Lepton Number Violation at the LHC

Jonathan Kriewald

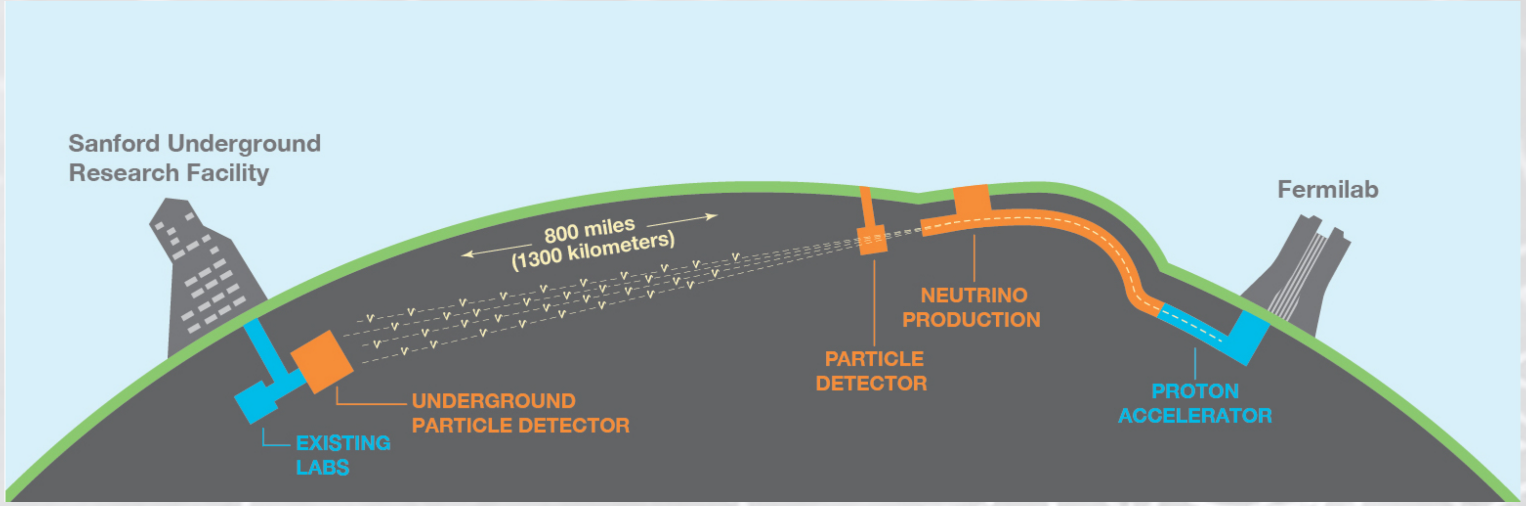
Jožef Stefan Institute

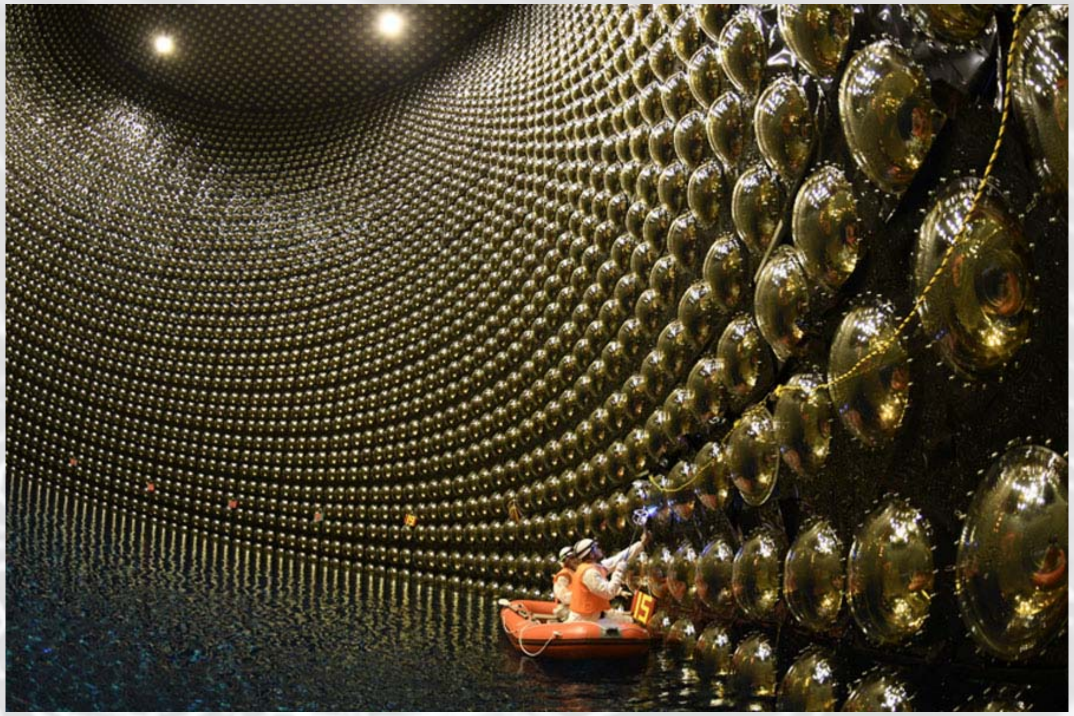
F1 Seminar

Based on 2403.07756, 2408.00833, 2503.21354

with Patrick D. Bolton, Benjamin Fuks, Miha Nemevšek, Fabrizio Nesti & Juan Carlos Vasquez

Strong arguments in f(l)avour of New Physics!





ν-oscillations 1st laboratory *evidence* of New Physics!

- New mechanism of mass generation? Majorana fields?
- Neutral lepton flavour violation ⇒ charged LFV?

Several experimental puzzles remain:

- Absolute mass scale?
- Mass ordering? (NO vs IO)
- Leptonic CP violation?

Making neutrino masses

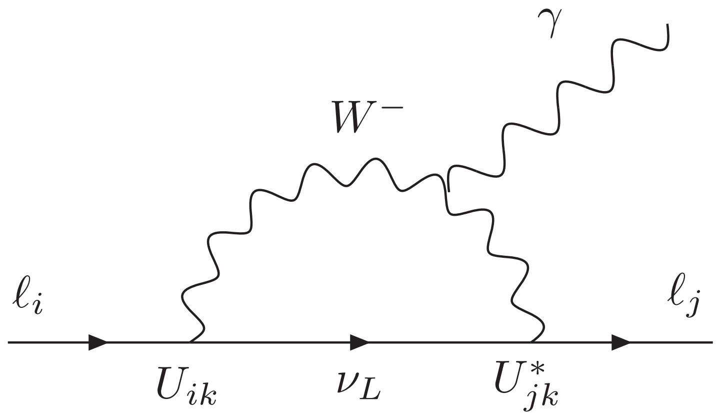
Extend SM to accommodate $\nu_\alpha \leftrightsquigarrow \nu_\beta$: ad-hoc 3 $\nu_R \Rightarrow$ Dirac masses, "SM$_{m_\nu}$", U_{PMNS}

In SM$_{m_\nu}$: **flavour-universal** lepton couplings, lepton number conserved

[Petcov '77]

cLFV possible … but not observable! $\mathrm{BR}(\mu \to e\gamma) \propto |\sum U^*_{\mu i} U_{ei} m^2_{\nu_i}/m^2_W| \simeq 10^{-54}$

EDMs still tiny… (2-loop from δ_{CP}, $|d_\ell| \sim 10^{-35} e\mathrm{cm}$)



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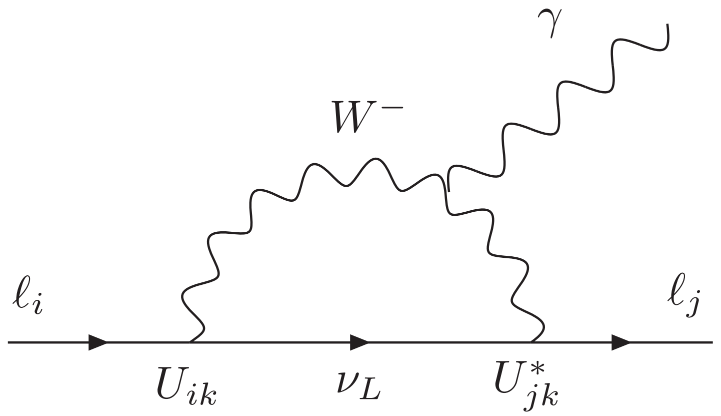
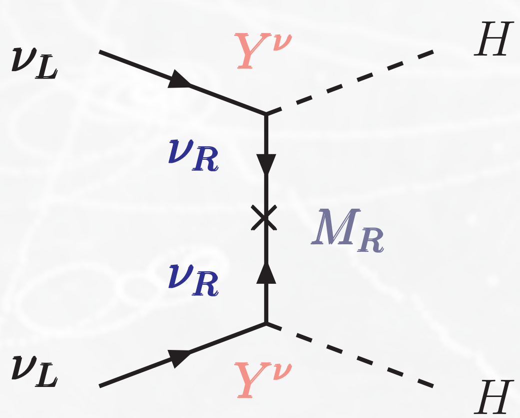


Nothing forbids an additional mass term of the form $\mathscr{L} \supseteq \frac{m_{RR}}{2} \bar{\nu}_R \nu_R^C$!

$\Rightarrow$ Neutrinos become **Majorana** particles – also SM-like neutrinos: $\mathscr{L}_{\text{eff}} \sim \frac{m_{LL}}{2} \bar{\nu}_L \nu_L^C$

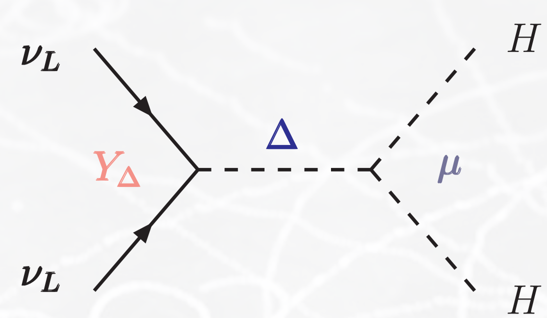
Making Majorana neutrinos

Effective mass term $\mathscr{L}_{\text{eff}} \sim \frac{m_{LL}}{2}\,\bar{\nu}_L\,\nu_L^C$ from Weinberg operator: $\mathscr{L}^{d=5} \sim \frac{h_{ij}}{2\Lambda}(H\,L_i\,H\,L_j)$



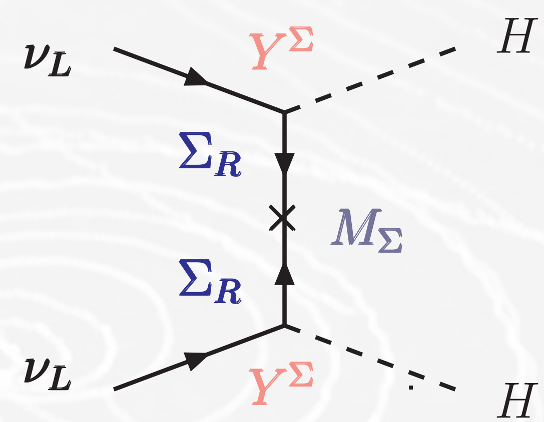
Type I (fermion singlet)

(Minkowski ’77)



Type II (scalar triplet)

(e.g. Schechter & Valle ’80)



Type III (fermion triplet)

(e.g. Foot et al. ’89)

Mass terms: $m_\nu^I \sim -\,v^2 Y_\nu^T \frac{1}{M_R} Y_\nu$, $\quad m_\nu^{II} \sim -\,v^2 Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} \sim -\,Y_\Delta v_\Delta$, $\quad m_\nu^{III} \sim -\,Y_\Sigma^T \frac{v^2}{2M_\Sigma} Y_\Sigma$

Countless more possibilities with higher odd-dimensional operators or loop-level realisations…

(Actually they are countable, see e.g. [John Gargalionis and Ray Volkas: 2009.13537]

Making neutrino masses

Effective ... $I\,L_j)$

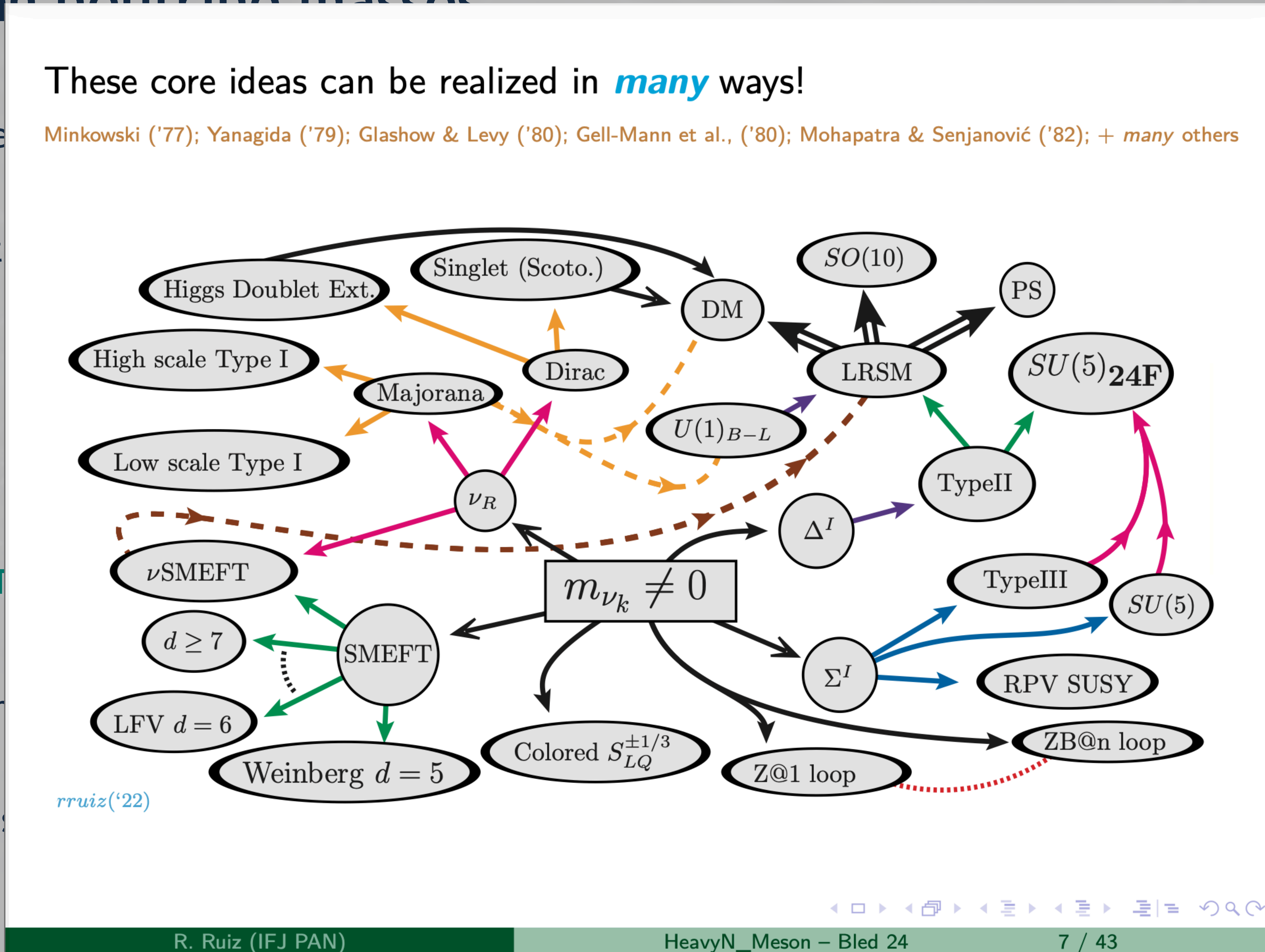
Different ... $)(L_j^T \sigma_a H)$

T ...)

Mass term ...

Countless ... tions...

09.13537]



Making neutrino masses

Effective mass ... $H L_i H L_j)$

Different realis... $L_i^T \sigma_a H)(L_j^T \sigma_a H)$

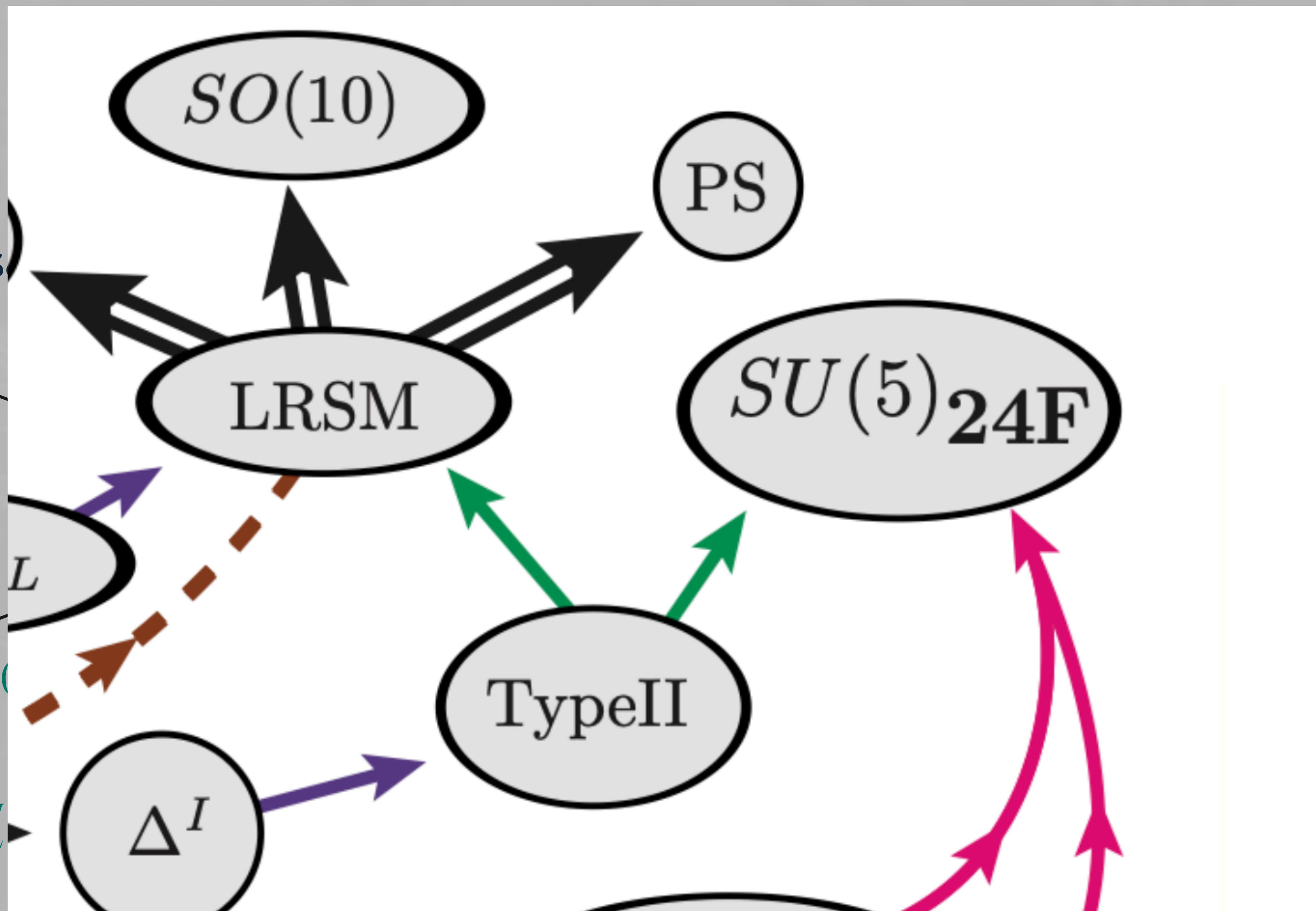
ν_L ... H

Σ

ν_L ... H

Type I (... n triplet)

Mass terms: m_ν^I ... $\frac{v^2}{2M_\Sigma} Y_\Sigma$



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Type II seesaw mechanism

Extend Standard Model with a **scalar** $Y = 1$, $SU(2)_L$-triplet

$$\Delta_L = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \frac{v_\Delta + \Delta^0 + i\chi_\Delta}{\sqrt{2}} & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

Assign lepton number $L = 2$ to Δ_L

$\Rightarrow$ Yukawa Lagrangian $\mathscr{L}_{\text{yuk}} \supset Y_\Delta^{ij} L_{Li}^T \mathscr{C} \, i\sigma_2 \, \Delta_L \, L_{Lj} + \text{h.c.}$

$\Rightarrow$ **Majorana** neutrino masses: $M_\nu = U_P^* \, m_\nu \, U_P^\dagger = \sqrt{2} v_\Delta Y_\Delta$

Yukawas fixed by oscillation data; $Y_\Delta \simeq \mathcal{O}(1)$ for $v_\Delta \simeq 10^{-10}\,\text{GeV}$

Lepton Number Violation @ low energies

Yukawa Lagrangian $\mathscr{L}_{\text{yuk}} \supset Y_\Delta^{ij} L_{Li}^T \mathscr{C} i\sigma_2 \Delta_L L_{Lj} + \text{h.c.}$

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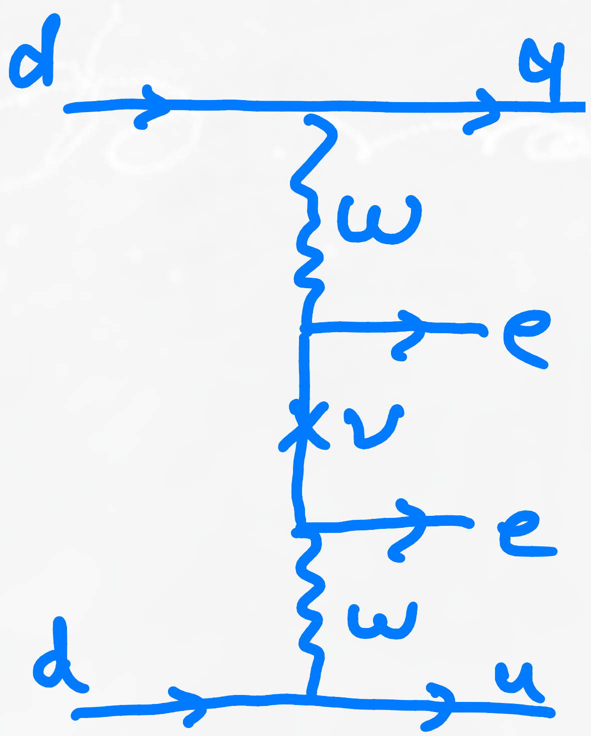
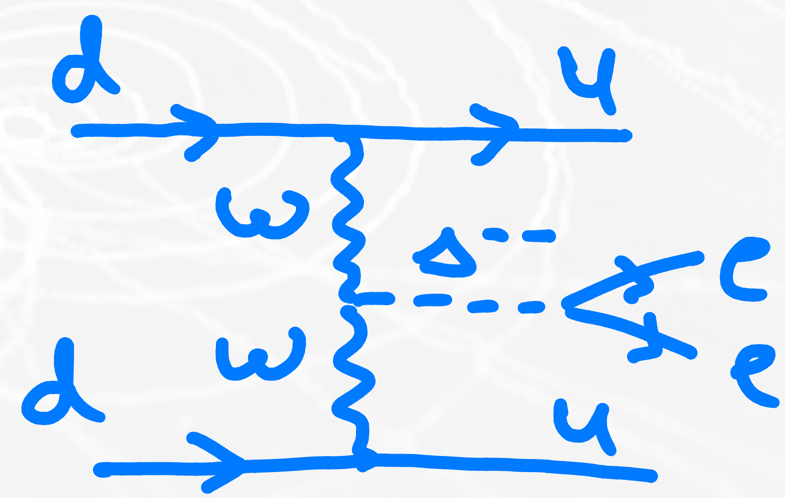
Neutrinoless double beta decay ($0\nu 2\beta$):



Long range interaction from light **Majorana mass** insertion



Short range interaction strongly suppressed for $m_\Delta \gtrsim 100\,\text{GeV}$, vertex: $\Delta^{++}WW \propto v_\Delta / v$

Lepton Number Violation @ low energies

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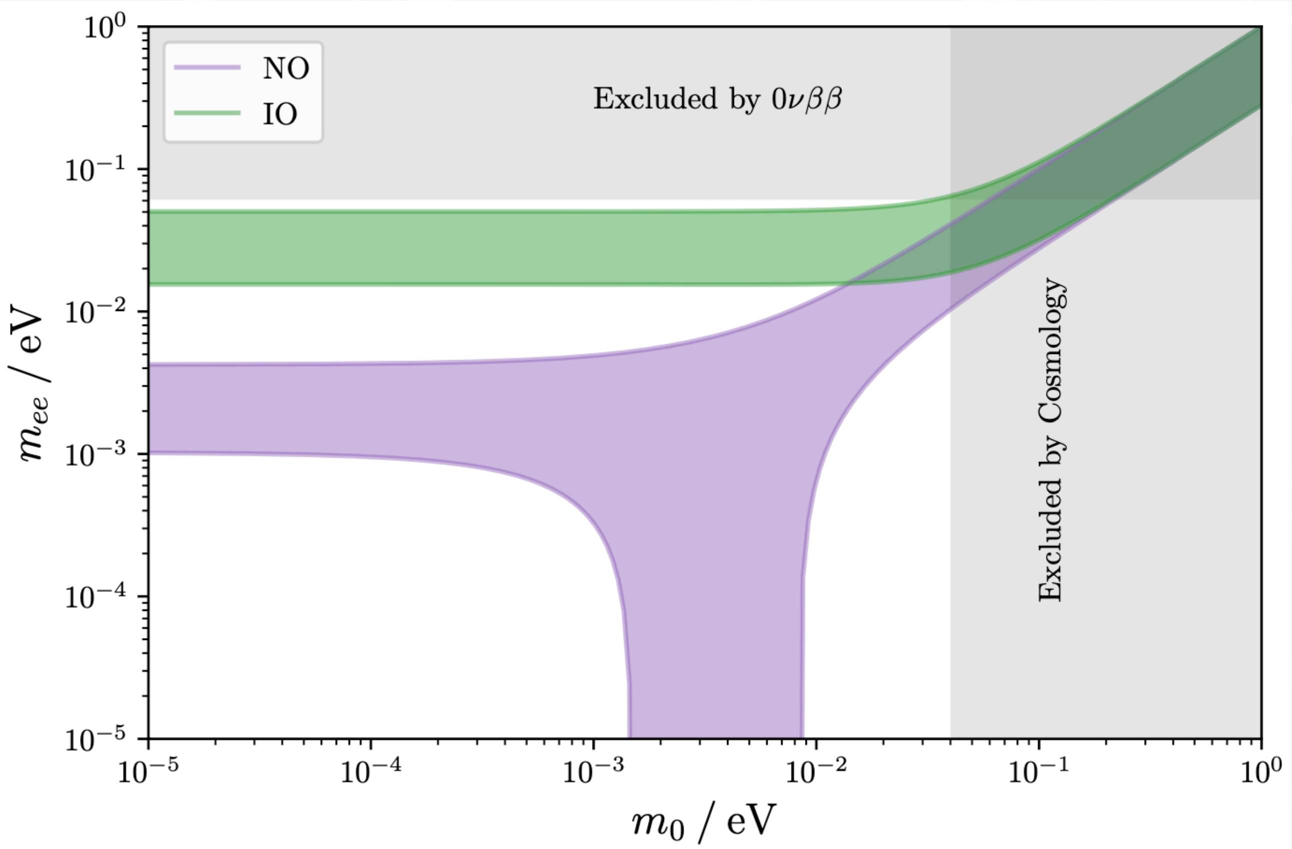
Neutrinoless double beta decay ($0\nu 2\beta$):

Long-range interaction fixed by U_P, m_{ν_i}



Constraints on a $Y = 1$ scalar triplet

Mixing with would-be Goldstones $\leftrightsquigarrow$ corrections to M_W, M_Z, ρ, **EWPO**

At **tree level**

$$\rho^0 = \frac{M_W^2}{\cos^2\theta_w M_Z^2} = \frac{v^2 + 2v_\Delta^2}{v^2 + 4v_\Delta^2} = 1.00031 \pm 0.00019 \quad \Rightarrow \text{upper limit on } v_\Delta \lesssim \mathcal{O}(\text{few GeV})$$

From electroweak fit (see PDG)

Oblique parameters S, T, U measure corrections to W, Z, γ **self-energies (one-loop)**

[Peskin, Takeuchi '91]

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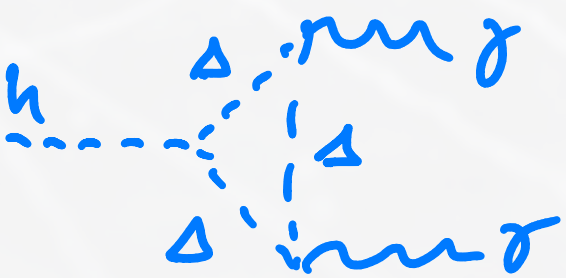
[Peskin, Takeuchi '91]

Computed for general $SU(2)_L$ multiplets in [hep-ph/9309262] for $v_\Delta = 0$

LEP measurements of Z line shape, Γ_Z : $\quad m_{\Delta^{++,+,0}} \gtrsim \frac{M_Z}{2}$



Higgs couplings: $\lambda_{h\Delta 1}\, \varphi^\dagger \varphi \mathrm{Tr}\left[\Delta^\dagger \Delta\right] + \lambda_{h\Delta 2}\, \mathrm{Tr}\left[\varphi\varphi^\dagger \Delta\Delta^\dagger\right] \Rightarrow$ corrections to $h \to \gamma\gamma$

Constraints on a $Y = 1$ scalar triplet

Yukawa fixed by oscillation data, $Y_\Delta \simeq \mathcal{O}(1)$ for $v_\Delta \simeq 10^{-10}\,\mathrm{GeV}$

$\Rightarrow$ **lepton flavour-violating** interactions:

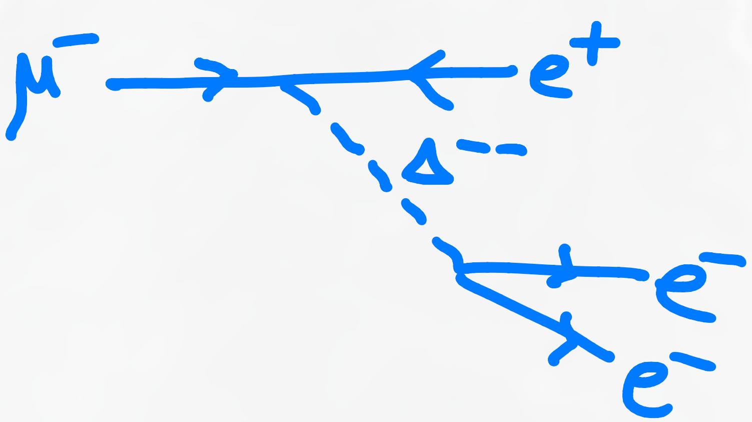
$\ell_\alpha^- \to \ell_i^+ \ell_j^- \ell_k^-$ Tree

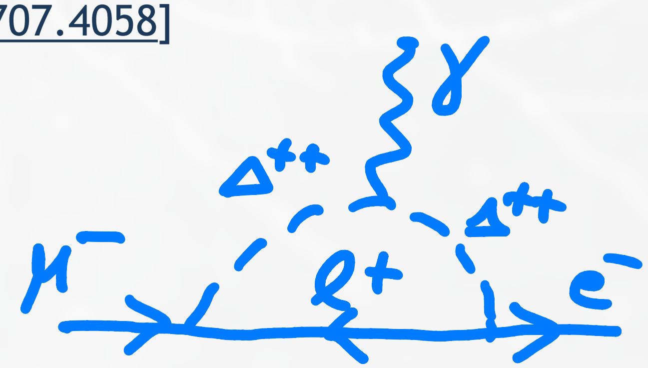
$$\Gamma \simeq \frac{m_{\ell_\alpha}^5}{(1+\delta_{jk})96\pi^3 m_{\Delta^{++}}^4} |Y_\Delta^{\alpha i}|^2 |Y_\Delta^{jk}|^2$$

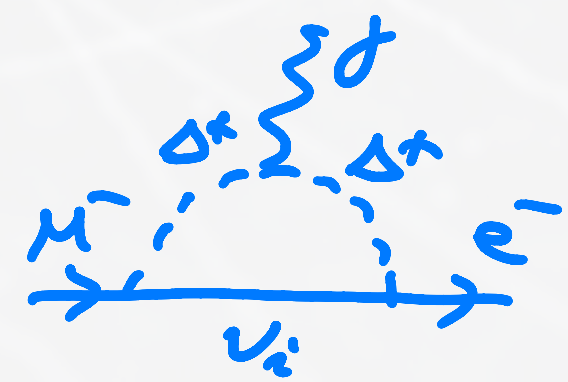
$\ell_\alpha \to \ell_\beta \gamma$ Loop

$$\Gamma \propto \frac{m_{\ell_\alpha}^5}{256\pi^3 m_{\Delta^{++}}^4} |\sum Y_\Delta^{\alpha i \dagger} Y_\Delta^{\beta i}|^2$$

See e.g. [0707.4058]

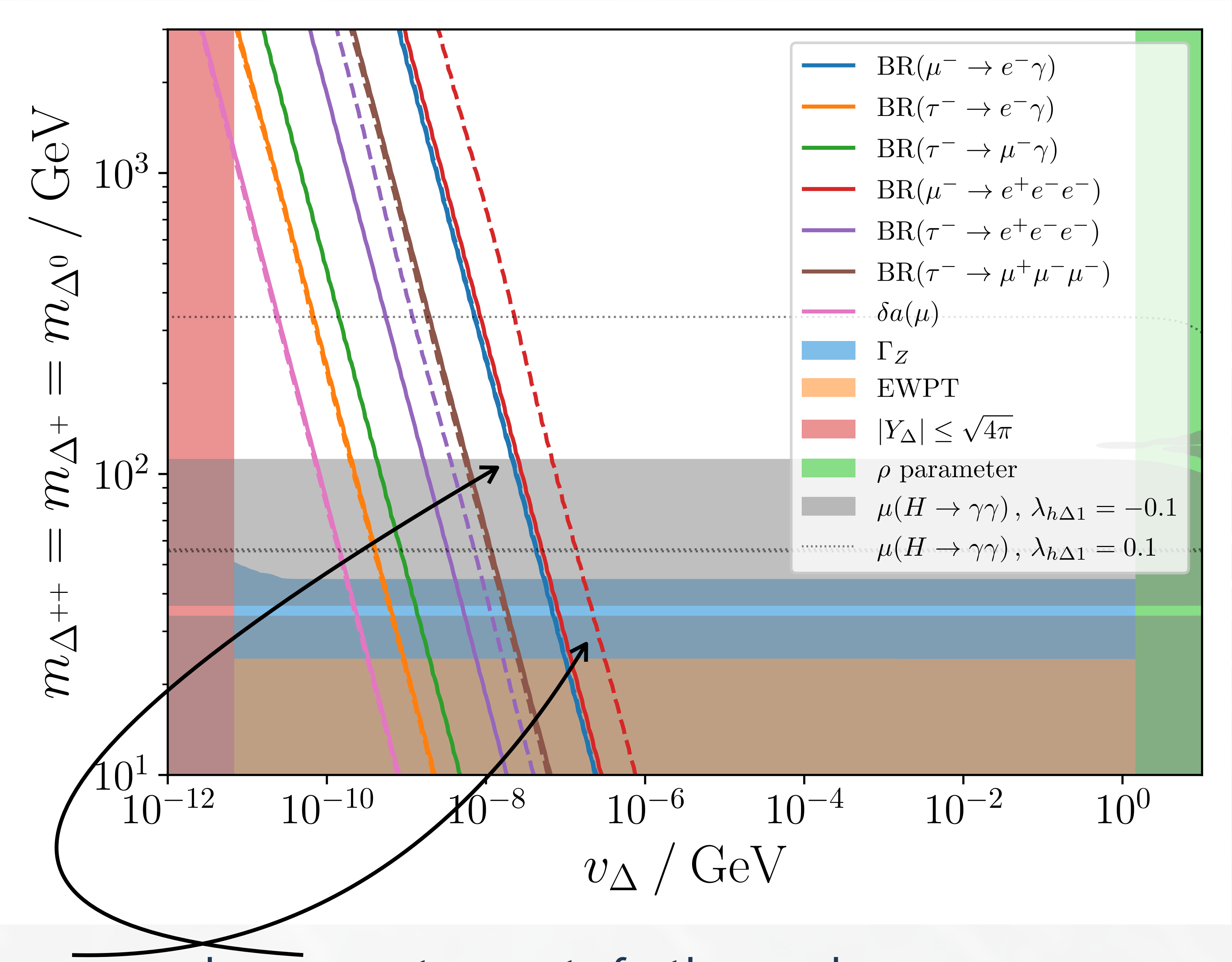






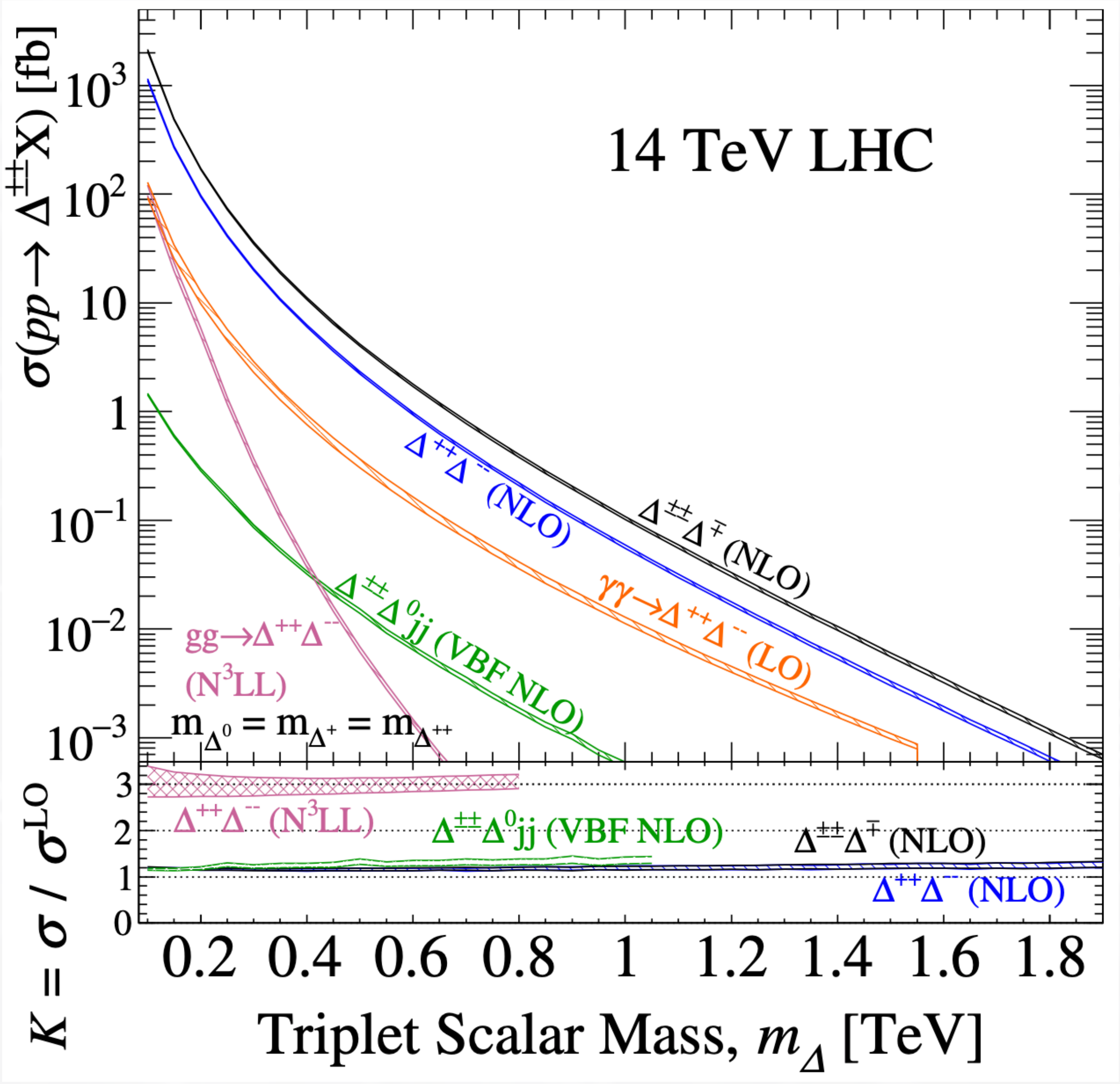
(Contributions to $(g-2)_\ell$ generically negative, weaker bound)

Indirect constraints on a $Y = 1$ scalar triplet



$\Rightarrow$ Bounds on $\mu \to eee$ and $\mu \to e\gamma$ strongest, further push v_Δ

Direct searches — production modes



[1912.08975]

⇒ @ LHC: **Drell-Yan** always dominant for $m_\Delta \gtrsim 100$ GeV

⇒ Production at LEP: $e^+e^- \to Z^*, \gamma^* \to \Delta^{++,+}\Delta^{--,-}$

Decay modes of the triplet components

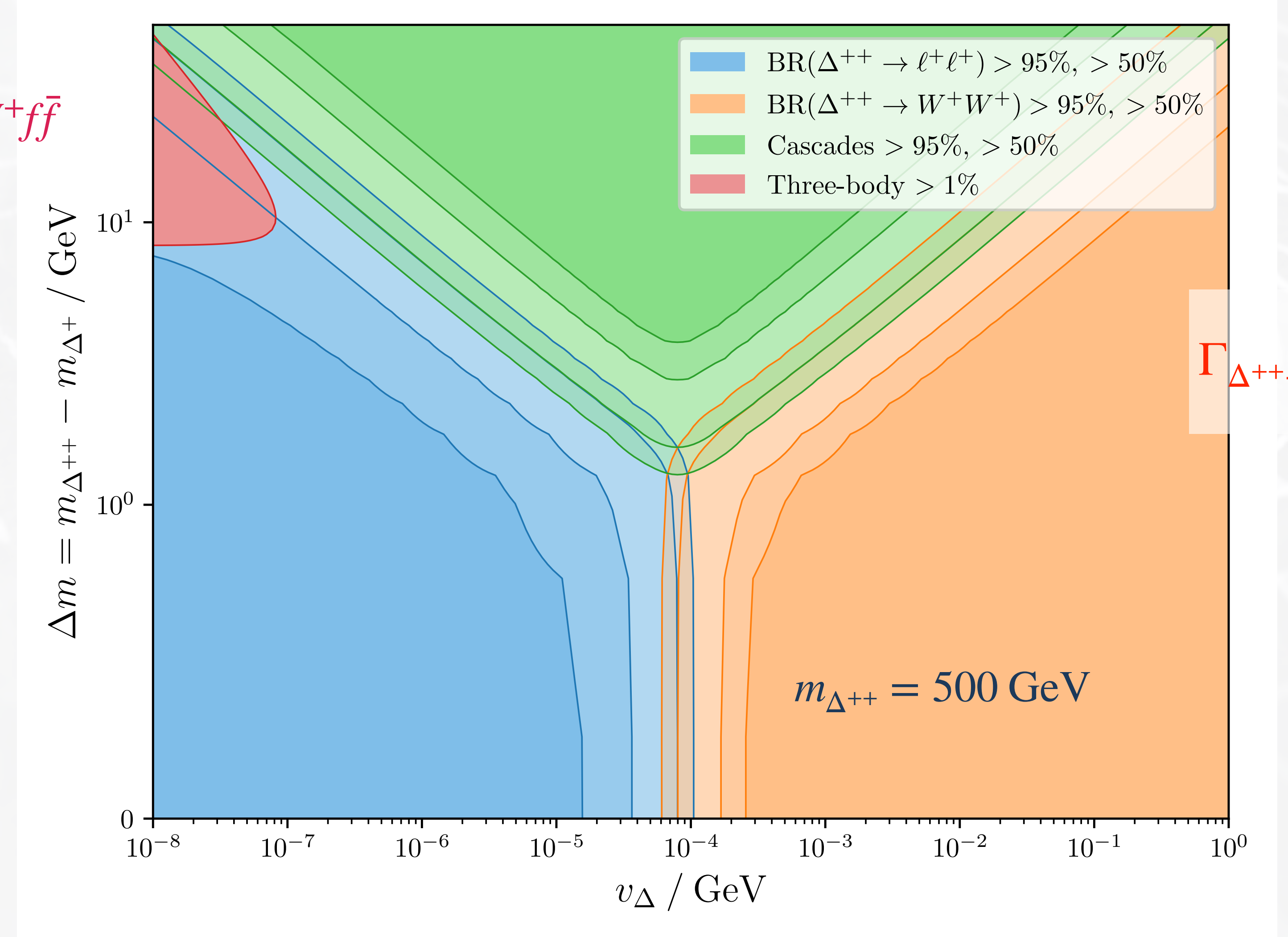
Smoking gun signal: resonance in the same-sign di-lepton invariant mass from $\Delta^{\pm\pm}$ decay

$v_\Delta \lesssim 10^{-4}$ GeV: $\Delta^{\pm\pm} \to \ell_i^\pm \ell_j^\pm$ **dominant**

Larger v_Δ: $\Delta^{\pm\pm} \to W^\pm W^\pm$ quickly dominates

Three-body decays subdominant $\Delta^{++} \to W^+ f\bar{f}$

$$\Gamma_{\Delta^{++}\to\ell_i^+\ell_j^+} = \frac{m_{\Delta^{++}}}{8\pi\left(1+\delta_{ij}\right)}\left|\frac{M_{\nu ij}}{v_\Delta}\right|^2$$



$$\Gamma_{\Delta^{++}\to W^+W^+} \propto \alpha_2^2 \frac{v_\Delta^2}{v^2}\frac{m_{\Delta^{++}}}{M_W^2}$$

If $m_{\Delta^{++}} > m_{\Delta^+}$: $\Delta^{\pm\pm} \to \Delta^\pm + X$ **cascades** dominate

$$\Gamma_{\Delta^{++}\to\Delta^+ f\bar{f}} \simeq \frac{3\alpha_2^2}{5\pi}\frac{\Delta m_\Delta^5}{M_W^4}$$

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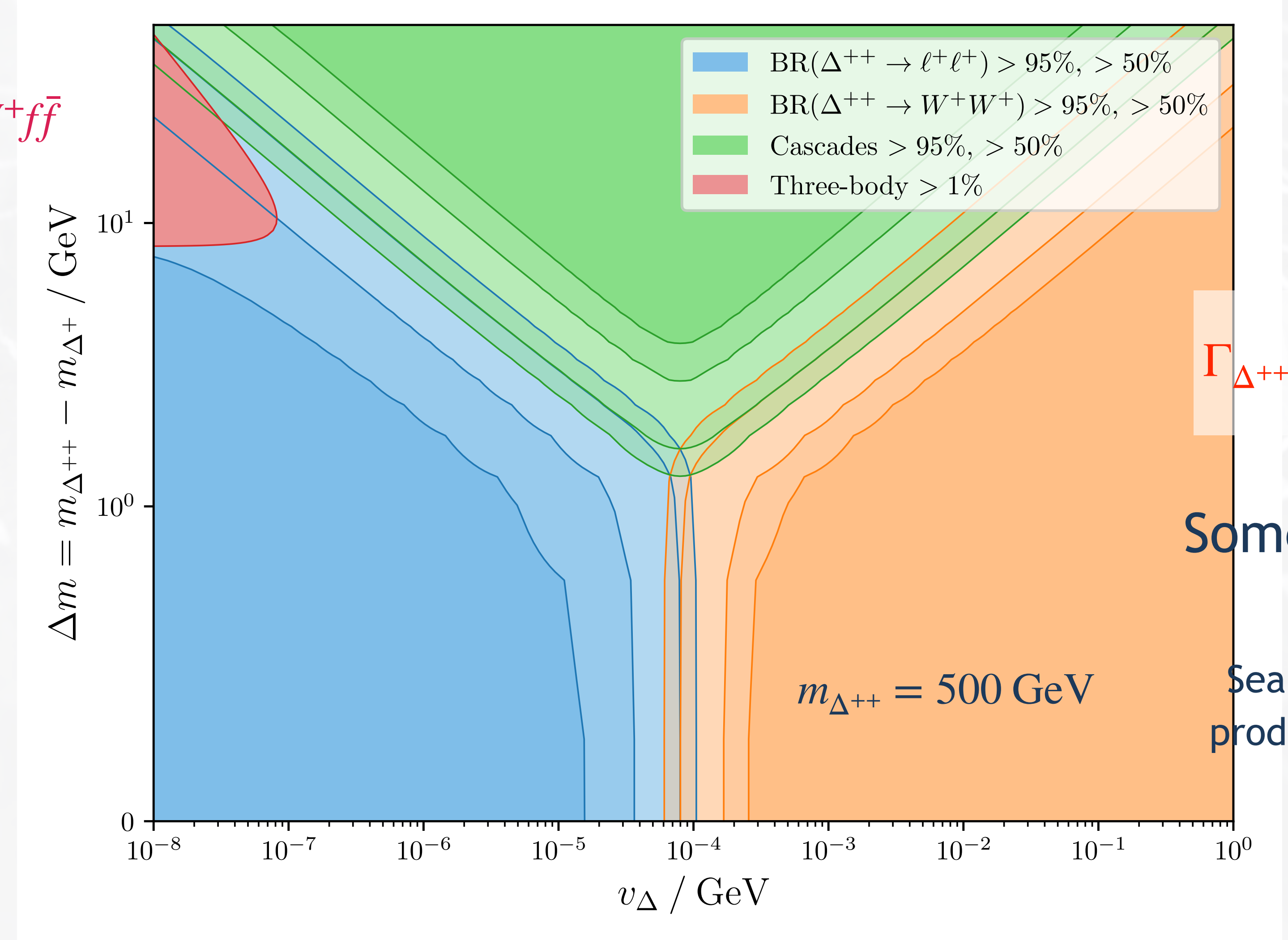
Searches for pair & associate production: $\ell^+\ell^+\ell^-\ell^-$ and $\ell^+\ell^+\ell^-\nu$ final states



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Some ATLAS searches for **di-boson**

Searches for pair & associate production: $W^+W^+W^-W^-$ and $W^+W^+W^-Z$

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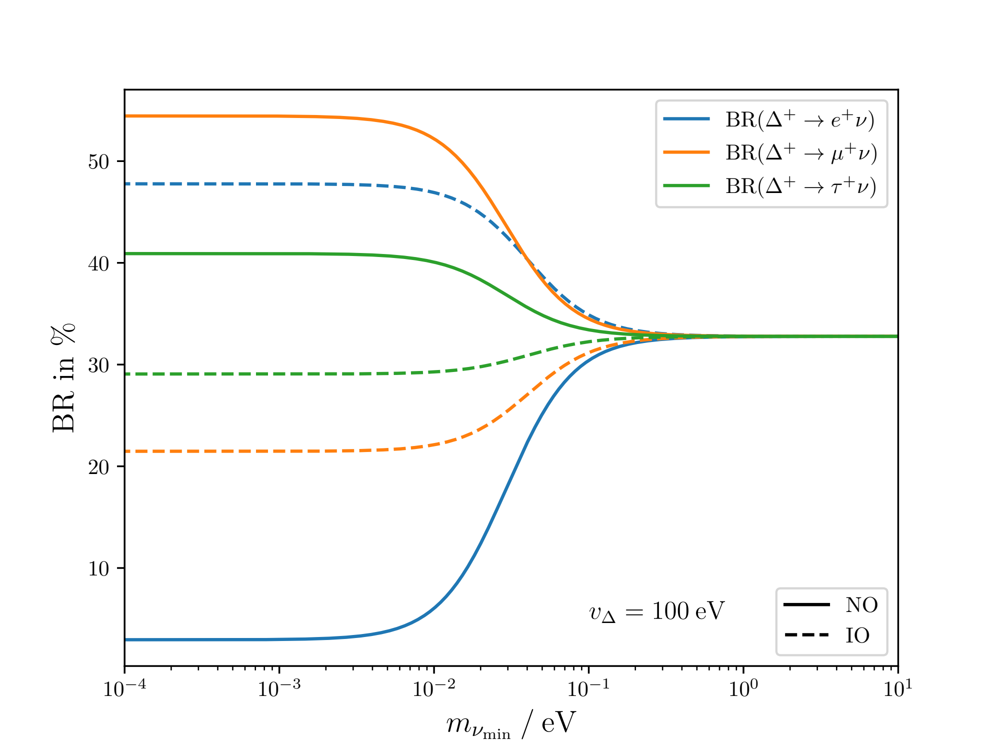
Decay modes of the triplet components

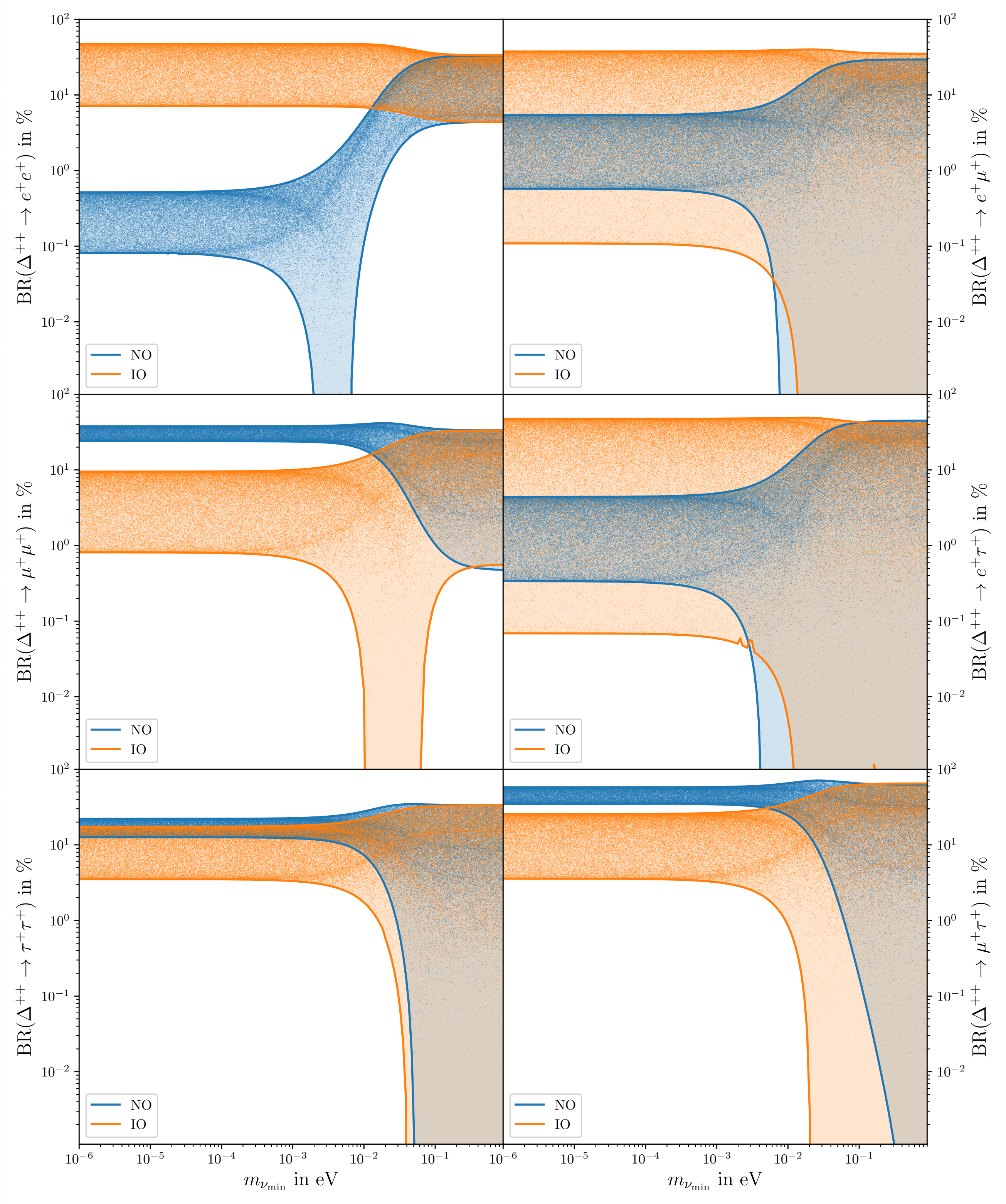
Flavour composition of $\Delta^{++} \to \ell_i^+ \ell_j^+$

strongly depends on the **PMNS input** and neutrino **mass spectrum/ordering**

$$\Gamma_{\Delta^{++}\to\ell_i^+\ell_j^+} = \frac{m_{\Delta^{++}}}{8\pi\left(1+\delta_{ij}\right)} \left|\frac{M_{\nu ij}}{v_\Delta}\right|^2$$

Interference of **PMNS phases** can lead to **funnel regions**





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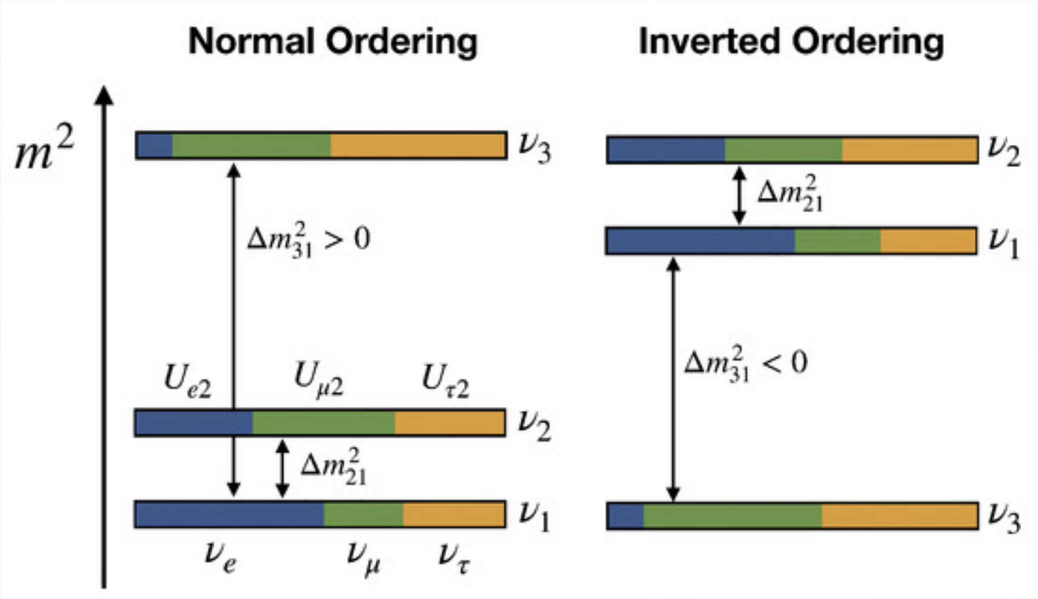
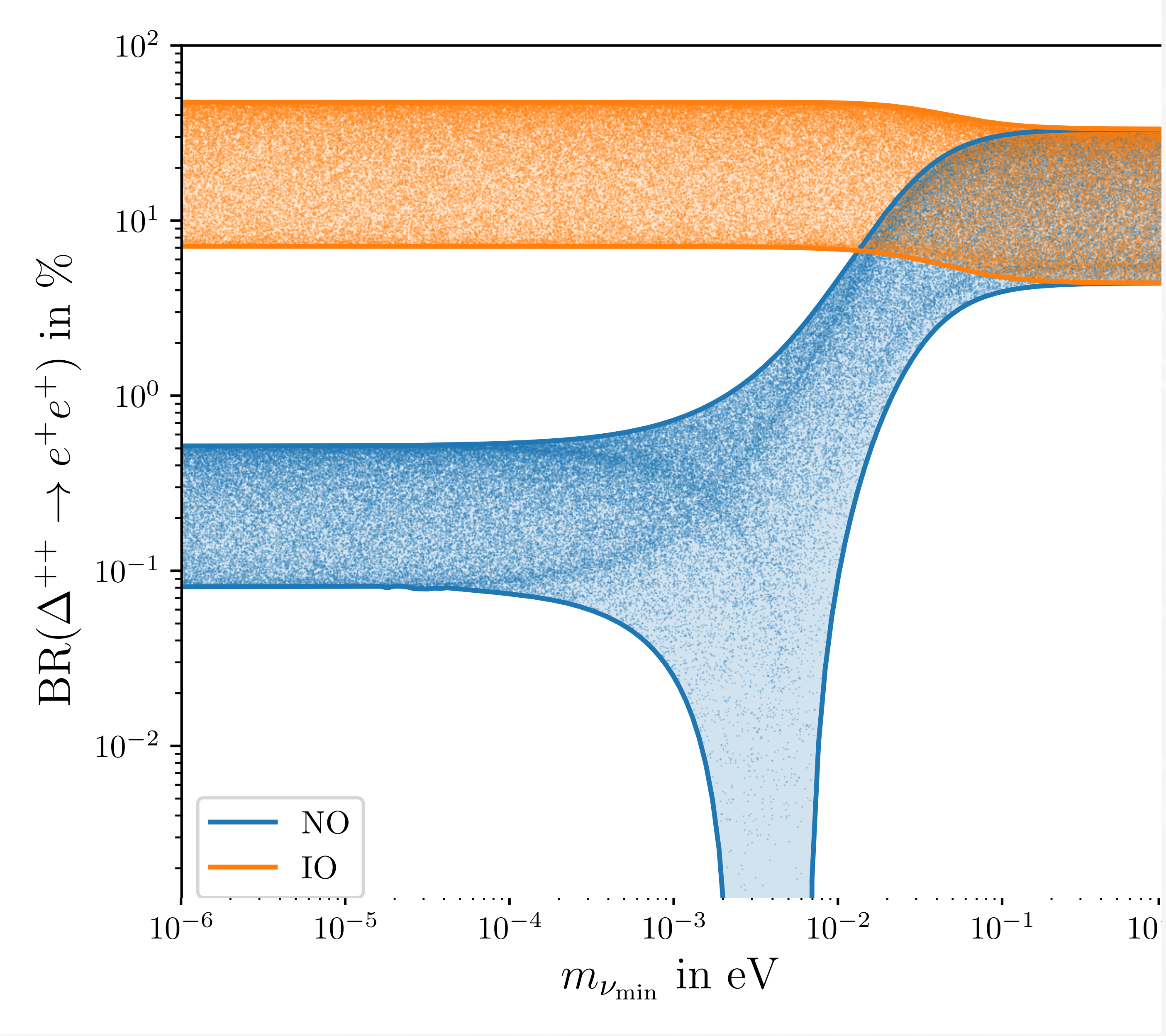
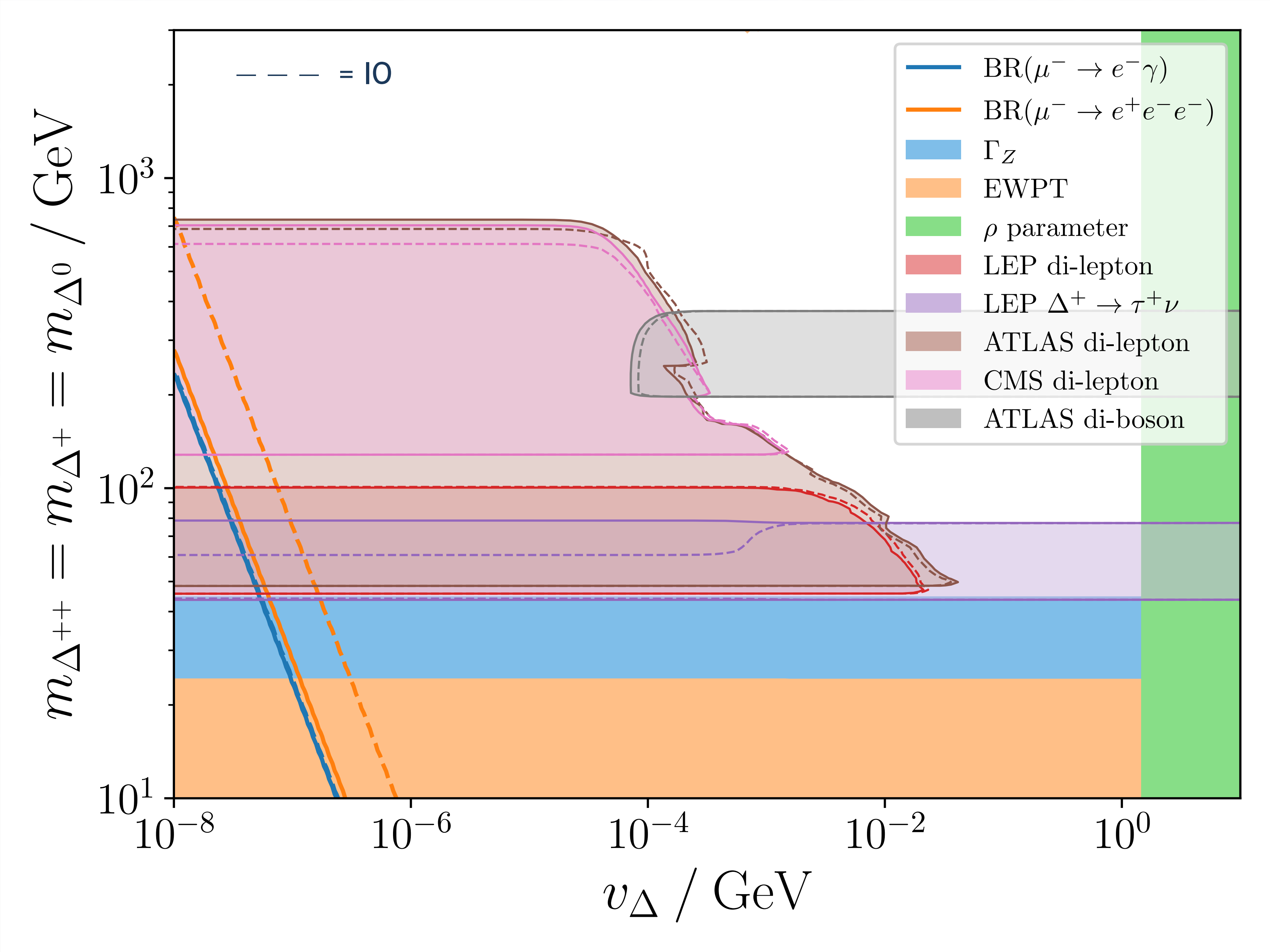




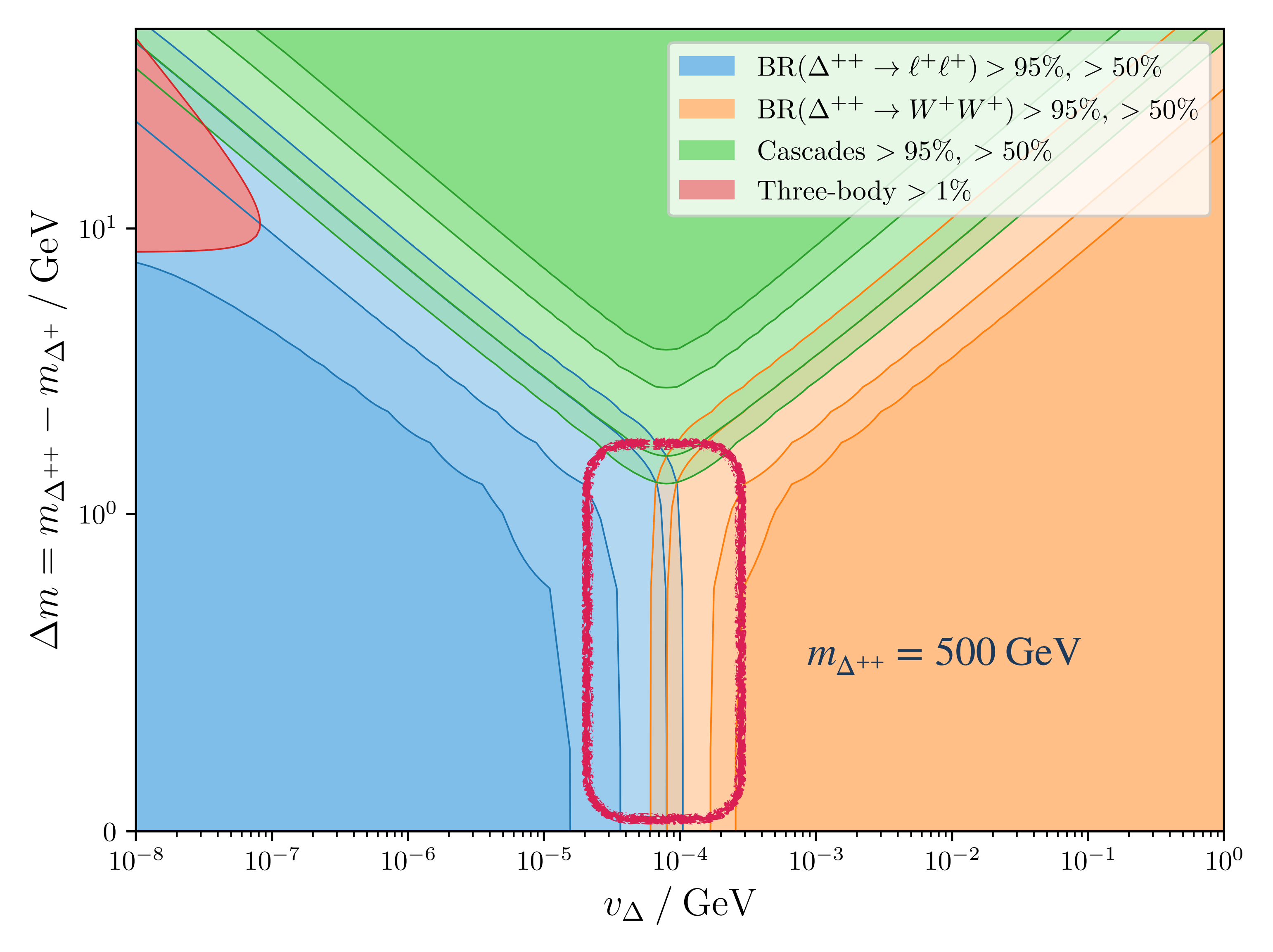
Current state of the art



⇒ LHC searches exclude $m_{\Delta^{++}} \lesssim 700$ GeV for small v_Δ

⇒ Di-boson final states harder to reconstruct, smaller efficiencies

Decay modes of the triplet components

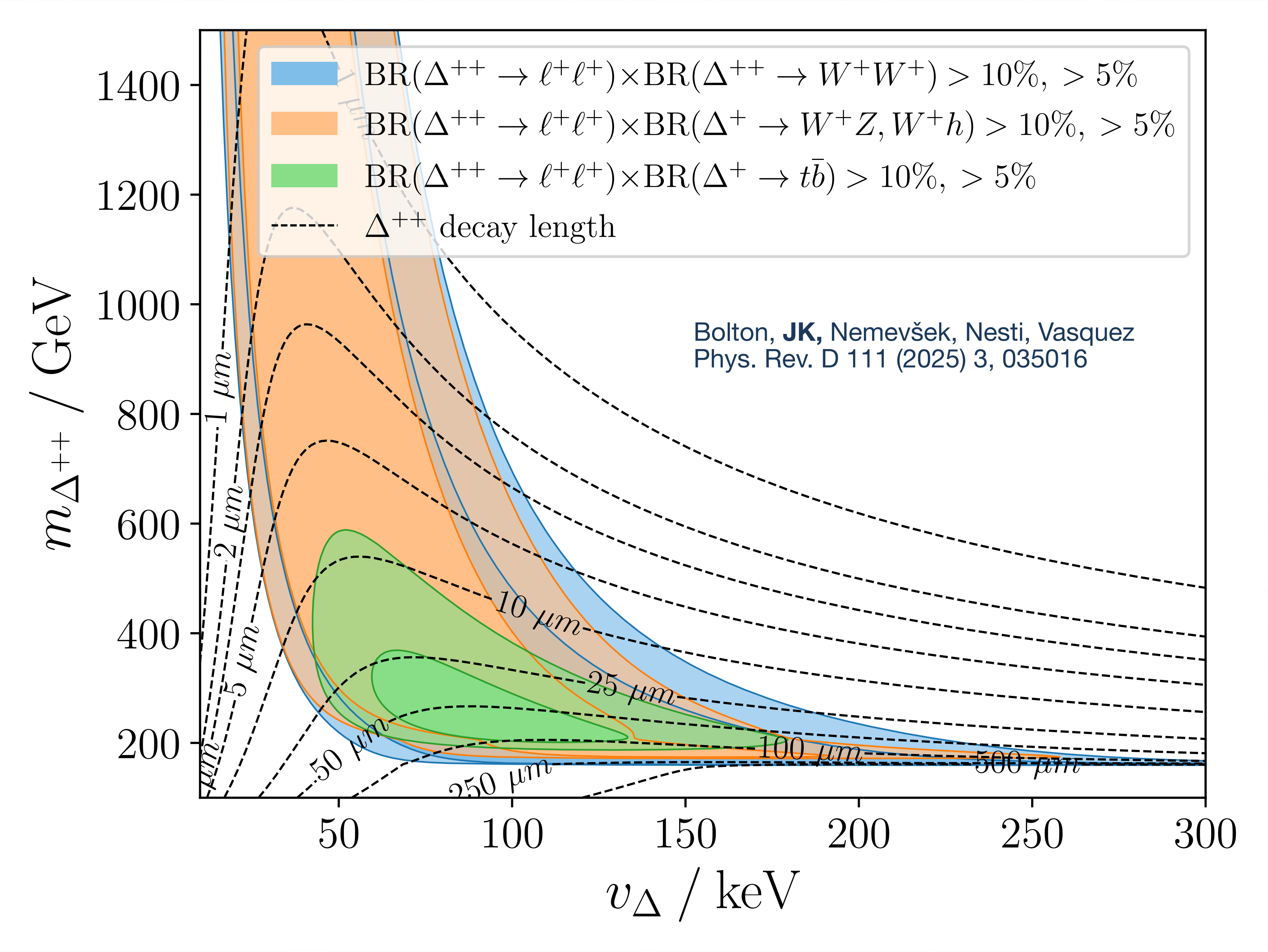


Intermediate region: "**LNV window**" Maeizza, Nemevšek, Nesti '16

Narrow window where $\mathrm{BR}(\Delta^{++} \to \ell_i^+ \ell_j^+) \simeq \mathrm{BR}(\Delta^{++} \to W^+ W^+)$

Leading to **manifestly lepton number violating** final states at colliders: $pp \to \ell_i^\pm \ell_j^\pm W^\mp W^\mp$

The LNV window



⇒ Identify three **different signal processes**

⇒ Mass reach maximal for $v_\Delta \simeq 40 - 50\,\mathrm{keV}$

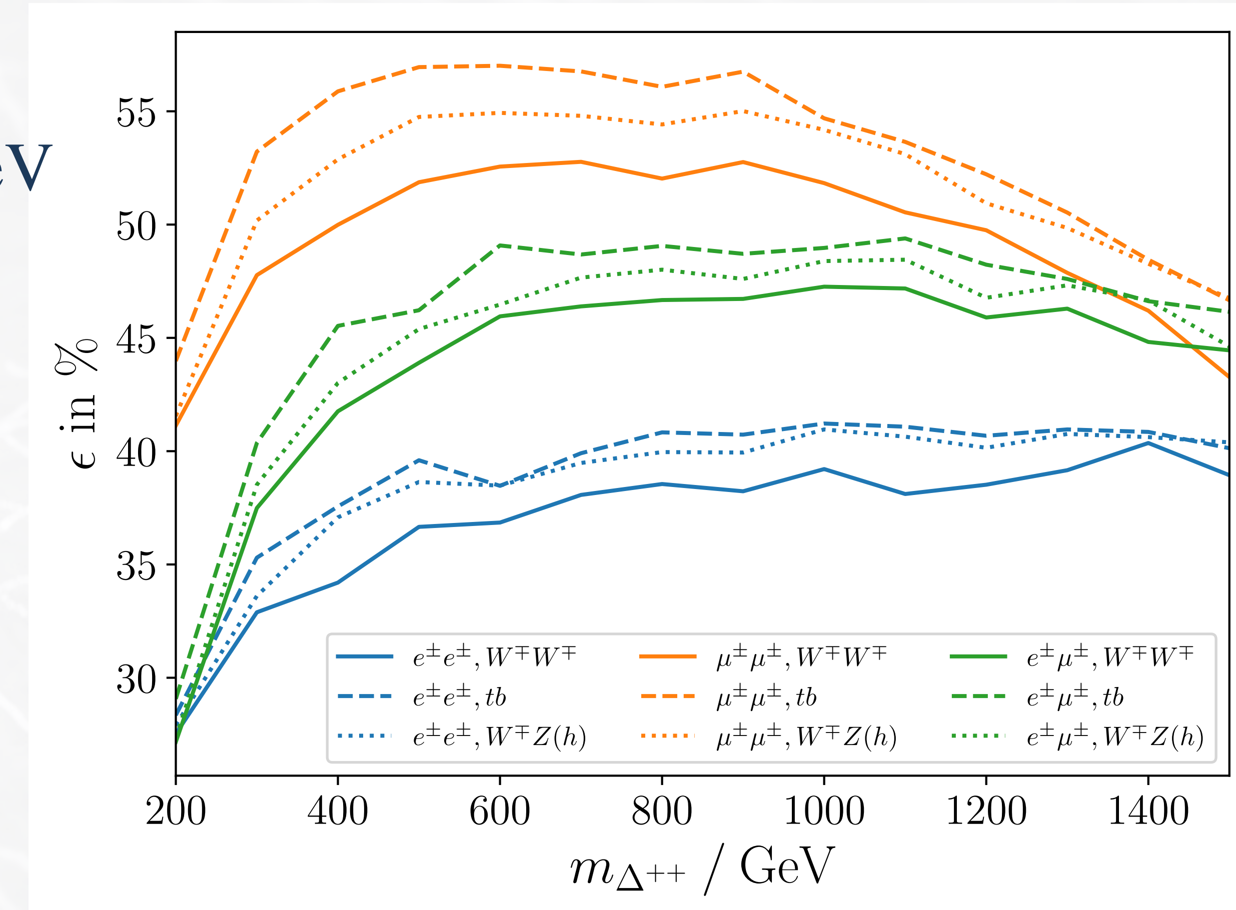
⇒ Decays mostly prompt (except at W threshold)

Accessing the LNV window at (HL)-LHC

Event selection:

- (At least) **2 same-sign leptons** $\ell^{\pm}\ell'^{\pm},\, \ell, \ell' = e, \mu$
- (At least) **2 matched jets** $\Delta R = 0.3\,, p_{Tj\mathrm{min}} = 20\ \mathrm{GeV}$
- Demand $p_{Tj,\ell} > 50\ \mathrm{GeV}$ on **leading lepton/jet**
- **Demand leading leptons** $m_{\ell\ell} \in [0.9{,}1.1]\, m_{\Delta^{++}}$
- Reject $m_{j_1 j_2} > 1.1\, m_{\Delta^{++}}$

Signal efficiencies after cuts



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Dominant backgrounds:

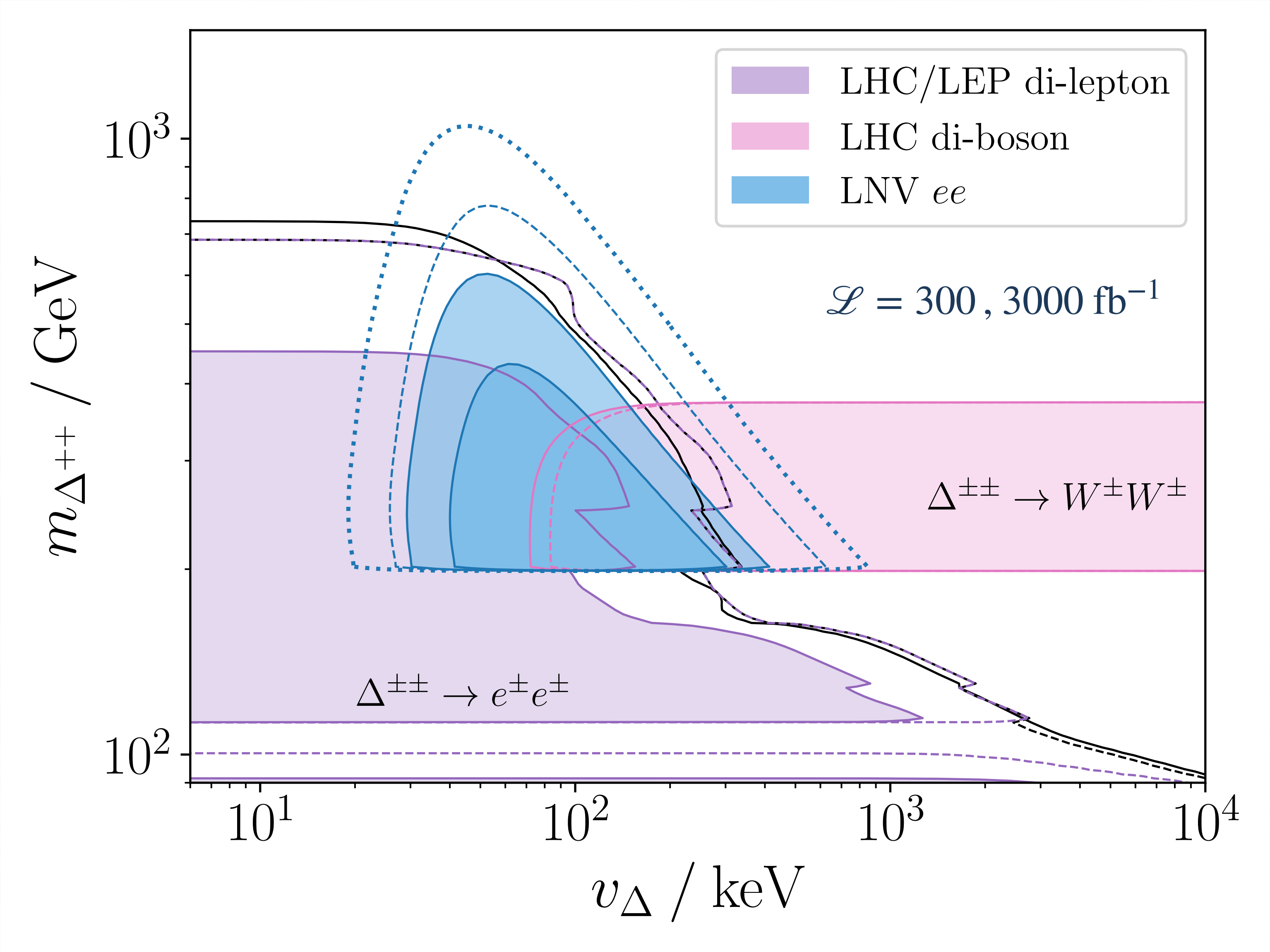
- $pp \to V + 012j,\, pp \to VV + 012j\,,\, V = W^{\pm}, Z$
- $pp \to t\bar{t} + 012j$, ($pp \to VVV + 012j$ found to subdominant)

$\Rightarrow$ Muon final state highest efficiency

Event simulation: Model file adapted from Fuks, Nemevšek, Ruiz [1912.08975]

- Use ***MadGraph5*** (at LO) + ***Pythia8*** + ***Delphes*** (default card) + ***MadAnalysis5*** chain
- Rescaled to NLO in QCD, signals and backgrounds simulated to $100\ \mathrm{fb}^{-1}$

The LNV window — sensitivities



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$e^{\pm}e^{\pm}/\mu^{\pm}\mu^{\pm}$ reach strongly depends on **ordering**

Cover region towards larger m_Δ and v_Δ

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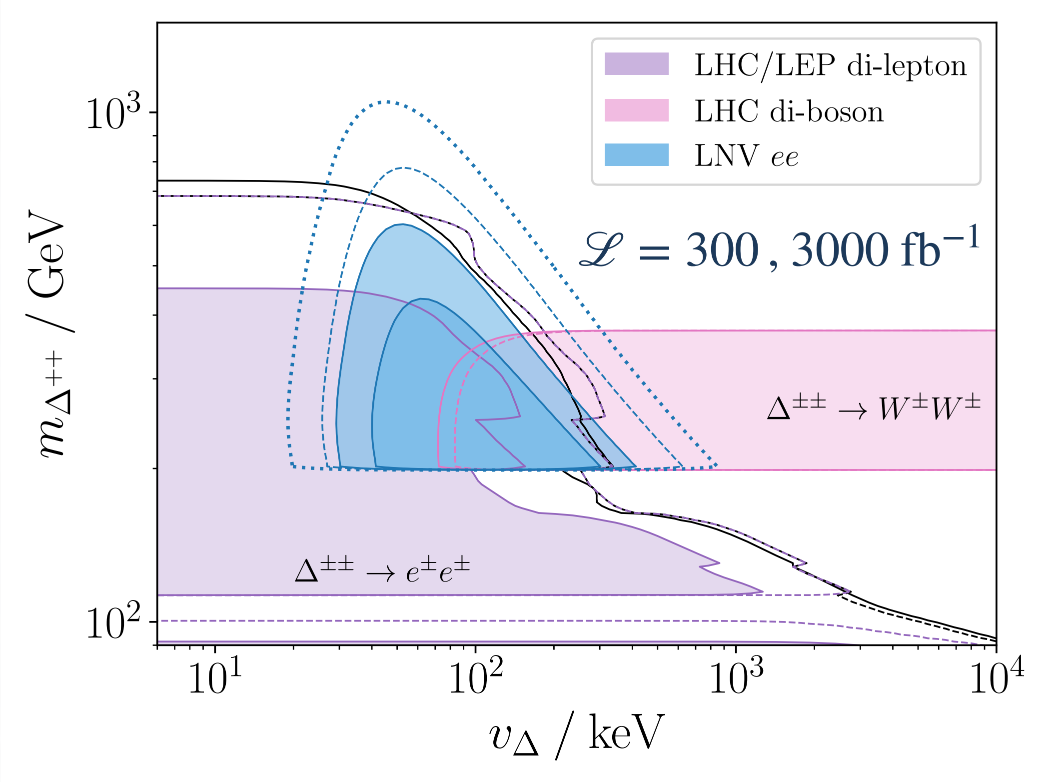

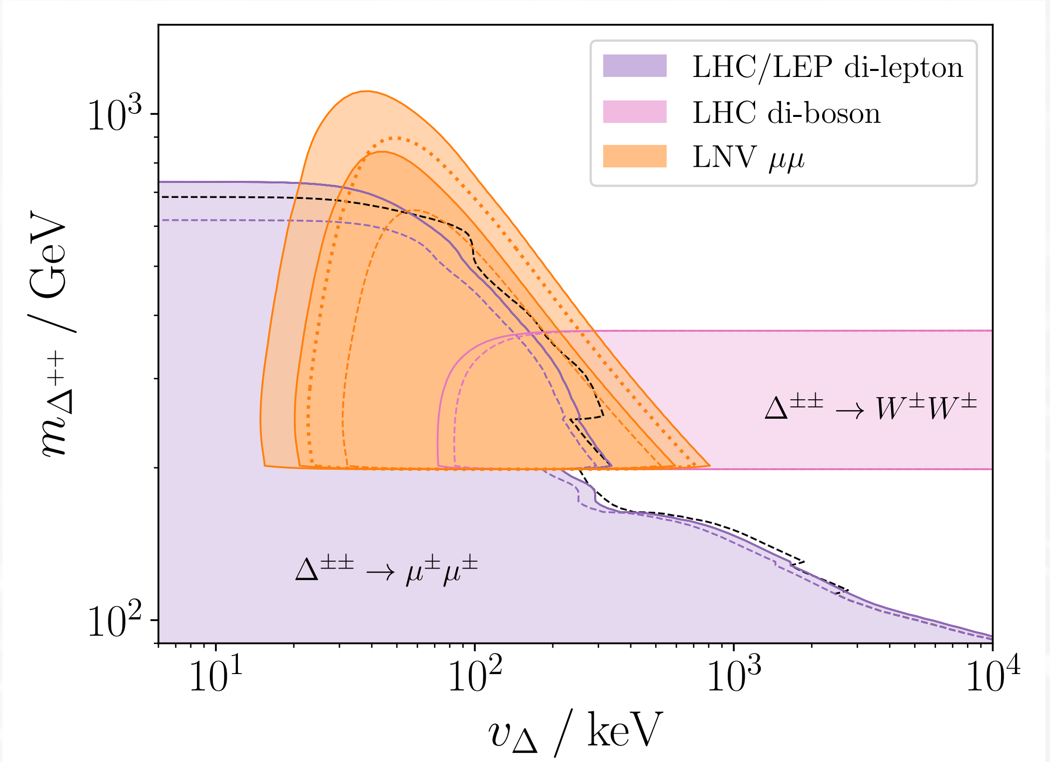

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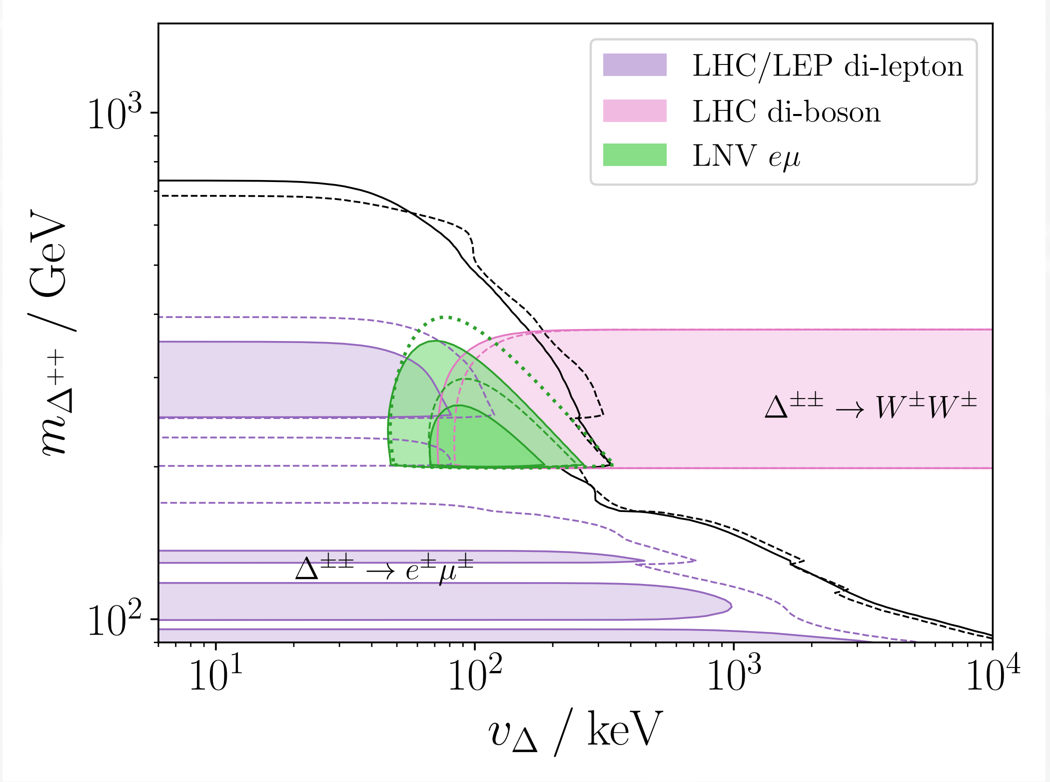

Cover region towards larger m_Δ and v_Δ

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$e\mu$ final state suffers from larger backgrounds

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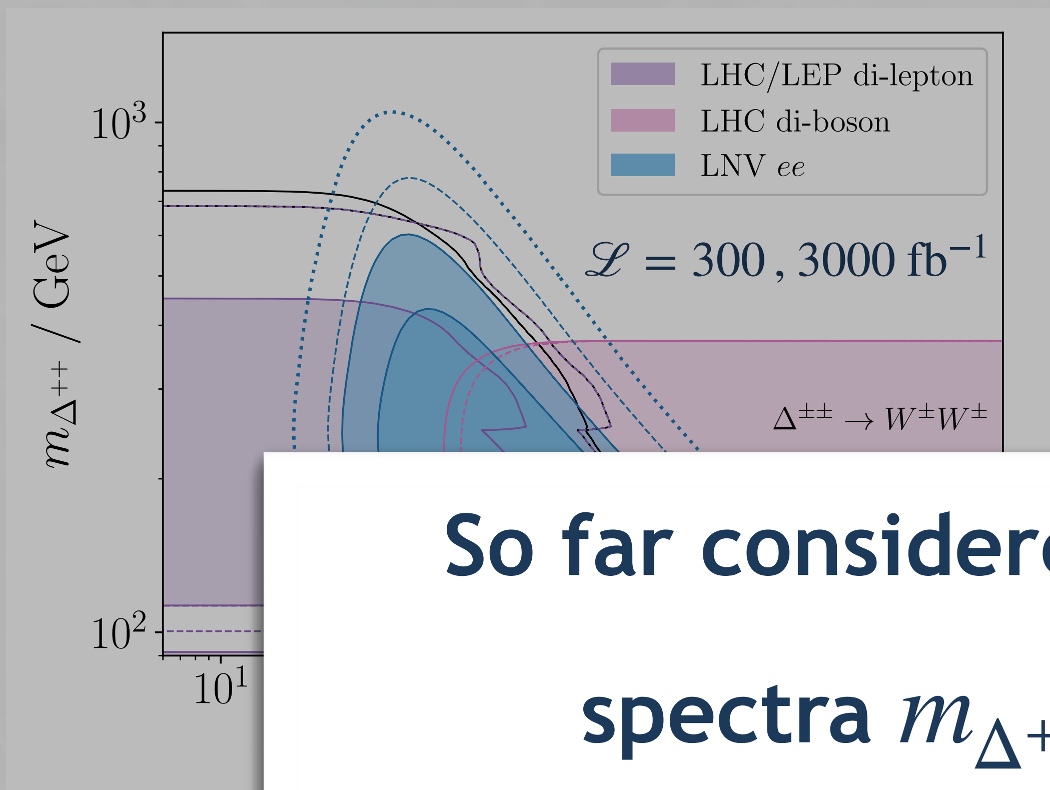
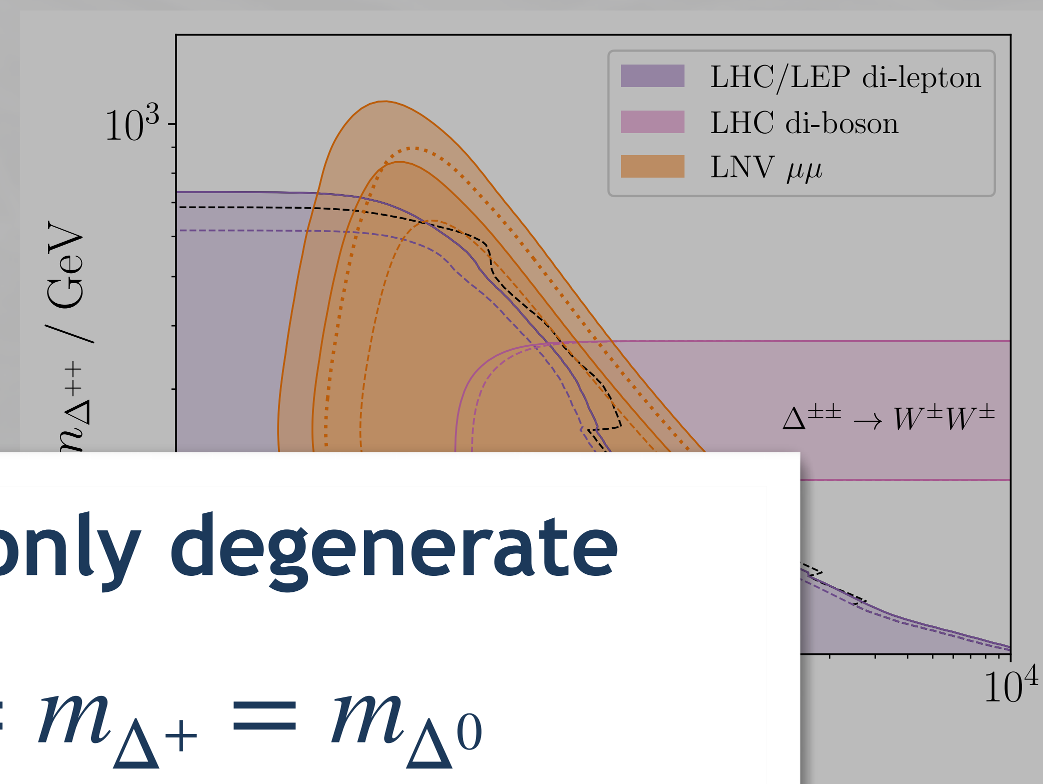




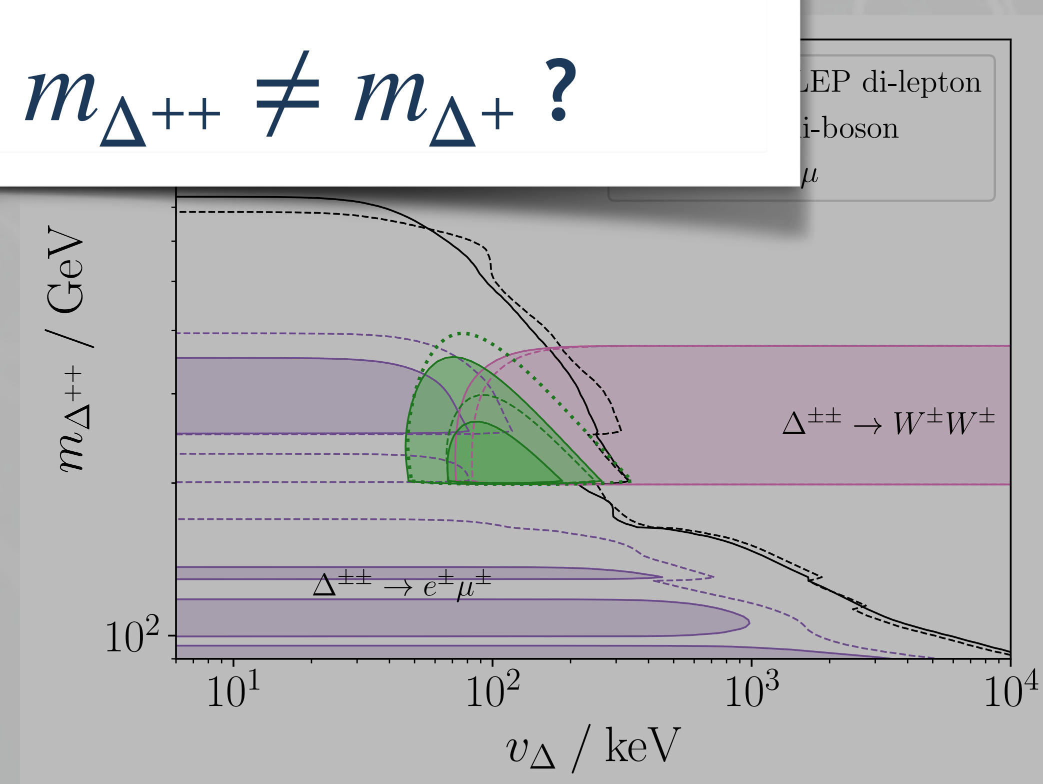
So far considered only degenerate spectra $m_{\Delta^{++}} = m_{\Delta^{+}} = m_{\Delta^{0}}$

What happens for $m_{\Delta^{++}} \neq m_{\Delta^{+}}$?

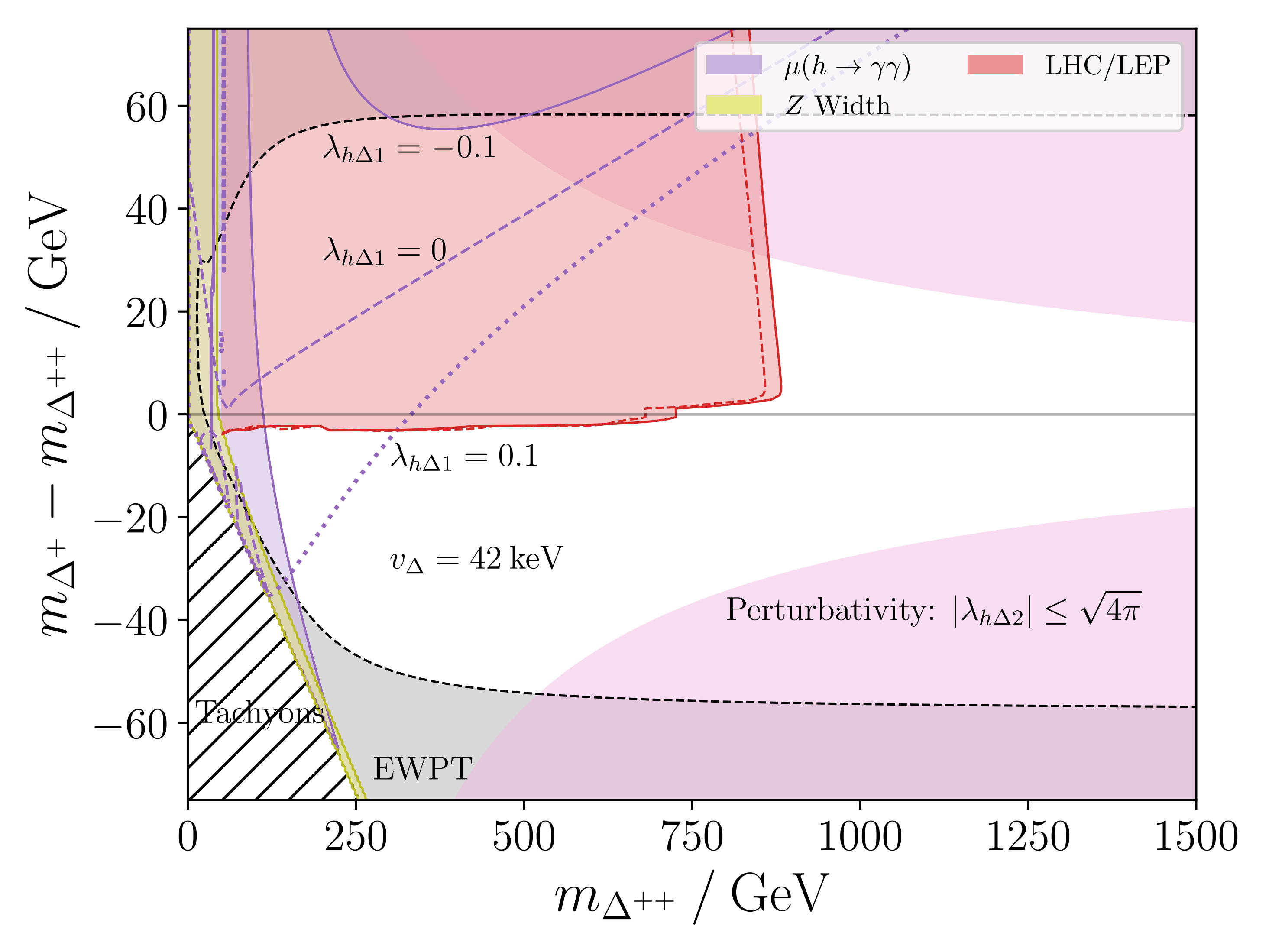
Cover region

$e^{\pm}e^{\pm}/\mu^{\pm}\mu^{\pm}$ reach strongly depends on **ordering**

$e\mu$ final state suffers from larger backgrounds



Switching on cascades



⇒ EWPO and $h \to \gamma\gamma$ strongly depend on **mass splitting**

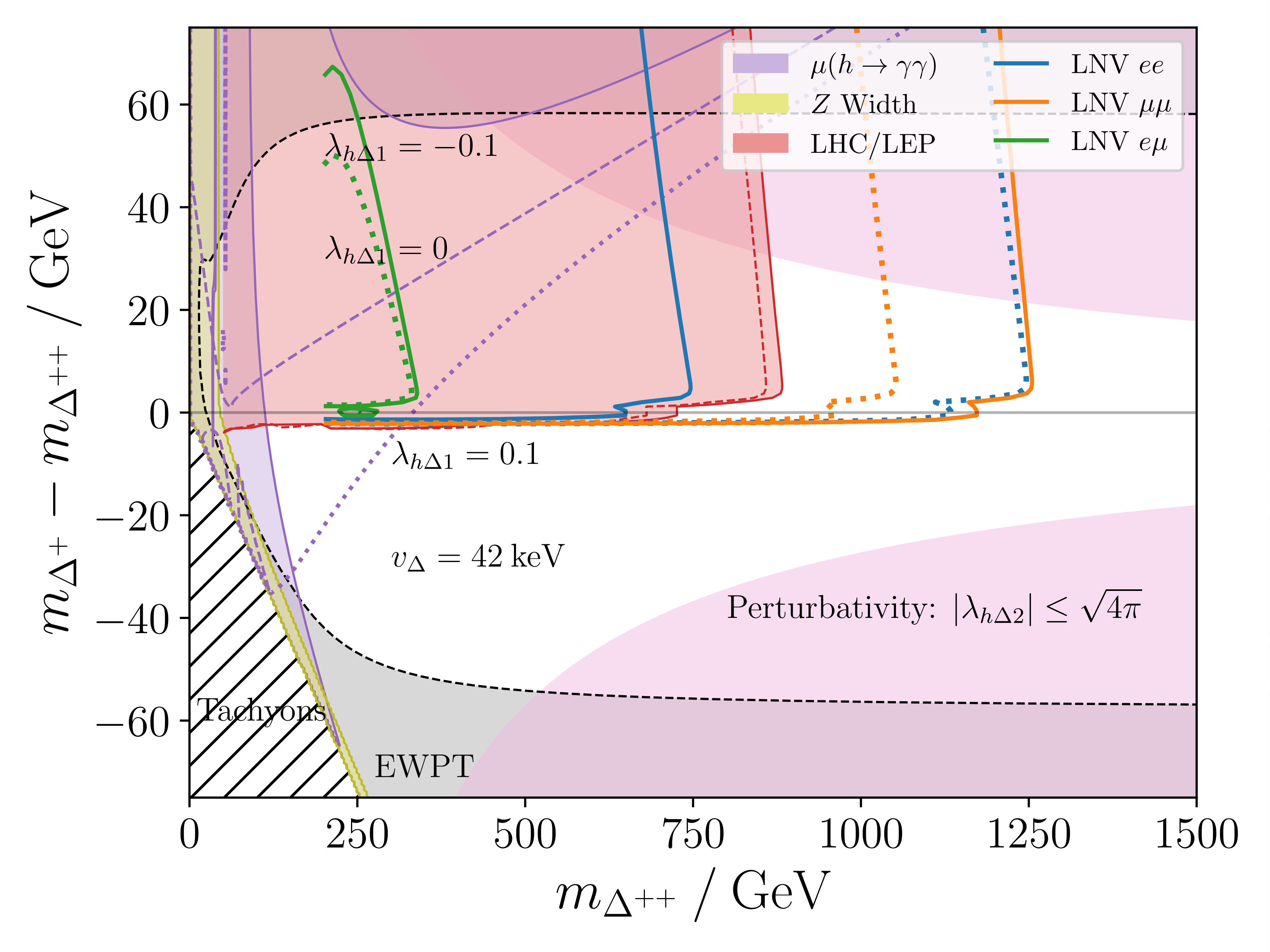
(additional jets mostly soft)

⇒ **New production channels**: e.g. $pp \to \Delta^0(\to \Delta^- jj \to \Delta^{--} jjjj)\, \Delta^+(\to \Delta^{++} jj)$

⇒ **Increase or decrease mass reach:** $\sigma \times \mathrm{BR}$ **tends quickly to 0** for $m_{\Delta^{++}} > m_{\Delta^+}$

Switching on cascades

Bolton, **JK,** Nemevšek, Nesti, Vasquez Phys. Rev. D 111 (2025) 3, 035016



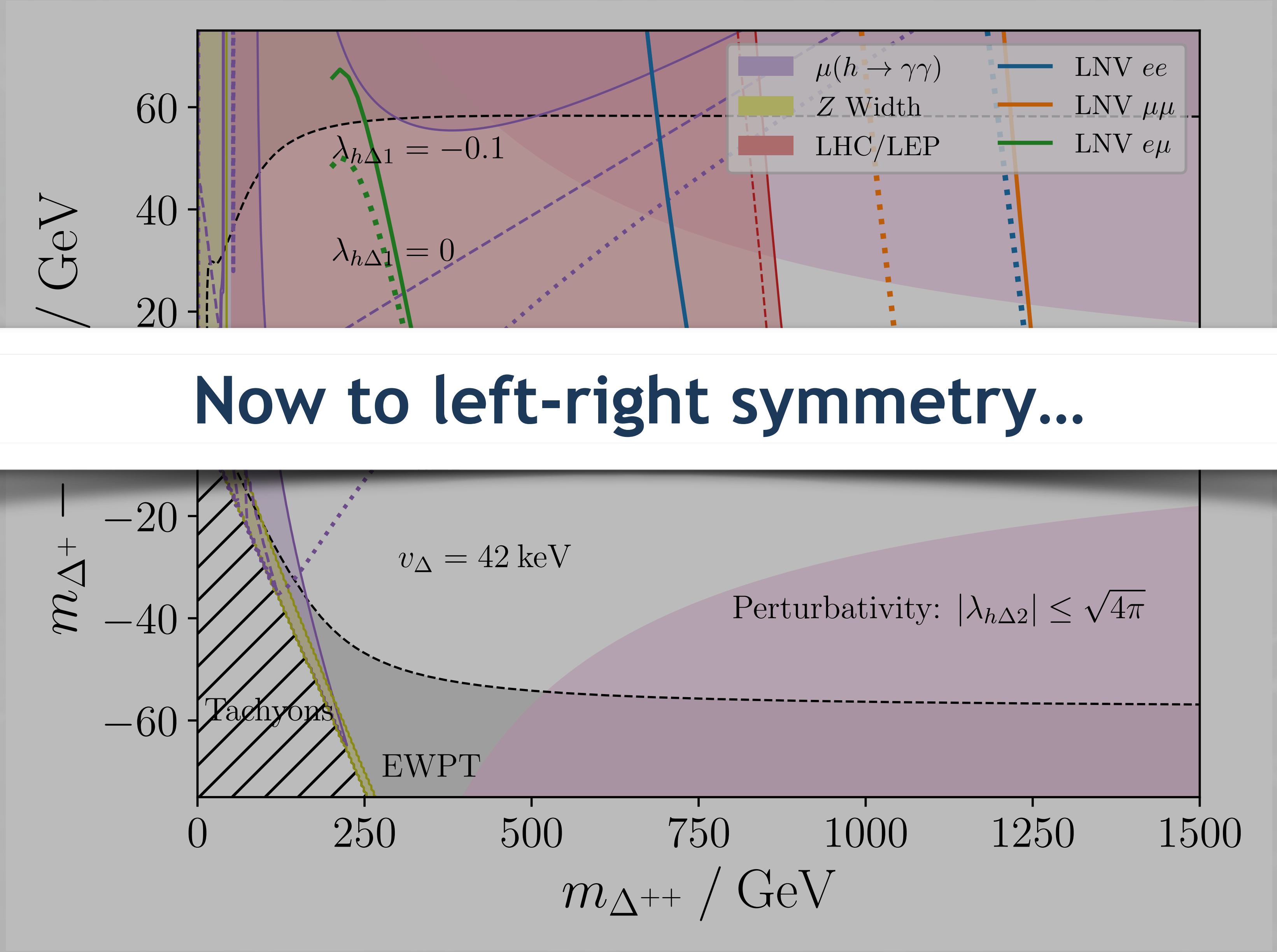
$\Rightarrow$ **Increase or decrease mass reach:** $\sigma \times \mathrm{BR}$ **tends quickly to 0** for $m_{\Delta^{++}} > m_{\Delta^+}$

Existing searches: $m_{\Delta^{++}} \gtrsim 900$ GeV

LNV window: $m_{\Delta^{++}} \gtrsim 1300$ GeV

Switching on cascades

Bolton, **JK,** Nemevšek, Nesti, Vasquez Phys. Rev. D 111 (2025) 3, 035016



Now to left-right symmetry…

⇒ **Increase or decrease mass reach:** $\sigma \times \mathrm{BR}$ **tends quickly to 0** for $m_{\Delta^{++}} > m_{\Delta^+}$

Existing searches: $m_{\Delta^{++}} \gtrsim 900$ GeV

LNV window: $m_{\Delta^{++}} \gtrsim 1300$ GeV

Introducing Left-Right: Motivation

Features:

Mohapatra, Senjanović '75

- Combination of **type I** & **type II** seesaw mechanism, new states $\sim \mathcal{O}(\mathrm{TeV})$
- Can address the **strong CP problem** (see e.g. [2107.10852])
- Lightest **right-handed neutrino** can be a **Dark Matter candidate** [2312.00129]
- Low(ish)-scale **leptogenesis** can be implemented [C. Hati et al. '18]
- Left-right symmetry $\mathcal{G}_{\mathrm{LR}} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

appears in the breaking of **GUTs**, e.g.:

$$SO(10) \to SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \to \mathcal{G}_{LR} \to \mathcal{G}_{\mathrm{SM}}$$

Introducing Left-Right: Model overview

SM Gauge group is extended: $\mathcal{G}_{\mathrm{LR}} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

Right-handed SM fermion singlets are promoted to $SU(2)_R$**-doublets**

$\Rightarrow$ **Add RH neutrinos,** $U(1)_{B-L}$-anomalies automatically cancelled

(E6 models)

Scalar sector: different (minimal) possibilities, bi-doublet + 2 doublets

or (like here) bi-doublet + 2 triplets

Physical spectrum: **SM +** $N_R\,, W_R^{\pm}\,, Z_R\,, \Delta_{R,L}^{\pm\pm}\,, \Delta_L^{+}\,, \Delta_L^{0}\,, \chi_L^{0}\,, \Delta_R^{0}\,, A^0\,, H^0\,, H^{\pm}$

Field content: $\mathcal{G}_{\mathrm{LR}} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

Fermions: $Q_{L,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}$, $L_{L,R} \begin{pmatrix} \nu \\ \ell \end{pmatrix}$ **(3 generations)**

Gauge Fields: $SU(2)_{L,R}$-gauge fields, $A_{L,R} = A^a_{L,R}\frac{\sigma^a}{2}$, $A^{\pm}_{L,R} = \frac{A^1_{L,R} \mp iA^2_{L,R}}{\sqrt{2}}$

$U(1)_{B-L}$-gauge field B + **QCD** $SU(3)_c$

Scalar Fields: $SU(2)_{L,R}$ triplets, $\Delta_{L,R} = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$, $(1,3,1,2)_L$ & $(1,1,3,2)_R$

$SU(2)_{L,R}$ bi-doublet, $\phi = \begin{pmatrix} \phi_1^{0*} & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}$, **(1, 2, 2, 0)**

Electrical charge: $Q = T^3_L + T^3_R + \frac{B-L}{2}$

Making Neutrino Masses

Majorana Mass from $SU(2)_R$ breaking

Discrete $\mathscr{C}$-symmetry:

$\mathscr{C} : \phi \leftrightarrow \phi^T \,, \Delta_L \leftrightarrow \Delta_R^*$

$$\mathscr{L}_Y^\ell \supseteq \bar{L}_L'(Y_\ell \phi + \tilde{Y}_\ell \tilde{\phi}) L_R' + \bar{L}_L'^c i\sigma_2 \, \Delta_L \, Y_L^M L_L' + L_R'^c i\sigma_2 \, \Delta_R \, Y_R^M L_R'$$

$$\Rightarrow Y_\ell = Y_\ell^T \,, \; \tilde{Y}_\ell = \tilde{Y}_\ell^T \,, \; Y_L^M = Y_R^M \,, \; M_D = M_D^T \,, \; M_L = \frac{v_L}{v_R} M_R$$

From the light and heavy masses

$$M_\nu \simeq M_L - M_D M_R^{-1} M_D^T \,, \qquad M_N \simeq M_R$$

All Yukawas **fully determined** by measurable inputs $(m_{\nu_i}, m_{N_i}, \mathscr{U}_L, \mathscr{U}_R)$

$$M_D = M_N \sqrt{\frac{v_L}{v_R} \mathbb{1} - M_N^{-1} M_\nu}$$

Nemevšek, Senjanović, Tello PRL'13

Analytical matrix square-root

JK, Nemevšek, Nesti EPJC'24

$$\sqrt{A} = c_0 \, \mathbb{1} + c_1 \, A + c_2 \, A.A$$

c_i are functions of invariants of A

Diagonalising the Lagrangian: Scalar sector

The most general $\mathscr{P}$- (and $\mathscr{C}$-) symmetric potential is given by:

$$\begin{aligned}
\mathcal{V} = & -\mu_1^2\left[\phi^\dagger\phi\right] - \mu_2^2\left(\left[\tilde{\phi}\phi^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\right) - \mu_3^2\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right) \\
& + \lambda_1\left[\phi^\dagger\phi\right]^2 + \lambda_2\left(\left[\tilde{\phi}\phi^\dagger\right]^2 + \left[\tilde{\phi}^\dagger\phi\right]^2\right) + \lambda_3\left[\tilde{\phi}\phi^\dagger\right]\left[\tilde{\phi}^\dagger\phi\right] + \lambda_4\left[\phi^\dagger\phi\right]\left(\left[\tilde{\phi}\phi^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\right) \\
& + \rho_1\left(\left[\Delta_L\Delta_L^\dagger\right]^2 + \left[\Delta_R\Delta_R^\dagger\right]^2\right) + \rho_2\left(\left[\Delta_L\Delta_L\right]\left[\Delta_L^\dagger\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right]\right) + \rho_3\left[\Delta_L\Delta_L^\dagger\right]\left[\Delta_R\Delta_R^\dagger\right] \\
& + \rho_4\left(\left[\Delta_L\Delta_L\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right] + \left[\Delta_L^\dagger\Delta_L^\dagger\right]\left[\Delta_R\Delta_R\right]\right) + \alpha_1\left[\phi^\dagger\phi\right]\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right) \\
& + \left(\alpha_2\left(\left[\tilde{\phi}\phi^\dagger\right]\left[\Delta_L\Delta_L^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\left[\Delta_R\Delta_R^\dagger\right]\right) + \text{h.c.}\right) + \alpha_3\left(\left[\phi\phi^\dagger\Delta_L\Delta_L^\dagger\right] + \left[\phi^\dagger\phi\Delta_R\Delta_R^\dagger\right]\right) \\
& + \beta_1\left(\left[\phi\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_2\left(\left[\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_3\left(\left[\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger\right]\right)
\end{aligned}$$

The minimisation conditions $\frac{\partial\mathscr{V}}{\partial S_i} = 0$ and $\frac{\partial^2\mathscr{V}}{\partial S_i\partial S_j} > 0$ lead us to:

$$\begin{aligned}
\mu_1^2 &= 2\left(\lambda_1 + s_{2\beta}c_\alpha\lambda_4\right)v^2 + \left(\alpha_1 - \alpha_3\frac{s_\beta^2}{c_{2\beta}}\right)v_R^2\,, \\
\mu_2^2 &= \left(s_{2\beta}\left(2c_{2\alpha}\lambda_2 + \lambda_3\right) + \lambda_4\right)v^2 \\
&\quad + \frac{1}{2c_\alpha}\left(2c_{\alpha+\delta_2}\alpha_2 + \alpha_3\frac{t_{2\beta}}{2c_\alpha}\right)v_R^2\,, \\
\mu_3^2 &= \left(\alpha_1 + \left(2c_{\alpha+\delta_2}\alpha_2 s_{2\beta} + \alpha_3 s_\beta^2\right)\right)v^2 + 2\rho_1 v_R^2 \\
\alpha_2 s_{\delta_2} &= \frac{s_\alpha}{4}\left(\alpha_3 t_{2\beta} + 4\left(\lambda_3 - 2\lambda_2\right)s_{2\beta}\epsilon^2\right).
\end{aligned}$$

$$\begin{aligned}
v_L = &\frac{\epsilon^2 v_R}{\left(1+t_\beta^2\right)\left(2\rho_1 - \rho_3\right)}\Big(-\beta_1 t_\beta\cos(\alpha-\theta_L) \\
&+ \beta_2\cos(\theta_L) + \beta_3 t_\beta^2\cos(2\alpha-\theta_L)\Big)\,.
\end{aligned}$$

For exact solvability we assume $\beta_i = v_L = 0$ and keep only the phase δ_2 (no impact on collider pheno)

In any case: $v_L \ll v \ll v_R \simeq \mathscr{O}(\text{TeV})$

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

Let's start with the "easy" ones that don't mix (in units of v_R) :

$$m^2_{\Delta_R^{++}} = 4\rho_2 + \frac{c_{2\beta}}{c_\beta^4}\alpha_3\epsilon^2\,,$$

$v_L = 0 \Rightarrow$ no mixing of $\Delta_L\,, \Delta_R^{++}$

$$m^2_{\Delta_L^{++}} = (\rho_3 - 2\rho_1) - \frac{t_\beta^4 - 2c_{2\alpha}t_\beta^2 + 1}{t_\beta^4 - 1}\alpha_3\epsilon^2\,,$$

$$m^2_{\Delta_L^{+}} = (\rho_3 - 2\rho_1) - \frac{\left(t_\beta^2 + 1\right)^2 - 4t_\beta^2 c_{2\alpha}}{2\left(t_\beta^4 - 1\right)}\alpha_3\epsilon^2\,,$$

$$m^2_{\Delta_L^{0}} = m^2_{\chi_L^0} = (\rho_3 - 2\rho_1) + s_{2\beta}t_{2\beta}s_\alpha^2\alpha_3\epsilon^2\,,$$

Take as input parameters: $m_{\Delta_R^{++}}$, $m_{\Delta_L^0}$, (and $\tan\beta$ and α), solve for $\rho_{2,3}$

ρ_1 and α_3 are fixed by other masses

$\Rightarrow$ Mass spectrum of Δ_L follows a sum rule:

$$m^2_{\Delta_L^{++}} - m^2_{\Delta_L^{+}} = m^2_{\Delta_L^{+}} - m^2_{\Delta_L^{0}} = v^2\alpha_3\frac{c_{2\beta}}{2}$$

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

Now the singly charged scalars:

$$(\phi_1^-, \phi_2^-, \Delta_R^-)\, M_+^2 \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \Delta_R^+ \end{pmatrix} \quad \text{with} \quad M_+^2 = \alpha_3 v_R^2 \begin{pmatrix} \frac{s_\beta^2}{c_{2\beta}} & -e^{-i\alpha}\frac{t_{2\beta}}{2} & -\epsilon e^{-i\alpha}\frac{s_\beta}{\sqrt{2}} \\ -e^{i\alpha}\frac{t_{2\beta}}{2} & \frac{c_\beta^2}{c_{2\beta}} & \epsilon\frac{c_\beta}{\sqrt{2}} \\ -\epsilon e^{i\alpha}\frac{s_\beta}{\sqrt{2}} & \epsilon\frac{c_\beta}{\sqrt{2}} & \epsilon^2\frac{c_{2\beta}}{2} \end{pmatrix}$$

M_+ is diagonalised with a unitary rotation (up to $\mathscr{O}(\epsilon^2)$:

$$\begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \Delta_R^+ \end{pmatrix} = U_+ \begin{pmatrix} \varphi_L^+ \\ H^+ \\ \varphi_R^+ \end{pmatrix}$$

$$\begin{aligned} U_+ &= \begin{pmatrix} c_\beta & -e^{-i\alpha}s_\beta & 0 \\ e^{i\alpha}s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1+\frac{1}{4}\epsilon^2 c_{2\beta}^2 & \frac{\epsilon c_{2\beta}}{\sqrt{2}} \\ 0 & -\frac{\epsilon c_{2\beta}}{\sqrt{2}} & -1+\frac{1}{4}\epsilon^2 c_{2\beta}^2 \end{pmatrix} \begin{pmatrix} 1-\frac{1}{4}\epsilon^2 s_{2\beta}^2 & 0 & \frac{\epsilon s_{2\beta}}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{\epsilon s_{2\beta}}{\sqrt{2}} & 0 & 1-\frac{1}{4}\epsilon^2 s_{2\beta}^2 \end{pmatrix} \\ &\simeq \begin{pmatrix} c_\beta & e^{-i\alpha}s_\beta & 0 \\ e^{i\alpha}s_\beta & -c_\beta & 0 \\ 0 & 0 & -1 \end{pmatrix} + \frac{\epsilon}{\sqrt{2}} \begin{pmatrix} 0 & 0 & e^{-i\alpha}s_\beta \\ 0 & 0 & c_\beta \\ e^{i\alpha}s_{2\beta} & -c_{2\beta} & 0 \end{pmatrix} + \frac{\epsilon^2}{2} \begin{pmatrix} -4c_\beta s_\beta^4 & -e^{-i\alpha}s_\beta c_{2\beta}^2 & 0 \\ -4e^{i\alpha}s_\beta c_\beta^4 & c_\beta c_{2\beta}^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

$\Rightarrow$ $\varphi_{L,R}^\pm$ are the goldstones of $W_{L,R}^\pm$ and remain massless

The remaining (fixed) mass is

$$m_{H^+} \simeq \sqrt{\frac{\alpha_3}{c_{2\beta}}}\, v_R \left(1 + \epsilon^2 \frac{c_{2\beta}^{3/2}}{4}\right)$$

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

Goldstones are decoupled in an independent rotation

We are left with 4 neutral states in the basis $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})$

$$m_0^2 = \begin{pmatrix} 4\epsilon^2\left(\lambda_1 + \frac{4tc_\alpha\left(\lambda_4\left(t^2+1\right)+4\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2}\right) & 2\epsilon\left(\alpha_1 - \frac{t^2X\left(t^2-s_{2\alpha+\delta_2}/s_{\delta_2}\right)}{\left(t^2+1\right)^2}\right) & \frac{4\epsilon^2\left(t^2c_{2\alpha}-1\right)\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} & \frac{4t^2\epsilon^2 s_{2\alpha}\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} \\ 2\epsilon\left(\alpha_1 - \frac{t^2X\left(t^2-s_{2\alpha+\delta_2}/s_{\delta_2}\right)}{\left(t^2+1\right)^2}\right) & Y & \frac{2tX\epsilon\left(t^2c_{2\alpha}-1\right)s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} & \frac{2t^3X\epsilon s_{2\alpha}s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} \\ \frac{4\epsilon^2\left(t^2c_{2\alpha}-1\right)\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} & \frac{2tX\epsilon\left(t^2c_{2\alpha}-1\right)s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} & X + \frac{16\lambda_2\epsilon^2\left(t^2c_{2\alpha}-1\right)^2}{\left(t^2+1\right)^2} & \frac{16\lambda_2 t^2\epsilon^2 s_{2\alpha}\left(t^2c_{2\alpha}-1\right)}{\left(t^2+1\right)^2} \\ \frac{4t^2\epsilon^2 s_{2\alpha}\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} & \frac{2t^3X\epsilon s_\alpha s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} & \frac{16\lambda_2 t^2\epsilon^2 s_{2\alpha}\left(t^2c_{2\alpha}-1\right)}{\left(t^2+1\right)^2} & X + \frac{16\lambda_2 t^4\epsilon^2 s_{2\alpha}^2}{\left(t^2+1\right)^2} \end{pmatrix}$$

First we decouple the **SM**-like Higgs h from the rest via a 2-1 rotation around θ:

$$m_h^2 = v^2\left(4\lambda_1 + \frac{64\lambda_2 t^2 c_\alpha^2}{\left(t^2+1\right)^2} + \frac{16\lambda_4 t c_\alpha}{t^2+1} - Y\tilde{\theta}^2\right)$$

$$\tilde{\theta} = \frac{\theta}{\epsilon} = \left(\frac{2\alpha_1}{Y} + \frac{X}{Y}\frac{t^2(1-t^2)}{1+t^2}\frac{\sin(2\alpha+\delta_2)}{\sin(\delta_2)}\right)$$

m_h and θ will be taken as input to solve for λ_1 and α_1

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

We are left with 4 neutral states in the basis $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})$

Remarkably, setting $\lambda_3 = 2\lambda_2$ allows to determine the remaining rotations *exactly*:

We rotate $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})^T = O_N(h, \Delta_R, H, A)^T$

$$\theta \equiv \epsilon\,\tilde{\theta} \equiv \theta_{21} = \epsilon\left[\frac{2\alpha_1}{Y} - \frac{2X\left(t^4 - t^2\, s_{2\alpha+\delta_2}/s_{\delta_2}\right)}{Y\left(t^2+1\right)^2}\right],$$

$$\phi \equiv \epsilon^2\tilde{\phi} \equiv \theta_{31} = \epsilon^2\frac{\left(t^2 c_{2\alpha}-1\right)}{(1+t^2)^2}\left[\frac{32tc_\alpha\lambda_2 + 4\lambda_4(1+t^2)}{X} - 2\,t\,\tilde{\theta}\,s_{\alpha+\delta_2}/s_{\delta_2}\right],$$

$$\theta_{41} = \epsilon^2\frac{t^2 s_{2\alpha}}{(1+t^2)^2}\left[\frac{32tc_\alpha\lambda_2 + 4\lambda_4(t^2+1)}{X} - 2\,t\,\tilde{\theta}\,s_{\alpha+\delta_2}/s_{\delta_2}\right],$$

$$\theta_{34} = \cot^{-1}\left[\cot(2\alpha) - \frac{\csc(2\alpha)}{t^2}\right],$$

$$\eta \equiv \theta_{23} = -\frac{1}{2}\tan^{-1}\left[\frac{4tX\epsilon\sqrt{t^4-2c_{2\alpha}t^2+1}\,s_{\alpha+\delta_2}/s_{\delta_2}}{(t^2+1)^2\left(Y\tilde{\theta}^2\epsilon^2 - \frac{16(t^4-2c_{2\alpha}t^2+1)\lambda_2\epsilon^2}{(t^2+1)^2} - X + Y\right)}\right]$$

$$h \text{ part of } \Re\Delta_R: \quad \theta \equiv \theta_{21} \simeq -(O_N)_{2,1}\,,$$

$$H \text{ part of } \Re\Delta_R: \quad \eta \equiv \theta_{23} = \arcsin[(O_N)_{2,3}/c_\theta]\,,$$

$$h \text{ part of } \Re\phi_{20}: \quad \phi \equiv \theta_{31} \simeq -(O_N)_{3,1}\,,$$

θ, ϕ, η can be taken as **input** parameters!

Mixing angles also control scalar couplings to SM-gauge bosons & quarks!

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

We rotate $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})^T = O_N(h, \Delta_R, H, A)^T$ and get the mass eigenvalues:

$$m_h^2 = \epsilon^2 \left(4\lambda_1 + \frac{64\lambda_2 t^2 c_\alpha^2}{(t^2+1)^2} + \frac{16\lambda_4 t c_\alpha}{t^2+1} - Y\tilde{\theta}^2\right),$$

$$m_\Delta^2 = Y + \sec(2\eta)\left[(Y-X)s_\eta^2 + \epsilon^2\left(Y\tilde{\theta}^2 c_\eta^2 - \frac{16\lambda_2\left(t^4 - 2c_{2\alpha}t^2+1\right)}{(t^2+1)^2}s_\eta^2\right)\right]$$

$$m_H^2 = X - \sec(2\eta)\left[(Y-X)s_\eta^2 + \epsilon^2\left(Y\tilde{\theta}^2 s_\eta^2 - \frac{16\lambda_2\left(t^4 - 2c_{2\alpha}t^2+1\right)}{(t^2+1)^2}c_\eta^2\right)\right]$$

$$m_A^2 = X\,,$$

The masses $m_h\,, m_\Delta\,, m_H,\ m_A$ are taken as input parameters to determine the potential

And we get another sum rule: $$m_{H^+}^2 - m_A^2 = v^2\alpha_3\frac{c_{2\beta}}{2} \sim O(150\,\mathrm{GeV})^2$$

Mass splitting $|m_H^2 - m_A^2|$ must be small to ensure perturbativity of λ_2 : $|m_H^2 - m_A^2| \lesssim 16v^2$

Scalar Sector: the bottom line

All potential parameters are cast as physical masses and mixings

(see **JK**, Nemevšek, Nesti 2403.07756 for the gruesome details)

Scalar Sector: the bottom line

All potential parameters are cast as physical masses and mixings

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New model file (FeynRules/UFO):

- **All mixings** are calculated
- New parameter inversion: cast all parameters in **physical (measurable) parameters**
- Includes full **QCD NLO corrections** for the first time
- Also a parity violating version of the model file where $g_L \neq g_R$

Scalar Sector: the bottom line

All potential parameters are cast as physical masses and mixings

(see **JK**, Nemevšek, Nesti 2403.07756 for the gruesome details)

Better to look at some pheno :)

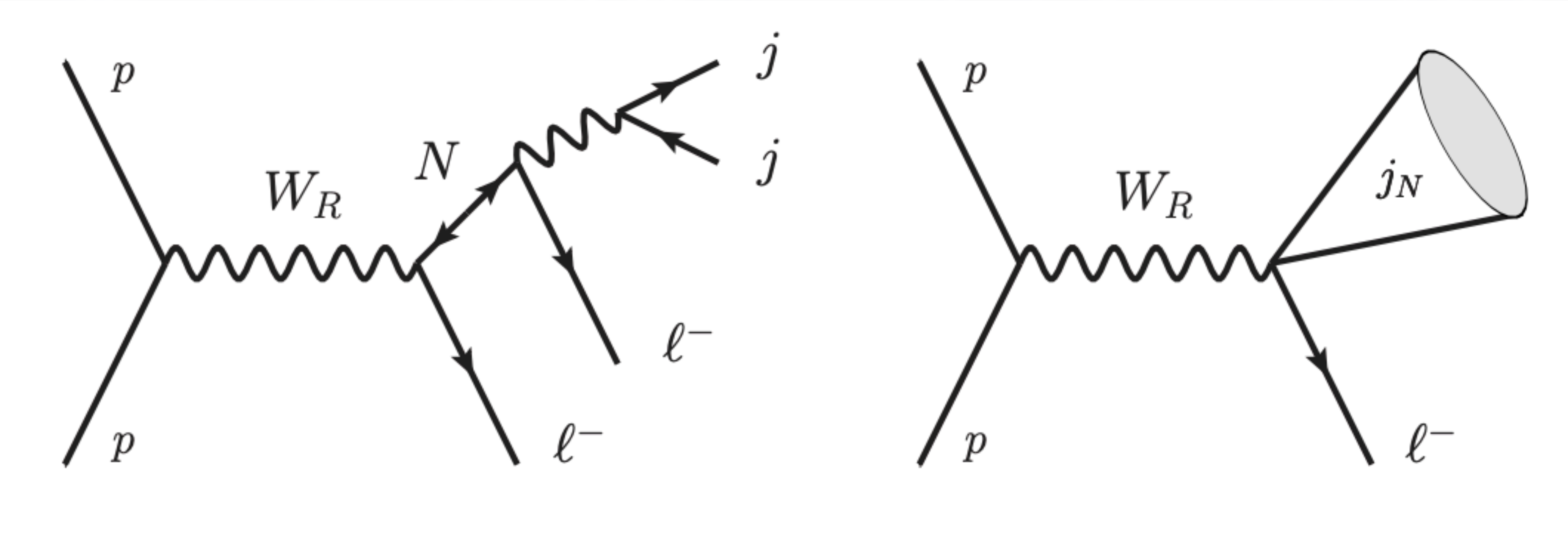
New model file (FeynRules/UFO):

- **All mixings** are calculated
- New parameter inversion: cast all parameters in **physical (measurable) parameters**
- Includes full **QCD NLO corrections** for the first time
- Also a parity violating version of the model file where $g_L \neq g_R$

LNV at LHC in Left-Right: Keung Senjanović process

Production and decay of N via W_R

Keung, Senjanović PRL'83



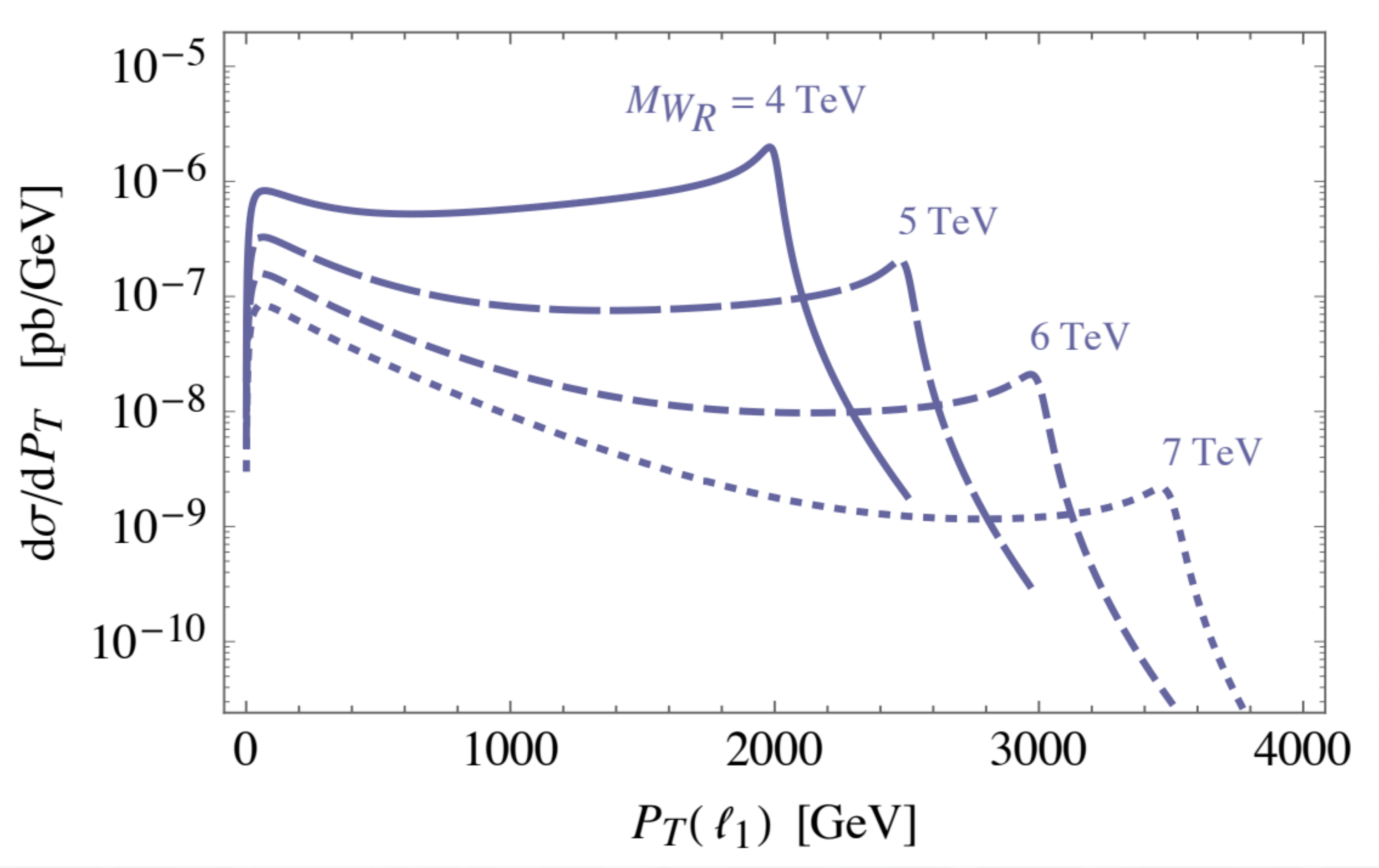
- Same-sign di-leptons + jets, 1 very high-p_T lepton

- If N lightish $\Rightarrow$ boosted/merged signatures

LNV at LHC in Left-Right: Keung Senjanović process

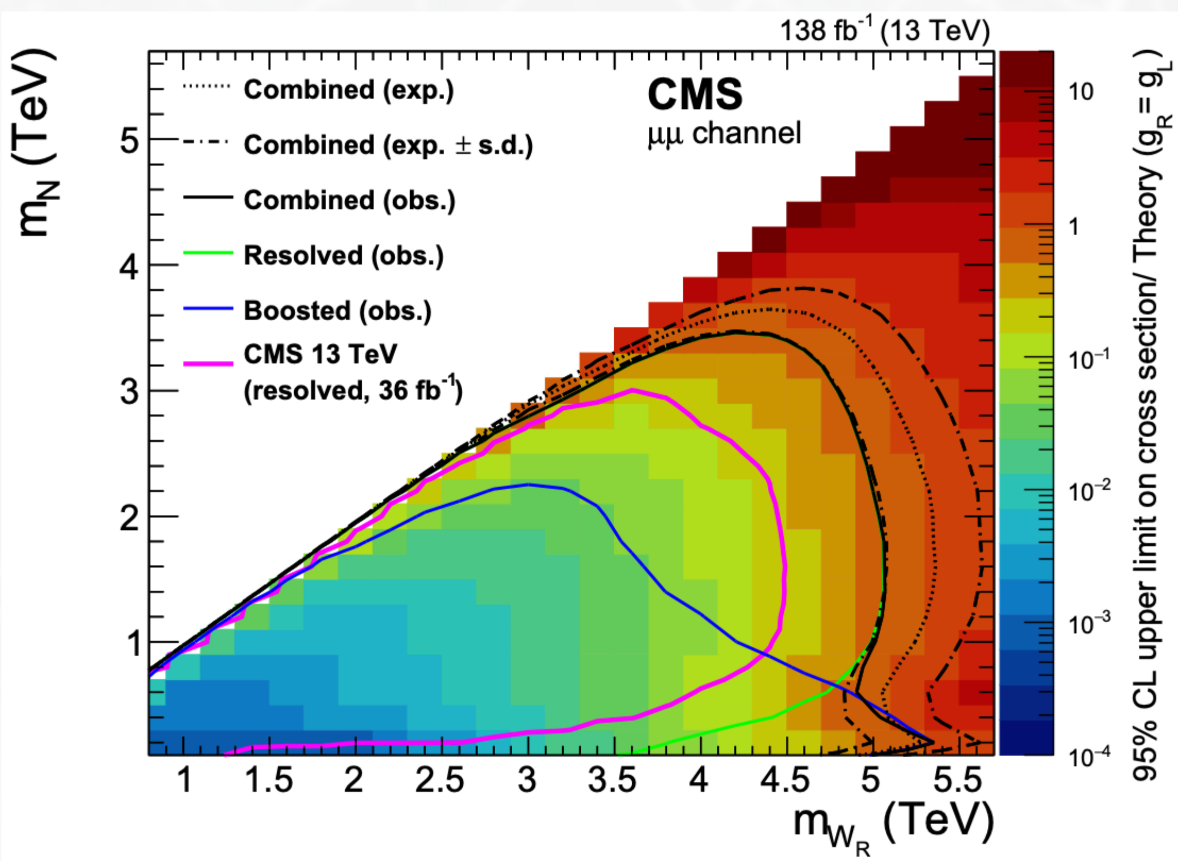
Production and decay of N via W_R

Nemevšek, Nesti, Popara PRD'18

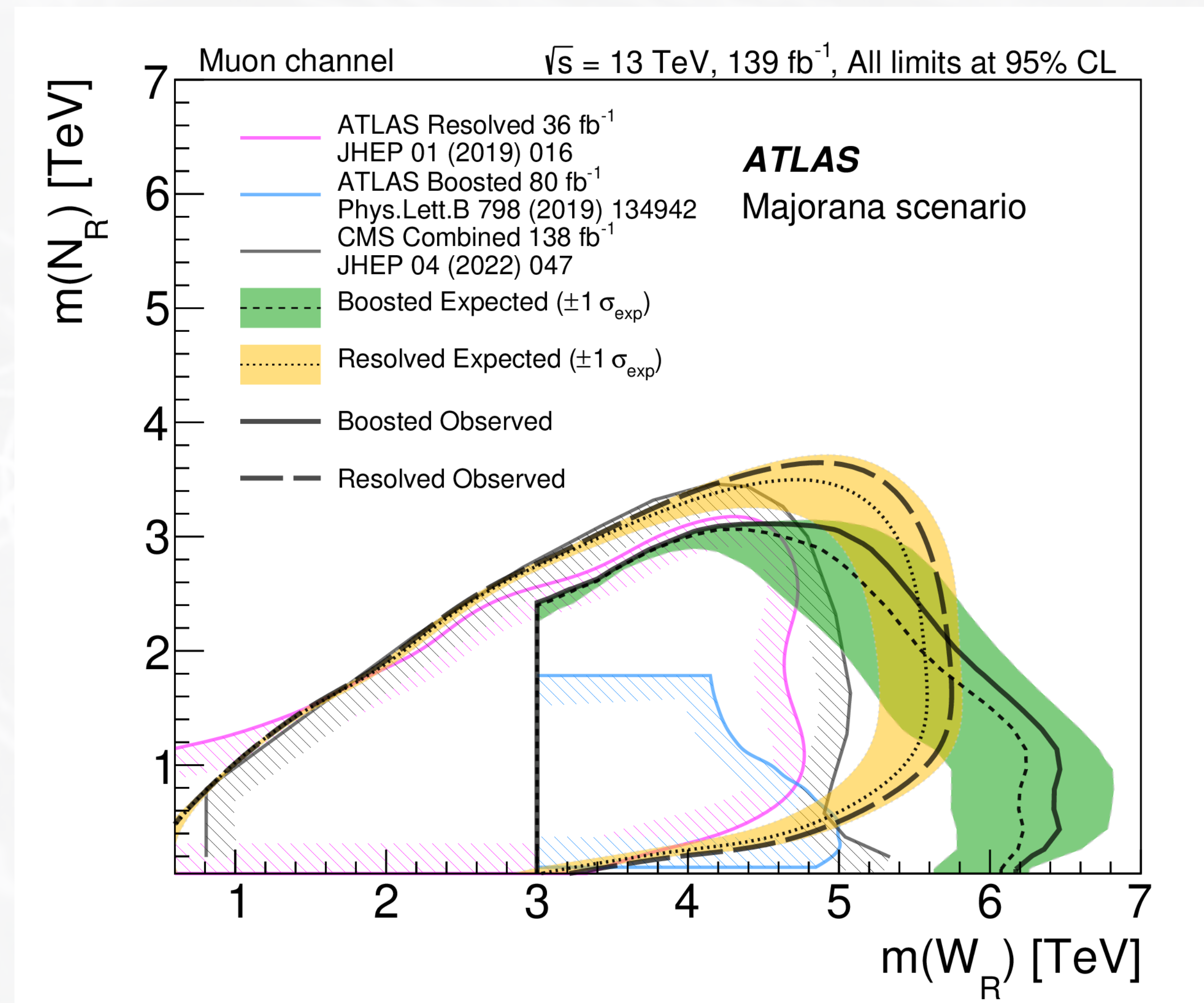


LNV at LHC in Left-Right: Keung Senjanović process

[CMS: 2112.03949]



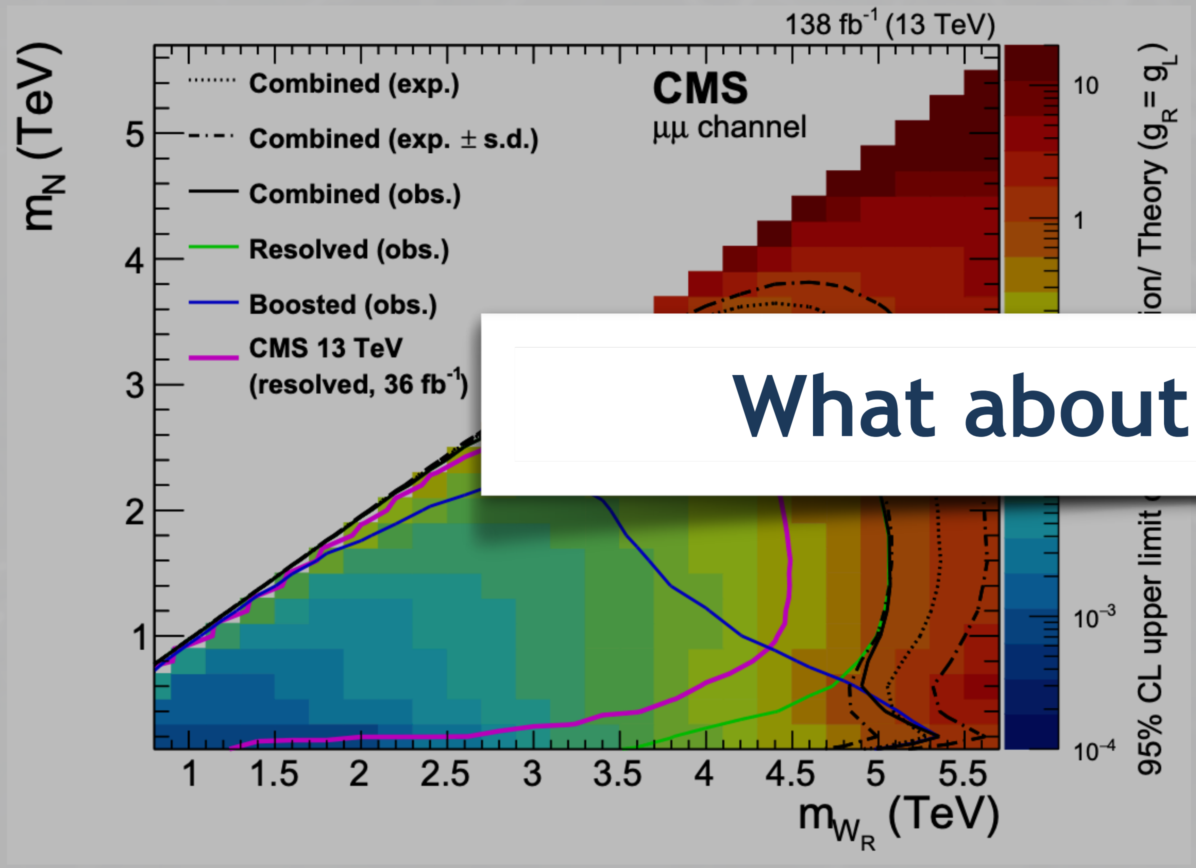
[ATLAS: 2304.09553]



- Exclusion $m_{W_R} \gtrsim 6 - 7\,\mathrm{TeV}$
  - Di-jet searches $m_{W_R} \gtrsim 4.5\,\mathrm{TeV}$

LNV at LHC in Left-Right: Keung Senjanović process

[CMS: 2112.03949]



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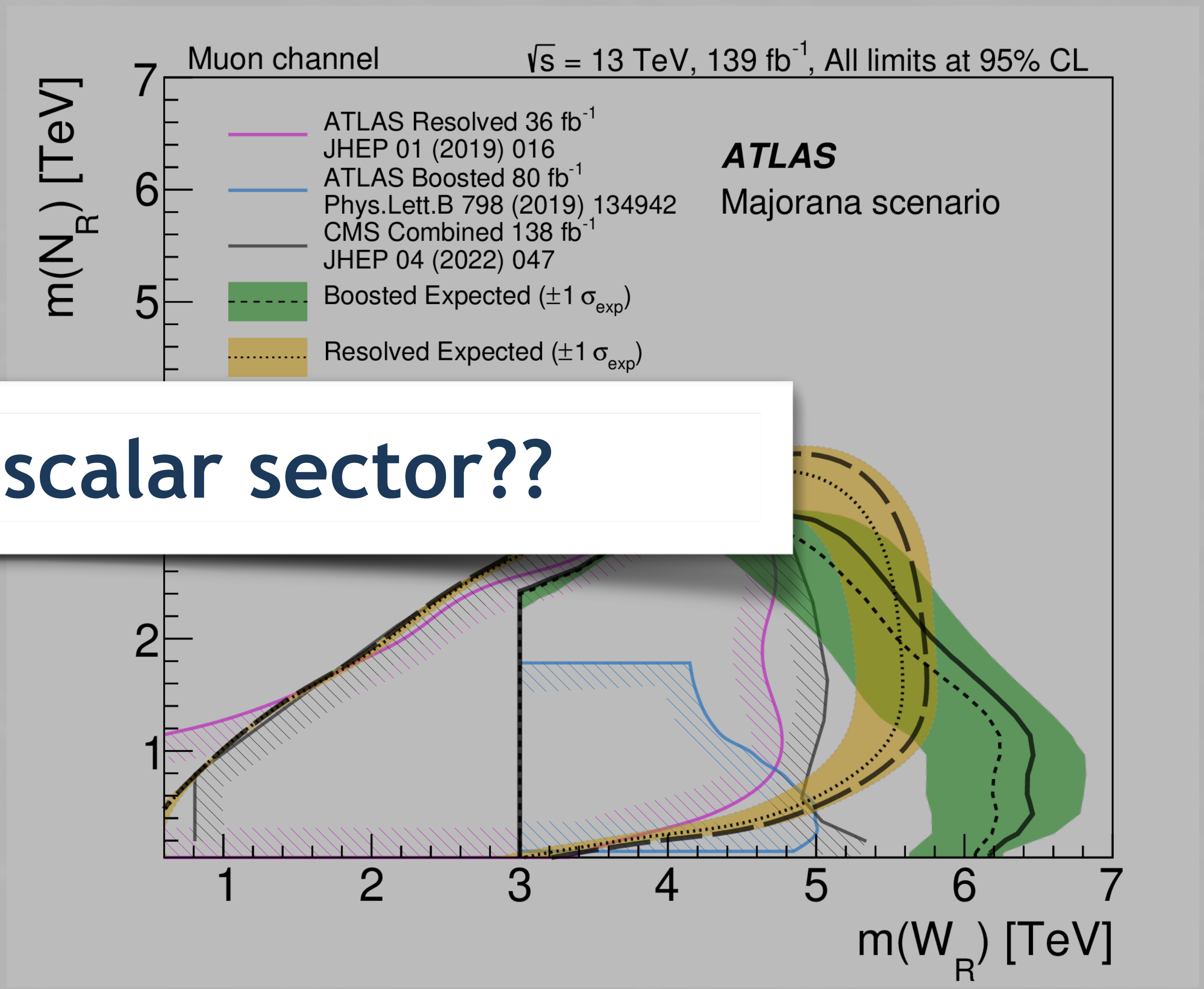
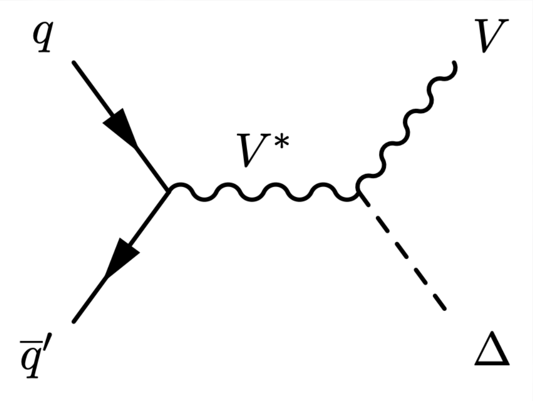


What about the scalar sector??

- Exclusion $m_{W_R} \gtrsim 6-7$ TeV
  - Di-jet searches $m_{W_R} \gtrsim 4.5$ TeV

LNV at LHC in Left-Right: “Majorana Higgs”



Here: production and decay of $\Delta_R^0 \to NN$

$$\mathscr{L}_Y^\ell \supseteq \bar{L}_L^{\prime}(Y_\ell \phi + \tilde{Y}_\ell \tilde{\phi})L_R^{\prime} + \bar{L}_L^{\prime c} i\sigma_2 \, \Delta_L \, Y_L^M L_L^{\prime} + L_R^{\prime c} i\sigma_2 \, \Delta_R \, Y_R^M L_R^{\prime}$$

$$\Gamma(\Delta_R^0 \to NN) \propto m_{\Delta_R^0} \frac{m_N^2}{m_{W_R}^2}$$

$$\Delta_R = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta_R^{++} \\ \Delta_R^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

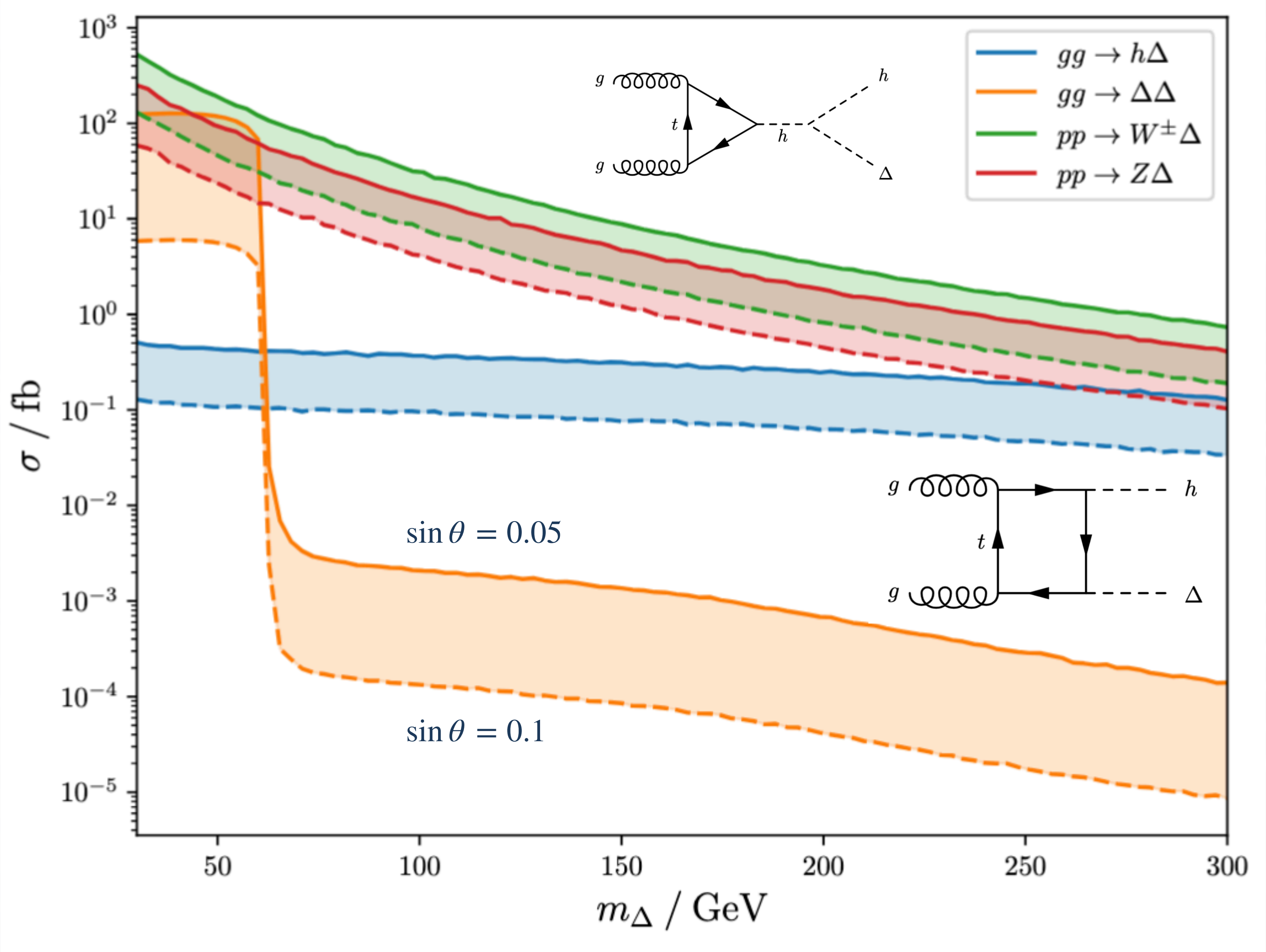
Δ_R^0 mixes with **SM-like Higgs** $\propto \sin\theta$

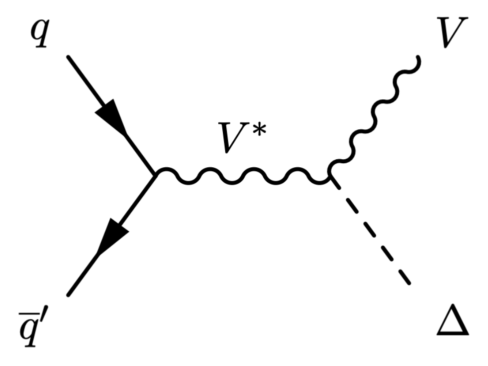
$\Rightarrow \Delta_R^0$ decays into **SM states**

$$\Gamma(\Delta_R^0 \to VV^{(*)}) \simeq \sin^2\theta \, \Gamma(h \to VV^{(*)}) \qquad \Gamma(\Delta_R^0 \to f\bar{f}) \simeq \sin^2\theta \, \Gamma(h \to f\bar{f})$$

LNV at LHC in Left-Right: Production of Δ

Fuks, **JK,** Nemevšek, Nesti arXiv:2503.soon





Sizeable roduction in "**Δ-strahlung**" and **gluon fusion**

Resonant production via Higgs decay for very light Δ

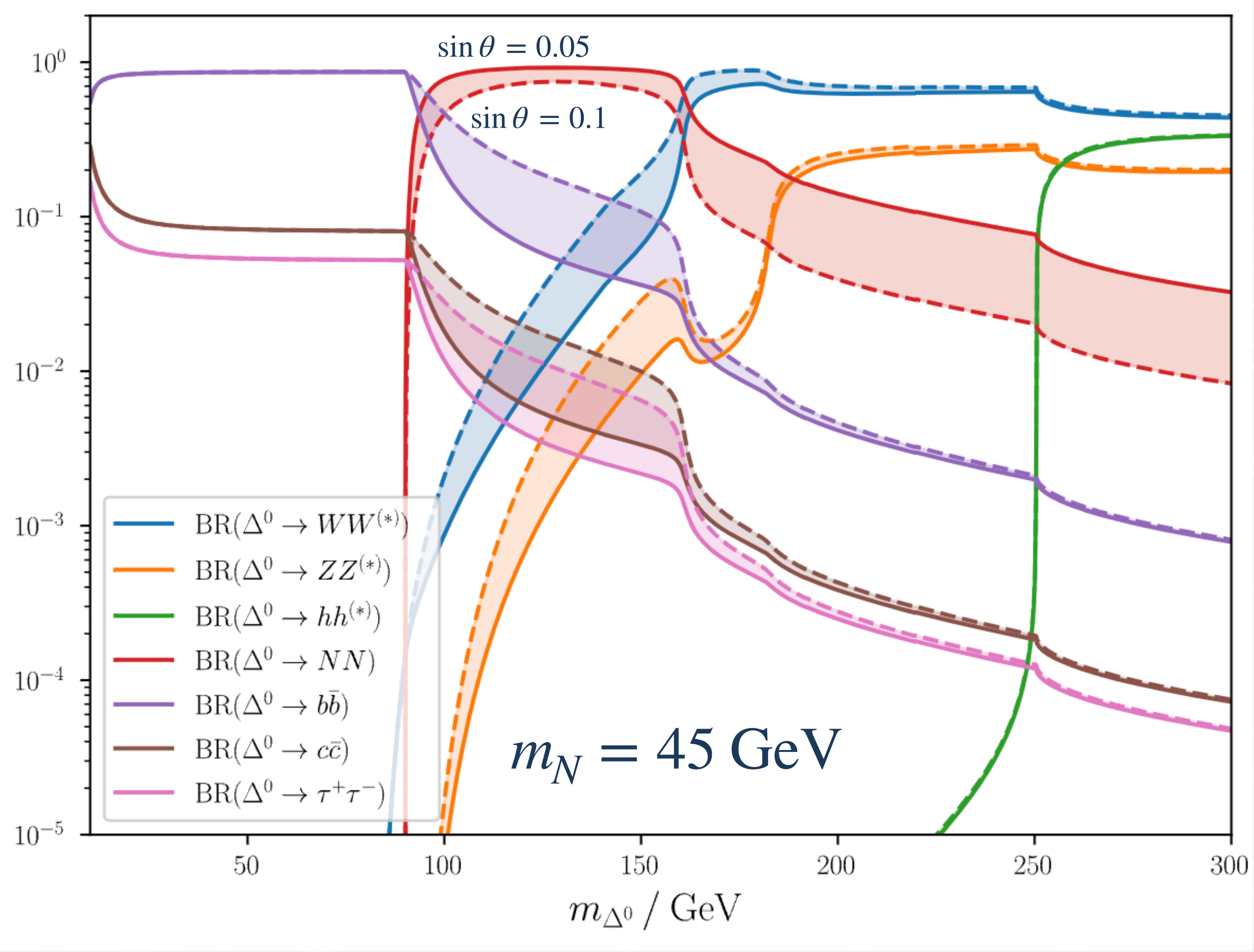
NLO model-file **JK**, Nemevšek, Nesti EPJC'24

See also: https://sites.google.com/site/leftrighthep

LNV at LHC in Left-Right: Decays of the Δ

$$\Gamma(\Delta_R^0 \to NN) \propto m_{\Delta_R^0} \frac{m_N^2}{m_{W_R}^2}$$

Fuks, **JK,** Nemevšek, Nesti arXiv:2503.21354



Ample room for $\Delta_R^0 \to NN$!

$$\Gamma(\Delta_R^0 \to VV^{(*)}) \simeq \sin^2\theta\, \Gamma(h \to VV^{(*)})$$

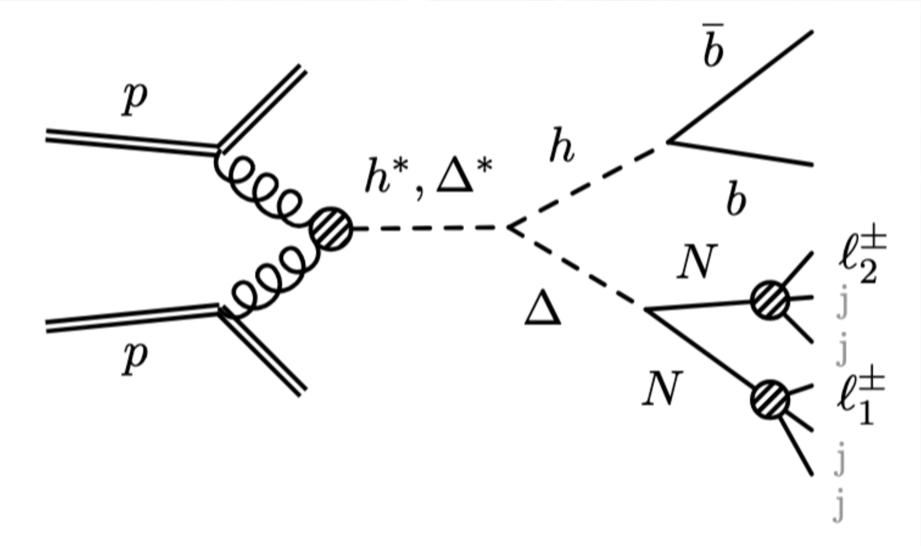
$$\Gamma(\Delta_R^0 \to f\bar{f}) \simeq \sin^2\theta\, \Gamma(h \to f\bar{f})$$

(Transverse) displacement of N

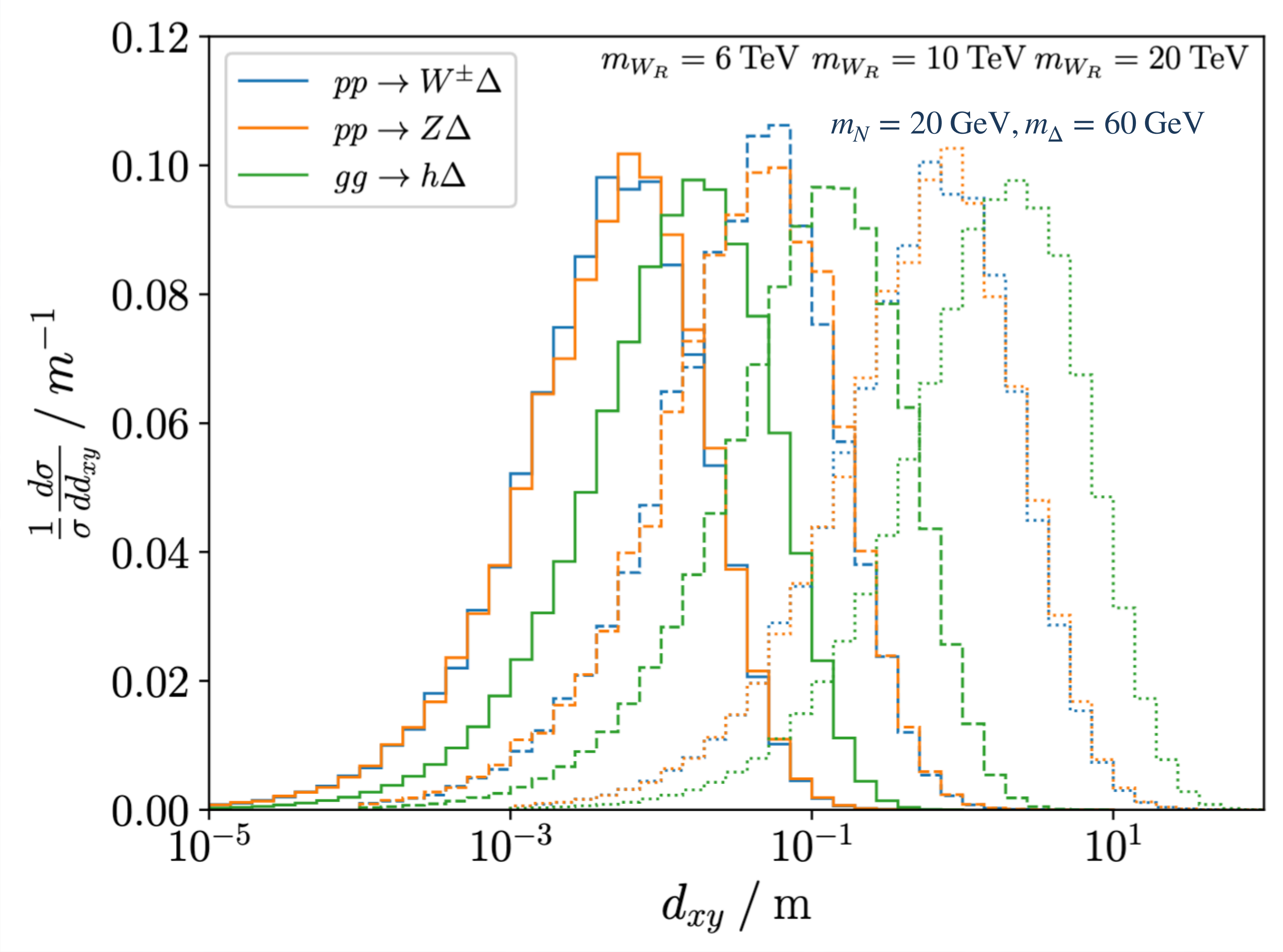
Fuks, **JK,** Nemevšek, Nesti arXiv:2503.21354

Sizeable **displacement** from N decay

$\Rightarrow$ Focus on **displaced** leptons/jets



$\Rightarrow$ Decay of associated boson triggers event



N lifetime $\approx 2.5\text{mm}\,\dfrac{(m_{W_R}/3\text{ TeV})^4}{(m_N/10\text{ GeV})^5}$

Analysis outline

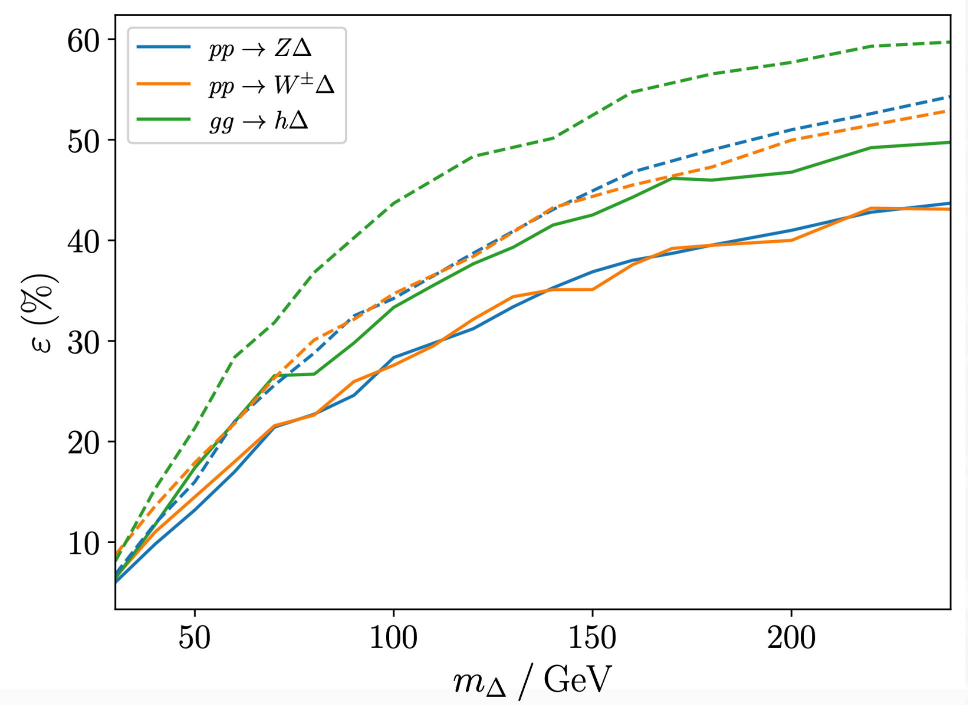
Select events with 2 **same-sign** leptons with

$\Delta R(\ell, j_c) > 0.25$ (lose most events due to soft-lepton isolation)

$|\eta(\ell)| < 2.4$ kinematic/isolation efficiencies: $20 - 40\,\%$

$p_T(\ell) > 10\ \mathrm{GeV}$

$0.1\ \mathrm{mm} < d_{xy} < 30\ \mathrm{cm}$ (Decay in **inner tracker**)



Simulation with MadGraph5, Pythia, Delphes, MadAnalysis tool-chain

NLO model-file **JK,** Nemevšek, Nesti EPJC'24

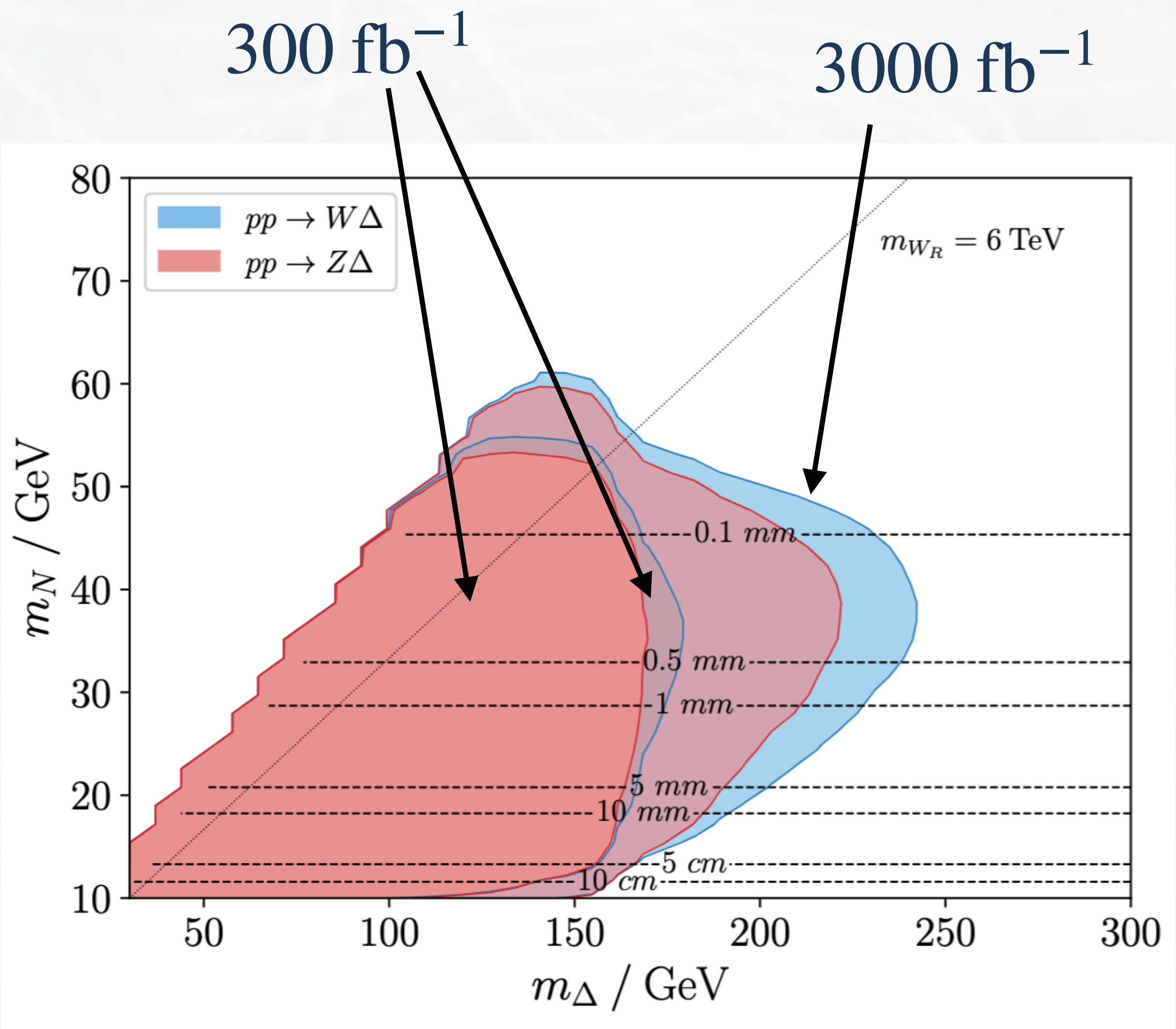
See also: https://sites.google.com/site/leftrighthep

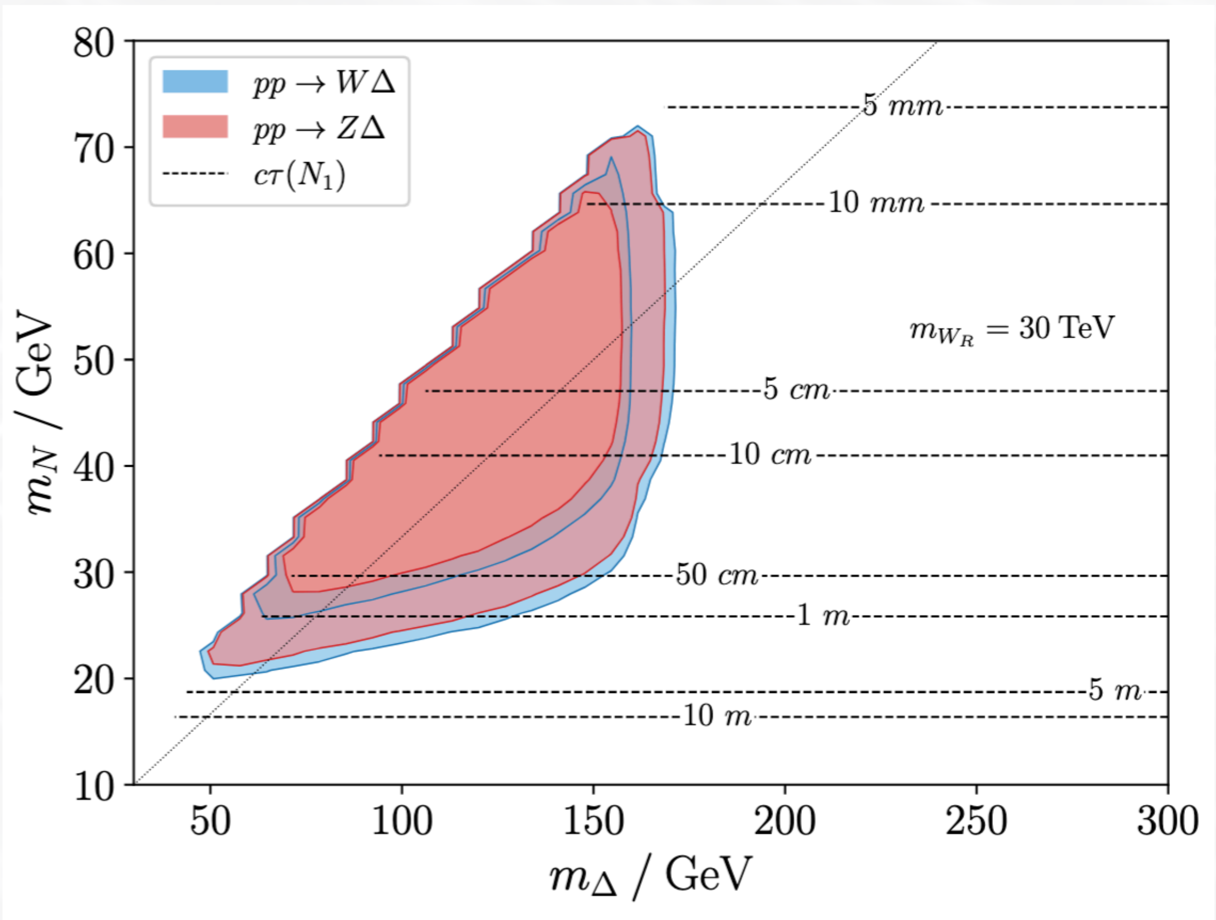
Large displacements, same-sign leptons, $m_V(\ell jj) \gtrsim 10\ \mathrm{GeV}$: no prompt backgrounds

Sensitivities at (HL)-LHC

Detection/Reconstruction of at least 3 Events

Fuks, **JK,** Nemevšek, Nesti arXiv:2503.21354





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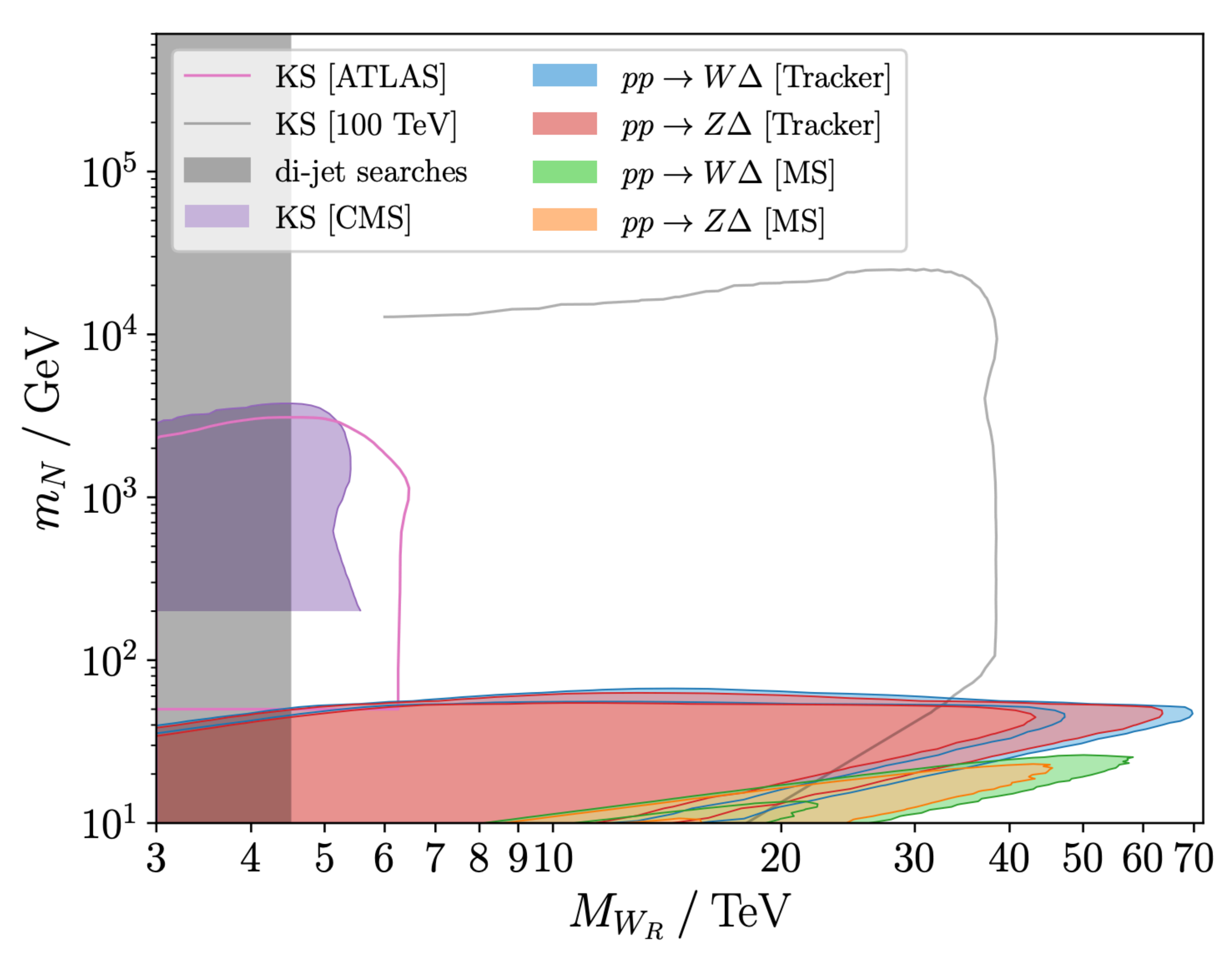
Fuks, **JK,** Nemevšek, Nesti arXiv:2503.21354



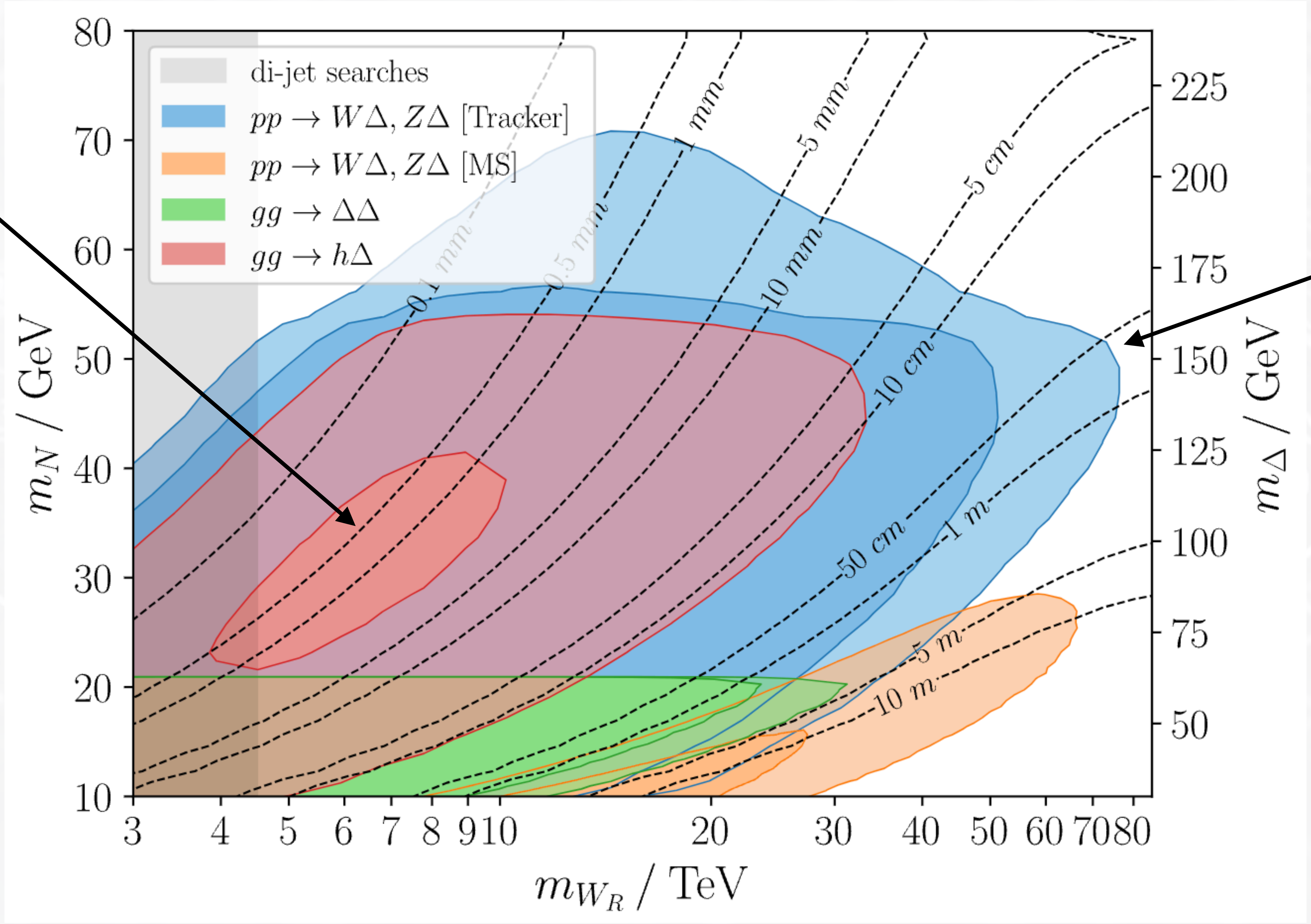
$\Rightarrow$ complementary parameter space,
exclusion up to $m_{W_R} \gtrsim 70 - 80$ TeV!

Sensitivities at (HL)-LHC

Detection/Reconstruction of at least 3 Events

Fuks, **JK,** Nemevšek, Nesti arXiv:2503.21354

$300\ \mathrm{fb}^{-1}$



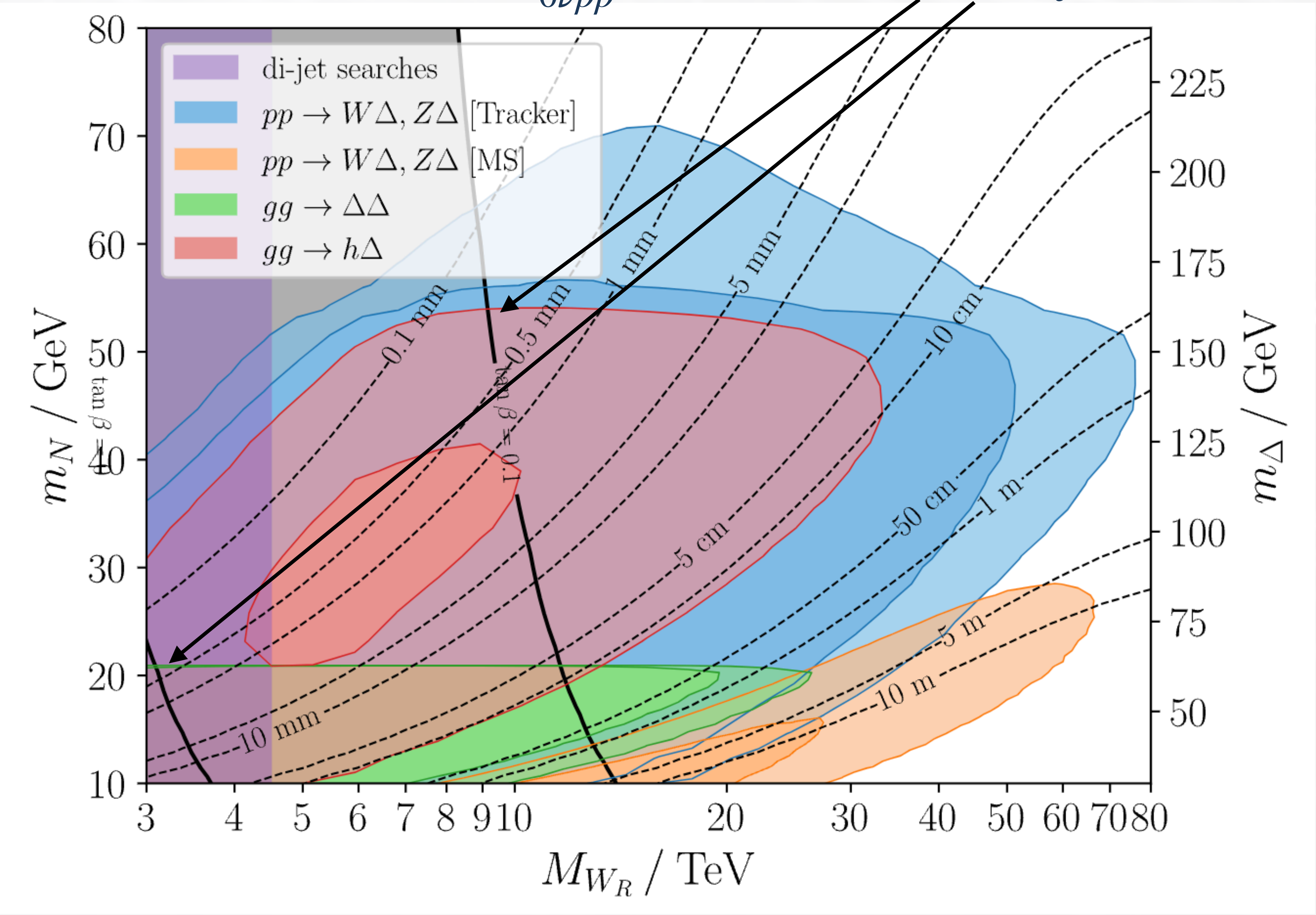
$3000\ \mathrm{fb}^{-1}$

⇒ complementary parameter space, exclusion up to $m_{W_R} \gtrsim 70 - 80$ TeV!

⇒ Large displacements up to Muon Spectrometer [MS] possible! $8\ \mathrm{m} < d_{xy} < 13\ \mathrm{m}$

Interplay with $0\nu\beta\beta$

KamLAND-Zen: $T^{1/2}_{0\nu\beta\beta}(^{136}\text{Xe}) \gtrsim 2.3 \times 10^{26}\text{yrs}$



Conclusions & Outlook

- Minimal **Type II seesaw** is a *cool* model that gives an origin to neutrino masses

  Appears e.g. in the left-right symmetric model on the way to GUTs

- Collider searches start to gradually **exclude the low-scale** parameter space

  Small v_Δ: di-lepton   Large v_Δ: di-boson

- Suggest new search strategy for intermediate v_Δ region: the **LNV window**

  Complementary search for **Lepton Number Violation** (vs $0\nu2\beta$)

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- **Same-sign leptons** from $\Delta_R^0 \to NN \to \ell^\pm\ell^\pm + \text{jets decay} \Rightarrow$ **LNV**

- Dedicated **displaced vertex analysis**: $\Rightarrow m_{W_R} \gtrsim 70 - 80\,\text{TeV}$

Conclusions & Outlook

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- Suggest new search str... the **LNV window**

  Complementary sea... β)

Thanks for your attention!

- Suggest new search for (light) Δ_R^0 in **Left-Right symmetric model**

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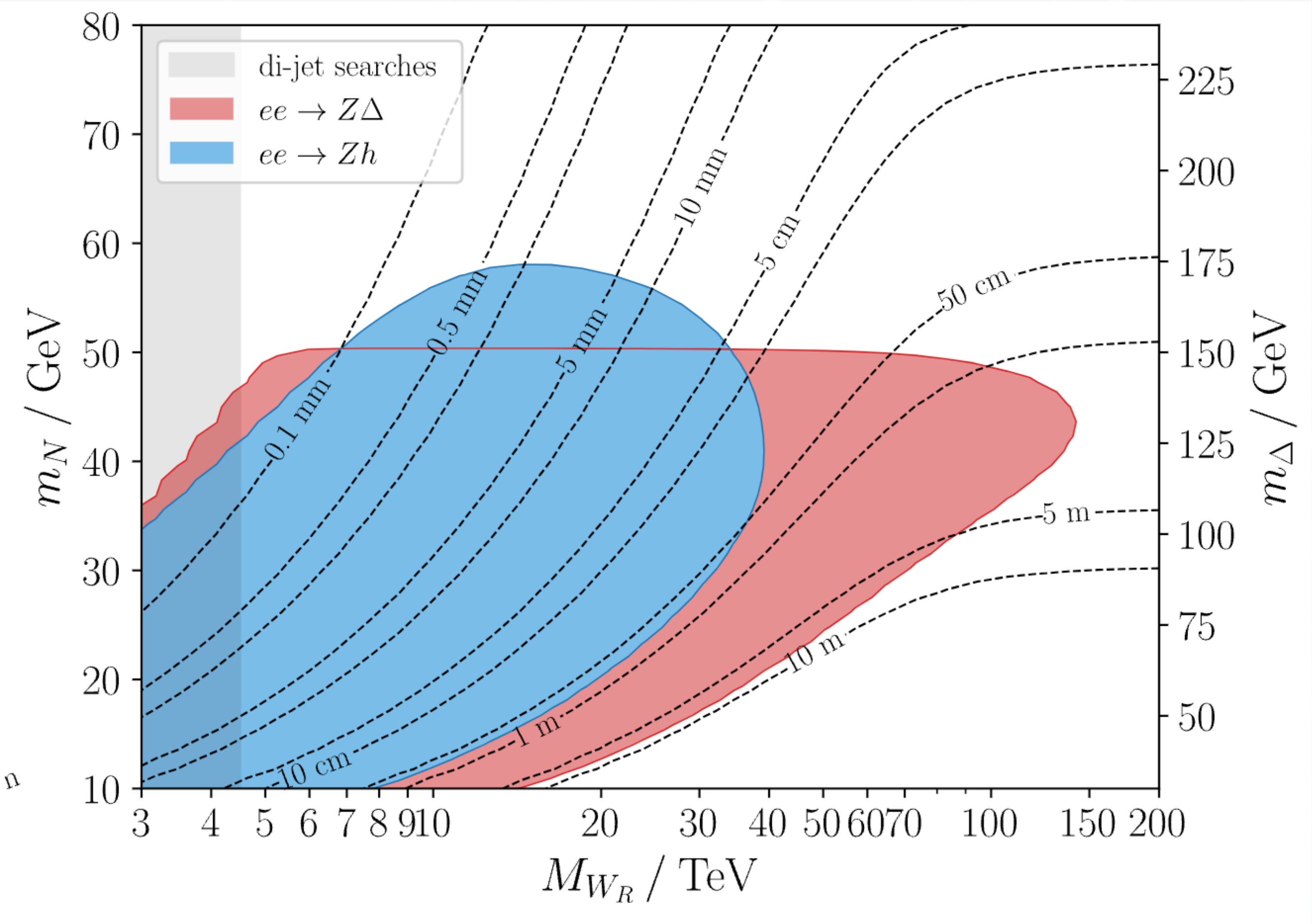
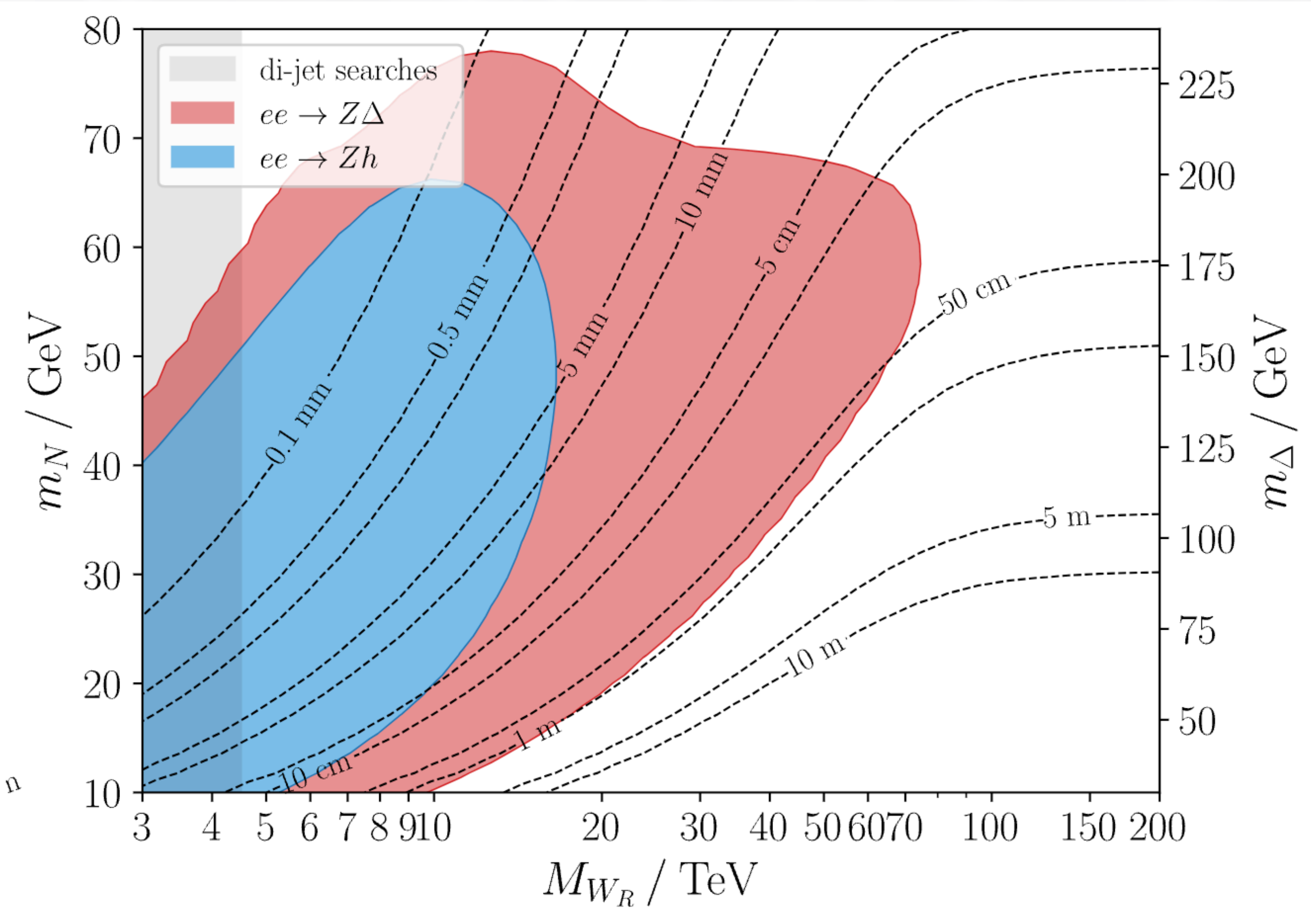
Bonus content

Signals @ FCC-ee

$\sqrt{s} = 240\,\mathrm{GeV}\,,\, \mathscr{L}_{\mathrm{int}} = 5.1\,\mathrm{ab}^{-1}$



$\sqrt{s} = 365\,\mathrm{GeV}\,,\, \mathscr{L}_{\mathrm{int}} = 1.7\,\mathrm{ab}^{-1}$



Type II seesaw mechanism: the induced vev

$$\Delta_L = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \frac{v_\Delta + \Delta^0 + i\chi_\Delta}{\sqrt{2}} & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

Extend Standard Model with a **scalar** $Y = 1$, $SU(2)_L$-triplet

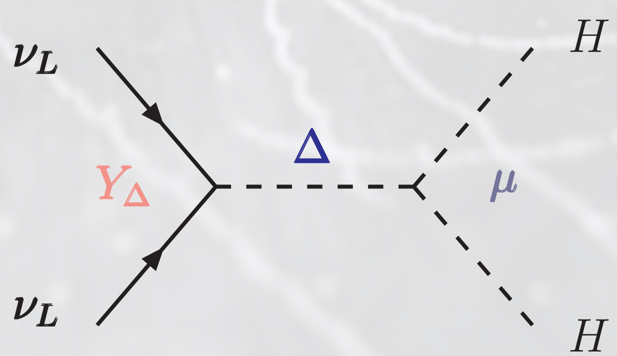
$$V(\varphi, \Delta) = -\mu_h^2 \varphi^\dagger \varphi + m_\Delta^2 \mathrm{Tr}[\Delta^\dagger \Delta] + \lambda_h (\varphi^\dagger \varphi)^2 + \lambda_{\Delta 1} \mathrm{Tr}\left[\Delta^\dagger \Delta\right]^2 + \lambda_{\Delta 2}\left[(\Delta^\dagger \Delta)^2\right]$$
$$+ \mu_{h\Delta}\left(\varphi^T i\sigma_2 \Delta^\dagger \varphi + \mathrm{h.c.}\right) + \lambda_{h\Delta 1}\, \varphi^\dagger \varphi \mathrm{Tr}\left[\Delta^\dagger \Delta\right] + \lambda_{h\Delta 2}\, \mathrm{Tr}\left[\varphi \varphi^\dagger \Delta \Delta^\dagger\right]$$

φ = **SM-like $SU(2)_L$-doublet**

Minimise **potential**: $\mu_h^2 \simeq v^2 \lambda_h$, $\mu_{h\Delta} \simeq \frac{v_\Delta (2m_\Delta^2 + v^2 \lambda_{h\Delta})}{\sqrt{2} v^2}$

$\Rightarrow$Triplet vev v_Δ **induced** by SM-like electroweak vev and $\mu_{h\Delta} \neq 0$ (stability condition $\mu_{h\Delta} > 0$)

See e.g. [1105.1925]



$\Rightarrow$ Combined presence of **Yukawa** and $\mu_{h\Delta}$ leads to **Lepton Number violating** interactions

$\Rightarrow$ small $\mu_{h\Delta}$ & v_Δ technically natural

Type II seesaw mechanism: the scalar spectrum

$$\Delta_L = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \frac{v_\Delta + \Delta^0 + i\chi_\Delta}{\sqrt{2}} & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

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$$+ \mu_{h\Delta} \left(\varphi^T i\sigma_2 \Delta^\dagger \varphi + \mathrm{h.c.}\right) + \lambda_{h\Delta 1} \varphi^\dagger \varphi \mathrm{Tr}\left[\Delta^\dagger \Delta\right] + \lambda_{h\Delta 2} \mathrm{Tr}\left[\varphi \varphi^\dagger \Delta \Delta^\dagger\right]$$

Components of Δ_L have mass terms:

$$m_h^2 = 2\lambda_h v^2 \qquad m_{\Delta^0}^2 = m_{\chi_\Delta}^2 = m_{\Delta^{++}}^2 + \frac{\lambda_{h\Delta 2}}{2} v^2 \qquad m_{\Delta^+}^2 = m_{\Delta^{++}}^2 + \frac{\lambda_{h\Delta 2}}{4} v^2 \qquad m_{\Delta^{++}}^2 = m_\Delta^2 + \frac{\lambda_{h\Delta 1}}{2} v^2$$

And mix with the SM-like doublet φ:

$$\sin\theta_{h\Delta} \simeq \frac{2m_\Delta^2}{m_h^2 - m_{\Delta_0}^2} \left(\frac{v_\Delta}{v}\right) \qquad \sin\theta_{\Delta^+\varphi^+} \simeq \sqrt{2} \left(\frac{v_\Delta}{v}\right) \qquad \sin\theta_{\chi\varphi^0} \simeq 2 \left(\frac{v_\Delta}{v}\right)$$

Mixing induces couplings to pairs of quarks, W, Z

Mixing with would-be Goldstones ⇝ corrections to M_W, M_Z, ρ, EWPO

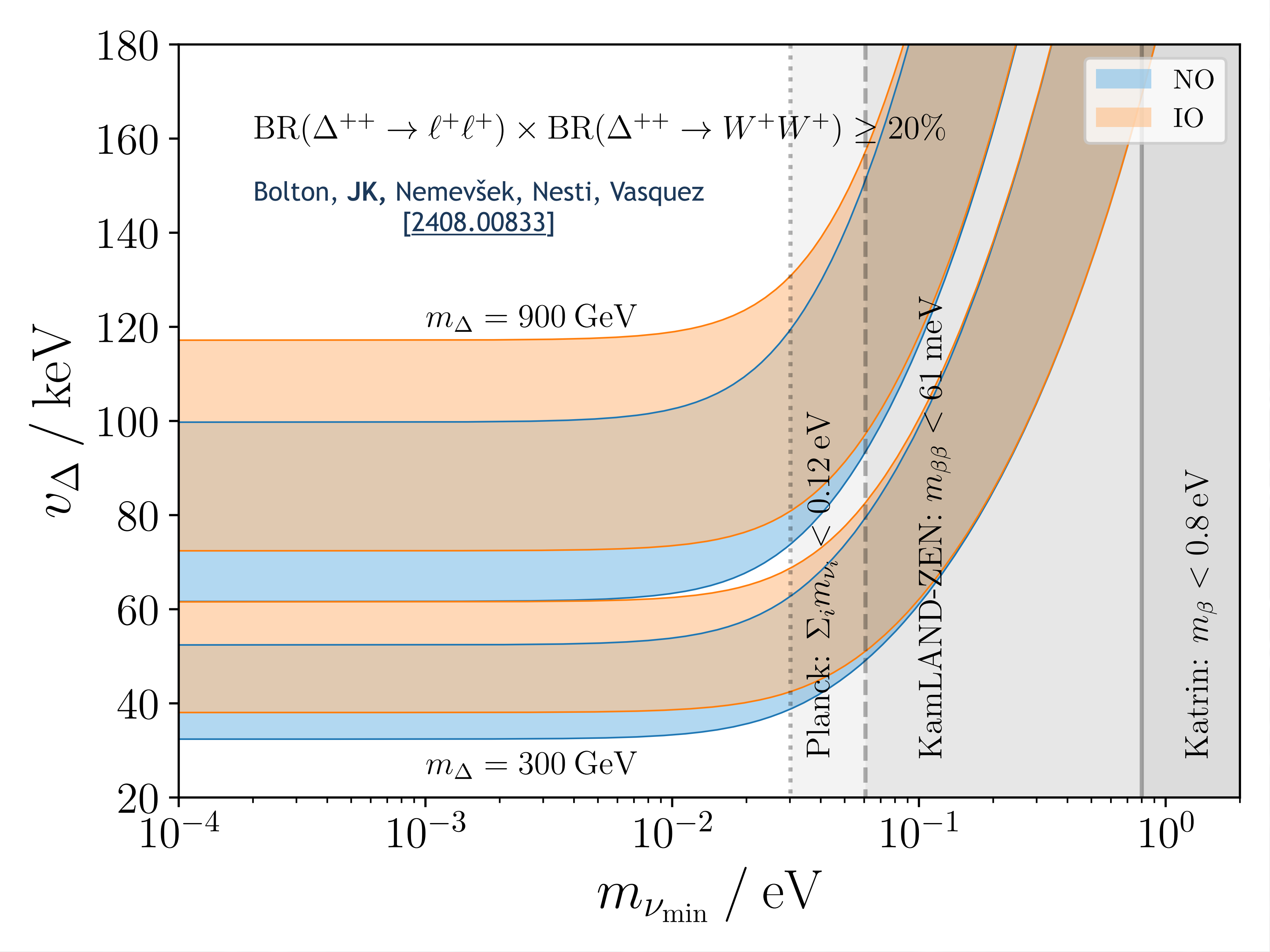
Mass splittings follow sum-rule:

$$m_{\Delta^0}^2 - m_{\Delta^+}^2 = m_{\Delta^+}^2 - m_{\Delta^{++}}^2 = \frac{\lambda_{h\Delta 2}}{4} v^2$$

Mass splittings limited by Tachyon conditions & perturbative unitarity

See [1105.1925] for comprehensive analysis of the potential

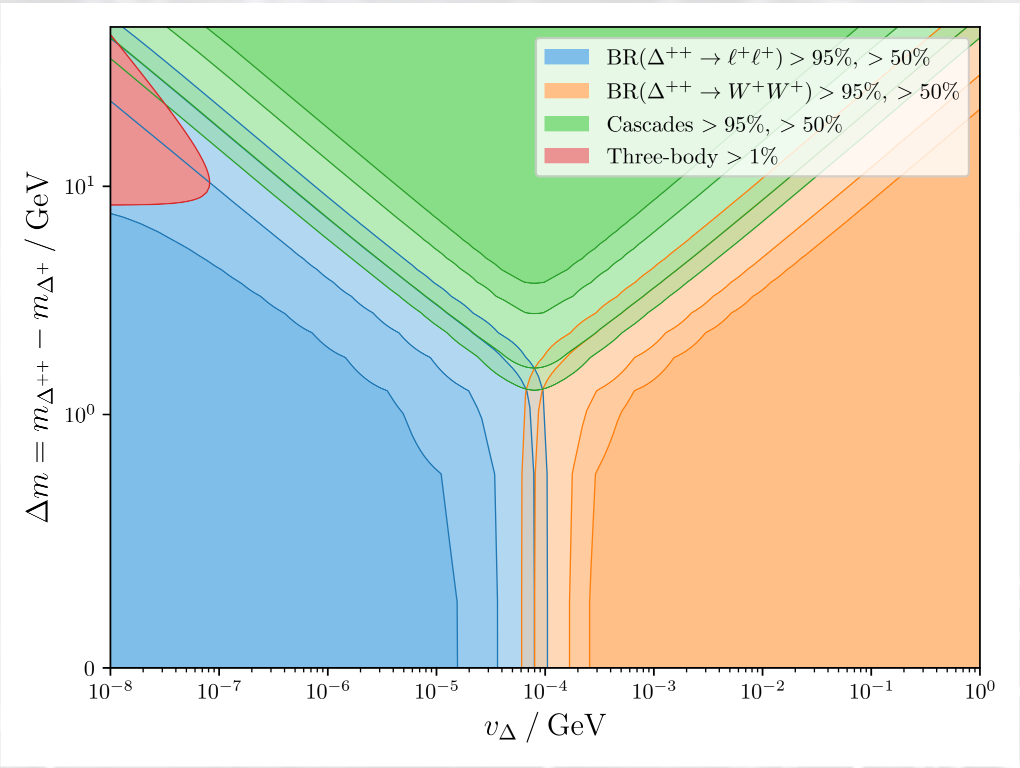
The LNV window

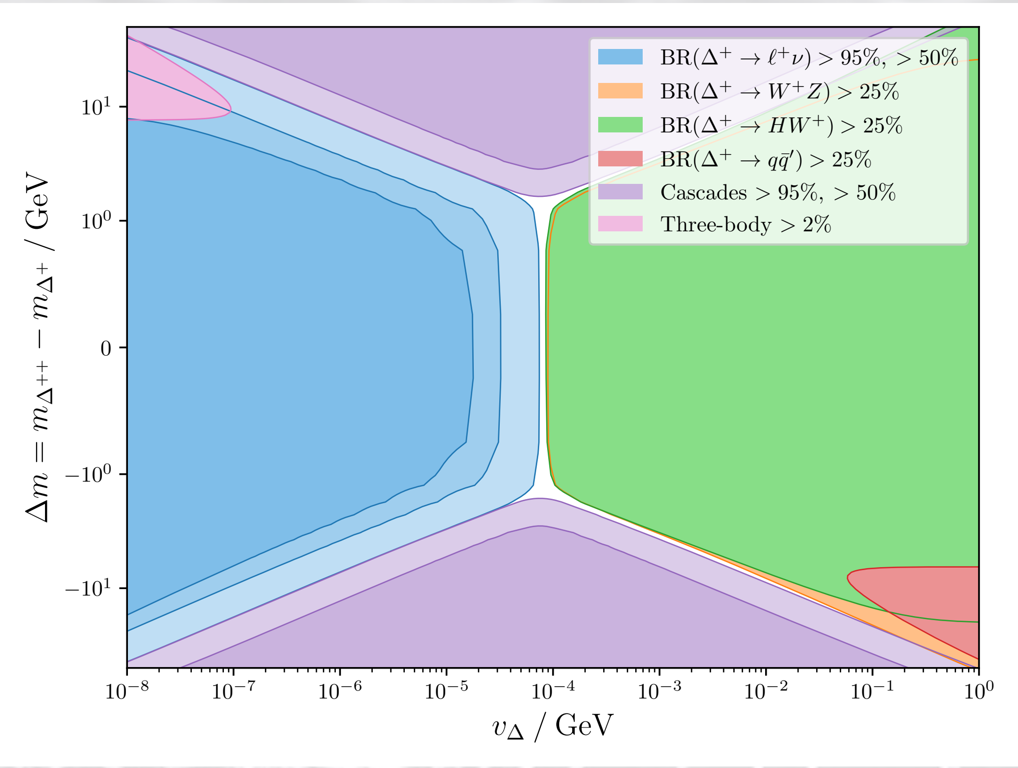


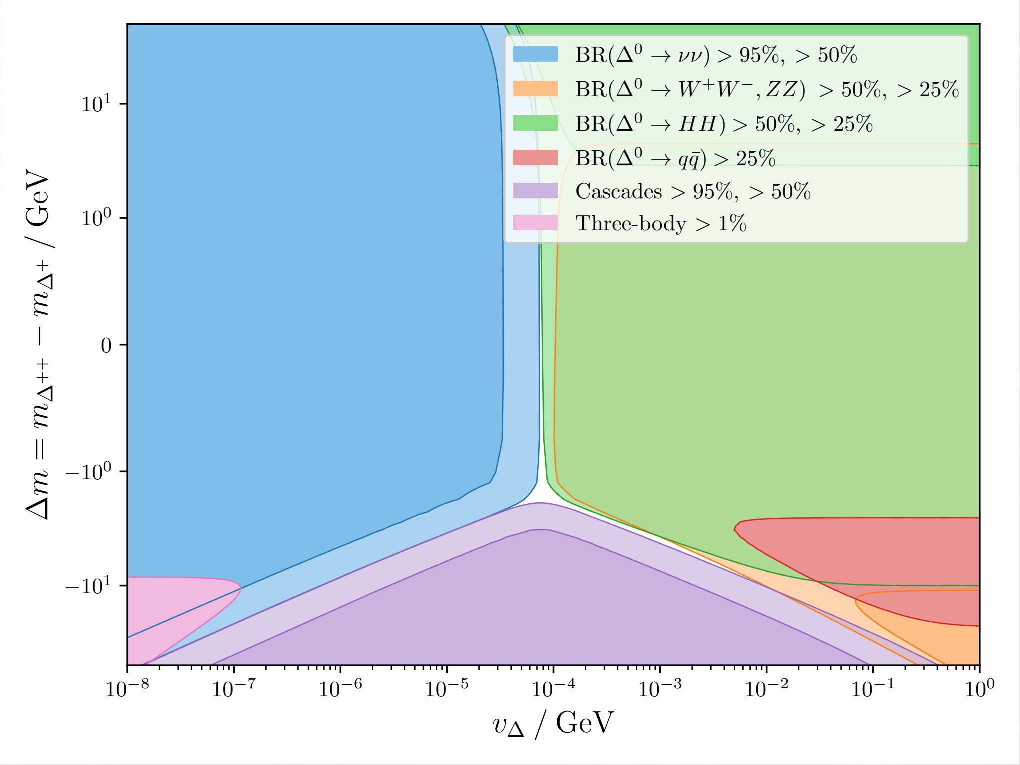
⇒ In phenomenologically viable region: only mild dependence on $m_{\nu_{\text{min}}}$ and ordering

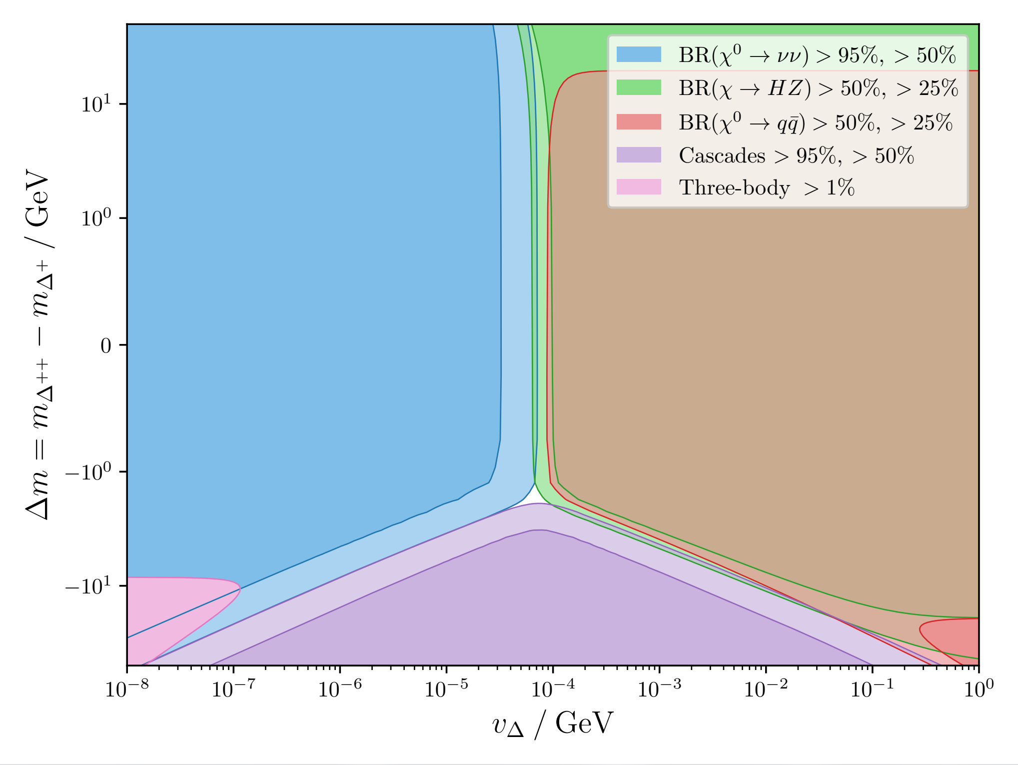
(Stronger dependence on ordering in flavour channels)

Decay modes of the triplet components









Diagonalising the Lagrangian: Gauge sector

Higgs Mechanism: Scalar multiplets acquire vevs at

$$\langle\phi\rangle = \begin{pmatrix} v_1 & 0 \\ 0 & -e^{i\alpha}v_2 \end{pmatrix}, \langle\Delta_{L,R}\rangle = \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix} \text{ with } v = \sqrt{v_1^2 + v_2^2} = 246\,\text{GeV}, \tan\beta = \frac{v_2}{v_1}$$

Leading to masses for gauge bosons from the scalar-kinetic terms:

$$\mathscr{L}_{\text{kin}} = |D\phi|^2 + |D\Delta_L|^2 + |D\Delta_R|^2$$

$$D^\mu\phi = \partial^\mu - ig(A_L^\mu\phi - \phi A_R^\mu)$$

$$D^\mu\Delta_{L,R} = \partial\Delta_{L,R} - ig[A^\mu_{L,R}, \Delta_{L,R}] - ig'B^\mu\Delta_{L,R}$$

(Order of fields matters due to matrix representation)

Manifest left-right model: $g_L = g_R \equiv g$ leads to conservation of $\mathscr{C}$ or $\mathscr{P}$

Diagonalising the Lagrangian: Gauge sector

Charged gauge fields mass matrix:

$$(A_L^-, A_R^-)\, M_{W_{L,R}}^2 \begin{pmatrix} A_L^+ \\ A_R^+ \end{pmatrix} \quad \Rightarrow M_{W_{L,R}}^2 = \frac{g^2}{2} v_R^2 \begin{pmatrix} \epsilon^2 & e^{-i\alpha}\epsilon^2 \sin 2\beta \\ e^{i\alpha}\epsilon^2 \sin 2\beta & 2+\epsilon^2 \end{pmatrix} \quad \epsilon = \frac{v}{v_R}$$

$M_{W_{L,R}}$ is diagonalised with a **unitary** rotation: $\begin{pmatrix} A_L^+ \\ A_R^+ \end{pmatrix} = U_W \begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix}$

with $U_W = \begin{pmatrix} c_\xi & s_\xi e^{-i\alpha} \\ -s_\xi e^{i\alpha} & c_\xi \end{pmatrix}$ and $s_\xi \simeq \frac{\epsilon^2}{2} s_{2\beta} \simeq \frac{M_{W_L}^2}{M_{W_R}^2} s_{2\beta}$ (Up to $\mathcal{O}(\epsilon^2)$)

Mixing controlled by $\beta = \arctan(v_2/v_1)$

The mass eigenvalues then become:

$$M_{W_L} \simeq \frac{gv}{\sqrt{2}}, \; M_{W_R} = g v_R \left(1 + \frac{\epsilon^2}{4}\right)$$

Input parameters: $M_{W_L}, M_{W_R}, \tan\beta, \alpha, g$

Diagonalising the Lagrangian: Gauge sector

$$(A_{3L}, A_{3R}, B)\, M^2_{Z_{L,R}} \begin{pmatrix} A_{3L} \\ A_{3R} \\ B \end{pmatrix} \Rightarrow M^2_{Z_{L,R}} = \frac{g^2}{2} v_R^2 \begin{pmatrix} \epsilon^2 & -\epsilon^2 & 0 \\ -\epsilon^2 & 4+\epsilon^2 & -4r \\ 0 & -4r & 4r^2 \end{pmatrix} \qquad r = \frac{g'}{g}$$

$M_{Z_{L,R}}$ is diagonalised with an **orthogonal** rotation: $\begin{pmatrix} A_{3L} \\ A_{3R} \\ B \end{pmatrix} = O_Z \begin{pmatrix} A \\ Z_L \\ Z_R \end{pmatrix}$

The mass eigenvalues then become:

$$M_A = 0\,,\ M_{Z_L} = \frac{gv}{\sqrt{1 + \frac{1}{1+2r^2}}}\,, \quad \text{(Up to } \mathcal{O}(\epsilon^2))$$

$$M_{Z_R} \simeq g v_R \sqrt{2(1+r^2)} \left(1 + \frac{\epsilon^2}{8(1+r^2)^2} \right)$$

Mixing is fixed:

$$O_Z = \begin{pmatrix} s_w & -c_w & 0 \\ s_w & s_w t_w & -\frac{\sqrt{c_{2w}}}{c_w} \\ \sqrt{c_{2w}} & \sqrt{c_{2w}}\, t_w & t_w \end{pmatrix} + \frac{\epsilon^2}{4} \begin{pmatrix} 0 & 0 & \frac{c_{2w}^{3/2}}{c_w^3} \\ 0 & -\frac{c_{2w}^2}{c_w^5} & -\frac{c_{2w}^{3/2} t_w^2}{c_w^3} \\ 0 & \frac{c_{2w}^{3/2} t_w}{c_w^4} & -\frac{c_{2w}^2 t_w}{c_w^4} \end{pmatrix}.$$

Diagonalising the Lagrangian: Gauge sector

From the mass eigenvalues:

$$M_A = 0\,,\ M_{Z_L} \simeq \frac{gv}{\sqrt{1+\frac{1}{1+2r^2}}}\,,$$

$$M_{Z_R} \simeq g v_R \sqrt{2(1+r^2)}\left(1+\frac{\epsilon^2}{8(1+r^2)^2}\right)$$

We can fix c_w and therefore r in the on-shell scheme: $c_w = \frac{M_{W_L}}{M_{Z_L}} \Rightarrow r = \frac{s_w}{\sqrt{c_{2w}}}$

Using M_{W_L}, M_{Z_L} and g as input parameters: $r \simeq \sqrt{\frac{M_{W_L}^2}{2M_{W_L}^2 - M_{Z_L}^2} - 1} \simeq 0.63$

$$\Rightarrow M_{Z_R} = \frac{\epsilon^2 M_{W_R}}{4\sqrt{2}} \frac{M_{W_L}^2}{(2M_{W_L}^2 - M_{Z_L}^2)^{3/2}} + \sqrt{2}\frac{M_{W_R}}{2M_{W_L}^2 - M_{Z_L}^2} \simeq 1.67 M_{W_R}$$

Diagonalising the Lagrangian: Gauge sector

Input parameters: $M_{Z_L}\,,M_{W_L}\,,M_{W_R}\,,\ \tan\beta\,,\alpha\,,g$

M_{W_L}, M_{Z_L} and g take their measured values and define $s_w\,,\alpha_e$, etc…

M_{W_R} is limited by direct searches to be $M_{W_R} \gtrsim 6\,\mathrm{TeV}$

Measurements of the neutron EDM d_n limit $\sin\alpha\tan 2\beta \lesssim 5.8\times 10^{-12}$ [2107.10852]

$$d_n \propto \bar{\theta} \simeq \frac{m_t}{2m_b}\sin\alpha\tan 2\beta$$

Dominant decay channels: $\mathrm{BR}(W_R^\pm \to q_u q_d) \simeq 75\,\%\,,\ \mathrm{BR}(W_R^\pm \to \ell^\pm N) \simeq 24\,\%$

$$\mathrm{BR}(Z_R \to q\bar{q}) \simeq 55\,\%\,,\ \mathrm{BR}(Z_R \to NN) \simeq 17\,\%\,,\ \mathrm{BR}(Z_R \to W_L^+ W_L^- h) \simeq 12\,\%$$

(mg5_aMC with default parameters)

Diagonalising the Lagrangian: Scalar sector

Scalars are complex

$$\Rightarrow \Delta_{L,R} = \begin{pmatrix} \frac{\Delta^+_{L,R}}{\sqrt{2}} & \Delta^{++}_{L,R} \\ v_{L,R} + \mathrm{Re}\Delta^0_{L,R} + i\mathrm{Im}\Delta^0_{L,R} & -\frac{\Delta^+_{L,R}}{\sqrt{2}} \end{pmatrix}, \quad \phi = \begin{pmatrix} v_1 + \mathrm{Re}\phi^0_1 - i\mathrm{Im}\phi^0_1 & \phi^+_2 \\ \phi^-_1 & v_2 + \mathrm{Re}\phi^0_2 + i\mathrm{Im}\phi^0_2 \end{pmatrix}$$

$\Rightarrow$ some of the **pseudo-scalar** excitations are eaten by the (massive) gauge bosons

The most general $\mathscr{P}$- (and $\mathscr{C}$-) symmetric potential is given by:

$$\mathscr{P} : \phi \to \phi^\dagger\,,\ \Delta_L \leftrightarrow \Delta_R \qquad \mathscr{C} : \phi \to \phi^T\,,\ \Delta_L \leftrightarrow \Delta_R^*$$

$$\begin{aligned}
\mathcal{V} = &-\mu_1^2\left[\phi^\dagger\phi\right] - \mu_2^2\left(\left[\tilde\phi\phi^\dagger\right] + \left[\tilde\phi^\dagger\phi\right]\right) - \mu_3^2\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right) \\
&+ \lambda_1\left[\phi^\dagger\phi\right]^2 + \lambda_2\left(\left[\tilde\phi\phi^\dagger\right]^2 + \left[\tilde\phi^\dagger\phi\right]^2\right) + \lambda_3\left[\tilde\phi\phi^\dagger\right]\left[\tilde\phi^\dagger\phi\right] + \lambda_4\left[\phi^\dagger\phi\right]\left(\left[\tilde\phi\phi^\dagger\right] + \left[\tilde\phi^\dagger\phi\right]\right) \\
&+ \rho_1\left(\left[\Delta_L\Delta_L^\dagger\right]^2 + \left[\Delta_R\Delta_R^\dagger\right]^2\right) + \rho_2\left(\left[\Delta_L\Delta_L\right]\left[\Delta_L^\dagger\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right]\right) + \rho_3\left[\Delta_L\Delta_L^\dagger\right]\left[\Delta_R\Delta_R^\dagger\right] \\
&+ \rho_4\left(\left[\Delta_L\Delta_L\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right] + \left[\Delta_L^\dagger\Delta_L^\dagger\right]\left[\Delta_R\Delta_R\right]\right) + \alpha_1\left[\phi^\dagger\phi\right]\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right) \\
&+ \left(\alpha_2\left(\left[\tilde\phi\phi^\dagger\right]\left[\Delta_L\Delta_L^\dagger\right] + \left[\tilde\phi^\dagger\phi\right]\left[\Delta_R\Delta_R^\dagger\right]\right) + \text{h.c.}\right) + \alpha_3\left(\left[\phi\phi^\dagger\Delta_L\Delta_L^\dagger\right] + \left[\phi^\dagger\phi\Delta_R\Delta_R^\dagger\right]\right) \\
&+ \beta_1\left(\left[\phi\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_2\left(\left[\tilde\phi\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\tilde\phi^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_3\left(\left[\phi\Delta_R\tilde\phi^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\tilde\phi\Delta_R^\dagger\right]\right)
\end{aligned}$$

In the case of $\mathscr{C}$, additional phases appear:

$\Rightarrow$ the parameters $\mu_2\,, \lambda_2\,, \lambda_4\,, \rho_4$ and β_i can now be complex, in $\mathscr{P}$ only α_2 carries the phase δ_2

Diagonalising the Lagrangian: Scalar sector

The most general $\mathscr{P}$- (and $\mathscr{C}$-) symmetric potential is given by:

$$\begin{aligned}\mathcal{V} = &-\mu_1^2\left[\phi^\dagger\phi\right] - \mu_2^2\left(\left[\tilde{\phi}\phi^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\right) - \mu_3^2\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right)\\
&+ \lambda_1\left[\phi^\dagger\phi\right]^2 + \lambda_2\left(\left[\tilde{\phi}\phi^\dagger\right]^2 + \left[\tilde{\phi}^\dagger\phi\right]^2\right) + \lambda_3\left[\tilde{\phi}\phi^\dagger\right]\left[\tilde{\phi}^\dagger\phi\right] + \lambda_4\left[\phi^\dagger\phi\right]\left(\left[\tilde{\phi}\phi^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\right)\\
&+ \rho_1\left(\left[\Delta_L\Delta_L^\dagger\right]^2 + \left[\Delta_R\Delta_R^\dagger\right]^2\right) + \rho_2\left(\left[\Delta_L\Delta_L\right]\left[\Delta_L^\dagger\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right]\right) + \rho_3\left[\Delta_L\Delta_L^\dagger\right]\left[\Delta_R\Delta_R^\dagger\right]\\
&+ \rho_4\left(\left[\Delta_L\Delta_L\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right] + \left[\Delta_L^\dagger\Delta_L^\dagger\right]\left[\Delta_R\Delta_R\right]\right) + \alpha_1\left[\phi^\dagger\phi\right]\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right)\\
&+ \left(\alpha_2\left(\left[\tilde{\phi}\phi^\dagger\right]\left[\Delta_L\Delta_L^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\left[\Delta_R\Delta_R^\dagger\right]\right) + \text{h.c.}\right) + \alpha_3\left(\left[\phi\phi^\dagger\Delta_L\Delta_L^\dagger\right] + \left[\phi^\dagger\phi\Delta_R\Delta_R^\dagger\right]\right)\\
&+ \beta_1\left(\left[\phi\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_2\left(\left[\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_3\left(\left[\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger\right]\right)\end{aligned}$$

The minimisation conditions $\dfrac{\partial\mathscr{V}}{\partial S_i} = 0$ and $\dfrac{\partial^2\mathscr{V}}{\partial S_i\partial S_j} > 0$ lead us to:

$$\begin{aligned}\mu_1^2 &= 2\left(\lambda_1 + s_{2\beta}c_\alpha\lambda_4\right)v^2 + \left(\alpha_1 - \alpha_3\frac{s_\beta^2}{c_{2\beta}}\right)v_R^2\,,\\
\mu_2^2 &= \left(s_{2\beta}\left(2c_{2\alpha}\lambda_2 + \lambda_3\right) + \lambda_4\right)v^2\\
&\quad + \frac{1}{2c_\alpha}\left(2c_{\alpha+\delta_2}\alpha_2 + \alpha_3\frac{t_{2\beta}}{2c_\alpha}\right)v_R^2\,,\\
\mu_3^2 &= \left(\alpha_1 + \left(2c_{\alpha+\delta_2}\alpha_2 s_{2\beta} + \alpha_3 s_\beta^2\right)\right)v^2 + 2\rho_1 v_R^2\\
\alpha_2 s_{\delta_2} &= \frac{s_\alpha}{4}\left(\alpha_3 t_{2\beta} + 4\left(\lambda_3 - 2\lambda_2\right)s_{2\beta}\epsilon^2\right).\end{aligned}$$

$$\begin{aligned}v_L = &\frac{\epsilon^2 v_R}{\left(1+t_\beta^2\right)\left(2\rho_1 - \rho_3\right)}\Big(-\beta_1 t_\beta\cos(\alpha-\theta_L)\\
&+ \beta_2\cos(\theta_L) + \beta_3 t_\beta^2\cos(2\alpha - \theta_L)\Big)\,.\end{aligned}$$

For exact solvability we assume $\beta_i = v_L = 0$ and keep only the phase δ_2 (no impact on collider pheno)

In any case: $v_L \ll v \ll v_R \simeq \mathcal{O}(\text{TeV})$

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

Let's start with the "easy" ones that don't mix (in units of v_R) :

$$m^2_{\Delta_R^{++}} = 4\rho_2 + \frac{c_{2\beta}}{c_\beta^4}\alpha_3\epsilon^2 \,,$$

$v_L = 0 \Rightarrow$ no mixing of $\Delta_L \,, \Delta_R^{++}$

$$m^2_{\Delta_L^{++}} = (\rho_3 - 2\rho_1) - \frac{t_\beta^4 - 2c_{2\alpha}t_\beta^2 + 1}{t_\beta^4 - 1}\alpha_3\epsilon^2 \,,$$

$$m^2_{\Delta_L^{+}} = (\rho_3 - 2\rho_1) - \frac{\left(t_\beta^2 + 1\right)^2 - 4t_\beta^2 c_{2\alpha}}{2\left(t_\beta^4 - 1\right)}\alpha_3\epsilon^2 \,,$$

$$m^2_{\Delta_L^0} = m^2_{\chi_L^0} = (\rho_3 - 2\rho_1) + s_{2\beta}t_{2\beta}s_\alpha^2\alpha_3\epsilon^2 \,,$$

Take as input parameters: $m_{\Delta_R^{++}}$, $m_{\Delta_L^0}$, (and $\tan\beta$ and α), solve for $\rho_{2,3}$

ρ_1 and α_3 are fixed by other masses

$\Rightarrow$ Mass spectrum of Δ_L follows a sum rule:

$$m^2_{\Delta_L^{++}} - m^2_{\Delta_L^{+}} = m^2_{\Delta_L^{+}} - m^2_{\Delta_L^{0}} = v^2\alpha_3\frac{c_{2\beta}}{2}$$

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

Now the singly charged scalars:

$$(\phi_1^-, \phi_2^-, \Delta_R^-)\, M_+^2 \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \Delta_R^+ \end{pmatrix} \quad \textbf{with} \quad M_+^2 = \alpha_3 v_R^2 \begin{pmatrix} \frac{s_\beta^2}{c_{2\beta}} & -e^{-i\alpha}\frac{t_{2\beta}}{2} & -\epsilon e^{-i\alpha}\frac{s_\beta}{\sqrt{2}} \\ -e^{i\alpha}\frac{t_{2\beta}}{2} & \frac{c_\beta^2}{c_{2\beta}} & \epsilon\frac{c_\beta}{\sqrt{2}} \\ -\epsilon e^{i\alpha}\frac{s_\beta}{\sqrt{2}} & \epsilon\frac{c_\beta}{\sqrt{2}} & \epsilon^2\frac{c_{2\beta}}{2} \end{pmatrix}$$

M_+ is diagonalised with a unitary rotation (up to $\mathscr{O}(\epsilon^2)$:

$$\begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \Delta_R^+ \end{pmatrix} = U_+ \begin{pmatrix} \varphi_L^+ \\ H^+ \\ \varphi_R^+ \end{pmatrix}$$

$$\begin{aligned} U_+ &= \begin{pmatrix} c_\beta & -e^{-i\alpha}s_\beta & 0 \\ e^{i\alpha}s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1+\frac{1}{4}\epsilon^2 c_{2\beta}^2 & \frac{\epsilon c_{2\beta}}{\sqrt{2}} \\ 0 & -\frac{\epsilon c_{2\beta}}{\sqrt{2}} & -1+\frac{1}{4}\epsilon^2 c_{2\beta}^2 \end{pmatrix} \begin{pmatrix} 1-\frac{1}{4}\epsilon^2 s_{2\beta}^2 & 0 & \frac{\epsilon s_{2\beta}}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{\epsilon s_{2\beta}}{\sqrt{2}} & 0 & 1-\frac{1}{4}\epsilon^2 s_{2\beta}^2 \end{pmatrix} \\ &\simeq \begin{pmatrix} c_\beta & e^{-i\alpha}s_\beta & 0 \\ e^{i\alpha}s_\beta & -c_\beta & 0 \\ 0 & 0 & -1 \end{pmatrix} + \frac{\epsilon}{\sqrt{2}} \begin{pmatrix} 0 & 0 & e^{-i\alpha}s_\beta \\ 0 & 0 & c_\beta \\ e^{i\alpha}s_{2\beta} & -c_{2\beta} & 0 \end{pmatrix} + \frac{\epsilon^2}{2} \begin{pmatrix} -4c_\beta s_\beta^4 & -e^{-i\alpha}s_\beta c_{2\beta}^2 & 0 \\ -4e^{i\alpha}s_\beta c_\beta^4 & c_\beta c_{2\beta}^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

$\Rightarrow \varphi_{L,R}^\pm$ are the goldstones of $W_{L,R}^\pm$ and remain massless

The remaining (fixed) mass is

$$m_{H^+} \simeq \sqrt{\frac{\alpha_3}{c_{2\beta}}}\, v_R \left(1 + \epsilon^2 \frac{c_{2\beta}^{3/2}}{4}\right)$$

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

Goldstones are decoupled in an independent rotation

We are left with 4 neutral states in the basis $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})$

$$m_0^2 = \begin{pmatrix} 4\epsilon^2\left(\lambda_1 + \frac{4tc_\alpha\left(\lambda_4\left(t^2+1\right)+4\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2}\right) & 2\epsilon\left(\alpha_1 - \frac{t^2X\left(t^2-s_{2\alpha+\delta_2}/s_{\delta_2}\right)}{\left(t^2+1\right)^2}\right) & \frac{4\epsilon^2\left(t^2c_{2\alpha}-1\right)\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} & \frac{4t^2\epsilon^2 s_{2\alpha}\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} \\ 2\epsilon\left(\alpha_1 - \frac{t^2X\left(t^2-s_{2\alpha+\delta_2}/s_{\delta_2}\right)}{\left(t^2+1\right)^2}\right) & Y & \frac{2tX\epsilon\left(t^2c_{2\alpha}-1\right)s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} & \frac{2t^3X\epsilon s_{2\alpha}s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} \\ \frac{4\epsilon^2\left(t^2c_{2\alpha}-1\right)\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} & \frac{2tX\epsilon\left(t^2c_{2\alpha}-1\right)s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} & X + \frac{16\lambda_2\epsilon^2\left(t^2c_{2\alpha}-1\right)^2}{\left(t^2+1\right)^2} & \frac{16\lambda_2 t^2\epsilon^2 s_{2\alpha}\left(t^2c_{2\alpha}-1\right)}{\left(t^2+1\right)^2} \\ \frac{4t^2\epsilon^2 s_{2\alpha}\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} & \frac{2t^3X\epsilon s_\alpha s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} & \frac{16\lambda_2 t^2\epsilon^2 s_{2\alpha}\left(t^2c_{2\alpha}-1\right)}{\left(t^2+1\right)^2} & X + \frac{16\lambda_2 t^4\epsilon^2 s_{2\alpha}^2}{\left(t^2+1\right)^2} \end{pmatrix}$$

First we decouple the **SM**-like Higgs h from the rest via a 2-1 rotation around θ:

$$m_h^2 = v^2\left(4\lambda_1 + \frac{64\lambda_2 t^2 c_\alpha^2}{\left(t^2+1\right)^2} + \frac{16\lambda_4 tc_\alpha}{t^2+1} - Y\tilde{\theta}^2\right)$$

$$\tilde{\theta} = \frac{\theta}{\epsilon} = \left(\frac{2\alpha_1}{Y} + \frac{X}{Y}\frac{t^2(1-t^2)}{1+t^2}\frac{\sin(2\alpha+\delta_2)}{\sin(\delta_2)}\right)$$

m_h and θ will be taken as input to solve for λ_1 and α_1

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

We are left with 4 neutral states in the basis $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})$

Remarkably, setting $\lambda_3 = 2\lambda_2$ allows to determine the remaining rotations *exactly*:

We rotate $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})^T = O_N(h, \Delta_R, H, A)^T$

$$\theta \equiv \epsilon\,\tilde{\theta} \equiv \theta_{21} = \epsilon \left[\frac{2\alpha_1}{Y} - \frac{2X\left(t^4 - t^2\, s_{2\alpha+\delta_2}/s_{\delta_2}\right)}{Y\left(t^2+1\right)^2}\right],$$

$$\phi \equiv \epsilon^2 \tilde{\phi} \equiv \theta_{31} = \epsilon^2 \frac{\left(t^2 c_{2\alpha} - 1\right)}{(1+t^2)^2}\left[\frac{32 t c_\alpha \lambda_2 + 4\lambda_4(1+t^2)}{X} - 2\,t\,\tilde{\theta}\, s_{\alpha+\delta_2}/s_{\delta_2}\right],$$

$$\theta_{41} = \epsilon^2 \frac{t^2 s_{2\alpha}}{\left(1+t^2\right)^2}\left[\frac{32 t c_\alpha \lambda_2 + 4\lambda_4(t^2+1)}{X} - 2\,t\,\tilde{\theta}\, s_{\alpha+\delta_2}/s_{\delta_2}\right],$$

$$\theta_{34} = \cot^{-1}\left[\cot(2\alpha) - \frac{\csc(2\alpha)}{t^2}\right],$$

$$\eta \equiv \theta_{23} = -\frac{1}{2}\tan^{-1}\left[\frac{4tX\epsilon\sqrt{t^4 - 2c_{2\alpha}t^2 + 1}\, s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2\left(Y\tilde{\theta}^2\epsilon^2 - \frac{16(t^4 - 2c_{2\alpha}t^2+1)\lambda_2\epsilon^2}{(t^2+1)^2} - X + Y\right)}\right]$$

$$h \text{ part of } \Re\Delta_R: \quad \theta \equiv \theta_{21} \simeq -(O_N)_{2,1},$$

$$H \text{ part of } \Re\Delta_R: \quad \eta \equiv \theta_{23} = \arcsin[(O_N)_{2,3}/c_\theta],$$

$$h \text{ part of } \Re\phi_{20}: \quad \phi \equiv \theta_{31} \simeq -(O_N)_{3,1},$$

θ, ϕ, η can be taken as **input** parameters!

Mixing angles also control scalar couplings to SM-gauge bosons & quarks!

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

We rotate $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})^T = O_N(h, \Delta_R, H, A)^T$ and get the mass eigenvalues:

$$
\begin{aligned}
m_h^2 &= \epsilon^2 \left(4\lambda_1 + \frac{64\lambda_2 t^2 c_\alpha^2}{(t^2+1)^2} + \frac{16\lambda_4 t c_\alpha}{t^2+1} - Y\tilde{\theta}^2\right), \\
m_\Delta^2 &= Y + \sec(2\eta)\left[(Y-X)s_\eta^2 + \epsilon^2\left(Y\tilde{\theta}^2 c_\eta^2 - \frac{16\lambda_2\left(t^4 - 2c_{2\alpha}t^2+1\right)}{(t^2+1)^2} s_\eta^2\right)\right] \\
m_H^2 &= X - \sec(2\eta)\left[(Y-X)s_\eta^2 + \epsilon^2\left(Y\tilde{\theta}^2 s_\eta^2 - \frac{16\lambda_2\left(t^4 - 2c_{2\alpha}t^2+1\right)}{(t^2+1)^2} c_\eta^2\right)\right] \\
m_A^2 &= X\,,
\end{aligned}
$$

The masses m_h, m_Δ, m_H, m_A are taken as input parameters to determine the potential

And we get another sum rule:

$$
m_{H^+}^2 - m_A^2 = v^2 \alpha_3 \frac{c_{2\beta}}{2} \sim O(150\,\mathrm{GeV})^2
$$

Mass splitting $|m_H^2 - m_A^2|$ must be small to ensure perturbativity of λ_2 : $|m_H^2 - m_A^2| \lesssim 16v^2$

Diagonalising the Lagrangian: Fermions

The various vevs in the model induce tree-level mass terms for all fermions:

Only the bi-doublet couples to quarks, giving masses to up- and down-type quarks

$$\mathcal{L}_Y^q = \bar{Q}_L' \left(Y_q \, \phi + \tilde{Y}_q \, \tilde{\phi} \right) Q_R' + \text{H.c.} \, ,$$
$$\mathcal{L}_Y^\ell = \bar{L}_L' \left(Y_\ell \, \phi + \tilde{Y}_\ell \, \tilde{\phi} \right) L_R' +$$
$$+ \bar{L}_L'^c i\sigma_2 \Delta_L Y_L^M L_L' + \bar{L}_R'^c i\sigma_2 \Delta_R Y_R^M L_R' + \text{H.c.} \, .$$

$$M_u = Y_q \, v_1 + \tilde{Y}_q \, e^{-i\alpha} v_2$$
$$M_d = -Y_q \, e^{i\alpha} v_2 - \tilde{Y}_q \, v_1$$

Which are diagonalised as:

$$M_u = U_{uL} \, m_u \, U_{uR}^\dagger \, , \qquad M_d = U_{dL} \, m_d \, U_{dR}^\dagger$$

From these mixings we can define the **CKM** and its **right-handed** (measurable) analogue:

$V_L^{\text{CKM}} \equiv U_{uL}^\dagger U_{dL}$, $V_R^{\text{CKM}} \equiv U_{uR}^\dagger U_{dR}$ (V_R can has additional phases in the case of $\mathscr{C}$)

The quark Yukawas are then fully determined from measurable inputs: quark masses and mixings

$$Y_q = \frac{1}{v_1^2 - v_2^2} \left(M_u \, v_1 + e^{-i\alpha} M_d \, v_2 \right)$$
$$\tilde{Y}_q = -\frac{1}{v_1^2 - v_2^2} \left(M_d \, v_1 + e^{i\alpha} M_u \, v_2 \right)$$

Diagonalising the Lagrangian: Fermions

Both triplets and the bi-doublet induce mass-terms for the leptons:

$$M_\ell = -Y_\ell v_2 e^{i\alpha} + \tilde{Y}_\ell v_1\,, \quad M_D = Y_\ell v_1 - \tilde{Y}_\ell v_2 e^{-i\alpha}\,, \quad M_L = v_L Y_L^M\,, \quad M_R = v_R Y_R^M$$

In which M_D is a mass-term between **LH** and RH neutrinos, M_L and M_R are Majorana

The charged lepton mass M_ℓ is easily diagonalised: $M_\ell = U_{\ell L}\, m_\ell\, U_{\ell R}^\dagger$

And the Yukawas of the bi-doublet are given by:

$$Y_\ell = \frac{1}{v_1^2 - v_2^2}\left(M_D\, v_1 + M_\ell\, e^{-i\alpha} v_2\right)$$

$$\tilde{Y}_\ell = -\frac{1}{v_1^2 - v_2^2}\left(M_\ell\, v_1 + M_D\, e^{i\alpha} v_2\right)$$

Due to M_D, **LH** and **RH** neutrinos mix with each other, we need to diagonalise the mass matrix:

$$\bar{n}_L' M_n n_L^{'c} = \left(\bar{\nu}_L' \; \bar{\nu}_R^{'c}\right) \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^{'c} \\ \nu_R' \end{pmatrix}$$

Diagonalising the Lagrangian: Fermions

Due to M_D, **LH** and **RH** neutrinos mix with each other, we need to diagonalise the mass matrix:

$$\bar{n}'_L M_n n'^c_L = (\bar{\nu}'_L \; \bar{\nu}'^c_R) \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu'^c_L \\ \nu'_R \end{pmatrix}$$

Majorana mass matrix is complex symmetric: (block-) diagonalised via Autonne-Takagi factorisation:

$$\tilde{W}^T M_n \tilde{W} = \begin{pmatrix} m_{\text{light}} & \mathbb{0} \\ \mathbb{0} & m_{\text{heavy}} \end{pmatrix}$$

Perturbative diagonalisation (expand in M_R^{-1}) gives us:

$$M_\nu \simeq M_L - M_D M_R^{-1} M_D^T \,, \qquad M_N \simeq M_R$$

In which the blocks are diagonalised via the unitary matrices V_ν and V_N:

$$V_\nu^T M_\nu V_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \,, \quad V_N^\dagger M_N V_N^* = \text{diag}(m_{N_1}, m_{N_2}, m_{N_3})$$

Diagonalising the Lagrangian: Fermions

The full rotation matrix is approximately given by (up to M_R^{-1}):

$$W = \begin{pmatrix} \sqrt{\mathbb{1} - BB^\dagger} V_\nu & BV_N^* \\ -B^\dagger V_\nu & \sqrt{\mathbb{1} - B^\dagger B} V_N^* \end{pmatrix} \simeq \begin{pmatrix} V_\nu & B_1 V_N^* \\ -B_1^\dagger V_\nu & V_N^* \end{pmatrix} .$$

With $B_1 = M_D^\dagger M_R^{-1\dagger}$

Charged lepton currents can be cast as:

$$\mathcal{L}_{cc}^\ell = \frac{g_L}{\sqrt{2}} \bar{\ell}_L \gamma^\mu \mathcal{U}_L n_L W_L^\mu + \frac{g_R}{\sqrt{2}} \bar{\ell}_R \gamma^\mu \mathcal{U}_R n_R W_R^\mu$$

With the 3×6 mixing matrices given by:

$$(\mathcal{U}_L)_{\alpha i} = \sum_{k=1}^{3} (V_{\ell L}^\dagger)_{\alpha k} W_{ki} ,$$

$$(\mathcal{U}_R)_{\alpha i} = \sum_{k=1}^{3} (V_{\ell L}^\dagger)_{\alpha k} W_{(k+3)i} .$$

The first 3×3 block of $\mathcal{U}_L$ can be identified as the **LH** would-be PMNS, the second 3×3 block of $\mathcal{U}_R$ as its **RH** analogue

$\mathcal{U}_R$ could be measured in $W_R^\pm \to \ell N$ decays