Electromagnetic form factors and structure of the T_{bb} tetraquark

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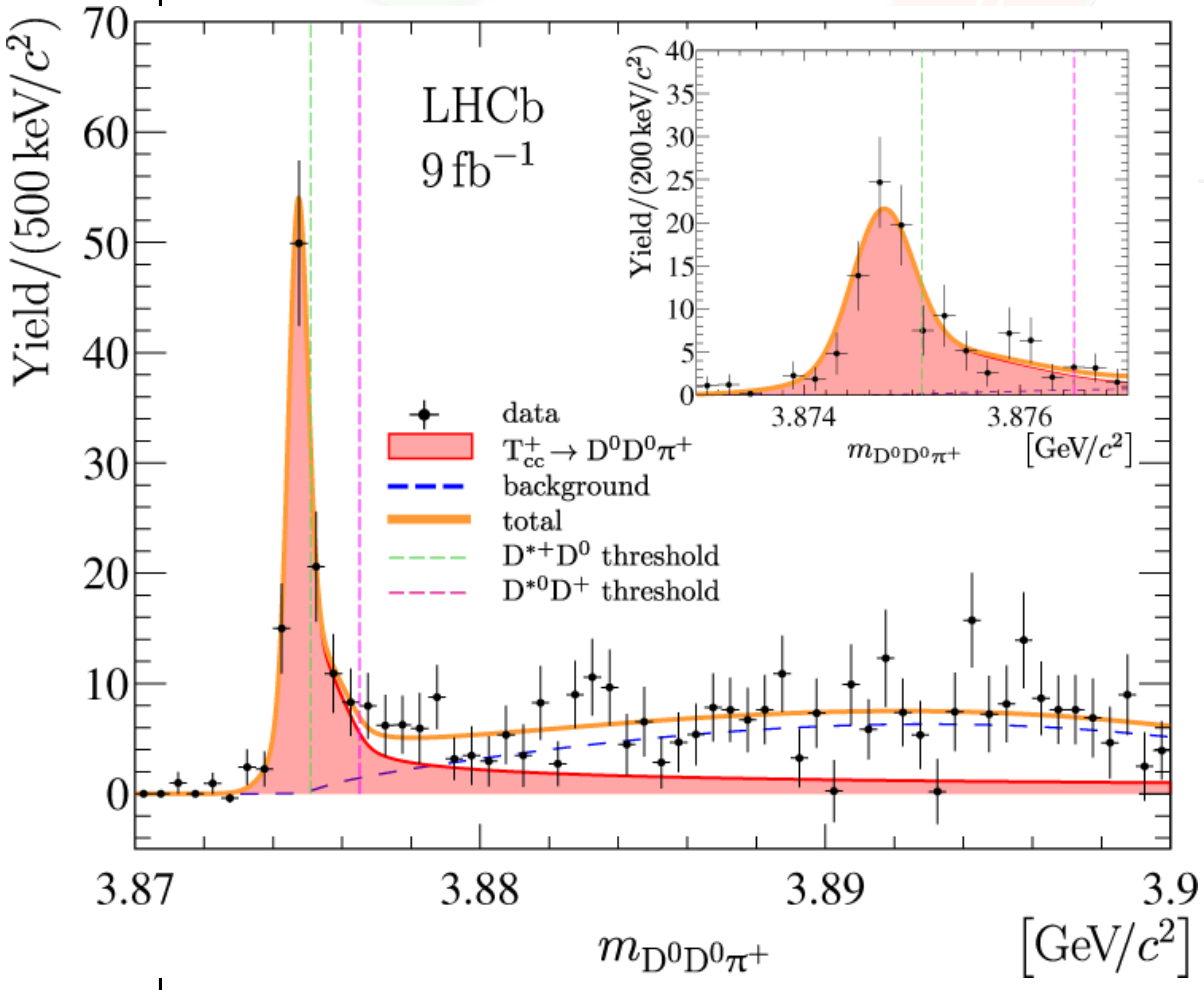
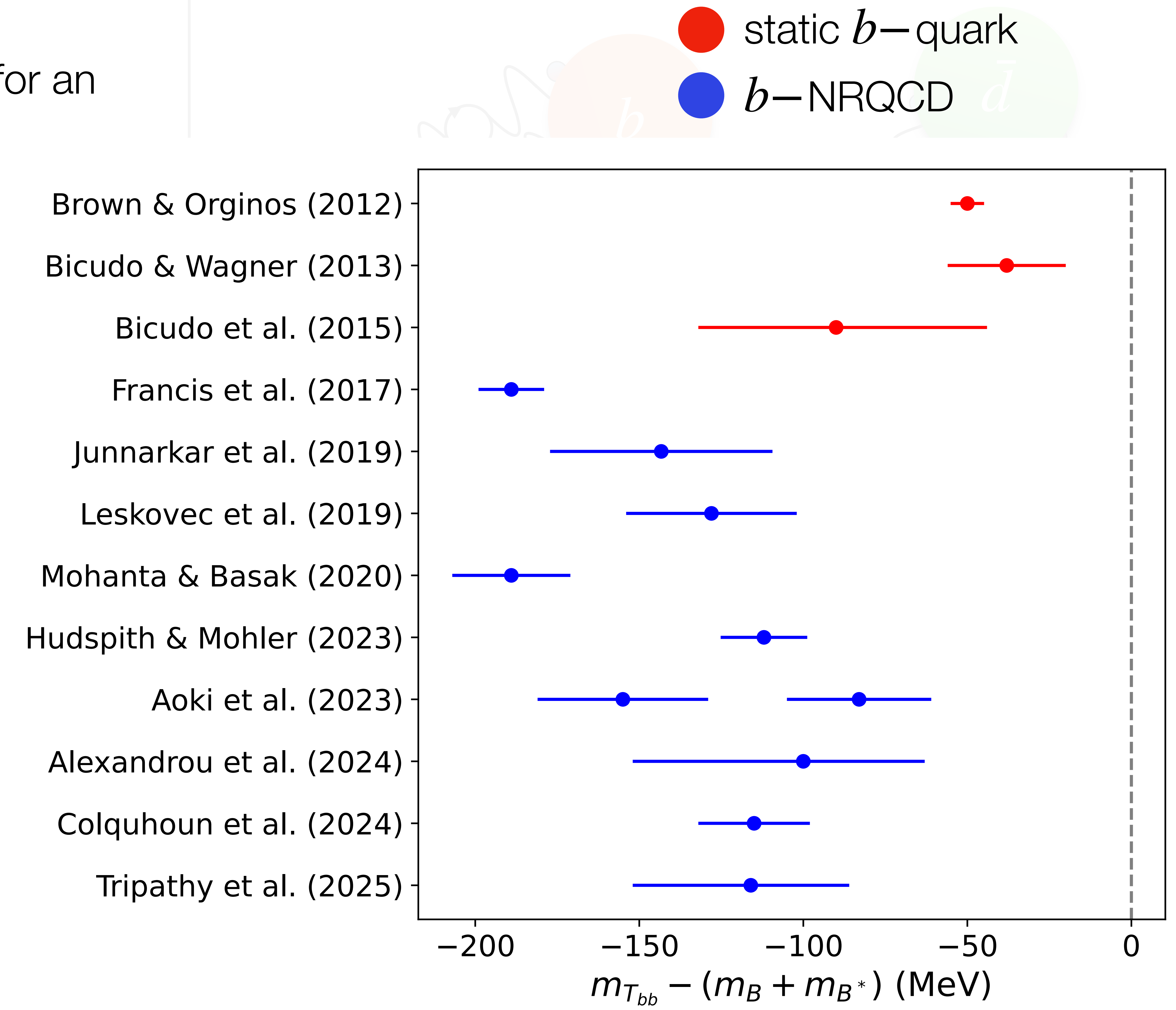
Introduction

- $T_{bb} = bb\bar{u}\bar{d},\ I(J^P) = 0(1^+)$
- yet-experimentally-undiscovered: one of the most promising candidates for an exotic QCD bound state
    - $T_{cc} = cc\bar{u}\bar{d}, I(J^P) = 0(?^?)$ found just below DD^* thr. (LHCb)



Introduction

- $T_{bb} = bb\bar{u}\bar{d}$, $I(J^P) = 0(1^+)$
- yet-experimentally-undiscovered: one of the most promising candidates for an exotic QCD bound state
- quark models + **<u>lattice QCD (LQCD)</u>**
  - state-of-the-art LQCD spectroscopy:
    - large operator basis
      - local, bilocal meson-meson ops.: $(b\bar{u})(b\bar{d})$
      - local diquark-antidiquark ops.: $[bb][\bar{u}\bar{d}]$
    - infinite-volume ($L \to \infty$) + continuum ($a \to 0$) + physical pion mass ($m_\pi^{lat} \to m_\pi^{phys}$) limit
- **<u>bound state $\mathcal{O}(100\ \mathrm{MeV})$ below BB^* threshold</u>**



Introduction

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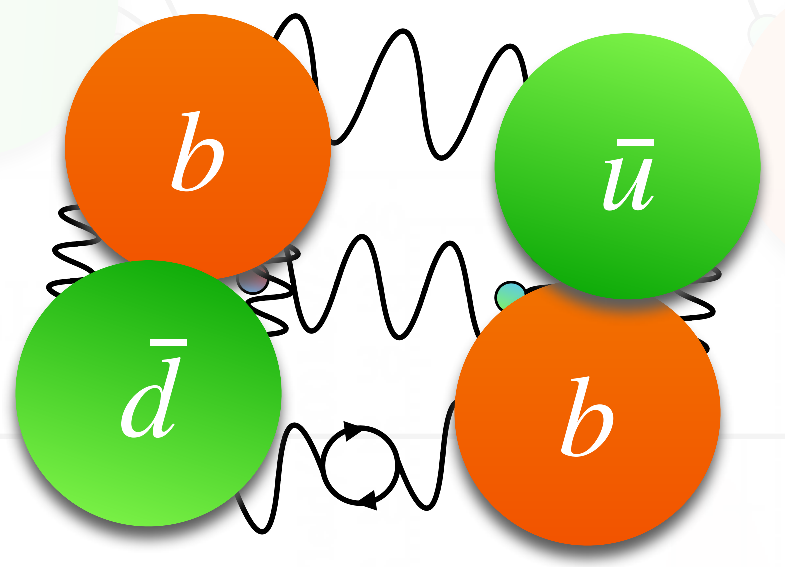
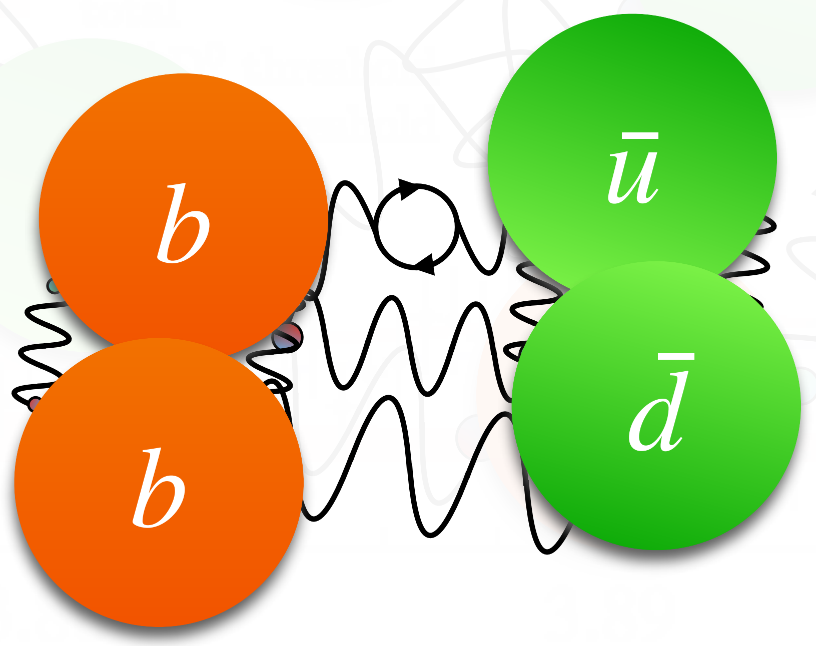
      - local diquark-antidiquark ops.: $[bb][\bar{u}\bar{d}]$
- well-motivated ideas about its structure exist
  - was never before studied using fully *ab-initio* approach
- in this talk:
  - first glimpse of the T_{bb} internal structure via its electromagnetic (EM) form factors

- molecular binding: $(b\bar{u})_1(b\bar{d})_1,\ (b\bar{u})_8(b\bar{d})_8$



- diquark-antidiquark binding: $[bb]_{\bar{3}}[\bar{u}\bar{d}]_3,\ [bb]_6[\bar{u}\bar{d}]_{\bar{6}}$



Detour: intro to Lattice QCD

- numerical evaulation of QCD path integrals in a box

$$t \to t_E \equiv it$$

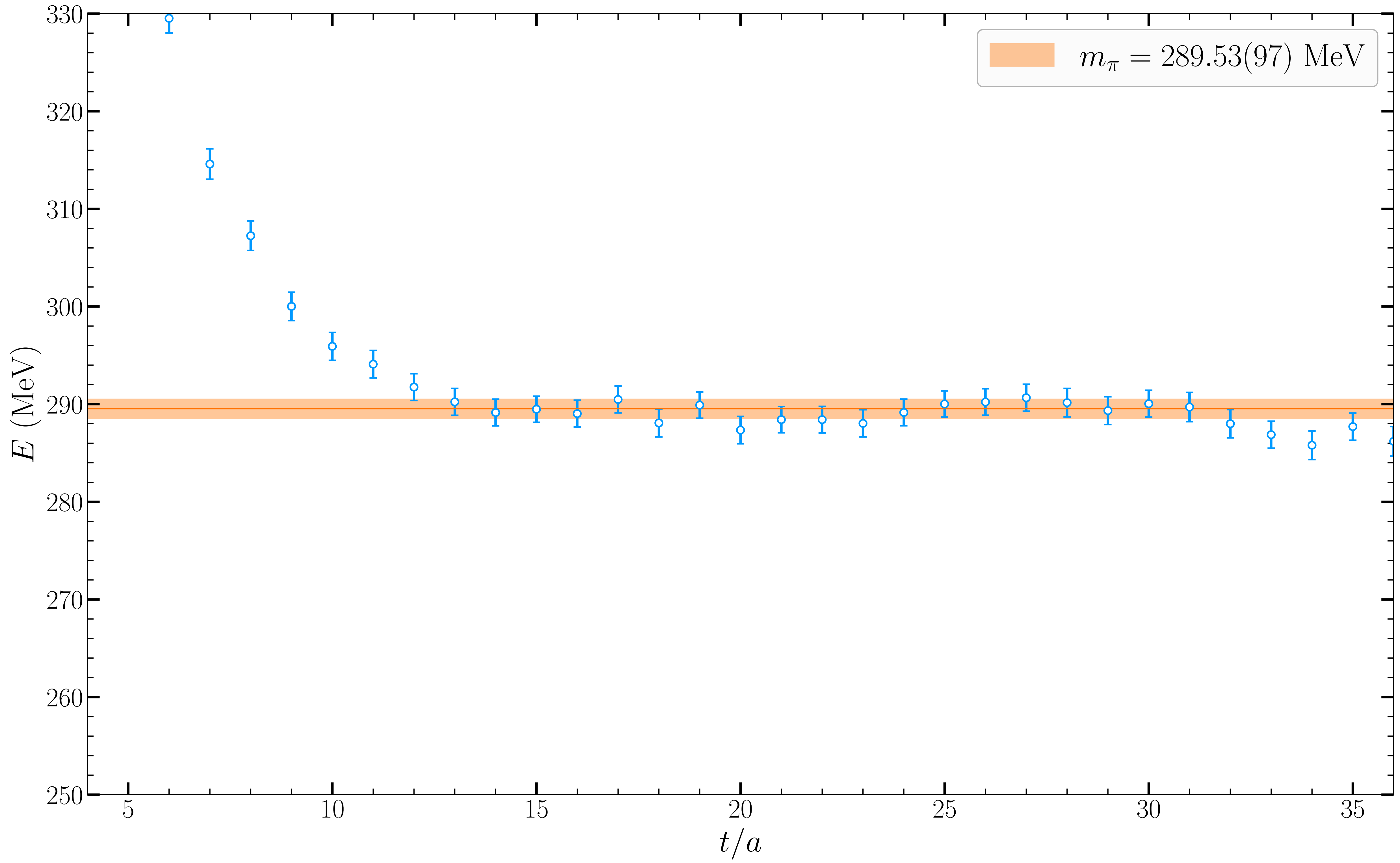
$$\langle \Omega | \mathcal{O} | \Omega \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}q \ \mathcal{D}\bar{q} \ \mathcal{D}G \ \mathcal{O}[q, \bar{q}, G] e^{-S_E(q,\bar{q},G)}$$

- $n-$point correlation functions
  - two-point correlators → **energy spectrum of the system**

$$C_2(t, t_0) = \langle \Omega | \mathcal{O}(t) \mathcal{O}^\dagger(t_0) | \Omega \rangle =$$

$$= \sum_n \langle \Omega | \mathcal{O}(t) | n \rangle \langle n | \mathcal{O}^\dagger(t_0) | \Omega \rangle = \sum_n |\mathcal{Z}_n|^2 e^{-E_n(t-t_0)}$$

$$\hat{H}_{QCD} | n \rangle = E_n | n \rangle$$



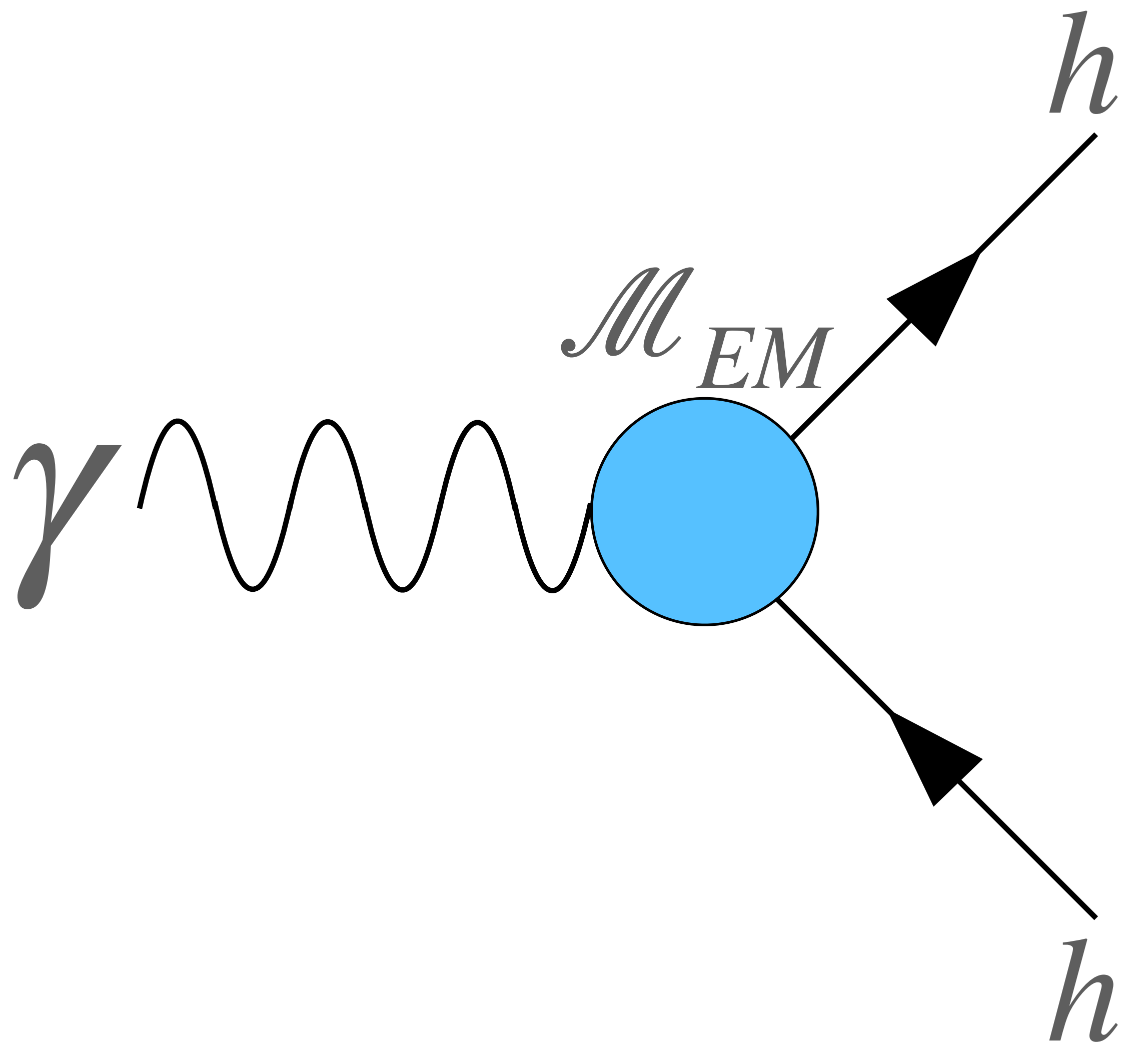
- $\mathcal{O}^\dagger, \mathcal{O}$ create/annihilate a tower of states with desired quantum numbers
  - in practice: compute $N \times N$ Hermitian matrix $\langle \Omega | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(t_0) | \Omega \rangle \to E_n, n = 1,\ldots N$

Elastic EM form factors

- matrix elements: $\mathscr{M}^{\mu}_{EM} \equiv \langle h(p_2, \lambda_2) | \hat{j}^{\mu}_{EM} | h(p_1, \lambda_1) \rangle = \sum_j K_j(p_{1,2}, \lambda_{1,2}) \cdot F_j(Q^2)$

$$\hat{j}^{\mu}_{EM} = \sum_q e_q \bar{q} \gamma^{\mu} q = \sum_q \hat{j}^{\mu}_q \qquad Q^2 \equiv -(p_2 - p_1)^2 > 0$$

- additive form factors → flavor decomposition: $F_j = \sum_q F^q_j(Q^2)$
    - can probe distributions of individual quarks
- $F_j \equiv$ form factors of electromagnetic multipoles
- constrained by hadron spin ($0 \leq j \leq 2s$), current parity and conservation



Elastic EM form factors

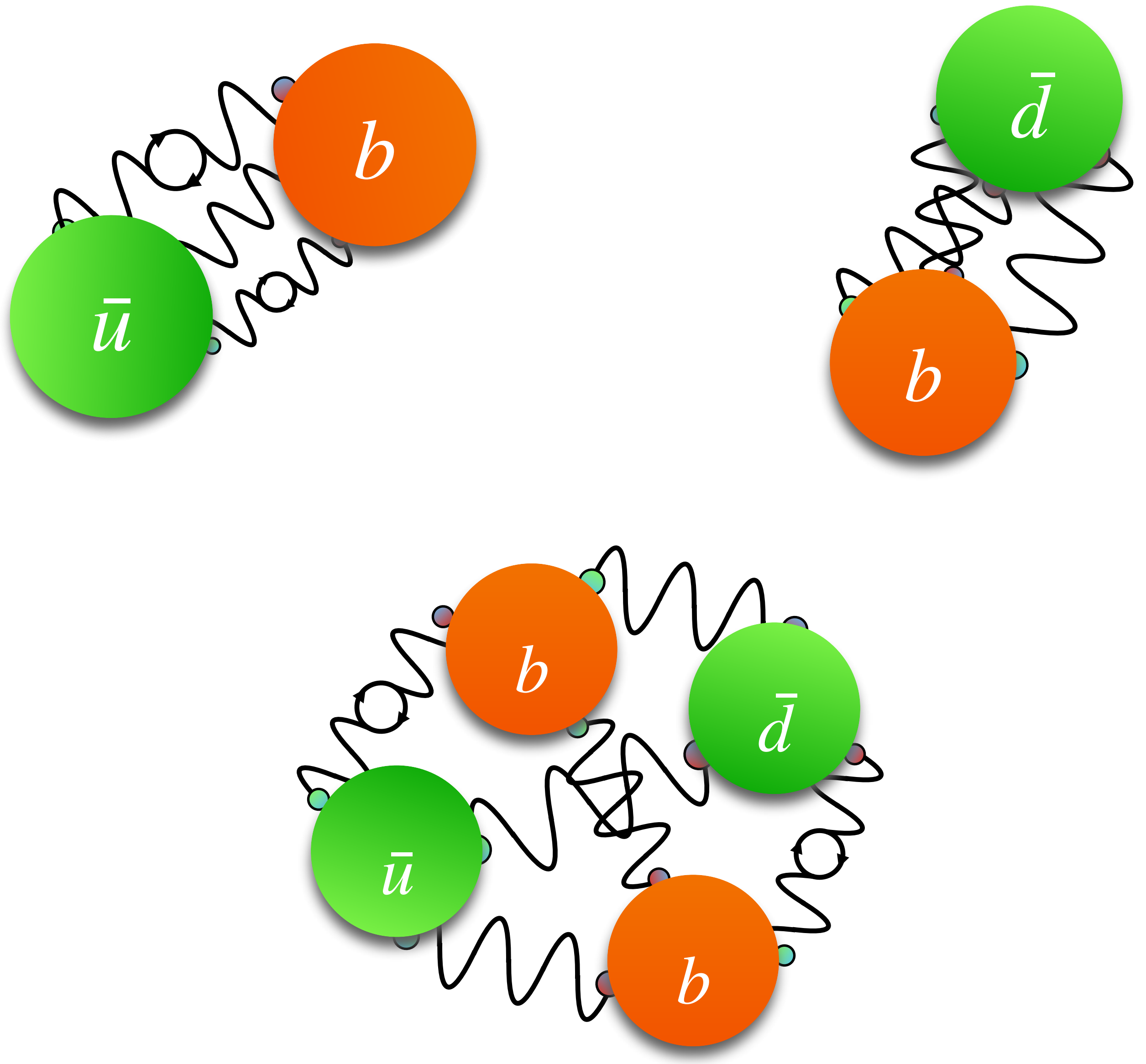
- relevant states h:
  - T_{bb}
  - $B, \quad B^*$ $\longrightarrow$ set the scale of the problem
  - π $\longrightarrow$ measured experimentally, can be compared to

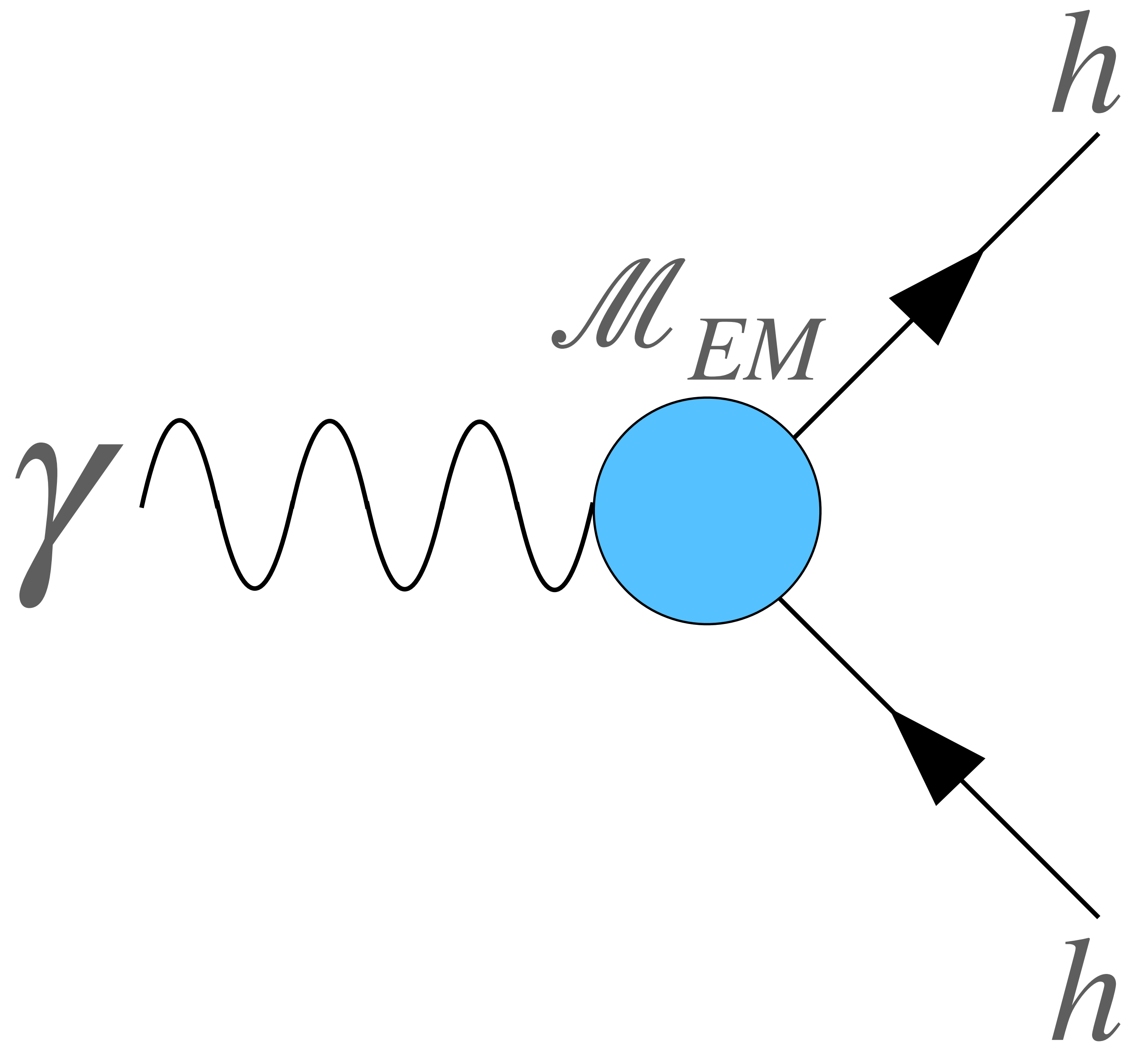
$J^P = 0^- : \quad \mathcal{M}^{\mu}_{EM} \sim F_C(Q^2)$ ◉ γ reveals **charge monopole** distribution

$J^P = 1^{\pm} : \mathcal{M}^{\mu}_{EM} \sim F_C(Q^2), F_M(Q^2), F_{\mathcal{Q}}(Q^2)$ ◉ γ reveals **charge monopole, magnetic dipole and electric quadrupole** distributions

- infinite-volume, continuum normalizations:

$$F_C(0) = -1 \qquad F_M(0) = 2m \cdot \mu \qquad F_{\mathcal{Q}}(0) = m^2 \cdot \mathcal{Q}$$



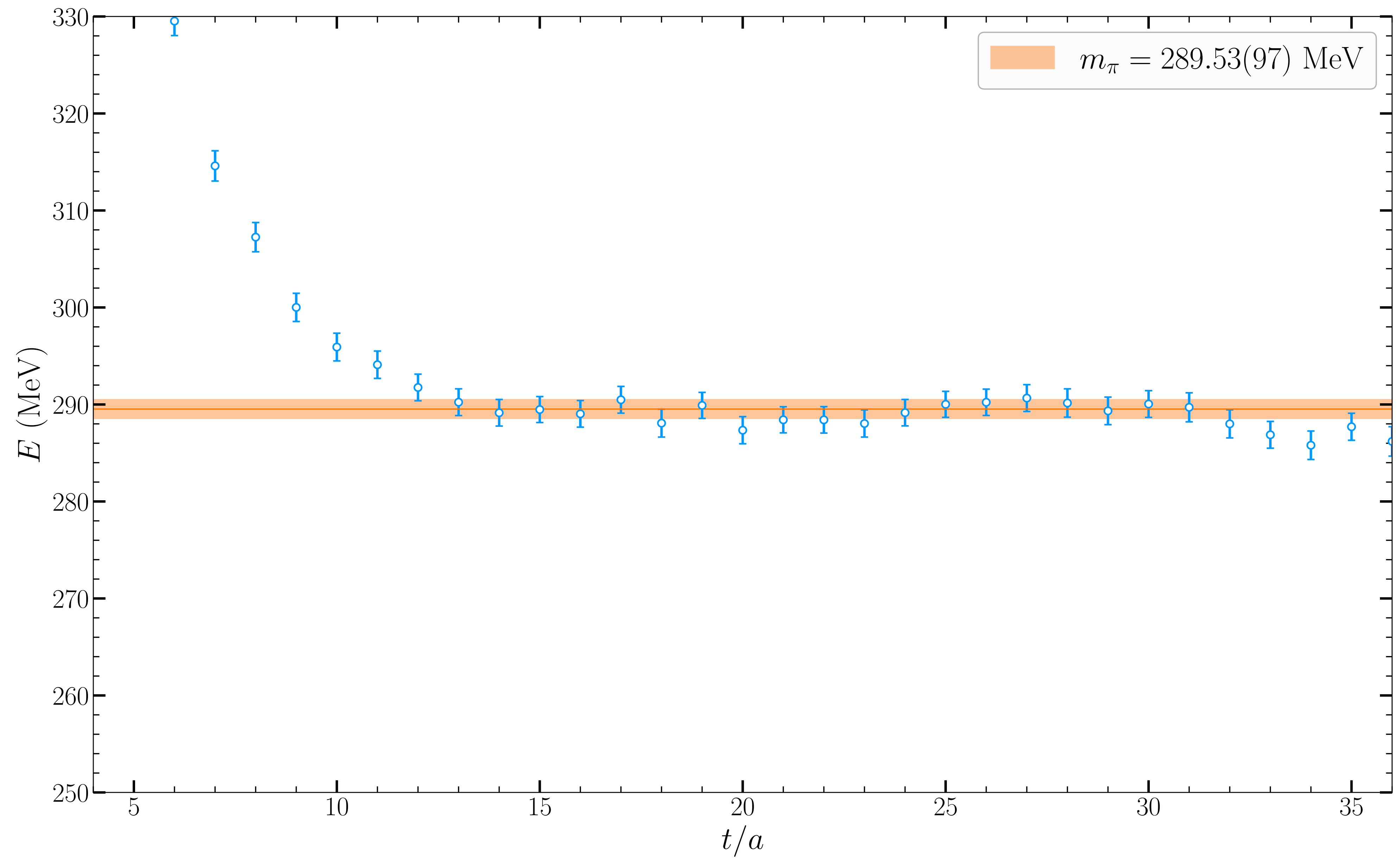


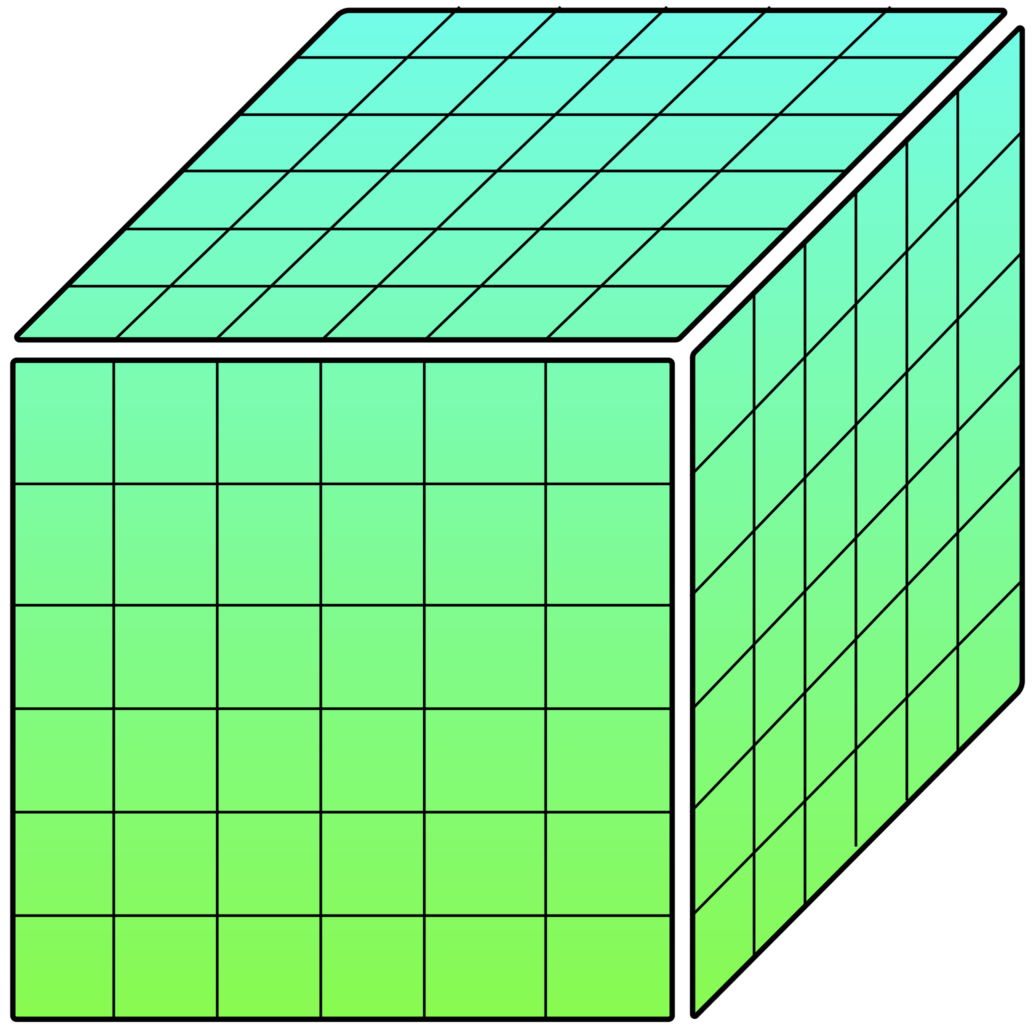
Lattice setup

- single $N_f = 2 + 1$ gauge ensemble, generated by CLS
  - $V = N_L^3 \times N_T = 40^3 \times 128$
  - lattice spacing $a \approx 0.064\ \text{fm}$
  - u/d quarks: isospin symmetry; isotropic, $\mathrm{O}(a)$ improved Wilson action

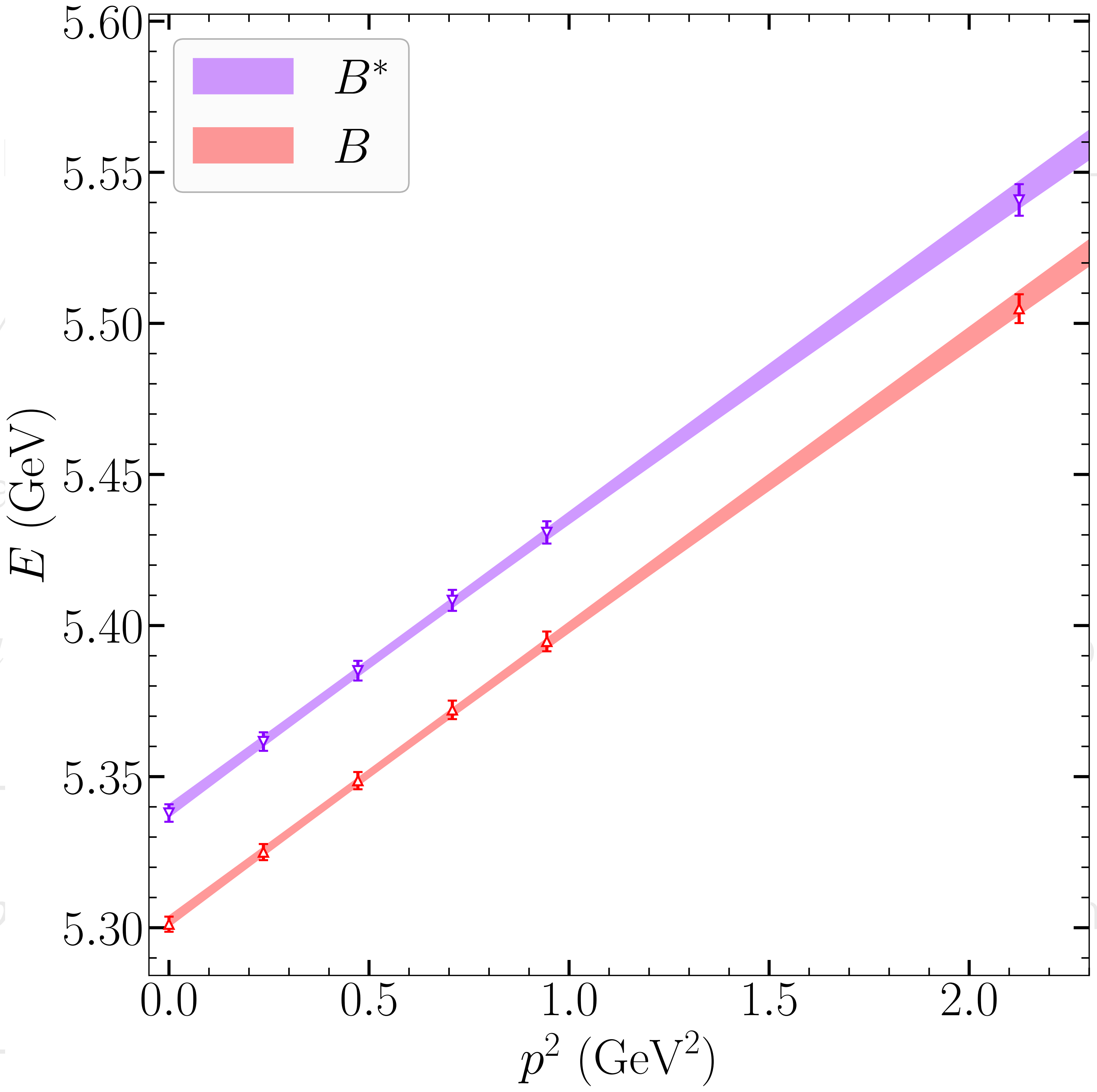
    $\rightarrow$ pion mass $m_\pi \approx 290\ \text{MeV}$
  - b quarks: anisotropic relativistic heavy quark action

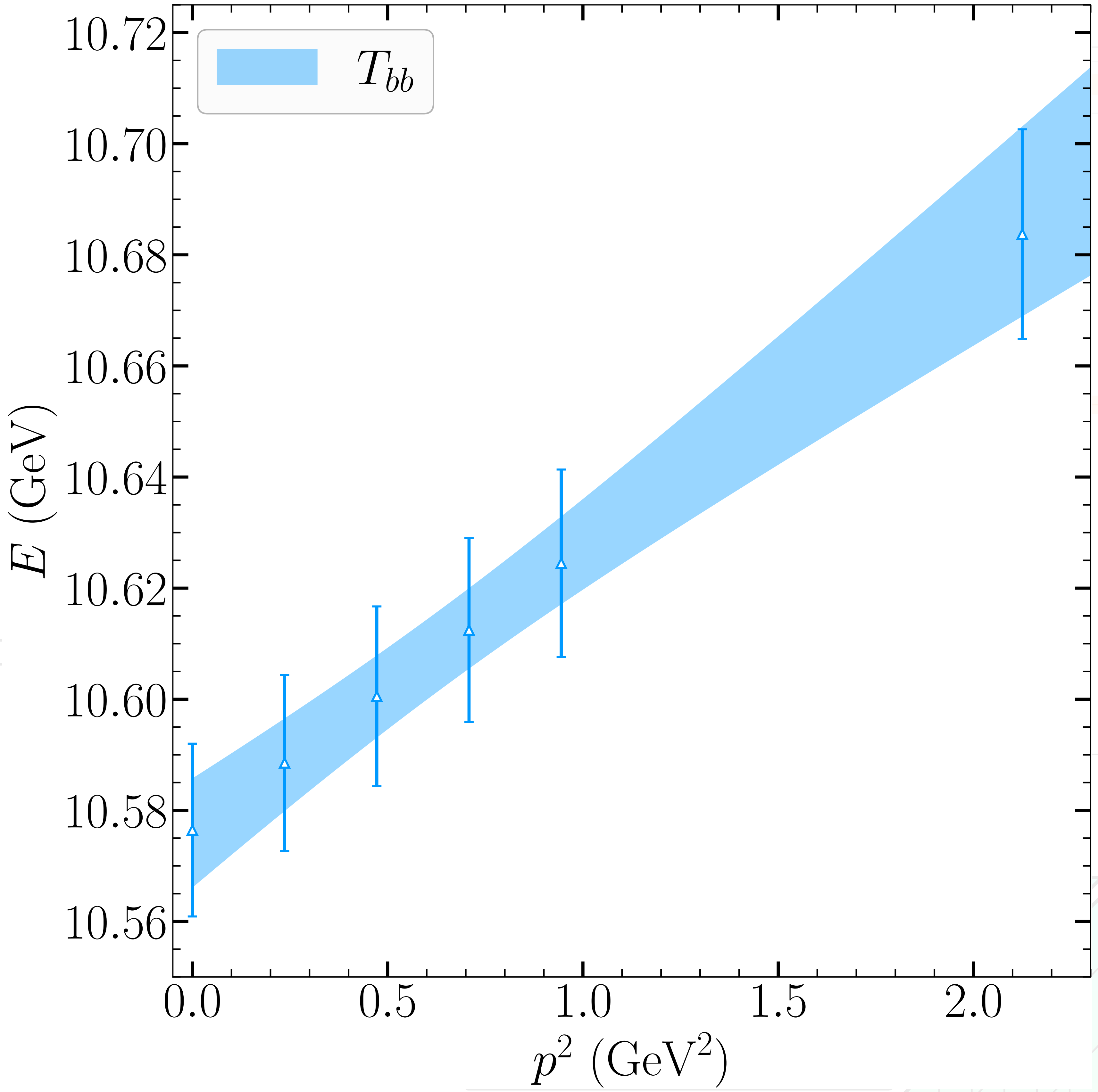
    $\rightarrow$ tuning observables: m_B, m_{B^*} and energy-momentum disp. relations





Lattice setup





$$E = \sqrt{m^2 + (c\vec{p})^2}$$

h	m_h (GeV)	c
B	5.3020(17)	1.020(12)
B^*	5.3387(20)	1.021(14)
T_{bb}	10.5765(98)	1.04(11)

B, B^* lattice masses well within 1% of PDG values

$$m_{T_{bb}} - (m_B + m_{B^*}) = -64(10)\ \text{MeV}$$

Lattice observables

- <u>what is sought</u>: $\langle h(\vec{p}_2)\,|\,\hat{j}^\mu_q\,|\,h(\vec{p}_1)\rangle$, $h = T_{bb}, B, B^*, \pi$

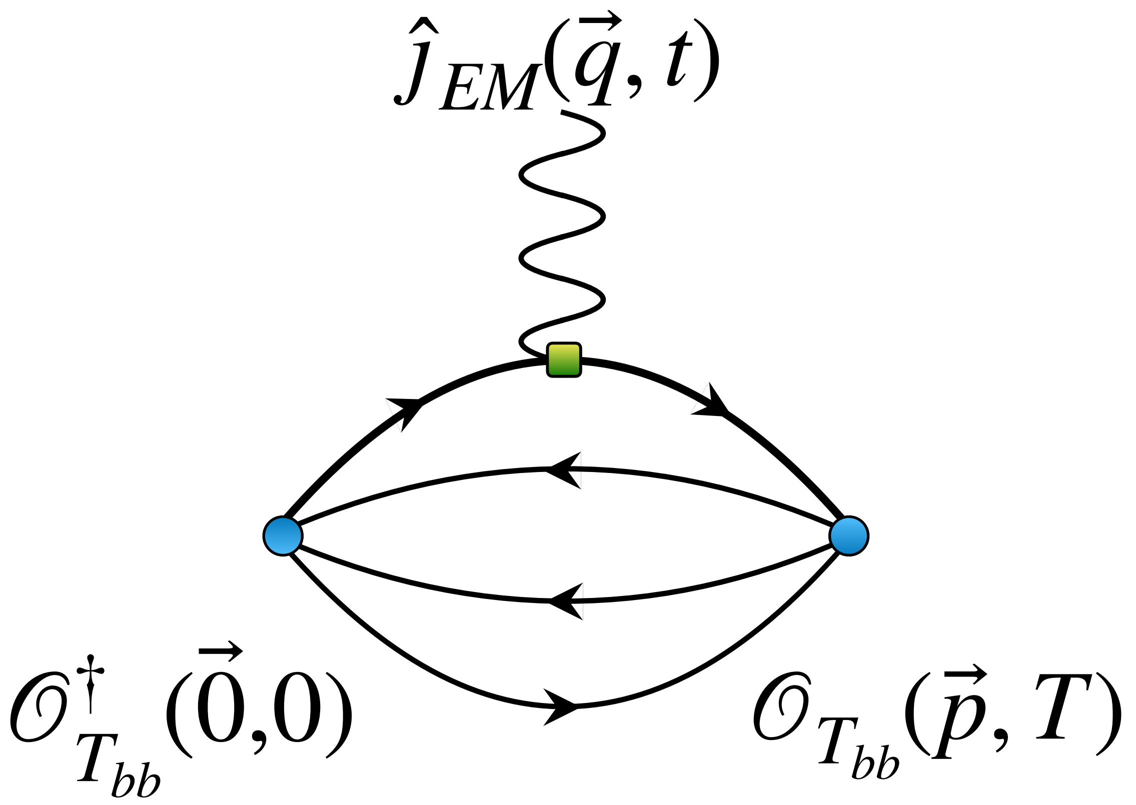
$$q = u, d, b$$

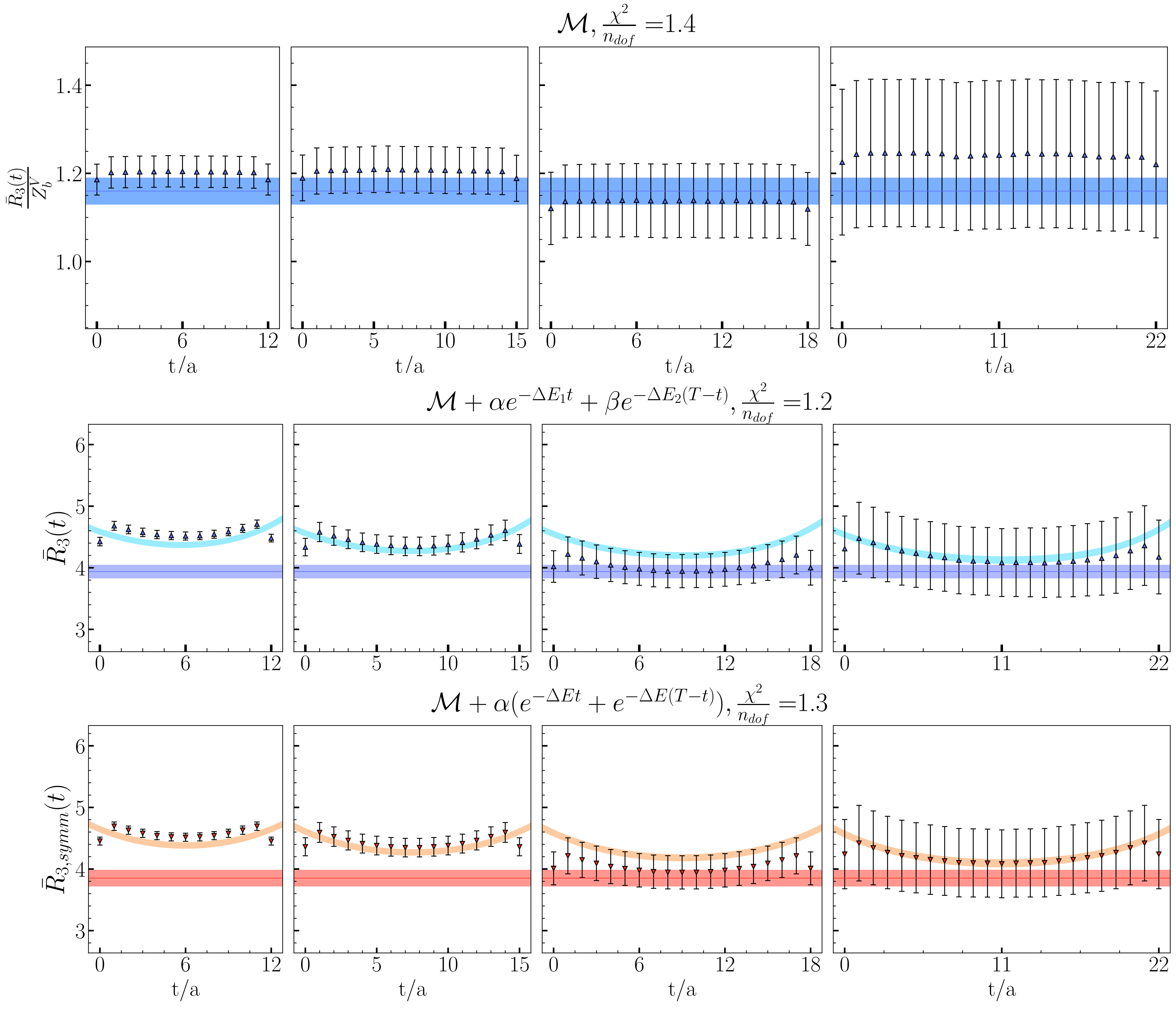
- <u>three-pt. correlators</u>:

$$1 = \sum_n |n\rangle\langle n|$$

$$C_3(\vec{p}_2, \vec{q}, T, t) = \langle \Omega\,|\,\mathcal{O}_h(\vec{p}_2, T)\hat{j}_{EM}(\vec{q}, t)\mathcal{O}^\dagger_h(0)\,|\,\Omega\rangle =$$

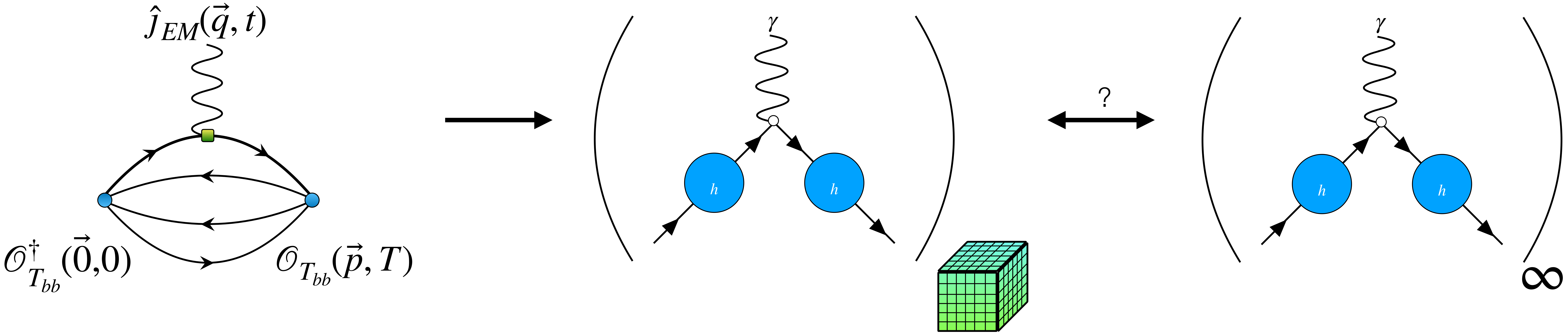
$$= \sum_{n,m} \frac{\mathcal{Z}^{f*}_n \mathcal{Z}^i_m}{(2E^f_n)(2E^i_m)} \cdot \mathcal{M}_{nm} \cdot e^{-E^f_n(T-t)} e^{-E^i_m t}$$



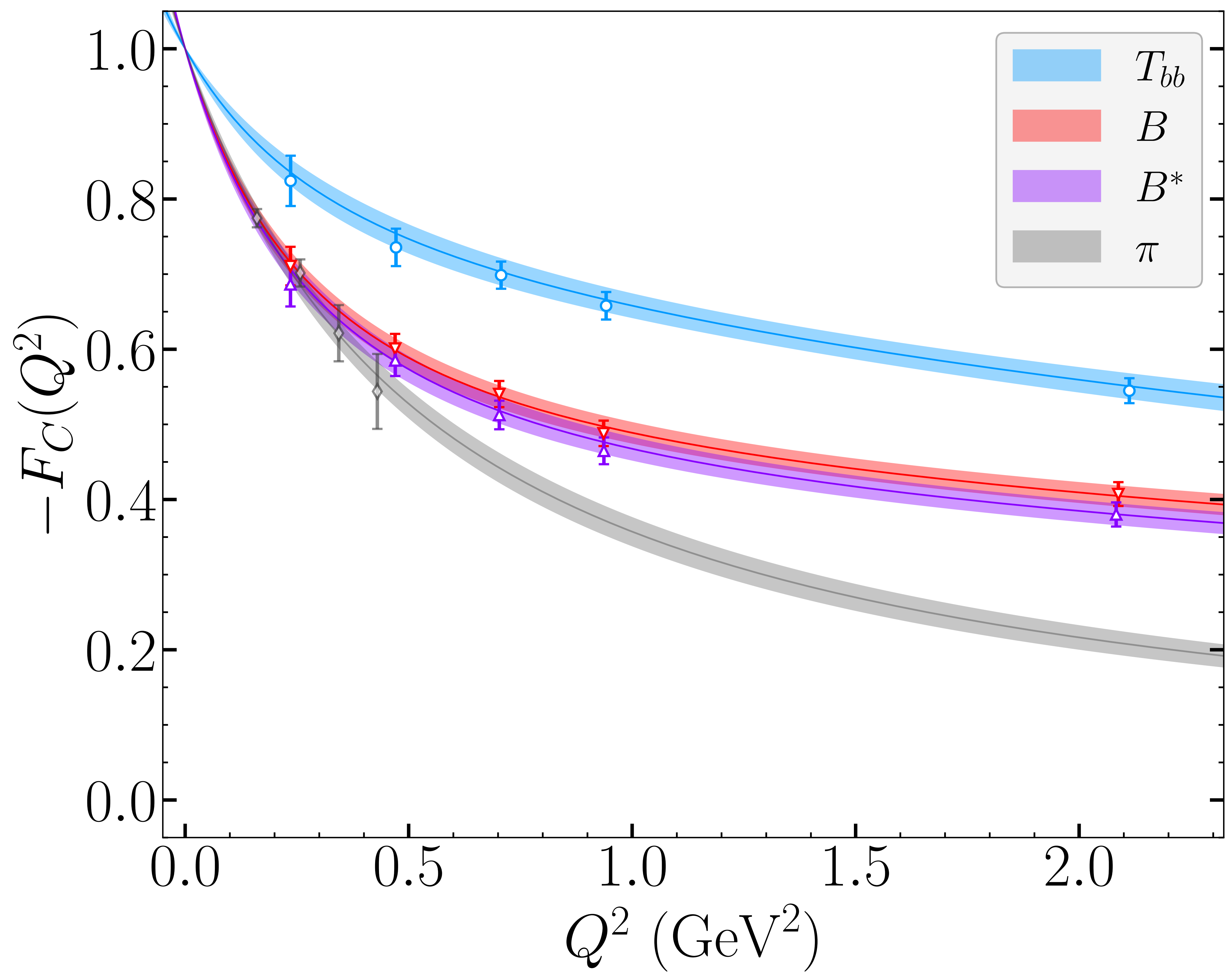


Finite-volume effects

- $h = T_{bb}, B, B^*, \pi \rightarrow$ all stable against the strong decay on the lattice
    - $\langle h \,|\, \hat{j} \,|\, h \rangle$ accessible directly from the lattice
    - exponentially suppressed finite-volume corrections
        - $\langle h \,|\, \hat{j} \,|\, h \rangle_L - \langle h \,|\, \hat{j} \,|\, h \rangle_\infty \sim \mathcal{O}(e^{-m_\pi L})$



Results: charge distributions of $T_{bb}, B^{(*)}, \pi$



- general parametrisation: $z-$expansion
  - captures the effect of the closest branch cut t_+ and resonance pole m_r

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{m_r^2}} \sum_{n=0}^{\infty} a_n z^n(Q^2, t_+, t_0) \qquad z(Q^2, t_+, t_0) = \frac{\sqrt{t_+ + Q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ + Q^2} + \sqrt{t_+ - t_0}}$$

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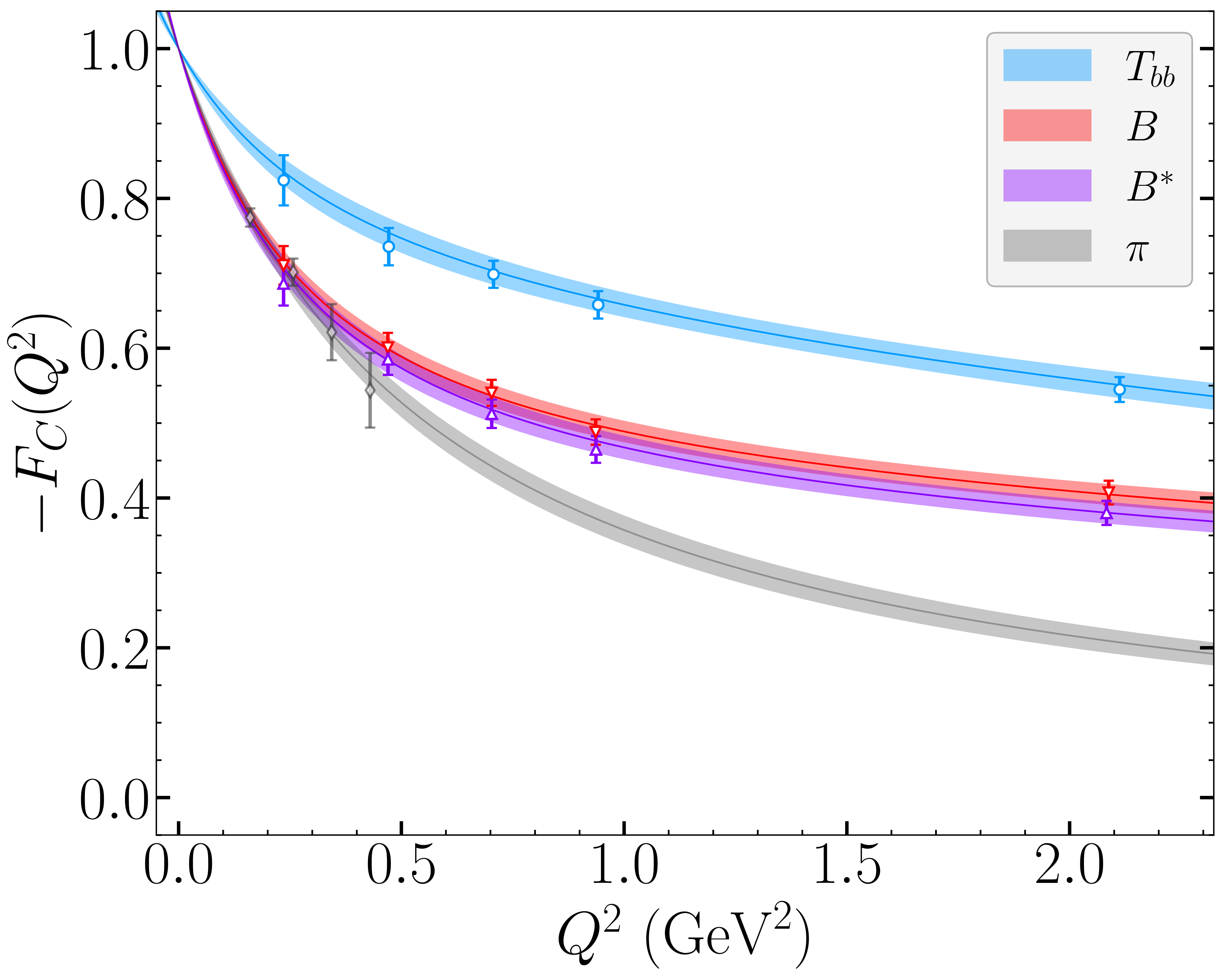
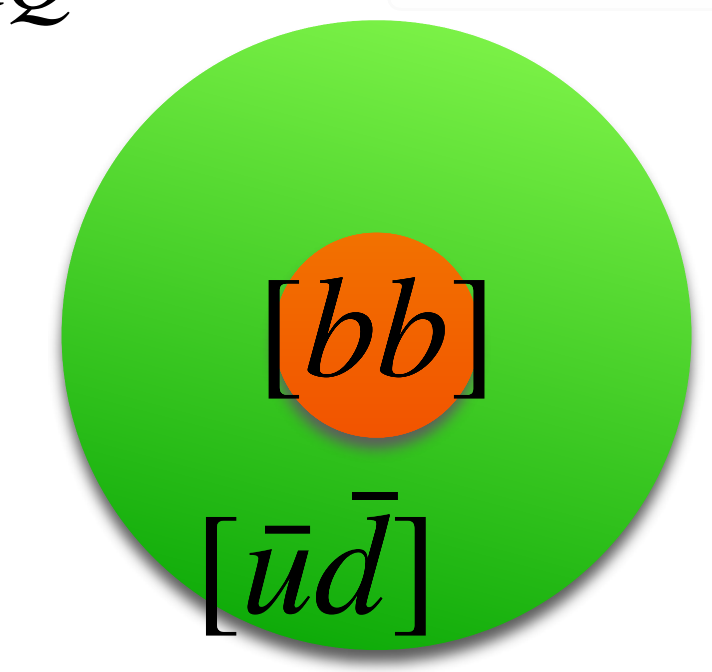


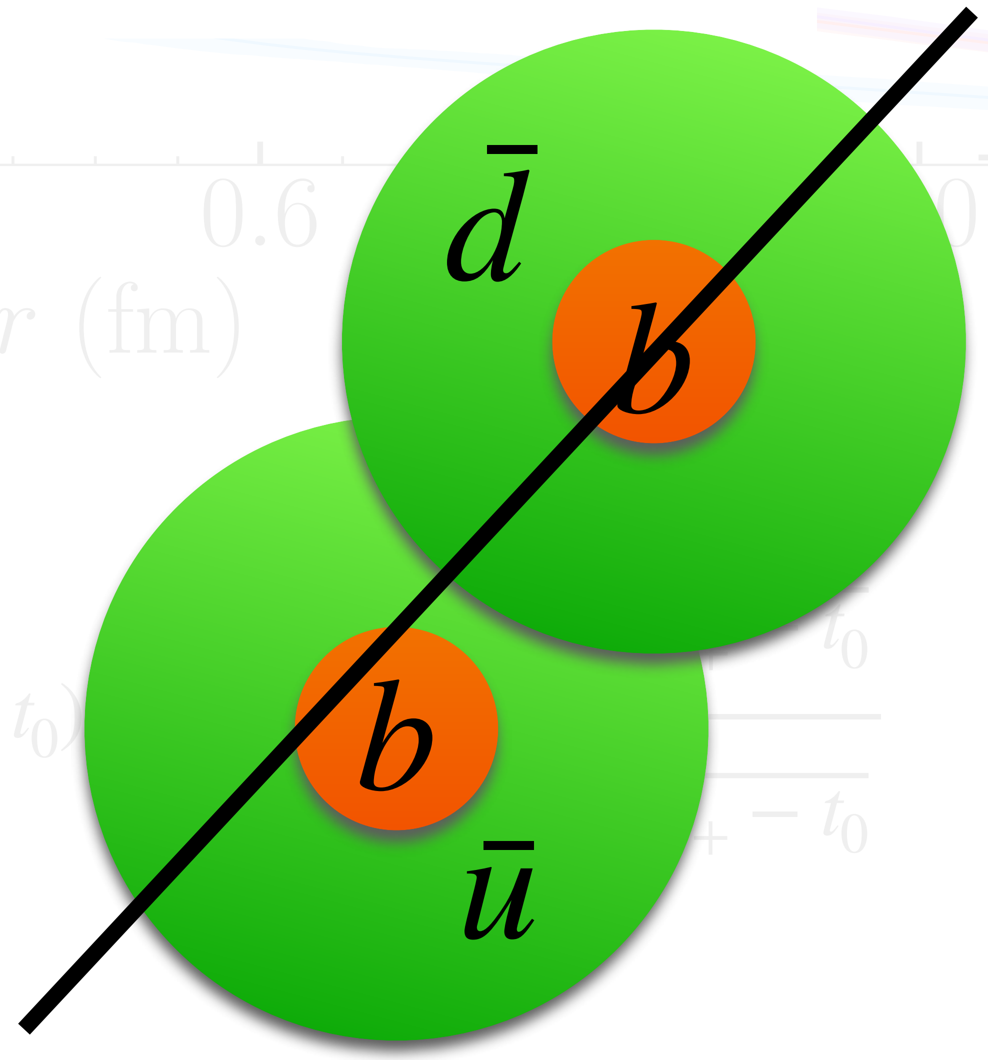
- charge form factors in momentum-space
- primary observable: $r_C \equiv \sqrt{\langle r_C^2 \rangle} = 6\dfrac{\mathrm{d}F_C}{\mathrm{d}Q^2}(0)$

h	$\sqrt{\langle r_C^2 \rangle}$ (fm)
T_{bb}	0.499(31)
B	0.692(21)
B^*	0.698(23)
π	0.652(20)



$$r_C^{T_{bb}} \ll r_C^B + r_C^{B^*}$$

- disfavors $B - B^*$ structure



Results: charge distributions of $T_{bb}, B^{(*)}, \pi$

- position-space charge densities
  - non-relativistic approximation
  - justified for hadrons with heavy quarks

$$\rho(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} F_C(|\vec{q}|^2)$$

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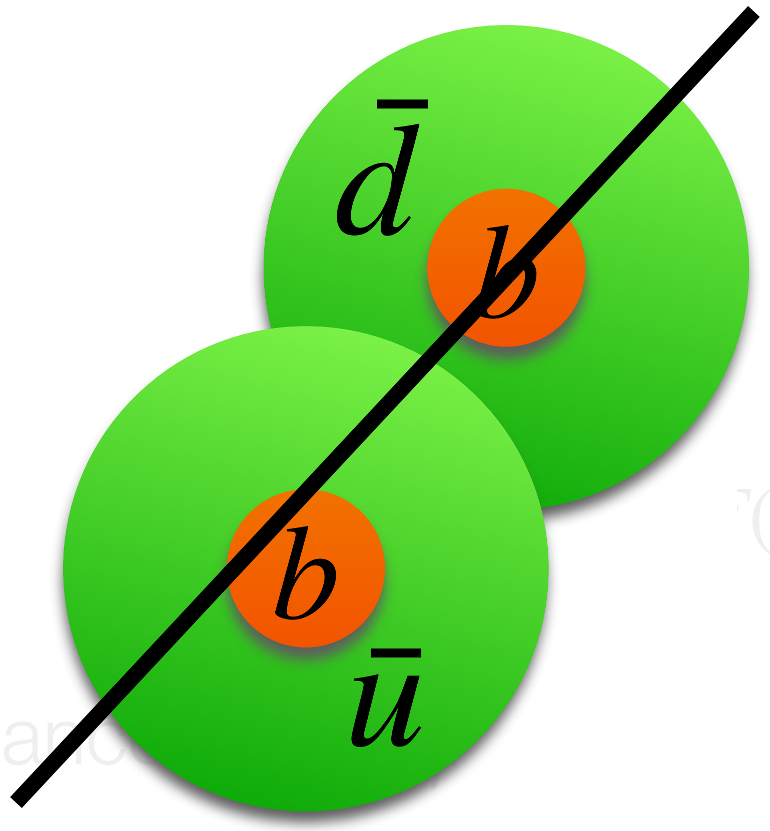
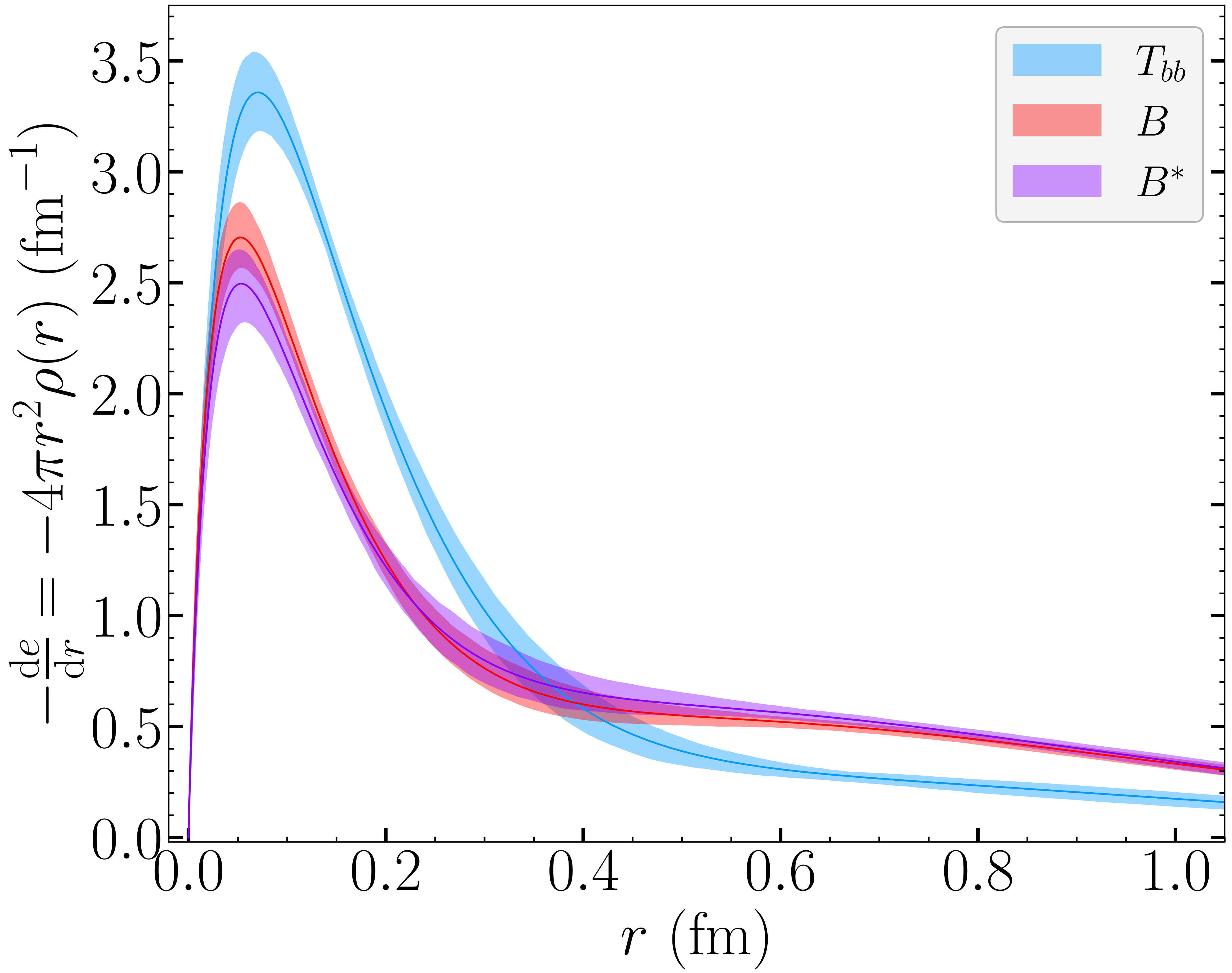
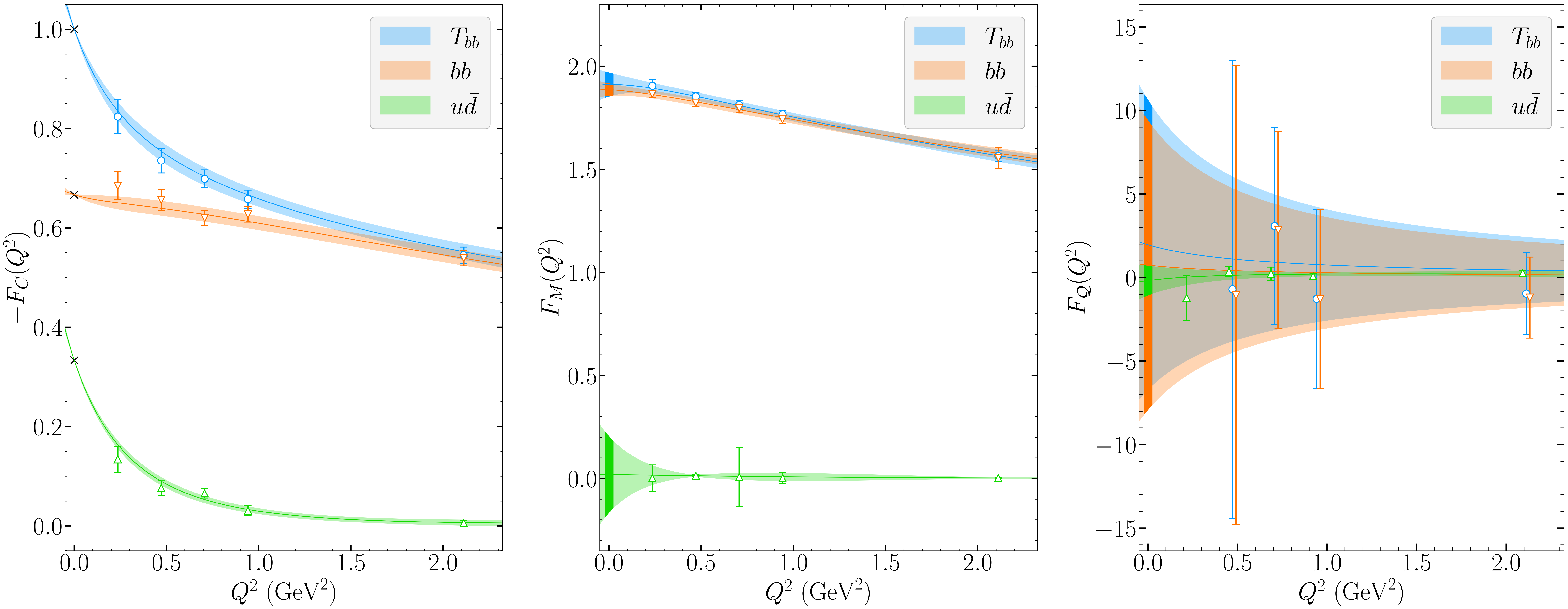




Results: T_{bb} form factors



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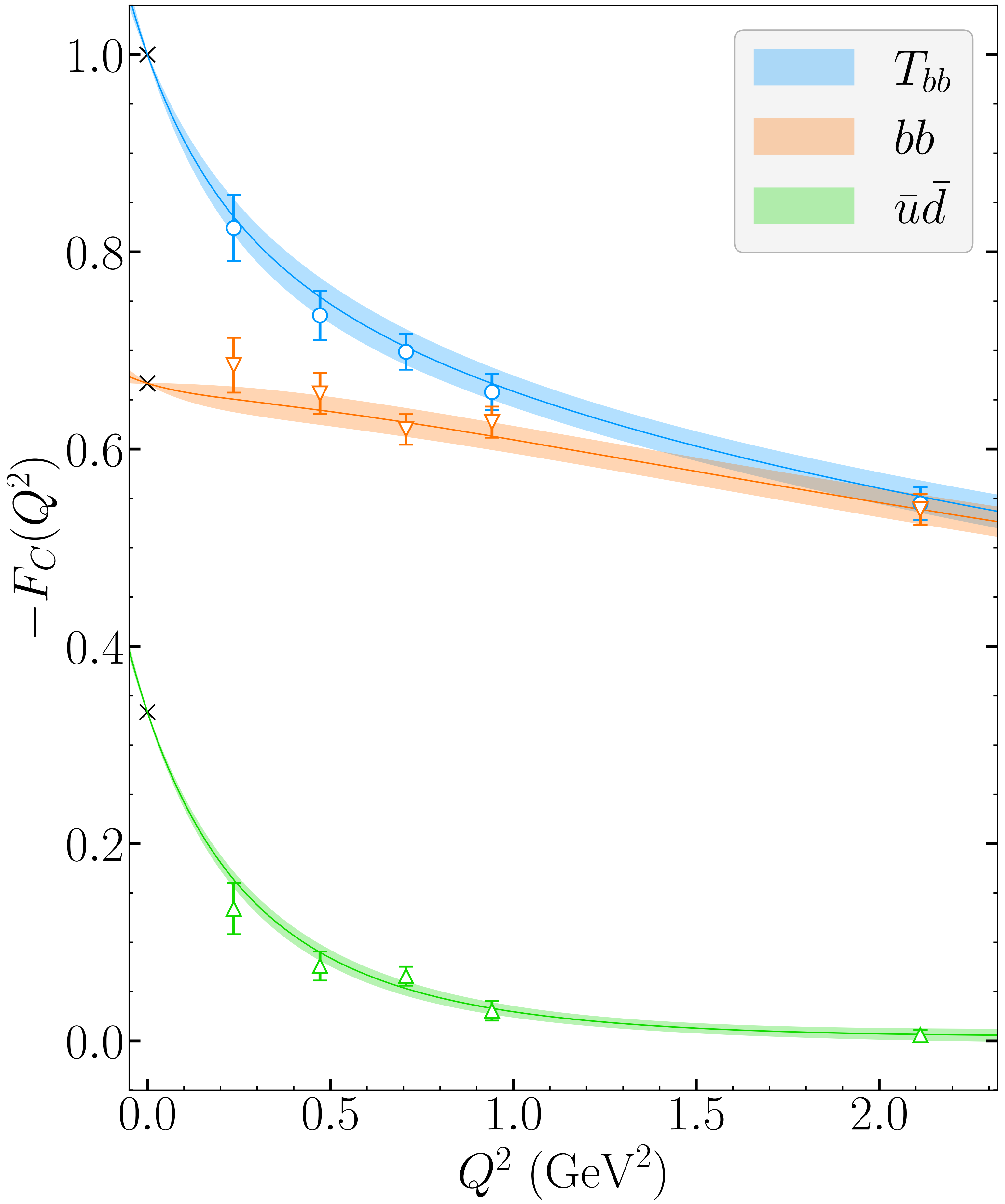
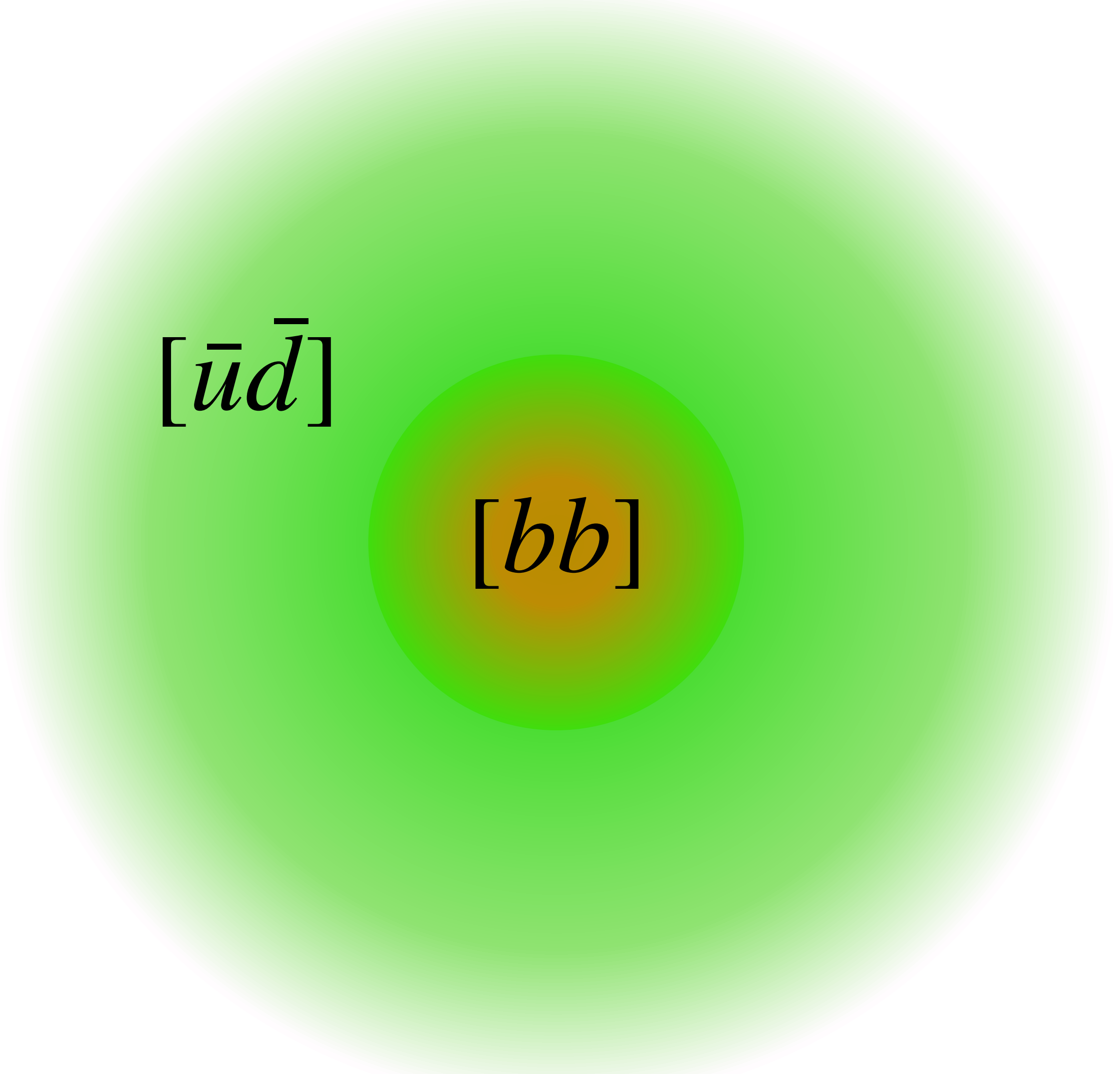


$$r_C \equiv \sqrt{\langle r_C^2 \rangle} = 6 \frac{\mathrm{d}F_C}{\mathrm{d}Q^2}(0)$$

- $r_C^{bb} \ll r_C^{\bar{u}\bar{d}}$
- strongly favors diquark-antidiquark structure

	T_{bb}	$[bb]$	$[\bar{u}\bar{d}]$
$\sqrt{\langle r_C^2 \rangle}$ (fm)	0.499(31)	0.174(59)	0.511(14)



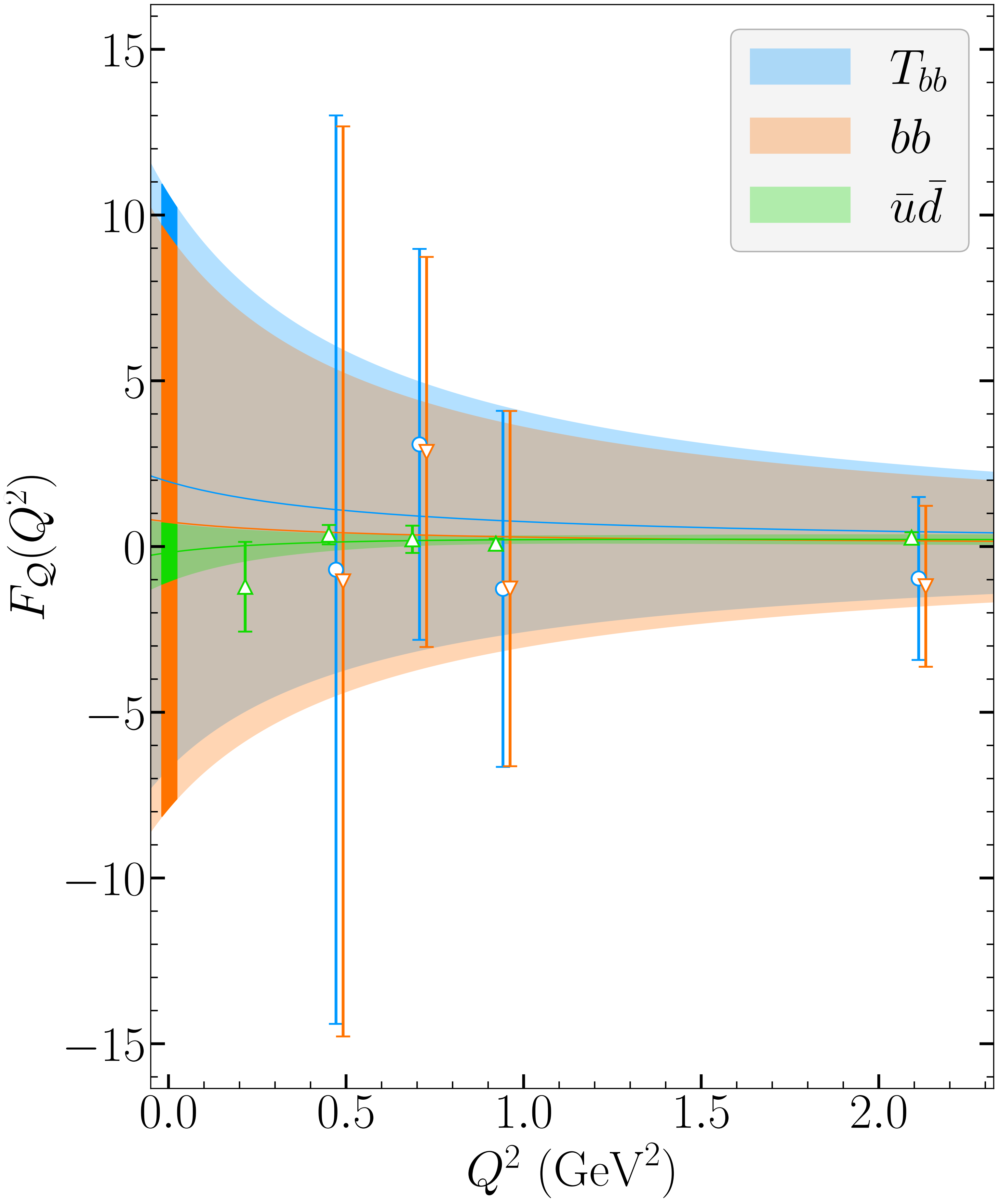
- $\left\{[bb]_{\bar{c}}^{l_{bb},s_{bb}}[\bar{u}\bar{d}]_{c}^{I=0,l_{\bar{u}\bar{d}},s_{\bar{u}\bar{d}}}\right\}_{l_{12}}^{J^P=1^+}$

Results: T_{bb} form factors

- all three components components consistent with zero at $Q^2 = 0$
- can only happen with lowest config. of orbital angular momenta: $l_{bb} = l_{\bar{u}\bar{d}} = l_{12} = 0$

	T_{bb}	$[bb]$	$[\bar{u}\bar{d}]$
$\sqrt{\langle r_C^2 \rangle}$ (fm)	0.499(31)	0.174(59)	0.511(14)
$2m_{T_{bb}} \cdot \mu$	1.912(57)	1.887(29)	0.02(18)
$m_{T_{bb}}^2 \cdot \mathcal{Q}$	1.9(8.7)	0.6(8.8)	−0.18(86)

- $\{[bb]_{\bar{c}}^{l_{bb}=0, s_{bb}} [\bar{u}\bar{d}]_{c}^{I=0, l_{\bar{u}\bar{d}}=0, s_{\bar{u}\bar{d}}}\}_{l_{12}=0}^{J^P=1^+}$



Results: T_{bb} form factors

- heavy- and light-quark pair spin correlation
- total magnetic dipole moment saturated by $b-$quarks

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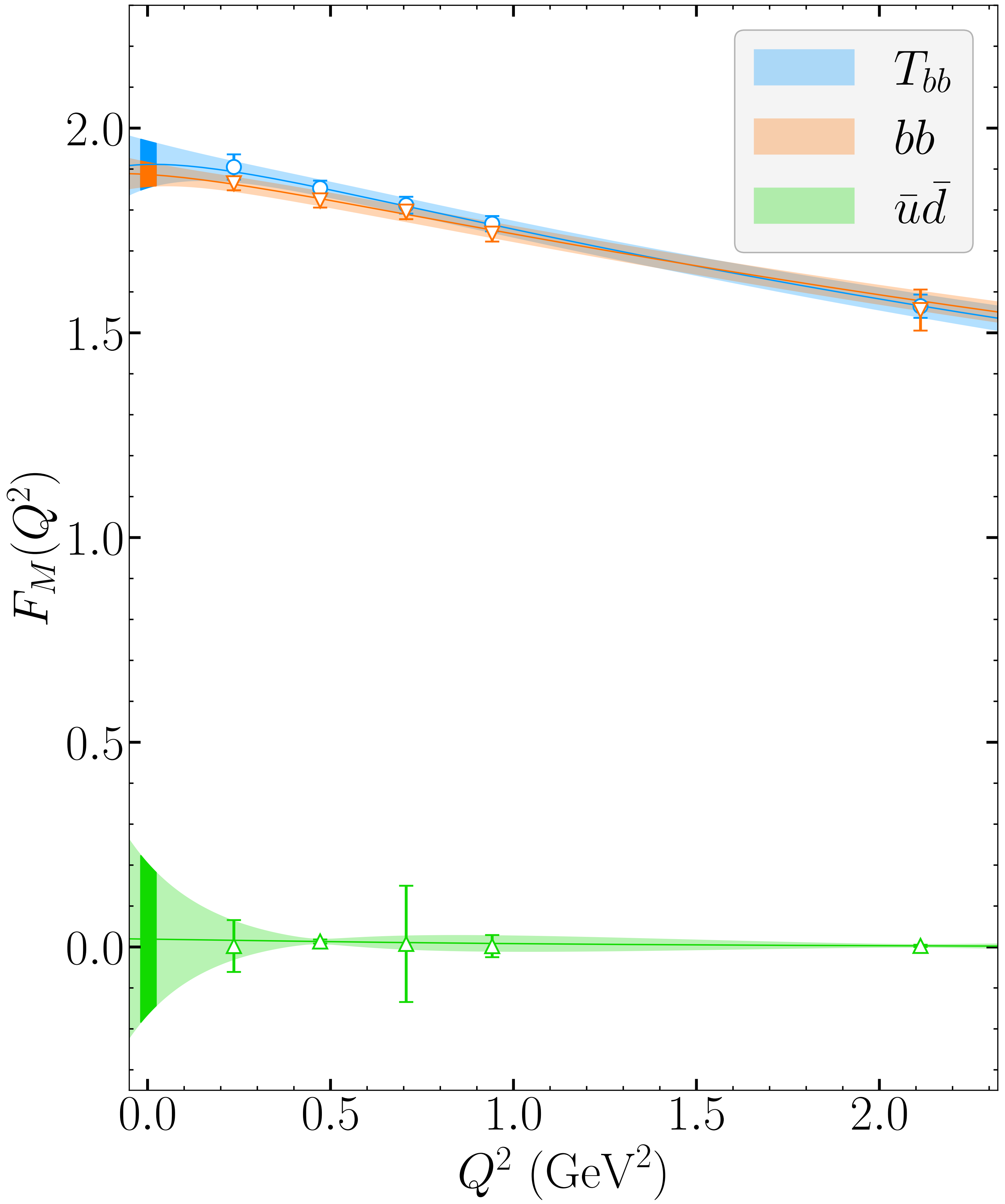
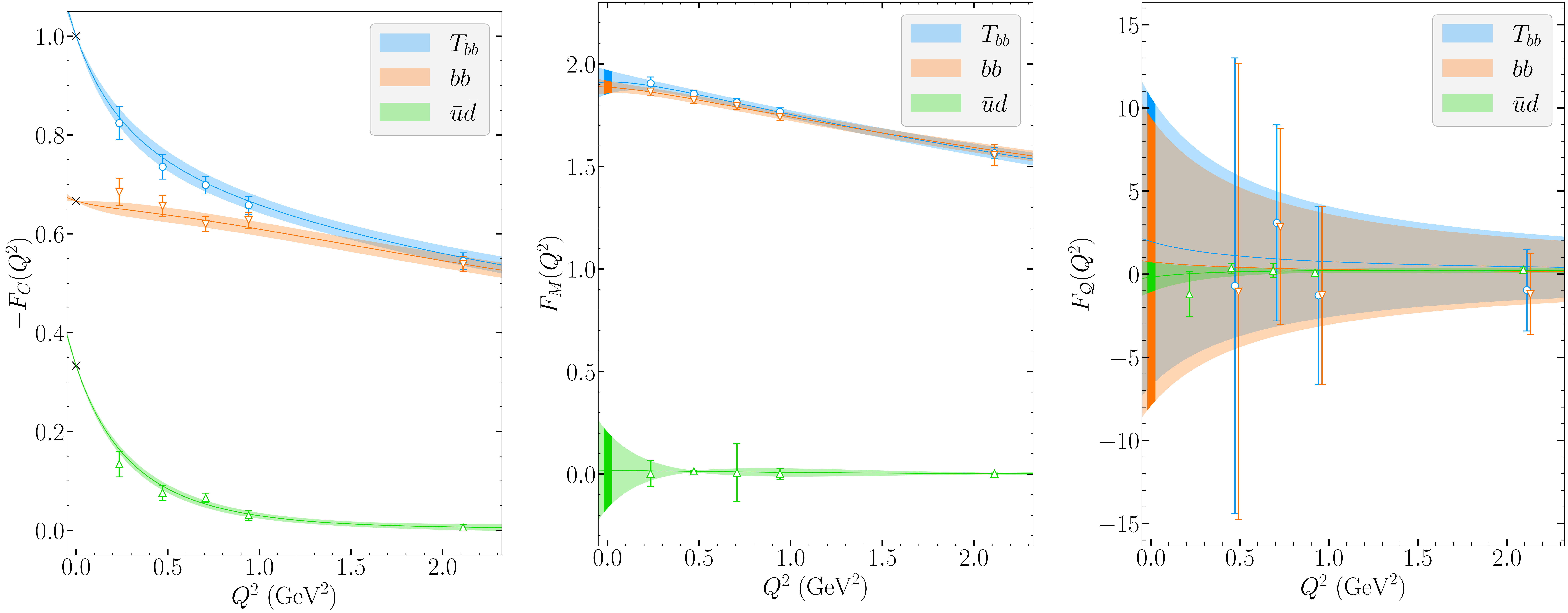
$2m_{T_{bb}} \cdot \mu$	1.912(57)	1.887(29)	0.02(18)
$m_{T_{bb}}^2 \cdot Q$	1.9(8.7)	0.6(8.8)	−0.18(86)

- $\mu_{T_{bb}} = \langle T_{bb} | \sum_{q=u,d,b} \frac{e_q}{2m_q}(\hat{l}_q + g_q \hat{s}_q) | T_{bb} \rangle = \frac{F_M(0)}{2m_{T_{bb}}}$
- $s_{bb} = 1, \quad s_{\bar{u}\bar{d}} = 0$



- $\{[bb]_{\bar{c}}^{l_{bb}=0, s_{bb}=1} [\bar{u}\bar{d}]_c^{I=0, l_{\bar{u}\bar{d}}=0, s_{\bar{u}\bar{d}}=0}\}_{l_{12}=0}^{J^P=1^+}$

Results: T_{bb} form factors



$$\odot\ \{[bb]_{\bar{c}}^{l_{bb}=0,s_{bb}=1}[\bar{u}\bar{d}]_{c}^{I=0,l_{\bar{u}\bar{d}}=0,s_{\bar{u}\bar{d}}=0}\}_{l_{12}=0}^{J^P=1^+}$$

$$c \xrightarrow{?} 3,\bar{6}$$

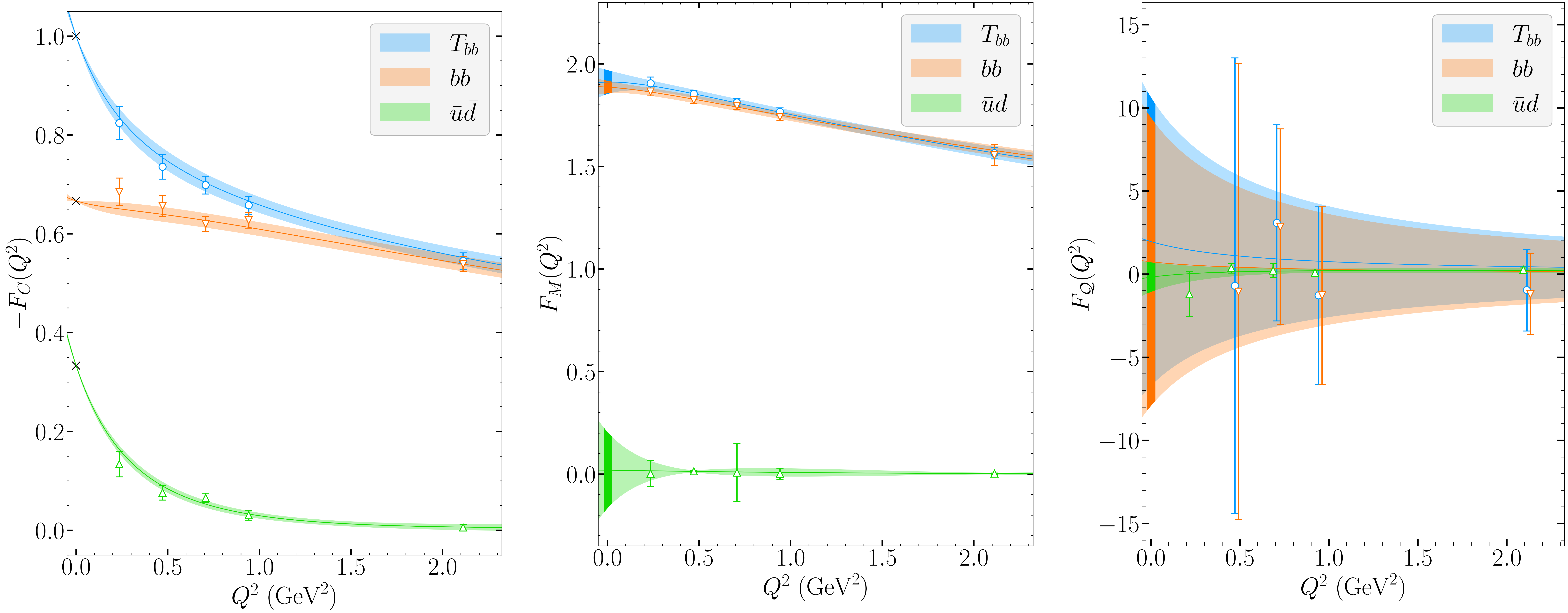
Results: T_{bb} form factors

- $\{[bb]_{\bar{c}}^{l_{bb}=0,s_{bb}=1}[\bar{u}\bar{d}]_{c}^{I=0,l_{\bar{u}\bar{d}}=0,s_{\bar{u}\bar{d}}=0}\}_{l_{12}=0}^{J^P=1^+}$

$c \xrightarrow{?} 3,\bar{6}$

- color config. constrained by Pauli principle
- $(-1)^{s_{bb}+l_{bb}+c} = 1, \quad (-1)^{s_{\bar{u}\bar{d}}+l_{\bar{u}\bar{d}}+c} = -1 \quad \longrightarrow \quad c = 3$

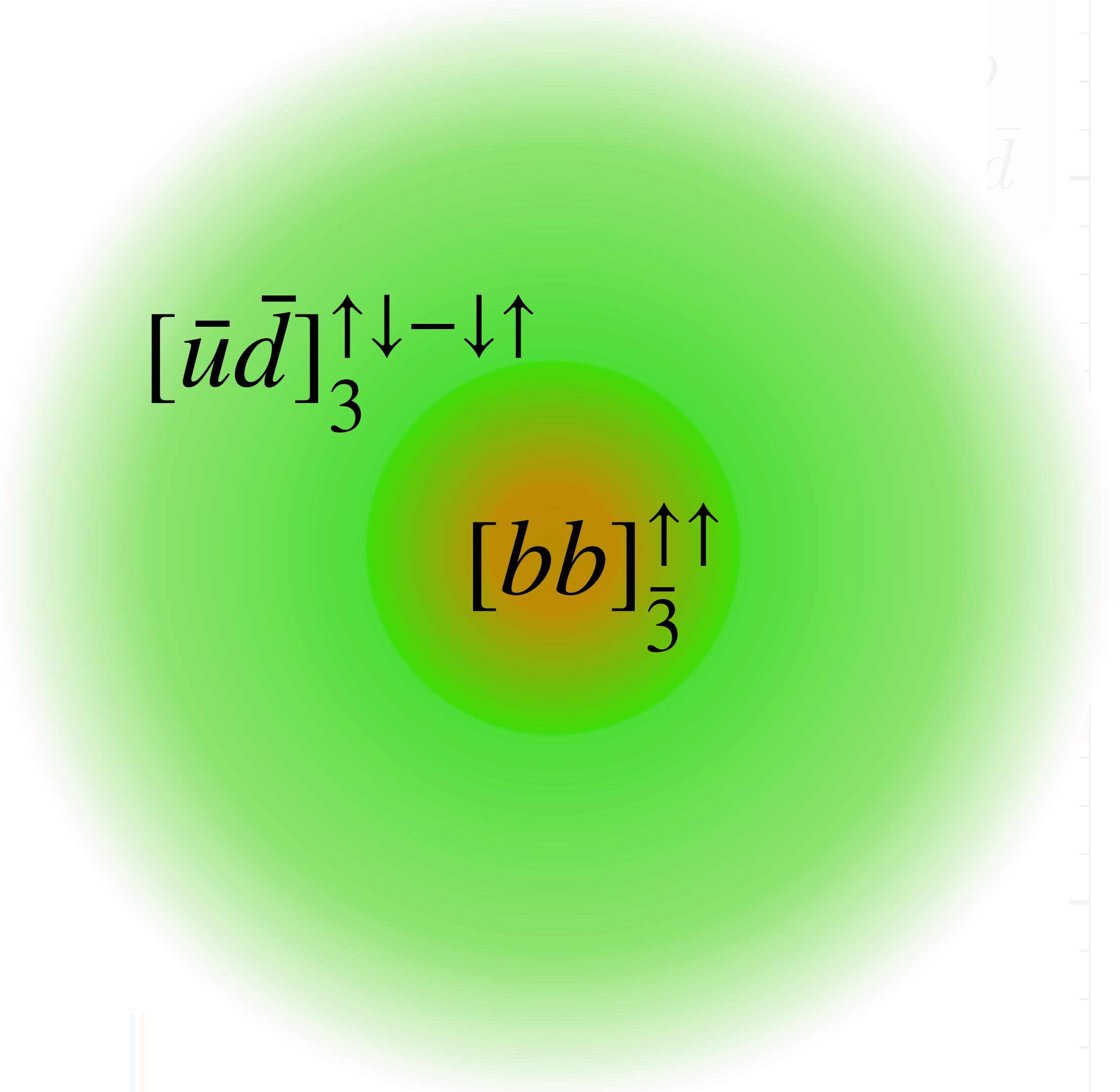
Results: T_{bb} form factors



$$\{[bb]_{\bar{3}}^{l_{bb}=0,s_{bb}=1}[\bar{u}\bar{d}]_{3}^{I=0,l_{\bar{u}\bar{d}}=0,s_{\bar{u}\bar{d}}=0}\}_{l_{12}=0}^{J^P=1^+}$$

Summary

- first glimpse of T_{bb} structure via its EM form factors
- extracted ground-state electromagnetic matrix elements
    - five values of $Q^2 = -(p_2 - p_1)^2$
    - four states: T_{bb}, B, B^*, π
        - obtained all available EM form factors for each state
- lattice three-pt data + Pauli sufficiently constrain structure

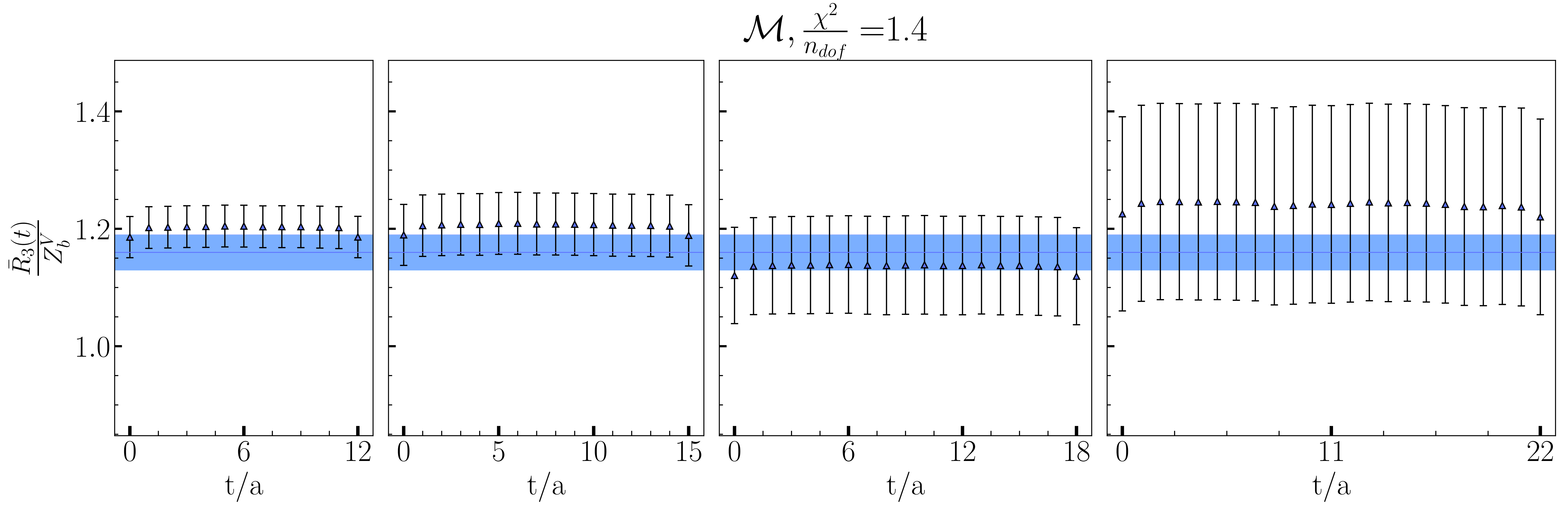


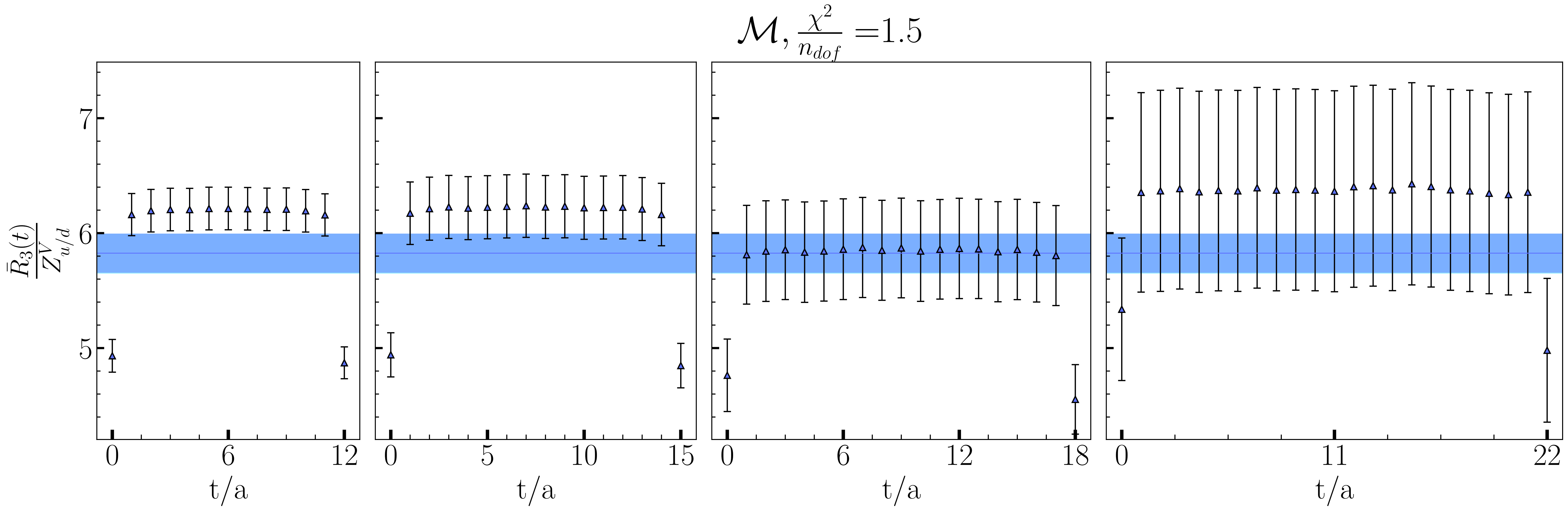
- $\left\{ [bb]_{\bar{3}}^{l_{bb}=0, s_{bb}=1} [\bar{u}\bar{d}]_3^{I=0, l_{\bar{u}\bar{d}}=0, s_{\bar{u}\bar{d}}=0} \right\}_{l_{12}=0}^{J^P=1^+}$

Backup: vector current renormalization

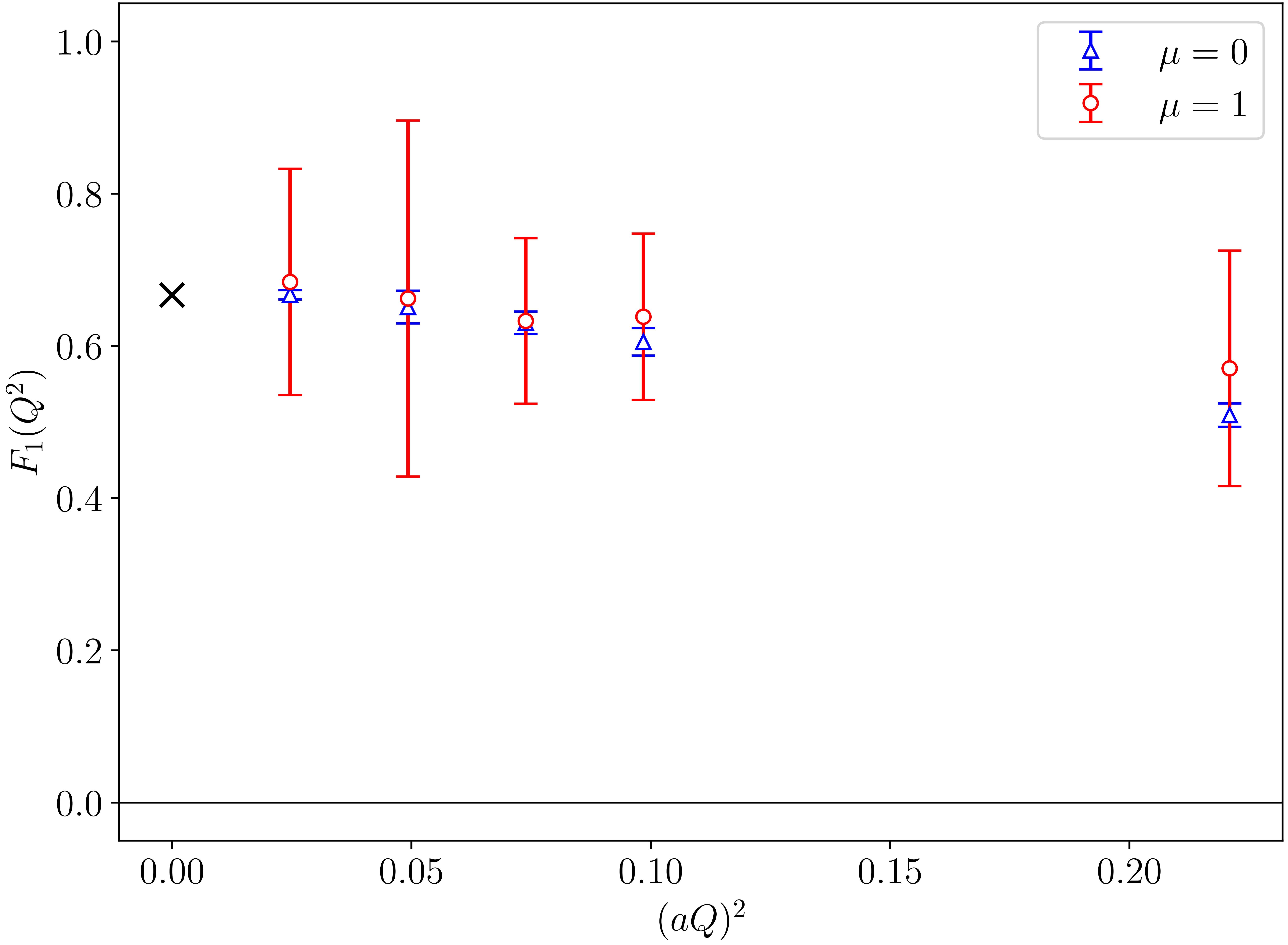
$$\hat{j}^{\mu}_{EM} = Z^V_{u/d}\left(\frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d\right) - Z^V_b \cdot \frac{1}{3}\bar{b}\gamma^\mu b$$

- renormalization conditions:
  - $F_C^{b,T_{bb}} \equiv -\dfrac{2}{3}$
  - $F_C^{\bar{u},T_{bb}} \equiv -\dfrac{2}{3}$
- $Z_b^V = 3.97(11), \quad Z_{u/d}^V = 0.725(16)$





Backup: current renormalization anisotropy



Backup: T_{bb} quadrupole contribution to charge density $\rho(\vec{r})$