Shadow Matter

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Gauge Theories and Gravity without Constraints

The QFT of the world produces the classical theories that we discovered first (GR, EM, Newton's laws)

We explore EM and GR without standard constraints on the initial states and see some new effects

Pre(r)amble

Classical Equations of Motion

$$S = \int dt\, L(q, \dot{q})$$

$$\frac{\delta S}{\delta q} = 0$$

Hamiltonian can be build from: $p \equiv \frac{\delta L}{\delta \dot{q}}$

$$H(p, q) = p\dot{q} - L\,|_{\dot{q}(q,p)}$$

Classical: Ehrenfest

$x \equiv \langle \hat{x} \rangle$ $\qquad$ $\left[\hat{x}, \hat{p}\right] = i$

From Schrödinger's Eq $\qquad$ for $H = \dfrac{p^2}{2m} + V(x)$

$$\partial_t x = i\,[H, x] = \frac{\partial H}{\partial p} \qquad \partial_t\, x = \frac{p}{m}$$

$$\partial_t p = i\left[H, p\right] = -\,\frac{\partial H}{\partial x} \qquad \partial_t p = -\,\partial_x V(x)$$

Choose a Hamiltonian to get the classical equations you want

Non-dynamical d.o.f.

Simple example: $$\hat{H} = \frac{\hat{p}_1^2}{2m} + \frac{\hat{p}_2^2}{2m} + V(\hat{q}_2 - \hat{q}_1)$$

$$Q = \frac{q_1 + q_2}{2} \qquad q = q_2 - q_1$$

$$P = p_1 + p_2 \qquad p = \frac{p_2 - p_1}{2}$$

$$\rightarrow H = \frac{P^2}{2M} + \frac{p^2}{2\mu} + V(q)$$

$$[P, H] = 0 \qquad \hat{P}\,|\,P\rangle = P\,|\,P\rangle$$

Think of eigenstates of P as super-selection sectors

Can choose
$$\hat{P}\,|\,\psi\rangle_{\text{phys}} = 0$$
$$\hat{P}\,|\,\psi\rangle_{\text{phys}} = P\,|\,\psi\rangle_{\text{phys}}$$
$$\langle\psi\,|\,\hat{P}\,|\,\psi\rangle_{\text{phys}} = P$$

Non-dynamical d.o.f.

Next example: $$H = \frac{P^2}{2M} + \frac{p^2}{2\mu} + V(q) + \frac{P}{M}\overline{V}(q)$$

Still, $[P, H] = 0$

Again, can choose

$$\hat{P}\,|\psi\rangle_{\text{phys}} = 0$$

$$\hat{P}\,|\psi\rangle_{\text{phys}} = P\,|\psi\rangle_{\text{phys}}$$

$$\langle\psi|\,\hat{P}\,|\psi\rangle_{\text{phys}} = P$$

Measurement allows us to tell what super-selection sector we are in

EM

EM

classical: $$S = \int d^4x \mathscr{L} = \int d^4x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A_\mu J^\mu + \cdots \right]$$

Gauss' Law $$\frac{\delta S}{\delta A_0} = \underbrace{\partial_\mu F^{\mu 0}}_{\nabla \cdot E} - J^0 = 0$$

Ampere's Law $$\frac{\delta S}{\delta A_i} = \underbrace{\partial_\mu F^{\mu i}}_{\dot{E} + \nabla \times B} - J^i = 0$$

conjugate momenta:

$$\pi^i \equiv \frac{\delta \mathscr{L}}{\delta \dot{A}_i} = \partial^0 A^i - \partial^i A^0 \equiv -E^i$$

$$\pi^0 \equiv \frac{\delta \mathscr{L}}{\delta \dot{A}_0} = 0$$

EM

Hamiltonian: $$H = \int d^3x \mathscr{H}$$

$$\mathscr{H} = \frac{1}{2}\mathbf{E}^2 + \frac{1}{2}(\nabla \times \mathbf{A})^2 - A_0(\nabla \cdot E - J^0) + \mathbf{A} \cdot \mathbf{J} + \cdots$$

Make this quantum — choose A_0 — gauge freedom makes it plausible that this choice doesn't affect dynamics.

Simple choice: $A_0 = 0$ (Weyl gauge)

Commutators canonical: $$\left[E^i(\mathbf{x}), A_j(\mathbf{y})\right] = i\,\delta^i_j\,\delta(\mathbf{x} - \mathbf{y})$$

EM

From the Schrödinger Eq:

$$\partial_t \langle \mathbf{A} \rangle = i \langle \left[H, \mathbf{A} \right] \rangle = \langle \mathbf{E} \rangle$$

$$\partial_t \langle \mathbf{E} \rangle = i \langle \left[H, \mathbf{E} \right] \rangle = \langle \nabla \times \mathbf{B} + \mathbf{J} \rangle$$ Ampere's Law

Gauss' Law? $G \equiv \nabla \cdot \mathbf{E} - J^0$

Note: $\left[H, G \right] = 0$

Can require: $G \, | \psi \rangle_{\text{phys}} = 0$

EM

Gauss' Law — Can require:

$$G \equiv \nabla \cdot \mathbf{E} - J^0$$

$$G\,|\,\psi\rangle_{\text{phys}} = 0$$

Could instead require: $G\,|\,\psi\rangle_{\text{phys}} = \rho_{\text{sh}}(\mathbf{x})\,|\,\psi\rangle_{\text{phys}}$

Or even: $\langle\psi\,|\,G\,|\,\psi\rangle = 0$ or $\rho_{\text{sh}}(\mathbf{x})$

like a static charge density b.g.

$$\langle\, \nabla \cdot \mathbf{E} - J^0 - \rho_{\text{sh}}(\mathbf{x})\rangle = 0$$

equivalent physics to infinite mass charge distribution

Gravity (toy)

Gravity: minisuperspace

zero-mode only (FRW): $ds^2 = -N(t)^2 dt^2 + a(t)^2 d\mathbf{x}^2$

classical: $S = \int d^4x \sqrt{-g}\,\Big(M_{\rm pl}^2 R + \underbrace{\mathscr{L}_{\rm matter}(\phi)}_{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \cdots} \Big) + S_{GHY}$

$$\left.\frac{\delta S}{\delta N}\right|_{N=1} = a^3 \left(6M_{\rm pl}^2 \frac{\dot{a}^2}{a^2} - \frac{1}{2}\dot{\phi}^2 - V(\phi) \right) = 0$$

1st Friedmann Eq

$$\left.\frac{\delta S}{\delta a}\right|_{N=1} = 3a^2 \left[4M_{\rm pl}^2 \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{2a^2} \right) + \frac{1}{2}\dot{\phi}^2 - V(\phi) \right] = 0$$

2nd Friedmann Eq

$$\left.\frac{\delta S}{\delta \phi}\right|_{N=1} = -a^3 \left(\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} \right)$$

matter EOM

Gravity: minisuperspace

classical:
$$S = \int d^4x\sqrt{-g}\,(\;M_{\rm pl}^2 R \;+\; \underbrace{\mathscr{L}_{\rm m}(\phi)}_{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+\cdots}\;) \;+\; S_{GHY}$$

conjugate momenta:

$$\pi_a \equiv \frac{\delta\mathscr{L}}{\delta\dot{a}} = -\,6M_{\rm pl}^2\frac{a\dot{a}}{N} \qquad \pi_N \equiv \frac{\delta\mathscr{L}}{\delta\dot{N}} = 0 \qquad \pi_\phi = \frac{\delta\mathscr{L}}{\delta\dot{\phi}} = \frac{a^3}{N}\dot{\phi}$$

Hamiltonian:

$$H = \left[\pi\dot{a} + \pi_\phi\dot{\phi} - \mathscr{L}\right]_{\dot{a}=\cdots,\dot{\phi}=\cdots} = -\frac{N}{24M_{pl}^2 a}\pi^2 + \frac{N}{2a^3}\pi_\phi^2 + Na^3V(\phi)$$

$$= N\tilde{H}(\pi_a, a, \pi_\phi, \phi)$$

Gravity: minisuperspace

Schrödinger eq:

$$i\frac{d}{dt}|\psi\rangle = N(t)\,\tilde{H}\,|\psi\rangle \quad \rightarrow i\frac{d}{N(t)dt}|\psi\rangle = \tilde{H}\,|\psi\rangle$$

N maintains time reparameterization invariance.

N is a like a gauge d.o.f. Simple gauge choice: $N = 1$

$$\partial_t\langle a\rangle = i\langle[H, a]\rangle \rightarrow \langle\pi_a\rangle = \langle -6M_{\rm pl}a\dot{a}\rangle$$

(choosing operator ordering wisely)

$$\partial_t\langle\pi_a\rangle = i\langle\left[H, \pi_a\right]\rangle \rightarrow$$

$$\longrightarrow \frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{M_{\rm pl}^2}p$$

2nd Friedmann Eq

(replacing π_a's with $\dot{a}$'s, assuming classical states)

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Gravity: minisuperspace

1st Friedmann Eq? From classical:

$$0 = \frac{\delta S}{\delta N} = \frac{\delta \int \pi_a \dot{a} + \pi_\phi \dot{\phi} - N\tilde{H}}{\delta N} \longrightarrow \tilde{H} = 0$$

The 'Hamiltonian Constraint' *is* the 1st Friedmann Eq

Standard treatment (like Gauss): $\tilde{H}\,|\,\psi\rangle_{\text{phys}} = 0$ (Wheeler-deWitt)

??? No Schrödinger equation ???

The "problem of time" in quantum gravity

Simple fix: $\langle\psi\,|\,H\,|\,\psi\rangle = 0$

Gravity: minisuperspace

1st Friedmann Eq:

$$\langle \psi | \tilde{H} | \psi \rangle = 0$$

For classical states:

$$\tilde{H} = a^3 \left(3M_{\rm pl}^2 \left(\frac{\dot{a}}{a} \right)^2 - \rho \right) = 0$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

Gravity: minisuperspace

Could choose:

$$\langle \psi | \tilde{H} | \psi \rangle = \mathbb{C}$$

$\mathbb{C}$ is constant as $[\tilde{H}, \tilde{H}] = 0$

For classical states:

$$\tilde{H} = a^3 \left(3M_{\rm pl}^2 \left(\frac{\dot{a}}{a} \right)^2 - \rho \right) = \mathbb{C}$$

$$\longrightarrow \left(\frac{\dot{a}}{a} \right)^2 = \frac{1}{3M_{\rm pl}^2} \left(\rho + \frac{\mathbb{C}}{a^3} \right)$$

behaves as another matter component

Interlude

Point 1: The equations that are modified (Gauss, 1st Friedmann) are constraints on initial conditions.

Point 2: The Schrödinger equation will evolve any state you give it. These are simply a broader set of initial conditions.

Point 3: These are initial conditions — Inflation will redshift all of this stuff away. For rest of talk, assume inflation didn't happen.

GR

GR

$$S = \int d^4x\, \underbrace{(\sqrt{-g}\, M_{\rm pl}^2 R \;+\; \sqrt{-g}\, \mathscr{L}_{\rm matter}(\phi))}_{\mathscr{L}_g} \;+\; S_{GHY}$$

classical:

$$\frac{\delta S}{\delta g_{\mu\nu}} = \sqrt{-g}\, \left(M_{\rm pl}^2 G^{\mu\nu} - T^{\mu\nu} \right) = 0$$

conjugates:

$$\frac{\delta \mathscr{L}_g}{\delta \dot{g}_{ij}} \equiv \pi^{ij} \qquad \frac{\delta \mathscr{L}_g}{\delta \dot{g}_{0\mu}} = 0$$

construct Hamiltonian:

$$g_{0\mu} = -\,\delta_{0\mu} \qquad \gamma_{ij} \equiv g_{ij}$$

(synchronous gauge)

$$\left[\gamma_{ij}(\mathbf{x}), \pi^{kl}(\mathbf{y})\right] = i\,\delta^k_{(i}\delta^l_{j)}\,\delta(\mathbf{x}-\mathbf{y})$$

$$\mathscr{H} = \frac{1}{\sqrt{\gamma}}(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2) - \sqrt{\gamma}\;^{(3)}R + \cdots$$

GR

EOM: $$\partial_t \langle \pi^{ij} \rangle \sim - \left\langle \frac{\delta H}{\delta \gamma_{ij}} \right\rangle \longrightarrow G^{ij} = 8\pi G T^{ij} \qquad \left(\frac{\delta S}{\delta g_{ij}} = 0 \right)$$

(under rug: coherent states, operator ordering)

Remaining equations constraints — impose on the initial states

$$\langle \mathscr{H} \rangle = 0 \quad \text{Hamiltonian} \longrightarrow \sqrt{-g} \left(M_{\rm pl}^2\, G^{00} - T^{00} \right) = 0$$

$$\langle \chi^i \rangle = 0 \quad \text{Momentum} \longrightarrow \sqrt{-g} \left(M_{\rm pl}^2\, G^{0i} - T^{0i} \right) = 0$$

$$\langle \mathscr{H} \rangle = \mathbb{H} \longrightarrow \sqrt{-g} \left(M_{\rm pl}^2\, G^{00} - T^{00} \right) = \mathbb{H}$$

$$\langle \chi^i \rangle = \mathbb{P}^i \longrightarrow \sqrt{-g} \left(M_{\rm pl}^2\, G^{0i} - T^{0i} \right) = \mathbb{P}^i$$

GR

$$G^{00} = 8\pi G\left(T^{00} + \frac{\mathbb{H}}{\sqrt{-g}}\right)$$

$$G^{0i} = 8\pi G\left(T^{0i} + \frac{\mathbb{P}^i}{\sqrt{-g}}\right)$$

$$G^{ij} = 8\pi G T^{ij}$$

$\Bigg\}\longrightarrow$ $G^{\mu\nu} = 8\pi G(T^{\mu\nu} + T^{\mu\nu}_{\text{sh}})$

$$T^{\mu\nu}_{\text{sh}} \equiv \frac{1}{\sqrt{-g}}\begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

GR

$$G^{\mu\nu} = 8\pi G(T^{\mu\nu} + T^{\mu\nu}_{\rm sh})$$

$$T^{\mu\nu}_{\rm sh} \equiv \frac{1}{\sqrt{-g}} \begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

$\mathbb{H}, \mathbb{P}^i$ are constrained functions

(identity) $\nabla_\mu G^{\mu\nu} = 0$

(EOM) $\nabla_\mu T^{\mu\nu} = 0$

$$\longrightarrow \nabla_\mu T^{\mu\nu}_{\rm sh} = 0 \longrightarrow$$

$$\partial_0 \mathbb{H} + \partial_i \mathbb{P}^i = 0$$

$$\partial_0 (g_{ij} \mathbb{P}^j) = 0$$

GR

$$G^{\mu\nu} = 8\pi G(T^{\mu\nu} + T^{\mu\nu}_{\text{sh}})$$

What do these source terms do?

$$T^{\mu\nu}_{\text{sh}} \equiv \frac{1}{\sqrt{-g}} \begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

$$\partial_0 \mathbb{H} + \partial_i \mathbb{P}^i = 0$$

$$\partial_0 (g_{ij} \mathbb{P}^j) = 0$$

look at limits

GR

$$\partial_0 \mathbb{H} + \partial_i \mathbb{P}^i = 0$$

$$\partial_0 (g_{ij} \mathbb{P}^j) = 0$$

limit: $\mathbb{P}^i = 0 \quad \rightarrow \partial_0 \mathbb{H} = 0$

$$G^{00} = 8\pi G \; (T^{00} + \frac{\mathbb{H}(\mathbf{x})}{\sqrt{-g}})$$

write as:

$$u^\mu = \{1, 0, 0, 0\} \qquad \rho_{\rm sh} = \mathbb{H}(\mathbf{x})/\sqrt{-g}$$

$$T^{\mu\nu}_{\rm sh} = \rho_{\rm sh} u^\mu u^\nu$$

Looks like pressure-less dust!

GR

how does this behave? look at dust:

$$T^{\mu\nu}_{\rm dust} = \rho u^\mu u^\nu \qquad\qquad u^\mu = \{1,0,0,0\}$$

$$\Gamma^0_{00} = \Gamma^i_{00} = \Gamma^0_{0i} = \Gamma^0_{i0} = 0$$ in synchronous gauge

thus: $$0 = \sqrt{-g}\,\nabla_\mu T^{\mu\nu}_{\rm dust} = \sqrt{-g}(\partial_\mu(\rho u^\mu u^\nu) + \Gamma^\mu_{\mu\lambda}\rho u^\lambda u^\nu + \Gamma^\nu_{\mu\lambda}\rho u^\mu u^\lambda)$$

$$= \sqrt{-g}\; u^\nu \partial_0 \rho + \sqrt{-g}\;\Gamma^\mu_{\mu 0}\rho u^\nu + \sqrt{-g}\;\Gamma^\nu_{00}\rho$$

$$= \sqrt{-g}\; u^\nu \partial_0 \rho + u^\nu(\partial_0 \sqrt{-g})\rho$$ using $\partial_\lambda \sqrt{-g} = \sqrt{-g}\;\Gamma^\mu_{\mu\lambda}$

$$0 = u^\nu \partial_0(\rho\sqrt{-g}) \longrightarrow \rho = \mathbb{H}(\mathbf{x})/\sqrt{-g}$$

This is the general form of pressure-less dust!

GR

limit: $\mathbb{P}^i = 0$

if $\mathbb{H}(\mathbf{x}) > 0$ everywhere, then can do a spatial coordinate redefinition:

$$\frac{\mathbb{H}(\mathbf{x})}{\sqrt{-g}} \longrightarrow \frac{\overline{\mathbb{H}}}{\sqrt{-g}}$$

$$ds^2 = -dt^2 + a(t)^2(\delta_{ij} + h_{ij})dx^i dx^j \qquad h = h_{ij}\delta^{ij}$$

at linear order $\quad T^{00}_{\rm sh} = (\bar{\rho}_{\rm sh} + \delta\rho_{\rm sh}) = \dfrac{\overline{\mathbb{H}}}{\sqrt{-g}} \simeq \dfrac{\overline{\mathbb{H}}}{a^3}(1 - h/2)$

or $\dot{\delta} = -\dot{h}/2$

GR

limit: $\mathbb{H}(\mathbf{x}) = \overline{\mathbb{H}} + \delta\mathbb{H}(\mathbf{x})$ $\mathbb{P}^i \equiv \delta\mathbb{P}^i$

Perturbative expansion around homogeneity

constraints:
$$\partial_0\mathbb{H} + \partial_i\mathbb{P}^i = 0$$
$$\partial_0(g_{ij}\mathbb{P}^j) = 0$$

at linear order: $\partial_0(a^2\,\delta\mathbb{P}^j) = 0$

$\delta\mathbb{P}^i \sim a^{-2}$ and $\delta\mathbb{P}^i/\sqrt{-g} \sim a^{-5}$

Redshift quickly away outside horizon
May be there in smaller structures

GR

general: $\mathbb{H}(x),\ \mathbb{P}^i(x)$

$$T^{\mu\nu}_{\rm sh} \equiv \frac{1}{\sqrt{-g}} \begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

$$q^\mu = (0, \mathbb{P}^1, \mathbb{P}^2, \mathbb{P}^3)/\sqrt{-g}$$

$$T^{\mu\nu}_{\rm sh} = \rho_{\rm sh}\, u^\mu u^\nu + q^\mu u^\nu + u^\mu q^\nu$$

q^μ produces a ‘heat flux’ that differs from normal particle dynamics could appear in smaller structures

GR

Crazy things — $\mathbb{H}(x)$ could be negative in some places mimicking negative mass particles!

In the early universe, if $2\,|\,\mathbb{P}\,| > \mathbb{H}$, shadow matter violates the NEC

Possibilities of wormholes or bouncing cosmology

GR + EM

$$D_\mu F^{\mu\nu} = (J^\nu + J^\nu_{\rm sh}) \qquad\qquad D_\mu J^\mu_{\rm sh} = 0$$

$$\sqrt{-g}\,J^\mu_{\rm sh} \equiv \{\mathbb{J}(\mathbf{x}), 0, 0, 0\}$$

↑

time independent

$$J^\mu_{\rm sh} = \rho^{ch}_{\rm sh} v^\mu \qquad \rho^{ch}_{\rm sh} = \mathbb{J}/\sqrt{g} \qquad v^\mu = \{1, 0, 0, 0\}$$

synchronous gauge

shadow charge density follows geodesics and does not respond to electromagnetic fields directly

GR + EM

additional modification to Einstein's equations

$$T^{\mu\nu} = \mathscr{E}^{\mu\nu} + T^{\mu\nu}_{matter} \equiv F^{\mu\lambda}F^{\nu}_{\lambda} - \frac{1}{4}g^{\mu\nu}F^{\lambda\sigma}F_{\lambda\sigma} + T^{\mu\nu}_{matter}$$

$$\begin{aligned} \nabla_\mu T^{\mu\nu} &= \nabla_\mu \mathscr{E}^{\mu\nu} + \nabla_\mu T^{\mu\nu}_{matter} \\ &= F^{\nu}{}_{\lambda}(J^\lambda + J^\lambda_{\rm sh}) - F^{\nu}{}_{\lambda}J^\lambda \end{aligned}$$

modified constraints

$$\partial_0 \mathbb{H} + \partial_i \mathbb{P}^i = 0$$

$$\partial_0 (g_{ij}\mathbb{P}^j) = -F_{i0}\,\mathbb{J}$$

Shadow Matter

Loosening constraints of GR allows for source terms that could explain why we think there is dark matter

New source terms for EM produce a charged component of the fake dark matter.
Could effect the CMB, BBN, galactic dynamics, and direct detection.
Challenging pheno (plasma dynamics)

New source terms could violate NEC with no microscopic instabilities.
New phenomena possible

If Shadow Matter is most or all of dark matter, it is in conflict with inflation.
Worth exploring new ideas for initial conditions.

Thank you!