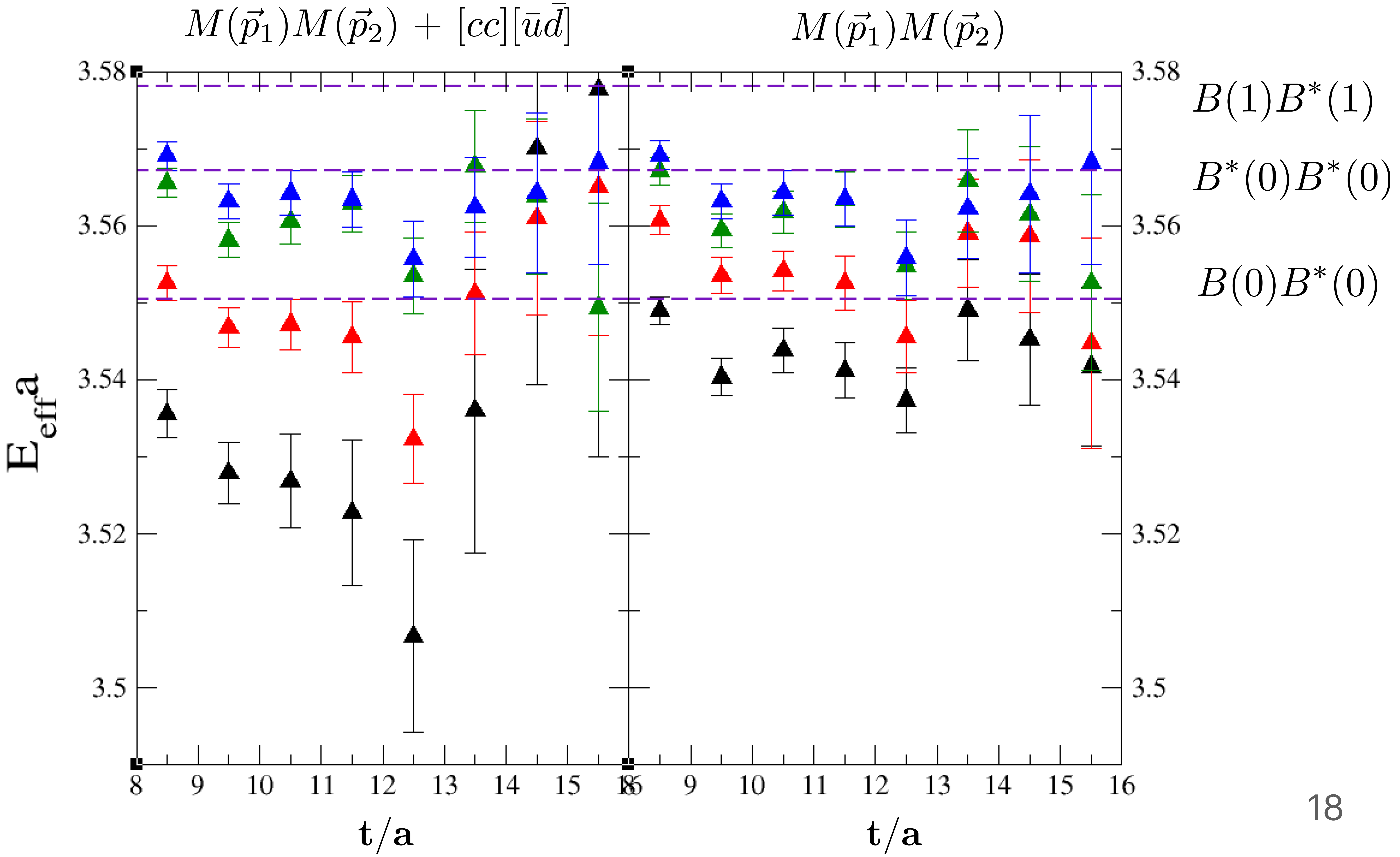

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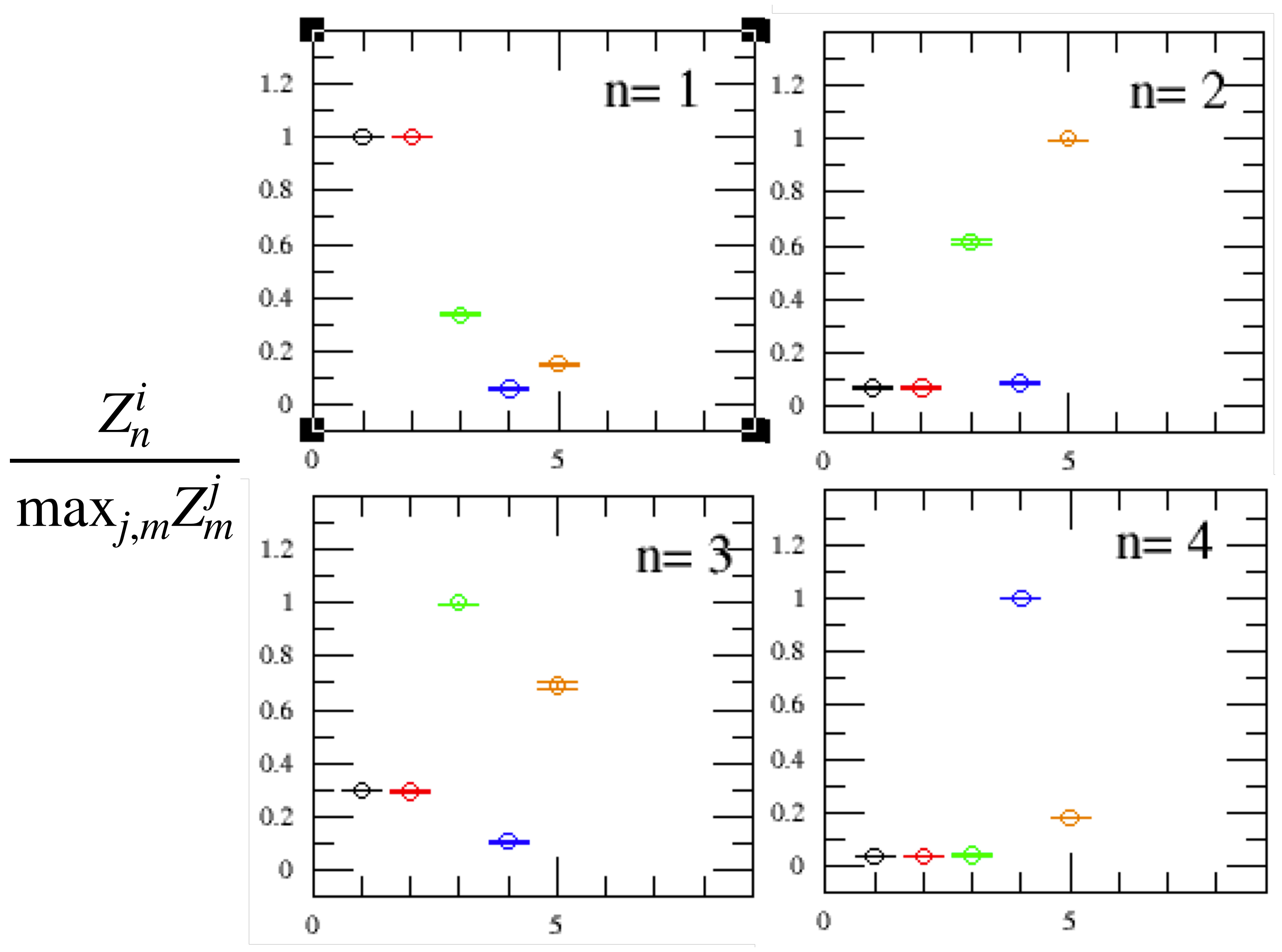

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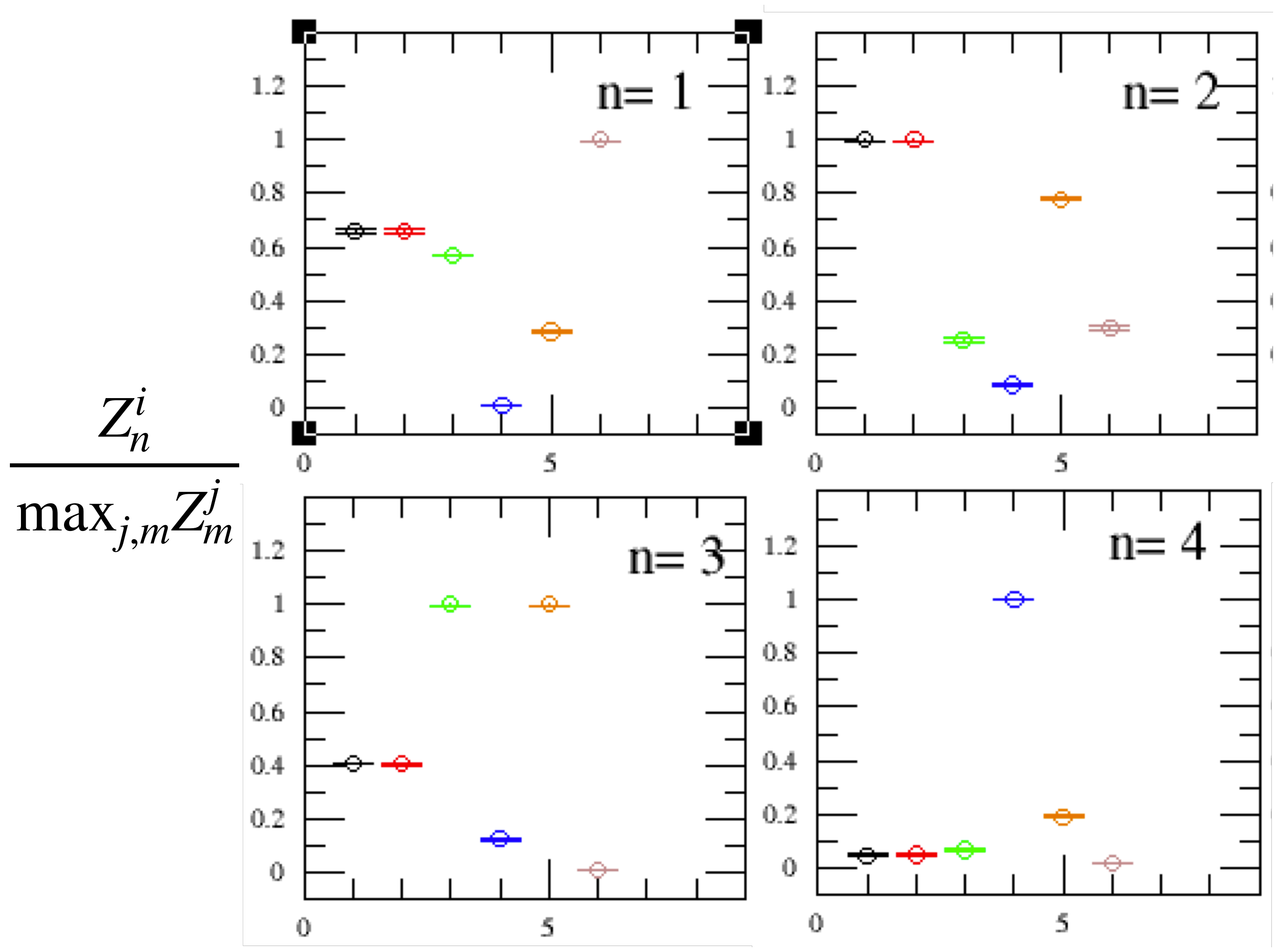

U101, T_1^+

$$m_b^{lat} \sim m_b^{phys}$$

$\mathcal{O}_1, \mathcal{O}_2 \sim B(0)B^*(0)$
$\mathcal{O}_3 = \left[B(1)B^*(-1)\right]_{l=0}$
$\mathcal{O}_4 = \left[B(1)B^*(-1)\right]_{l=2}$
$\mathcal{O}_5 = B^{*0}(0)B^{*+}(0)$
$\mathcal{O}_6 = [bb]\,[\bar{u}\bar{d}]_{I=0}$