SOLVING THE INVERSE PROBLEM FOR HADRONIZATION

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based on 2203.04983, 2308.13459, 2311.09296, 2410.nnnnn,; in collaboration with Christian Bierlich, Phil Ilten, Tony Menzo, Steve Mrenna, Manuel Szewc, Michael K. Wilkinson, Ahmed Youssef

Belica, Oct 3 2024

MONTE CARLO HEP EVENT

- block structure of HEP Monte Carlo
 - hard process
 - shower/evolution
 - hadronization
 - (detector simulation)

under good perturbative control and systematically improvable

modeling of nonperturbative physics



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MONTE CARLO HEP EVENT

• block structure of HEP Monte Carlo



OUTLINE

- should one care about hadronization?
- hadronization models
- ML hadronization (Mlhad/HadML)
- results of immediate relevance to Pythia

- if observables/measurements inclusive enough no need for modeling hadronization
 - for the past ~50 years observables have been constructed explicitly to remove any depend. on hadronization
- not the situation in the real world
 - experimental cuts, detectors not perfect, resonances decay in different ways
 - modeling well hadronization step essential for precision studies
- some measurements more sensitive than others
 - e.g., number of charged particles, correlations between exclusive states, etc.

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• understanding hadronization for precision measurements

• top mass:

VALUE (GeV)	pdglive.lbl.gov	DOCUMENT ID		TECN	COMMENT
$\textbf{172.69} \pm \textbf{0.30}$	OUR AVERAGE Error inclu	des scale factor of 1.3.	See the id	eogram belo	w.
$172.13 \ {}^{+0.76}_{-0.77}$		¹ TUMASYAN	2021G	CMS	t-channel single top production
172.6 ± 2.5		² SIRUNYAN	2020AR	CMS	jet mass from boosted top
$172.69 \pm 0.25 \pm 0.41$		³ AABOUD	2019AC	ATLS	7, 8 TeV ATLAS combination
$172.26 \pm 0.07 \pm 0.61$		⁴ SIRUNYAN	2019AP	CMS	lepton+jets, all-jets channels
$172.33 \pm 0.14 ~^{+0.66}_{-0.72}$		⁵ SIRUNYAN	2019AR	CMS	dilepton channel ($e\mu$, 2 e , 2 μ)
$172.44 \pm 0.13 \pm 0.47$		⁶ KHACHATRYA	2016AK	CMS	7, 8 TeV CMS combination
$174.30 \pm 0.35 \pm 0.54$		⁷ TEVEWWG	2016	TEVA	Tevatron combination

b quark hadronization model uncert. on indiv. measurements ~0.1-0.2 GeV

SHOULD ON ABOUT HADRO $e^+e^- \rightarrow b\overline{b}, \sqrt{s} = m_z$ 3.5⊢

 understanding hadronization for precision measurements

top mass:

F	VALUE (GeV)	pdglive.lbl.gov	DOCUMENT ID		Batilities 1941		////////////////////////////////
	$\textbf{172.69} \pm \textbf{0.30}$	OUR AVERAGE Error inclu	ides scale factor of 1.3.	See the		0.2 0.3	0.4 0.5 0
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SLD Data

1.5

0.5

o to Data

ATLAS Pythia8, A14-*r*b

H Data uncertainty

0.6 0.7 0.8

0.9

X_B

---- ATLAS PYTHIA8, A14

--- ATLAS HERWIG7.1.3

- understanding hadronization for precision measurements
- $\alpha_S(M_Z)$ determinations

hadronization corr. scale as $\sim \Lambda/Q$, can be dominant uncertainties

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 τ decays

&

low Q^2

BDP 2008-16

Boito 2018

PDG 2020

Boito 2021

Mateu 2018

Peset 2018



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see 2203.11110 for more examples

see 2203.11110

for more examples



SHOULD ONE CARE See 2203.1110 for more examples ABOUT HADRONIZATION strangeness enhancement in high



HADRONIZATION MODELS

HADRONIZATION

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- two main models for hadronization
 - Lund string model (Pythia)
 - cluster hadronization model (Herwig)
- both have as a starting point stage the final stage of QCD shower
 - stop shower at some scale Q_0
 - in large $N_c \rightarrow \infty$ limit planar graphs
 - groups final q, q
 , g in QCD singlet clusters/string pieces
 - either "color preconfinement" or "leading color dipoles/strings"

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CLUSTER VS. STRING MODEL



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CLUSTER MODEL

- assign mass to gluons, decay them to qq̄ pairs (in large N_c limit)
 - these are color singlets: *primary clusters*
 - primary clusters have universal mass distrib
- heavier clusters are decayed to lighter ones (fission/model dep. step)
- relatively small set of params, $\mathcal{O}(30)$





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LUND STRING MODEL

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- strings connect $q\bar{q}$ systems
 - gluons kinks in strings
- strings break into hadrons
 - controlled by Lund symmetric string fragmentation function f(z)
 - flavor selection modeled with tunable parameters
- Pythia Lund string model: many parameters, O(100)
 - 4 params for kinematics
 - most params related to flavor selection and color reconnection





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MACHINE LEARNING HADRONIZATION

MLhad: Bierlich, Ilten, Menzo, Mrenna, Wilkinson, Youssef, JZ, 2203.04983, https://gitlab.com/uchep/mlhad

see also HadML: (Chan, Ghosh,) Ju, (Kania), Nachman, (Sangli,) Siodmok, 2203.12660, 2305.17169



- MLhad: the long term goal
 - use ML to "parametrize our ignorance" about hadronization, use data

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MORE IMMEDIATE GOALS

- ML hadronization
 - develop ML architectures
 - reproduce simplified
 Pythia model
 - learn how to train on data

- Pythia focused
 - easier / faster
 parameter variation
 - improve on Lund
 string fragmentation
 function determin.

we are here

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ML HADRONIZATION

ML HADRONIZATION: MLHAD ROADMAP

- a series of progressive steps to be done before practically useful in Pythia/MC simulations
 - ML architecture that mimicks a simplified Lund string hadronization model
 - train ML on truth level Pythia output (not obs. in exp)
 - develop a framework to propagate errors
 - improved ML architecture with full hadron flavor selector
 - train on mock data (i.e. just observable information)
 - train on real data (i.e. just already measured information)

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replace/supplement Pythia string model

we are here

MLHAD/HADML STATUS

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• MLhad: two architectures

- MLhad cSWAE (conditional Sliced Wasserstein Auto Encoder)
 - latent space distribution need not be analytically known ⇒ could use Pythia output
- MLhad NF (Bayesian Norm. Flow)
 - incorporation of errors
- trained on a simplif. Pythia string model
- trained on first emissions
 - uses hadronization history ⇒
 information that cannot be measured
 - present work: relaxing this assumption

MLhad: 2203.04983, 2311.09296, 2410.nnnn

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- HadML: single architecture
- GAN (Generative Adversarial Network)
 - latent space need not be analytically known
 - two implementations HadML_vl and HadML_v2
- trained on a simplifed Herwig cluster model
- HadML_vl trained on first emissions
- HadML_v2 trained on particle flow (point clouds)
 - information that can in principle be measured

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THE STRATEGY

- MLhad architectures capture well the (simplified) Pythia Lund string model
- proof of principle need to see how this ports to training on data
- we want to achieve this in steps
 - modify only parts of Pythia Lund string model

PYTHIA FOCUSED

TWO DIFFERENT GOALS

- solving the inverse problem for hadronization
 - extracting string fragmentation function from data
- speeding up Pythia hadronization when varying parameters
 - reweighting method ⇒ backup slides

SIMPLIFIED STRING HADRONIZATION MODEL

- assume that color flow done correctly by Pythia
 - including splitting gluons, so that only strings with q, \bar{q} ends
- hadron emission from a string piece controlled by fragmentation function f(z)
 - the whole hadronization chain is then reproduced by iterating
 - the string is labeled by q, \bar{q} flavor and its energy in cms, 2E
- for now only *u*,*d* quarks, uses Pythia flavor selector



0.025

0.020

0.005

0.000

0.172

0.27

0.015 Densit

NF

Pythia

STRING FRAGMENTATION FUNCTION

- in c.o.m. frame of the string:
 - hadron emission described by p_z, p_T + uniform distrib. in azimuthal angle
- p_z distribution from Lund string fragmentation function

ng
$$z = (p_{h,z} + E_h)/E_{\text{string}}$$

$$f(z) \propto \frac{(1-z)^{a}}{z} \exp\left(-\frac{b m_{\perp}^{2}}{z}\right)$$

$$z = (p_{h,z} + D_h)/D_{\text{strin}}$$

 $m_{\perp}^2 \equiv m^2 + p_T^2$

- for light quark flavored hadrons only three params.: *a*,*b* and mass, *m*
- p_T from random Gaussian distributions; width another param.

INVERSE PROBLEM

- inverse problem for hadronization
 - can one learn f(z) from data*?
 - *without asking for its parametric form
- compare: NNPDF determinations of PDFs from data

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A HARDER PROBLEM

• PDFs: appear at most in quadratic form in observables

$$\sigma_X(s, M_X^2) = \sum_{a, b} \int_{x_{\min}}^1 dx_1 dx_2 f_{a/h_1}(x_1, M_X^2) f_{b/h_2}(x_2, M_X^2) \hat{\sigma}_{ab \to X} \left(x_1 x_2 s, M_X^2 \right).$$

- string fragmentation function: each event a different number of hadrons
 - different number of samplings of f(z)

Prob.
$$\sim \prod_{i=1}^{N_{\text{hadr.}}} f(z_i)$$

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A HARDER PROBLEM

- further complications
 - a complicated permutation symmetry: swapping orders of emissions + appropriate boosts ⇒ lead to the same event
 - in Pythia the final emission may or may not lead to viable kinematics ⇒ O(1) fraction of simulated fragmentation chains is rejected



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MLhad: 2410.nnnn

THE HOMER METHOD

MLhad: 2410.nnnnn

- the HOMER (Histories and Observables for Monte-Carlo Event Reweighting) method
- 3 step approach
 - Step 1: train a classifier on events ⇒ likelihood of an event
 - Step 2: can use this on simulated events to find a neural-net representation of *f*(*z*)
 - Step 3: simulate using this new f(z)
- in all steps use ratios of probabilities (weights)
 - always reweighting from base Pythia simulation results



- Step 2: can use this on simulated events to find a neural-net representation of *f*(*z*)
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HOMER RESULTS

MLhad: 2410.nnnnn

- Note: HOMER uses only measurable quantities three versions
 - (i) binned high-level obs.;
 - (ii) unbinned high level obs.;
 - (iii) point cloud
- so far a simplified case: qq
 string of fixed energy

HOMER RESULTS

MLhad: 2410.nnnnn

- in this simplified case
 - binned high-level observables (multiplicities, shape observ.,...) suffice
 - additional gain for unbinned highlevel obs. case
 - point cloud harder to train on



MLhad: 2410.nnnnn

- additional gain for unbinned highlevel obs. case
- point cloud harder to train on

ADDING GLUONS

MLhad: 2412.nnnnn

- working in progress: adding gluons to the string
 - energies and number of gluons as one would get from parton shower
- approximation that went into HOMER for $q\bar{q}$ string breaks down
 - one needs to calculate averages over several simulated fragmentation chains to obtain the estimate for an event weight
 - works well for a single gluon, even if strings with many energies in a sample

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• for many gluons further work required



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- works well for a single gluon, even if strings with many energies in a sample

31

• for many gluons further work required

PYTHIA-8 PLUGIN MODULE

- not directly related to hadronization, but is an output of MLhad effort:
 - Pythia8 user contribution plugin platform Pythia8-contrib will be available
 - similar to FastJet-contrib in concept
 - MLhad NF and MLhad cSWAE as test packages
 - contact me, if you are interested in submitting a package/beta test

CONCLUSIONS

- MLhad/HadML: first steps in creating ML based hadronization description
- of immediate use but not shown in the talk
 - reweighting algorithms in Pythia for faster variation of hadronization params.

BACKUP SLIDES

CLUSTER MODEL

- assign mass to gluons, decay them to $q\bar{q}$ pairs
 - these are color singlets: *primary clusters*
 - primary clusters have universal mass distrib
- heavier clusters are decayed to lighter ones (model dep. step)
- relatively small set of params, $\mathcal{O}(30)$



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LUND STRING MODEL

- strings connect $q\bar{q}$ systems
- gluons kinks in strings
 - split gluons to a collinear $q\bar{q}$ pair \Rightarrow string pieces
- string pieces break into hadrons (model dep.)
 - controlled by Lund string fragmentation function
- Pythia Lund string model: many parameters, $\mathcal{O}(100)$



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COLOR RECONNECTION

- all perturbative predictions in leading color approximation ($N_c \rightarrow \infty$ with $\alpha_s N_c$ fixed)
 - direct mapping of color flow to strings
- color reconnection: inclusion of $1/N_c$ suppressed terms (model dep.)
 - reassing colors, no change in parton momenta
 - several examples where important
 - first historic mention: for charmonium production in *B* decays
 - for multiple parton interactions (Pythia MPI model) Sjöstrand, Zijl, 1987
 - $e^+e^- \rightarrow W^+W^- \rightarrow 4j$ at LEP 2 excludes no CR hypothesis 1302.3415
 - top quark mass determination from hadronic tops
 - several color reconnection models in Pythia
- computationally expensive, especially at high multiplicities

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Fritzch, 1977; Ali et al, 1979

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CHALLENGES FOR HADRONIZATION MODELS

Fischer, Sjostrand, 1610.09818

- in general out of the box hadronizations models work within 20-50%
- some challenges for Pythia
 - change of flavor composition with event multiplicity
 - high multiplicity events have higher strangenesss content
 - no mechanism in Pythia to mimic it
 - average $\langle p_T \rangle$ larger for heavier particles, trend ok in Pythia, but numerically not large enough
 - charge particle p_T spectrum not correctly modelled at low p_T
 - partially can be fixed by tunes, but then a problem at interm. p_T
 - there is a peak in $\Lambda/K p_T$ spectrum at $p_T \sim 2.5$ GeV, not reproduced by Pythia
 - the observation of the ridge in *pp* requires collective effects
- at least some of them addressed in Pythia 8.3 by introducing more involved models of string interactions, thermodynamical string fragmentation model, etc.
- Herwig has a different set of challenges, e.g., predicting heavy baryon distributions

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MLHAD

- right now trained directly on Pythia first emission output
 - hadron mom. described by p_z, p_T
- the IR cut-off has two effects
 - p_z and p_T distributions are uncorellated
 - makes the problem scale invariant in p_Z
 - enough to train at one string mass, $2E_{ref}$
 - for other energies can rescale

$$p'_z \equiv E_{\rm ref} \frac{p}{E},$$

• this is relaxed in the end, *E* dependence can be recovered



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 - enough to train at one string mass, $2E_{ref}$
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$$p'_z \equiv E_{\rm ref} \frac{p}{E},$$

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CSWAE

- use conditional Sliced-Wasserstein Autoencoder
 - SW gives flexibility in the use of latent space distributions



• string energy E_i is encoded in a label \bar{c}_i

$$\bar{c}_i = \frac{E_{\max} - E_i}{E_{\max} - E_{\min}},$$

- training data: \mathbf{x}_i sorted vector of 100 first emission
 - either p_z or p_T values
- loss function

$$\mathcal{L}(\psi,\phi) = \mathcal{L}_{
m rec} + \mathcal{L}_{
m SW},$$

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MLHAD AS A GENERATOR

• MLhad as a generator of the hadronization chains



RESULTS - FIRST EMISSIONS

- three different latent space distributions used
- cSWAE training configurations

		latent space dim		sw vs. I	L _{rec} #	t of S
Variable \boldsymbol{x}	Target \boldsymbol{z}	$\mid t \; (ext{epochs}) \; $	d_z	λ	L	-
p_z'	Pythia	150	35	35	15	-
	Trapezoidal	300	2	20	30	
	Triangular	150	2	30	25	
p_T	Pythia	100	20	30	30	
	Skew-norm	120	4	20	25	
	Triangular	120	4	15	25	



RESULTS - FIRST EMISSION

• MLhad generated *p*_z distribs.



RESULTS - FIRST EMISSION

• MLhad generated p_T distribs.



E DEPENDENT DISTRIBUTIONS

- train on first hadron emissions at $E = \{5, 30, 700, 1000\}$ GeV
- generate at a different set of string energies



GENERATING HADRONIZATION CHAINS

 number of hadrons produced in hadronization of 50 GeV string



GENERATING HADRONIZATION CHAINS

• the distributions match over a range of string energies



Densit

STRING FRAGMENTATION FUNCTION

- in c.o.m. frame of the string:
 - hadron emission described by p_z, p_T + uniform distrib. in azimuthal angle
- p_z distribution from Lund string fragmentation function

$$f(z) \propto \frac{(1-z)^a}{z} \exp\left(-\frac{b m_{\perp}^2}{z}\right)$$

$$z = (p_{h,z} + E_h) / E_{\text{string}}$$
$$m_{\perp}^2 \equiv m^2 + p_T^2$$

- for light quark flavored hadrons only three params.: *a*,*b* and mass, *m*
- p_T from random Gaussian distributions; width another param.

- NF: analytic transform. from latent to feature space
 - for us feature space 2D: $\mathbf{x}_i = (p_z, p_T)$



- BNF: NN params. θ are normal random vars.
 - mean and widths trained such that statist. errors reproduced

MLhad: 2310.nnnn



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 - mean and widths trained such that statist. errors reproduced

SIMPLIFIED STRING HADRONIZATION MODEL

- assume that color flow done correctly by Pythia
 - including splitting gluons, so that only strings with q, \bar{q} ends
- want to reproduce first hadron emission from a string piece
 - the whole hadronization chain is then reproduced by iterating
 - the string is labeled by q, \bar{q} flavor and its energy in cms, 2E
- only *u,d* quarks, uses Pythia flavor selector
- have an IR cut-off of 25 GeV, at which hadronization chain terminates



• when used as a generator repeat single emissions in

c.m.s.'s +boosts



• in general conditioned on string eng. (c_i), hadron mass /



- single hadron emissions well reproduced
 - note: in simplified Pythia string model



- charge multiplicities well reproduced
 - note: in simplified Pythia string model



Bierlich et al [MLhad], 2308.13459

- event generation is time-consuming
 - want to reweight events without regenerating
- in Pythia the Lund string fragm. function sampled via standard accept/reject algorithm
 - if rejected instances are kept ⇒ a modified accept/reject algorithm
 - ⇒ new event weights for diff. hadronization params.

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Bell graphics by M. K. Wilkinson

 $e^+e^- \rightarrow Z \rightarrow \text{jets}$

Bierlich et al [MLhad], 2308.13459



 $e^+e^- \rightarrow Z \rightarrow \text{jets}$ Bierlich et al [MLhad], 2308.13459 a = 0.55a = 0.76a = 0.30 $a^{\text{base}} = 0.68$ A.U w' reweighted exact calc. 0.2 0.1 0.0 *a*/*A* 2.5 0.0 25 50 25 50 25 50 charge multiplicity charge multiplicity charge multiplicity Zupan inverse problem for nagronizat
REWEIGHTING HADRONIZED PYTHIA EVENTS

Bierlich et al [MLhad], 2308.13459

- implemented for
 - *a*, *b* Lund string fragmentation params.
 - also for heavy flavor param. r_b , and the width parameter for Gaussian sampling of p_T
- for full detector simulations can expect up to several orders of magnitude speed-ups
 - if many variations of hadroniz. params are needed,
 e.g., in *m_t* measurements
- caveat: new and old *f*(*z*) need to have large enough overlap/area of support

CAVEAT: LARGE PARAM. VARIATIONS

Bierlich et al [MLhad], 2308.13459

• if $f(z)_{\text{new}}$ nonzero where $f(z)_{\text{old}}$ ~vanishes \Rightarrow large errors



• mean weight μ can be a useful diagnostics tool

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• mean weight μ can be a useful diagnostics tool

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FLAVOR REWEIGHTING

Bierlich et al [MLhad], 2411.nnnn

- next step
 - reweighting for
 flavor parameter
 variations in Pythia
 hadronization
- baryons make
 everything more
 complicated



FLAVOR REWEIGHTING

Bierlich et al [MLhad], 2311.nnnn

• simple flavor decision flow for string breaks if no diquarks



becomes much more involved with diquarks





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