On the *D*oublet-*T*riplet Splitting Problem*

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*I.D., Mijo Matković, and Shaikh Saad, Phys.Rev.D 111 (2025) 11, 115039, arXiv:2504.06022.

*I.D. and Shaikh Saad, Phys.Rev.D 110 (2024) 7, 075025, arXiv:2404.09021.

OUTLINE

- **THE *D*OUBLET-*T*RIPLET SPLITTING PROBLEM**

- **G^2EFT**

- $SU(5)$ **WITH** 75_H

- **CONCLUSIONS**

THE *D*OUBLET-*T*RIPLET SPLITTING PROBLEM

GEORGI-GLASHOW MODEL*

FERMION SECTOR

$$\overline{5}_F^\alpha = \begin{pmatrix} d^C_{1,\alpha} \\ d^C_{2,\alpha} \\ d^C_{3,\alpha} \\ e_\alpha \\ -\nu_\alpha \end{pmatrix} \qquad \overline{5}^\alpha_{Fi}$$

$SU(5)$	$SU(3)\times SU(2)\times U(1)$
$\overline{5}_F^\alpha$	$L_\alpha \left(1, 2, -\frac{1}{2}\right)$ $d^C_\alpha \left(\overline{3}, 1, \frac{1}{3}\right)$

i = 1,2,3,4,5 - $SU(5)$ INDEX $\qquad$ α = 1,2,3 - FLAVOR INDEX

*H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438–441.

GEORGI-GLASHOW MODEL*

FERMION SECTOR

$$10_F^\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^C_{3,\alpha} & -u^C_{2,\alpha} & -u_{1,\alpha} & -d_{1,\alpha} \\ -u^C_{3,\alpha} & 0 & u^C_{1,\alpha} & -u_{2,\alpha} & -d_{2,\alpha} \\ u^C_{2,\alpha} & -u^C_{1,\alpha} & 0 & -u_{3,\alpha} & -d_{3,\alpha} \\ u_{1,\alpha} & u_{2,\alpha} & u_{3,\alpha} & 0 & -e^C_\alpha \\ d_{1,\alpha} & d_{2,\alpha} & d_{3,\alpha} & e^C_\alpha & 0 \end{pmatrix} \qquad 10_F^{\alpha ij} = -10_F^{\alpha ji}$$

$SU(5)$	$SU(3)\times SU(2)\times U(1)$
10_F^α	$Q_\alpha\ \left(3,2,\frac{1}{6}\right)$ $u^C_\alpha\ \left(\overline{3},1,-\frac{2}{3}\right)$ $e^C_\alpha\ (1,1,1)$

*H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438–441.

GEORGI-GLASHOW MODEL*

SCALAR SECTOR

24^i_{Hj} $\qquad \langle 24_H \rangle = v_{24}\text{diag}\,(-1,-1,-1,+3/2,+3/2) + v_0\text{diag}\,(0,0,0,+1,-1)$

$SU(5)$	$SU(3) \times SU(2) \times U(1)$
24_H	$(1,1,0)$ $(1,3,0)$ $\left(3,2,-\frac{5}{6}\right)$ $\left(\overline{3},2,\frac{5}{6}\right)$ $(8,1,0)$

*H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438–441.

GEORGI-GLASHOW MODEL*

SCALAR SECTOR

$$5_H = \begin{pmatrix} T \\ D \end{pmatrix} = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ D^+ \\ D^0 \end{pmatrix} \qquad 5_H^i \qquad \langle 5_H \rangle = (0 \quad 0 \quad 0 \quad 0 \quad v_5/\sqrt{2})^T$$

$SU(5)$	$SU(3) \times SU(2) \times U(1)$
5_H	$T\left(3, 1, -\frac{1}{3}\right)$ $D\left(1, 2, \frac{1}{2}\right)$

*H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438–441.

GEORGI-GLASHOW MODEL*

SYMMETRY BREAKING

24^i_{Hj} $\qquad \langle 24_H \rangle = v_{24}\mathrm{diag}\,(-1,-1,-1,+3/2,+3/2) + v_0\mathrm{diag}\,(0,0,0,+1,-1)$

$$\Big\downarrow \; SU(3) \times SU(2) \times U(1)$$

X, Y

$$5_H = \begin{pmatrix} T \\ D \end{pmatrix} = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ D^+ \\ D^0 \end{pmatrix}$$

5^i_H $\qquad \langle 5_H \rangle = (0 \quad 0 \quad 0 \quad 0 \quad v_5/\sqrt{2})^T$

$$\Big\downarrow \; SU(3) \times U(1)_{\mathrm{em}}$$

W^+, W^-, Z, γ

*H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438–441.

WHAT ARE THE *D*OUBLET COUPLINGS?

$$\mathcal{L}_Y = 10_F^{\alpha ij} Y_d^{\alpha\beta} \bar{5}_{Fi}^{\beta} 5_{Hj}^{*} + 10_F^{\alpha ij} 10_F^{\beta kl} 5_H^m Y_u^{\alpha\beta} \epsilon_{ijklm} + \text{h.c.}$$

$Y_d^{\alpha\beta}$, $Y_u^{\alpha\beta}$ - YUKAWA COUPLINGS

$$M_E = v_5 \left\{ \frac{1}{2} Y_d \right\}$$

$$M_D = v_5 \left\{ \frac{1}{2} Y_d^T \right\}$$

$$M_U = v_5 \left\{ \sqrt{2} \left(Y_u + Y_u^T \right) \right\}$$

WHAT ARE THE *D*OUBLET COUPLINGS?

$$\mathcal{L}_Y = 10_F^{\alpha ij} Y_d^{\alpha\beta} \overline{5}_{Fi}^{\beta} 5_{Hj}^{*} + 10_F^{\alpha ij} 10_F^{\beta kl} 5_H^m Y_u^{\alpha\beta} \epsilon_{ijklm} + \text{h.c.}$$

FLAVOR BASIS:

$$M_E = v_5 \left\{ \frac{1}{2} Y_d \right\}$$

$$M_D = v_5 \left\{ \frac{1}{2} Y_d^T \right\}$$

$$M_U = v_5 \left\{ \sqrt{2} \left(Y_u + Y_u^T \right) \right\}$$

MASS EIGENSTATE BASIS:

$$E_c^T M_E E = M_E^{\text{diag}}$$

$$D_c^T M_D D = M_D^{\text{diag}}$$

$$U_c^T M_U U = M_U^{\text{diag}}$$

E_c, E, D_c, D, U_c, U - UNITARY MATRICES

WHAT ARE THE *T*RIPLET COUPLINGS?

$$\mathcal{L}_Y = \underline{10_F^{\alpha ij} Y_d^{\alpha\beta} \overline{5}_{Fi}^{\beta} 5_{Hj}^*} + 10_F^{\alpha ij} 10_F^{\beta kl} 5_H^m Y_u^{\alpha\beta} \epsilon_{ijklm} + \text{h.c.}$$

$$u_{k,\alpha}^T C^{-1} e_\beta T_k^* : \quad \left\{ U^T \left[-\frac{Y_d}{\sqrt{2}} \right] E \right\}_{\alpha\beta}$$

$$d_{k,\alpha}^T C^{-1} \nu_\beta T_k^* : \quad \left\{ D^T \left[\frac{Y_d}{\sqrt{2}} \right] N \right\}_{\alpha\beta}$$

$$\epsilon_{ijk} u_{i,\alpha}^{C,T} C^{-1} d_{j,\beta}^C T_k^* : \quad \left\{ U_c^\dagger \left[\frac{Y_d}{\sqrt{2}} \right] D_c^* \right\}_{\alpha\beta}$$

$$M_E = v_5 \left\{ \frac{1}{2} Y_d \right\}$$

$$M_D = v_5 \left\{ \frac{1}{2} Y_d^T \right\}$$

WHAT ARE THE *T*RIPLET COUPLINGS?

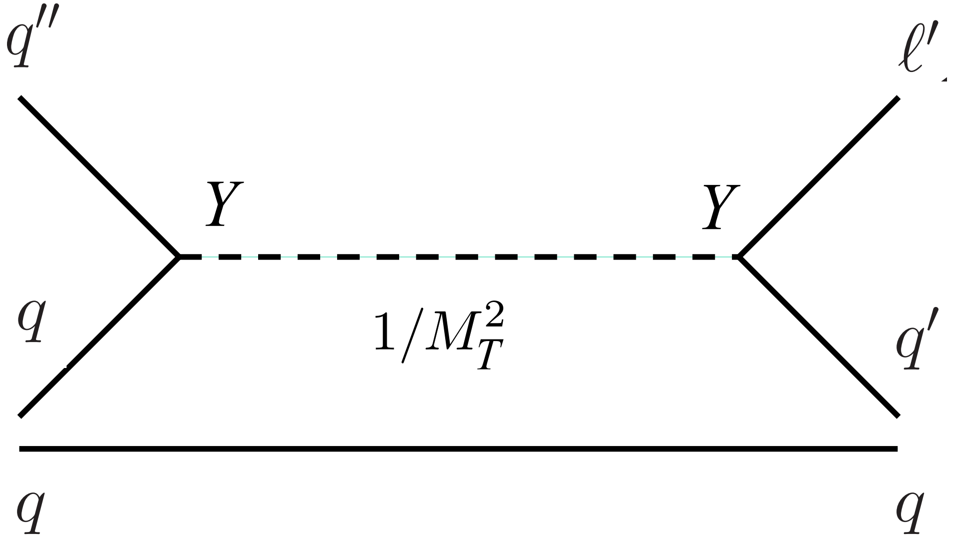
$$\mathcal{L}_Y = 10_F^{\alpha ij} Y_d^{\alpha\beta} \overline{5}_{Fi}^{\beta} 5_{Hj}^{*} + \underline{10_F^{\alpha ij} 10_F^{\beta kl} 5_H^{m} Y_u^{\alpha\beta} \epsilon_{ijklm}} + \text{h.c.}$$

$$\boxed{\begin{array}{ll} u_{k,\alpha}^{C,T} C^{-1} e_{\beta}^{C} T_k : & \left\{ U_c^{\dagger} \left[2 \left(Y_u + Y_u^T \right) \right] E_c^{*} \right\}_{\alpha\beta} \\ \epsilon_{ijk} u_{i,\alpha}^{T} C^{-1} d_{j,\beta} T_k : & \left\{ U^{T} \left[-2 \left(Y_u + Y_u^T \right) \right] D \right\}_{\alpha\beta} \end{array}}$$

$$\boxed{M_U = v_5 \left\{ \sqrt{2} \left(Y_u + Y_u^T \right) \right\}}$$

PROTON DECAY VIA *T*RIPLET MEDIATION

T (scalar leptoquark) induced two-body proton decay:



$p \to \pi^0 e^+$

$p \to \pi^0 \mu^+$

$p \to \eta^0 e^+$

$p \to \eta^0 \mu^+$

$p \to K^0 e^+$

$p \to K^0 \mu^+$

$p \to \pi^+ \bar{\nu}$

$p \to K^+ \bar{\nu}$

Y - GENERIC YUKAWA COUPLINGS

WHAT ARE THE T AND D MASSES IN $SU(5)$?

$$m^2 5^i_H 5^*_{Hi} \quad m' 5^i_H 24^j_{Hi} 5^*_{Hj} \quad \lambda 5^i_H 24^j_{Hi} 24^k_{Hj} 5^*_{Hk} \quad \lambda' 5^i_H 5^*_{Hi} 24^j_{Hk} 24^k_{Hj}$$

$$m^2 DD^* + m^2 TT^*$$

$$m' v_{24} \frac{3}{2} DD^* - m' v_{24} TT^*$$

G^2EFT

WHAT IF WE ADD HIGHER-DIMENSIONAL TERMS?[*]

$$\mathcal{L}_Y = \underline{10_F^{\alpha ij} Y_d^{\alpha\beta} \overline{5}_{Fi}^{\beta} 5_{Hj}^*} + 10_F^{\alpha ij} 10_F^{\beta kl} 5_H^m Y_u^{\alpha\beta} \epsilon_{ijklm} + \text{h.c.}$$

$$\Big\downarrow \; M_E, M_D$$

$$\underline{10_F^{\alpha ij} \left\{ \underbrace{Y_d^{\alpha\beta} \overline{5}_{Fi}^{\beta} 5_{Hj}^*}_{d=4} + \underbrace{\frac{1}{\Lambda} Y_1^{\alpha\beta} \overline{5}_{Fi}^{\beta} 5_{Hk}^* 24_{Hj}^k + \frac{1}{\Lambda} Y_2^{\alpha\beta} \overline{5}_{Fk}^{\beta} 5_{Hi}^* 24_{Hj}^k}_{d=5} \right\} +} \; \underbrace{\qquad\quad}_{d>5}$$

Λ - cutoff scale

[*]Borut Bajc, Pavel Fileviez Perez, Goran Senjanović, hep-ph/0210374.
I.D. and Shaikh Saad, Phys.Rev.D 110 (2024) 7, 075025, arXiv:2404.09021.

<u>WHAT IF WE ADD HIGHER-DIMENSIONAL TERMS?</u>

$$\mathcal{L}_Y = 10_F^{\alpha ij} Y_d^{\alpha\beta} \overline{5}_{Fi}^{\beta} 5^*_{Hj} + \underline{10_F^{\alpha ij} 10_F^{\beta kl} 5_H^m Y_u^{\alpha\beta} \epsilon_{ijklm}} + \text{h.c.}$$

$$\Big\downarrow M_U$$

$$\underline{10_F^{\alpha ij} 10_F^{\beta kl} 5_H^m \left\{ \underbrace{Y_u^{\alpha\beta} \epsilon_{ijklm}}_{d=4} + \underbrace{\frac{1}{\Lambda} Y_3^{\alpha\beta} 24^n_{Hm} \epsilon_{ijkln} + \frac{1}{\Lambda} Y_4^{\alpha\beta} 24^n_{Hk} \epsilon_{ijlmn}}_{d=5} \right\}} + \underbrace{\quad}_{d>5}$$

WHAT ARE THE *D*OUBLET COUPLINGS? *

$$M_E = v_5 \left\{ \frac{1}{2} Y_d \right\}$$

$$M_D = v_5 \left\{ \frac{1}{2} Y_d^T \right\}$$

$\Big\downarrow$ $d = 5$ OPERATORS

$$M_E = v_5 \left\{ \frac{1}{2} Y_d + \frac{3}{4} Y_1 \epsilon - \frac{3}{4} Y_2 \epsilon \right\}$$

$$M_D = v_5 \left\{ \frac{1}{2} Y_d^T + \frac{3}{4} Y_1^T \epsilon + \frac{1}{2} Y_2^T \epsilon \right\}$$

$\epsilon = v_{24}/\Lambda$ - dimensionless parameter

*I.D. and Shaikh Saad, Phys.Rev.D 110 (2024) 7, 075025, arXiv:2404.09021.

WHAT ARE THE *D*OUBLET COUPLINGS? *

$$\boxed{M_U = v_5\left\{\sqrt{2}\left(Y_u + Y_u^T\right)\right\}}$$

$$\Big\downarrow \; d = 5 \text{ OPERATORS}$$

$$\boxed{M_U = v_5\left\{\sqrt{2}\left(Y_u + Y_u^T\right) + \frac{3}{\sqrt{2}}\left(Y_3 + Y_3^T\right)\epsilon + \left(\frac{1}{2\sqrt{2}}Y_4 - \sqrt{2}Y_4^T\right)\epsilon\right\}}$$

$\epsilon = v_{24}/\Lambda$ - dimensionless parameter

*I.D. and Shaikh Saad, Phys.Rev.D 110 (2024) 7, 075025, arXiv:2404.09021.

WHAT ARE THE *T*RIPLET COUPLINGS?*

$$u^T_{k,\alpha} C^{-1} e_\beta T^*_k : \quad \left\{ U^T \left[-\frac{Y_d}{\sqrt{2}} + \frac{Y_1}{\sqrt{2}}\epsilon + \frac{3}{2\sqrt{2}} Y_2 \epsilon \right] E \right\}_{\alpha\beta}$$

$$d^T_{k,\alpha} C^{-1} \nu_\beta T^*_k : \quad \left\{ D^T \left[\frac{Y_d}{\sqrt{2}} - \frac{Y_1}{\sqrt{2}}\epsilon - \frac{3}{2\sqrt{2}} Y_2 \epsilon \right] N \right\}_{\alpha\beta}$$

$$\epsilon_{ijk} u^{C,T}_{i,\alpha} C^{-1} d^C_{j,\beta} T^*_k : \quad \left\{ U^\dagger_c \left[\frac{Y_d}{\sqrt{2}} - \frac{Y_1}{\sqrt{2}}\epsilon + \frac{Y_2}{\sqrt{2}} \epsilon \right] D^*_c \right\}_{\alpha\beta}$$

$$M_E = v_5 \left\{ \frac{1}{2} Y_d + \frac{3}{4} Y_1 \epsilon - \frac{3}{4} Y_2 \epsilon \right\}$$

$$M_D = v_5 \left\{ \frac{1}{2} Y_d^T + \frac{3}{4} Y_1^T \epsilon + \frac{1}{2} Y_2^T \epsilon \right\}$$

*I.D. and Shaikh Saad, Phys.Rev.D 110 (2024) 7, 075025, arXiv:2404.09021.

ONE IMPORTANT OBSERVATION *

$$u^T_{k,\alpha} C^{-1} e_\beta T^*_k : \quad \left\{ U^T \left[-\frac{Y_d}{\sqrt{2}} + \frac{Y_1}{\sqrt{2}} \epsilon \right] E \right\}_{\alpha\beta}$$

$$d^T_{k,\alpha} C^{-1} \nu_\beta T^*_k : \quad \left\{ D^T \left[\frac{Y_d}{\sqrt{2}} - \frac{Y_1}{\sqrt{2}} \epsilon \right] N \right\}_{\alpha\beta}$$

$$\epsilon_{ijk} u^{C,T}_{i,\alpha} C^{-1} d^C_{j,\beta} T^*_k : \quad \left\{ U^\dagger_c \left[\frac{Y_d}{\sqrt{2}} - \frac{Y_1}{\sqrt{2}} \epsilon \right. \right\}_{\alpha\beta}$$

$$M_E = v_5 \left\{ \frac{1}{2} Y_d + \frac{3}{4} Y_1 \epsilon \right\}$$

$$M_D = v_5 \left\{ \frac{1}{2} Y_d^T + \frac{3}{4} Y_1^T \epsilon \right\}$$

*Borut Bajc, Pavel Fileviez Perez, Goran Senjanović, hep-ph/0210374.
I.D. and Shaikh Saad, Phys.Rev.D 110 (2024) 7, 075025, arXiv:2404.09021.

ONE IMPORTANT OBSERVATION *

$$u^T_{k,\alpha} C^{-1} e_\beta T^*_k : \quad \left\{ U^T \left[-\frac{Y_d}{\sqrt{2}} \qquad + \frac{3}{2\sqrt{2}} Y_2 \epsilon \right] E \right\}_{\alpha\beta}$$

$$d^T_{k,\alpha} C^{-1} \nu_\beta T^*_k : \quad \left\{ D^T \left[\frac{Y_d}{\sqrt{2}} \qquad - \frac{3}{2\sqrt{2}} Y_2 \epsilon \right] N \right\}_{\alpha\beta}$$

$$\epsilon_{ijk} u^{C,T}_{i,\alpha} C^{-1} d^C_{j,\beta} T^*_k : \quad \left\{ U^\dagger_c \left[\frac{Y_d}{\sqrt{2}} \qquad + \frac{Y_2}{\sqrt{2}} \epsilon \right] D^*_c \right\}_{\alpha\beta}$$

$$M_E = v_5 \left\{ \frac{1}{2} Y_d \qquad - \frac{3}{4} Y_2 \epsilon \right\}$$

$$M_D = v_5 \left\{ \frac{1}{2} Y^T_d \qquad + \frac{1}{2} Y^T_2 \epsilon \right\}$$

*I.D. and Shaikh Saad, Phys.Rev.D 110 (2024) 7, 075025, arXiv:2404.09021.

ONE IMPORTANT OBSERVATION *

$$u^T_{k,\alpha} C^{-1} e_\beta T^*_k : \quad \left\{ U^T \left[\quad + \frac{Y_1}{\sqrt{2}}\epsilon + \frac{3}{2\sqrt{2}} Y_2 \epsilon \right] E \right\}_{\alpha\beta}$$

$$d^T_{k,\alpha} C^{-1} \nu_\beta T^*_k : \quad \left\{ D^T \left[\quad - \frac{Y_1}{\sqrt{2}}\epsilon - \frac{3}{2\sqrt{2}} Y_2 \epsilon \right] N \right\}_{\alpha\beta}$$

$$\epsilon_{ijk} u^{C,T}_{i,\alpha} C^{-1} d^C_{j,\beta} T^*_k : \quad \left\{ U^\dagger_c \left[\quad - \frac{Y_1}{\sqrt{2}}\epsilon + \frac{Y_2}{\sqrt{2}} \epsilon \right] D^*_c \right\}_{\alpha\beta}$$

$$M_E = v_5 \left\{ \quad + \frac{3}{4} Y_1 \epsilon - \frac{3}{4} Y_2 \epsilon \right\}$$

$$M_D = v_5 \left\{ \quad + \frac{3}{4} Y_1^T \epsilon + \frac{1}{2} Y_2^T \epsilon \right\}$$

*I.D. and Shaikh Saad, Phys.Rev.D 110 (2024) 7, 075025, arXiv:2404.09021.

WHAT ARE THE *T*RIPLET COUPLINGS?*

$$u^{C,T}_{k,\alpha} C^{-1} e^{C}_{\beta} T_k : \quad \left\{ U_c^{\dagger} \left[2\left(Y_u + Y_u^T\right) - 2\left(Y_3 + Y_3^T\right)\epsilon + \left(3Y_4 - 2Y_4^T\right)\epsilon \right] E_c^{*} \right\}_{\alpha\beta}$$

$$\epsilon_{ijk} u^{T}_{i,\alpha} C^{-1} d_{j,\beta} T_k : \quad \left\{ U^{T} \left[-2\left(Y_u + Y_u^T\right) + 2\left(Y_3 + Y_3^T\right)\epsilon - \frac{1}{2}\left(Y_4 + Y_4^T\right)\epsilon \right] D \right\}_{\alpha\beta}$$

$$M_U = v_5 \left\{ \sqrt{2}\left(Y_u + Y_u^T\right) + \frac{3}{\sqrt{2}}\left(Y_3 + Y_3^T\right)\epsilon + \left(\frac{1}{2\sqrt{2}} Y_4 - \sqrt{2} Y_4^T\right)\epsilon \right\}$$

*I.D. and Shaikh Saad, Phys.Rev.D 110 (2024) 7, 075025, arXiv:2404.09021.

ONE IMPORTANT OBSERVATION *

$$u_{k,\alpha}^{C,T} C^{-1} e_{\beta}^{C} T_k : \quad \left\{ U_c^{\dagger} \left[2\left(Y_u + Y_u^T\right) - 2\left(Y_3 + Y_3^T\right)\epsilon \qquad \right] E_c^* \right\}_{\alpha\beta}$$

$$\epsilon_{ijk} u_{i,\alpha}^{T} C^{-1} d_{j,\beta} T_k : \quad \left\{ U^{T} \left[-2\left(Y_u + Y_u^T\right) + 2\left(Y_3 + Y_3^T\right)\epsilon \qquad \right] D \right\}_{\alpha\beta}$$

$$M_U = v_5 \left\{ \sqrt{2}\left(Y_u + Y_u^T\right) + \frac{3}{\sqrt{2}}\left(Y_3 + Y_3^T\right)\epsilon \qquad \right\}$$

*I.D. and Shaikh Saad, Phys.Rev.D 110 (2024) 7, 075025, arXiv:2404.09021.

SUMMARY – THE *T*RIPLET COUPLINGS*

$d = 4$ & $d = 5$

$$u^T_{k,\alpha} C^{-1} e_\beta T^*_k : \quad \left\{ U^T \left[-\frac{Y_d}{\sqrt{2}} + \frac{Y_1}{\sqrt{2}}\epsilon + \frac{3}{2\sqrt{2}} Y_2 \epsilon \right] E \right\}_{\alpha\beta}$$

$$d^T_{k,\alpha} C^{-1} \nu_\beta T^*_k : \quad \left\{ D^T \left[\frac{Y_d}{\sqrt{2}} - \frac{Y_1}{\sqrt{2}}\epsilon - \frac{3}{2\sqrt{2}} Y_2 \epsilon \right] N \right\}_{\alpha\beta}$$

$$\epsilon_{ijk} u^{C,T}_{i,\alpha} C^{-1} d^C_{j,\beta} T^*_k : \quad \left\{ U^\dagger_c \left[\frac{Y_d}{\sqrt{2}} - \frac{Y_1}{\sqrt{2}}\epsilon + \frac{Y_2}{\sqrt{2}} \epsilon \right] D^*_c \right\}_{\alpha\beta}$$

$$u^{C,T}_{k,\alpha} C^{-1} e^C_\beta T_k : \quad \left\{ U^\dagger_c \left[2\left(Y_u + Y_u^T\right) - 2\left(Y_3 + Y_3^T\right)\epsilon + \left(3Y_4 - 2Y_4^T\right)\epsilon \right] E^*_c \right\}_{\alpha\beta}$$

$$\epsilon_{ijk} u^T_{i,\alpha} C^{-1} d_{j,\beta} T_k : \quad \left\{ U^T \left[-2\left(Y_u + Y_u^T\right) + 2\left(Y_3 + Y_3^T\right)\epsilon - \frac{1}{2}\left(Y_4 + Y_4^T\right)\epsilon \right] D \right\}_{\alpha\beta}$$

*I.D. and Shaikh Saad, Phys.Rev.D 110 (2024) 7, 075025, arXiv:2404.09021.

$SU(5)$ WITH 75_H

$SU(5)$ WITH 75_H^*

SCALAR SECTOR

$$75^{ij}_{H\,kl} = -75^{ji}_{H\,kl} = -75^{ij}_{H\,lk} = +75^{ji}_{H\,lk} \qquad 75^{ij}_{H\,il} = 0$$

$SU(5)$	$SU(3)\times SU(2)\times U(1)$
75_H	$(1,1,0)$
	$(1,3,0)$
	$\left(3,2,-\frac{5}{6}\right)$
	$\left(\overline{3},2,\frac{5}{6}\right)$
	$(8,1,0)$
	$(8,3,0)$
	$(3,1,5/3)$
	$(3,1,-5/3)$
	$(\overline{6},2,-5/6)$
	$(6,2,5/6)$

$$\begin{aligned}\langle 75_H\rangle &= \left(75^{12}_{H\,12}, 75^{13}_{H\,13}, 75^{23}_{H\,23}, 75^{14}_{H\,14}, 75^{15}_{H\,15}, 75^{24}_{H\,24}, 75^{25}_{H\,25}, 75^{34}_{H\,34}, 75^{35}_{H\,35}, 75^{45}_{H\,45}\right)\\ &= \frac{v_{75}}{3\sqrt{2}}(1,1,1,-1,-1,-1,-1,-1,-1,3)\end{aligned}$$

*T. Hubsch and S. Pallua, Phys. Lett. B 138 (1984) 279–282..

WHAT IF WE ADD HIGHER-DIMENSIONAL TERMS?[*]

$$\mathcal{L}_Y = 10_F^{\alpha ij}\bar{5}_{Fk}^{\beta}5_{Hl}^{*}\left\{\overbrace{Y_{a\alpha\beta}\delta_i^k\delta_j^l}^{d=4} + \overbrace{\frac{1}{\Lambda}Y_{b\alpha\beta}75_{Hij}^{kl}}^{d=5} + \overbrace{\frac{1}{\Lambda^2}Y_{c\alpha\beta}75_{Hin}^{km}75_{Hjm}^{ln} + \frac{1}{\Lambda^2}Y_{d\alpha\beta}75_{Hij}^{mn}75_{Hmn}^{kl}}^{d=6}\right\}$$

$$+\,10_F^{\alpha ij}10_F^{\beta kl}5_H^{m}\left\{\underbrace{Y_{A\alpha\beta}\epsilon_{ijklm}}_{d=4} + \underbrace{\frac{1}{\Lambda}Y_{B\alpha\beta}\epsilon_{ijnom}75_{Hkl}^{no} + \frac{1}{\Lambda}Y_{C\alpha\beta}\epsilon_{jklno}75_{Him}^{no} + \frac{1}{\Lambda}Y_{D\alpha\beta}\epsilon_{jlmno}75_{Hik}^{no}}_{d=5}\right\}$$

$Y_a, Y_b, Y_c, Y_d, Y_A, Y_B, Y_C, Y_D$ - YUKAWA COUPLING MATRICES

$\epsilon_{75} = v_{75}/\Lambda$ - dimensionless parameter

[*]I.D., Mijo Matković, and Shaikh Saad, Phys.Rev.D 111 (2025) 11, 115039, arXiv:2504.06022.

SUMMARY – THE *T*RIPLET COUPLINGS

$$M_E = v_5 \left\{ \frac{1}{2} Y_a + \frac{1}{\sqrt{2}} Y_b \epsilon_{75} - \frac{1}{6} Y_c \epsilon_{75}^2 + Y_d \epsilon_{75}^2 \right\}$$

$$M_D = v_5 \left\{ \frac{1}{2} Y_a^T - \frac{1}{3\sqrt{2}} Y_b^T \epsilon_{75} - \frac{1}{6} Y_c^T \epsilon_{75}^2 + \frac{1}{9} Y_d^T \epsilon_{75}^2 \right\}$$

$$M_U = v_5 \left\{ \sqrt{2} \left(Y_A + Y_A^T \right) - \frac{2}{3} \left(Y_B - Y_B^T \right) \epsilon_{75} + \frac{2}{3} \left(Y_C - Y_C^T \right) \epsilon_{75} \right\}$$

$$u_{k,\alpha}^T C^{-1} e_\beta T_k^* : \quad - \left\{ U^T \left[\frac{Y_a}{\sqrt{2}} - \frac{1}{3} Y_b \epsilon_{75} - \frac{1}{3\sqrt{2}} Y_c \epsilon_{75}^2 + \frac{\sqrt{2}}{9} Y_d \epsilon_{75}^2 \right] E \right\}_{\alpha\beta}$$

$$d_{k,\alpha}^T C^{-1} \nu_\beta T_k^* : \quad \left\{ D^T \left[\frac{Y_a}{\sqrt{2}} - \frac{1}{3} Y_b \epsilon_{75} - \frac{1}{3\sqrt{2}} Y_c \epsilon_{75}^2 + \frac{\sqrt{2}}{9} Y_d \epsilon_{75}^2 \right] N \right\}_{\alpha\beta}$$

$$\epsilon_{ijk} u_{i,\alpha}^{C,T} C^{-1} d_{j,\beta}^C T_k^* : \left\{ U_c^\dagger \left[\frac{Y_a}{\sqrt{2}} + \frac{1}{3} Y_b \epsilon_{75} + \frac{1}{9\sqrt{2}} Y_c \epsilon_{75}^2 + \frac{\sqrt{2}}{9} Y_d \epsilon_{75}^2 \right] D_c^* \right\}_{\alpha\beta}$$

$$\epsilon_{ijk} u_{i,\alpha}^T C^{-1} d_{j,\beta} T_k : \quad \left\{ U^T \left[-2 \left(Y_A + Y_A^T \right) + \frac{2\sqrt{2}}{3} \left(Y_B + Y_B^T \right) \epsilon_{75} + \frac{\sqrt{2}}{3} \left(Y_D + Y_D^T \right) \epsilon_{75} \right] D \right\}_{\alpha\beta}$$

$$u_{k,\alpha}^{C,T} C^{-1} e_\beta^C T_k : \quad \left\{ U_c^\dagger \left[2 \left(Y_A + Y_A^T \right) + \frac{\sqrt{8}}{3} \left(3Y_B + Y_B^T \right) \epsilon_{75} - \frac{\sqrt{8}}{3} \left(Y_C - Y_C^T \right) \epsilon_{75} + \frac{\sqrt{8}}{3} \left(Y_D + Y_D^T \right) \epsilon_{75} \right] E_c^* \right\}_{\alpha\beta}$$

CONCLUSIONS

The proposed approach accommodates a light color triplet, of the doublet-triplet splitting problem notoriety, that can still couple to either quark-quark or quark-lepton pairs.

The proposed mechanism can work in various unification frameworks such as $SU(5)$, $SO(10)$, or flipped $SU(5)$ and it is compatible with both supersymmetric and non-supersymmetric scenarios.

THANK YOU