

Functional EFT Methods: New Developments and the Matchete Tool

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UNIVERSITÄT
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FOR FUNDAMENTAL PHYSICS



Swiss National
Science Foundation

EFTs in BSM physics

What they are and how to use them

Direct searches for new physics

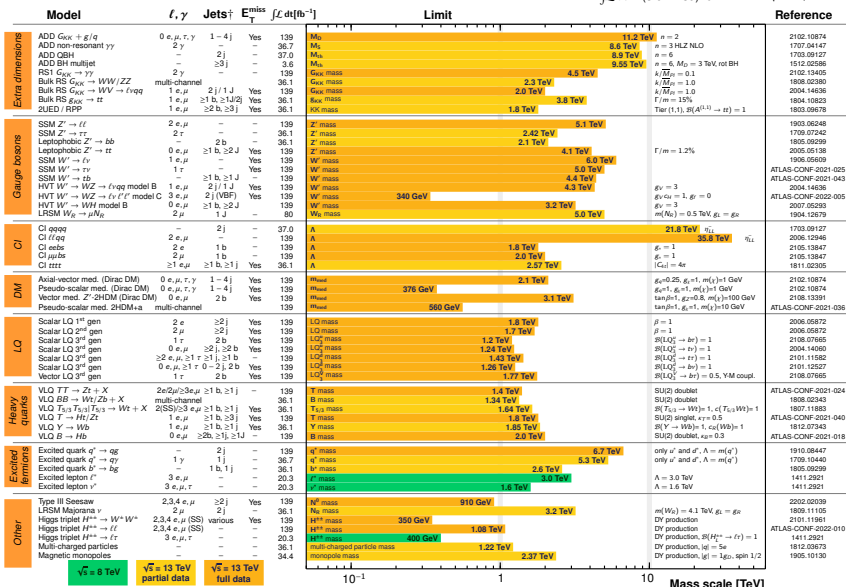
ATLAS Heavy Particle Searches* - 95% CL Upper Exclusion Limits

Status: March 2022

ATLAS Preliminary

$$\int \mathcal{L} dt = (3.6 - 139) \text{ fb}^{-1}$$

$$\sqrt{s} = 8, 13 \text{ TeV}$$



Direct searches for new physics

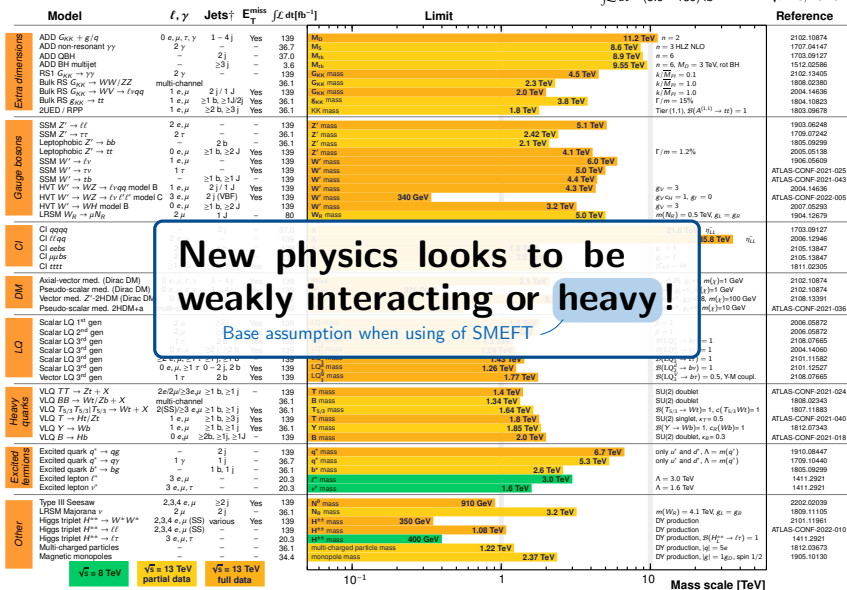
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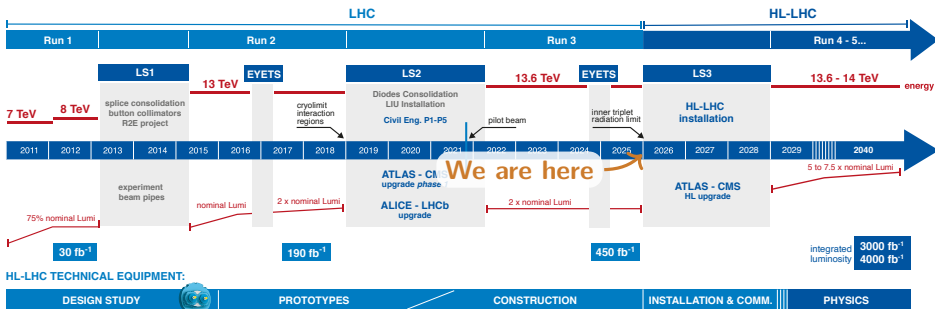
New physics looks to be weakly interacting or heavy!

Base assumption when using of SMEFT

Lots of luminosity

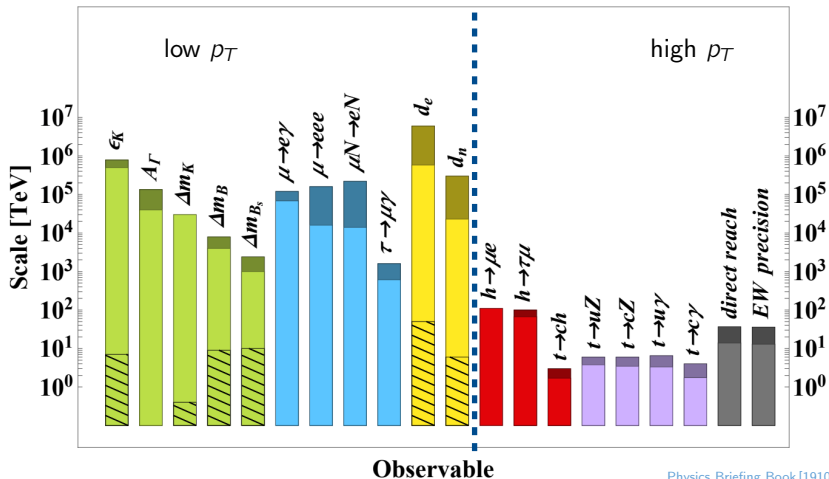


LHC / HL-LHC Plan



- Marginal increase in energy, but $\sim 20\times$ more int. luminosity!
- Rather than looking for resonances, we can look for traces of new physics

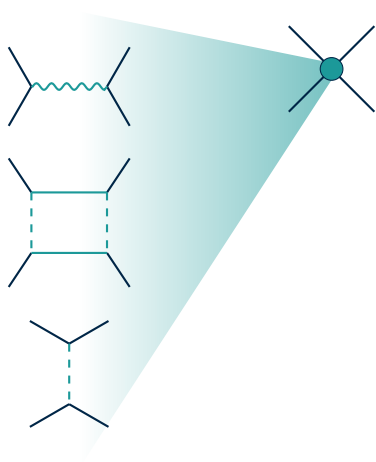
Probing high-scales through precision



Physics Briefing Book [1910.11775]

Effective field theory

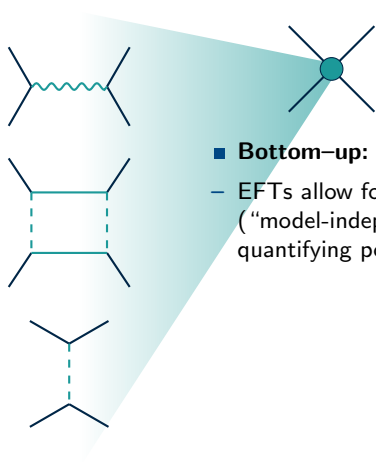
High-energy BSM physics manifests as contact interactions in the SMEFT


$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{d=5} \sum_k \frac{C_{d,k}}{\Lambda^{d-4}} \mathcal{O}_{d,k}$$

UV Physics

Effective field theory

High-energy BSM physics manifests as contact interactions in the SMEFT

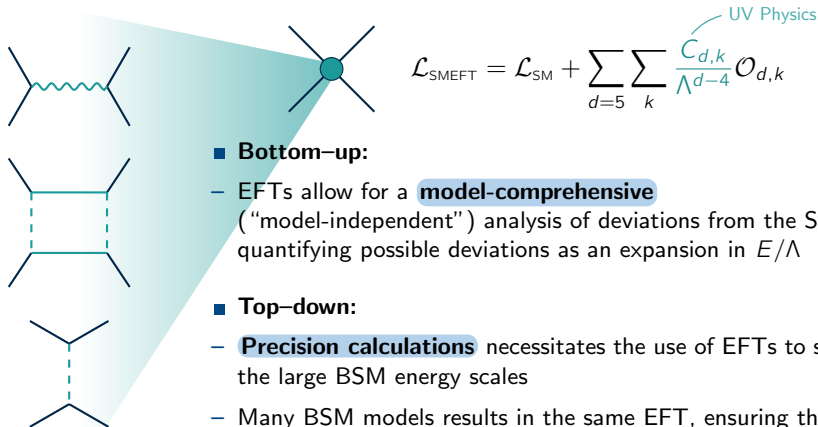

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UV Physics

- **Bottom-up:**
 - EFTs allow for a **model-comprehensive** (“model-independent”) analysis of deviations from the SM, quantifying possible deviations as an expansion in E/Λ

Effective field theory

High-energy BSM physics manifests as contact interactions in the SMEFT



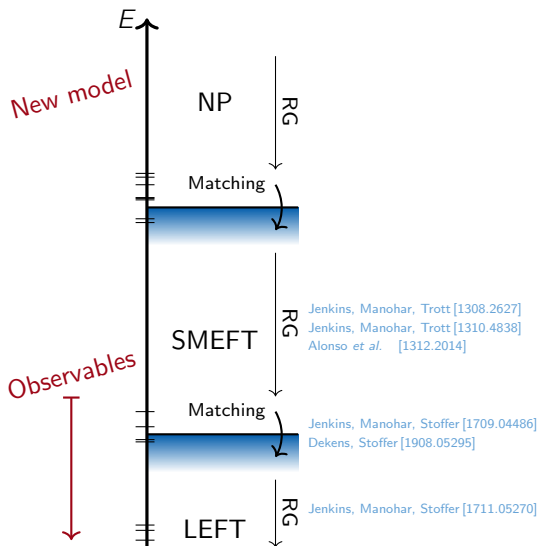
■ Bottom-up:

- EFTs allow for a **model-comprehensive** (“model-independent”) analysis of deviations from the SM, quantifying possible deviations as an expansion in E/Λ

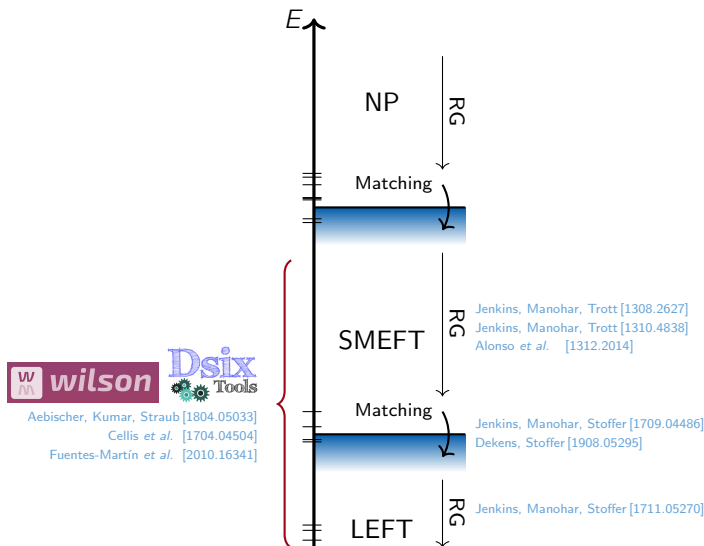
■ Top-down:

- **Precision calculations** necessitates the use of EFTs to separate the large BSM energy scales
- Many BSM models results in the same EFT, ensuring that computation are **reusable**: you only need to compute once in the EFT

Top-down EFT workflow

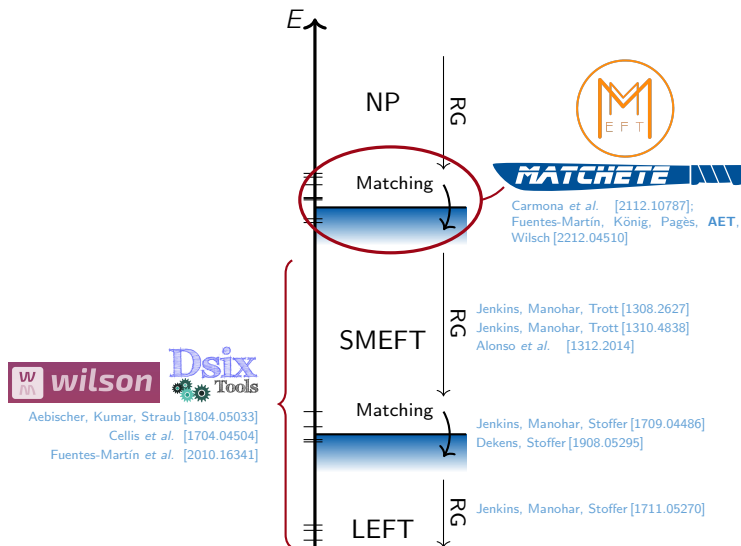


Top-down EFT workflow



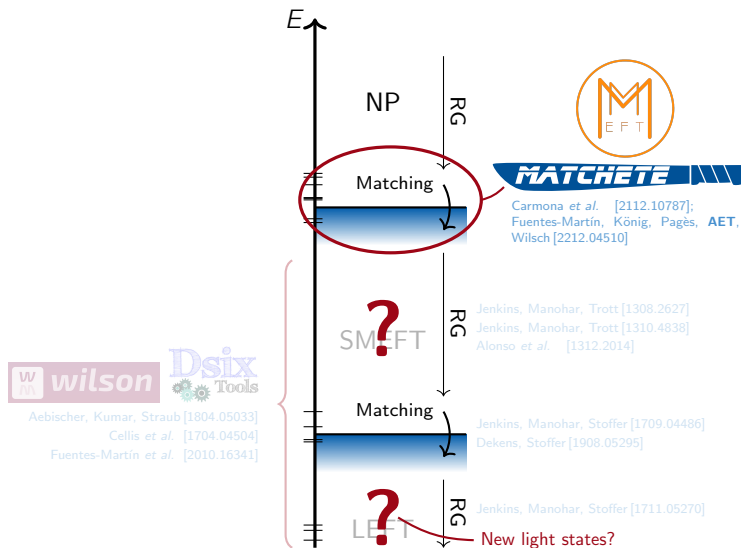
The repetitive nature of EFT computations calls for **automated tools!**

Top-down EFT workflow



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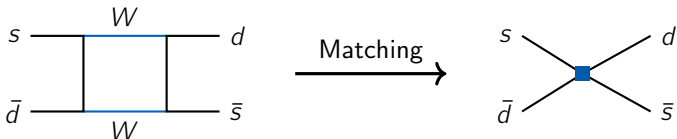


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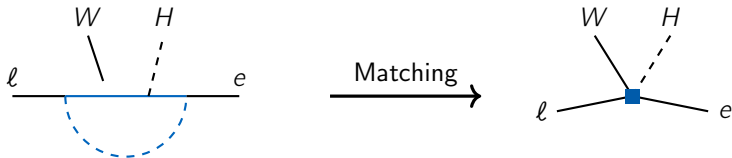
Matching beyond tree level

1-loop matching is often the **leading contribution** from high-scale physics

- FCNCs in the SM

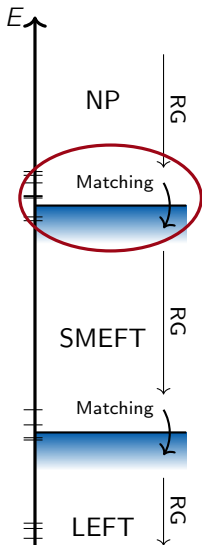


- In BSM models: dipoles, FCNCs, EW precision, ...



Matching weakly coupled theories

$\mathcal{L}_{\text{EFT}}$ should reproduce the physics of $\mathcal{L}_{\text{UV}}$ at energies $E \ll \Lambda$:



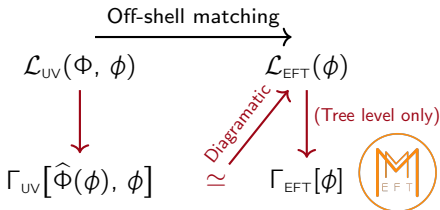
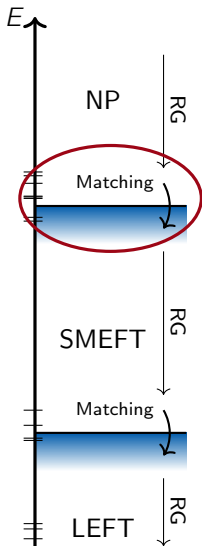
$$\mathcal{L}_{\text{UV}}(\Phi, \phi) \xrightarrow{\text{Off-shell matching}} \mathcal{L}_{\text{EFT}}(\phi)$$

$$\mathcal{L}_{\text{EFT}}(\phi) = \mathcal{L}_{d=4}(\phi) + \sum_{d=5}^{\infty} \sum_{\ell=0}^{\infty} \sum_k \underbrace{\frac{C_{d,k}^{(\ell)}}{(16\pi^2)^\ell \Lambda^{d-4}}}_{\text{double expansion}} \mathcal{O}_{d,k}$$

the goal!

Matching weakly coupled theories

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Hard-region matching formula

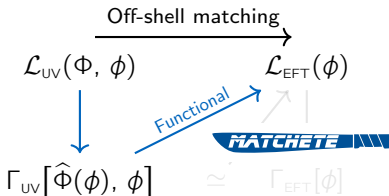
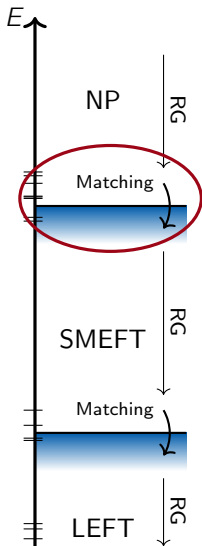
$$S_{\text{EFT}}[\phi] = \Gamma_{\text{UV}}[\hat{\Phi}, \phi]_{\text{hard}}, \quad \frac{\delta \Gamma_{\text{UV}}|_{\text{hard}}}{\delta \Phi}[\hat{\Phi}, \phi] = 0$$

"hard" denotes the part without *any* soft loop momenta (it includes all tree-level contributions)

Fuentes-Martin *et al.* [1607.02142]; Zhang [1610.00710]; Fuentes-Martin, Palavrić, *AET* [2311.13630];
Fuentes-Martín, Moreno, Palavrić, *AET* [2412.12270]

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Hard-region matching formula

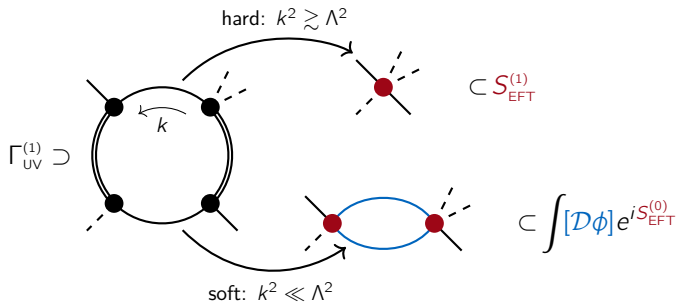
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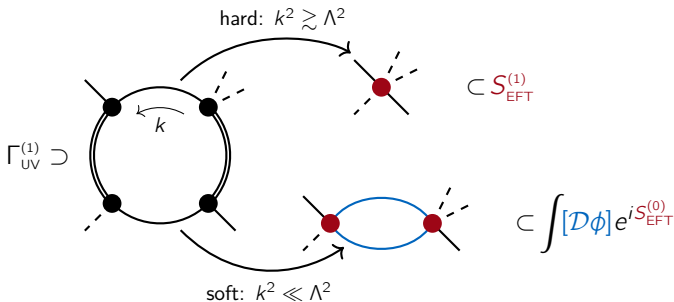
Separation of scales: hard-region matching

Mixed (heavy–light) loop example:



Separation of scales: hard-region matching

Mixed (heavy–light) loop example:



- $\Gamma_{UV}^{(1)}|_{\text{soft}}$: long-distance contributions included in 1-loop matrix elements of tree-level EFT operators

$$\Gamma_{UV}^{(1)}|_{\text{soft}} = \Gamma_{EFT}^{(1)}$$

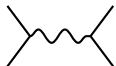
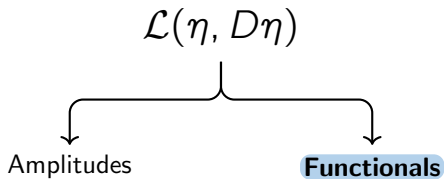
- $\Gamma_{UV}^{(1)}|_{\text{hard}}$: short-distance contributions going into the EFT operators

$$\Gamma_{UV}^{(1)}|_{\text{hard}} = S_{EFT}^{(1)}$$

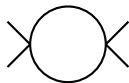
Functional Methods

For matching and RG calculations

What are functional methods anyway?



$$\mathcal{W}[\mathcal{J}]$$



$$\Gamma[\hat{\eta}]$$

Quantum information

Functional methods for counterterm calculations

■ EFT matching—extraction of EFT couplings

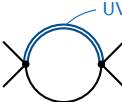
$$\text{UV theory} \sim \int_k \frac{1}{(k^2 - M^2)} \frac{1}{(k+p)^2 - m^2} \sim \frac{-1}{M^2} \int_k \frac{1}{(k+p)^2 - m^2} + \text{EFT vertex}$$

Functional methods @ one-loop order:

Aitchison, Fraser '84; Cheyette '85; Gaillard '86; Dittmaier, Grosse-Knetter [hep-ph/9501285]; Henning, Lu, Murayama [1412.1837], Drozd, Ellis, Quevillon, You [1512.03003]; Aguila, Kunzst, Santiago [1602.00126]; Cohen, Lu, Zhang [2011.02484]... (and many more)

Functional methods for counterterm calculations

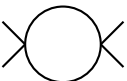
■ EFT matching—extraction of EFT couplings



$$\sim \int_k \frac{1}{(k^2 - M^2)} \frac{1}{(k+p)^2 - m^2} \xrightarrow{\text{Changed high-}k \text{ int}} \frac{-1}{M^2} \int_k \frac{1}{(k+p)^2 - m^2} + \text{EFT vertex}$$

The diagram shows a bubble loop with two external lines. A blue arrow points to it with the label "UV theory". The integral is shown as $\int_k \frac{1}{(k^2 - M^2)} \frac{1}{(k+p)^2 - m^2}$. A blue arrow points from the second denominator to the next term, labeled "Changed high- k int". The next term is $\frac{-1}{M^2} \int_k \frac{1}{(k+p)^2 - m^2}$. A blue arrow points from the final term to a blue circle with four external lines, labeled "EFT vertex".

■ Renormalization/RG functions—extraction of counterterms



$$\sim \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - M^2)^2} \longrightarrow \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - M^2)^2} + \delta\lambda$$

The diagram shows a bubble loop with two external lines. The integral is shown as $\int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - M^2)^2}$. A blue arrow points from the integral to the next term, labeled "Gives RG functions". The next term is $\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - M^2)^2} + \delta\lambda$.

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Functional methods for counterterm calculations

- EFT matching—extraction of EFT couplings

$$\text{UV theory loop} \sim \int_k \frac{1}{(k^2 - M^2)} \frac{1}{(k+p)^2 - m^2} \xrightarrow{\text{Changed high-}k \text{ int}} \frac{-1}{M^2} \int_k \frac{1}{(k+p)^2 - m^2} + \text{EFT vertex}$$

- Renormalization/RG functions—extraction of counterterms

$$\text{Loop diagram} \sim \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - M^2)^2} \longrightarrow \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - M^2)^2} + \delta\lambda$$

Gives RG functions

- Computation of the effective potential: derivative expansion

Functional methods @ one-loop order:

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Functional methods for counterterm calculations

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- Renormalization/RG functions—extraction of counterterms

$$\sim \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - M^2)^2} \longrightarrow \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - M^2)^2} + \delta\lambda \quad \text{Gives RG functions}$$

- Computation of the effective potential: derivative expansion

- Amplitudes? Why not?!

But there might be little point... and some *trouble* $\leftrightarrow$ non-local

Functional methods @ one-loop order:

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Semi-classical approximation

The **quantum effective action** is the generating functional of 1PI functions

$$e^{i\Gamma_{\text{UV}}[\hat{\eta}]} = \int \mathcal{D}\eta \exp(iS_{\text{UV}}[\eta + \hat{\eta}] + iJ_I \eta^I), \quad \frac{\delta \Gamma[\hat{\eta}]}{\delta \hat{\eta}^I} = -J_I$$

$\searrow \int_x J_a(x) \eta^a(x)$

The action is expanded by legs and loop order (including counterterms)

$$S[\eta + \hat{\eta}] = \hat{S} + \sum_{\ell=0}^{\infty} \hbar^{\ell} \left(\eta_I \hat{\mathcal{V}}_I^{(\ell)} + \frac{1}{2} \eta_I \hat{\mathcal{V}}_{IJ}^{(\ell)} \eta_J + \frac{1}{6} \eta_I \eta_J \eta_K \hat{\mathcal{V}}_{IJK}^{(\ell)} + \frac{1}{24} \eta_I \eta_J \eta_K \eta_L \hat{\mathcal{V}}_{IJKL}^{(\ell)} + \dots \right)$$

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With a **saddlepoint approximation**

$$e^{i\Gamma_{\text{UV}}[\hat{\eta}]} = e^{i\hat{S}} \int \mathcal{D}\eta \exp\left(\frac{i}{2} \eta^I \hat{\mathcal{Q}}_{IJ} \eta^J\right) \left[1 + \frac{i}{24} \eta_I \eta_J \eta_K \eta_L \hat{\mathcal{V}}_{IJKL}^{(0)} - \frac{1}{72} (\eta_I \eta_J \eta_K \hat{\mathcal{V}}_{IJK}^{(0)})^2 + \dots \right]$$

$\nearrow \mathcal{V}_{IJ}^{(0)}$

Semi-classical approximation

The loop-expansion is a polynomial of tensor contractions

$$\begin{aligned} \Gamma[\hat{\eta}] = & \hat{S} + \frac{i\hbar}{2} \overbrace{\zeta_{II} (\log \hat{Q})_{II}}^{\text{STr log } Q} + \frac{i\hbar^2}{2} \zeta_{II} \hat{Q}_{IJ}^{-1} \hat{V}_{JI}^{(1)} - \frac{\hbar^2}{8} \hat{Q}_{IJ}^{-1} \hat{Q}_{KL}^{-1} \hat{V}_{IJKL}^{(0)} \\ & + \frac{\hbar^2}{12} \zeta_{IM} \zeta_{JN} \zeta_{IN} \hat{Q}_{LI}^{-1} \hat{Q}_{MJ}^{-1} \hat{Q}_{NK}^{-1} \hat{V}_{IJK}^{(0)} \hat{V}_{LMN}^{(0)} + \mathcal{O}(\hbar^3) \end{aligned}$$

Spin statistics: ± 1

or as supergraphs

$$\Gamma = S + \frac{i}{2} \log \text{[circle]} + \frac{i}{2} \overset{(1)}{\text{[circle with green dot]}} - \frac{1}{8} \text{[two circles connected by a green dot]} + \frac{1}{12} \text{[circle with two blue dots connected by a dashed line]} + \mathcal{O}(\hbar^3)$$

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$$\Gamma = S + \frac{i}{2} \log \text{[circle]} + \frac{i}{2} \text{[circle with top vertex]} - \frac{1}{8} \text{[two circles connected]} + \frac{1}{12} \text{[circle with two vertices]} + \mathcal{O}(\hbar^3)$$

All **vertices and propagators are fully dressed** with background fields

$$Q_{IJ}[\hat{\eta}] = \frac{\delta^2 S}{\delta \eta^I \delta \eta^J}[\hat{\eta}] = \left(\text{[diagram: dashed line with orange wavy lines]} \right)^{-1}$$

$$\mathcal{V}_{IJK}^{(0)}[\hat{\eta}] = \text{[diagram: blue vertex with dashed lines and orange wavy lines]}$$

What does this look like at one-loop order

Locality of the action ensures the form of the 2-point function

$$\mathcal{Q}_{IJ} = \underbrace{Q_{ac}(x, P_x)}_{\text{manifestly covariant}} \delta^c_b(x, y)$$

(Note: In the original image, a blue arrow points from the text iD_μ to the P_x term in the equation.)

What does this look like at one-loop order

Locality of the action ensures the form of the 2-point function

$$\mathcal{Q}_{IJ} = \underbrace{Q_{ac}(x, P_x)^{iD_\mu}}_{\text{manifestly covariant}} \delta^c_b(x, y)$$

$$\mathcal{Q} = \Delta - \mathcal{X}, \quad \Delta(P_x) = \begin{cases} \pm(P_x^2 - m^2) & \text{for scalars (ghosts)} \\ \not{P}_x - m & \text{for fermions} \\ -g_{\mu\nu}(P_x^2 - m^2) & \text{for gauge fields} \end{cases}$$

The **tricky one-loop** effective action

$$\Gamma^{(1)} \supset \frac{i\hbar}{2} \text{Tr} \log \mathcal{Q}$$

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Locality of the action ensures the form of the 2-point function

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The **tricky one-loop** effective action

$$\Gamma^{(1)} \supset \frac{i\hbar}{2} \text{Tr} \log Q = \frac{i\hbar}{2} \underbrace{\text{Tr} \log \Delta}_{p(F_{\mu\nu})} - \frac{i\hbar}{2} \sum_{n=1}^{\infty} \frac{1}{n} \text{Tr} (\Delta^{-1} \mathcal{X})^n \equiv \Gamma_{\log}^{(1)} + \Gamma_{\text{power}}^{(1)}$$

$$\log \text{ (dashed circle) } = \log \text{ (solid circle) } - \sum_n \frac{1}{n} \text{ (green loop with four dots) }$$

A master formula for the one-loop effective action

$$\Gamma_{\text{power}}^{(1)} = -\frac{i\hbar}{2} \sum_{n=1}^{\infty} \frac{1}{n} \int_{x,y} \delta^b_a(y,x) \left(\Delta^{-1}(x, P_x) X(x, P_x) \right)^n_{ac} \delta^c_b(x,y)$$

func. Tr
($\Delta^{-1} X$)ⁿ

A master formula for the one-loop effective action

$$\Gamma_{\text{power}}^{(1)} = -\frac{i\hbar}{2} \sum_{n=1}^{\infty} \frac{1}{n} \int_{x,y} \delta^b{}_a(y,x) \left(\Delta^{-1}(x, P_x) X(x, P_x) \right)_{ac}^n \delta^c{}_b(x,y)$$

And in momentum space

$$\Gamma_{\text{power}}^{(1)} = -\frac{i\hbar}{2} \sum_{n=1}^{\infty} \frac{1}{n} \int_{x,y} \int_k e^{-ik \cdot (x-y)} \delta^b{}_a(y,x) \left(\Delta^{-1}(x, P_x + k) X(x, P_x + k) \right)_{ac}^n U^c{}_b(x,y)$$

Wilson line

A master formula for the one-loop effective action

$$\Gamma_{\text{power}}^{(1)} = -\frac{i\hbar}{2} \sum_{n=1}^{\infty} \frac{1}{n} \int_{x,y} \delta^b_a(y,x) (\Delta^{-1}(x, P_x) X(x, P_x))_{ac}^n \delta^c_b(x,y)$$

And in momentum space

$$\begin{aligned} \Gamma_{\text{power}}^{(1)} &= -\frac{i\hbar}{2} \sum_{n=1}^{\infty} \frac{1}{n} \int_{x,y} \int_k e^{-ik \cdot (x-y)} \delta^b_a(y,x) (\Delta^{-1}(x, P_x + k) X(x, P_x + k))_{ac}^n U^c_b(x,y) \\ &= -\frac{i\hbar}{2} \sum_{n=1}^{\infty} \frac{1}{n} \int_x \int_k (\Delta^{-1}(x, k + P_x) X(x, k + P_x))_{ab}^n U^b_a(x,y) \Big|_{y=x} \end{aligned}$$

Wilson line

The master formula for the one-loop quantum effective action

- is **manifestly gauge invariant**, simplifying calculations
- is unwieldy and non-local in the general case
- matching (and RG) calculations care only about a local piece:

$$\mathbf{S}_{\text{EFT}} = \mathbf{R}_{\text{hard}} \mathbf{\Gamma}_{\text{UV}} \quad (\text{and } \delta S = \bar{\mathbf{R}}^* \mathbf{\Gamma})$$

Evaluating the one-loop effective action

$$\Gamma_{\text{power}}^{(1)} = -\frac{i\hbar}{2} \sum_{n=1}^{\infty} \frac{1}{n} \int_{x,y} \delta^b_a(y,x) (\Delta^{-1}(x, P_x) X(x, P_x))_{ac}^n \delta^c_b(x,y)$$

And in momentum space

$$\begin{aligned} \Gamma_{\text{power}}^{(1)} &= -\frac{i\hbar}{2} \sum_{n=1}^{\infty} \frac{1}{n} \int_{x,y} \int_k e^{-ik \cdot (x-y)} \delta^b_a(y,x) (\Delta^{-1}(x, P_x+k) X(x, P_x+k))_{ac}^n U^c_b(x,y) \\ &= -\frac{i\hbar}{2} \sum_{n=1}^{\infty} \frac{1}{n} \int_x \int_k (\Delta^{-1}(x, k+P_x) X(x, k+P_x))_{ab}^n U^b_a(x,y) \Big|_{y=x} \end{aligned}$$

RG calculations (with $\mathbf{R}_{\text{hard}}$ or $\bar{\mathbf{R}}^*$) call for expansion of the propagators:

- Typical propagator: $\frac{1}{(k+P_x)^2 - m^2} = \frac{1}{k^2} - \frac{2k \cdot P_x}{(k^2 - m^2)^2} + \dots$
- Typical X -term: $X_{\phi A_\mu} \supset 2g\phi(P_x^\mu + k^\mu)$
- Closed(-ish) formula $P_x^{\mu_1} \dots P_x^{\mu_m} U(x,y) \Big|_{y=x} = p_m(P_x, G_{\mu\nu})$ polynomial

Evaluating the one-loop effective action

$$\Gamma_{\text{power}}^{(1)} = -\frac{i\hbar}{2} \sum_{n=1}^{\infty} \frac{1}{n} \int_{x,y} \delta^b{}_a(y,x) \left(\Delta^{-1}(x, P_x) X(x, P_x) \right)_{ac}^n \delta^c{}_b(x,y)$$

And in momentum space

$$\begin{aligned} \Gamma_{\text{power}}^{(1)} &= -\frac{i\hbar}{2} \sum_{n=1}^{\infty} \frac{1}{n} \int_{x,y} \int_k e^{-ik \cdot (x-y)} \delta^b{}_a(y,x) \left(\Delta^{-1}(x, P_x+k) X(x, P_x+k) \right)_{ac}^n U^c{}_b(x,y) \\ &= -\frac{i\hbar}{2} \sum_{n=1}^{\infty} \frac{1}{n} \int_x \int_k \left(\Delta^{-1}(x, k+P_x) X(x, k+P_x) \right)_{ab}^n \left. U^b{}_a(x,y) \right|_{y=x} \end{aligned}$$

RG calculations (with $\mathbf{R}_{\text{hard}}$ or $\bar{\mathbf{R}}^*$) call for expansion of the propagators:

- Typical propagator: $\frac{1}{(k+P_x)^2 - m^2} = \frac{1}{k^2} - \frac{2k \cdot P_x}{(k^2 - m^2)^2} + \dots$
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Summary: to evaluate simply **tensor-contract, expand, loop-integrate**

Two-loop formulas

2-loop contributions introduce 3- and 4-point functional tensors, e.g.,

$$\mathcal{V}_{IJK}^{(0)} \equiv V_{ade}(x, P_y, P_z) \delta^d_b(x, y) \delta^e_c(x, z) \quad \text{— Locality of the action}$$

$$V_{abc}(x, P_y, P_z) = \sum_{m,n=0} V_{abc}^{(\underline{m}, \underline{n})}(x) P_y^{\underline{m}} P_z^{\underline{n}} \equiv \sum_{m,n=0} V_{abc}^{(\mu_1 \dots \mu_m, \nu_1 \dots \nu_n)}(x) P_y^{\mu_1} \dots P_y^{\mu_m} P_z^{\nu_1} \dots P_z^{\nu_n}$$

Looks more scary than it is

Two-loop formulas

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The remaining **basic ingredients repeat** beyond 1-loop order

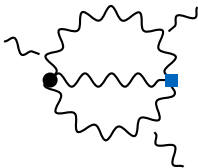
$$\begin{aligned} \text{Diagram: a dashed circle with two dots on a horizontal line} &= \sum_{m,n,m',n'} (-1)^{m+n} \int_x \int_{k\ell}^{\text{2-loop integral}} V_{abc}^{(\underline{m}, \underline{n})}(x) Q_{aa'}^{-1}(y, P_y - k - \ell) V_{a'b'c'}^{(\underline{m}', \underline{n}')} (y) \\ &\quad \times [(P_x + k) \overleftarrow{Q}_{be}^m(x, P_x + k) (P_x + k)^{\underline{m}'} U^e{}_{b'}(x, y)] \\ &\quad \times [(P_x + \ell) \overleftarrow{Q}_{cf}^n(x, P_x + \ell) (P_x + \ell)^{\underline{n}'} U^f{}_{c'}(x, y)] \Big|_{y=x} \end{aligned}$$

$$Q^{-1} = \Delta^{-1} + \Delta^{-1} \chi \Delta^{-1} + \Delta^{-1} \chi \Delta^{-1} \chi \Delta^{-1} + \dots$$

The figure-8 and the counterterm topologies are (even) simpler

RGs in the bosonic SMEFT

Private version
of Matchete



```
L = FreeLag[] + cGt[] Gtilde;
L // NiceForm
```

iceForm=

$$-\frac{1}{4} G^{\mu\nu X^2} + (D_\mu \bar{c}_6^X \cdot D_\mu c_6^X) + g G^{\mu\nu} (\bar{c}_6^X \cdot D_\mu c_6^Z) f^{XYZ} + \frac{1}{6} cGt G^{\mu\rho\gamma} G^{\nu\rho Z} G^{\alpha\kappa X} f^{XYZ} \epsilon^{\mu\nu\alpha\kappa}$$

```
ctlag = CountertermLagrangian[L, EFTOrder -> 6] // EchoTiming;
ctlag // NiceForm
```

» Added new CG cg1 with indices {SU3c[adj], SU3c[adj], SU3c[adj], SU3c[adj]}

» Added new CG cg2 with indices {SU3c[adj], SU3c[adj], SU3c[adj], SU3c[adj], SU3c[adj]}

 13 695.1 — Mathematica, single core

/NiceForm=

$$-\frac{11}{4} \hbar \frac{1}{\epsilon} g^2 G^{\mu\nu X^2} - \frac{51}{4} \hbar^2 \frac{1}{\epsilon} g^4 G^{\mu\nu X^2} - 3 \hbar \frac{1}{\epsilon} cGt g^2 G^{\mu\nu X} G^{\rho\sigma\gamma} G^{\alpha\kappa Z} f^{XYZ} \epsilon^{\mu\nu\rho\kappa} - \\ \frac{21}{2} \hbar^2 \frac{1}{\epsilon^2} cGt g^4 G^{\mu\nu X} G^{\rho\sigma\gamma} G^{\alpha\kappa Z} f^{XYZ} \epsilon^{\mu\nu\rho\kappa} - \frac{425}{24} \hbar^2 \frac{1}{\epsilon} cGt g^4 G^{\mu\nu X} G^{\rho\sigma\gamma} G^{\alpha\kappa Z} f^{XYZ} \epsilon^{\mu\nu\rho\kappa}$$

Born, Fuentes-Martin, Kvedaraitė, AET [2410.07320]

New challenges in full SMEFT

Born, Fuentes-Martín, AET [2601.xxxxx]

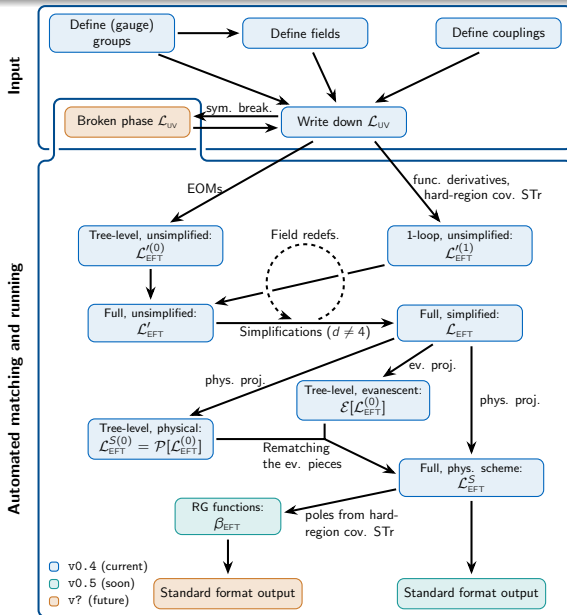
- Evanescent operators (leveraging Matchete features)
- Non-trivial interplay with evanescent operators and R^* -method
- Treatment of γ_5 : non-cyclicity of the anti-commuting γ_5

$$\begin{aligned}
 \beta_{C_{\text{eff}}}^{(2)} = & \frac{1}{12} g_L^2 (22 Y_e^{\text{pr}} T_{\text{CH}} - 243 (C_{\text{LHHH}}^\dagger C_{\text{HHHH}} Y_e)^{\text{pr}}) + \frac{1}{12} g_Y^2 (38 Y_e^{\text{pr}} T_{\text{CH}} + 51 (C_{\text{LHHH}}^\dagger C_{\text{HHHH}} Y_e)^{\text{pr}}) + \frac{1}{2} (7 (C_{\text{LHHH}}^\dagger C_{\text{HHHH}} Y_e Y_e^\dagger Y_e)^{\text{pr}} \\
 & + 7 (Y_e Y_e^\dagger C_{\text{LHHH}}^\dagger C_{\text{HHHH}} Y_e)^{\text{pr}} + 30 (Y_e Y_e^\dagger Y_e)^{\text{pr}} T_{\text{CH}} - 7 (C_{\text{LHHH}}^\dagger Y_e^\dagger Y_e C_{\text{HHHH}} Y_e)^{\text{pr}} - 10 T_2 (C_{\text{LHHH}}^\dagger C_{\text{HHHH}} Y_e)^{\text{pr}}) \\
 & - \lambda (5 (C_{\text{LHHH}}^\dagger C_{\text{HHHH}} Y_e)^{\text{pr}} + 24 Y_e^{\text{pr}} T_{\text{CH}}) - 6 Y_e^{\text{pr}} \text{Tr} [C_{\text{LHHH}}^\dagger C_{\text{HHHH}} Y_e Y_e^\dagger] + \left\{ 24 \lambda Y_e^{\text{pr}} - 48 (Y_e Y_e^\dagger Y_e)^{\text{pr}} \right\} C_{\text{CH}} + \left\{ \frac{1}{2} \lambda^2 Y_e^{\text{pr}} + \frac{47}{54} g_Y^2 g_L^2 Y_e^{\text{pr}} \right. \\
 & + \lambda \left(\frac{9}{4} g_L^2 Y_e^{\text{pr}} + \frac{163}{36} g_Y^2 Y_e^{\text{pr}} + 5 T_2 Y_e^{\text{pr}} - \frac{53}{2} (Y_e Y_e^\dagger Y_e)^{\text{pr}} \right) + g_Y^2 \left(\frac{1}{12} T_2 Y_e^{\text{pr}} + \frac{71}{16} (Y_e Y_e^\dagger Y_e)^{\text{pr}} \right) + \frac{25}{8} (Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e)^{\text{pr}} + \frac{45}{2} Y_e^{\text{pr}} \text{Tr} [Y_d^\dagger Y_d Y_u^\dagger Y_u] \\
 & - g_L^2 \left(\frac{17}{12} T_2 Y_e^{\text{pr}} + \frac{23}{16} (Y_e Y_e^\dagger Y_e)^{\text{pr}} \right) - \frac{74}{27} g_Y^4 Y_e^{\text{pr}} - \frac{71}{24} g_L^4 Y_e^{\text{pr}} - \frac{13}{4} T_4 Y_e^{\text{pr}} - \frac{9}{2} T_2 (Y_e Y_e^\dagger Y_e)^{\text{pr}} \left. \right\} C_{\text{HD}}^{(0)} + \left\{ \lambda^2 Y_e^{\text{pr}} + \lambda \left(\frac{287}{36} g_Y^2 Y_e^{\text{pr}} \right. \right. \\
 & + \frac{51}{4} g_L^2 Y_e^{\text{pr}} - 31 (Y_e Y_e^\dagger Y_e)^{\text{pr}} \left. \right) + g_Y^2 \left(\frac{1}{6} T_2 Y_e^{\text{pr}} + \frac{17}{4} (Y_e Y_e^\dagger Y_e)^{\text{pr}} \right) + g_L^2 \left(\frac{3}{2} T_2 Y_e^{\text{pr}} + \frac{43}{8} (Y_e Y_e^\dagger Y_e)^{\text{pr}} \right) + \frac{133}{96} g_L^2 Y_e^{\text{pr}} + 2 (Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e)^{\text{pr}} \\
 & - \frac{7}{4} T_4 Y_e^{\text{pr}} - \frac{1955}{864} g_Y^4 Y_e^{\text{pr}} - 3 T_2 (Y_e Y_e^\dagger Y_e)^{\text{pr}} - \frac{2143}{432} g_Y^2 g_L^2 Y_e^{\text{pr}} - \frac{27}{2} Y_e^{\text{pr}} \text{Tr} [Y_d^\dagger Y_d Y_u^\dagger Y_u] \left. \right\} C_{\text{HD}}^{(1)} + \left\{ g_Y^4 \left(\frac{39i}{2} Y_e^{\text{pr}} \text{Tr} [Y_d^\dagger Y_d] - \frac{79}{6} Y_e^{\text{pr}} \text{Tr} [Y_u^\dagger Y_u] \right. \right. \\
 & - \frac{43}{2} Y_e^{\text{pr}} \text{Tr} [Y_e^\dagger Y_e] - \frac{375}{4} (Y_e Y_e^\dagger Y_e)^{\text{pr}} \left. \right) + \frac{49}{2} \lambda g_Y^4 Y_e^{\text{pr}} - \frac{49}{3} g_Y^4 g_L^2 Y_e^{\text{pr}} - 242 g_Y^6 Y_e^{\text{pr}} \left. \right\} C_{\text{HB}} + \left\{ g_Y^4 \left(\frac{39i}{2} Y_e^{\text{pr}} \text{Tr} [Y_e^\dagger Y_e] + \frac{477i}{4} (Y_e Y_e^\dagger Y_e)^{\text{pr}} \right. \right. \\
 & - \frac{3i}{2} Y_e^{\text{pr}} \text{Tr} [Y_d^\dagger Y_d] - \frac{33i}{2} Y_e^{\text{pr}} \text{Tr} [Y_u^\dagger Y_u] \left. \right) + \frac{153i}{2} \lambda g_Y^4 Y_e^{\text{pr}} + \frac{1469i}{12} g_Y^6 Y_e^{\text{pr}} - \frac{21i}{4} g_Y^4 g_L^2 Y_e^{\text{pr}} \left. \right\} C_{\text{HB}} + \left\{ g_L^4 \left(\frac{3}{2} T_2 Y_e^{\text{pr}} - \frac{243}{4} (Y_e Y_e^\dagger Y_e)^{\text{pr}} \right. \right. \\
 & + \frac{77}{3} g_Y^2 Y_e^{\text{pr}} + \frac{147}{2} \lambda g_L^4 Y_e^{\text{pr}} - 19 g_Y^2 g_L^2 Y_e^{\text{pr}} \left. \right\} C_{\text{HW}} + \left\{ g_L^4 \left(\frac{9i}{2} Y_e^{\text{pr}} \text{Tr} [Y_e^\dagger Y_e] + \frac{27i}{2} Y_e^{\text{pr}} \text{Tr} [Y_d^\dagger Y_d] + \frac{213i}{4} (Y_e Y_e^\dagger Y_e)^{\text{pr}} - \frac{27i}{2} Y_e^{\text{pr}} \text{Tr} [Y_u^\dagger Y_u] \right. \right. \\
 & + \frac{51i}{2} \lambda g_L^4 Y_e^{\text{pr}} - \frac{21i}{4} g_Y^2 g_L^2 Y_e^{\text{pr}} - \frac{341i}{4} g_L^6 Y_e^{\text{pr}} \left. \right\} C_{\text{HW}} - g_s^4 \left\{ 176 Y_e^{\text{pr}} \text{Tr} [Y_d^\dagger Y_d] + 176 Y_e^{\text{pr}} \text{Tr} [Y_u^\dagger Y_u] \right\} C_{\text{HG}} + g_s^4 \left\{ 144 i Y_e^{\text{pr}} \text{Tr} [Y_d^\dagger Y_d] \right. \\
 & - 144 i Y_e^{\text{pr}} \text{Tr} [Y_u^\dagger Y_u] \left. \right\} C_{\text{HG}} + \left\{ g_Y^2 g_L^2 \left(\frac{23}{4} Y_e^{\text{pr}} \text{Tr} [Y_e^\dagger Y_e] + \frac{25}{4} Y_e^{\text{pr}} \text{Tr} [Y_d^\dagger Y_d] + \frac{33}{4} (Y_e Y_e^\dagger Y_e)^{\text{pr}} + \frac{47}{4} Y_e^{\text{pr}} \text{Tr} [Y_u^\dagger Y_u] \right) + \frac{23}{6} g_Y^2 g_L^4 Y_e^{\text{pr}} \right. \\
 & + \frac{63}{4} \lambda g_Y^2 g_L^2 Y_e^{\text{pr}} - \frac{181}{12} g_Y^4 g_L^2 Y_e^{\text{pr}} \left. \right\} C_{\text{HWB}} + \left\{ g_Y^2 g_L^2 \left(\frac{27i}{4} Y_e^{\text{pr}} \text{Tr} [Y_u^\dagger Y_u] - \frac{9i}{4} Y_e^{\text{pr}} \text{Tr} [Y_d^\dagger Y_d] - \frac{15i}{4} Y_e^{\text{pr}} \text{Tr} [Y_e^\dagger Y_e] - 12i (Y_e Y_e^\dagger Y_e)^{\text{pr}} \right. \right. \\
 & + \frac{11i}{3} g_Y^4 g_L^2 Y_e^{\text{pr}} - \frac{13i}{12} g_Y^2 g_L^4 Y_e^{\text{pr}} - \frac{81i}{4} \lambda g_Y^2 g_L^2 Y_e^{\text{pr}} \left. \right\} C_{\text{HWB}} + \left\{ g_L^6 \left(3 T_2 Y_e^{\text{pr}} - \frac{9}{4} (Y_e Y_e^\dagger Y_e)^{\text{pr}} \right) + \frac{103}{12} g_L^2 Y_e^{\text{pr}} - 6 \lambda g_L^6 Y_e^{\text{pr}} \right.
 \end{aligned}$$

Preliminary

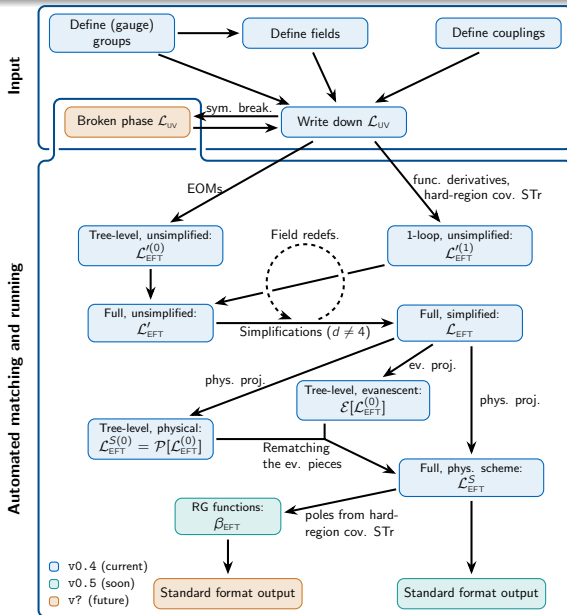


To make your way through the BSM jungle



- **Matchete v0.4** is a Mathematica package
- Matching of **any model** with heavy scalars/fermions
- Simple and intuitive input/output
- Handles arbitrary* group theory
- Simplifies to EFT basis
- Handles of evanescent contributions

Fuentes-Martín, König, Pagès, AET, Wilsch [2212.04510]



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Future plans:

- Interface with EFT tool chain
- 1-loop RG computations
- SSB and heavy vectors
- Many more...

Example: SM + Vector-like lepton

Setup

SM Lagrangian

```
In[3]:= LSM = LoadModel["SM"];
```

Define new field

```
In[4]:= DefineField[EE, Fermion, Charges -> {U1Y[-1]}, Mass -> {Heavy, ME}]
```

Define new coupling

```
In[5]:= DefineCoupling[yE, EFTOrder -> 0, Indices -> {Flavor}]
```

Write interactions

```
In[6]:= Lint = -yE[p] × Bar@l[i, p] ** PR ** EE[] × H[i] // PlusHc;
Lint // NiceForm
```

Out[7]//NiceForm=

$$-\bar{y}E^p H_i (EE \cdot P_L \cdot l^{1p}) - yE^p H^i (l_1^p \cdot P_R \cdot EE)$$

Define full UV Lagrangian

```
In[8]:= LUV = LSM + FreeLag[EE] + Lint;
LUV // NiceForm
```

Out[9]//NiceForm=

$$\begin{aligned} & -\frac{1}{4} B^{\mu\nu 2} - \frac{1}{4} G^{\mu\nu A 2} - \frac{1}{4} W^{\mu\nu I 2} + D_\mu H_i D_\mu H^i + \mu^2 H_i H^i + i (\bar{d}_a^p \cdot \gamma_\mu P_R \cdot D_\mu d^{ap}) + i (\bar{e}^p \cdot \gamma_\mu P_R \cdot D_\mu e^p) + \\ & i (EE \cdot \gamma_\mu \cdot D_\mu EE) - ME (EE \cdot EE) + i (\bar{l}_1^p \cdot \gamma_\mu P_L \cdot D_\mu l^{1p}) + i (\bar{q}_{a1}^p \cdot \gamma_\mu P_L \cdot D_\mu q^{a1p}) + i (\bar{u}_a^p \cdot \gamma_\mu P_R \cdot D_\mu u^{ap}) - \\ & \frac{1}{2} \lambda H_i H_j H^i H^j - \bar{V} d^{pr} H_i (\bar{d}_a^r \cdot P_L \cdot q^{a1p}) - \bar{V} e^{pr} H_i (\bar{e}^r \cdot P_L \cdot l^{1p}) - y d^{pr} H^i (\bar{l}_1^p \cdot P_R \cdot e^r) - y d^{pr} H^i (\bar{q}_{a1}^p \cdot P_R \cdot d^{ar}) - \\ & y u^{pr} H_i (\bar{q}_{a1}^p \cdot P_R \cdot u^{ar}) \bar{e}^{j1} - y \bar{u}^{pr} H^j (\bar{u}_a^r \cdot P_L \cdot q^{a1p}) \bar{e}_{i1} - \bar{y} E^p H_i (EE \cdot P_L \cdot l^{1p}) - y E^p H^i (l_1^p \cdot P_R \cdot EE) \end{aligned}$$

Example: SM + Vector-like lepton

Main matching routine

```
In[9]:= LEFT = Match[LUV, LoopOrder -> 1, EFTOrder -> 6] /. e^-1 -> 0;
```

Simplification to on-shell basis

```
In[10]:= LEFTOnShell = LEFT // EOMSimplify;
Length%
```

» The Lagrangian contains terms of lower power than dimension 4. Defining effective couplings and assuming these terms to be dimension 4. Use 'PrintEffectiveCouplings' and 'ReplaceEffectiveCouplings' to recover explicit expressions.

» Added new CG cgl with indices {Bar[SU2L[fund]], SU2L[adj], Bar[SU2L[fund]]}

```
Out[11]:= 66
```

Select Higgs-lepton current operator

```
In[12]:= SelectOperatorClass[LEFTOnShell, {e, Bar@e, H, Bar@H}, 1] // GreensSimplify // NiceForm
```

```
Out[12]//NiceForm=
```

$$\frac{i}{360} \hbar \frac{1}{ME^2} \left(48 g Y^4 \delta^{pr} + 5 \bar{Y} E^5 \left(3 Y E^t \bar{Y} e^{tr} Y e^{sp} \left(1 + 6 \text{Log} \left[\frac{\mu^2}{ME^2} \right] \right) - 2 Y E^5 g V^2 \left(13 + 6 \text{Log} \left[\frac{\mu^2}{ME^2} \right] \right) \delta^{pr} \right) \right. \\ \left. - D_\mu \bar{H}_1 H^1 (e^r \cdot \gamma_\mu P_R \cdot e^p) + \bar{H}_1 D_\mu H^1 (e^r \cdot \gamma_\mu P_R \cdot e^p) \right)$$

$$Q_{He}^{pr} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{e}_p \gamma^\mu e_r)$$

Example: SM + Vector-like lepton

LEFTOnShell // NiceForm

reForm

$$\begin{aligned}
 & -\frac{1}{4} G^{\nu A 2} - \frac{1}{4} W^{\nu I 2} + \left(-\frac{1}{4} - \frac{1}{3} h g Y^2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) B^{\nu I 2} + D_\mu H_1 D_\mu H_1 + \left(C_{H2} + \frac{1}{6} h \bar{Y}^E y^E P_{C_H 2} \frac{1}{\text{ME}^2} \left(2 C_{H2} - 3 \text{ME}^2 \left(1 + 2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) H_1 H^1 + i \left(\bar{d}_a^\dagger \cdot \gamma_\mu P_R \cdot D_\mu d^{aP} \right) \delta^{PR} + i \left(e^\dagger \cdot \gamma_\mu P_R \cdot D_\mu e^P \right) \delta^{PR} + \\
 & i \left(\bar{l}_i^\dagger \cdot \gamma_\mu P_L \cdot D_\mu l^{iP} \right) \delta^{Pi} + i \left(q_{a1}^\dagger \cdot \gamma_\mu P_L \cdot D_\mu q^{a1P} \right) \delta^{P1} + i \left(u_{a1}^\dagger \cdot \gamma_\mu P_R \cdot D_\mu u^{a1P} \right) \delta^{P1} + \left(-\frac{1}{2} \lambda + h \left(-\frac{1}{2} \bar{Y}^E P \left(4 y^E \bar{Y} e^{TS} Y e^{PS} \left(1 + \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) - y^E P \left(-2 \bar{Y} e^\dagger y^E \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] + \lambda \left(1 + 2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) \right. \right. \\
 & \left. \left. - \frac{1}{180} C_{H2} \frac{1}{\text{ME}^2} \left(12 g Y^4 - 5 \bar{Y}^E y^E P g Y^2 \left(13 + 6 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) + 5 \bar{Y}^E P \left(-12 \left(\bar{Y} e^\dagger y^E P y^E + 6 y^E \bar{Y} e^{TS} Y e^{PS} - 2 y^E P \lambda \right) + y^E P g L^2 \left(5 + 6 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) \right) \right) \\
 & H_1 H_2 H^1 H^1 + \left(-\bar{Y} d^{PR} + \frac{1}{12} h \bar{Y} e^S Y e^S \bar{Y} d^{PR} \frac{1}{\text{ME}^2} \left(-4 C_{H2} + 3 \text{ME}^2 \left(1 + 2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) H_1 \left(\bar{d}_a^\dagger \cdot P_L \cdot q^{a1P} \right) + \\
 & \left(-\bar{Y} e^{PR} + \frac{1}{24} h y^E \frac{1}{\text{ME}^2} \left(-3 \bar{Y}^E P \bar{Y} e^{SR} \left(2 C_{H2} - \text{ME}^2 \right) \left(3 + 2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) + 2 \bar{Y} e^S \bar{Y} e^{PR} \left(-4 C_{H2} + 3 \text{ME}^2 \left(1 + 2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) \right) H_1 \left(e^\dagger \cdot P_L \cdot l^{iP} \right) + \\
 & \left(-\bar{Y} e^{RP} + \frac{1}{24} h \bar{Y} e^S \frac{1}{\text{ME}^2} \left(3 \text{ME}^2 \left(2 y^E S \bar{Y} e^{RP} \left(1 + 2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) + y^E \bar{Y} e^{SP} \left(3 + 2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) - 2 C_{H2} \left(4 y^E S \bar{Y} e^{RP} + 3 y^E \bar{Y} e^{SP} \left(3 + 2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) \right) H^1 \left(\bar{l}_i^\dagger \cdot P_R \cdot e^P \right) + \\
 & \left(-\bar{Y} d^{RP} + \frac{1}{12} h \bar{Y} e^S Y e^S \bar{Y} d^{RP} \frac{1}{\text{ME}^2} \left(-4 C_{H2} + 3 \text{ME}^2 \left(1 + 2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) H^1 \left(q_{a1}^\dagger \cdot P_R \cdot d^{aP} \right) + \\
 & \left(-\bar{Y} u^{RP} + \frac{1}{12} h \bar{Y} e^S Y e^S \bar{Y} u^{RP} \frac{1}{\text{ME}^2} \left(-4 C_{H2} + 3 \text{ME}^2 \left(1 + 2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) H_1 \left(q_{a3}^\dagger \cdot P_R \cdot u^{aP} \right) \varepsilon^{J1} + \\
 & \left(-\bar{Y} u^{RP} + \frac{1}{12} h \bar{Y} e^S Y e^S \bar{Y} u^{RP} \frac{1}{\text{ME}^2} \left(-4 C_{H2} + 3 \text{ME}^2 \left(1 + 2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) H^1 \left(u_{a1}^\dagger \cdot P_L \cdot q^{a1P} \right) \bar{\varepsilon}_{ij} + \\
 & \frac{1}{180} h \frac{1}{\text{ME}^2} \left(12 \lambda g Y^4 + 5 \bar{Y}^E P \left(12 \bar{Y} e^\dagger y^E P \left(\bar{Y} e^\dagger y^E y^E + 6 y^E \bar{Y} e^{TS} Y e^{TS} - y^E \lambda \right) - 72 y^E \bar{Y} e^{TS} \left(Y e^{PS} \lambda + \bar{Y} e^{TS} Y e^{PP} Y e^{TS} \left(1 + \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) + y^E P \left(12 \lambda + g L^2 \left(5 + 6 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) - g Y^2 \left(13 + 6 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) \right) \right) \\
 & H_1 H_2 H_3 H^1 H^1 H^1 + \frac{1}{90} h \frac{1}{\text{ME}^2} \left(-12 g Y^4 + 5 \bar{Y}^E y^E P g Y^2 \left(13 + 6 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) + 45 \bar{Y}^E y^E \bar{Y} e^\dagger \left(-\bar{Y} e^\dagger y^E P + \bar{Y} e^{TS} Y e^{PS} \left(1 + 2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) H_1 D_\mu H_2 D_\mu H^1 H^1 + \\
 & \frac{1}{180} h \frac{1}{\text{ME}^2} \left(-12 g Y^4 + 5 \bar{Y}^E y^E P g Y^2 \left(13 + 6 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) - 15 \bar{Y}^E P \left(y^E P g L^2 \left(5 + 6 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) + 4 y^E \bar{Y} e^\dagger \left(2 \bar{Y} e^\dagger y^E P - 3 \bar{Y} e^{TS} Y e^{PS} \left(3 + 2 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) \right) H_1 D_\mu H_2 H^1 D_\mu H^1 + \\
 & \frac{1}{8} h \bar{Y}^E y^E P g Y^2 \frac{1}{\text{ME}^2} H_1 H^1 B^{\nu I 2} - \frac{1}{3} h g L g Y \bar{Y}^E y^E P \frac{1}{\text{ME}^2} H_1 H^1 B^{\nu I 2} W^{\nu I 2} T_3^I + \frac{1}{24} h \bar{Y}^E y^E P g L^2 \frac{1}{\text{ME}^2} H_1 H^1 W^{\nu I 2} + \\
 & \frac{1}{360} h \bar{Y} d^{PR} \frac{1}{\text{ME}^2} \left(12 g Y^4 - 5 \bar{Y} e^S Y e^S g Y^2 \left(13 + 6 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) + 5 \bar{Y} e^S \left(-12 \left(\bar{Y} e^\dagger y^E S y^E + 6 y^E \bar{Y} e^{TS} Y e^{TS} - 2 y^E S \lambda \right) + y^E S g L^2 \left(5 + 6 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) H_1 H_2 H^1 \left(\bar{d}_a^\dagger \cdot P_L \cdot q^{a1P} \right) + \\
 & \left(\frac{1}{2} \bar{Y}^E P y^E S \bar{Y} e^{SR} \frac{1}{\text{ME}^2} + \frac{1}{720} h \frac{1}{\text{ME}^2} \left(30 \bar{Y} e^S y^E S \bar{Y} e^{PT} \bar{Y} e^{SR} Y e^{TS} \left(37 + 18 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) - 45 \bar{Y}^E P \left(\bar{Y} e^\dagger y^E S y^E \bar{Y} e^{TR} \left(19 + 18 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) + 4 \bar{Y} e^{SR} \left(Y e^\dagger Y e^{TS} Y e^{SU} - 2 y^E S \lambda \left(5 + 4 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) \right) \right. \\
 & \left. + 2 \bar{Y} e^{PR} \left(12 g Y^4 - 5 \bar{Y} e^S Y e^S g Y^2 \left(13 + 6 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) + 5 \bar{Y} e^S \left(-12 \left(\bar{Y} e^\dagger y^E S y^E + 6 y^E \bar{Y} e^{TS} Y e^{TS} - 2 y^E S \lambda \right) + y^E S g L^2 \left(5 + 6 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) \right) H_1 H_2 H^1 \left(e^\dagger \cdot P_L \cdot l^{iP} \right) + \\
 & \left(\frac{1}{2} \bar{Y} e^\dagger y^E \bar{Y} e^{SP} \frac{1}{\text{ME}^2} + \frac{1}{720} h \frac{1}{\text{ME}^2} \left(24 Y e^{RP} g Y^4 - 10 \bar{Y} e^S Y e^S \bar{Y} e^{RP} g Y^2 \left(13 + 6 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) + 5 \bar{Y} e^S \left(2 y^E S \bar{Y} e^{RP} g L^2 \left(5 + 6 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) - 3 \bar{Y} e^\dagger y^E \left(8 Y e^S \bar{Y} e^{RP} + 3 y^E \bar{Y} e^{SP} \left(19 + 18 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) \right) \right. \\
 & \left. + 6 \left[4 \lambda \left(2 y^E S \bar{Y} e^{RP} + 3 y^E \bar{Y} e^{SP} \left(5 + 4 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) + \bar{Y} e^{TU} \left(-6 y^E \bar{Y} e^{SU} Y e^{TP} + y^E \bar{Y} e^\dagger \left(-24 Y e^{RP} Y e^{SU} + Y e^{TU} Y e^{SP} \left(37 + 18 \text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right] \right) \right) \right) \right] \right) H_1 H^1 H^1 \left(\bar{l}_i^\dagger \cdot P_R \cdot e^P \right) +
 \end{aligned}$$

Matching to a particular EFT basis

```
In[2]:= LEFT = Match@LoadModel@"E_VLL";
LTarget = LoadModel@"SMEFT";
MapEffectiveCouplings[LEFT, LTarget] // TableForm // NiceForm
```

- » Added new CG cgl with indices {SU2L[fund], SU2L[adj], SU2L[fund]}
- » The Lagrangian contains terms of lower power than dimension 4. Defining effective couplings and assuming these terms to be dimension 4. Use 'PrintEffectiveCouplings' and 'ReplaceEffectiveCouplings' to recover explicit expressions.

Out[4]//NiceForm=

```
g_L → g_L
g_s → g_s
g_Y → g_Y - 2/3 ħ 1/ε g_Y^3 - 2/3 ħ g_Y^3 Log[ p^2 / m_ε^2 ]
Y_d^{i1_i2-} → Y_d^{i1i2} - 1/4 ħ ȲEP yEP Y_d^{i1i2} - 1/2 ħ 1/ε ȲEP yEP Y_d^{i1i2} + 1/3 ħ C_H^2 1/m_ε^2 ȲEP yEP Y_d^{i1i2} - 1/2 ħ ȲEP
Y_e^{i1_i2-} → -3/8 ħ ȲEP yE^{i1} Y_e^{pi2} - 1/4 ħ 1/ε ȲEP yE^{i1} Y_e^{pi2} + 3/4 ħ C_H^2 1/m_ε^2 ȲEP yE^{i1} Y_e^{pi2} + 1/2 ħ 1/ε C_H^2 1/m_ε^2 Ȳ
Y_u^{i1_i2-} → Y_u^{i1i2} - 1/4 ħ ȲEP yEP Y_u^{i1i2} - 1/2 ħ 1/ε ȲEP yEP Y_u^{i1i2} + 1/3 ħ C_H^2 1/m_ε^2 ȲEP yEP Y_u^{i1i2} - 1/2 ħ ȲEP
λ → -5/9 ħ C_H^2 g_L^2 1/m_ε^2 ȲEP yEP - 2/3 ħ 1/ε C_H^2 g_L^2 1/m_ε^2 ȲEP yEP + 2 ħ 1/ε ȲEP ȲEP^r yEP yEP^r - 2 ħ C_H^2 1/m_ε^2 ȲEP Ȳ
μ2 → C_H^2 - 1/2 ħ C_H^2 ȲEP yEP - ħ 1/ε C_H^2 ȲEP yEP + 1/3 ħ C_H^2^2 1/m_ε^2 ȲEP yEP - ħ C_H^2 ȲEP yEP Log[ p^2 / m_ε^2 ]
C_{llHH}^{i1_i2-} → 0
C_{dB}^{i1_i2-} → 0
C_{dd}^{i1_i2-i3_i4-} → -2/135 ħ g_Y^4 1/m_ε^2 δ_{i1i2} δ_{i3i4}
C_{dG}^{i1_i2-} → 0
C_{dH}^{i1_i2-} → -5/36 ħ g_L^2 1/m_ε^2 ȲEP yEP Y_d^{i1i2} - 1/6 ħ 1/ε g_L^2 1/m_ε^2 ȲEP yEP Y_d^{i1i2} - 1/2 ħ 1/m_ε^2 ȲEP ȲEP^r yEP yEP^r Y_
C_{duq}^{i1_i2-i3_i4-} → 0
C_{uu}^{i1_i2-i3_i4-} → 0
```

Simplification and basis reduction

Number of SMEFT generators (1 gen., dim. 6):

$$\begin{array}{ccc} 80 & (1986) & \longrightarrow & 59 & (2017) \\ \text{Buchmüller, Wyler '86} & & & \text{Grzadkowski et al. [1008.4884]} & \end{array}$$

Simplification and basis reduction

$$\mathcal{L} = -\frac{1}{2}\phi\partial^2\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{24}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \frac{C_2}{\Lambda^2}\phi^3\partial^2\phi + \frac{C_3}{\Lambda^2}\phi^2(\partial_\mu\phi)^2$$

Simplification and basis reduction

$$\mathcal{L} = -\frac{1}{2}\phi\partial^2\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{24}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \frac{C_2}{\Lambda^2}\phi^3\partial^2\phi + \cancel{\frac{C_3}{\Lambda^2}\phi^2(\partial_\mu\phi)^2}$$

Exact simplification (linear):

IBP, Dirac algebra, group identities, commutation relations,...

$$\mathcal{L} = -\frac{1}{2}\phi\partial^2\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{24}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \cancel{\frac{3C_2 - C_3}{3\Lambda^2}\phi^3\partial^2\phi}$$

Simplification and basis reduction

$$\mathcal{L} = -\frac{1}{2}\phi\partial^2\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{24}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \frac{C_2}{\Lambda^2}\phi^3\partial^2\phi + \cancel{\frac{C_3}{\Lambda^2}\phi^2(\partial_\mu\phi)^2}$$

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On-shell equivalence (non-linear):

$$\text{Field redefinition: } \phi \longrightarrow \phi + \frac{3C_2 - C_3}{3\Lambda^2}\phi^3$$

$$\mathcal{L} \longrightarrow -\frac{1}{2}\phi\partial^2\phi - \frac{1}{2}m^2\phi^2 - \left(\frac{\lambda}{24} + \frac{(3C_2 - C_3)m^2}{3\Lambda^2} \right) \phi^4 + \frac{18C_1 - \lambda(3C_2 - C_3)}{18\Lambda^2} \phi^6$$

Simplification and basis reduction

$$\mathcal{L} = -\frac{1}{2}\phi\partial^2\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{24}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \frac{C_2}{\Lambda^2}\phi^3\partial^2\phi + \cancel{\frac{C_3}{\Lambda^2}\phi^2(\partial_\mu\phi)^2}$$

Exact simplification (linear):

IBP, Dirac algebra, group identities, commutation relations,...

$$\mathcal{L} = -\frac{1}{2}\phi\partial^2\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{24}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \cancel{\frac{3C_2 - C_3}{3\Lambda^2}\phi^3\partial^2\phi}$$

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Removal of evanescent operators: complicated but **fully automated**

Buras, Weisz '90; Dugan, Grinstein '91; Herrlich, Nierste [hep-ph/9412375]; Aebischer *et al.* [2211.01379]; AET *et al.* [2211.09144];...

Linear simplifications the Matchete way

Example: Integrating out heavy fermion in the fundamental representation of SU(3)

```
In[12]:= LEFT // NiceForm
```

```
Out[12]//NiceForm=
```

$$\begin{aligned} & \frac{7}{540} \hbar g^2 \frac{1}{M_{\Psi}^2} (D_{\rho} G^{\mu\nu A})^2 + \\ & \frac{1}{40} \hbar g^2 \frac{1}{M_{\Psi}^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540} \hbar g^2 \frac{1}{M_{\Psi}^2} D_{\rho} G^{\mu\nu A} D_{\nu} G^{\mu\rho A} - \\ & \frac{1}{180} \hbar g^2 \frac{1}{M_{\Psi}^2} D_{\nu} G^{\mu\nu A} D_{\rho} G^{\mu\rho A} + \frac{1}{40} \hbar g^2 \frac{1}{M_{\Psi}^2} G^{\mu\nu A} D_{\nu} D_{\rho} G^{\mu\rho A} + \\ & \frac{1}{40} \hbar g^2 \frac{1}{M_{\Psi}^2} G^{\mu\nu A} D_{\rho} D_{\nu} G^{\mu\rho A} - \frac{1}{24} \hbar g^3 \frac{1}{M_{\Psi}^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC} \end{aligned}$$

Linear simplifications the Matchete way

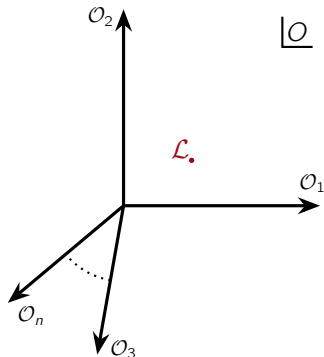
Example: Integrating out heavy fermion in the fundamental representation of SU(3)

In[12]:= LEFT // NiceForm

Out[12]//NiceForm=

$$\begin{aligned} & \frac{7}{540} \hbar g^2 \frac{1}{M_{\Psi}^2} (D_{\rho} G^{\mu\nu A})^2 + \\ & \frac{1}{40} \hbar g^2 \frac{1}{M_{\Psi}^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540} \hbar g^2 \frac{1}{M_{\Psi}^2} D_{\rho} G^{\mu\nu A} D_{\nu} G^{\mu\rho A} - \\ & \frac{1}{180} \hbar g^2 \frac{1}{M_{\Psi}^2} D_{\nu} G^{\mu\nu A} D_{\rho} G^{\mu\rho A} + \frac{1}{40} \hbar g^2 \frac{1}{M_{\Psi}^2} G^{\mu\nu A} D_{\nu} D_{\rho} G^{\mu\rho A} + \\ & \frac{1}{40} \hbar g^2 \frac{1}{M_{\Psi}^2} G^{\mu\nu A} D_{\rho} D_{\nu} G^{\mu\rho A} - \frac{1}{24} \hbar g^3 \frac{1}{M_{\Psi}^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC} \end{aligned}$$

$$\mathcal{L}_{\text{EFT}} = \sum_i C_i \mathcal{O}_i \in \mathcal{O}$$



Linear simplifications the Matchete way

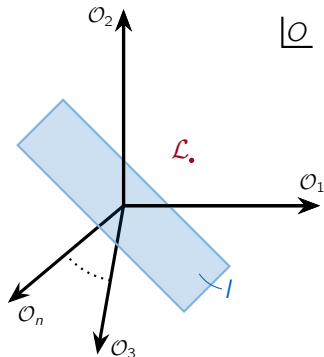
Example: Integrating out heavy fermion in the fundamental representation of SU(3)

In[12]:= LEFT // NiceForm

Out[12]//NiceForm=

$$\begin{aligned} & \frac{7}{540} \hbar g^2 \frac{1}{M^2} (D_\rho G^{\mu\nu A})^2 + \\ & \frac{1}{40} \hbar g^2 \frac{1}{M^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540} \hbar g^2 \frac{1}{M^2} D_\rho G^{\mu\nu A} D_\nu G^{\mu\rho A} - \\ & \frac{1}{180} \hbar g^2 \frac{1}{M^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} + \frac{1}{40} \hbar g^2 \frac{1}{M^2} G^{\mu\nu A} D_\nu D_\rho G^{\mu\rho A} + \\ & \frac{1}{40} \hbar g^2 \frac{1}{M^2} G^{\mu\nu A} D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24} \hbar g^3 \frac{1}{M^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC} \end{aligned}$$

$$\mathcal{L}_{\text{EFT}} = \sum_i C_i \mathcal{O}_i \in \mathcal{O}$$



$I \subseteq \mathcal{O}$ is the space of all operator identities, e.g., IBP relations such as

$$\mathcal{O}_1 + 2\mathcal{O}_3 = 0$$

is interpreted as

$$\mathcal{O}_1 + 2\mathcal{O}_3 \in I$$

Linear simplifications the Matchete way

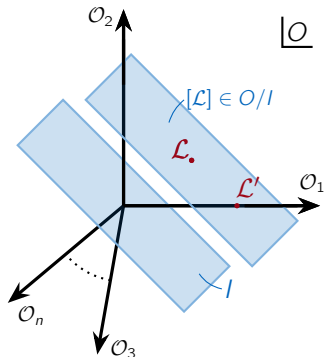
Example: Integrating out heavy fermion in the fundamental representation of SU(3)

```
In[12]:= LEFT // NiceForm
```

```
Out[12]//NiceForm=
```

$$\begin{aligned} & \frac{7}{540} \hbar g^2 \frac{1}{M^2} (D_\rho G^{\mu\nu A})^2 + \\ & \frac{1}{40} \hbar g^2 \frac{1}{M^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540} \hbar g^2 \frac{1}{M^2} D_\rho G^{\mu\nu A} D_\nu G^{\mu\rho A} - \\ & \frac{1}{180} \hbar g^2 \frac{1}{M^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} + \frac{1}{40} \hbar g^2 \frac{1}{M^2} G^{\mu\nu A} D_\nu D_\rho G^{\mu\rho A} + \\ & \frac{1}{40} \hbar g^2 \frac{1}{M^2} G^{\mu\nu A} D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24} \hbar g^3 \frac{1}{M^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC} \end{aligned}$$

$$\mathcal{L}_{\text{EFT}} = \sum_i C_i \mathcal{O}_i \in \mathcal{O}$$



$I \subseteq \mathcal{O}$ is the space of all operator identities, e.g., IBP relations such as

$$\mathcal{O}_1 + 2\mathcal{O}_3 = 0$$

is interpreted as

$$\mathcal{O}_1 + 2\mathcal{O}_3 \in I$$

Linear simplifications the Matchete way

Example: Integrating out heavy fermion in the fundamental representation of SU(3)

In[12]:= LEFT // NiceForm

Out[12]//NiceForm=

$$\begin{aligned} & \frac{7}{540} \hbar g^2 \frac{1}{M^2} (D_\rho G^{\mu\nu A})^2 + \\ & \frac{1}{40} \hbar g^2 \frac{1}{M^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540} \hbar g^2 \frac{1}{M^2} D_\rho G^{\mu\nu A} D_\nu G^{\mu\rho A} - \\ & \frac{1}{180} \hbar g^2 \frac{1}{M^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} + \frac{1}{40} \hbar g^2 \frac{1}{M^2} G^{\mu\nu A} D_\nu D_\rho G^{\mu\rho A} + \\ & \frac{1}{40} \hbar g^2 \frac{1}{M^2} G^{\mu\nu A} D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24} \hbar g^3 \frac{1}{M^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC} \end{aligned}$$

$$\mathcal{L}_{\text{EFT}} = \sum_i C_i \mathcal{O}_i \in \mathcal{O}$$

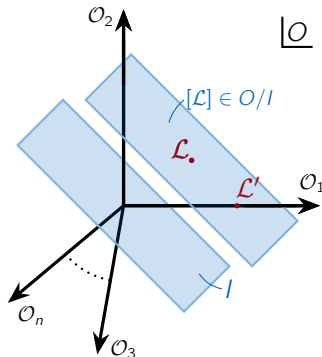
With **linear algebra** on the basis of $\mathcal{I}$ we find a simple representative element for $[\mathcal{L}_{\text{EFT}}] \in \mathcal{O}/\mathcal{I}$:

In[13]:= LEFT // GreensSimplify // NiceForm

Out[13]//NiceForm=

$$-\frac{1}{15} \hbar g^2 \frac{1}{M^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} - \frac{1}{180} \hbar g^3 \frac{1}{M^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$

Green's basis



$\mathcal{I} \subseteq \mathcal{O}$ is the space of all operator identities, e.g., IBP relations such as

$$\mathcal{O}_1 + 2\mathcal{O}_3 = 0$$

is interpreted as

$$\mathcal{O}_1 + 2\mathcal{O}_3 \in \mathcal{I}$$

Summary and outlook

- (Automatic) EFT matching and running is crucial to BSM phenomenology
- Functional methods provides a direct approach to automated matching and RG calculations
- **Matchete** is a public code for EFT matching. It greatly simplifies the matching task and many more features are planned for the near-future
- Keep an eye out for the 2-loop SMEFT RG-functions! [Born, Fuentes-Martín \[2601.—\]](#)

<https://gitlab.com/matchete/matchete>



Backup

Matchete SM implementation

Gauge Groups

```
DefineGaugeGroup[SU3c, SU@3, gs, G,  
  FundAlphabet → CharacterRange["a", "f"],  
  AdjAlphabet → CharacterRange["A", "F"]]  
DefineGaugeGroup[SU2L, SU@2, gL, W,  
  FundAlphabet → CharacterRange["i", "n"],  
  AdjAlphabet → CharacterRange["I", "N"]]  
DefineGaugeGroup[U1Y, U@1, gY, B]
```

Generation index

```
DefineFlavorIndex[Flavor, 3,  
  IndexAlphabet → {"p", "r", "s", "t", "u", "v"}]
```

Fermions

```
DefineField[q, Fermion,  
  Indices → {SU3c@fund, SU2L@fund, Flavor},  
  Charges → {U1Y[1/6]},  
  Chiral → LeftHanded,  
  Mass → 0]  
DefineField[u, Fermion,  
  Indices → {SU3c@fund, Flavor},  
  Charges → {U1Y[2/3]},  
  Chiral → RightHanded,  
  Mass → 0]  
DefineField[d, Fermion,  
  Indices → {SU3c@fund, Flavor},  
  Charges → {U1Y[-1/3]},  
  Chiral → RightHanded,  
  Mass → 0]
```

```
DefineField[l, Fermion,  
  Indices → {SU2L@fund, Flavor},  
  Charges → {U1Y[-1/2]},  
  Chiral → LeftHanded,  
  Mass → 0]  
DefineField[e, Fermion,  
  Indices → {Flavor},  
  Charges → {U1Y[-1]},  
  Chiral → RightHanded,  
  Mass → 0]
```

Higgs

```
DefineField[H, Scalar,  
  Indices → {SU2L@fund},  
  Charges → {U1Y[1/2]},  
  Mass → 0]
```

Couplings

```
DefineCoupling[λ, SelfConjugate → True]  
DefineCoupling[μ, SelfConjugate → True,  
  EFTorder → 1];  
DefineCoupling[Ye,  
  Indices → {Flavor, Flavor}]  
DefineCoupling[Yu,  
  Indices → {Flavor, Flavor}]  
DefineCoupling[Yd,  
  Indices → {Flavor, Flavor}]
```

Matchete SM implementation

Lagrangian

```

 $\mathcal{L}_{SM} = \text{FreeLag}[] +$ 
 $-\mu[]^2 \text{Bar}eH[i] \times H[i] -$ 
 $\frac{\lambda[]}{2} \text{Bar}eH[i] \times H[i] \times \text{Bar}eH[j] \times H[j] +$ 
 $\text{PlusHc}[$ 
 $-\text{Yu}[p, r] \times \text{CG}[\text{eps}eSU2L, \{i, j\}] \times$ 
 $\text{Bar}eH[i] \times \text{Bar}eQ[a, j, p] ** u[a, r]$ 
 $-\text{Yd}[p, r] \times H[i] \times \text{Bar}eQ[a, i, p] ** d[a, r]$ 
 $-\text{Ye}[p, r] \times H[i] \times \text{Bar}eL[i, p] ** e[r]$ 
 $] // \text{RelabelIndices};$ 

```

$\mathcal{L}_{SM}$ // NiceForm

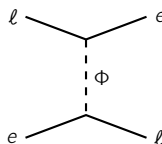
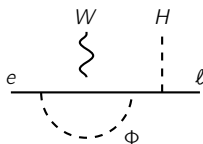
Form=

$$\begin{aligned}
 & -\frac{1}{4} B^{\mu\nu 2} - \frac{1}{4} G^{\mu\nu A 2} - \frac{1}{4} W^{\mu\nu I 2} + D_\mu H_i D_\mu H^i - \\
 & \mu^2 H_i H^i + i \left(\bar{d}_a^p \cdot \gamma_\mu P_R \cdot D_\mu d^{ap} \right) + i \left(\bar{e}^p \cdot \gamma_\mu P_R \cdot D_\mu e^p \right) + \\
 & i \left(\bar{l}_i^p \cdot \gamma_\mu P_L \cdot D_\mu l^{ip} \right) + i \left(\bar{q}_{ai}^p \cdot \gamma_\mu P_L \cdot D_\mu q^{aip} \right) + \\
 & i \left(\bar{u}_a^p \cdot \gamma_\mu P_R \cdot D_\mu u^{ap} \right) - \frac{1}{2} \lambda H_i H_j H^i H^j - \\
 & \bar{v} d^{pr} H_i \left(\bar{d}_a^r \cdot P_L \cdot q^{aip} \right) - \bar{v} e^{pr} H_i \left(\bar{e}^r \cdot P_L \cdot l^{ip} \right) - \\
 & \bar{y} e^{pr} H^i \left(\bar{l}_i^p \cdot P_R \cdot e^r \right) - \bar{y} d^{pr} H^i \left(\bar{q}_{ai}^p \cdot P_R \cdot d^{ar} \right) - \\
 & \bar{y} u^{pr} H_i \left(\bar{q}_{aj}^p \cdot P_R \cdot u^{ar} \right) \varepsilon^{ij} - \bar{v} u^{pr} H^i \left(\bar{u}_a^r \cdot P_L \cdot q^{ajp} \right) \bar{\varepsilon}_{ij}
 \end{aligned}$$

EFT from a 2HDM

Example: SM + leptophilic Higgs, $\Phi \sim (\mathbf{1}, \mathbf{2})_{1/2}$:

$$\mathcal{L} \supset \mathcal{L}_{\text{SM}} + D_\mu \Phi^\dagger D^\mu \Phi - M_\Phi^2 \Phi^\dagger \Phi - (y_{\Phi e}^{pr} \bar{\ell}_p \Phi e_r + \text{h.c.}) + \dots$$



Below the scale $M_\Phi \gg v_{\text{EW}}$

$$\mathcal{L}_{\text{EFT}} \supset C_{eW}^{pr} Q_{eW}^{pr} + C_{\ell e}^{prst} R_{\ell e}^{prst}$$

But the tree-level operator **$R_{\ell e}$ is not part of the Warsaw basis**

Changing basis in an EFT

In $d = 4$ dimensions, $\mathcal{L}_{\text{EFT}} = \tilde{\mathcal{L}}_{\text{EFT}}$, where

$$\mathcal{L}_{\text{EFT}} \supset C_{eW}^{pr} Q_{eW}^{pr} + C_{le}^{prst} R_{le}^{prst}$$

$$\tilde{\mathcal{L}}_{\text{EFT}} \supset C_{eW}^{pr} Q_{eW}^{pr} - \frac{1}{2} C_{le}^{ptsr} Q_{le}^{prst}$$

$$R_{le}^{prst} = (\bar{\ell}_p e_r)(\bar{e}_s \ell_t)$$

$$Q_{le}^{ptsr} = (\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r)$$

But the 1-loop EFT **amplitudes disagree!**

$$i(\mathcal{A}_{eH \rightarrow \ell W} - \tilde{\mathcal{A}}_{eH \rightarrow \ell W}) = \frac{g_2}{64\pi^2} [C_{le}]^{prst} y_e^{ts} (\bar{u} \tau^I \sigma_{\mu\nu} P_R u) q^\mu \varepsilon^{*\nu}$$



For $d \neq 4$, there is an **evanescent operator**:

$$R_{le}^{prst} = -\frac{1}{2} Q_{le}^{ptsr} + E_{le}^{prst},$$

$$E_{le}^{prst} \xrightarrow{d \rightarrow 4} 0$$