

New methods to probe Standard Model Extensions via Proton Decay and Neutrino Masses

Arnau Bas i Beneito

ABiB, J. Gargalionis, J. Herrero-García, M.A.Schmidt, [arXiv 2503.20928 \[hep-ph\]](https://arxiv.org/abs/2503.20928)

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- 1. Introduction**
2. EFT framework for Majorana neutrino masses and proton decay
3. Theoretical framework
4. Results
5. Conclusions and outlook

SM physics

The SM of Particle Physics (SM) $\longrightarrow$ **Most accurate** theories of the interactions of particles

• **Ingredients:** Particle content and local symmetries (gauge group)

• **Recipe:** interactions between the particles allowed by our gauge (and Lorentz) group

$$\mathcal{L}_{SM} = \mathcal{L}^{\text{kin}}(\psi, \phi, A) + \mathcal{L}^{\text{int}}(\psi, \phi, A)$$

Chiral fermions
Gauge bosons
 $\downarrow$
 $\downarrow$
 $\uparrow$
Higgs scalar

	1 st	2 nd	3 rd	
Quarks	u up	c charm	t top	γ photon
	d down	s strange	b beauty	
Leptons	e electron	μ muon	τ tau	$W^\pm$ W boson
	ν_e neutrino electron	ν_μ neutrino muon	ν_τ neutrino tau	
				Z^0 Z boson
				g gluon
				H Higgs Boson

- Anomalous magnetic dipole moment of the electron ($g - 2$)
- EWPTs, m_W and m_Z
- Existence of a (light) scalar
- Prediction and discovery of heavy top quark

...

SM physics

The SM of Particle Physics (SM) $\longrightarrow$ **Most accurate** theories of the interactions of particles

What lies beyond the SM?

What lies beyond the SM?

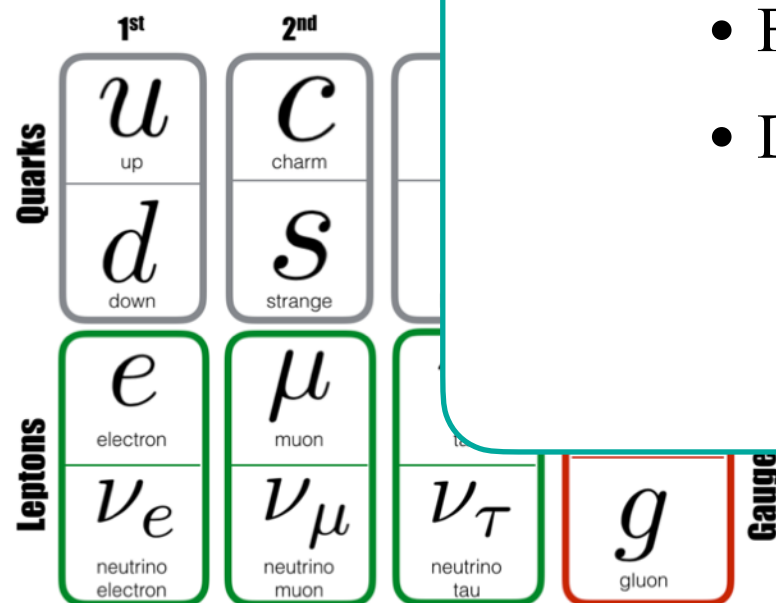
- Neutrino masses
- Matter over anti-matter antisymmetry
- Dark Matter
- Strong CP puzzle
- Flavour puzzle
- Dark Energy

• • •

oment of the

- Prediction and discovery of heavy top quark

• • •



SM physics

The SM of Particle Physics (SM) $\longrightarrow$ **Most accurate** theories of the interactions of particles

• **Ingredients:** Particles, forces, symmetries (gauge symmetries)

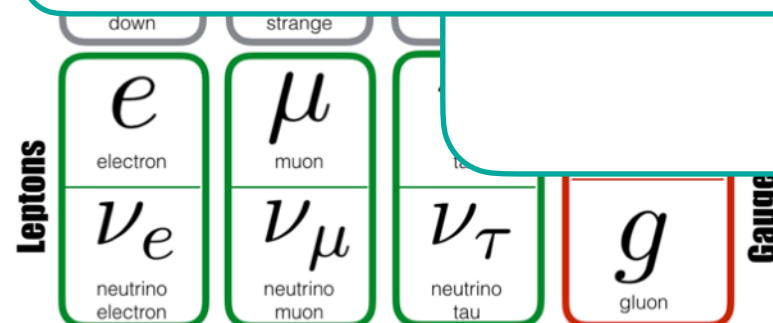
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- Neutrino masses
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- Strong CP puzzle

Connected to **Lepton Number**

Connected to **Baryon Number**

[A. Sakharov 1967]



- Prediction and discovery of heavy top quark

...

Lepton and Baryon Number

$$\mathcal{L}_{\text{SM}} \supset \bar{L}i\not{D}L + \bar{e}_Ri\not{D}e_R + \bar{Q}i\not{D}Q + \bar{u}_Ri\not{D}u_R + \bar{d}_Ri\not{D}d_R$$

Kinetic terms preserve largest flavour symmetry $U(3)^5$

Yukawa
couplings

$$\longrightarrow [Y_e]_{pq}\bar{L}^pe_R^qH \quad [Y_d]_{pq}\bar{Q}^pd_R^qH \quad [Y_u]_{pq}\bar{Q}^pu_R^qi\sigma_2H^*$$

$$U(3)^5 \rightarrow U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$$

[Super-Kamiokande 1999, SNO 2002, KamLAND 2003]

Quark sector

In the SM Lagrangian there are no **Baryon Number Violating (BNV)** interactions



B (perturbatively) conserved



The proton (lightest baryon) is **stable**

Lepton sector

Observance of neutrino oscillations break Flavour Lepton Number to **Total Lepton Number**

$$U(1)_{L_i}^3 \rightarrow U(1)_L$$



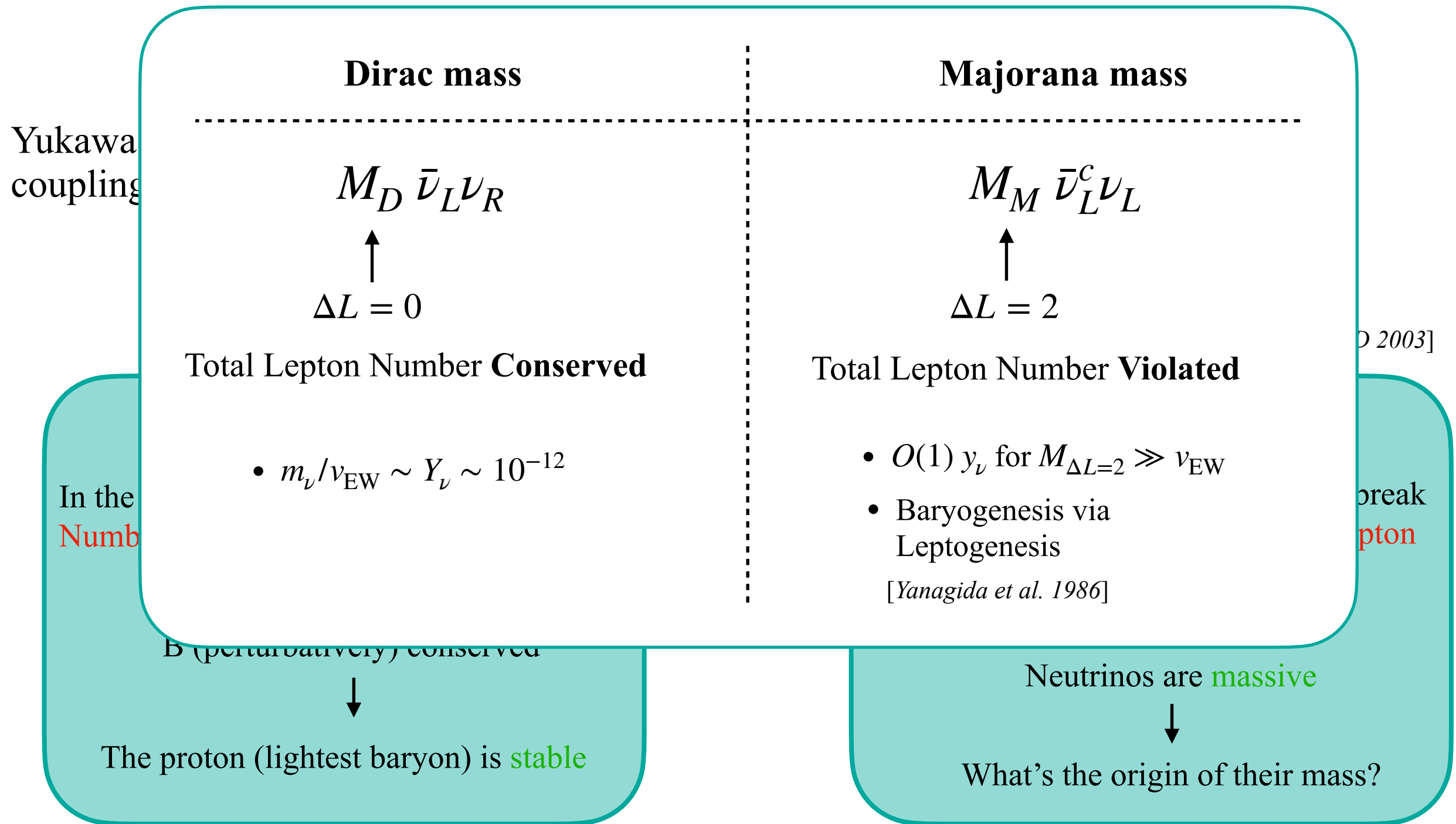
Neutrinos are **massive**



What's the origin of their mass?

Lepton and Baryon Number

$$\mathcal{L}_{\text{SM}} \supset \bar{L} i \not{D} L + \bar{e}_R i \not{D} e_R + \bar{Q} i \not{D} Q + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R$$



Lepton and Baryon Number Violation

B and L are accidental symmetries of the SM and seems artificial to forbid them in UV models



- No fundamental argument to have B and L conserved
- Explicit **BNV** and **LVN** in simple UV extensions, such as in:
 - GUTs *[Georgi et al. 1973, H. Fritzsch et al. 1975...]*
 - Leptoquark interactions *[Buchmüller et al. 1987, I. Doršner et al. 2016...]*
 - Seesaw *[P. Minkowski 1977, T. Yanagida 1979, R. N. Mohapatra et al. 1980...]*
- **BNV** needed to account for Baryogenesis *[A. Sakharov 1967]*

Lepton and Baryon Number Violation

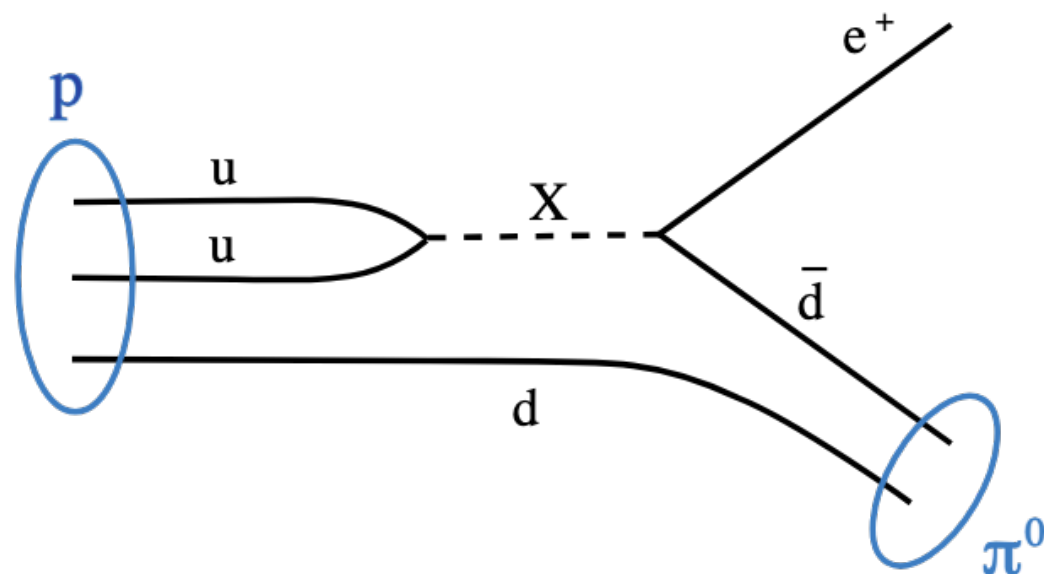
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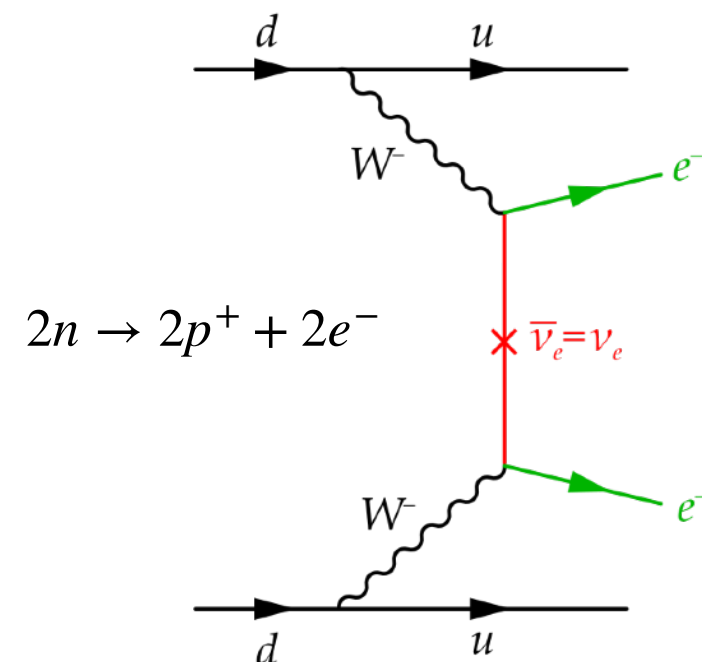
proton decay

($\Delta B = \pm \Delta L = 1$ processes)

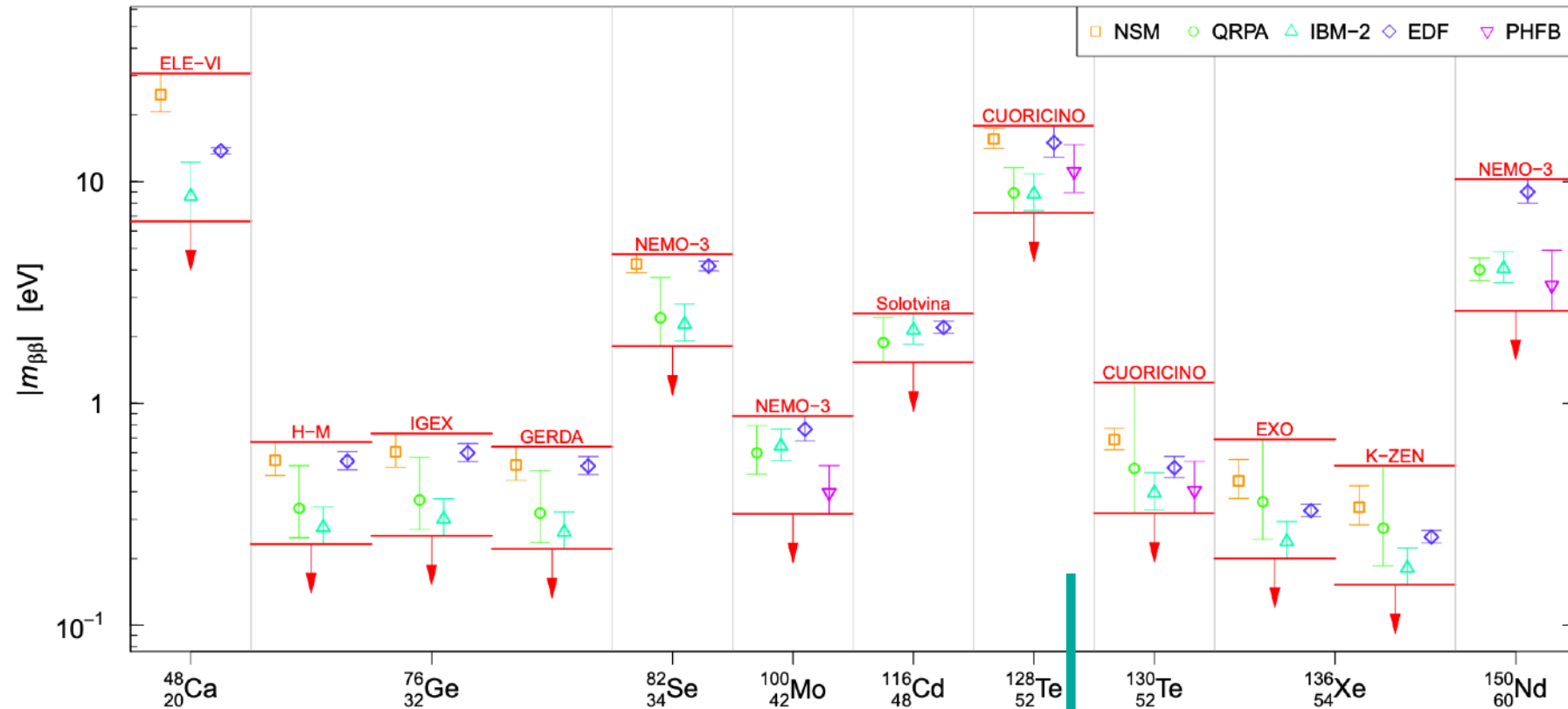


$0\nu\beta\beta$ -decay

($\Delta L = 2$ processes)



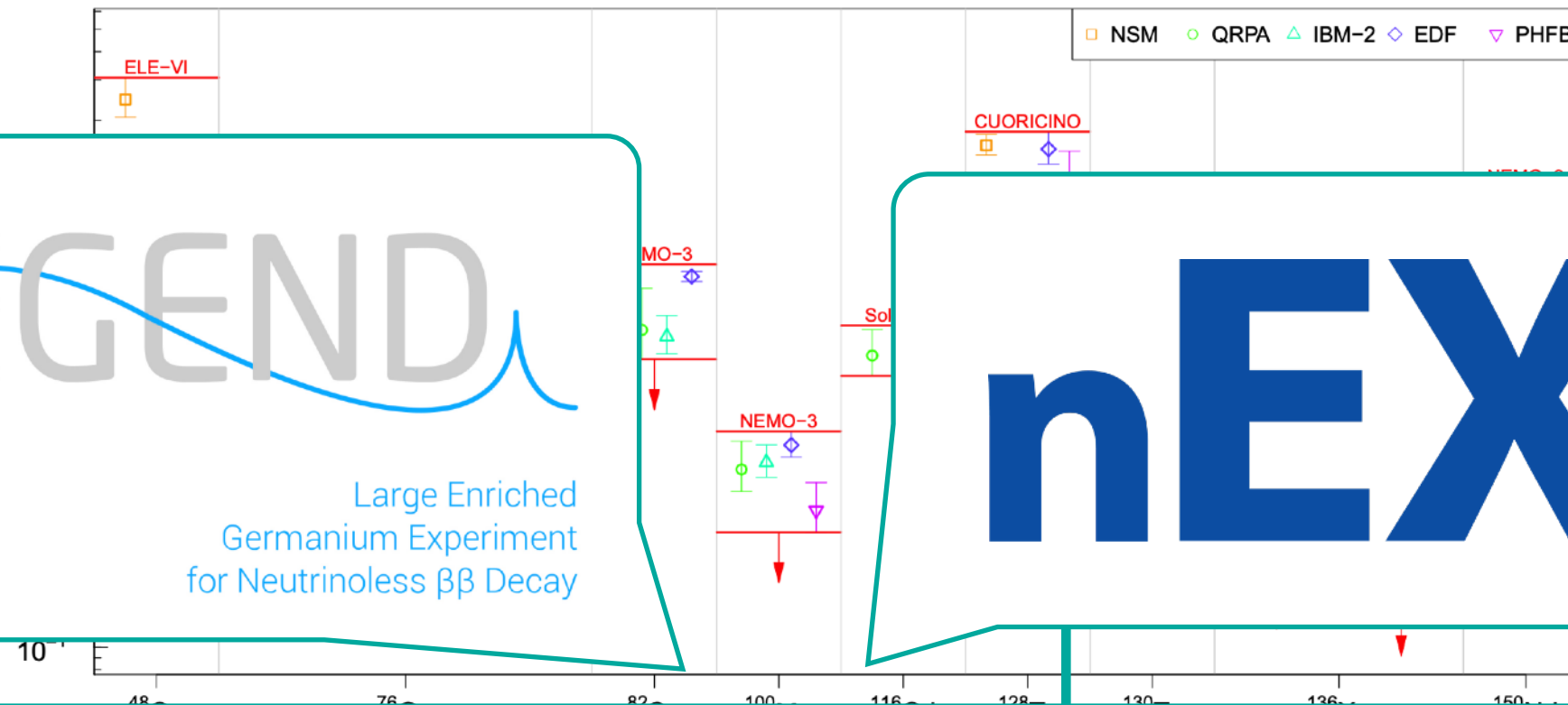
$0\nu\beta\beta$ -decay: searching for Majorana nature of ν



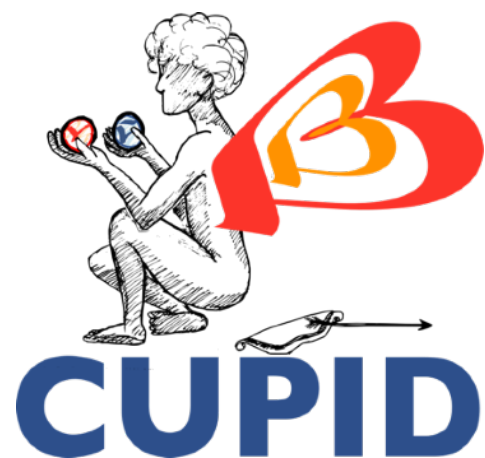
$\beta\beta^-$ decay	experiment	$T_{1/2}^{0\nu}$ [y]	$m_{\beta\beta}$ [eV]
$^{48}_{20}\text{Ca} \rightarrow ^{48}_{22}\text{Ti}$	ELEGANT-VI [119]	$> 1.4 \times 10^{22}$	$< 6.6 - 31$
$^{76}_{32}\text{Ge} \rightarrow ^{76}_{34}\text{Se}$	Heidelberg-Moscow [224]	$> 1.9 \times 10^{25}$	$< 0.23 - 0.67$
	IGEX [226]	$> 1.6 \times 10^{25}$	$< 0.25 - 0.73$
	GERDA [32]	$> 2.1 \times 10^{25}$	$< 0.22 - 0.64$
$^{82}_{34}\text{Se} \rightarrow ^{82}_{36}\text{Kr}$	NEMO-3 [120]	$> 1.0 \times 10^{23}$	$< 1.8 - 4.7$
$^{100}_{42}\text{Mo} \rightarrow ^{100}_{44}\text{Ru}$	NEMO-3 [121]	$> 2.1 \times 10^{25}$	$< 0.32 - 0.88$
$^{116}_{48}\text{Cd} \rightarrow ^{116}_{50}\text{Sn}$	Solotvina [234]	$> 1.7 \times 10^{23}$	$< 1.5 - 2.5$
$^{128}_{52}\text{Te} \rightarrow ^{128}_{54}\text{Xe}$	CUORICINO [235]	$> 1.1 \times 10^{23}$	$< 7.2 - 18$
$^{130}_{52}\text{Te} \rightarrow ^{130}_{54}\text{Xe}$	CUORICINO [236]	$> 2.8 \times 10^{24}$	$< 0.32 - 1.2$
$^{136}_{54}\text{Xe} \rightarrow ^{136}_{56}\text{Ba}$	EXO [239]	$> 1.1 \times 10^{25}$	$< 0.2 - 0.69$
	KamLAND-Zen [241]	$> 1.9 \times 10^{25}$	$< 0.15 - 0.52$
$^{150}_{60}\text{Nd} \rightarrow ^{150}_{62}\text{Sm}$	NEMO-3 [243]	$> 2.1 \times 10^{25}$	$< 2.6 - 10$

Images extracted from
S.M. Bilenky et al. 2025

$0\nu\beta\beta$ -decay: searching for Majorana nature of ν



LVN could be the next big discovery!

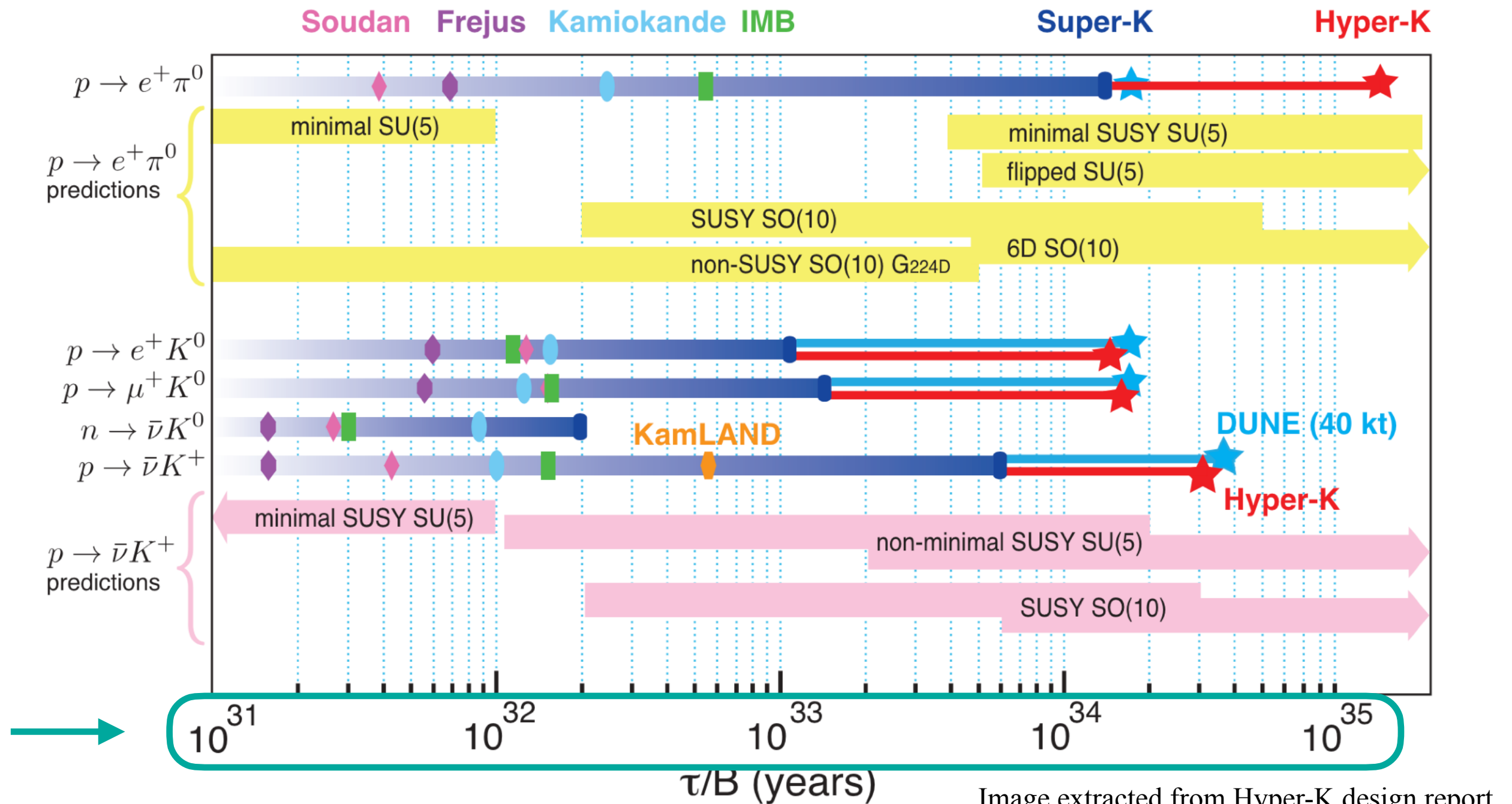


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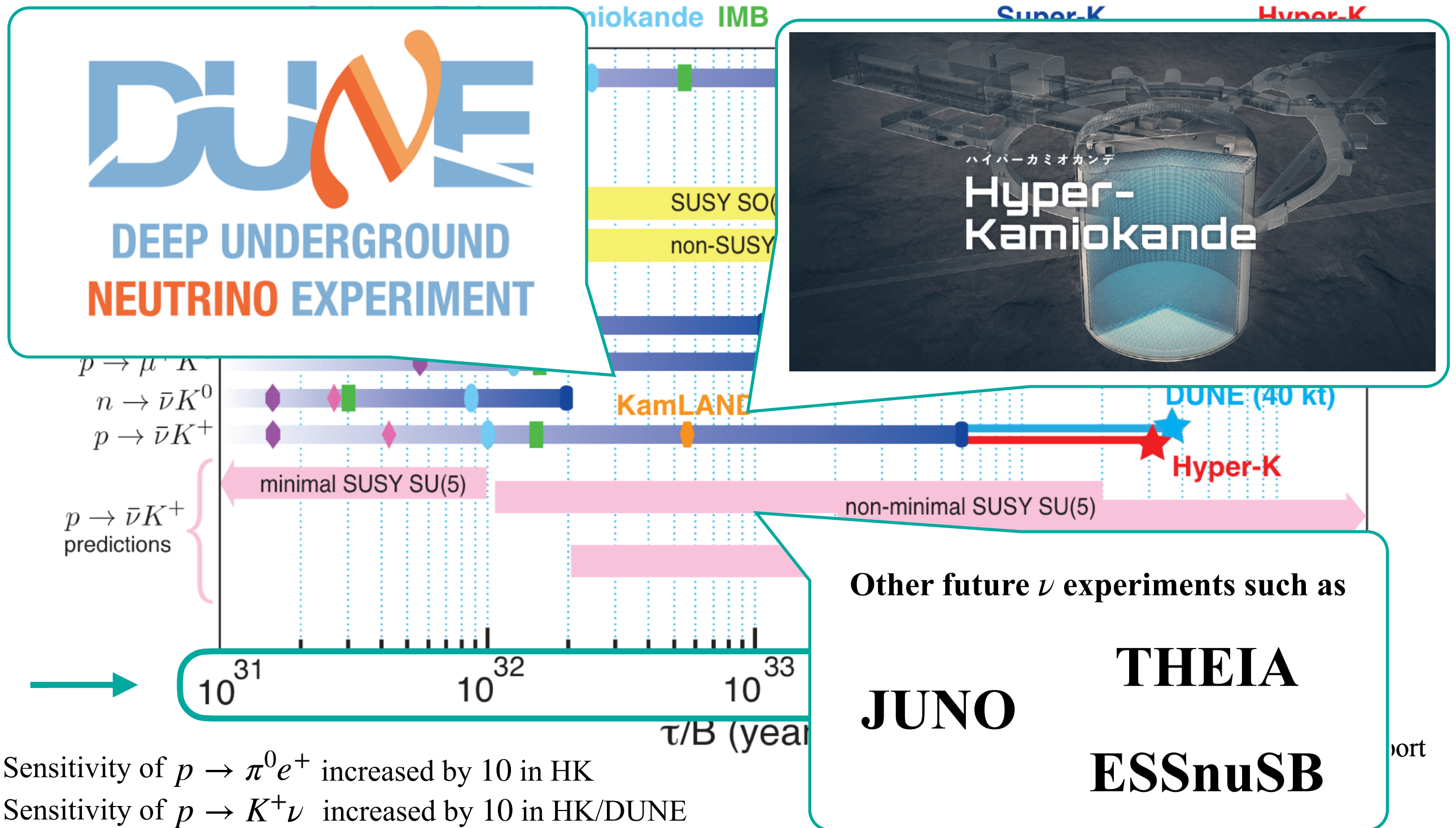


Images extracted from
S.M. Bilenky et al. 2025

Searches for Proton Decay



Searches for Proton Decay



Sensitivity of $p \rightarrow \pi^0 e^+$ increased by 10 in HK
Sensitivity of $p \rightarrow K^+ \nu$ increased by 10 in HK/DUNE
Sensitivity of $n \rightarrow K^+ e^-$ increased by 10^3 in HK

BNV nucleon decay could be the next big discovery!

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The SM is an effective theory → New Physics parametrised by higher-dimensional operators

[S. Weinberg 1979,
F. Wilczek et al 1979,
B. Grzadkowski et al. 2010,
W. Buchmuller et al. 1986,
I. Brivio et al. 2019,
B. Henning et al. 2016,
De Gouvea et al. 2014]

SM Effective Field Theory (SMEFT)

$$\mathcal{L} = \mathcal{L}_{SM} + \sum \frac{1}{\Lambda^{d-4}} \mathcal{O}^{(d)} \quad \begin{array}{l} [\mathcal{O}^{(d)}] = d \\ \text{Invariant under } G_{SM} \end{array}$$

Bounds on SMEFT WCs serve as a **bridge** to specific UV models

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c^{d=5}}{\Lambda} \mathcal{O}_W + \frac{c^{d=6}}{\Lambda^2} \mathcal{O}^{d=6} + \frac{c^{d=7}}{\Lambda^3} \mathcal{O}^{d=7} + \dots$$

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Bounds on SMEFT WCs serve as a **bridge** to specific UV models

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c^{d=5}}{\Lambda} \mathcal{O}_W + \frac{c^{d=6}}{\Lambda^2} \mathcal{O}^{d=6} + \frac{c^{d=7}}{\Lambda^3} \mathcal{O}^{d=7} + \dots$$

$\Delta L = 2$

↓

↑

Number of
independent
Components:

$\Delta(B - L) = 0$

↓

↑

$\Delta(B - L) = 2$

↓

↑

1 (6) for $N_f = 1$ (3)

63 (2772) for $N_f = 1$ (3)
Of which 4 (273) are BNV

18 (771) for $N_f = 1$ (3)
Of which 6 (297) are BNV

The SM is an effective field theory with $d=4$ operators. The SM Lagrangian is invariant under G_{SM} .

[S. Weinberg 1979, F. Wilczek et al. 1980, B. Grzadka et al. 1980, W. Buchmüller et al. 1980, I. Brivio et al. 1980, B. Henning et al. 2014, De Gouvea et al. 2014]

$$\begin{aligned} [\mathcal{O}_{\bar{l}dddH}]_{pqrs} &= (L_p^\dagger \bar{d}_q^\dagger)(\bar{d}_r^\dagger \bar{d}_s^\dagger)H, & [\mathcal{O}_{\bar{l}dq\tilde{H}}]_{pqrs} &= (L_p^\dagger \bar{d}_q^\dagger)(Q_r Q_s^i)\tilde{H}^j \epsilon_{ij}, \\ [\mathcal{O}_{\bar{e}qdd\tilde{H}}]_{pqrs} &= (\bar{e}_p Q_q^i)(\bar{d}_r^\dagger \bar{d}_s^\dagger)\tilde{H}^j \epsilon_{ij}, & [\mathcal{O}_{\bar{l}dud\tilde{H}}]_{pqrs} &= (L_p^\dagger \bar{d}_q^\dagger)(\bar{u}_r^\dagger \bar{d}_s^\dagger)\tilde{H}, \\ [\mathcal{O}_{\bar{l}qdDd}]_{pqrs} &= (L_p^\dagger \bar{\sigma}^\mu Q_q)(\bar{d}_r^\dagger i\overleftrightarrow{D}_\mu \bar{d}_s^\dagger), & [\mathcal{O}_{\bar{e}dddD}]_{pqrs} &= (\bar{e}_p \sigma^\mu \bar{d}_q^\dagger)(\bar{d}_r^\dagger i\overleftrightarrow{D}_\mu \bar{d}_s^\dagger). \end{aligned}$$

[S. Weinberg 1980]

$= d$

invariant under G_{SM}

$$\begin{aligned} [\mathcal{O}_{qqql}]_{pqrs} &= (Q_p^i Q_q^j)(Q_r^l L_s^k) \epsilon_{ik} \epsilon_{jl}, & [\mathcal{O}_{qq\bar{e}e}]_{pqrs} &= (Q_p^i Q_q^j)(\bar{u}_r^\dagger \bar{e}_s^\dagger) \epsilon_{ij}, \\ [\mathcal{O}_{duue}]_{pqrs} &= (\bar{d}_p^\dagger \bar{u}_q^\dagger)(\bar{u}_r^\dagger \bar{e}_s^\dagger), & [\mathcal{O}_{duql}]_{pqrs} &= (\bar{d}_p^\dagger \bar{u}_q^\dagger)(Q_r^i L_s^j) \epsilon_{ij}. \end{aligned}$$

[S. Weinberg 1979&1980, F. Wilczek et al. 1980, L. Abbott et al. 1980]

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c^{d=5}}{\Lambda} \mathcal{O}^{d=5} + \frac{c^{d=6}}{\Lambda^2} \mathcal{O}^{d=6} + \frac{c^{d=7}}{\Lambda^3} \mathcal{O}^{d=7} + \dots$$

Number of independent Components

$$[\mathcal{O}_5]_{pq} = (L_p^i L_q^j) H^k H^l \epsilon_{ik} \epsilon_{jl}$$

[S. Weinberg 1979]

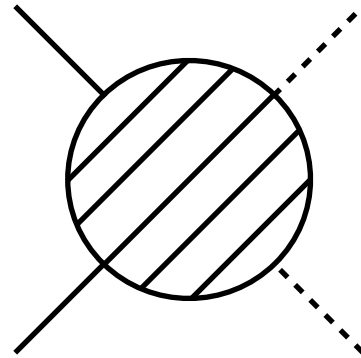
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Tree-level Proton decay and Neutrino masses

[S. Weinberg 1979 & 1980]

$$\Delta L = 2$$



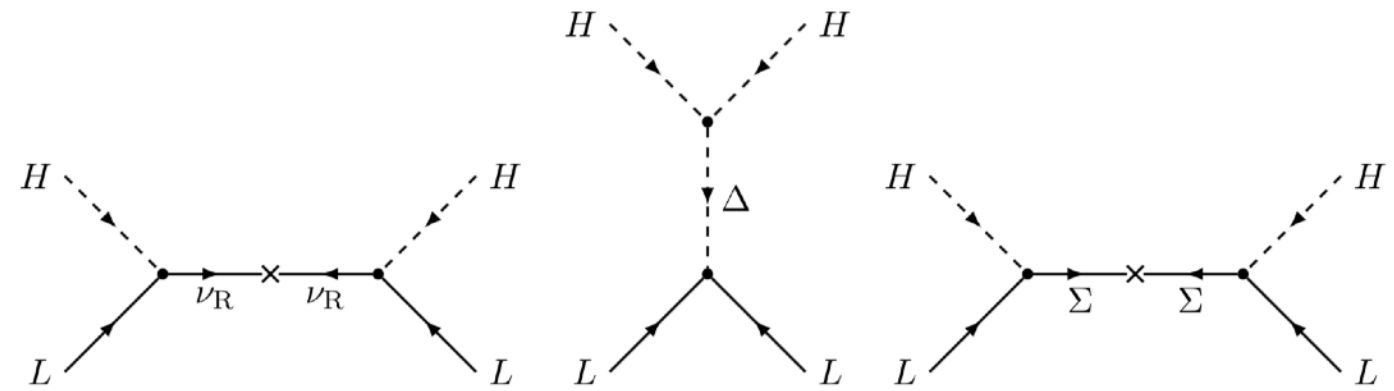
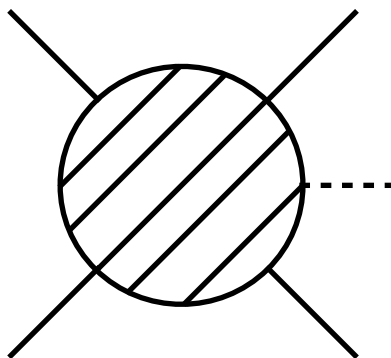
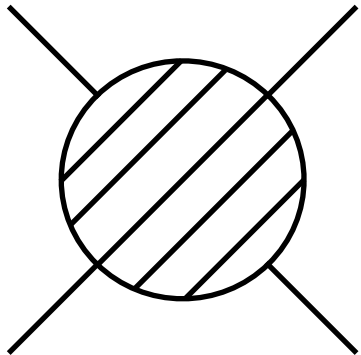
$$\Delta(B - L) = 0$$



Different
phenomenology

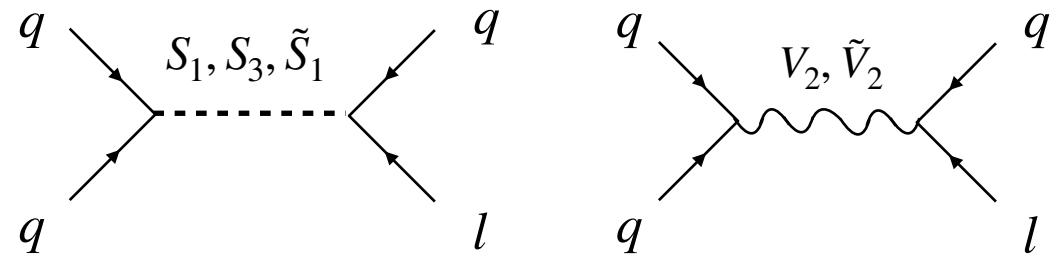


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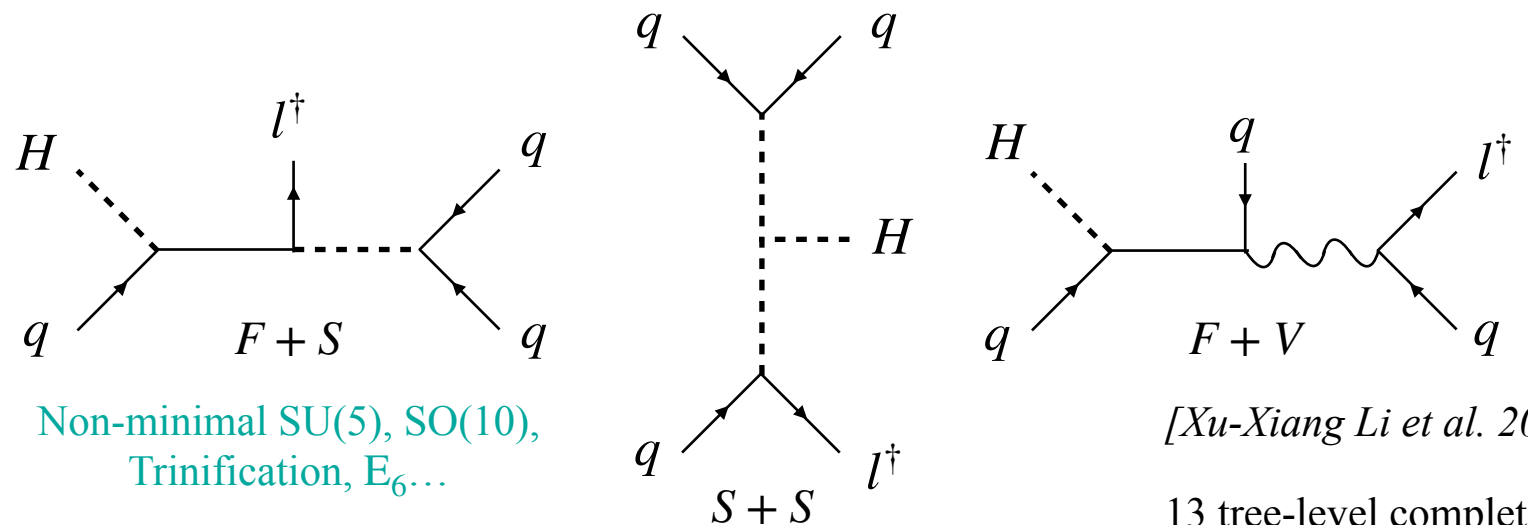
Seesaw mechanism

[P. Minkowski 1977, T. Yanagida 1979, M. Gell-Mann et al. 1979, R. N. Mohapatra et al. 1980, Glashow 1980...]



[J. de Blas et al. 2017, I. Doršner et al 2016, Xu-Xiang Li et al. 2023]

Minimal SU(5), SO(10), Trinification, E_6 ...



Non-minimal SU(5), SO(10),
Trinification, E_6 ...

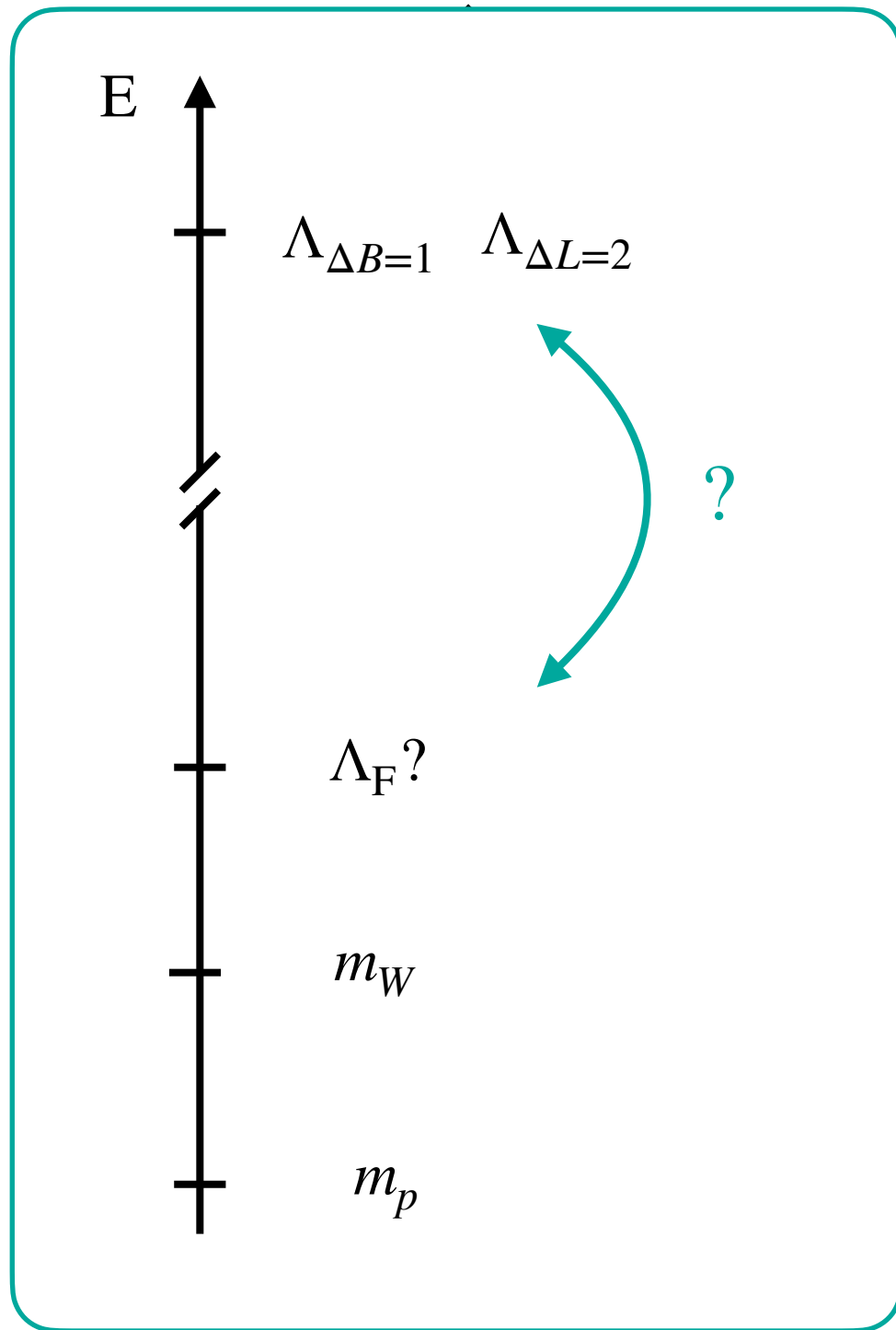
[Xu-Xiang Li et al. 2023]

13 tree-level completions

... and many more: $\Delta B = 2$, $\Delta(3B - L)$, $\Delta L = 4$...

Tree-level Proton decay and Neutrino masses

[S. Weinberg 1979 & 1980]



$$M \sim 10^{11} \text{ TeV} \quad \text{for O(1) WCs}$$

[Super-Kamiokande 1999]

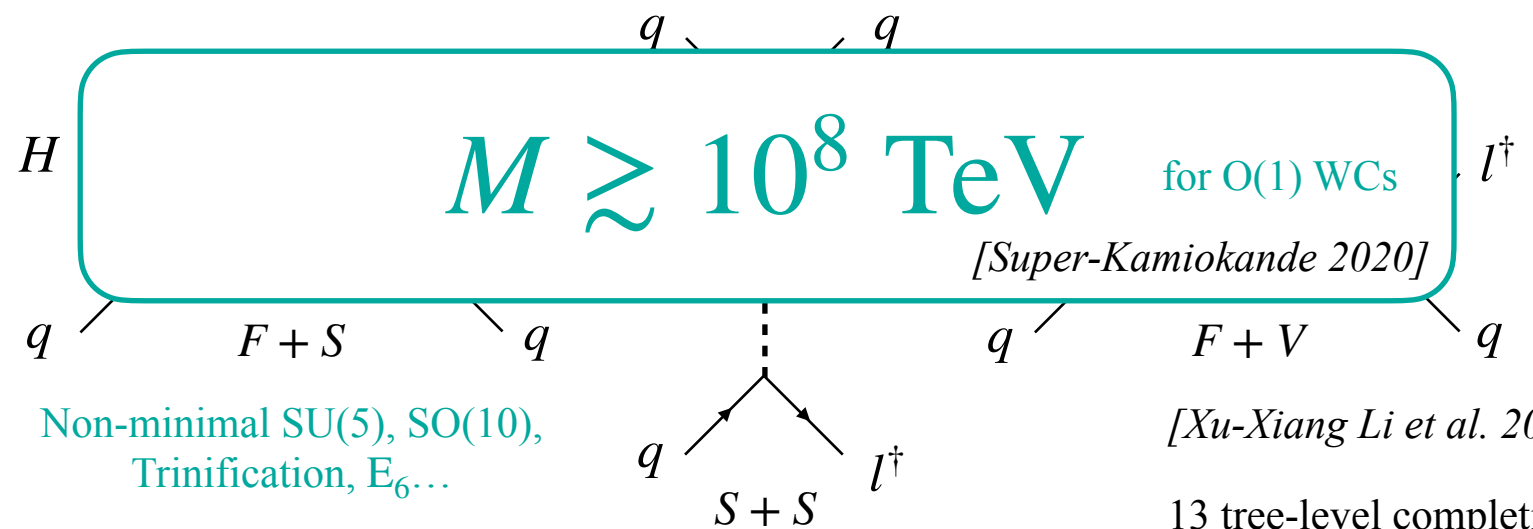
Seesaw mechanism [P. Minkowski 1977, T. Yanagida 1979, M. Gell-Mann et al. 1979, R. N. Mohapatra et al. 1980, Glashow 1980...]

$$M \gtrsim 10^{12} \text{ TeV} \quad \text{for O(1) WCs}$$

[Super-Kamiokande 2020]

[J. de Blas et al. 2017, I. Doršner et al 2016, Xu-Xiang Li et al. 2023]

Minimal SU(5), SO(10), Trinification, E_6 ...



$$M \gtrsim 10^8 \text{ TeV} \quad \text{for O(1) WCs}$$

[Super-Kamiokande 2020]

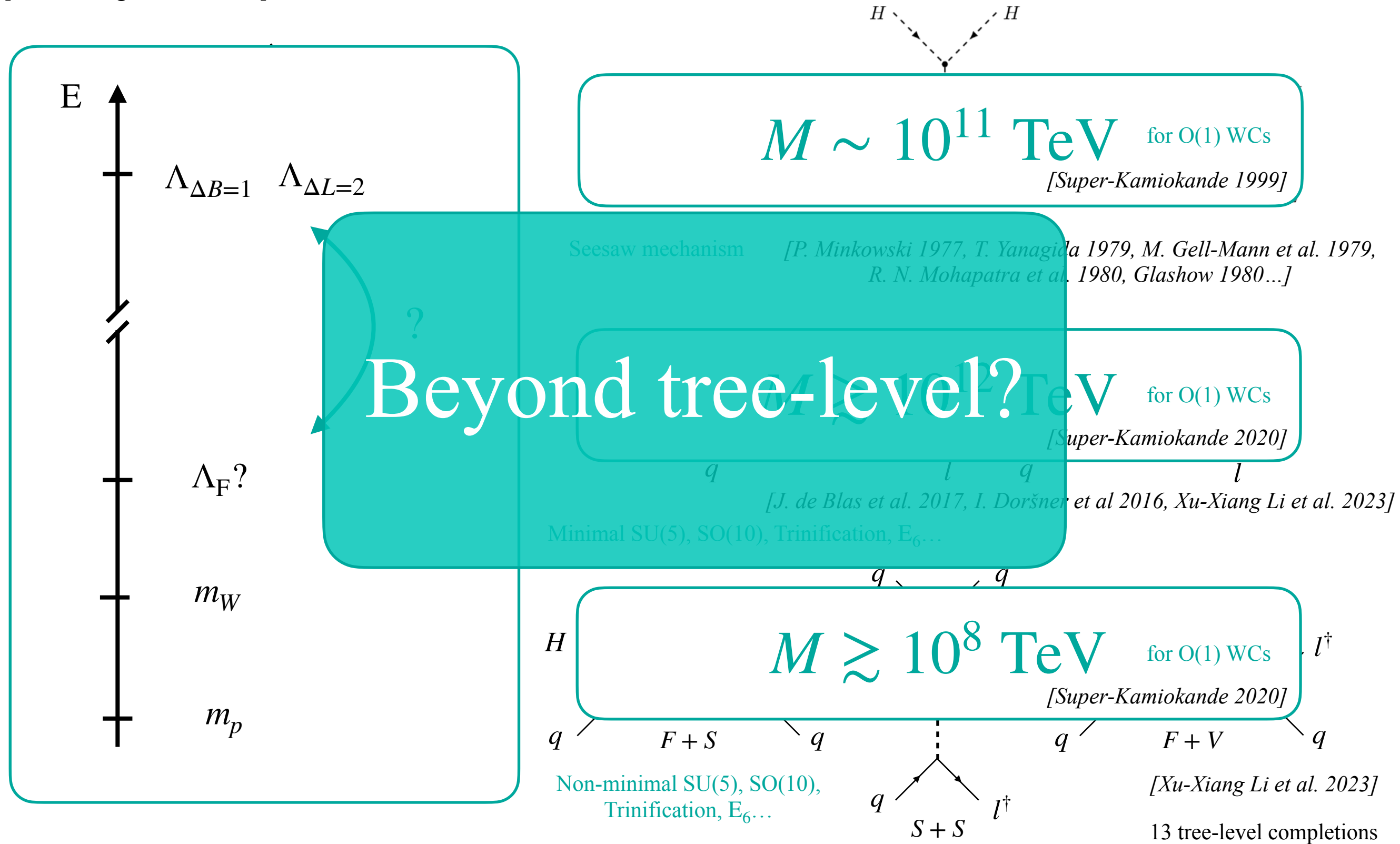
[Xu-Xiang Li et al. 2023]

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Tree-level Proton decay and Neutrino masses

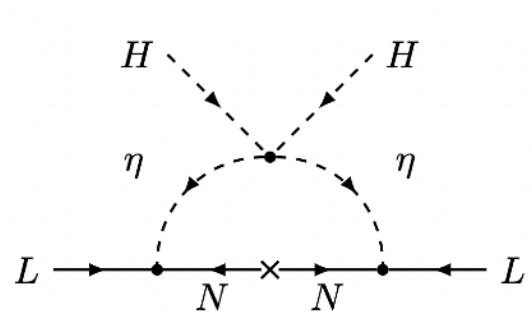
[S. Weinberg 1979 & 1980]



... and many more: $\Delta B = 2$, $\Delta(3B - L)$, $\Delta L = 4$...

Loop-level Proton decay and Neutrino masses

Neutrino masses



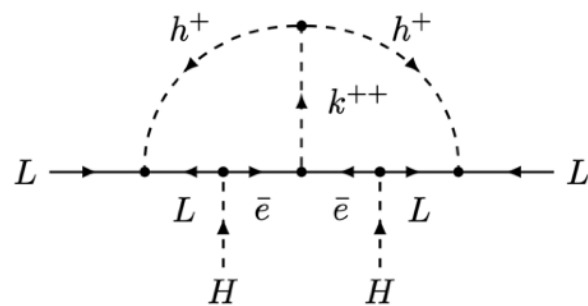
[E. Ma 2006]

- Scotogenic model (1-loop)
- Zee model (1-loop)
- Zee-Babu model (2-loop)
- KNT model (3-loop)

...

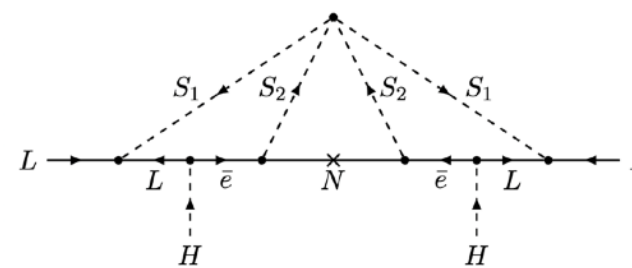


(Hundreds of models)



[A. Zee 1980, K. Babu 1980]

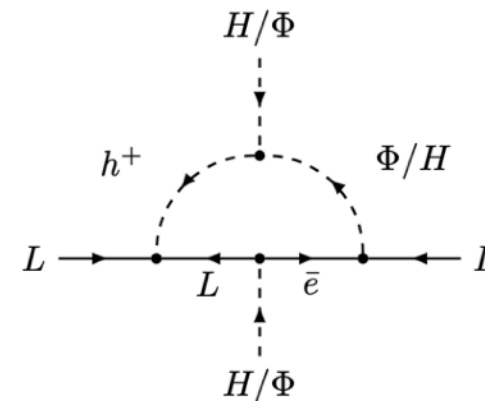
$$\Lambda^{n\text{-loop}} \sim \frac{\Lambda^{\text{tree}}}{(16\pi^2)^n}$$



[L. Krauss et al. 2003]

For a review see [J. Herrero-García et al. 2017]

Proton decay



[A. Zee 1980]

[S. Fajfer & N. Kosnik et al. 2012]

[M. Hirsch et al. 2019]

[S. Fajfer et al. 2022]

[O. Popov et al. 2024]

[J. Gargalionis et al. 2024]

[R. Srivastava et al. 2025]

...



(Dozens of models)

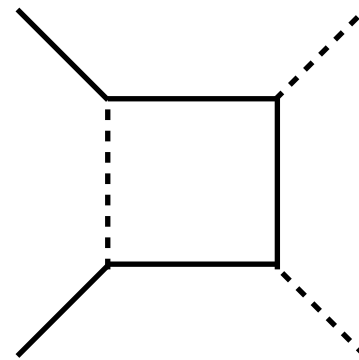
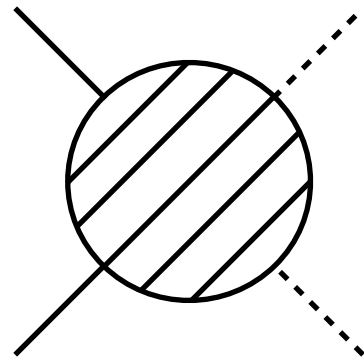
Typically 1-loop models

$$\Lambda^{1\text{-loop}} \sim \frac{\Lambda^{\text{tree}}}{4\pi}$$

Loop-level Proton decay and Neutrino masses

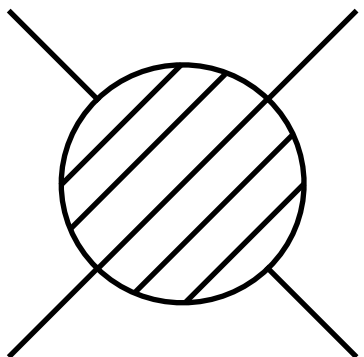
Loop-level completions of operators giving rise to m_ν and **proton decay** at tree level

$$\Delta L = 2$$



+ ...

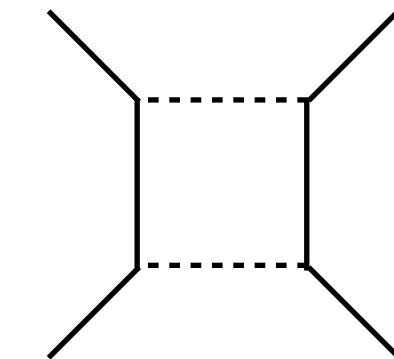
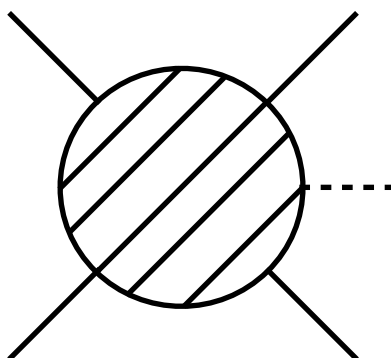
[F. Bonnet et al. 2012, M. A. Schmidt et al. 2013
M. Hirsch et al. 2012 & 2017 & 2019 ...]



$$\Delta(B - L) = 0$$

↑
Different
phenomenology

↓
$$\Delta(B - L) = 2$$



[M. Hirsch et al. 2019]
(Only S+F, not V)

?

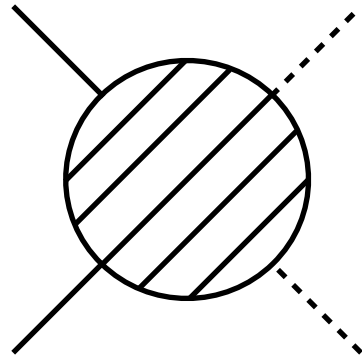
Main drawbacks:

1. ∞ loop-level completions
2. (Typically) 4 exotic particles (less economical UV models)
3. (Typically) do not generate dimension-6 SMEFT operators when integrated out at tree-level

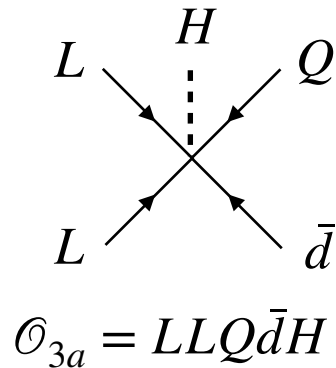
Loop-level Proton decay and Neutrino masses

Tree-level completions of higher-dimensional $\Delta L = 2$ and $\Delta B = 1$ operators that lead to m_ν and proton decay at loop level

$$\Delta L = 2$$



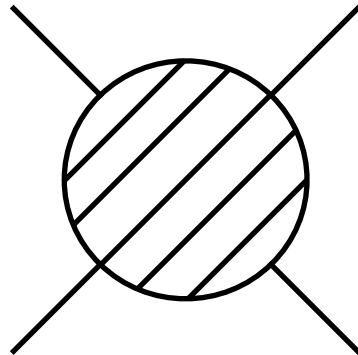
E.g.



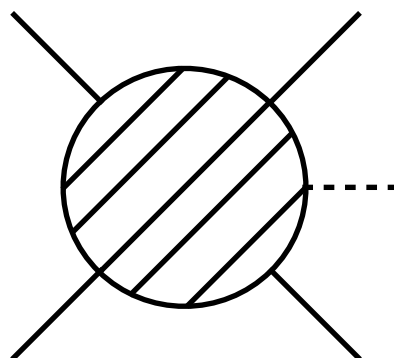
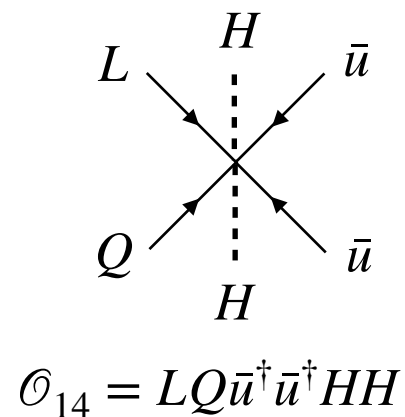
$$\Delta(B - L) = 0$$

Different
phenomenology

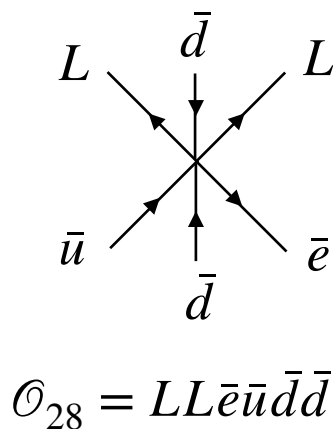
$$\Delta(B - L) = 2$$



E.g.



E.g.



Arising at even-d in the SMEFT

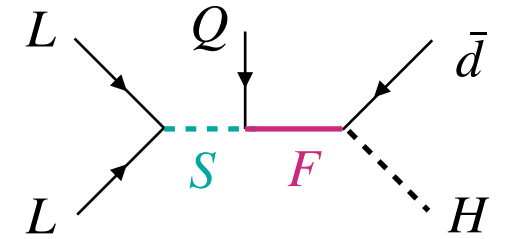
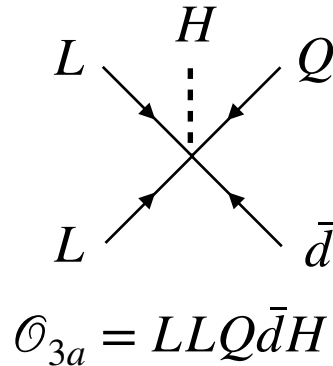
Arising at odd-d in the SMEFT [A. Kobach 2016]

Loop-level Proton decay and Neutrino masses

Tree-level completions of higher-dimensional $\Delta L = 2$ and $\Delta B = 1$ operators that lead to m_ν and proton decay at loop level

$$\Delta L = 2$$

E.g.

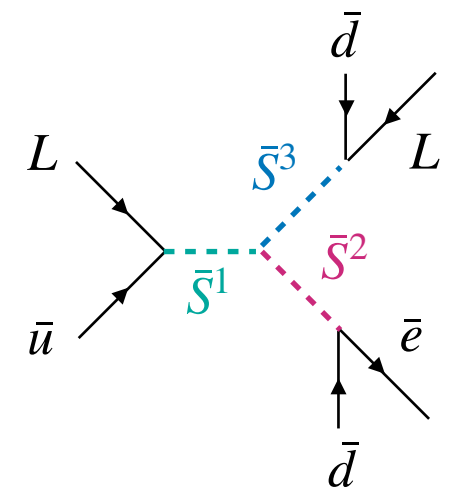
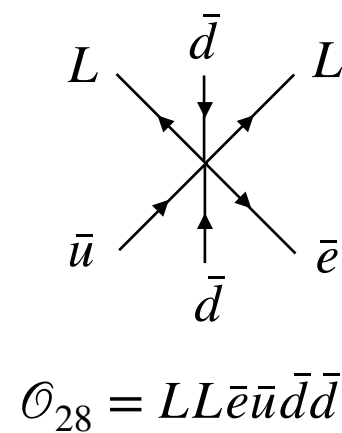
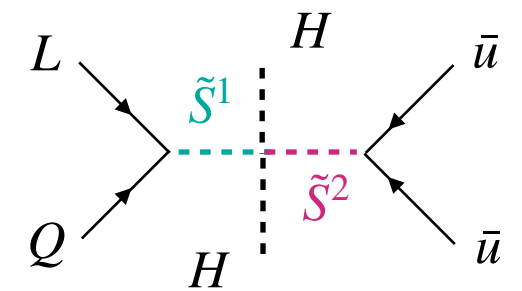
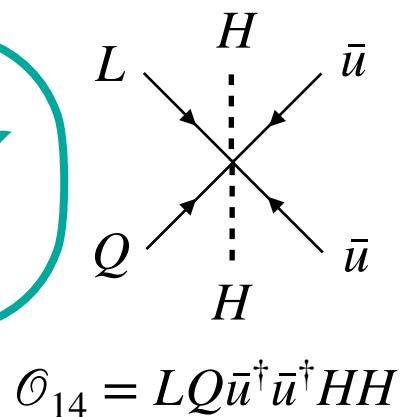


They admit a **finite number** of **tree-level completions** in the UV ✓

Different phenomenology

$$\Delta(B - L) = 2$$

E.g.



Arising at even-d in the SMEFT

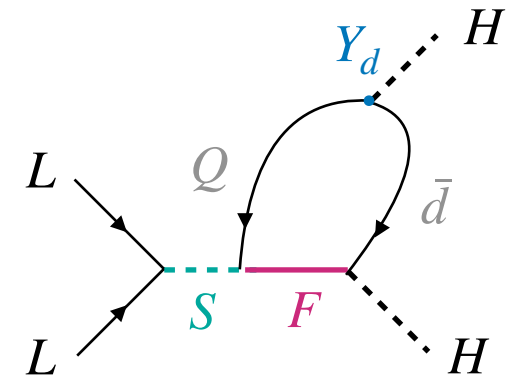
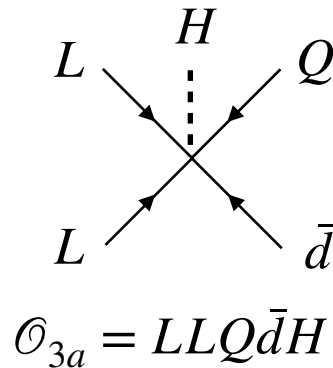
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Loop-level Proton decay and Neutrino masses

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E.g.

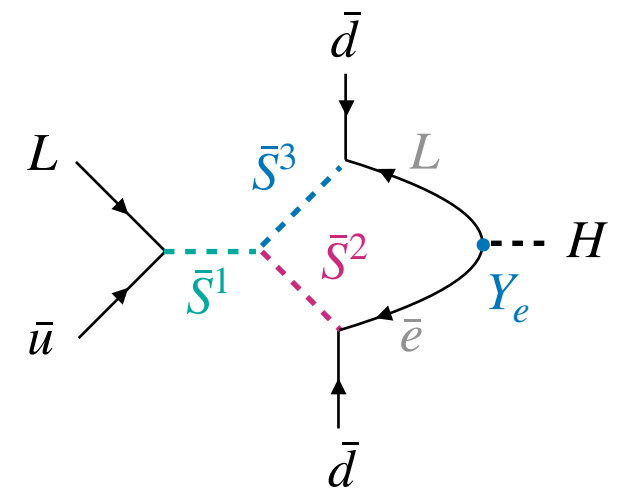
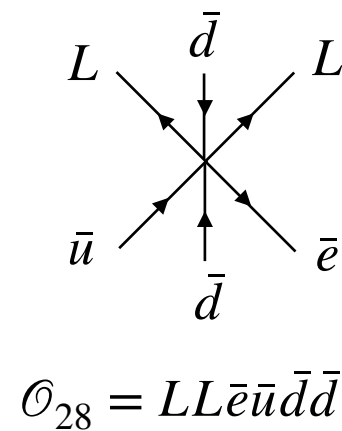
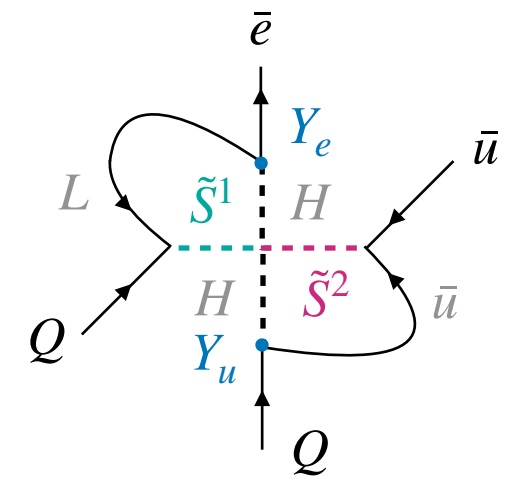
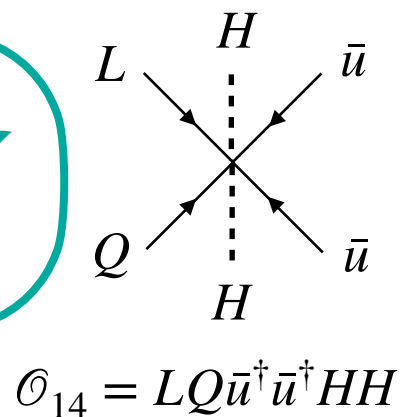


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E.g.



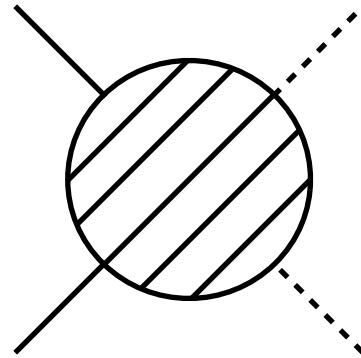
Arising at even-d in the SMEFT

Arising at odd-d in the SMEFT [A. Kobach 2016]

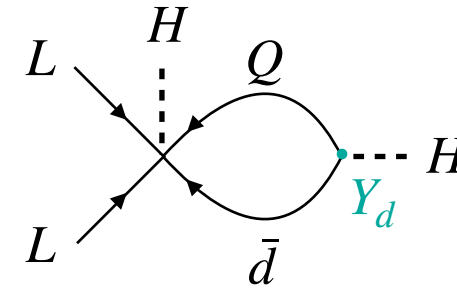
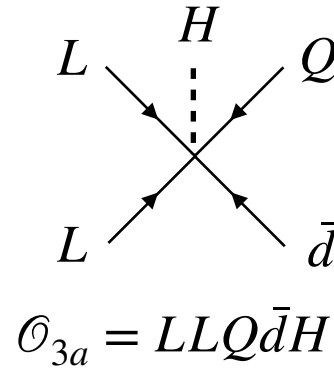
Loop-level Proton decay and Neutrino masses

Tree-level completions of higher-dimensional $\Delta L = 2$ and $\Delta B = 1$ operators that lead to m_ν and proton decay at loop level

$$\Delta L = 2$$



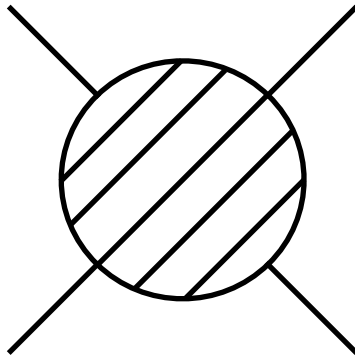
E.g.



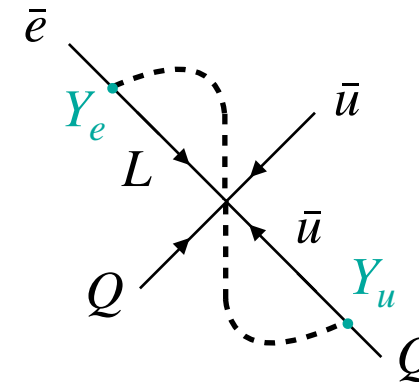
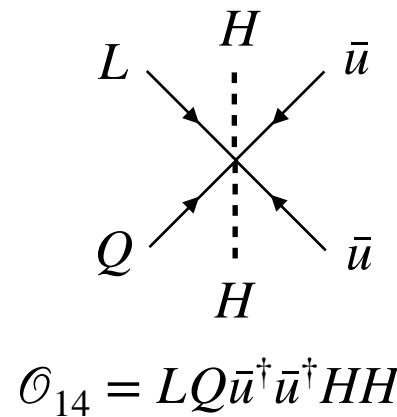
$$C_W \sim \frac{y_b}{16\pi^2} \frac{1}{\Lambda} c_{3a}$$

$$\Delta(B - L) = 0$$

Different
phenomenology

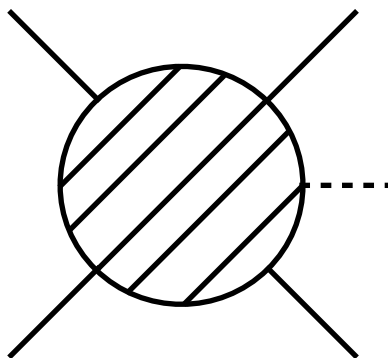


E.g.

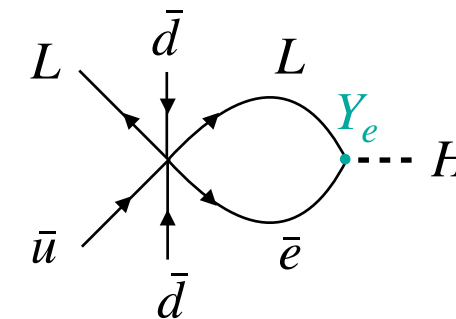
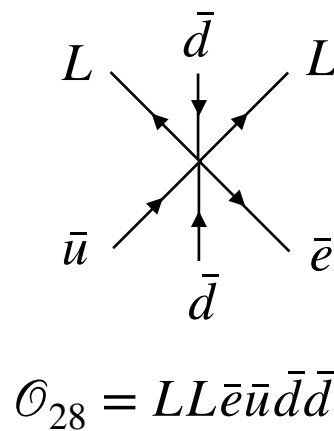


$$C_{qqe} \sim \frac{y_u y_e}{(16\pi^2)^2} \frac{1}{\Lambda^2} c_{14}$$

$$\Delta(B - L) = 2$$



E.g.



$$C_{\bar{l}dud\bar{H}} \sim \frac{y_e}{16\pi^2} \frac{1}{\Lambda^3} c_{28}$$

Arising at even-d in the SMEFT

Arising at odd-d in the SMEFT [A. Kobach 2016]

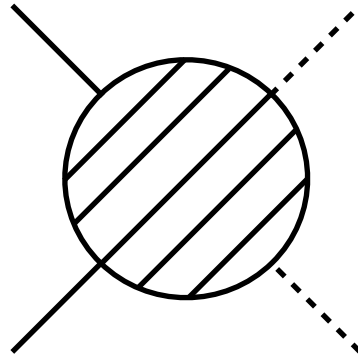
Loop-level estimates

(Without specifying the full UV completion)

Loop-level Proton decay and Neutrino masses

Tree-level completions of higher-dimensional $\Delta L = 2$ and $\Delta B = 1$ operators that lead to m_ν and proton decay at loop level

$$\Delta L = 2$$



Matching onto a unique operator

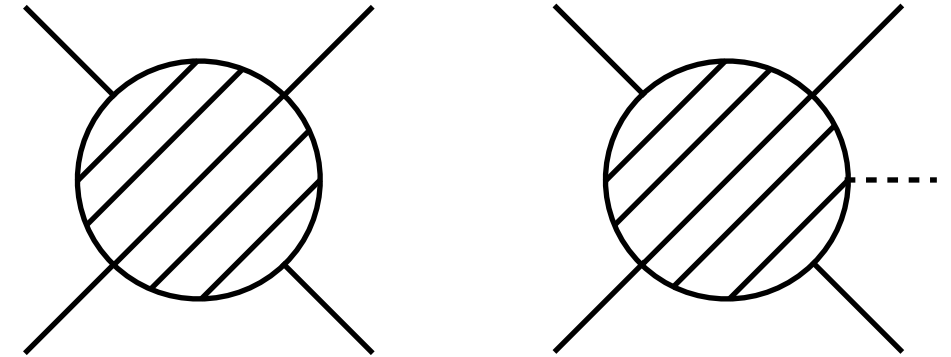
[J. Gargalionis et al. 2020]

Labels	Operator	Models	Filtered	Loops	Λ [TeV]
1	$L^i L^j H^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	3	3	0	$6 \cdot 10^{11}$
2	$L^i L^j L^k \bar{e} H^l \cdot \epsilon_{ik} \epsilon_{jl}$	8	2	1	$4 \cdot 10^7$
3a	$L^i L^j Q^k \bar{d} H^l \cdot \epsilon_{ij} \epsilon_{kl}$	9	2	2	$2 \cdot 10^5$
3b	$L^i L^j Q^k \bar{d} H^l \cdot \epsilon_{ik} \epsilon_{jl}$	14	5	1	$9 \cdot 10^7$
4a	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l \cdot \epsilon_{ik} \epsilon_{jl}$	5	0	1	$4 \cdot 10^9$
4b	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l \cdot \epsilon_{ij} \epsilon_{kl}$	4	2	2	$10 \cdot 10^6$
5a	$L^i L^j Q^k \bar{d} H^l H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	790	36	2	$6 \cdot 10^5$
5b	$\mathcal{O}_1 \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	492	14	1,2	$6 \cdot 10^5$
5c	$\mathcal{O}_{3a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	509	0	2,3	$1 \cdot 10^3$
5d	$\mathcal{O}_{3b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	799	16	1,2	$6 \cdot 10^5$
6a	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	289	14	2	$2 \cdot 10^7$
6b	$\mathcal{O}_1 \cdot \tilde{Q}^i \bar{u}^\dagger \tilde{H}^j \cdot \epsilon_{ij}$	177	0	1,2	$2 \cdot 10^7$
6c	$\mathcal{O}_{4a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	262	0	1,2	$2 \cdot 10^7$
6d	$\mathcal{O}_{4b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	208	0	2,3	$6 \cdot 10^4$
7	$L^i \bar{e}^\dagger Q^j \tilde{Q}^k H^l H^m H^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	240	15	2	$2 \cdot 10^5$

114 $\Delta L = 2$ operators up to **dimension-11** in the SMEFT

For similar studies see [K. S. Babu et al. 2001, A. de Gouvea et al. 2008]

$$\Delta(B - L) = 0, 2$$



Matching onto 4 + 6 operators

[J. Gargalionis et al. 2024]

#	Operator	Matching estimate	Flavour	Λ [GeV]	Process
Dimension 6					
1	$LQQQ$	—	1111	$3 \cdot 10^{15}$	$p \rightarrow \pi^0 e^+$
2	$\bar{e}^\dagger Q Q \bar{u}^\dagger$	—	1111	$4 \cdot 10^{15}$	$p \rightarrow \pi^0 e^+$
3	$\bar{e}^\dagger \bar{u}^\dagger \bar{u}^\dagger \bar{d}^\dagger$	—	1111	$3 \cdot 10^{15}$	$p \rightarrow \pi^0 e^+$
4	$LQ \bar{u}^\dagger \bar{d}^\dagger$	—	1111	$3 \cdot 10^{15}$	$p \rightarrow \pi^0 e^+$
Dimension 7					
5	$L \bar{d} \bar{d} \bar{d} H^\dagger$	—	1112	$2 \cdot 10^{10}$	$n \rightarrow K^+ e^-$
6	$DLQ^\dagger \bar{d} \bar{d}$	—	1112	$6 \cdot 10^9$	$p \rightarrow K^+ \nu$
7	$D \bar{e}^\dagger \bar{d} \bar{d} \bar{d}$	—	1111	$3 \cdot 10^9$	$n \rightarrow \pi^+ e^-$
8	$LQ^\dagger Q^\dagger \bar{d} H$	—	1111	$6 \cdot 10^{10}$	$n \rightarrow \pi^0 \nu$
9	$\bar{e}^\dagger Q^\dagger \bar{d} \bar{d} H$	—	1112	$3 \cdot 10^{10}$	$n \rightarrow K^+ e^-$
10	$L \bar{u} \bar{d} \bar{d} H$	—	1111	$6 \cdot 10^{10}$	$n \rightarrow \pi^0 \nu$
Dimension 8					
11	$DLQ Q \bar{d}^\dagger H$	$C_{qqql}^{pqrs} = \frac{1}{16\pi^2} V_{ru'}^*(y_d)^{u'} C_{11}^{spqu'}$	1113	$4 \cdot 10^{12}$	$p \rightarrow K^+ \nu$
12	$DL \bar{u}^\dagger \bar{d}^\dagger \bar{d}^\dagger H$	$C_{duql}^{pqrs} = \frac{1}{16\pi^2} V_{rt'}^*(y_d)^{t'} C_{12}^{sqtl'p}$	1131	$3 \cdot 10^{12}$	$p \rightarrow K^+ \nu$

Many possibilities!

50 $\Delta L = 2$ operators up to **dimension-9** in the SMEFT
(Both $\Delta(B - L) = 0, 2$ operators)

Can we say **anything** about the **mass of the mediator(s)** in these BSM processes?

Proton decay and Neutrino masses

Can we say **anything** about the **mass of the mediator(s)** in these BSM processes?

Neutrino masses

Yes, if we demand that the **atmospheric lower bound** on Δm_{atm}^2 be reproduced.

$$m_\nu \geq \sqrt{\Delta m_{\text{atm}}^2} \geq 0.05 \text{ eV} \implies M \leq \Lambda \leq \#^{\text{exp}}$$

$\uparrow$
[J. Herrero et al. 2019] $y \leq 1$

Proton decay

No, only lower bounds on the the combination $\Lambda/\sqrt{c}$ from current limits of Super-K...

Using $c \leq 1$ does not help...

$$\tau_p > \tau_p^{\text{exp}} \implies \frac{c}{\Lambda^2}, \frac{c}{\Lambda^3} \leq \#^{\text{exp}}$$

[A. Bas i Beneito et al. 2023]

If we saw proton decay, how could we establish what the underlying mechanism is?

Neutrino masses

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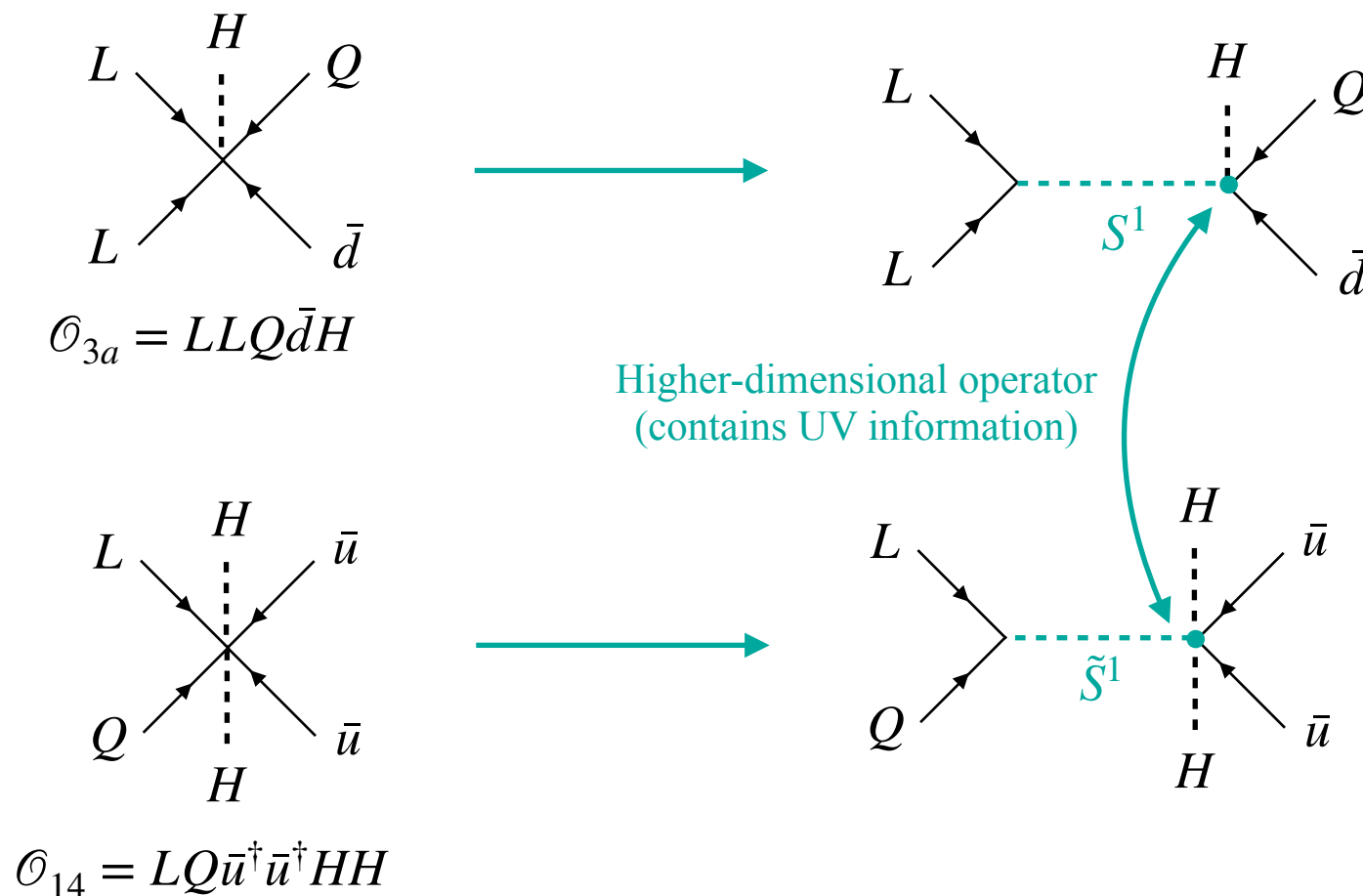
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[A. Bas i Beneito et al. 2023]

If we saw proton decay, how could we establish what the underlying mechanism is?



Assuming this particle to be the **lightest BSM particle**, can we say something about its **mass**?

1. Introduction
2. EFT framework for Majorana neutrino masses and proton decay
- 3. Theoretical framework**
4. Results
5. Conclusions and outlook

Theoretical framework

Tree-level completions of higher-dimensional $\Delta L = 2$ and $\Delta(B - L) = 0, 2$ operators



We choose to study the set of **48 exotic multiplets** that **couple linearly** to the SM at the **renormalisable level**, what we call Linear SM Extensions (**LSMEs**)

Theoretical framework

Tree-level completions of higher-dimensional $\Delta L = 2$ and $\Delta(B - L) = 0, 2$ operators



We choose to study the set of **48 exotic multiplets** that **couple linearly** to the SM at the **renormalisable level**, what we call Linear SM Extensions (**LSMEs**)

19 SCALARS	13 FERMIONS	16 VECTORS
$X (\psi_{\text{SM}} \psi_{\text{SM}})$	$(X \psi_{\text{SM}}) H$	$X^\mu (\psi_{\text{SM}}^\dagger \sigma_\mu \psi_{\text{SM}})$
$X \cdot f(H, H^\dagger) *$		$X^\mu D_\mu H (H)$

- These multiplets generate **dimension-5** and **dimension-6 operators in the SMEFT at tree level**.
- We aim to **constrain each LSME** by analysing their **contributions to $\Delta L = 2$ and $\Delta B = 1$ phenomena** using EFT.
- The analysis applies to the **simplest and most minimal UV models** in which the LSME appears, characterised by the **lowest-dimensional operator** we can write. To achieve this, we write down **effective operators** that include such exotic multiplets.

* Their electrically neutral component may acquire a VEV

Theoretical framework

“Granada” dictionary

Dimension-6 proton decay

Seesaw mediators

Scalars	Vectors	Fermions
$\Xi_1 \sim (1, 3, 1)_S$	$\mathcal{U}_2 \sim (3, 1, 2/3)_V$	$N \sim (1, 1, 0)_F$
$\mathcal{S} \sim (1, 1, 0)_S$	$\mathcal{X} \sim (3, 3, 2/3)_V$	$\Sigma \sim (1, 3, 0)_F$
$\varphi \sim (1, 2, 1/2)_S$	$\mathcal{Q}_1 \sim (3, 2, 1/6)_V$	$\Sigma_1 \sim (1, 3, -1)_F$
$\Xi \sim (1, 3, 0)_S$	$\mathcal{L}_1 \sim (1, 2, 1/2)_V$	$Q_7 \sim (3, 2, 7/6)_F$
$\Theta_1 \sim (1, 4, 1/2)_S$	$\mathcal{Y}_1 \sim (\bar{6}, 2, 1/6)_V$	$T_1 \sim (3, 3, -1/3)_F$
$\Theta_3 \sim (1, 4, 3/2)_S$	$\mathcal{Y}_5 \sim (\bar{6}, 2, -5/6)_V$	$Q_1 \sim (3, 2, 1/6)_F$
$\omega_1 \sim (3, 1, -1/3)_S$	$\mathcal{G}_1 \sim (8, 1, 1)_V$	$Q_5 \sim (3, 2, -5/6)_F$
$\zeta \sim (3, 3, -1/3)_S$	$\mathcal{H} \sim (8, 3, 0)_V$	$T_2 \sim (3, 3, 2/3)_F$
$\Pi_1 \sim (3, 2, 1/6)_S$	$\mathcal{B} \sim (1, 1, 0)_V$	$\Delta_1 \sim (1, 2, -1/2)_F$
$\mathcal{S}_1 \sim (1, 1, 1)_S$	$\mathcal{W} \sim (1, 3, 0)_V$	$\Delta_3 \sim (1, 2, -3/2)_F$
$\Omega_4 \sim (6, 1, 4/3)_S$	$\mathcal{G} \sim (8, 1, 0)_V$	$E \sim (1, 1, -1)_F$
$\Upsilon \sim (6, 3, 1/3)_S$	$\mathcal{Q}_5 \sim (3, 2, -5/6)_V$	$D \sim (3, 1, -1/3)_F$
$\Phi \sim (8, 2, 1/2)_S$	$\mathcal{U}_5 \sim (3, 1, 5/3)_V$	$U \sim (3, 1, 2/3)_F$
$\Omega_2 \sim (6, 1, -2/3)_S$	$\mathcal{B}_1 \sim (1, 1, 1)_V$	
$\omega_4 \sim (3, 1, -4/3)_S$	$\mathcal{W}_1 \sim (1, 3, 1)_V$	
$\Pi_7 \sim (3, 2, 7/6)_S$	$\mathcal{L}_3 \sim (1, 2, -3/2)_V$	
$\mathcal{S}_2 \sim (1, 1, 2)_S$		
$\omega_2 \sim (3, 1, 2/3)_S$		
$\Omega_1 \sim (6, 1, 1/3)_S$		

Quantum numbers under
 $G_{\text{SM}} = (\text{SU}(3)_C, \text{SU}(2)_L, \text{U}(1)_Y)$

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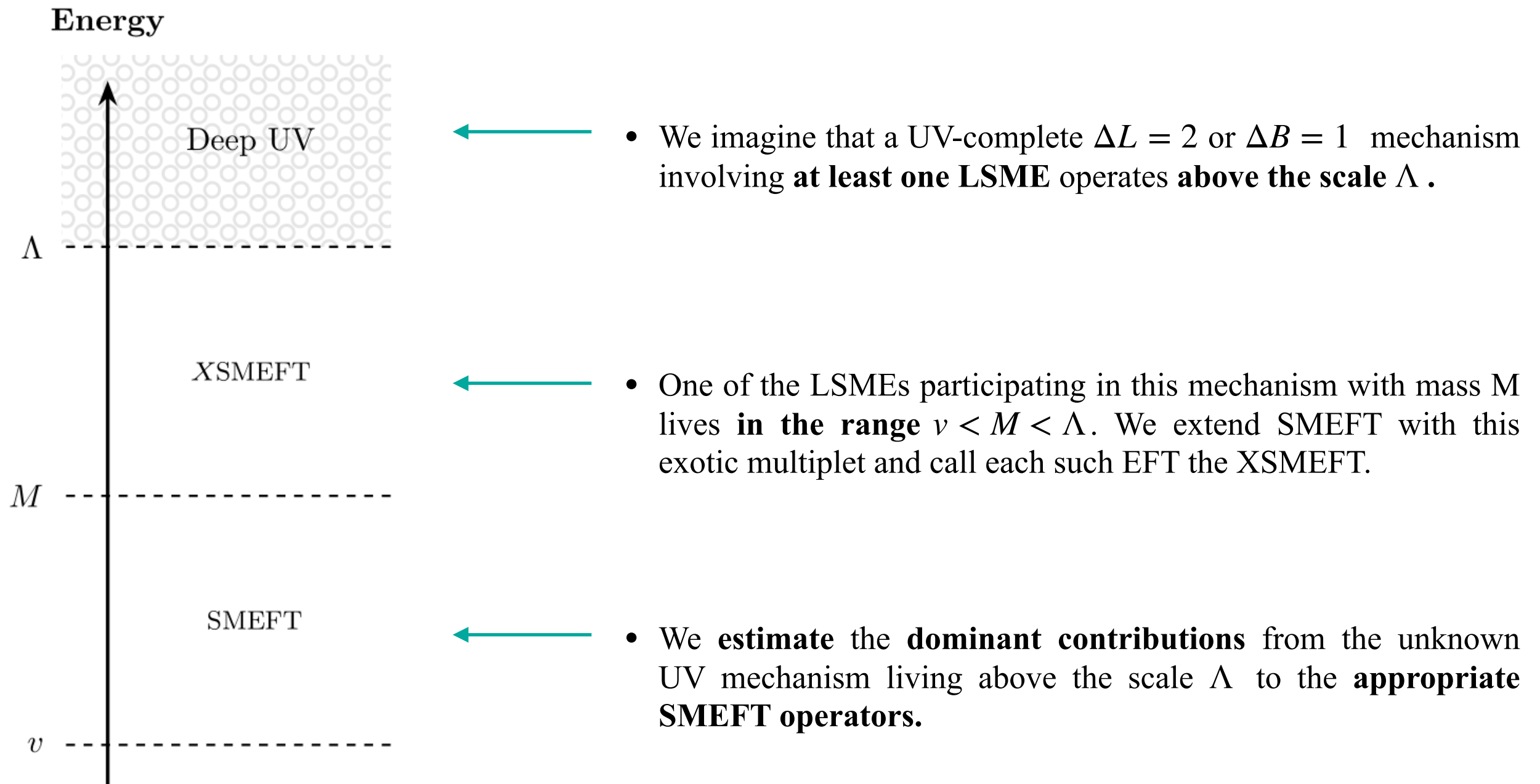
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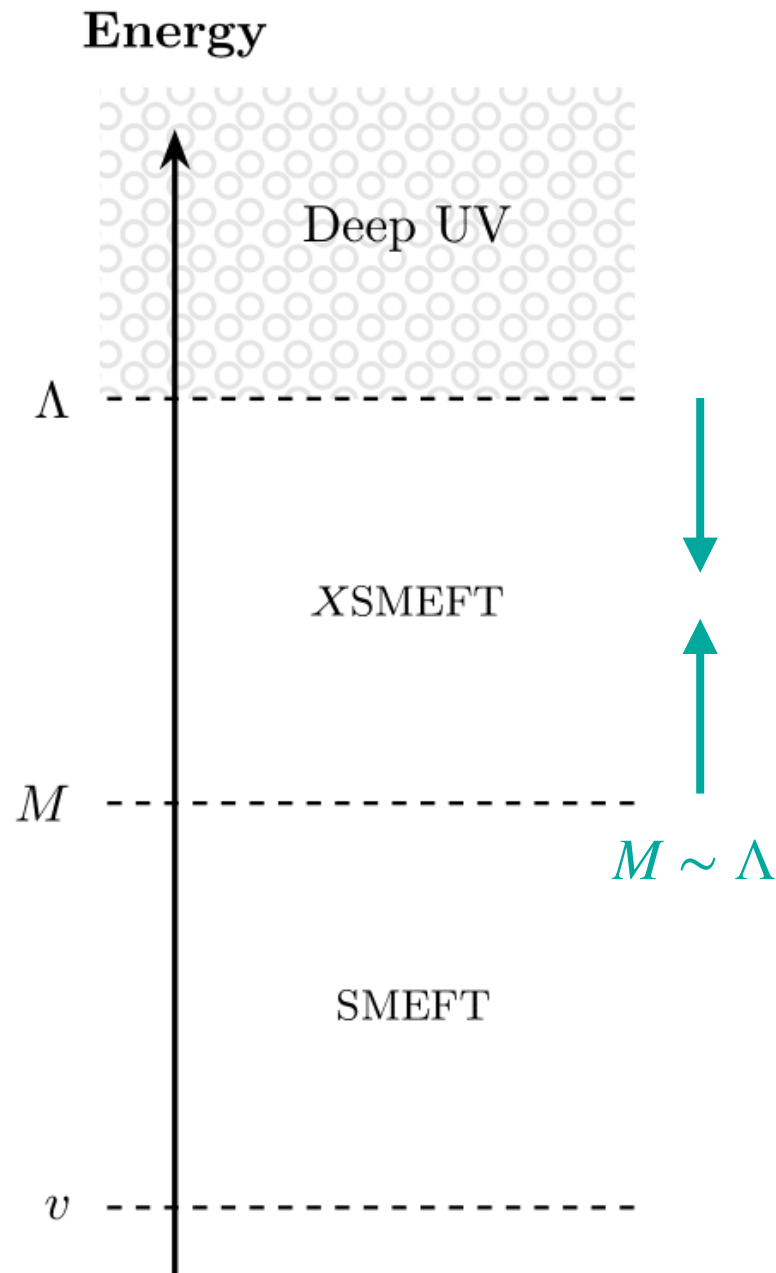
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Theoretical framework



Theoretical framework



In this setup we can derive **upper bounds** on M if

- We impose the lower bound $m_\nu > \sqrt{\Delta m_{\text{atm}}^2} = 0.05 \text{ eV}$
- We assume a **positive signal** in Hyper-K: $\tau_p \simeq 10^{35}$ years
- We select **third-family Yukawa couplings** to estimate the **largest contribution** to $\Delta L = 2$ and $\Delta B = 1$ processes.
- We set dimensionless WCs $y, c \leq 1$
- We saturate the EFT condition $M \rightarrow \Lambda$ *

Very conservative assumptions!

* All expressions will depend on the two energy scales of our set-up: M and Λ . However, to remain as conservative as possible, we saturate the limit $M \sim \Lambda$

Illustrative example: S_2 for m_ν

Partially-resolved UV theory:

$$S_2 \sim (1, 1, 2)_s$$

Information of the full UV model

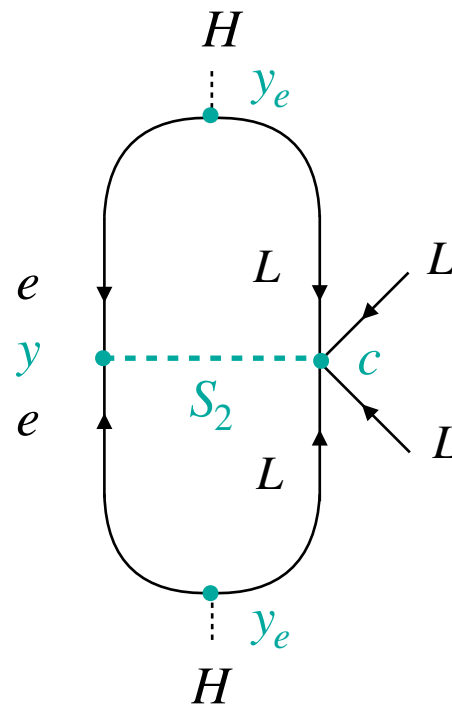
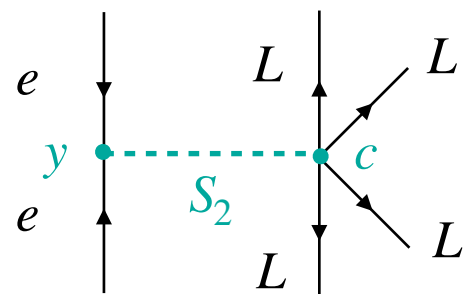
$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{S_2}^{\Delta L=2}$$



$$\mathcal{L}_{S_2}^{\Delta L=2} = y S_2 (\bar{e}^c e) + \frac{c}{\Lambda^3} S_2^\dagger (\bar{L}^c L) (\bar{L}^c L) + h.c.$$



d-6 SMEFT operators at tree-level



Two-loop generated m_ν

$$m_\nu \sim \frac{1}{(16\pi^2)^2} y c y_e^2 \frac{v^2}{\Lambda}$$

$$y, c \leq 1$$

$$M \rightarrow \Lambda$$

$$\sqrt{\Delta m_{\text{atm}}^2} < m_\nu \sim \frac{1}{(16\pi^2)^2} y c y_e^2 \frac{v^2}{\Lambda} \leq \frac{1}{(16\pi^2)^2} y_e^2 \frac{v^2}{\Lambda} \leq \frac{1}{(16\pi^2)^2} y_e^2 \frac{v^2}{M} \rightarrow M \leq \#$$

Illustrative example: S_2 for m_ν

Fully-resolved UV theory: **Zee-Babu model**

[A. Zee 1980, K. Babu 1980]

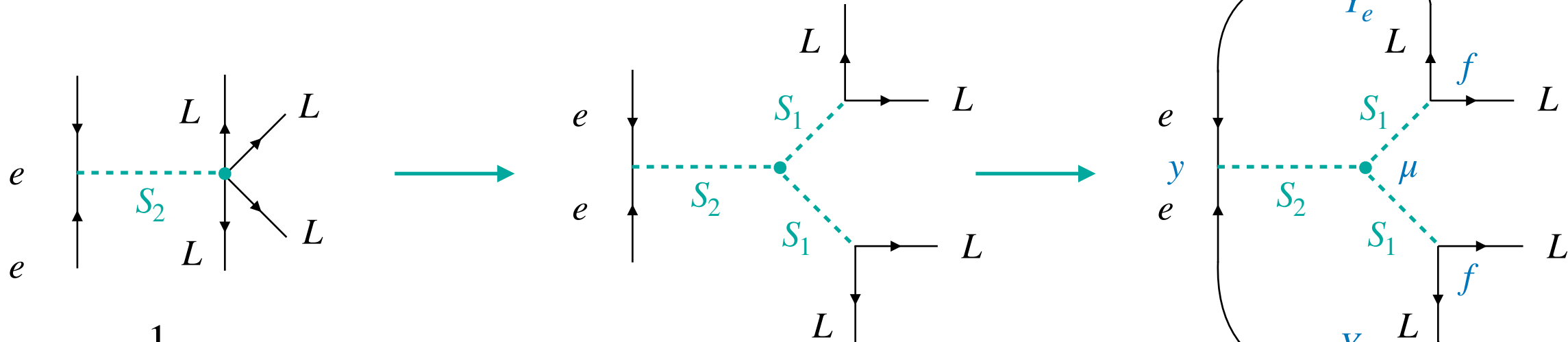
SM + S_1 and S_2

$$S_1 \sim (1, 1, 1)_s$$

$$S_2 \sim (1, 1, 2)_s$$

$$\mathcal{L} = \mathcal{L}_{SM} + y S_2 (\bar{e}^c e) + f S_1 (\bar{L}^c L) + \mu S_1^2 S_2^\dagger + h.c.$$

$$m_{S_2} \leq m_{S_1}$$



$$\mathcal{L}_\nu = -\frac{1}{2} \bar{\nu}_{Li}^c [m_\nu]_{ij} \nu_{Lj}$$

Two-loop integral $\longrightarrow$

$$[m_\nu]_{ij} \sim \mu f_{ia} [m_e]_a y_{ab} I_{ab} [m_e]_b f_{jb}$$

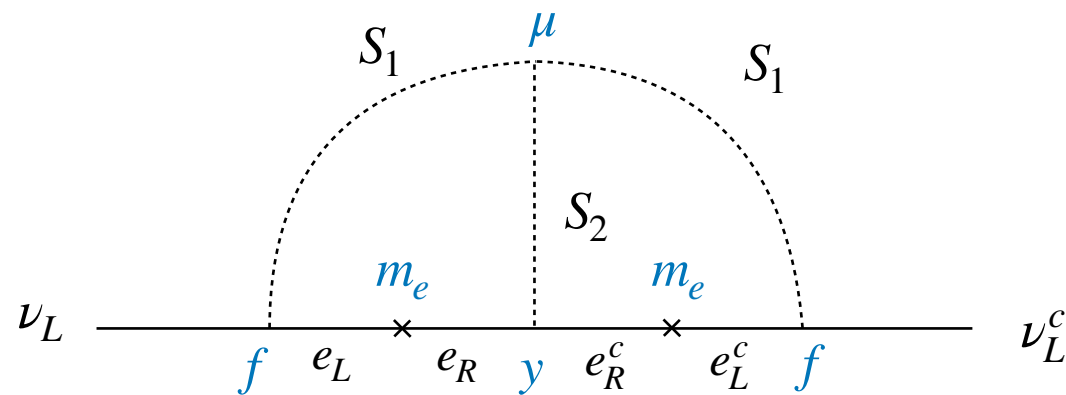
$$I_{ab} \sim \frac{1}{(16\pi^2)^2} \frac{1}{M^2}$$

$$M = \max[m_{S_1}, m_{S_2}] = m_{S_1}$$

$$m_\nu \sim \frac{1}{(16\pi^2)^2} f^2 m_e^2 \frac{\mu}{M_{S_1}^2} \equiv \frac{1}{(16\pi^2)^2} f^2 y_e^2 \frac{v^2}{\Lambda}$$

$$m_\nu > \sqrt{\Delta m_{\text{atm}}^2} \quad \downarrow \quad f \leq 1 \quad m_{S_2} \rightarrow m_{S_1}$$

$$\sqrt{\Delta m_{\text{atm}}^2} \leq \frac{1}{(16\pi^2)^2} y_e^2 \frac{v^2}{m_{S_2}} \longrightarrow m_{S_2} \leq \#$$



Genuineness procedure

- Verification that the **lowest-dimensional XSMEFT** operators **dominantly contribute** to neutrino masses or nucleon decay.
- The UV completions of the XSMEFT operators **do not include** a subset of particles that gives rise to the same phenomenon more dominantly.

E.g. $\Theta_1 \sim (1,4,1/2)_S \rightarrow$ dimension-5 $\Theta_{1ijk} \underbrace{L^i L^j}_{\Xi_1} \underbrace{H^k}_{\Sigma}$ ✗

↓
Dimension-7 with a more suppressed contribution to Neutrino masses

For $\Delta L = 2$ SMEFT operators see [J. Gargalionis et al. 2020]

We provide the **first steps** towards the exhaustive genuineness procedure for $\Delta B = 1$ SMEFT operators

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Antisymmetry in flavour space

- Specific LSMEs have fixed **flavour symmetries** in the renormalisable operator, that **impact the strongest constraints** on the operator in two different ways:
 - 1) For neutrino masses: we cannot always choose third family-Yukawa to get the would-be most dominant estimate
 - 2) For proton decay: we may not generate proton decay at tree-level

E.g. $\omega_4 \sim (3,1, -4/3)_S \rightarrow$ Dimension-6 $y^{[pq]} \omega_4(\bar{u}_{[p}^\dagger \bar{u}_{q]}^\dagger)$

↑
No 2-body proton decay at tree-level

See for example [S. Fajfer & N. Kosnik et al. 2012]

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Scalars with neutral components

- We also consider the possibility that the electrically neutral component of certain scalar multiplets **acquires a VEV, induced by EWSB**. Specifically those triplets and quadruplets under $SU(2)_L$:

E.g. $y \Xi H^\dagger H \rightarrow \langle \Xi^0 \rangle \sim \mu \frac{v^2}{M^2}$

E.g. $y \Theta_1 H^\dagger H H^\dagger \rightarrow \langle \Xi^0 \rangle \sim y \frac{v^3}{M^2}$

From EWPTs
 $\langle X^0 \rangle \lesssim 1 \text{ GeV}$

[PDG 2024] Custodial symmetry
 $\rho \sim 1$

See for example [P. Langacker 1981, S. Dawson et al. 2017...]

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No 2-body proton decay at tree-level

See for example [S. Fajfer & N. Kosnik et al. 2012]

Algorithm for the derivation of Upper bounds

Select 1 exotic heavy particle (LSMEs)

Neutrino masses

Write down the **combination of lowest-dimensional genuine** operators that induce $\Delta L = 2$ phenomena $\mathcal{L}^{\Delta L=2}$



Estimate the (loop) contribution to m_ν through SM Yukawa couplings + $\mathcal{L}^{\Delta L=2}$



* We do not aim to accommodate U_{PMNS} and mass patterns for m_ν

Reproduce the atmospheric neutrino mass scale
 $\sqrt{\Delta m_{\text{atm}}^2} > 0.05 \text{ eV}$

$M \rightarrow \Lambda$
 $y, c \leq 1$



$M \leq \#$

Central assumption



BSM phenomena **dominantly** generated by this heavy multiplet

Proton decay

Write down the **combination of lowest-dimensional genuine** operators that induce $\Delta B = 1$ phenomena $\mathcal{L}^{\Delta B=1}$



Estimate the (loop) contribution to proton decay through SM Yukawa couplings + $\mathcal{L}^{\Delta B=1}$



$\tau_p \simeq 10^{35} \text{ years (Hyper-K)}$

Future positive signal from proton decay induced by the action of $\mathcal{L}^{\Delta B=1}$



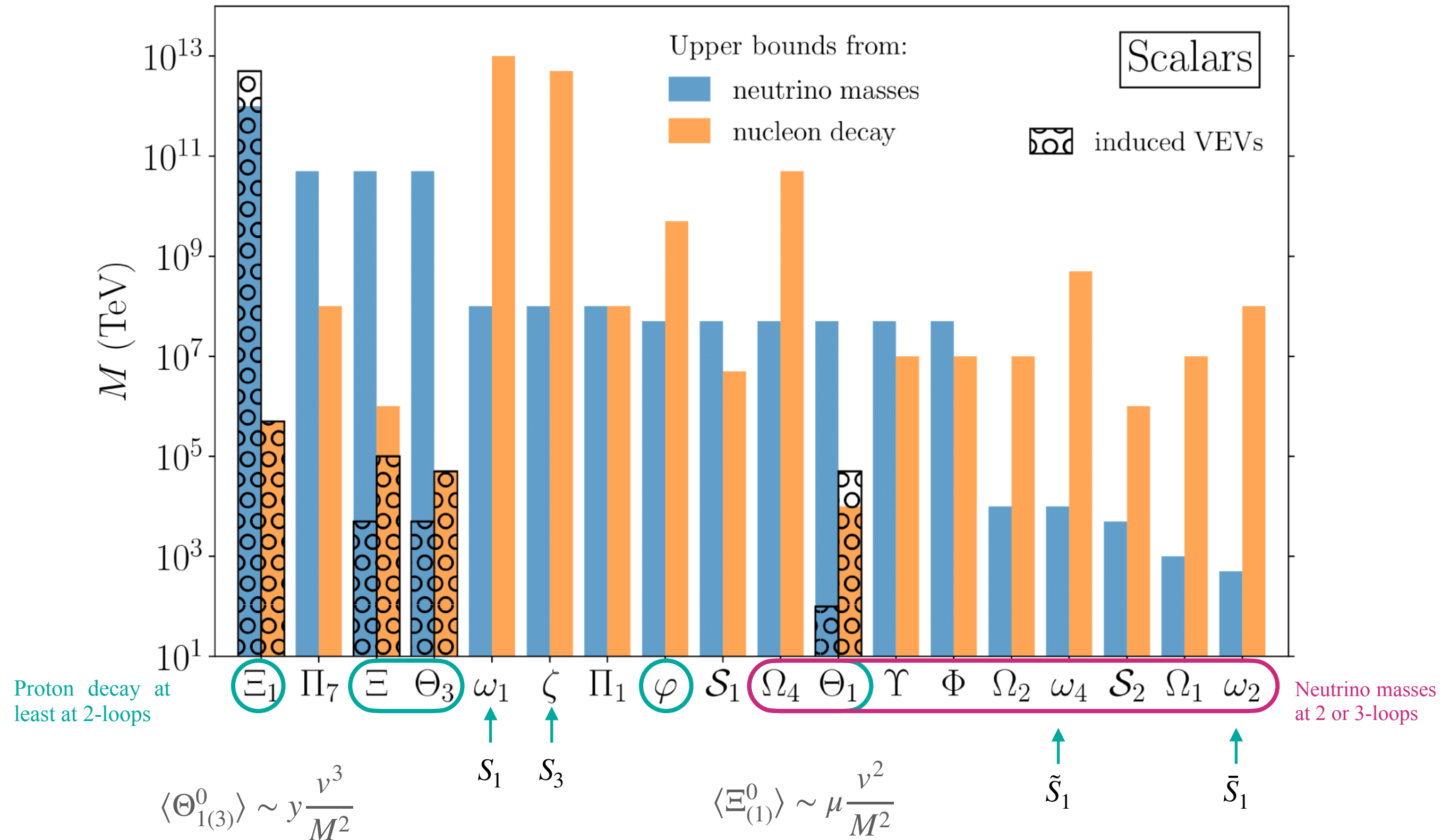
$M \rightarrow \Lambda$
 $y, c \leq 1$

$M \leq \#$

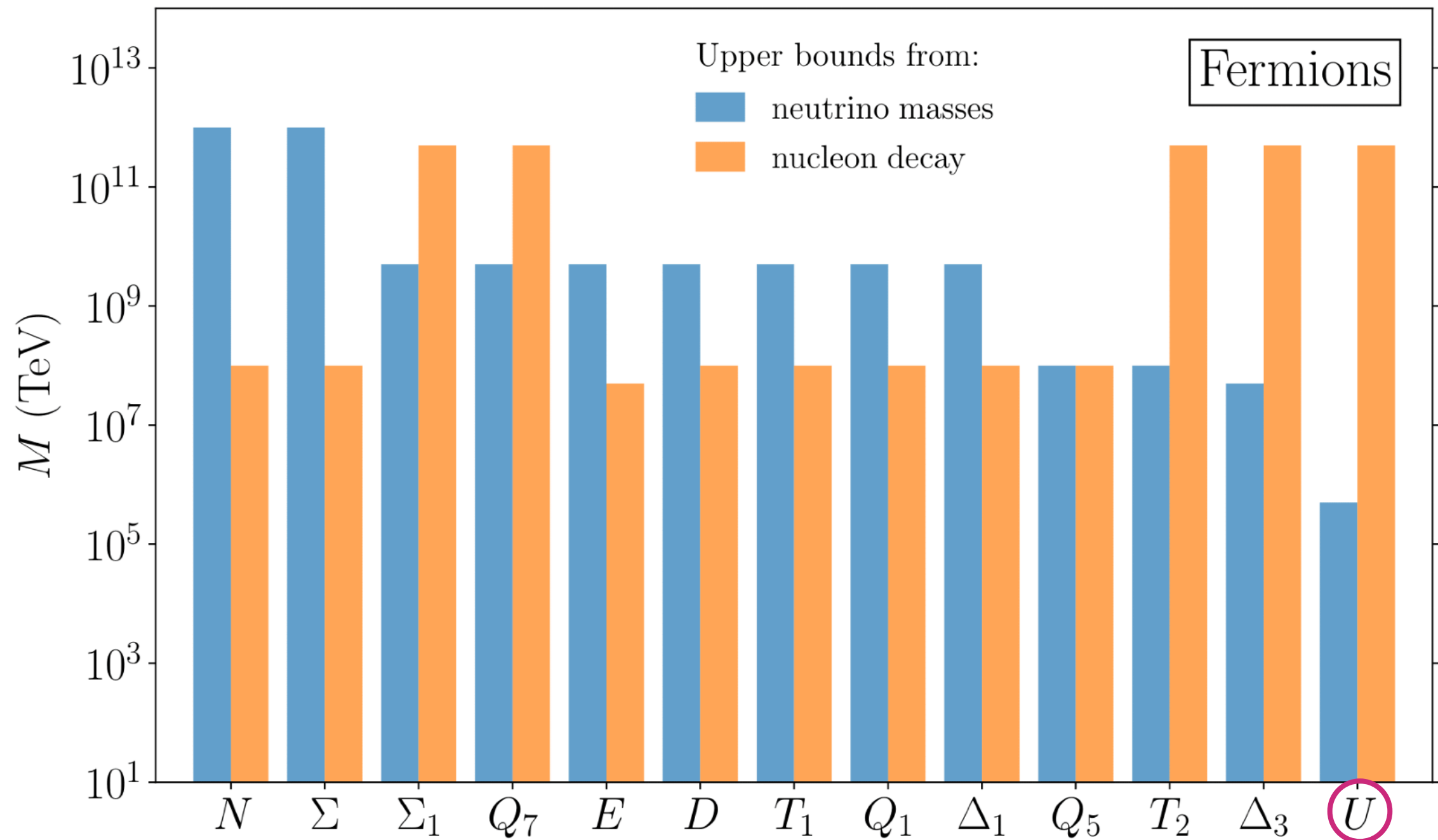
Exotic parameters are **flavour blind**

1. Introduction
2. EFT framework for Majorana neutrino masses and proton decay
3. Theoretical framework
- 4. Results**
5. Conclusions and outlook

Upper bounds for scalars



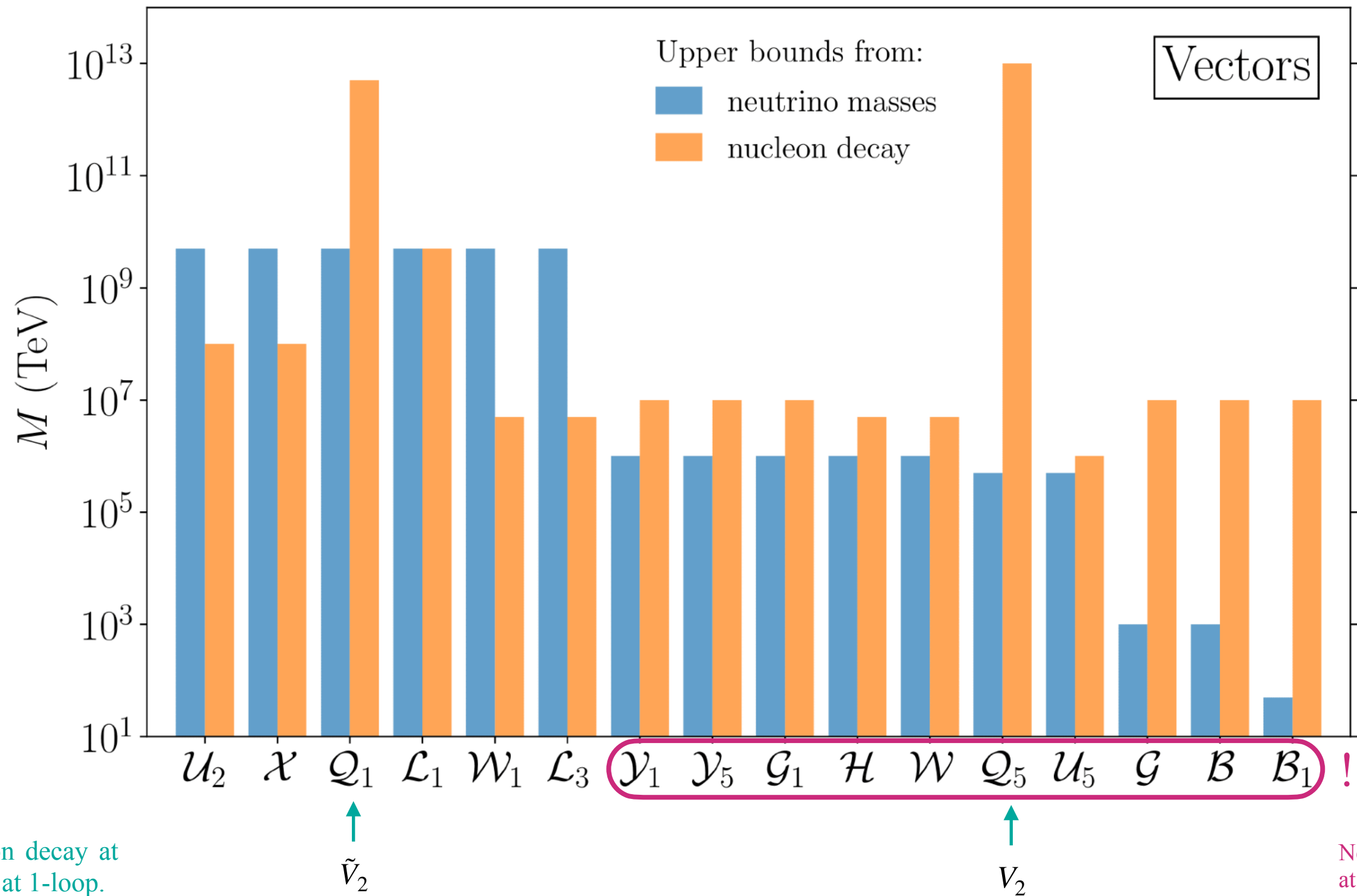
Upper bounds for fermions



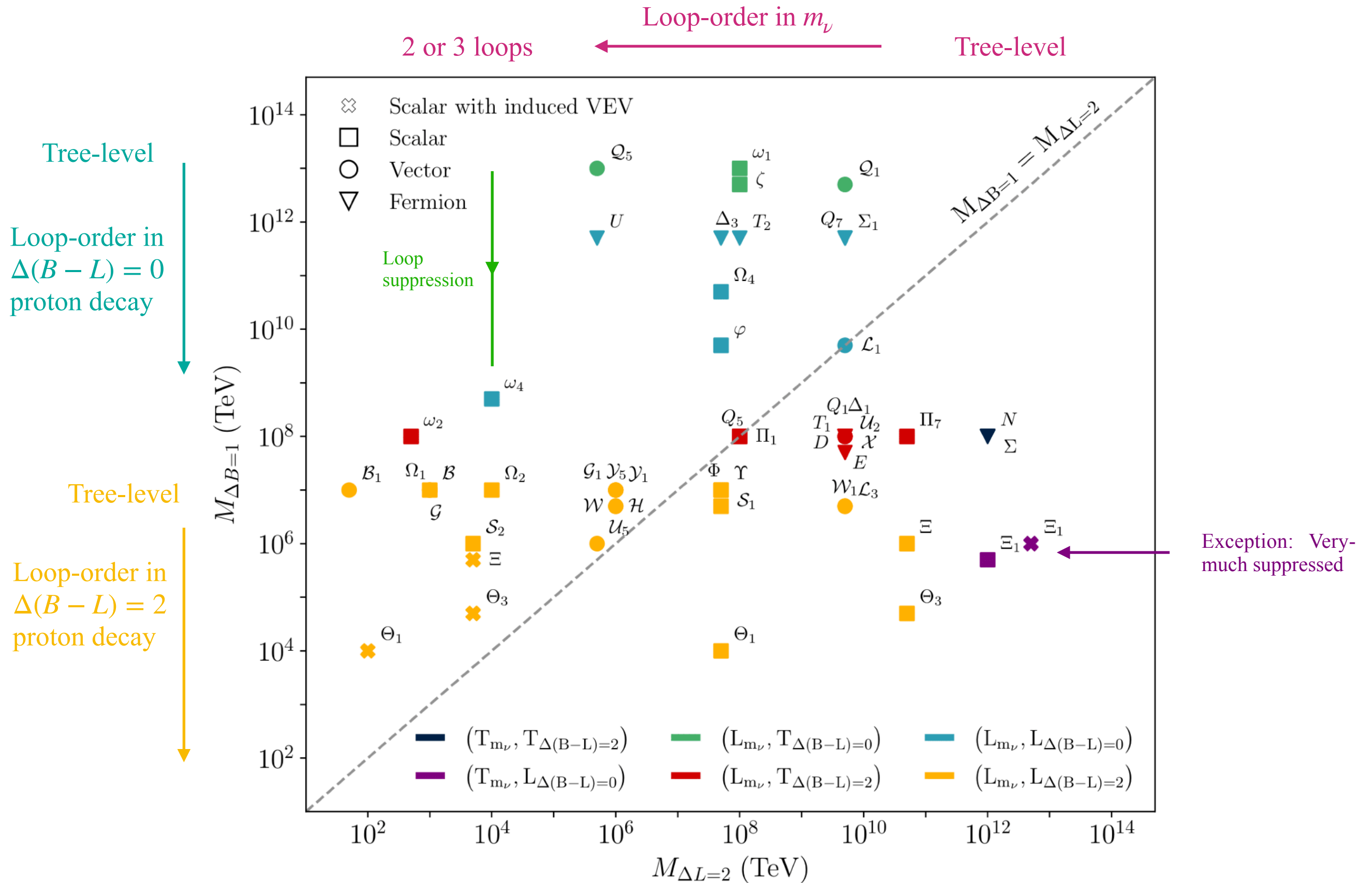
Both Neutrino masses and Proton decay at tree-level or at most 1-loop.

Exception: Neutrino masses at 2-loops

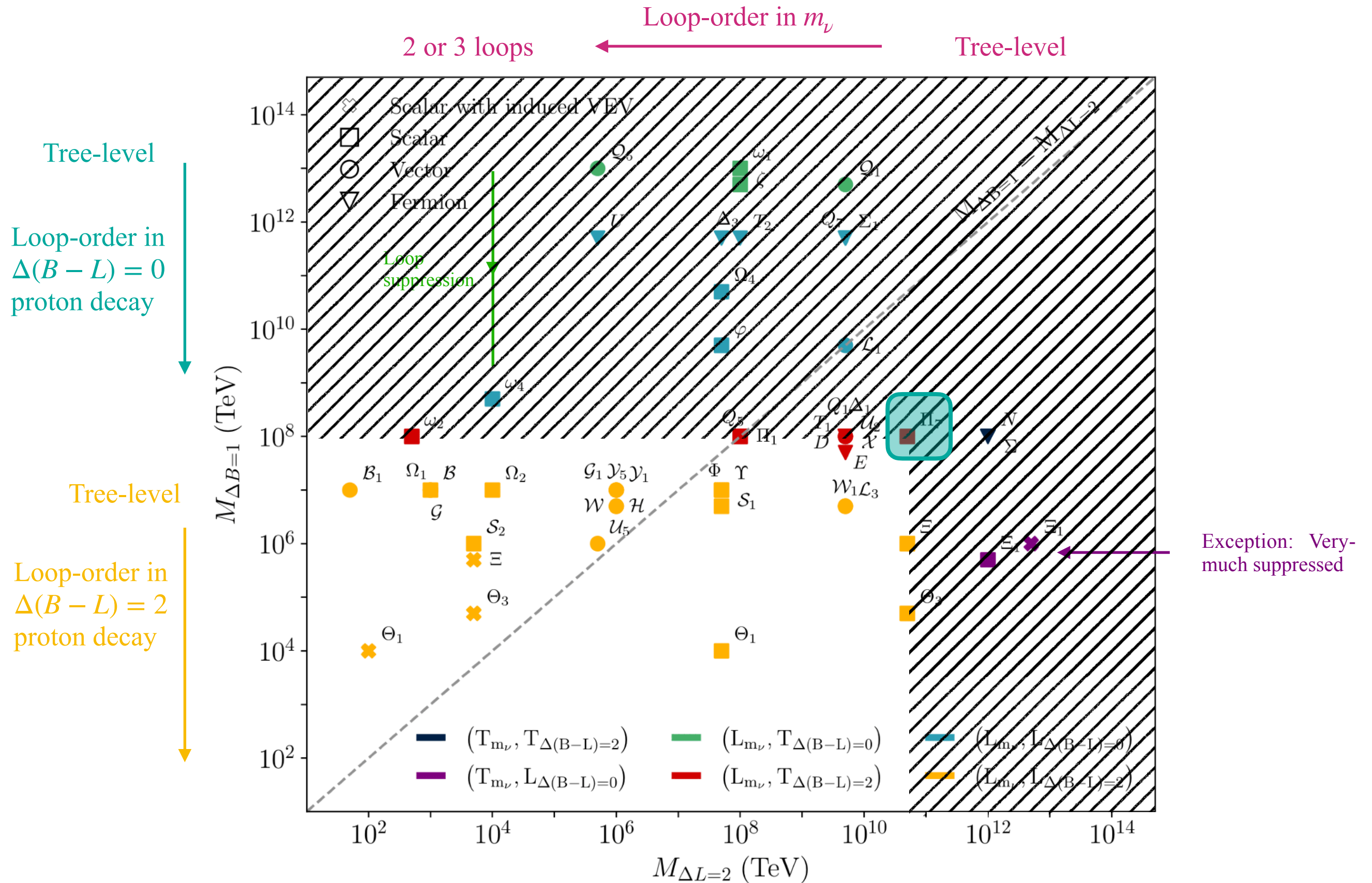
Upper bounds for vectors



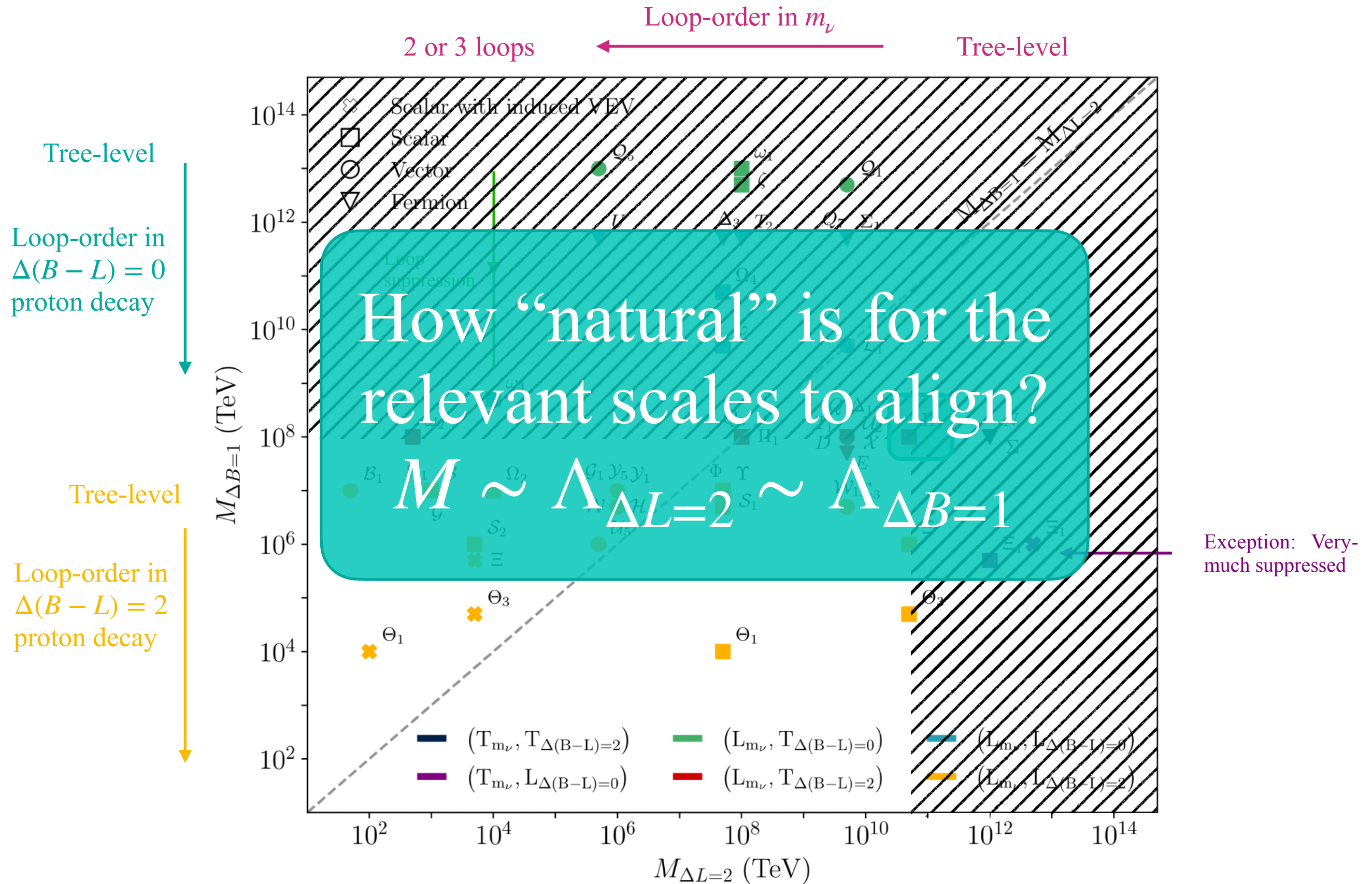
Neutrino masses and Proton decay



Neutrino masses and Proton decay

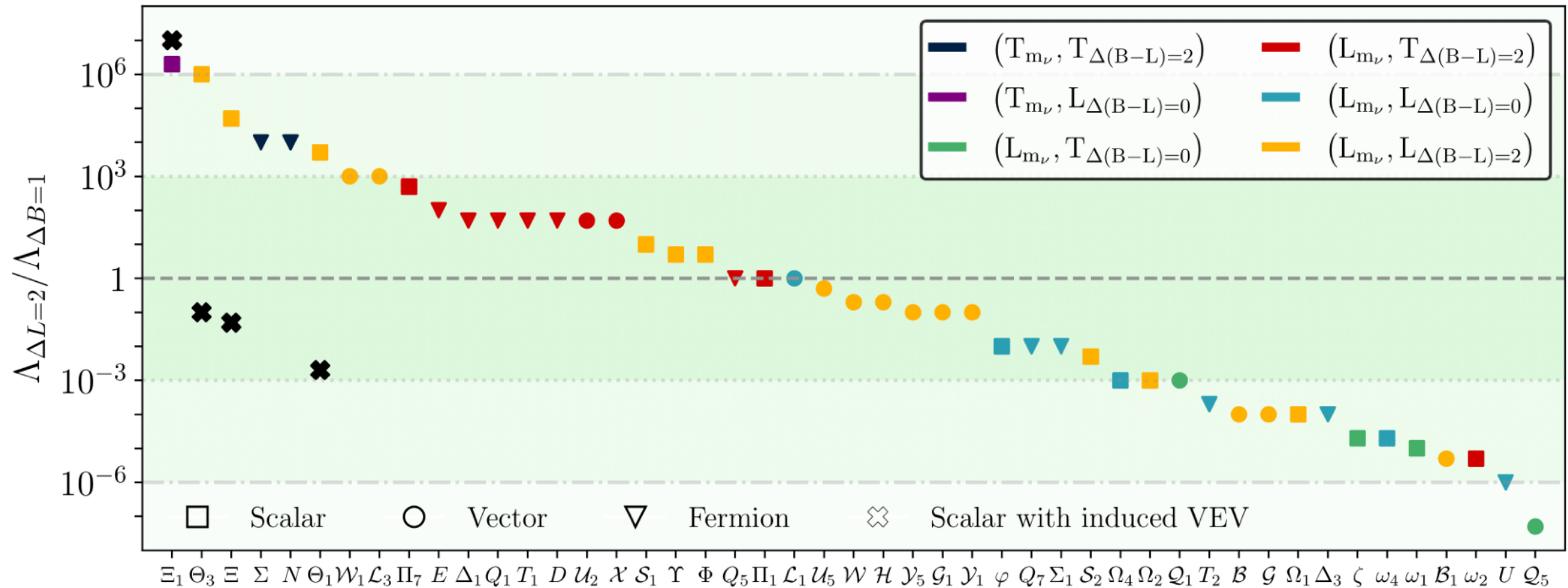


Neutrino masses and Proton decay



Neutrino masses and Proton decay

- Degree of **fine-tuning** required for the dimensionless WCs associated with either $\Delta L = 2$ and $\Delta B = 1$ operators to **account for all observed phenomena** within the same UV framework.



$$\Lambda_{\Delta L=2} \gg \Lambda_{\Delta B=1} \implies c_{\Delta L=2} \ll c_{\Delta B=1}$$

$$\Lambda_{\Delta B=1} \gg \Lambda_{\Delta L=2} \implies c_{\Delta B=1} \ll c_{\Delta L=2}$$

Or through another (more-suppressed) mechanism

1. Introduction
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- 5. Conclusions and outlook**

- **Conservative upper bounds** on the mass of the **lightest mediator** of Majorana neutrino masses and proton decay, both $B - L$ conserving and violating.
- **Framework to organise the space of UV models** generating $\Delta L = 2$ and $\Delta B = 1$ phenomena.
- **Tool for model builders** interested in explaining (radiative) Majorana neutrino masses and proton decay.
- First steps to build **UV-complete models** with *low* $\Lambda_{\Delta B=1}$ and $\Lambda_{\Delta L=2}$
- First steps to provide an exhaustive **genuineness program** in the $\Delta B = 1$ sector.

- **Specific UV Models** with relatively low-scale **BNV violation**?
- Look for **complementary searches** in the flavour sector?
- **Embedding** into GUT frameworks?
- **Connection** of the UV particles with **gauge coupling unification**?
- **Flavor hypothesis** implemented in the flavoured exotic couplings?
- Specific realisations of **Baryogenesis**?
- Similar program to $\Delta B = 2$ **processes**, e.g. $n - \bar{n}$ oscillations?

HVALA - GRÀCIES!

*(Happy) intruder from
Universitat de València*



Thanks for the hospitality!

Backup slides

Model class	References	Lifetime [years]	Ruled out?
Minimal SU(5)	Georgi & Glashow [21]	$10^{30} - 10^{31}$	yes
Minimal SUSY SU(5)	Dimopoulos & Georgi [22]; Sakai & Yanagida [23]	$10^{28} - 10^{34}$	yes
SUGRA SU(5)	Nath, Chamseddine & Arnowitt [24]	$10^{32} - 10^{34}$	yes
SUSY (MSSM/ESSM) SO(10)/G(224)	Babu, Pati & Wilczek [25]	$2 \cdot 10^{34}$	yes
SUSY (MSSM/ESSM, $d = 5$) SO(10)	Lucas & Raby [26]; Pati [27]	$10^{32} - 10^{35}$	partially
SUSY SO(10) + U(1) _f	Shafi & Tavartkiladze [28]	$10^{32} - 10^{35}$	partially
SUSY ($d = 5$) SU(5) – option I	Hebecker & March-Russell [29]	$10^{34} - 10^{35}$	partially
SUSY (MSSM, $d = 6$) SU(5) or SO(10)	Pati [27]	$\sim 10^{34.9 \pm 1}$	partially
Minimal non-SUSY SU(5)	Doršner & Fileviez-Pérez [30]	$10^{31} - 10^{38}$	partially
Minimal non-SUSY SO(10)		—	no
SUSY (CMSSM) flipped SU(5)	Ellis, Nanopoulos & Walker [31]	$10^{35} - 10^{36}$	no
GUT-like models from string theory	Klebanov & Witten [32]	$\sim 10^{36}$	no
Split SUSY SU(5)	Arkani-Hamed <i>et al.</i> [33]	$10^{35} - 10^{37}$	no
SUSY ($d = 5$) SU(5) – option II	Alciati <i>et al.</i> [34]	$10^{36} - 10^{39}$	no

[Image extracted from T. Ohlsson 2023]

Running and matching estimates

$$\Delta d \equiv d_{XSMEFT} - d_{SMEFT}$$

We distinguish two ways in which the Weinberg operator or the $d \leq 7$ baryon-number-violating operators may arise at the low scale:

- $\Delta d \leq 1$ 1. Renormalisation group mixing of low-dimensional lepton- and baryon-number-violating operators in the $XSMEFT$ featuring the exotic field X into the appropriate SMEFT operators between the scales Λ and M ;
- $\Delta d > 1$ 2. Loop-level matching at the scale Λ onto the relevant SMEFT operators.

First, we highlight that the running contributions to the dimension-5 Weinberg operator defined in Eq. (2.1) C_5 are fixed by dimensional analysis to be¹⁹

Running $C_{5,\text{EFT}} \sim \frac{1}{\Lambda} \left(\frac{M}{\Lambda} \right)^{d-5} .$ (A.1)

The matching contributions that might compete with this can also have a similar form, which we write schematically as

Matching $C_{5,\text{Match}} \sim \frac{1}{\Lambda} \left(\frac{M}{\Lambda} \right)^\delta ,$ (A.2)

where $\delta = 1$ if we can conclude that the neutrino mass must contain a massive parameter in the numerator, and otherwise $\delta = 0$. We distinguish two possible cases where the matching contribution to the Weinberg operator might take this form in our framework:

Running dominates: $\Delta d \leq 1$

Model 1 ($\Delta d = 1$)

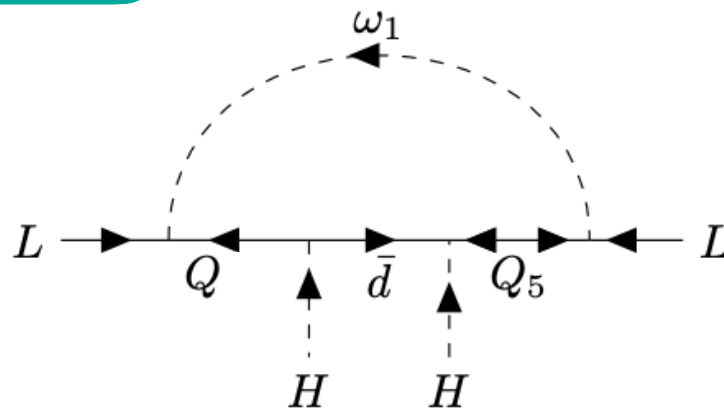


SM + ω_1 + Q_5 with $m_{Q_5} = M < m_{\omega_1} = \Lambda$

$$C_5 \propto y_b M \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} \frac{1}{q^2 - \Lambda^2} \frac{1}{q^2 - M^2}$$

$$\propto \frac{y_b}{16\pi^2} \frac{M}{M^2 - \Lambda^2} \log\left(\frac{M}{\Lambda}\right),$$

$$C_5 \propto \boxed{\frac{y_b}{16\pi^2} \frac{M}{\Lambda^2} \log\left(\frac{M}{\Lambda}\right)} + \mathcal{O}(M^3/\Lambda^3),$$



Model 1 ($\Delta d = 0$)



SM + ω_1 + Π_1 with $m_{\omega_1} = M < m_{\Pi_1} = \Lambda$

$$C_5 \propto i y_b \Lambda \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} \frac{1}{q^2 - \Lambda^2} \frac{1}{q^2 - M^2}$$

$$\propto \frac{y_b}{16\pi^2} \frac{\Lambda}{M^2 - \Lambda^2} \log\left(\frac{M}{\Lambda}\right),$$

$$C_5 \propto \boxed{\frac{y_b}{16\pi^2} \frac{1}{\Lambda} \log\left(\frac{M}{\Lambda}\right)} + \mathcal{O}(M^2/\Lambda^2),$$

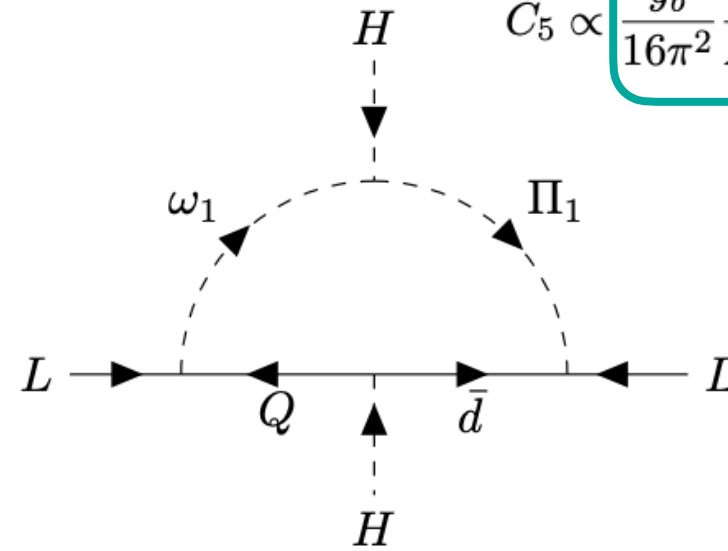


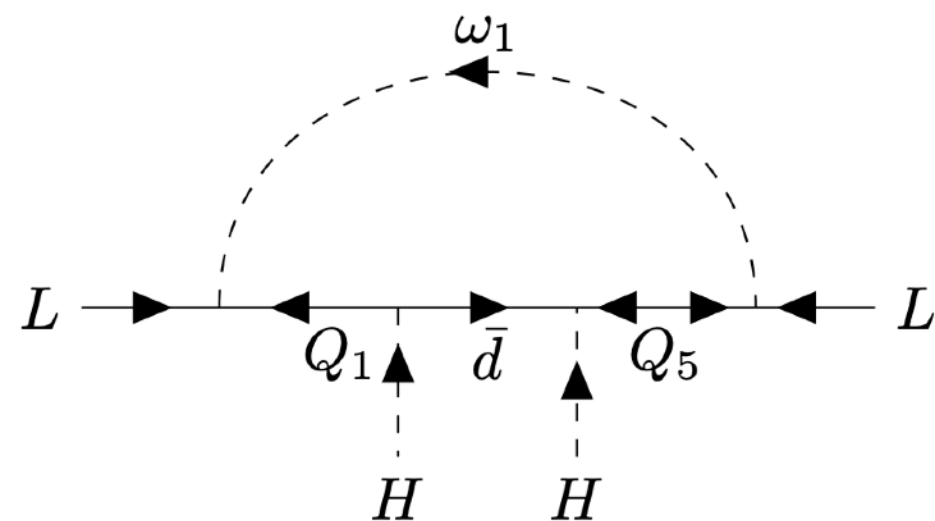
Figure 6. (Left) The neutrino mass diagram for the model presented in Ref. [42] for the choice of Q on the first internal fermion line. This is the example model presented in the $\Delta d = 1$ scenario of Sec. B.1. The choice of Q_1 is relevant for the $\Delta d > 1$ example model, presented in Sec. B.3. (Right) The neutrino-mass diagram for the leptoquark model of neutrino masses, as presented in Sec. B.2.

Matching dominates: $\Delta d > 1$



SM + ω_1 + Q_1 + Π_1 with $m_{Q_1} = M < m_{\omega_1} = m_{\Pi_1} = \Lambda$

$$\begin{aligned}
 I &\propto \Lambda \int \frac{d^4 q}{(2\pi)^4} \frac{\bar{\sigma} \cdot q}{q^2 - M^2} \frac{\sigma \cdot q}{q^2} \left(\frac{1}{q^2 - \Lambda^2} \right)^2 \\
 &\propto \frac{\Lambda}{16\pi^2} \left[\frac{1}{M^2 - \Lambda^2} - \frac{M^2}{(M^2 - \Lambda^2)^2} \log \frac{M^2}{\Lambda^2} \right] \\
 &\propto \boxed{\frac{1}{16\pi^2 \Lambda}} + \mathcal{O}(M^2/\Lambda^2),
 \end{aligned}$$



Induced VEVs

LSME	G_{SM}	$\mathcal{L}_{\Delta L=2}$	$[m_\nu]_{pq}$	Upper Bound (TeV)	$\langle X^0 \rangle$ (TeV)
Ξ_1	$(1, 3, 1)_S$	$\mu \Xi_1^\dagger H H + c_{\{pq\}} \Xi_1 (L_p L_q)$	$c_{\{pq\}} \left(\mu \frac{v^2}{M^2} \right)$	$5 \cdot 10^{12}$	$5 \cdot 10^{-15}$
Ξ	$(1, 3, 0)_S$	$\mu \Xi H^\dagger H + c_{\{pq\}} \Xi^{\{ij\}} (L_p^k L_q^l) H^m H^n \epsilon_{km} \epsilon_{il} \epsilon_{jn}$	$c_{\{pq\}} \frac{v^2}{\Lambda^2} \left(\mu \frac{v^2}{M^2} \right)$	$5 \cdot 10^3$	$6 \cdot 10^{-6}$
Θ_3	$(1, 4, 3/2)_S$	$y \Theta_3^\dagger H H H + c_{\{pq\}} \Theta_3 (L_p L_q) H^\dagger$	$c_{\{pq\}} \frac{v}{\Lambda} \left(y \frac{v^3}{M^2} \right)$	$5 \cdot 10^3$	$5 \cdot 10^{-10}$
Θ_1	$(1, 4, 3/2)_S$	$y \Theta_1^\dagger H H^\dagger H + c_{pq} \Theta_1^{\{ijk\}} (L_p^l L_q^m) H^n H_l^\dagger H^o \epsilon_{im} \epsilon_{jn} \epsilon_{ko}$	$c_{pq} \frac{v^3}{\Lambda^3} \left(y \frac{v^3}{M^2} \right)$	10^2	$5 \cdot 10^{-7}$

Table 5. Same as Tab. 2 for the generation of Majorana neutrino masses by the four scalar LSMEs that have a neutral component and may develop a VEV, as explained in Sec. 2.4. In the sixth column we quote the upper bound on Λ in the limit $\mu \sim M \sim \Lambda$, and in the last column, we display the value of the VEV induced by the SM Higgs doublet.

LSME	G_{SM}	$\mathcal{L}_{\Delta B=1}$	$[L_{q_1 q_2 q_3}^{S,XY}]_{pqrs}$ [79, 92]	Upper Bound (TeV)	$\langle X^0 \rangle$ (TeV)
Ξ_1	$(1, 3, 1)_S$	$\mu \Xi_1^\dagger H H + c_{pq[rs]} \Xi_1 (Q_p L_q) (\bar{d}_r^\dagger \bar{d}_s^\dagger)$	$[L_{ddu}^{S,RL}]_{pqrs} = c_{rs[pq]} \frac{1}{\Lambda^3} \left(\mu \frac{v^2}{M^2} \right)$	$5 \cdot 10^5$	$6 \cdot 10^{-8}$
Ξ	$(1, 3, 0)_S$	$\mu \Xi H^\dagger H + c_{pq[rs]} \Xi_{ij} (L_{pk}^\dagger \bar{d}_q^\dagger) (Q_r^i Q_s^j) H_l^\dagger \epsilon^{kl}$	$[L_{udd}^{S,LR}]_{pqrs} = c_{rs[pq]} \frac{v}{\Lambda^4} \left(\mu \frac{v^2}{M^2} \right)$	10^5	$3 \cdot 10^{-7}$
Θ_3	$(1, 4, 3/2)_S$	$y \Theta_3 H^\dagger H^\dagger H^\dagger + c_{[pq]rs} \Theta_3^\dagger (Q_p Q_q) (L_r^\dagger \bar{u}_s^\dagger)$	$[L_{ddu}^{S,LR}]_{pqrs} = c_{[pq]rs} \frac{1}{\Lambda^3} \left(y \frac{v^3}{M^2} \right)$	$5 \cdot 10^4$	$2 \cdot 10^{-8}$
Θ_1	$(1, 4, 1/2)_S$	$y \Theta_1 H^\dagger H H^\dagger + c_{pq[rs]} \Theta_1^\dagger (L_p^\dagger \bar{d}_q^\dagger) (Q_r Q_s)$	$[L_{udd}^{S,LR}]_{pqrs} = c_{rs[pq]} \frac{1}{\Lambda^3} \left(y \frac{v^3}{M^2} \right)$	$5 \cdot 10^4$	$2 \cdot 10^{-8}$

Table 6. Same as Tab. 2 for the generation of nucleon decays by the four scalar LSMEs that have a neutral component and may develop a VEV, as explained in Sec. 2.4. All the $\Delta B = 1$ LEFT WCs lead to the $p \rightarrow K^+ \nu$ decay channel, from which the limit quoted in the sixth column is obtained.

Tables for scalars in $\Delta L = 2$

LSME	G_{SM}	$\mathcal{L}_{\Delta L=2}$	Δd	Op.	$[m_\nu]_{pq}$	Upper Bound (TeV)
Ξ_1	$(1, 3, 1)_S$	$y_{\{pq\}} \Xi_1(L_p L_q) + \mu \Xi_1^\dagger H H$	-2	$\mathcal{O}_1$	$y_{\{pq\}} \frac{\mu}{M} \frac{v^2}{M}$	10^{12}
Ξ	$(1, 3, 0)_S$	$\mu \Xi H^\dagger H + c_{\{pq\}} \Xi^{\{ij\}} (L_p^k L_q^l) H^m H^n \epsilon_{km} \epsilon_{il} \epsilon_{jn}$	1	$\mathcal{O}'_1$	$c_{\{pq\}} \left(L + \frac{v^2}{\Lambda^2} \right) \frac{\mu}{\Lambda} \frac{v^2}{\Lambda}$	$5 \cdot 10^9$
Θ_3	$(1, 4, 3/2)_S$	$y \Theta_3^\dagger H H H + c_{\{pq\}} \Theta_3(L_p L_q) H^\dagger$	0	$\mathcal{O}'_1$	$y c_{\{pq\}} \left(L + \frac{v^2}{\Lambda^2} \right) \frac{v^2}{\Lambda}$	$5 \cdot 10^9$
Π_7	$(3, 2, 7/6)_S$	$y_{pr} \Pi_7(\bar{u}_r L_p) + c_{qs} \Pi_7^\dagger L_q^j \bar{u}_s^\dagger (D H)^i H^l \epsilon_{jl}$	2	$\mathcal{O}_{D12a}$	$y_{pr} c_{qr} \frac{v^2}{\Lambda} \epsilon$	$5 \cdot 10^9$
ω_1	$(3, 1, -1/3)_S$	$y_{pr} \omega_1^\dagger (L_p Q_r) + c_{qs} \omega_1(L_q \bar{d}_s) H$	0	$\mathcal{O}_{3b}$	$y_{pr} c_{qr} [y_d]_r \frac{v^2}{\Lambda} L$	10^8
ζ	$(3, 3, -1/3)_S$	$y_{pr} \zeta^\dagger (L_p Q_r) + c_{qs} \zeta(L_q \bar{d}_s) H$	0	$\mathcal{O}_{3b}$	$y_{pr} c_{qr} [y_d]_r \frac{v^2}{\Lambda} L$	10^8
Π_1	$(3, 2, 1/6)_S$	$y_{pr} \Pi_1(L_p \bar{d}_r) + c_{qs} \Pi_1^\dagger (L_q^j Q_s^i) H^l \epsilon_{jl}$	0	$\mathcal{O}_{3b}$	$y_{pr} c_{qr} [y_d]_r \frac{v^2}{\Lambda} L$	10^8
φ	$(1, 2, 1/2)_S$	$y_{rp} \varphi^\dagger (\bar{e}_r L_p) + c_{[qs]} \varphi^i H^j (L_q^k L_s^l) \epsilon_{ij} \epsilon_{kl}$	0	$\mathcal{O}_2$	$y_{sq} c_{[sp]} [y_e]_s \frac{v^2}{\Lambda} L$	$5 \cdot 10^7$
Θ_1	$(1, 4, 1/2)_S$	$y \Theta_1^\dagger H H^\dagger H + c_{pq} \Theta_1^{\{ijk\}} (L_p^l L_q^m) H^n H_l^\dagger H^o \epsilon_{im} \epsilon_{jn} \epsilon_{ko}$	2	$\mathcal{O}''_1$	$y c_{pq} \left[\epsilon^2 + \left(\frac{v^2}{\Lambda^2} \right)^2 \right] \frac{v^2}{\Lambda}$	$5 \cdot 10^7$
$\mathcal{S}_1$	$(1, 1, 1)_S$	$y_{[pr]} \mathcal{S}_1(L_p L_r) + c_{qs} \mathcal{S}_1^\dagger (L_q \bar{e}_s) H$	0	$\mathcal{O}_2$	$y_{[pr]} c_{qr} [y_e]_r \frac{v^2}{\Lambda} L$	$5 \cdot 10^7$
Ω_4	$(6, 1, 4/3)_S$	$y_{\{rs\}} \Omega_4^\dagger (\bar{u}_r^\dagger \bar{u}_s^\dagger) + c_{pqtu} \Omega_4(L_p L_q)^{\{ij\}} (Q_t^\dagger Q_u^\dagger)_{ij}$	2	$\mathcal{O}_{12a}$	$y_{\{rs\}} c_{pqrs} [y_u]_r [y_u]_s \frac{v^2}{\Lambda} \epsilon^2$	$5 \cdot 10^7$
Υ	$(6, 3, 1/3)_S$	$y_{\{rs\}} \Upsilon(Q_r^\dagger Q_s^\dagger) + c_{\{pq\}\{tu\}} \Upsilon^\dagger(L_p L_q) (\bar{u}_t^\dagger \bar{u}_u^\dagger)$	2	$\mathcal{O}_{12a}$	$y_{sr} c_{\{pq\}\{rs\}} [y_u]_r [y_u]_s \frac{v^2}{\Lambda} \epsilon^2$	$5 \cdot 10^7$
Φ	$(8, 2, 1/2)_S$	$y_{rs} \Phi^\dagger (Q_r^\dagger \bar{u}_s^\dagger) + c_{pqtu} \Phi^i (L_p^j L_q^k) (Q_{tk}^\dagger \bar{u}_u^\dagger) \epsilon_{ij}$	2	$\mathcal{O}_{12a}$	$y_{sr} c_{pqrs} [y_u]_r [y_u]_s \frac{v^2}{\Lambda} \epsilon^2$	$5 \cdot 10^7$
Ω_2	$(6, 1, -2/3)_S$	$y_{\{rs\}} \Omega_2(\bar{d}_r \bar{d}_s) + c_{pqtu} \Omega_2^\dagger (L_p^i Q_t^k) (L_q^j Q_u^l) \epsilon_{ik} \epsilon_{jl}$	2	$\mathcal{O}_{11b}$	$y_{\{rs\}} c_{pqrs} [y_d]_r [y_d]_s \frac{v^2}{\Lambda} \epsilon^2$	10^4
ω_4	$(3, 1, 4/3)_S$	$y_{rs} \omega_4(\bar{e}_r \bar{d}_s) + c_{pqrs} \omega_4^\dagger (L_p^i L_q^j) (L_r^k Q_s^l) \epsilon_{ik} \epsilon_{jl}$	2	$\mathcal{O}_{10}$	$y_{rs} c_{\{pqr\}} [y_e]_r [y_d]_s \frac{v^2}{\Lambda} \epsilon^2$	10^4
$\mathcal{S}_2$	$(1, 1, 2)_S$	$y_{\{rs\}} \mathcal{S}_2^\dagger (\bar{e}_r \bar{e}_s) + c_{[pq][tu]} \mathcal{S}_2(L_p^i L_t^j) (L_q^k L_u^l) \epsilon_{ij} \epsilon_{kl}$	2	$\mathcal{O}_9$	$y_{\{rs\}} c_{pqrs} [y_e]_r [y_e]_s \frac{v^2}{\Lambda} \epsilon^2$	$5 \cdot 10^5$
Ω_1	$(6, 1, 1/3)_S$	$y_{rs} \Omega_1^\dagger (\bar{u}_r^\dagger \bar{d}_s^\dagger) + c_{pqtu} \Omega_1(L_p \bar{d}_t) (L_q \bar{d}_u)$	2	$\mathcal{O}_{17}$	$y_{ru} c_{pqtu} [y_d]_r [y_u]_r g^2 \frac{v^2}{\Lambda} \epsilon^3$	10^3
ω_2	$(3, 1, 2/3)_S$	$y_{[rs]} \omega_2^\dagger (\bar{d}_r \bar{d}_s) + c_{pqtu} \omega_2(L_p^i Q_t^j) (L_q^k Q_u^l) \epsilon_{ij} \epsilon_{kl}$	2	$\mathcal{O}_{11b}$	$y_{[rs]} c_{pqrs} [y_d]_r [y_d]_s \frac{v^2}{\Lambda} \epsilon^2$	$5 \cdot 10^2$

Tables for scalars in $\Delta B = 1$

LSME	G_{SM}	$\mathcal{L}_{\Delta B=1}$	Δd	Op.	Matching	Process	Upper Bound (TeV)
ω_1	$(3, 1, -1/3)_S$	$y_{pq}\omega_1^\dagger(\bar{u}_p^\dagger\bar{e}_q^\dagger) + c_{\{rs\}}\omega_1(Q_rQ_s)$	-2	$\mathcal{O}_2^{1111}$	$C_{qqe}^{pqrs} = y_{rs}c_{pq}\frac{1}{M^2}$	$p \rightarrow \pi^0 e^+$	10^{13}
ζ	$(3, 3, -1/3)_S$	$y_{pq}\zeta^\dagger(Q_pL_q) + c_{[rs]}\zeta(Q_rQ_s)$	-2	$\mathcal{O}_1^{1112}$	$C_{qqql}^{pqrs} = y_{rs}c_{[pq]}\frac{1}{M^2}$	$p \rightarrow K^+\nu$	$5 \cdot 10^{12}$
Ω_4	$(6, 1, 4/3)_S$	$y_{\{pq\}}\Omega_4^\dagger(\bar{u}_p^\dagger\bar{u}_q^\dagger) + c_{rstu}\Omega_4(\bar{e}^\dagger Q^\dagger)(QQ)H^\dagger$	2	$\mathcal{O}_{\Omega_4}^{111313}$	$C_{qqe}^{pqrs} = y_{rw}c_{swpq}[y_u]_w\frac{1}{\Lambda^2}\epsilon^2$	$p \rightarrow \pi^0 e^+$	$5 \cdot 10^{10}$
φ	$(1, 2, 1/2)_S$	$y_{pq}\varphi^\dagger Q_p\bar{d}_q + c_{rs[tu]}\varphi^i(L_r^jQ_s^k)(\bar{d}_t^\dagger\bar{d}_u^\dagger)H^l\epsilon_{il}\epsilon_{jk}$	2	$\mathcal{O}_\varphi^{131131}$	$C_{qqql}^{pqrs} = y_{ws}c_{sq[vw]}[y_d]_vV_{vp}^*\frac{1}{\Lambda^2}\epsilon^2$	$p \rightarrow K^+\nu$	$5 \cdot 10^9$
ω_4	$(3, 1, -4/3)_S$	$y_{pq}\omega_4^\dagger(\bar{e}_p^\dagger\bar{d}_q^\dagger) + c_{[rs]}\omega_4(\bar{u}_r^\dagger\bar{u}_s^\dagger)$	-2	$\mathcal{O}_3^{1131}$	$C_{qqe}^{pqrs} = y_{sw}c_{[rq]}[y_u]_q[y_d]_wV_{wp}^*\frac{1}{M^2}L'$	$p \rightarrow \pi^0 e^+$	$5 \cdot 10^8$
Π_1	$(3, 2, 1/6)_S$	$y_{pq}\Pi_1^\dagger(L_p^\dagger\bar{d}_q^\dagger) + c_{rs}\Pi_1(Q_rQ_s)H^\dagger$	-2	$\mathcal{O}_8^{1112}$	$C_{ldqq\bar{H}}^{pqrs} = y_{pq}c_{rs}\frac{1}{M^2\Lambda}$	$p \rightarrow K^+\nu$	10^8
Π_7	$(3, 2, 7/6)_S$	$y_{pq}\Pi_7^\dagger(L_p^\dagger\bar{u}_q^\dagger) + c_{[rs]}\Pi_7H^\dagger(\bar{d}_r^\dagger\bar{d}_s^\dagger)$	-2	$\mathcal{O}_{10}^{1112}$	$C_{ldud\bar{H}}^{pqrs} = y_{pr}c_{[qs]}\frac{1}{M^2\Lambda}$	$p \rightarrow K^+\nu$	10^8
ω_2	$(3, 1, 2/3)_S$	$y_{[pq]}\omega_2(\bar{d}_p^\dagger\bar{d}_q^\dagger) + c_{rs}\omega_2^\dagger(L_r^\dagger\bar{u}_s^\dagger)H^\dagger$	-2	$\mathcal{O}_{10}^{1112}$	$C_{ldud\bar{H}}^{pqrs} = y_{[qs]}c_{pr}\frac{1}{M^2\Lambda}$	$p \rightarrow K^+\nu$	10^8
Υ	$(6, 3, 1/6)_S$	$y_{\{pq\}}\Upsilon^\dagger(Q_pQ_q) + c_{rstu}\Upsilon(Q_r\bar{u}_s)(L_t^\dagger\bar{d}_u^\dagger)$	0	$\mathcal{O}_{40}^{113132}$	$C_{ldqq\bar{H}}^{pqrs} = y_{\{ws\}}c_{rwpq}[y_u]_w\frac{1}{\Lambda^3}L$	$p \rightarrow K^+\nu$	10^7
Φ	$(8, 2, 1/2)_S$	$y_{pq}\Phi(Q_p\bar{u}_q) + c_{rstu}\Phi^\dagger(Q_rQ_s)(L_t^\dagger\bar{d}_u^\dagger)$	0	$\mathcal{O}_{40}^{111332}$	$C_{ldqq\bar{H}}^{pqrs} = y_{sw}c_{wrpq}[y_u]_w\frac{1}{\Lambda^3}L$	$p \rightarrow K^+\nu$	10^7
Ω_2	$(6, 1, -2/3)_S$	$y_{\{pq\}}\Omega_2^\dagger(\bar{d}_p^\dagger\bar{d}_q^\dagger) + c_{rstu}\Omega_2(Q_r\bar{u}_s)(L_t^\dagger\bar{u}_u^\dagger)$	0	$\mathcal{O}_{50}^{131321}$	$C_{ldud\bar{H}}^{pqrs} = y_{qs}c_{wvpr}[y_u]_w\frac{1}{\Lambda^3}L$	$p \rightarrow K^+\nu$	10^7
Ξ	$(1, 3, 0)_S$	$\mu\Xi H^\dagger H + c_{pq[rs]}\Xi_{ij}(L_{pk}^\dagger\bar{d}_q^\dagger)(Q_r^iQ_s^j)H_l^\dagger\epsilon^{kl}$	1	$\mathcal{O}_{45}^{1131}$	$C_{ldqq\bar{H}}^{pqrs} = c_{pq[rs]}[y_u]_s[y_u]_s\frac{\mu}{\Lambda}\frac{1}{\Lambda^3}L^2$	$n \rightarrow K^+e^-$	10^6
Ξ_1	$(1, 3, 1)_S$	$\mu\Xi_1^\dagger HH + c_{rs[tu]}\Xi_1(Q_rL_s)(\bar{d}_t^\dagger\bar{d}_u^\dagger)$	1	$\mathcal{O}_{16}^{1132}$	$C_{qqql}^{pqrs} = c_{qs[vw]}[y_d]_v[y_d]_wV_{vp}^*V_{wr}^*\frac{\mu}{\Lambda}\frac{1}{\Lambda^2}L^2$	$p \rightarrow K^+\nu$	$5 \cdot 10^5$
Θ_3	$(1, 4, 3/2)_S$	$y\Theta_3H^\dagger H^\dagger H^\dagger + c_{[pq]rs}\Theta_3^\dagger(Q_pQ_q)(L_r^\dagger\bar{u}_s^\dagger)$	0	$\mathcal{O}_{37}^{1123}$	$C_{ldqq\bar{H}}^{pqrs} = y c_{[rs]pw}[y_u]_w[y_d]_wV_{wq}\frac{1}{\Lambda^3}L^2$	$p \rightarrow K^+\nu$	$5 \cdot 10^4$
Θ_1	$(1, 4, 1/2)_S$	$y\Theta_1H^\dagger HH^\dagger + c_{pq[rs]}\Theta_1^\dagger(L_p^\dagger\bar{d}_q^\dagger)(Q_rQ_s)$	0	$\mathcal{O}_{45}^{1213}$	$C_{ldqq\bar{H}}^{pqrs} = y c_{pw[vr]}[y_d]_v[y_d]_wV_{ws}^*V_{vq}\frac{1}{\Lambda^3}L^2$	$p \rightarrow K^+\nu$	10^4
Ω_1	$(6, 1, 1/3)_S$	$y_{[pq]}\Omega_1^\dagger(Q_pQ_q) + c_{rstu}\Omega_1(L_r^\dagger\bar{d}_s^\dagger)(Q_t\bar{u}_u)$	0	$\mathcal{O}_{40}^{113132}$	$C_{ldqq\bar{H}}^{pqrs} = y_{[wr]}c_{pqsw}[y_u]_w\frac{1}{\Lambda^3}L$	$p \rightarrow K^+\nu$	10^7
$\mathcal{S}_1$	$(1, 1, 1)_S$	$y_{[pq]}\mathcal{S}_1^\dagger(L_p^\dagger L_q^\dagger) + c_{rstu}\mathcal{S}_1\bar{e}_p^\dagger\bar{u}_q^\dagger\bar{d}_r^\dagger\bar{d}_s^\dagger$	0	$\mathcal{O}_{28}^{133121}$	$C_{ldud\bar{H}}^{pqrs} = y_{[pw]}c_{wrqs}[y_e]_w\frac{1}{\Lambda^3}L$	$p \rightarrow K^+\nu$	$5 \cdot 10^6$
$\mathcal{S}_2$	$(1, 1, 2)_S$	$y_{\{pq\}}\mathcal{S}_2^\dagger(\bar{e}_p\bar{e}_q) + c_{rs[tu]}\mathcal{S}_2(\bar{e}_r^\dagger\bar{d}_s^\dagger)(\bar{d}_t^\dagger\bar{d}_u^\dagger)$	0	$\mathcal{O}_{25}^{133112}$	$C_{\bar{e}ddd}^{pqrs} = y_{\{pw\}}c_{wqrs}\frac{1}{\Lambda^3}L$	$n \rightarrow K^+e^-$	10^6

$M < \Lambda$ regime

	m_ν		Γ_p
$(T_{m_\nu}, L_{\Delta(B-L)=0})$	$\frac{\mu}{M} \frac{1}{M}$		$\frac{\mu}{\Lambda} \frac{1}{\Lambda^2} \left(\log \frac{\Lambda}{M} \right)^2$
$(T_{m_\nu}, T_{\Delta(B-L)=2})$	$\frac{1}{M}$		$\frac{1}{\Lambda^2 M}$
$(L_{m_\nu}, T_{\Delta(B-L)=0})$	$\frac{1}{\Lambda} \log \frac{\Lambda}{M}$	$\frac{M}{\Lambda} \frac{1}{\Lambda} \log \frac{\Lambda}{M}$	$\frac{1}{M^2}$
$(L_{m_\nu}, T_{\Delta(B-L)=2})$	$\frac{1}{\Lambda} \log \frac{\Lambda}{M}$	$\frac{M}{\Lambda} \frac{1}{\Lambda} \log \frac{\Lambda}{M}$	$\frac{1}{\Lambda^2 M}$
$(L_{m_\nu}, L_{\Delta(B-L)=0})$	$\frac{1}{\Lambda} \log \frac{\Lambda}{M}$	$\frac{M}{\Lambda} \frac{1}{\Lambda} \log \frac{\Lambda}{M}$	$\frac{1}{\Lambda^2} \log \frac{\Lambda}{M} \quad \left \quad \frac{M}{\Lambda} \frac{1}{\Lambda^2} \log \frac{\Lambda}{M} \right.$
$(L_{m_\nu}, L_{\Delta(B-L)=2})$	$\frac{1}{\Lambda} \log \frac{\Lambda}{M}$	$\frac{M}{\Lambda} \frac{1}{\Lambda} \log \frac{\Lambda}{M}$	$\frac{1}{\Lambda^3} \log \frac{\Lambda}{M}$

$M < \Lambda$ regime

$\alpha = M/\Lambda \leq 1$	m_ν		Γ_p
$(T_{m_\nu}, L_{\Delta(B-L)=0})$	$\frac{\mu}{M} \frac{1}{M}$	✓	$\alpha^3 \left[\log \left(\frac{1}{\alpha} \right) \right]^2 \frac{\mu}{M} \frac{1}{M^2}$ ✓
$(T_{m_\nu}, T_{\Delta(B-L)=2})$	$\frac{1}{M}$	✓	$\frac{1}{(M/\alpha)^2 M} = \frac{\alpha^2}{M^3}$ ✓
$(L_{m_\nu}, T_{\Delta(B-L)=0})$	$\alpha \log \left(\frac{1}{\alpha} \right) \frac{1}{M}$	$\alpha^2 \log \left(\frac{1}{\alpha} \right) \frac{1}{M}$ ✓	$\frac{1}{M^2}$ ✓
$(L_{m_\nu}, T_{\Delta(B-L)=2})$	$\alpha \log \left(\frac{1}{\alpha} \right) \frac{1}{M}$	$\alpha^2 \log \left(\frac{1}{\alpha} \right) \frac{1}{M}$ ✓	$\frac{1}{(M/\alpha)^2 M} = \frac{\alpha^2}{M^3}$ ✓
$(L_{m_\nu}, L_{\Delta(B-L)=0})$	$\alpha \log \left(\frac{1}{\alpha} \right) \frac{1}{M}$	$\alpha^2 \log \left(\frac{1}{\alpha} \right) \frac{1}{M}$ ✓	$\alpha^2 \log \left(\frac{1}{\alpha} \right) \frac{1}{M^2}$ $\alpha^3 \log \left(\frac{1}{\alpha} \right) \frac{1}{M^2}$ ✓
$(L_{m_\nu}, L_{\Delta(B-L)=2})$	$\alpha \log \left(\frac{1}{\alpha} \right) \frac{1}{M}$	$\alpha^2 \log \left(\frac{1}{\alpha} \right) \frac{1}{M}$ ✓	$\alpha^3 \log \left(\frac{1}{\alpha} \right) \frac{1}{M^3}$ ✓

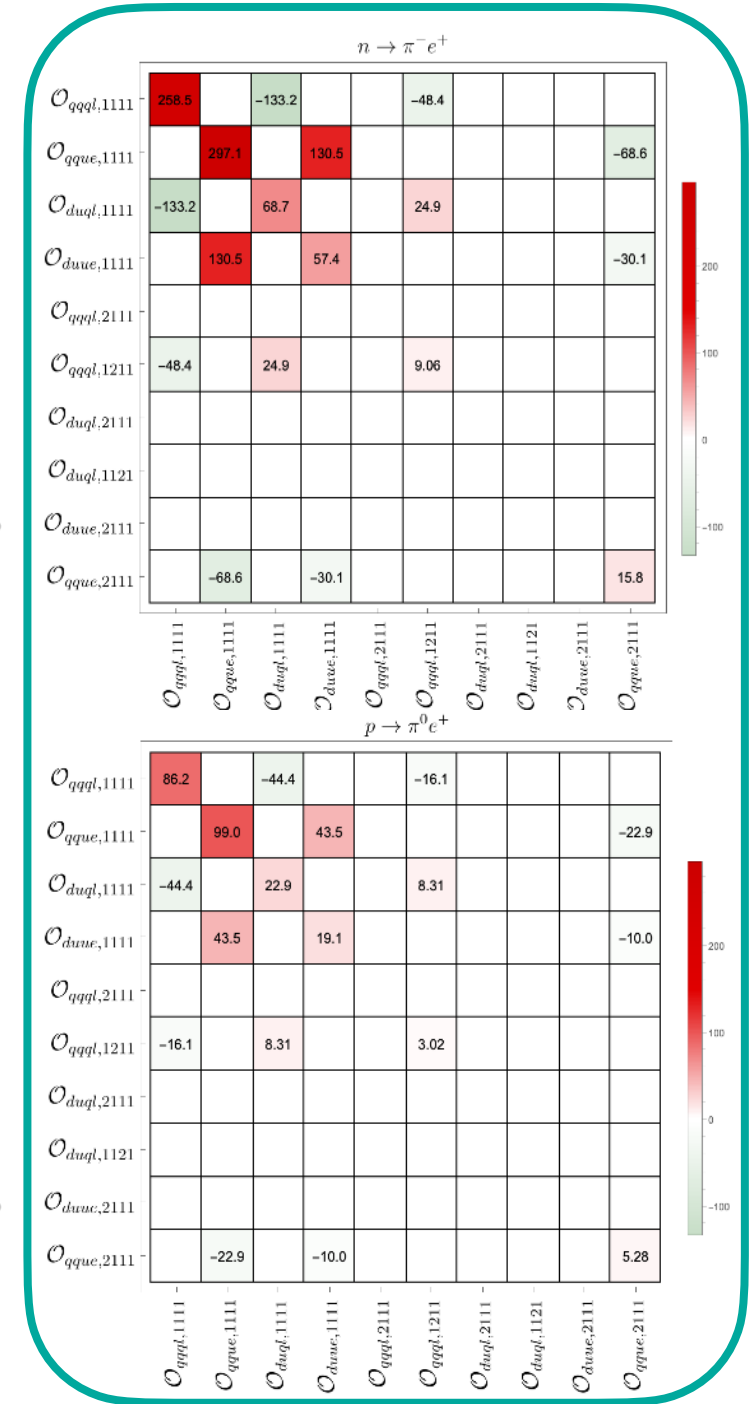
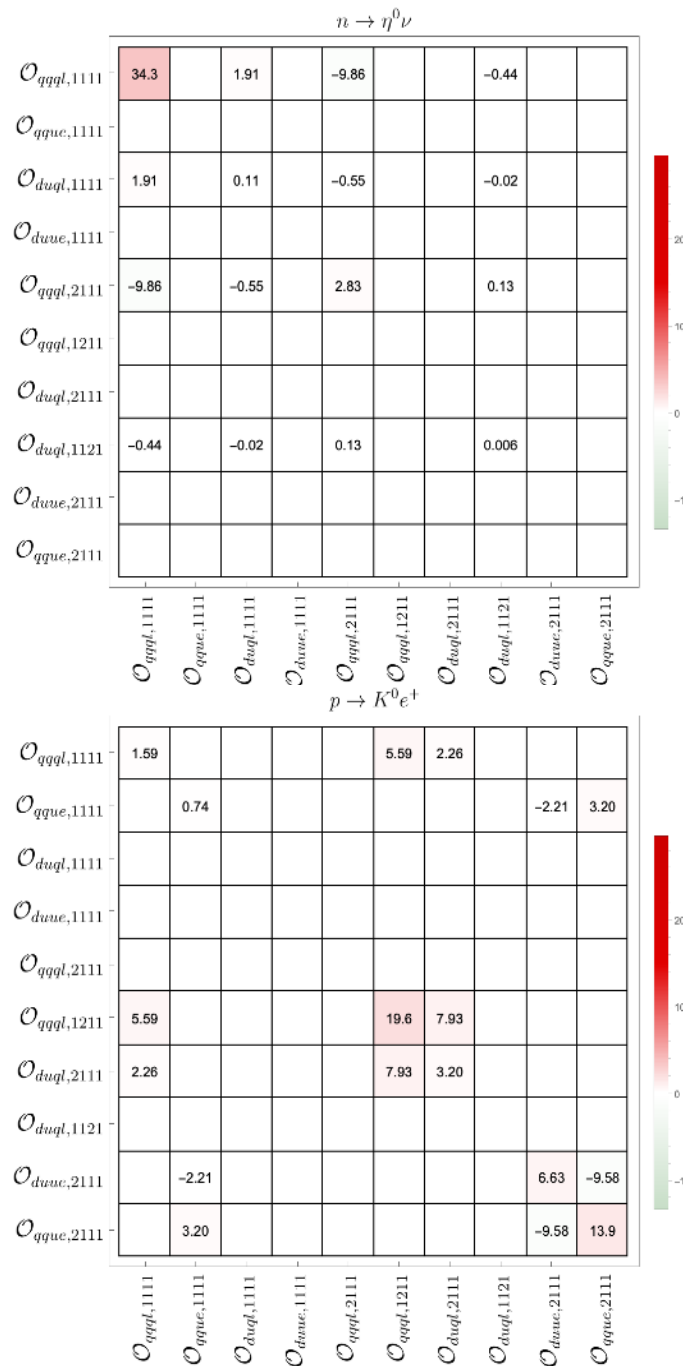
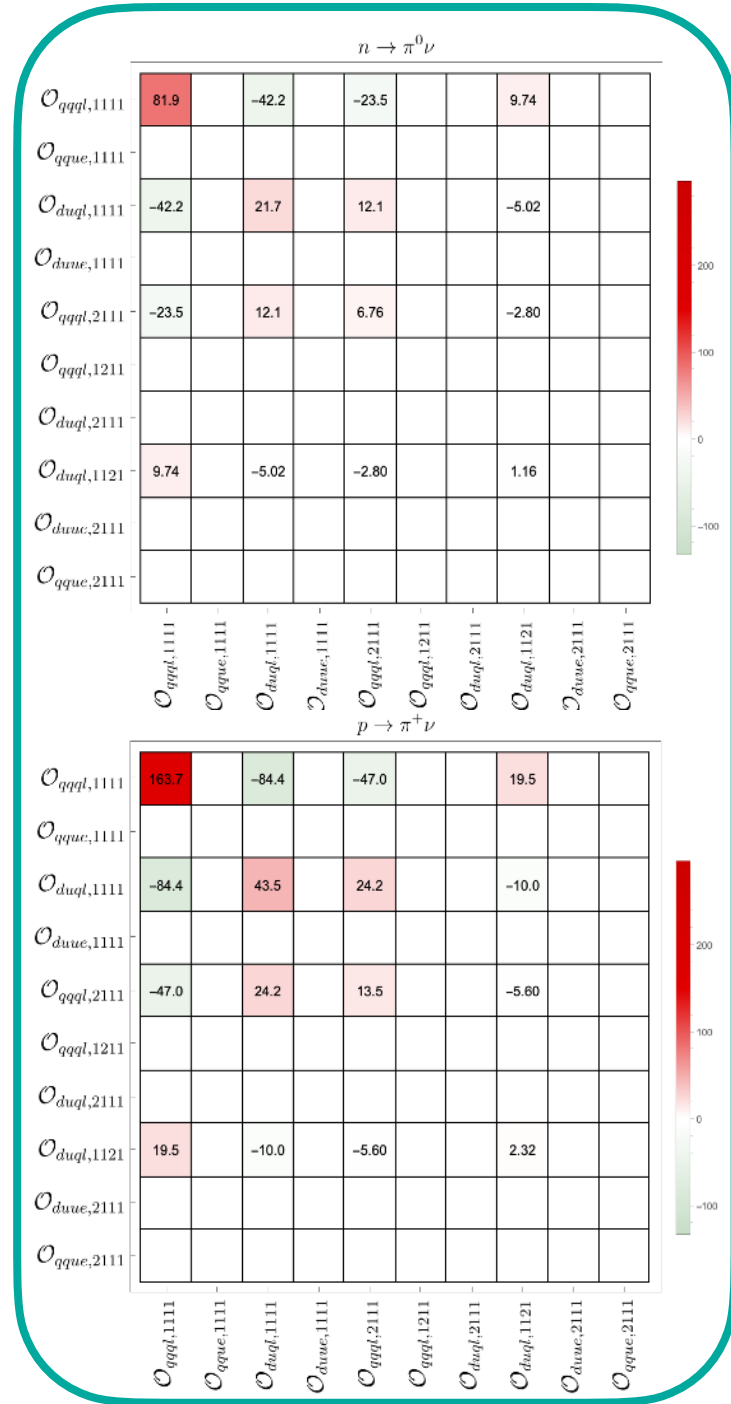
Phenomenological matrices for $\Delta(B - L) = 2$ proton decay

$$\Gamma_{(i)}^{\Delta(B-L)=0} \equiv 10^{-4} c_j^* \kappa_{(i)}^{jk} c_k \frac{m_p^5}{\Lambda^4} \quad \text{for} \quad i = p \rightarrow \pi^0 e^+, p \rightarrow K^+ \nu \dots \quad (9 \text{ matrices}) \quad [A. Bas i Beneito et al. 2023]$$

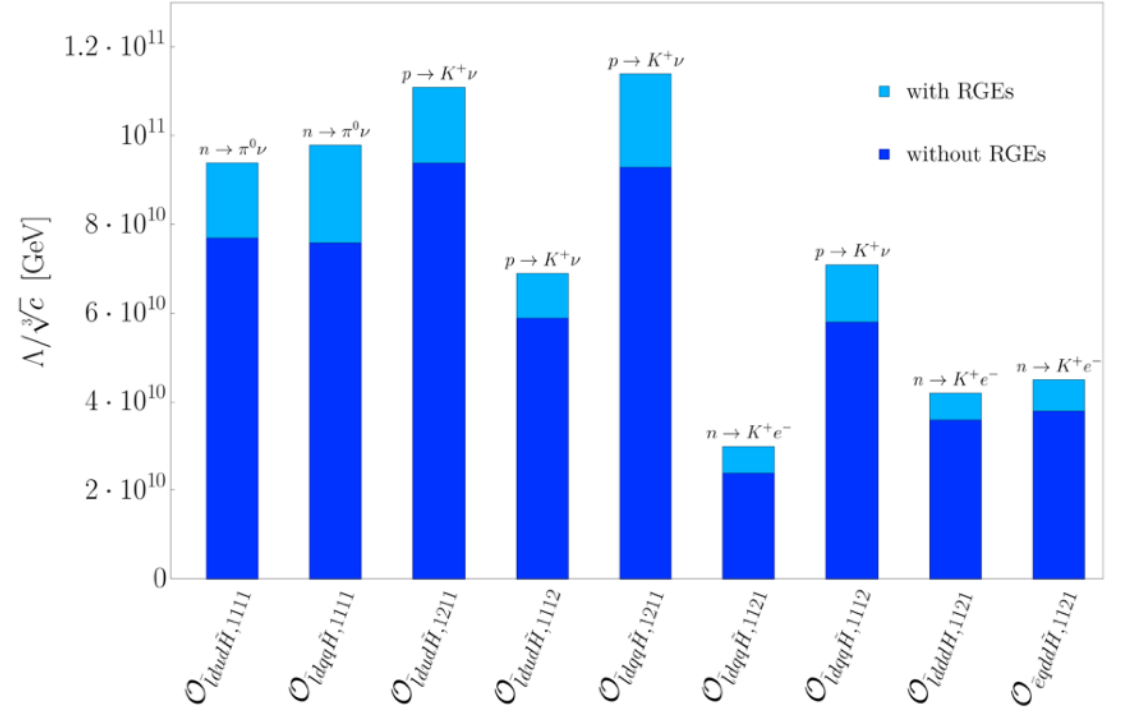
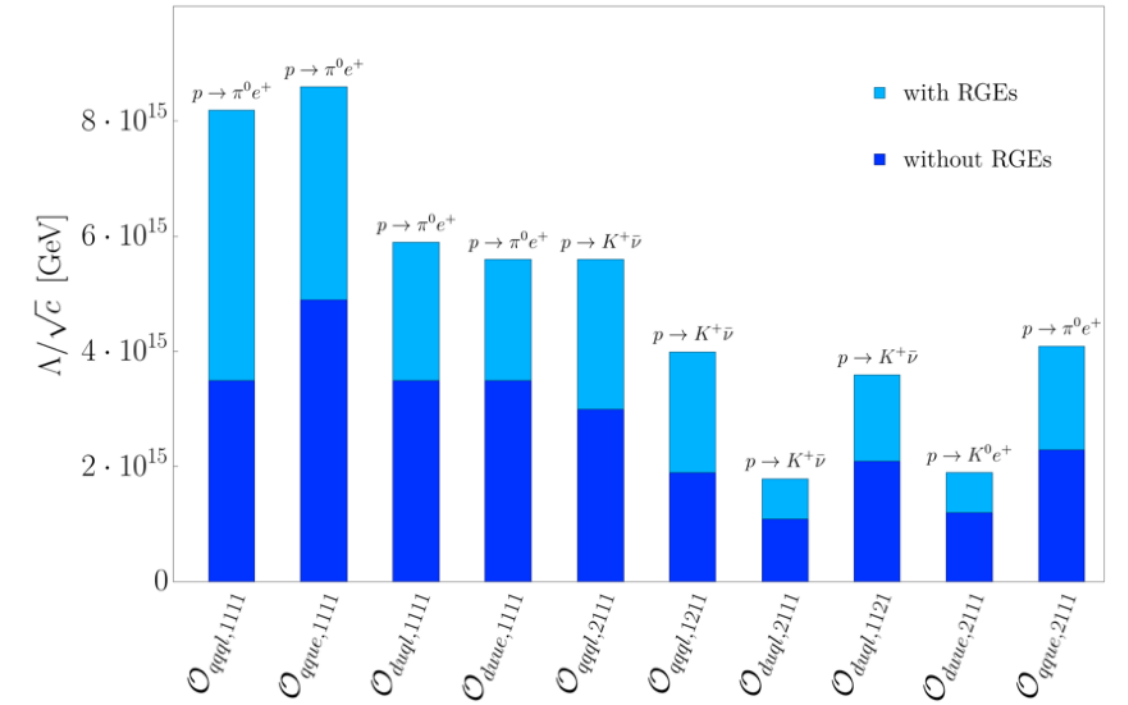
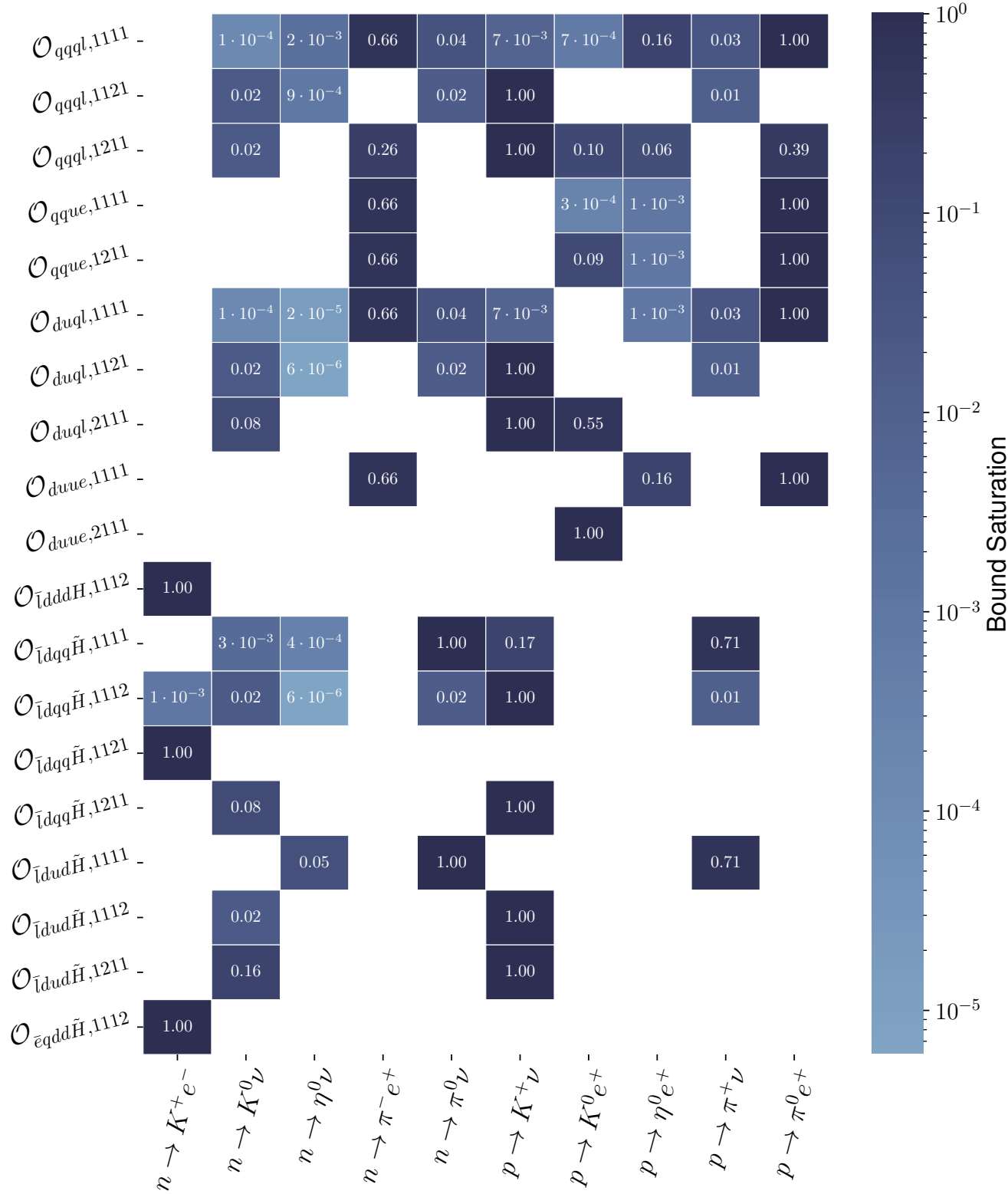
$$\Gamma_{(i)}^{|\Delta(B-L)|=2} \equiv c_j^* \kappa_{(i)}^{jk} c_k \frac{m_p^7}{\Lambda^6} \quad \text{for} \quad i = p \rightarrow K^+ \nu, n \rightarrow K^+ e^- \dots \quad (6 \text{ matrices})$$

$$\Gamma(p \rightarrow \pi^+ \nu) = 2 \Gamma(n \rightarrow \pi^0 \nu)$$

$$\Gamma(n \rightarrow \pi^- e^+) = 3 \Gamma(p \rightarrow \pi^0 e^+)$$



Correlations & tree-level bounds on proton decay



[A. Bas i Beneito et al. 2023]

Matching onto B χ PT

$$\begin{aligned} \mathcal{L}_0 \supset & \left(\frac{D-F}{f_\pi} \bar{\Sigma}^+ \gamma^\mu \gamma_5 p - \frac{D+3F}{\sqrt{6}f_\pi} \bar{\Lambda}^0 \gamma^\mu \gamma_5 n - \frac{D-F}{\sqrt{2}f_\pi} \bar{\Sigma}^0 \gamma^\mu \gamma_5 n \right) \partial_\mu \bar{K}^0 \\ & + \left(\frac{D-F}{\sqrt{2}f_\pi} \bar{\Sigma}^0 \gamma^\mu \gamma_5 p - \frac{D+3F}{\sqrt{6}f_\pi} \bar{\Lambda}^0 \gamma^\mu \gamma_5 p + \frac{D-F}{f_\pi} \bar{\Sigma}^- \gamma^\mu \gamma_5 n \right) \partial_\mu K^- \\ & + \frac{3F-D}{2\sqrt{6}f_\pi} (\bar{p} \gamma^\mu \gamma_5 p + \bar{n} \gamma^\mu \gamma_5 n) \partial_\mu \eta \\ & + \frac{D+F}{f_\pi} \bar{p} \gamma^\mu \gamma_5 n \partial_\mu \pi^+ \\ & + \frac{D+F}{2\sqrt{2}f_\pi} (\bar{p} \gamma^\mu \gamma_5 p - \bar{n} \gamma^\mu \gamma_5 n) \partial_\mu \pi^0 + \text{h.c.} . \end{aligned}$$

$$\xi B \xi \rightarrow L \xi B \xi R^\dagger$$

$$\xi^\dagger B \xi^\dagger \rightarrow R \xi^\dagger B \xi^\dagger L^\dagger$$

$$\xi B \xi^\dagger \rightarrow L \xi B \xi^\dagger L^\dagger$$

$$\xi^\dagger B \xi \rightarrow R \xi^\dagger B \xi R^\dagger$$

$$\xi B \xi \sim (\mathbf{3}, \bar{\mathbf{3}}), \quad \xi^\dagger B \xi^\dagger \sim (\bar{\mathbf{3}}, \mathbf{3}), \quad \xi B \xi^\dagger \sim (\mathbf{8}, \mathbf{1}), \quad \xi^\dagger B \xi \sim (\mathbf{1}, \mathbf{8})$$

$$\alpha \cdot \nu \text{ tr}(\xi B \xi^\dagger P_{32}) = - (du)(d\nu) = [\mathcal{O}_{udd}]_{1111}^{S,LL}$$

$$\langle 0 | \epsilon^{abc} (\bar{u}_a^\dagger \bar{d}_b^\dagger) u_c | p^{(s)} \rangle = \alpha P_L u_p^{(s)}$$

$$\langle 0 | \epsilon^{abc} (u_a d_b) u_c | p^{(s)} \rangle = \beta P_L u_p^{(s)}$$

Name	LEFT	Flavour/B χ PT
$[\mathcal{O}_{udd}^{S,LL}]_{rstu}$	$(u_r d_s)(d_t \nu_u)$	$(\mathbf{8}, \mathbf{1})$
$[\mathcal{O}_{udd}^{S,LL}]_{111r}$	$(ud)(d\nu_r)$	$-\beta \bar{\nu}_L^c \text{tr}(\xi B \xi^\dagger P_{32}) \supset -\beta \bar{\nu}_L^c n - \frac{i\beta}{f_\pi} \bar{\nu}_L^c \left(\sqrt{\frac{3}{2}} n\eta - \frac{1}{\sqrt{2}} n\pi^0 + p\pi^- \right)$
$[\mathcal{O}_{udd}^{S,LL}]_{121r}$	$(us)(d\nu_r)$	$-\beta \bar{\nu}_L^c \text{tr}(\xi B \xi^\dagger \tilde{P}_{22}) \supset -\beta \bar{\nu}_L^c \left(-\frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} \right) - \frac{i\beta}{f_\pi} \bar{\nu}_L^c n \bar{K}^0$
$[\mathcal{O}_{udd}^{S,LL}]_{112r}$	$(ud)(s\nu_r)$	$-\beta \bar{\nu}_L^c \text{tr}(\xi B \xi^\dagger P_{33}) \supset \beta \sqrt{\frac{2}{3}} \bar{\nu}_L^c \Lambda^0 - \frac{i\beta}{f_\pi} \bar{\nu}_L^c (n \bar{K}^0 + p K^-)$
$[\mathcal{O}_{duu}^{S,LL}]_{rstu}$	$(d_r u_s)(u_t e_u)$	$(\mathbf{8}, \mathbf{1})$
$[\mathcal{O}_{duu}^{S,LL}]_{111r}$	$(du)(ue_r)$	$-\beta \bar{e}_L^c \text{tr}(\xi B \xi^\dagger \tilde{P}_{31}) \supset \beta \bar{e}_L^c p + \frac{i\beta}{f_\pi} \bar{e}_L^c \left(\sqrt{\frac{3}{2}} p\eta + \frac{1}{\sqrt{2}} p\pi^0 + n\pi^+ \right)$
$[\mathcal{O}_{duu}^{S,LL}]_{211r}$	$(su)(ue_r)$	$-\beta \bar{e}_L^c \text{tr}(\xi B \xi^\dagger P_{21}) \supset -\beta \bar{e}_L^c \Sigma^+ + \frac{i\beta}{f_\pi} \bar{e}_L^c p \bar{K}^0$
$[\mathcal{O}_{uud}^{S,LR}]_{[rs]tu}$	$(u_r u_s)(\bar{d}_t^\dagger \bar{e}_u^\dagger)$	—
$[\mathcal{O}_{duu}^{S,LR}]_{rstu}$	$(d_r u_s)(\bar{u}_t^\dagger \bar{e}_u^\dagger)$	$(\bar{\mathbf{3}}, \mathbf{3})$
$[\mathcal{O}_{duu}^{S,LR}]_{111r}$	$(du)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$\alpha \bar{e}_{Rr}^c \text{tr}(\xi^\dagger B \xi^\dagger \tilde{P}_{31}) \supset -\alpha \bar{e}_{Rr}^c p + \frac{i\alpha}{f_\pi} \bar{e}_{Rr}^c \left(-\frac{1}{\sqrt{6}} p\eta + \frac{1}{\sqrt{2}} p\pi^0 + n\pi^+ \right)$
$[\mathcal{O}_{duu}^{S,LR}]_{211r}$	$(su)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$\alpha \bar{e}_{Rr}^c \text{tr}(\xi^\dagger B \xi^\dagger P_{21}) \supset \alpha \bar{e}_{Rr}^c \Sigma^+ - \frac{i\alpha}{f_\pi} \bar{e}_{Rr}^c p \bar{K}^0$
$[\mathcal{O}_{uud}^{S,RL}]_{[rs]tu}$	$(\bar{u}_r^\dagger \bar{u}_s^\dagger)(d_t e_u)$	—
$[\mathcal{O}_{duu}^{S,RL}]_{rstu}$	$(\bar{d}_r^\dagger \bar{u}_s^\dagger)(u_t e_u)$	$(\mathbf{3}, \bar{\mathbf{3}})$
$[\mathcal{O}_{duu}^{S,RL}]_{111r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(ue_r)$	$-\alpha \bar{e}_L^c \text{tr}(\xi B \xi^\dagger \tilde{P}_{31}) \supset \alpha \bar{e}_L^c p + \frac{i\alpha}{f_\pi} \bar{e}_L^c \left(-\frac{1}{\sqrt{6}} p\eta + \frac{1}{\sqrt{2}} p\pi^0 + n\pi^+ \right)$
$[\mathcal{O}_{duu}^{S,RL}]_{211r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(ue_r)$	$-\alpha \bar{e}_L^c \text{tr}(\xi B \xi^\dagger P_{21}) \supset -\alpha \bar{e}_L^c \Sigma^+ - \frac{i\alpha}{f_\pi} \bar{e}_L^c p \bar{K}^0$
$[\mathcal{O}_{dud}^{S,RL}]_{rstu}$	$(\bar{d}_r^\dagger \bar{u}_s^\dagger)(d_t \nu_u)$	$(\mathbf{3}, \bar{\mathbf{3}})$
$[\mathcal{O}_{dud}^{S,RL}]_{111r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(d\nu_r)$	$\alpha \bar{\nu}_L^c \text{tr}(\xi B \xi^\dagger \tilde{P}_{32}) \supset -\alpha \bar{\nu}_L^c n + \frac{i\alpha}{f_\pi} \bar{\nu}_L^c \left(\frac{1}{\sqrt{6}} n\eta + \frac{1}{\sqrt{2}} n\pi^0 - p\pi^- \right)$
$[\mathcal{O}_{dud}^{S,RL}]_{211r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(d\nu_r)$	$\alpha \bar{\nu}_L^c \text{tr}(\xi B \xi^\dagger P_{22}) \supset \alpha \bar{\nu}_L^c \left(\frac{\Lambda^0}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} \right) + \frac{i\alpha}{f_\pi} \bar{\nu}_L^c n \bar{K}^0$
$[\mathcal{O}_{dud}^{S,RL}]_{112r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(s\nu_r)$	$\alpha \bar{\nu}_L^c \text{tr}(\xi B \xi^\dagger \tilde{P}_{33}) \supset \alpha \bar{\nu}_L^c \sqrt{\frac{2}{3}} \Lambda^0 - \frac{i\alpha}{f_\pi} \bar{\nu}_L^c (n \bar{K}^0 + p K^-)$
$[\mathcal{O}_{dud}^{S,RL}]_{212r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(s\nu_r)$	$\alpha \bar{\nu}_L^c \text{tr}(\xi B \xi^\dagger P_{23}) \supset \alpha \bar{\nu}_L^c \Xi^0$
$[\mathcal{O}_{ddu}^{S,RL}]_{[rs]tu}$	$(\bar{d}_r^\dagger \bar{d}_s^\dagger)(u_t \nu_u)$	$(\mathbf{3}, \bar{\mathbf{3}})$
$[\mathcal{O}_{ddu}^{S,RL}]_{[12]1r}$	$(\bar{d}^\dagger \bar{s}^\dagger)(u\nu_r)$	$-\alpha \bar{\nu}_L^c \text{tr}(\xi B \xi^\dagger P_{11}) \supset \alpha \bar{\nu}_L^c \left(\frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} \right) - \frac{i\alpha}{f_\pi} \bar{\nu}_L^c p K^-$
$[\mathcal{O}_{duu}^{S,RR}]_{rstu}$	$(\bar{d}_r^\dagger \bar{u}_s^\dagger)(\bar{u}_t^\dagger \bar{e}_u^\dagger)$	$(\mathbf{1}, \mathbf{8})$
$[\mathcal{O}_{duu}^{S,RR}]_{111r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$\beta \bar{e}_{Rr}^c \text{tr}(\xi^\dagger B \xi^\dagger \tilde{P}_{31}) \supset -\beta \bar{e}_{Rr}^c p + \frac{i\beta}{f_\pi} \bar{e}_{Rr}^c \left(\sqrt{\frac{3}{2}} p\eta + \frac{1}{\sqrt{2}} p\pi^0 + n\pi^+ \right)$
$[\mathcal{O}_{duu}^{S,RR}]_{211r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$\beta \bar{e}_{Rr}^c \text{tr}(\xi^\dagger B \xi^\dagger P_{21}) \supset \beta \bar{e}_{Rr}^c \Sigma^+ + \frac{i\beta}{f_\pi} \bar{e}_{Rr}^c p \bar{K}^0$

BNV Nucleon decay channels

$$\begin{array}{l}
 \Gamma(N \rightarrow M\ell_\alpha) \\
 \Delta(B - L) = 0
 \end{array}
 \left\{
 \begin{array}{l}
 n \rightarrow \eta^0 \nu \\
 n \rightarrow \pi^0 \nu \\
 p \rightarrow \pi^+ \nu \\
 n \rightarrow \pi^- e^+ \\
 p \rightarrow \eta^0 e^+ \\
 p \rightarrow \pi^0 e^+ \\
 p \rightarrow K^0 e^+ \\
 n \rightarrow K^0 \nu \\
 p \rightarrow K^+ \nu
 \end{array}
 \right.
 \qquad
 \begin{array}{l}
 \Gamma(N \rightarrow M\ell_\alpha) \\
 |\Delta(B - L)| = 2
 \end{array}
 \left\{
 \begin{array}{l}
 n \rightarrow \eta^0 \nu \\
 n \rightarrow \pi^0 \nu \\
 p \rightarrow \pi^+ \nu \\
 n \rightarrow K^0 \nu \\
 p \rightarrow K^+ \nu \\
 n \rightarrow K^+ e^-
 \end{array}
 \right.$$

· All 2-body PS decays except for $p \rightarrow \bar{K}^0 e^+$ $n \rightarrow \bar{K}^0 \nu$ $n \rightarrow K^- e^+$ $n \rightarrow \pi^+ e^-$