Axion String Source Modelling

with

Adaptive Mesh Refinement

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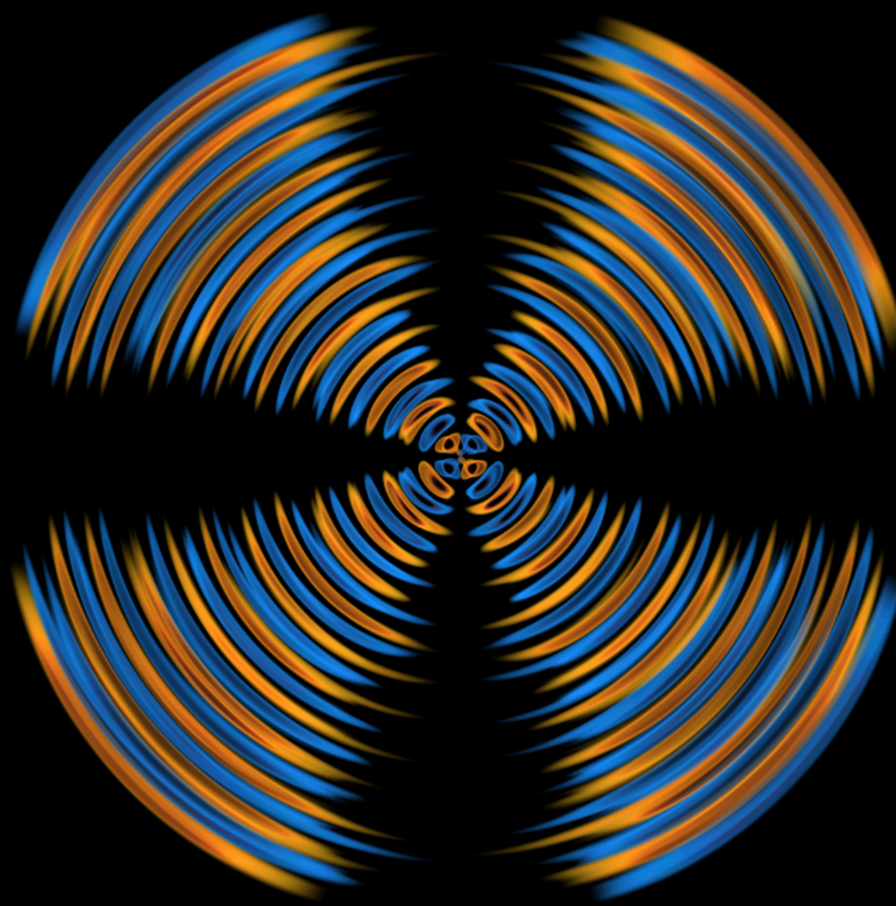
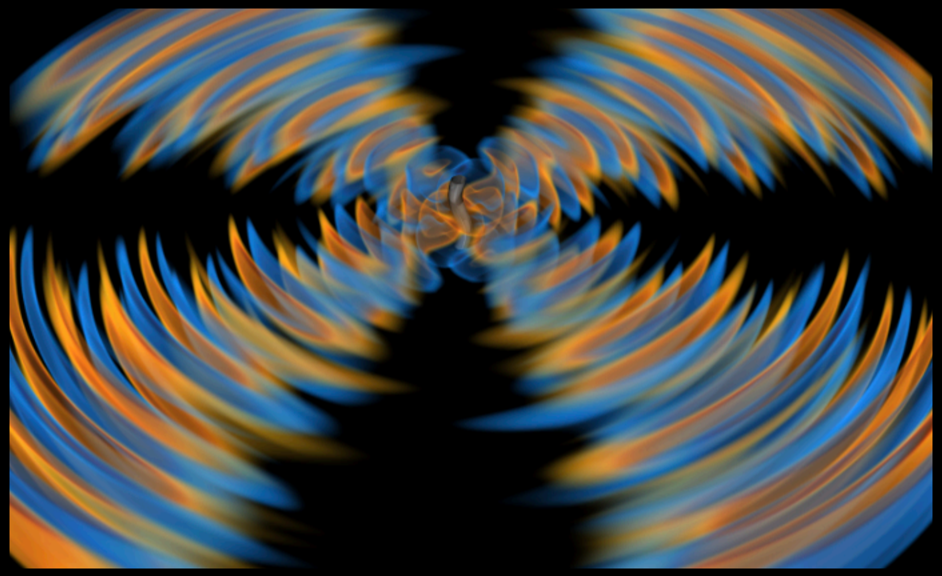
Based on:
A.D., E. P. S. Shellard [1910.01718]
A.D., E. P. S. Shellard [2211.10184]
A.D., T. Kinowski, E. P. S. Shellard [2312.07701]

Overview

- Why Axions?
- Axion Strings
- Analytic Modelling
- Numerical Implementation
- Conclusion





Why Axions?

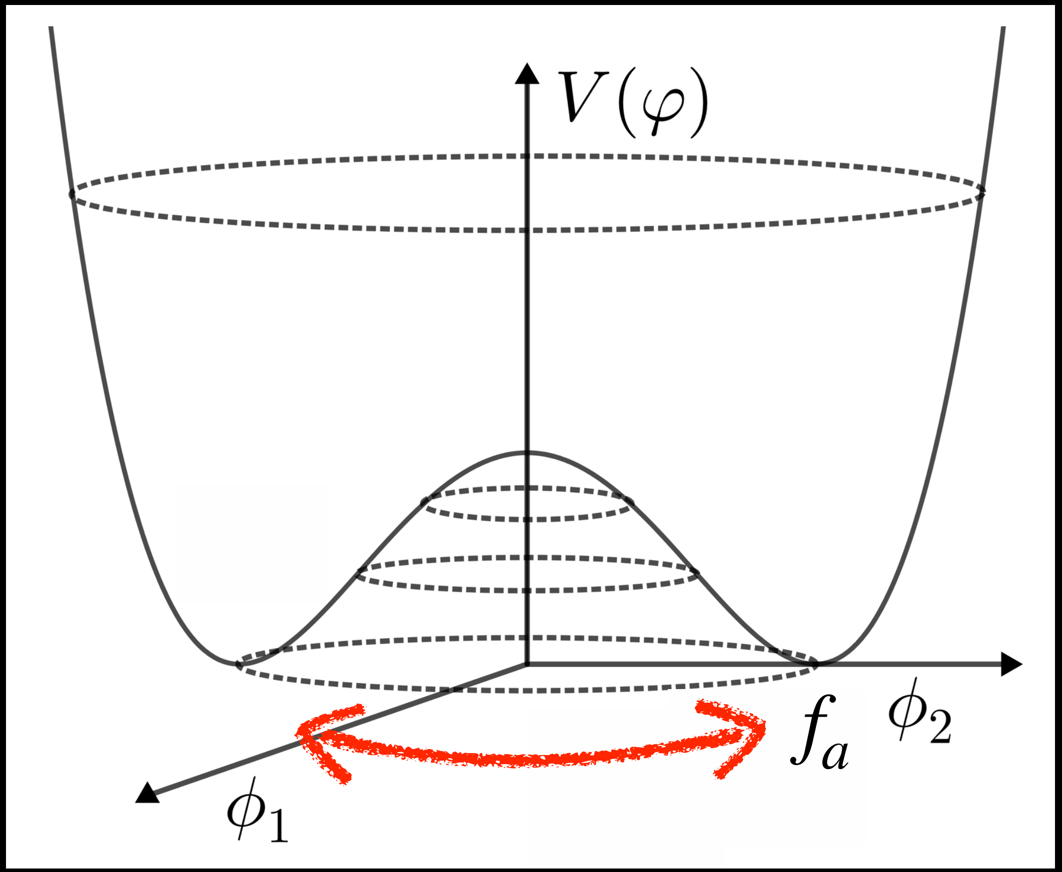
- Well-motivated dark matter candidate
  - Stable, cold, weakly coupled
  - Solve eg. Strong CP problem
- Rich phenomenology
- Current target of experiments

Axion Strings

- Pseudo Nambu-Goldstone boson of global $U(1)_{\rm PQ}$ spontaneously broken at energy scale f_a

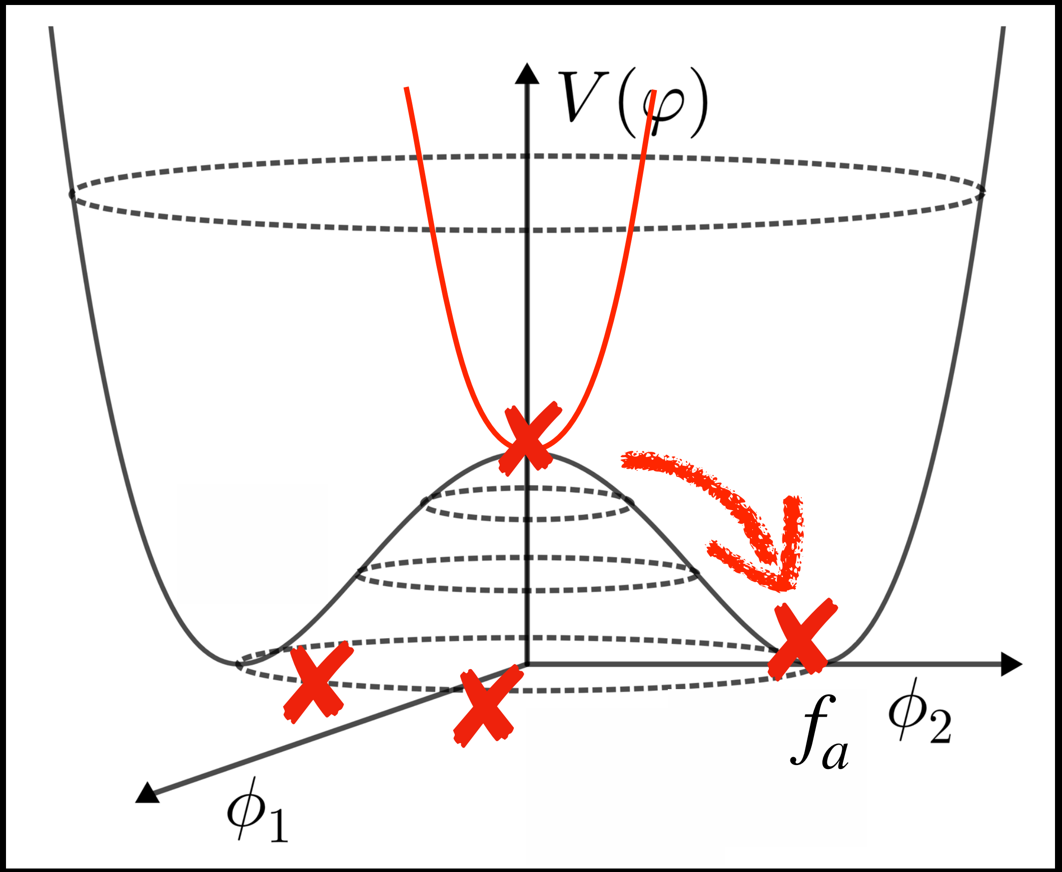
- Represented by phase $\vartheta(x)$ of (classical) complex scalar

$$\varphi(x) = |\varphi|(x)\, e^{i\vartheta(x)/f_a}$$



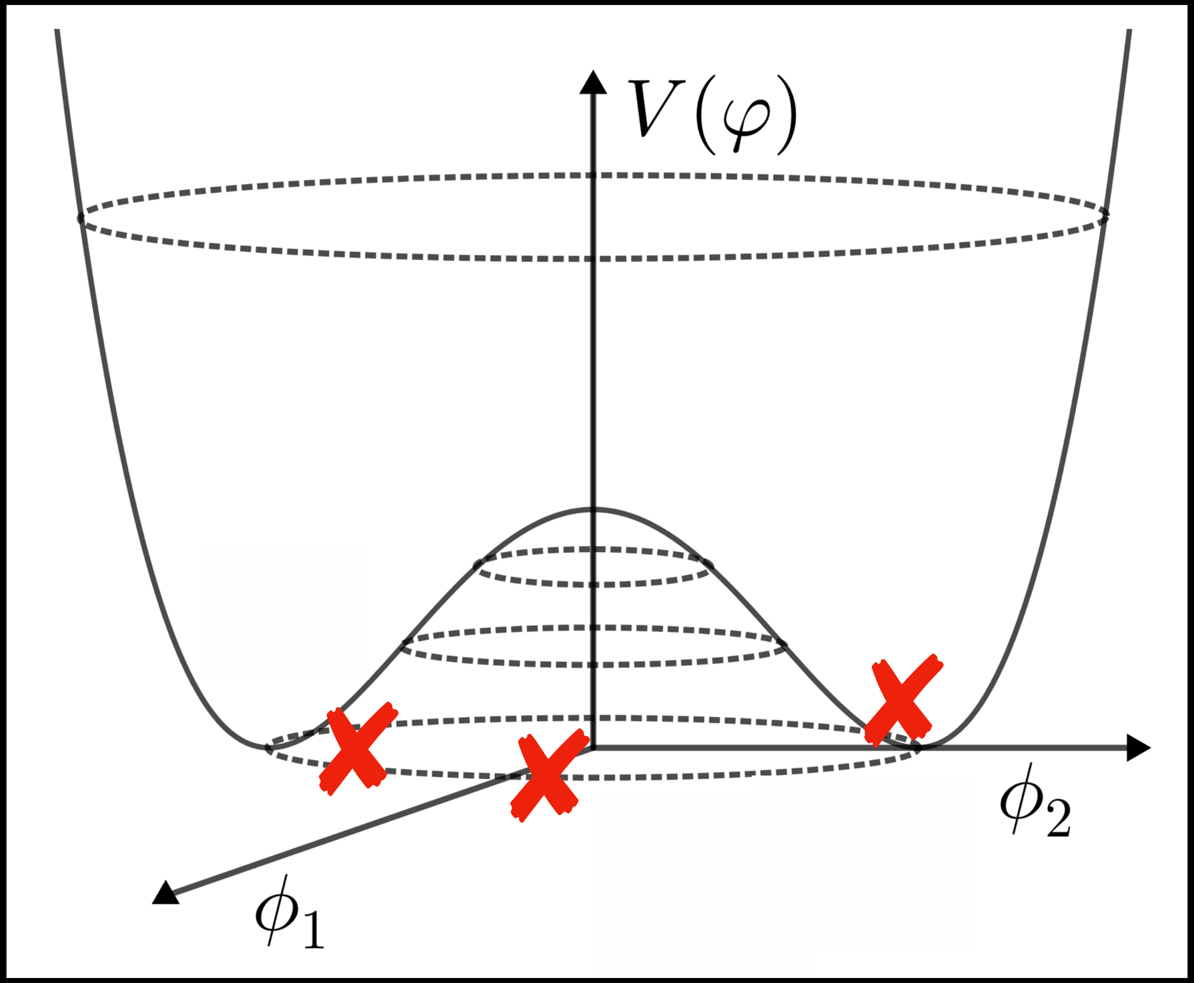
Axion Strings

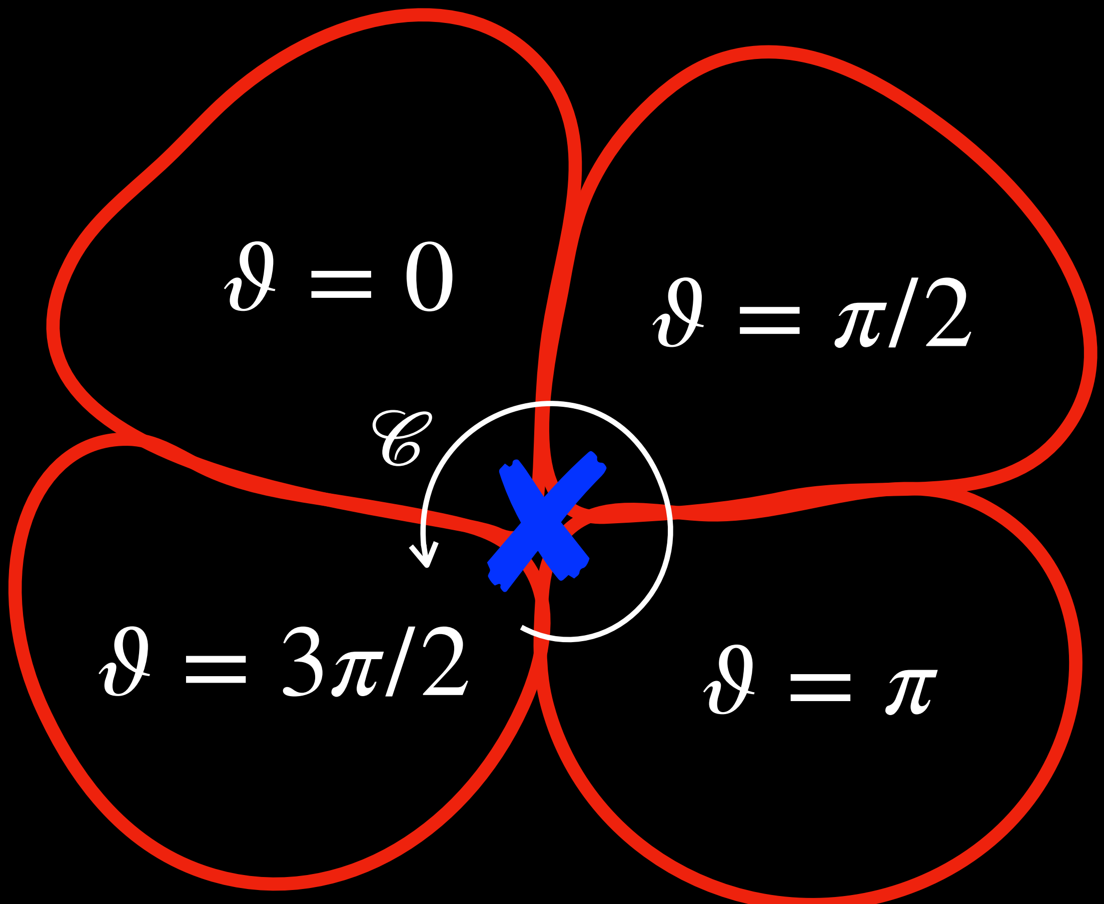
- Vacuum manifold $\mathscr{M} = S^1$ is non-contractible
- Process of spontaneous symmetry breaking $\rightarrow$ string defects
- Held in place by topology - cannot decay
- Cosmologically, form network of axion strings



Axion Strings

String = original, higher energy (unbroken phase)





$$\oint_{\mathscr{C}} \frac{\mathrm{d}\vartheta}{\mathrm{d}\theta}\,\mathrm{d}\theta = 2\pi n \neq 0$$

Axion Strings

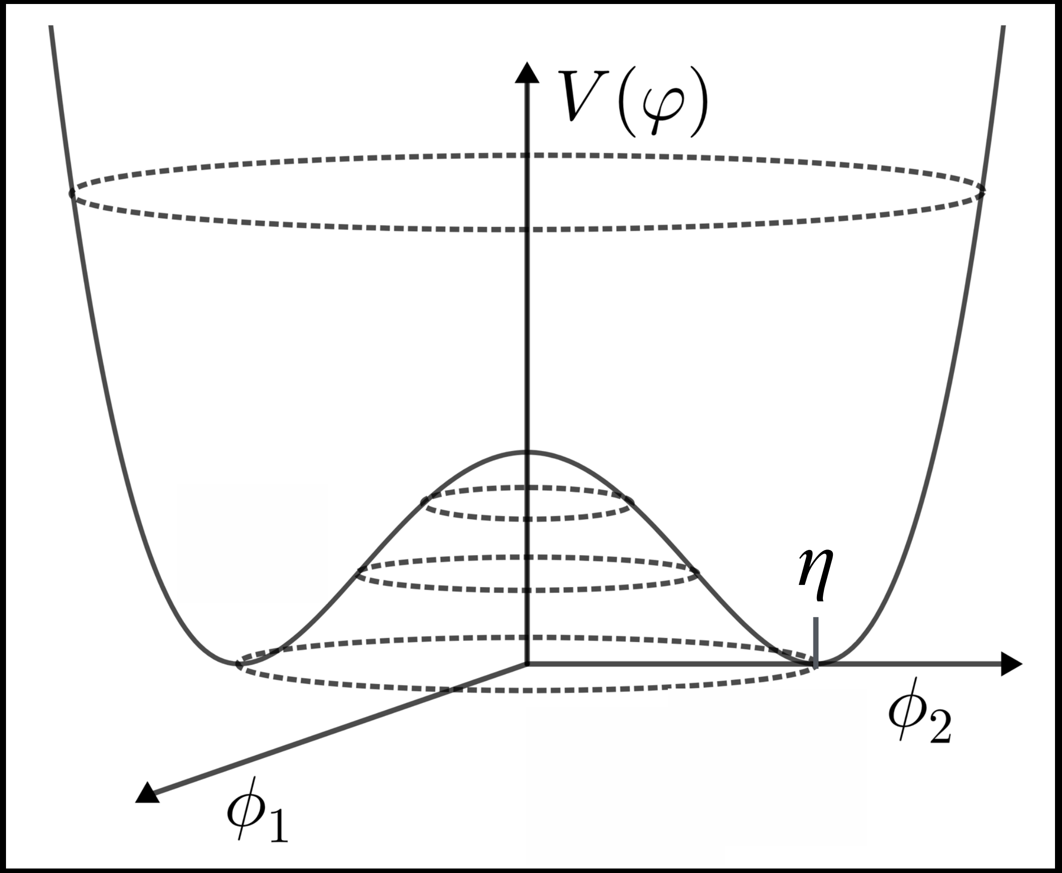
- 2D profile has circular symmetry
- Static ansatz

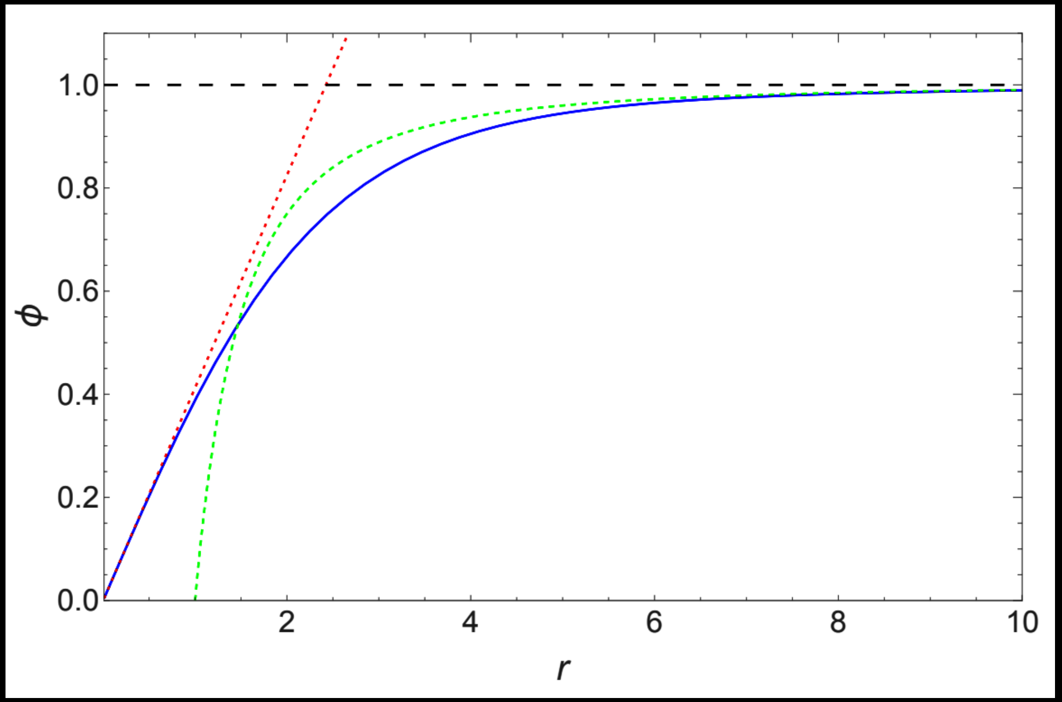
$$\varphi = \phi(r)\, e^{in\theta}$$

$$\downarrow$$

$$\partial_\mu \partial^\mu \varphi + \frac{\lambda}{2}\varphi(|\varphi|^2 - \eta^2) = 0$$

$$\varphi(0) = 0 \qquad \varphi(r) \to \eta \text{ as } r \to \infty$$





$$\longleftrightarrow \qquad \delta \sim (\sqrt{\lambda}\eta)^{-1}$$

Axion Strings

- Characterised by mass per unit length or 'tension' μ

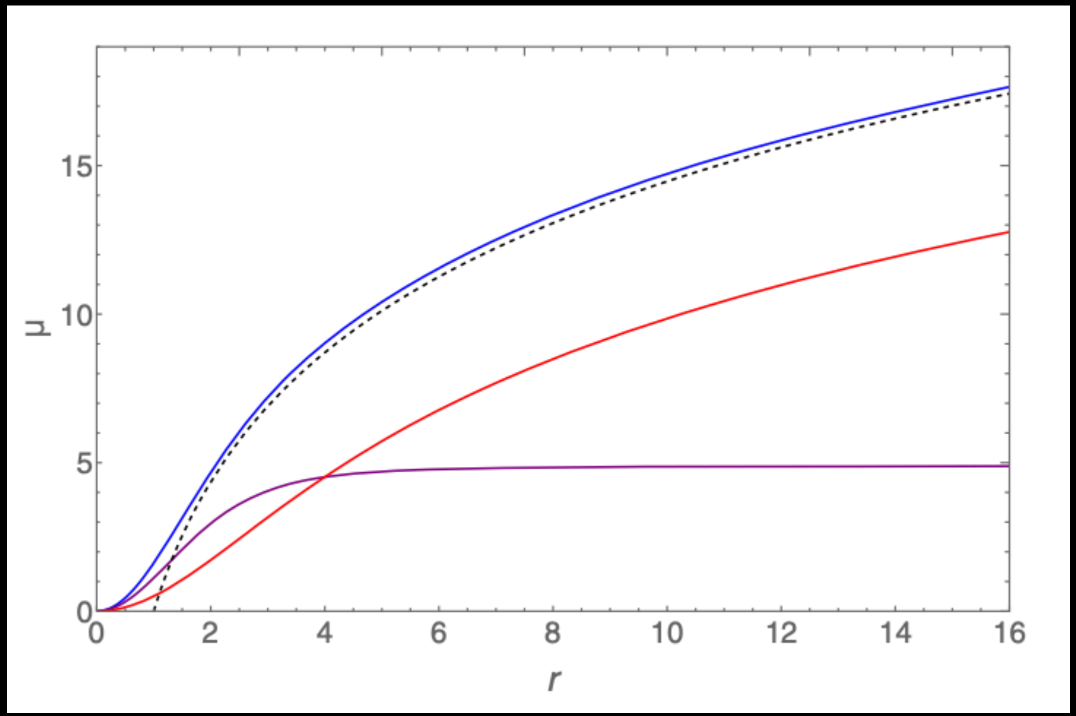
$$\rho(r) = \left(\frac{d\phi}{dr}\right)^2 + \frac{\lambda}{4}\left(\phi^2 - \eta^2\right)^2 + \left(\frac{n\phi}{r}\right)^2 \qquad \left(\frac{1}{r}\left|\frac{\partial\varphi}{\partial\theta}\right|\right)^2 = \left(\frac{n\phi}{r}\right)^2$$

Long-range 'winding'

Massive core μ_0

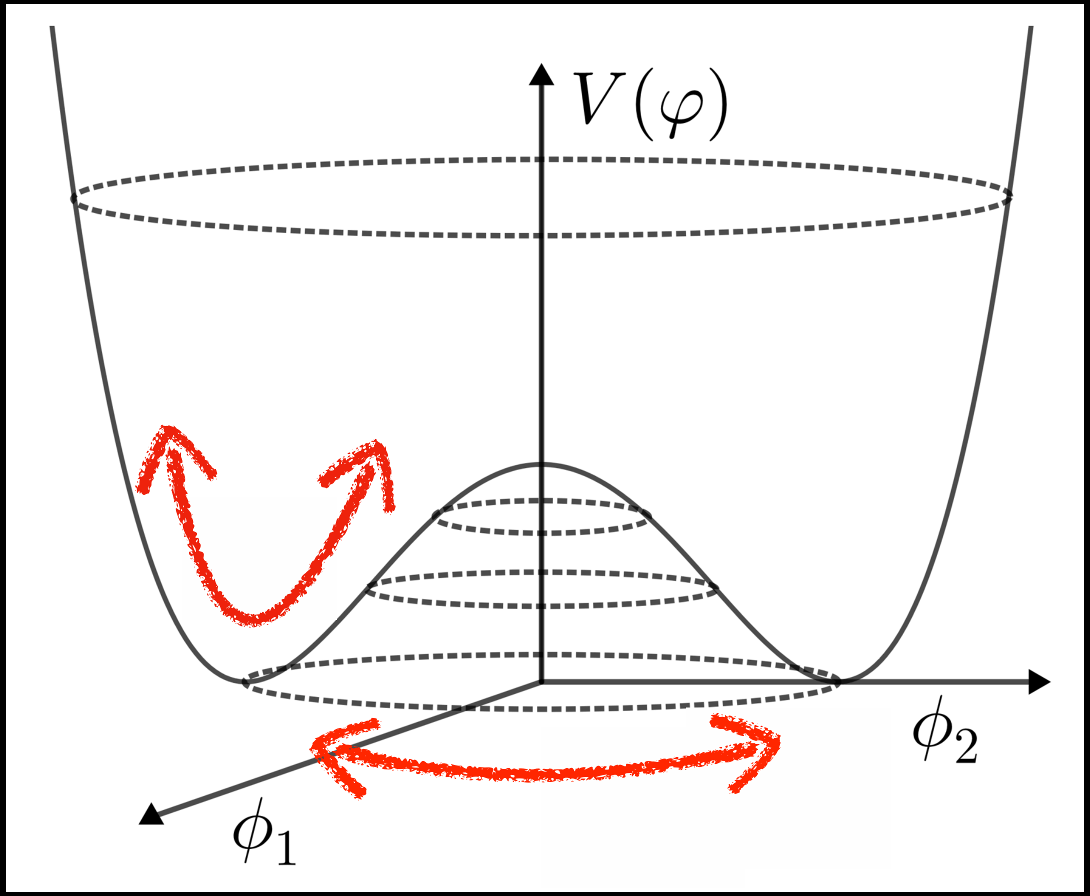
$$\mu(R) = \mu_0 + \mu_\theta(R)$$

$$\mu_\theta(R) \approx \int_\delta^R \left(\frac{n\phi}{r}\right)^2 2\pi r\,\mathrm{d}r = \underline{2\pi\eta^2 \ln(R/\delta)}$$



Axion Strings

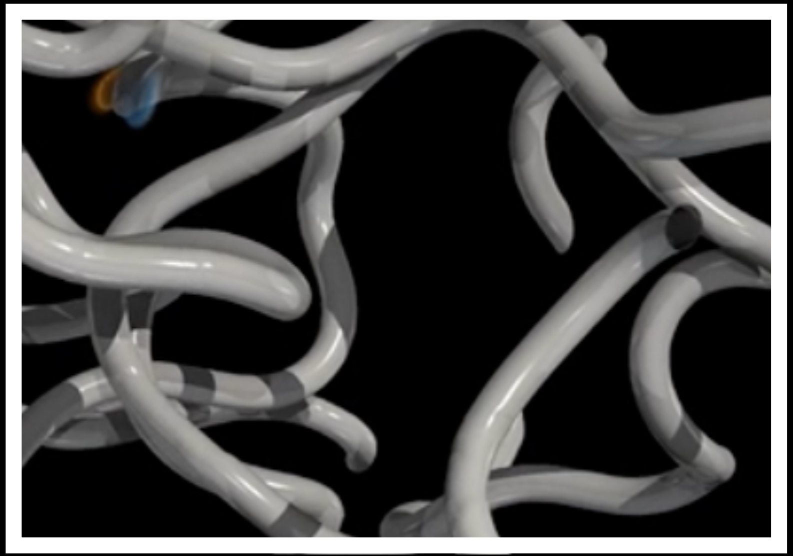
- When perturbed away from ground state, strings emit:
  - Massless radiation (axions)
  - Massive modes $\delta^{-1} \approx m_H = \sqrt{\lambda}\eta$
  - Gravitational waves (sourced by $T_{\mu\nu}$)

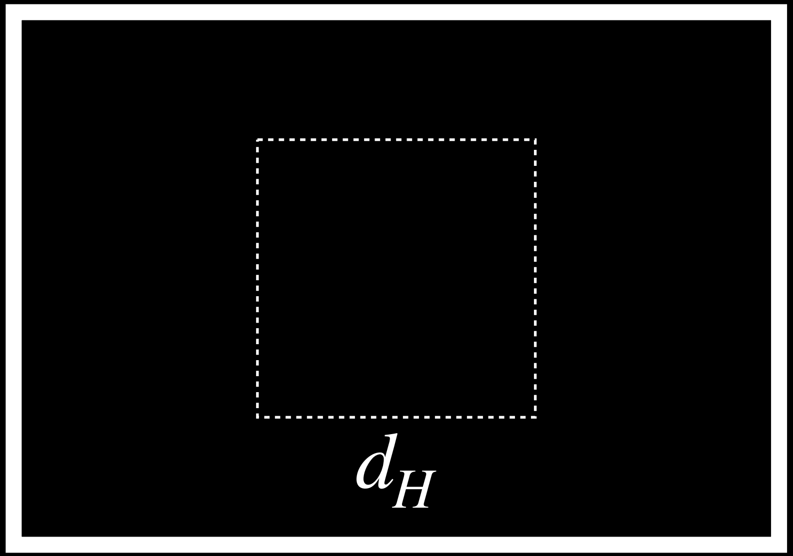


Axion Strings

Time

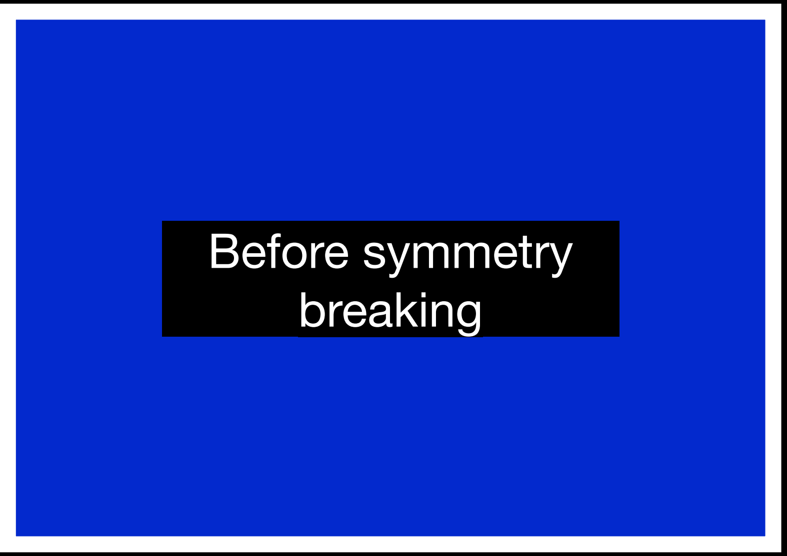
Pre-Inflationary
Symmetry
Breaking
$f_a > H/2\pi$

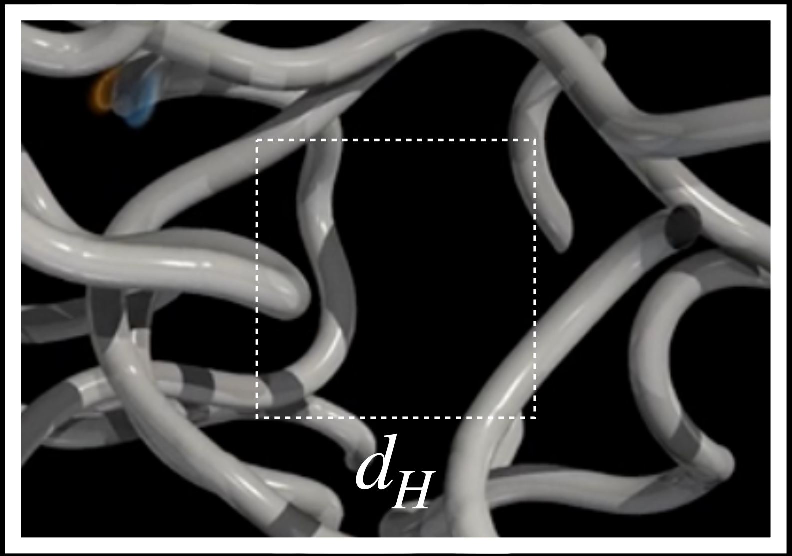




Inflation

Post-Inflationary
Symmetry
Breaking
$f_a < H/2\pi$

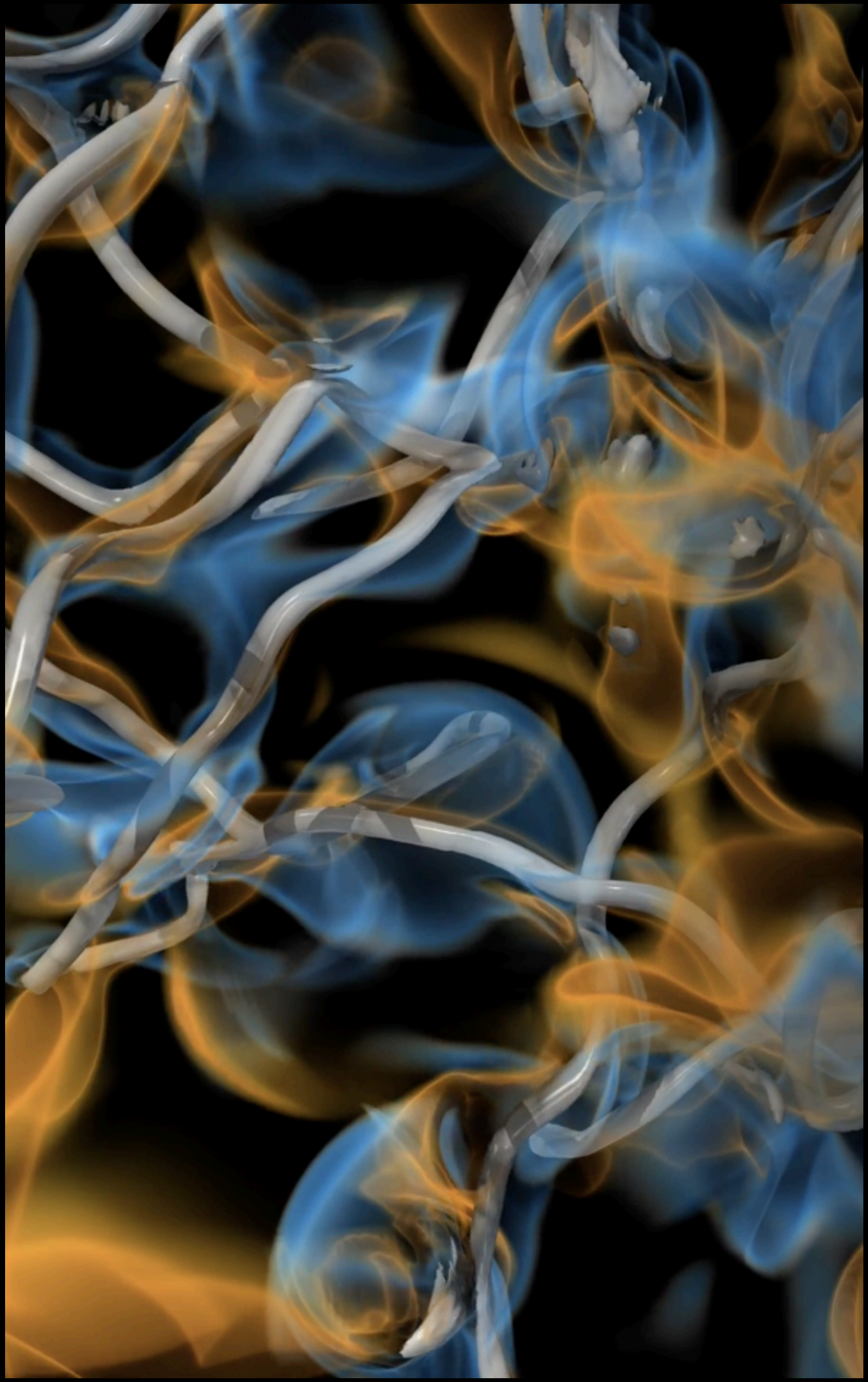




Reproduced from Ciaran O’Hare 2024

Axion Strings

- Two important scales: UV (core width) and IR (string curvature/ separation)
- Important for eg. axion emission spectrum
- Analytic modelling possible in some approximation
- Otherwise require numerical simulations



Analytic Modelling

- Infinitely thin strings can be modelled using the Nambu-Goto string action

$$S = -\mu_0 \int \sqrt{-\gamma}\, d^2\zeta$$

$$\mu_0 = 2\pi\eta^2$$

$$\gamma_{ab} = X^{\mu}_{,a} X^{\nu}_{,b} g_{\mu\nu}$$

2D worldsheet metric

$$X(\zeta)$$

Position of the string

$$\zeta^a \equiv (\tau, \sigma)$$

- Vary action → general solution for 4 components of $X(\zeta)$

Analytic Modelling

- Axion strings have long-range interactions
- Couple Nambu-Goto action to long-range field

$$S = -\mu_0 \int \sqrt{-\gamma}\, d^2\zeta - \frac{1}{6}\int H^2 d^4x - 2\pi\eta \int B_{\mu\nu} d\sigma^{\mu\nu}$$

Massless field

Coupling to string worldsheet

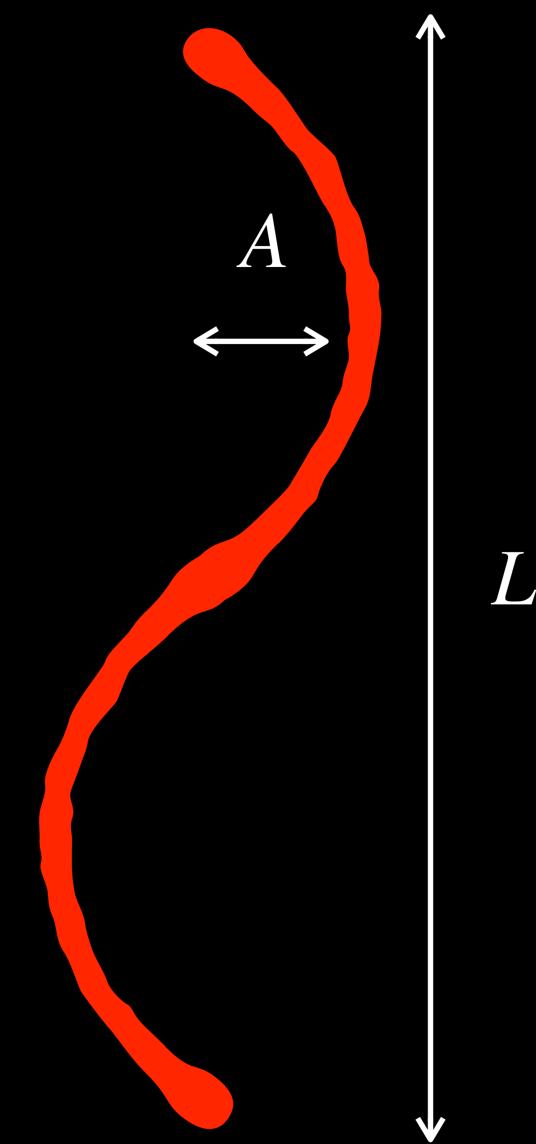
- Some string equations of motion analytically tractable
- Approximation valid when $R \gg \delta$

Analytic Modelling

- Eg. sinusoidal string
  - Amplitude will decay due to massless backreaction on string

$$\frac{1}{\varepsilon^2} \propto \frac{t}{\bar{\mu} L} \qquad \varepsilon = \frac{2\pi A}{T}$$

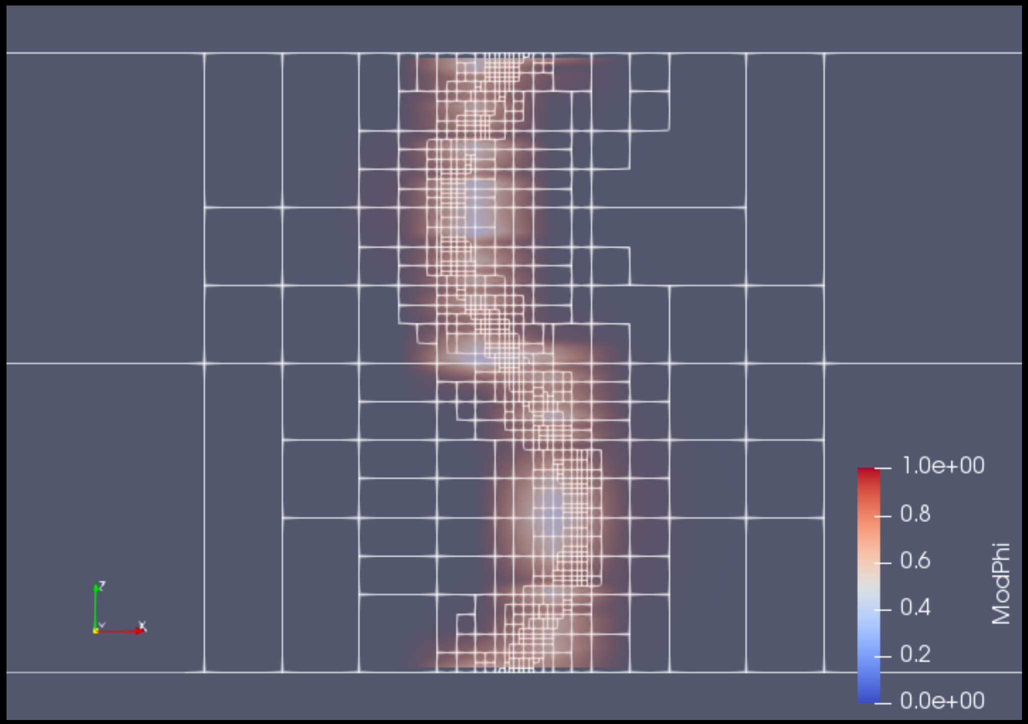
- Does this match field theory?
- Where does the approximation break down?



Battye and Shellard `93

Numerical Implementation

- Implement field theory strings on numerical grid
- Evolve in time (fourth order Runge-Kutta)
- Measure damping rate, extract signals



Numerical Implementation

- Measure energy flux from different channels

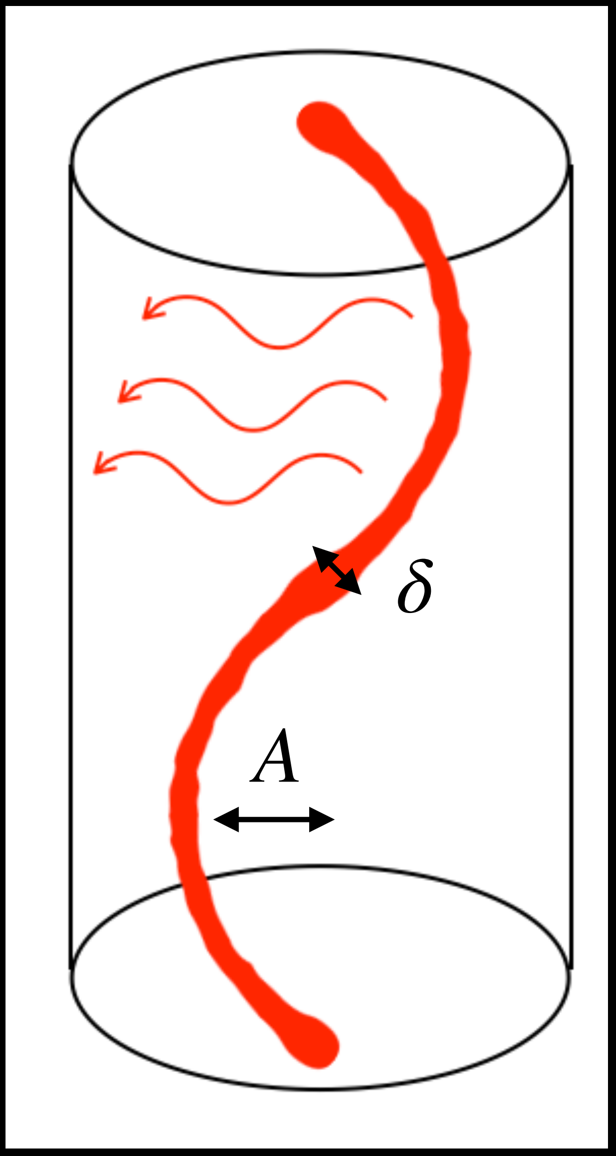
$$\varphi \equiv \phi\, e^{i\vartheta}$$

$$T^{0i} \sim \dot{\phi}\,\nabla\phi + \phi^2(\dot{\vartheta}\,\nabla\vartheta)$$

Massive

Massless

$$\rightarrow \quad E_{\text{massless}} \sim \int \phi^2(\dot{\vartheta}\,\nabla\vartheta)\cdot\hat{\mathbf{r}}\,\mathrm{d}S\,\mathrm{d}t$$



- Also measure mode decomposition of components eg. $\dot{\phi}$, $\nabla\vartheta$ and core position

Numerical Implementation

- Improve performance with adaptive mesh refinement

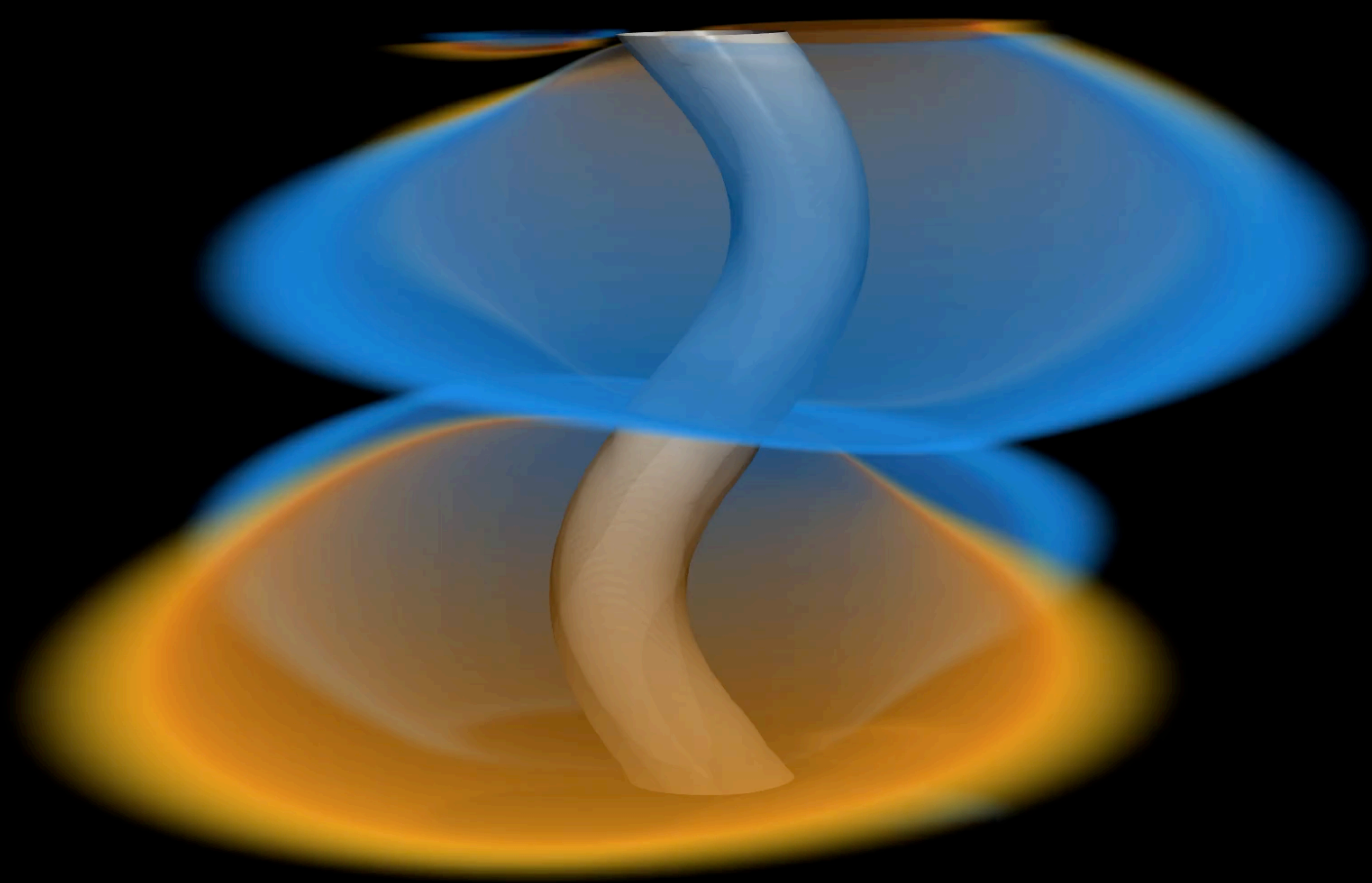
Fixed Grid

- Δx fixed throughout
- Whole grid must be resolved at required core resolution

AMR

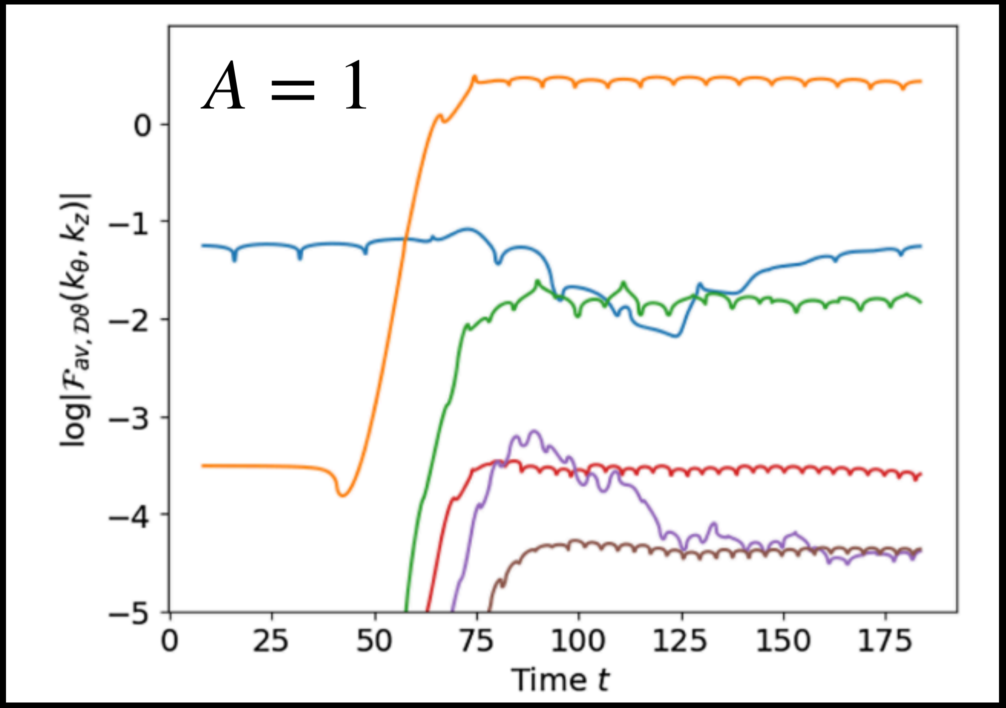
- Δx adapts to scale of interesting physics
- Simulation runs much faster, quicker parameter scans
- E.g. equivalent $\ln(R/\delta) \sim 8$ simulation can run on 32x fewer processors AND multiple 10x faster using AMR

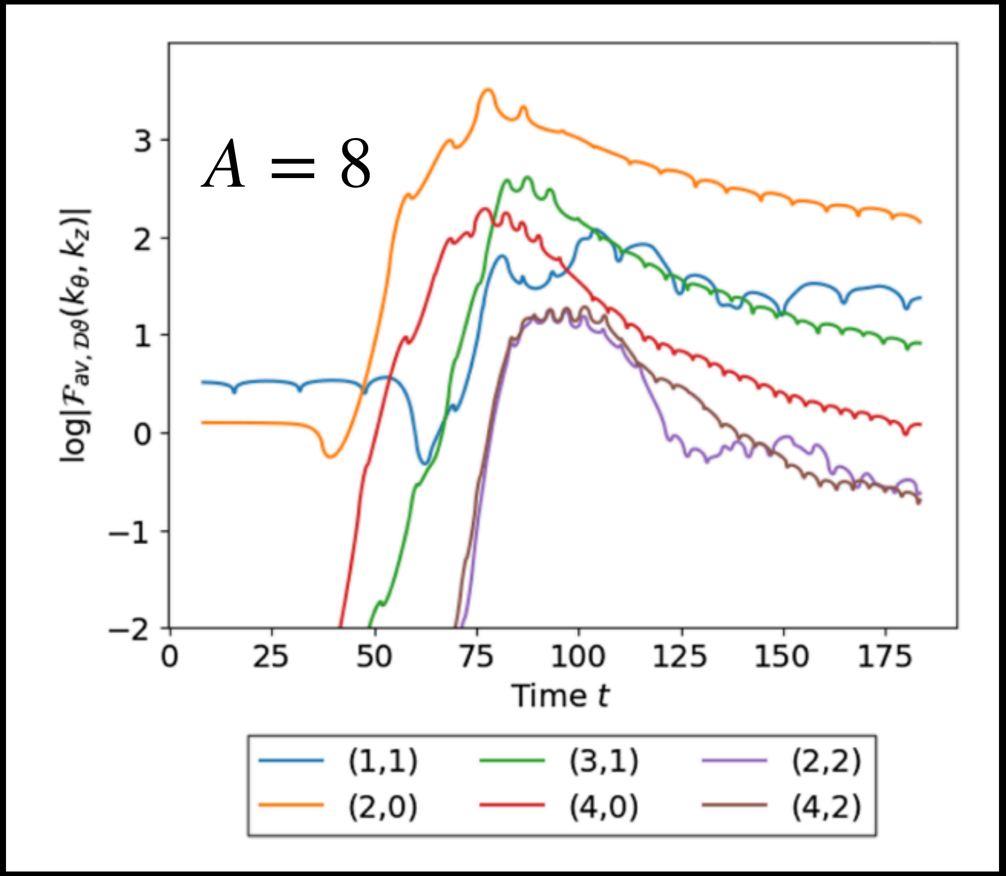
$$\mathcal{O}(\text{days}) \rightarrow \mathcal{O}(\text{hours})$$



Numerical Implementation

- Axion modes are cylindrical harmonics of the fundamental frequency $\Omega_z = 2\pi/L$
- Axion dominated by $(2,0)$ quadrupole
- Higher curvature $\rightarrow$ higher contribution from UV modes

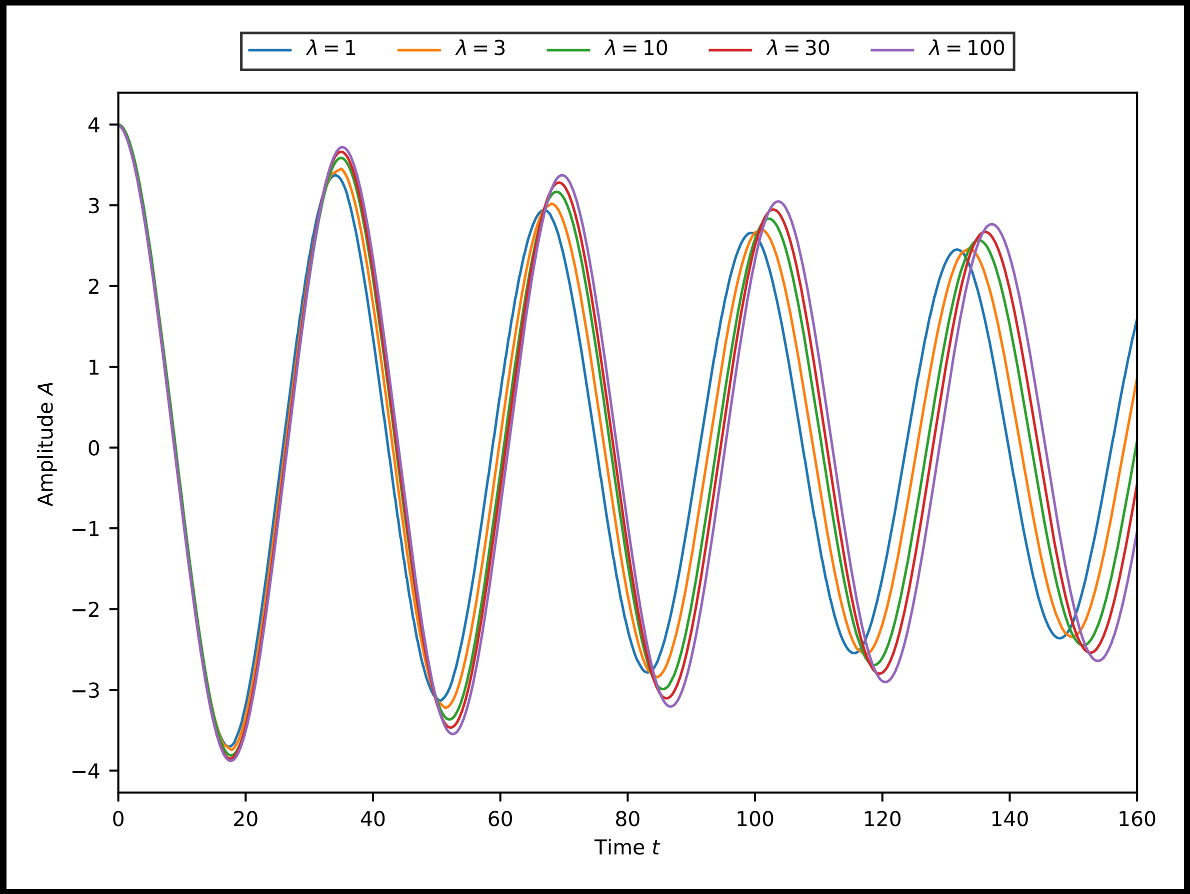


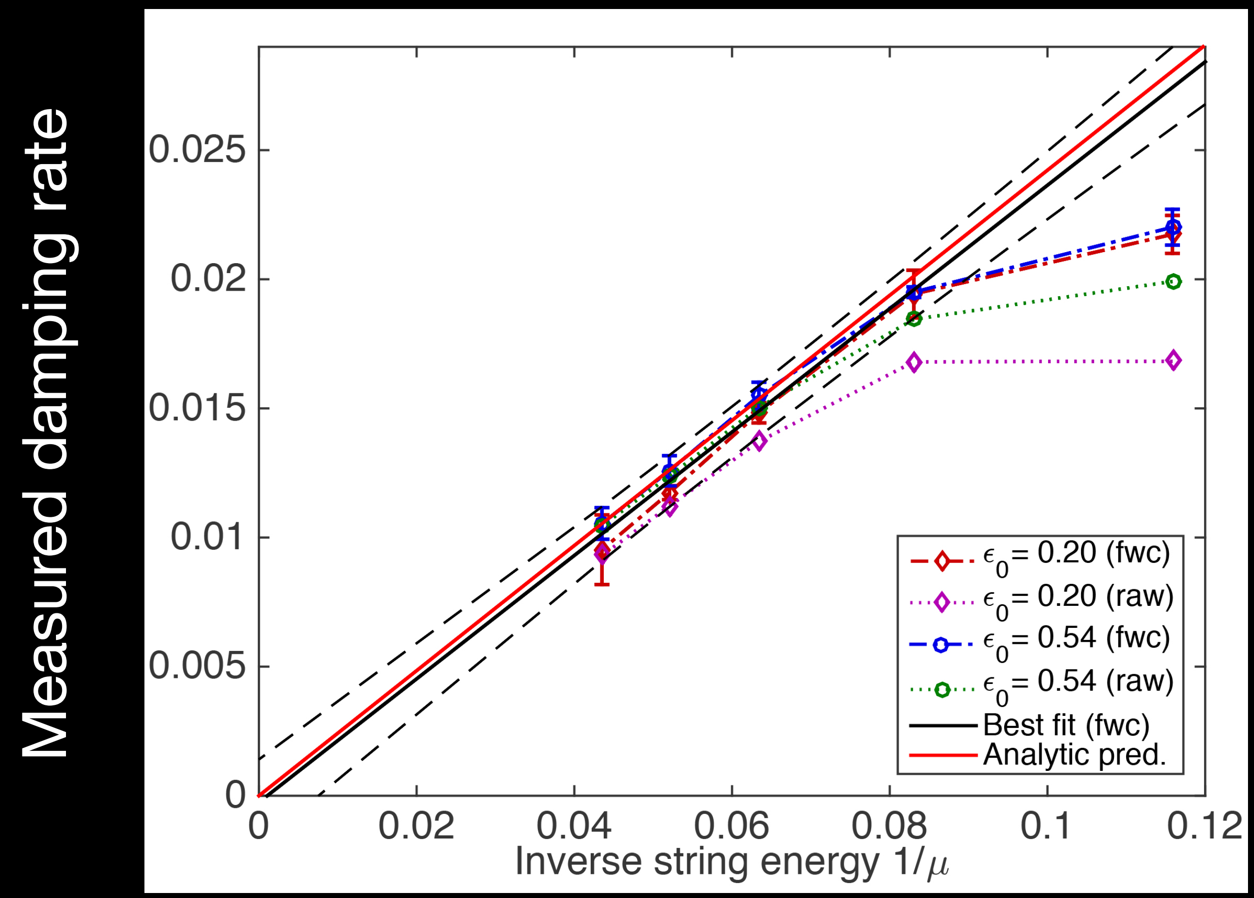


Numerical Implementation

- Measure dependence of damping rate on μ
- Thin strings agree with backreaction model

$$\frac{1}{\varepsilon^2} \propto \frac{t}{\bar{\mu} L}$$



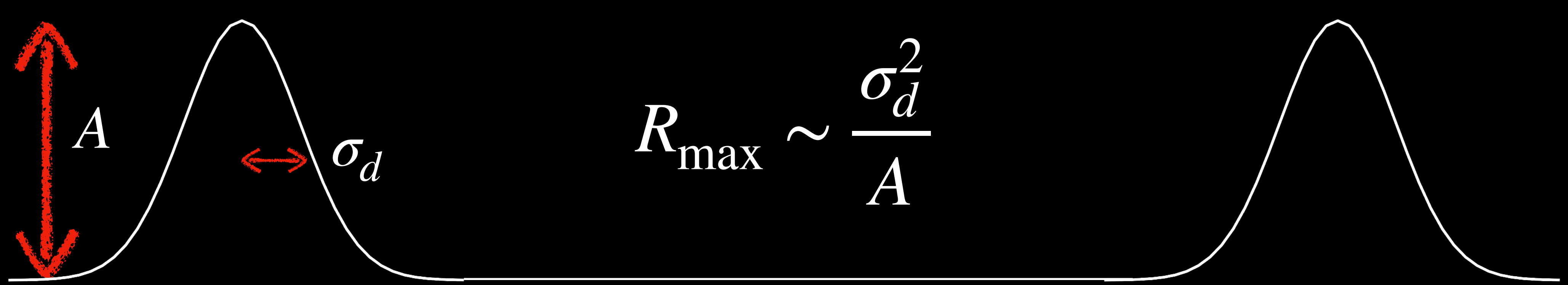


String tension

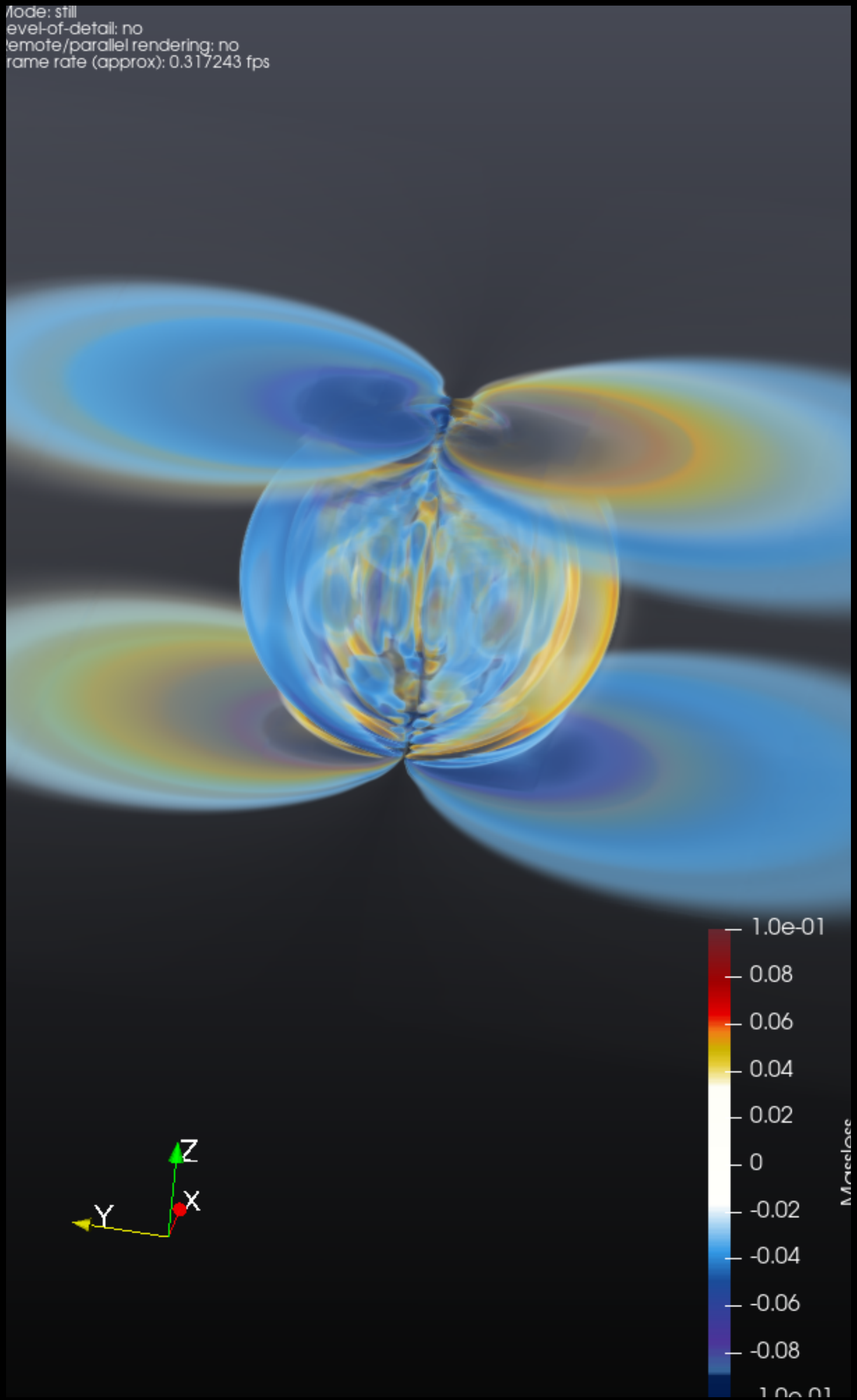
Numerical Implementation

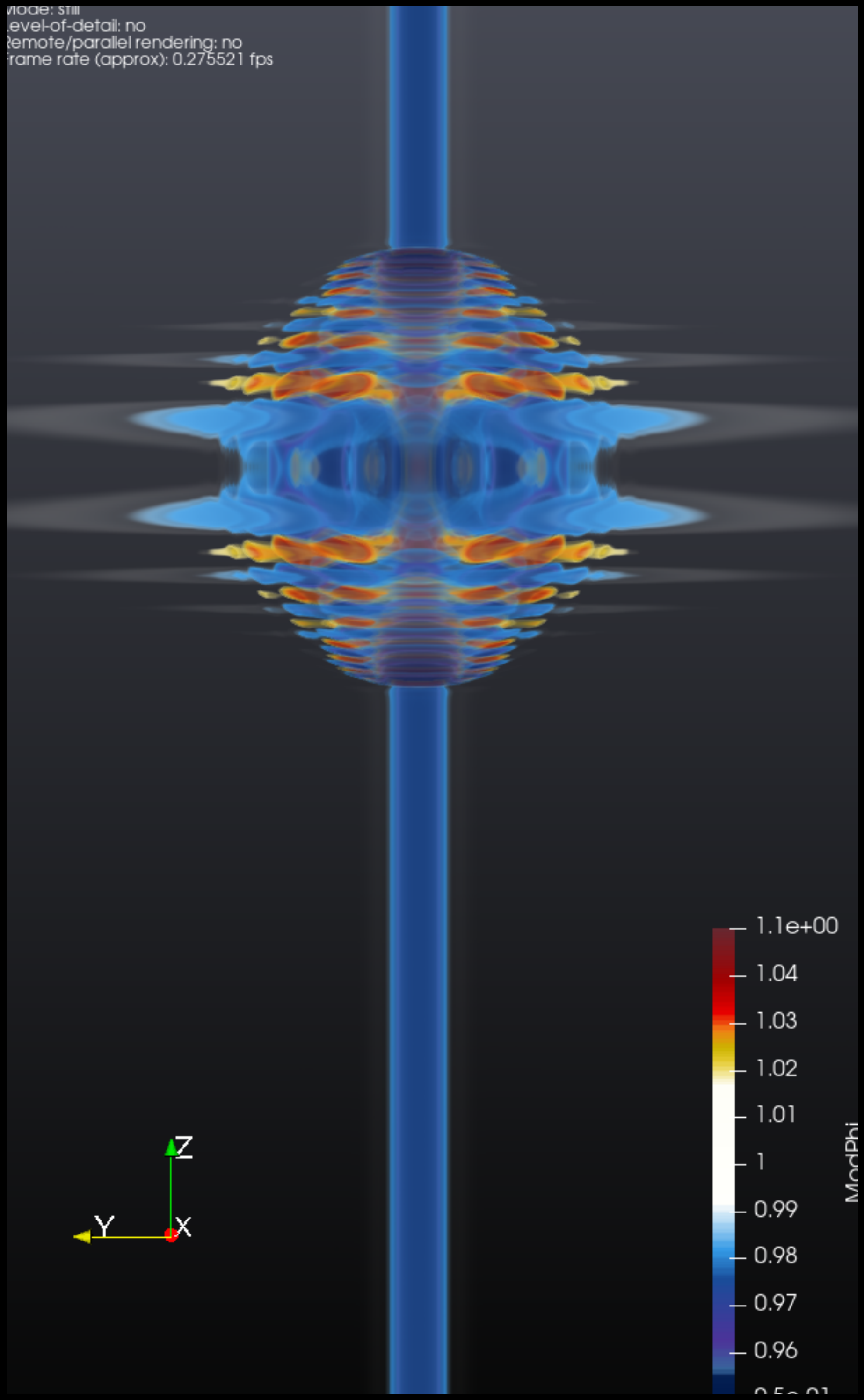
- Cosmologically, we expect to see `burst’ signals
- Can investigate with colliding travelling waves (vacuum solutions)
- Implement for different A and σ_{d}, evolve and measure modes

Vachaspati, Vachaspati `90



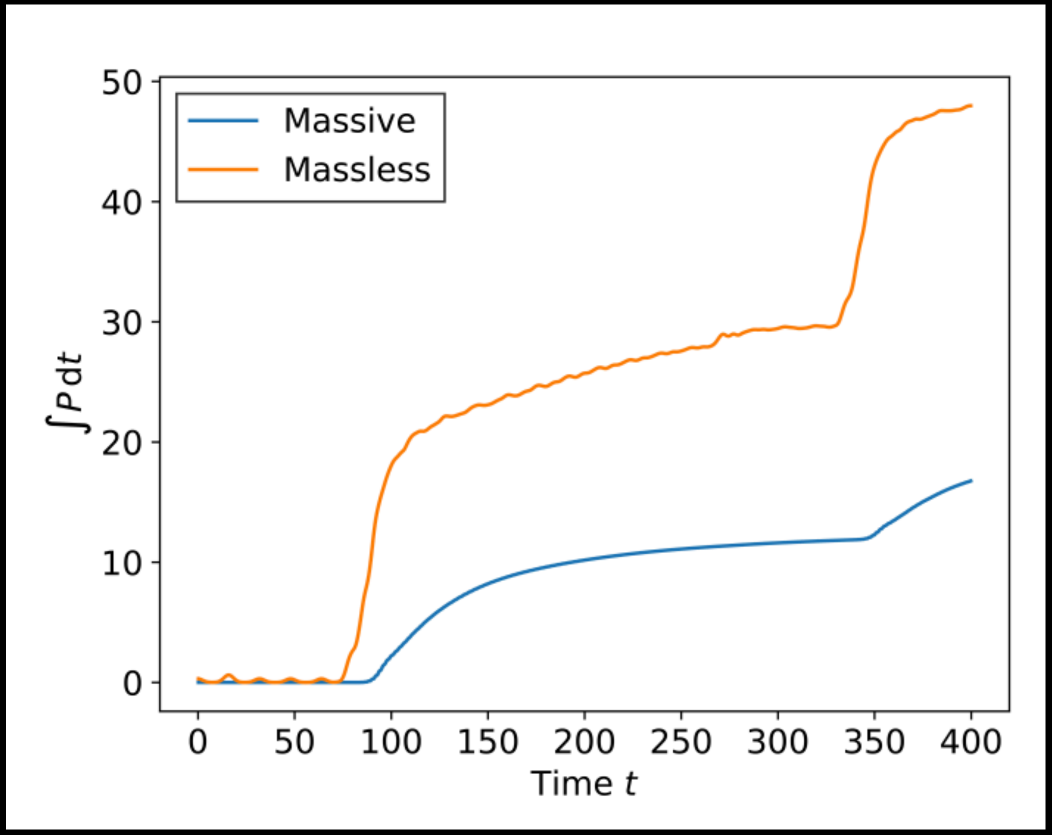
Numerical Implementation





Numerical Implementation

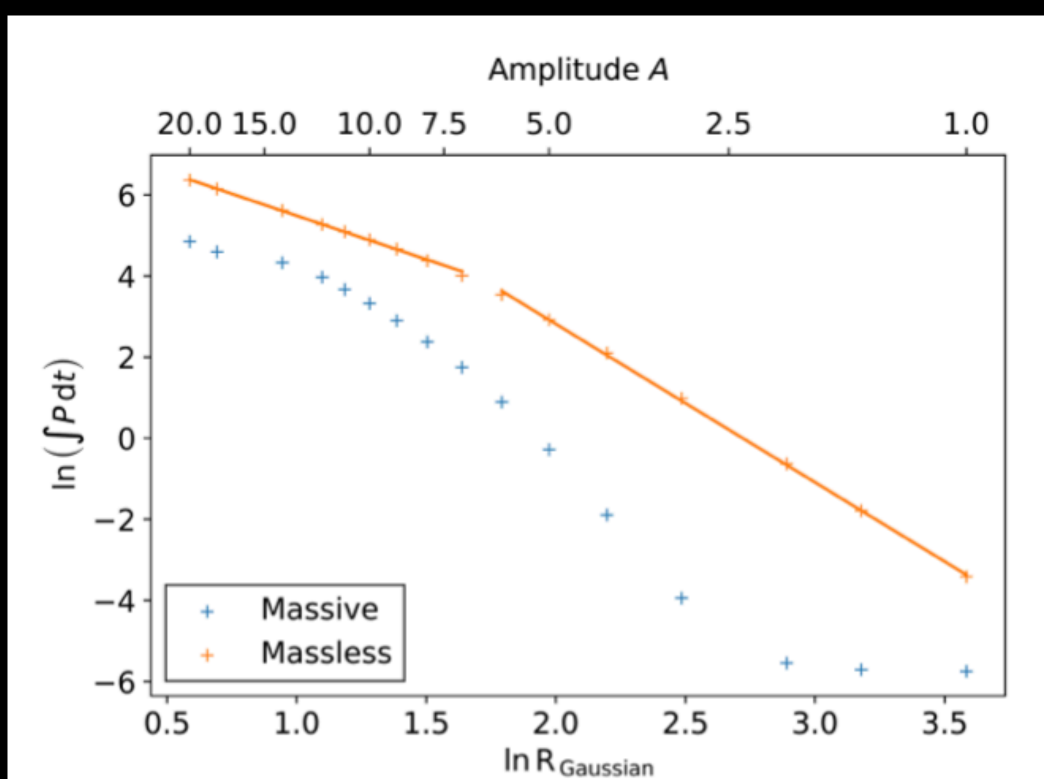
- Measure total energy released in massless and massive modes for a range of curvatures
- Integrate up to start of second collision (with periodic boundary)



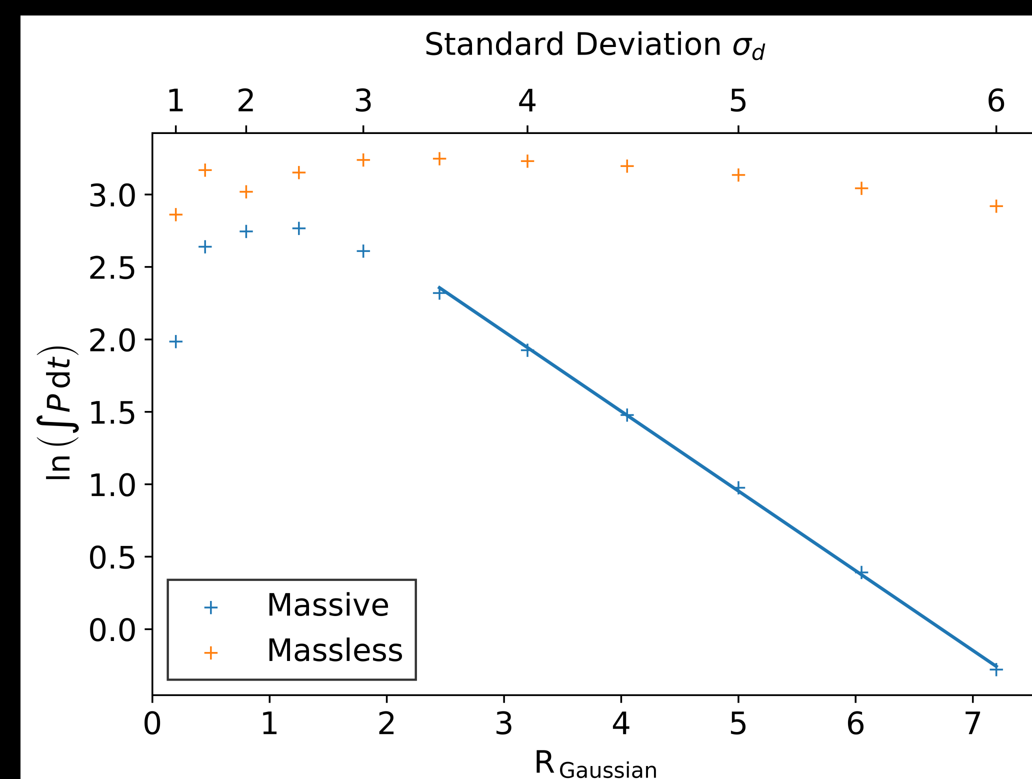
$A = 5,\ \sigma_{\mathrm{d}} = 3.5$

Numerical Implementation

- $E_{\text{massless}} \propto A^{\gamma}$ with different γ in different regimes
- Massive modes exponentially suppressed for small $R_{\max}$ $E_{\text{massive}} \propto e^{-\zeta R_{\max}}$, power law for $E_{\text{massive}} \propto R_{\max}^{-\gamma}$
- For extreme $R_{\max}$, axion/radial modes are comparable



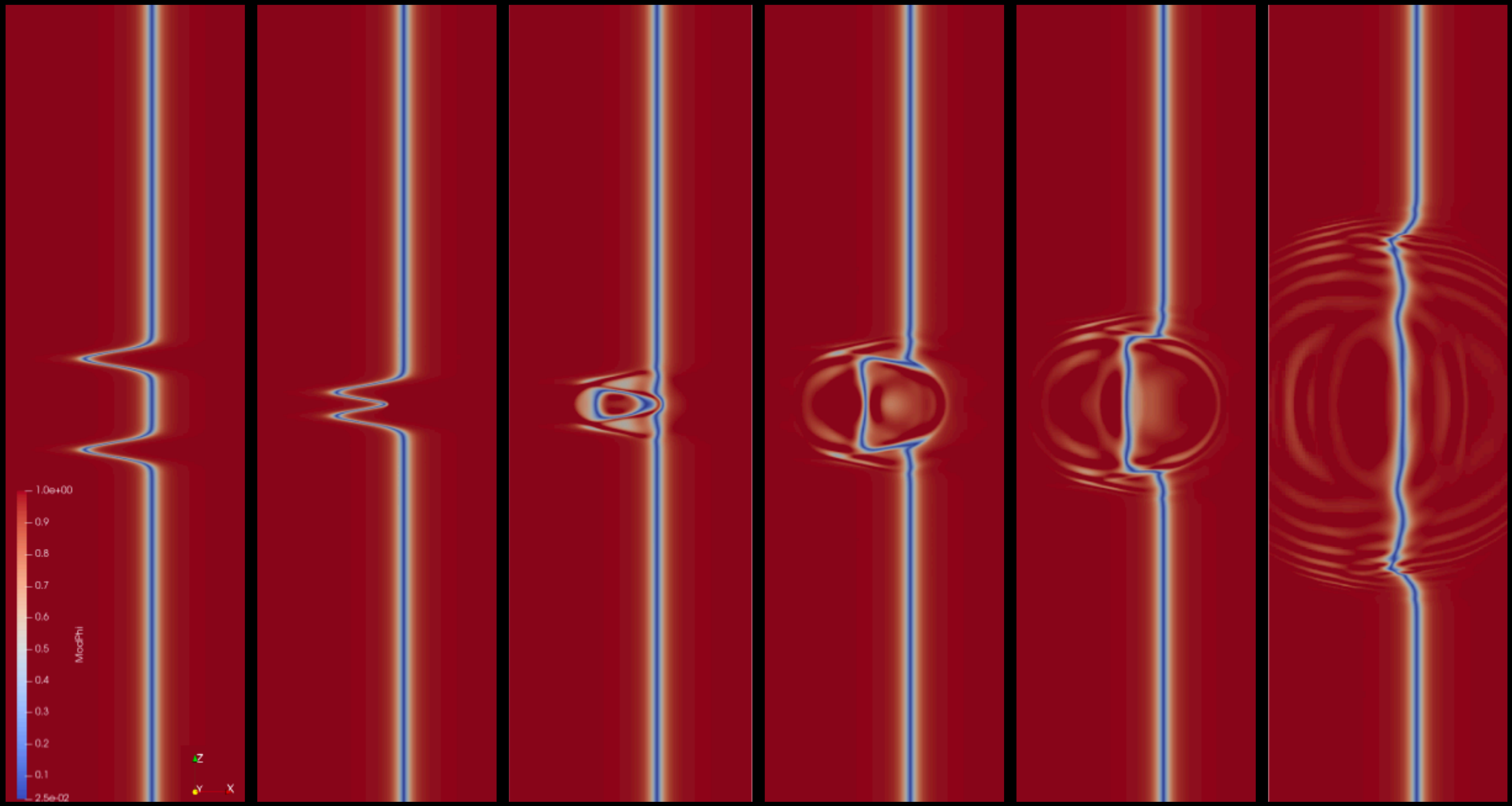
$\sigma_{\text{d}} = 6$



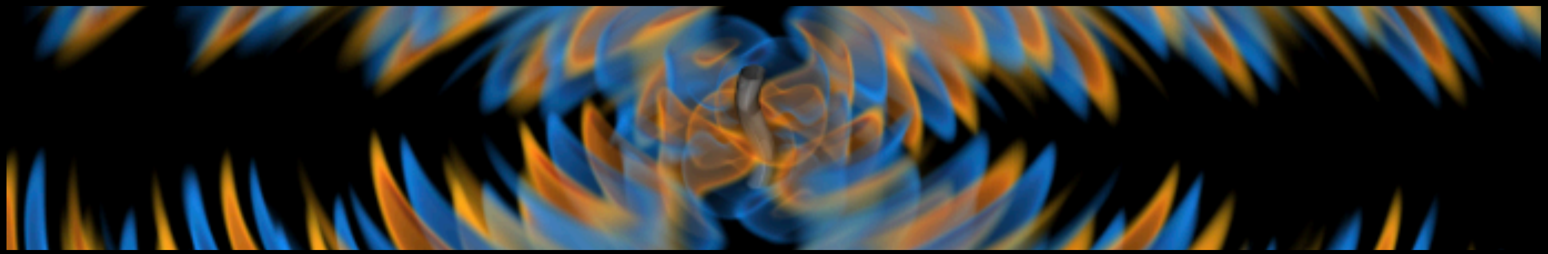
$A = 5$

Numerical Implementation

- Also observe highly nonlinear dynamics



Conclusion



- Axion strings form generically in axion models
- Cosmologically important in post-inflationary symmetry breaking
- Can model analytically in certain scenarios by coupling Nambu-Goto action to long-range massless field
- Analytic modelling agrees with field theory simulations for low curvature/width ratio
- Can investigate high curvature or otherwise complicated configurations eg. travelling waves, networks, massive modes, with field theory simulations


Thank you!