Machine Learning the Likelihoods

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in collaboration with

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BRDA 2025: Selected topics in (B)SM, 30-10-2025

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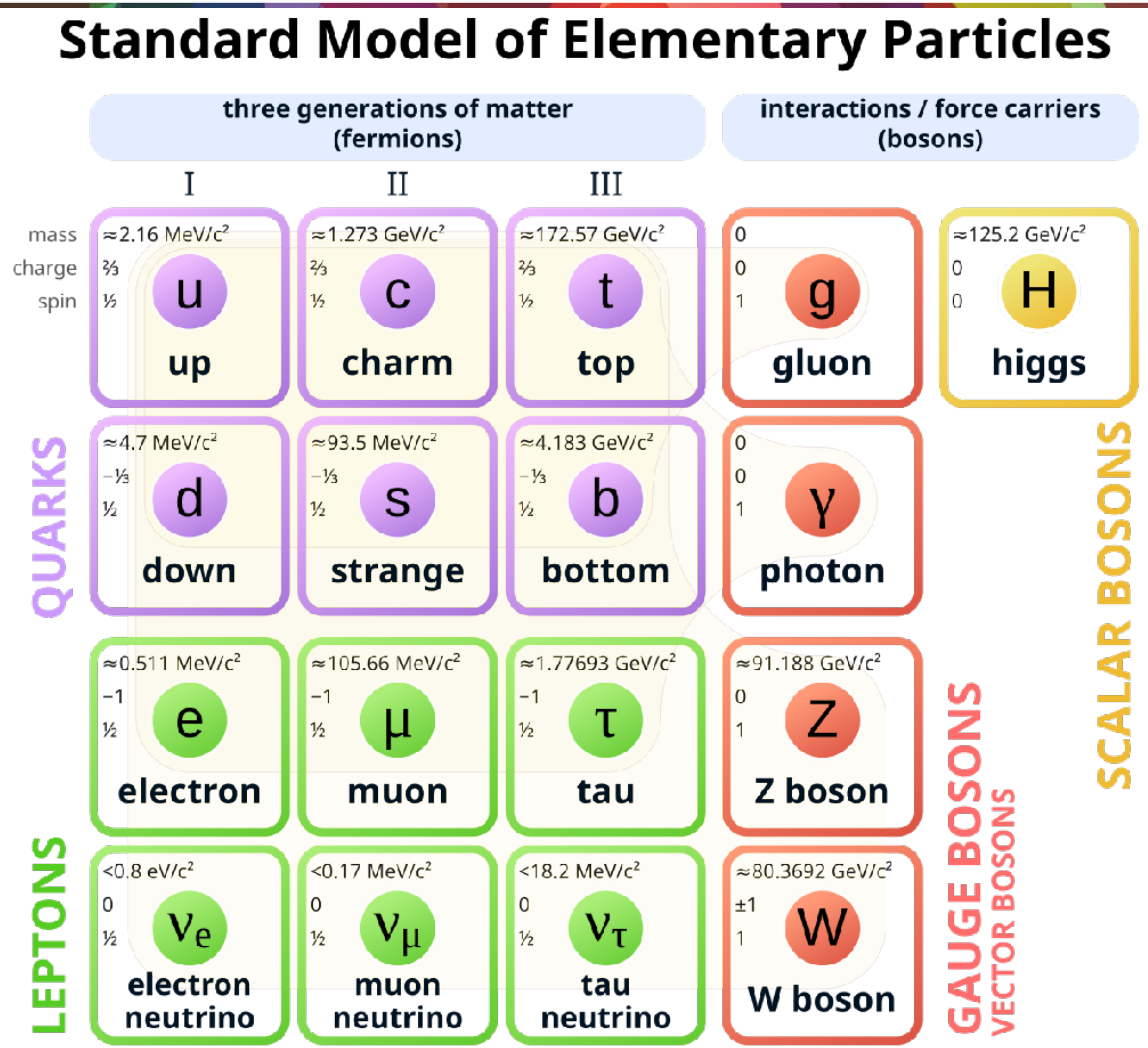
Status of the Particle Physics



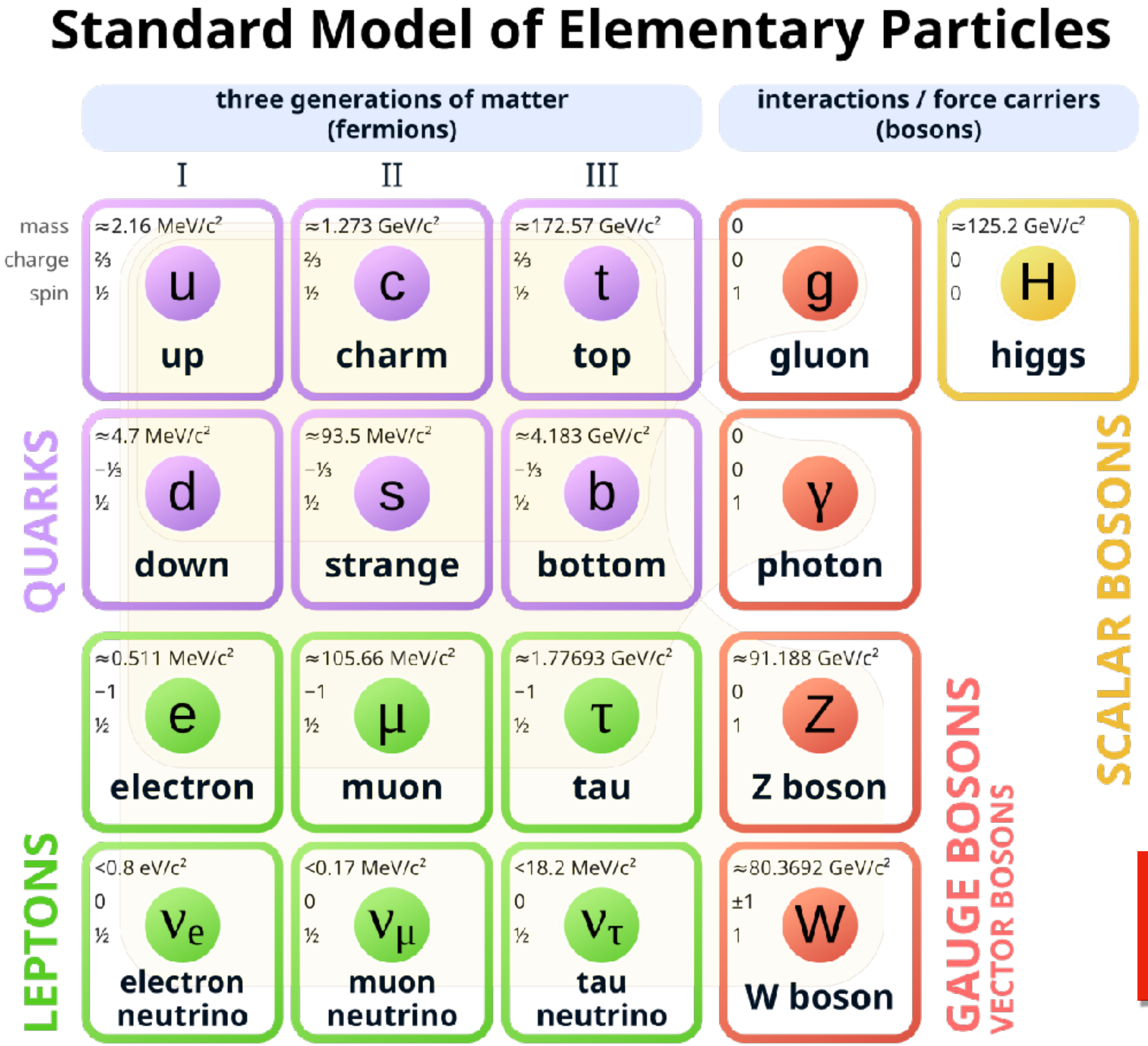
IMG source: https://en.wikipedia.org/wiki/Standard_Model

Status of the Particle Physics

⚠ Dark Matter

⚠ matter-antimatter asymmetry

⚠ neutrino masses



⚠ hierarchy problem

⚠ strong CP problem

⚠ quantum gravity?

IMG source: https://en.wikipedia.org/wiki/Standard_Model

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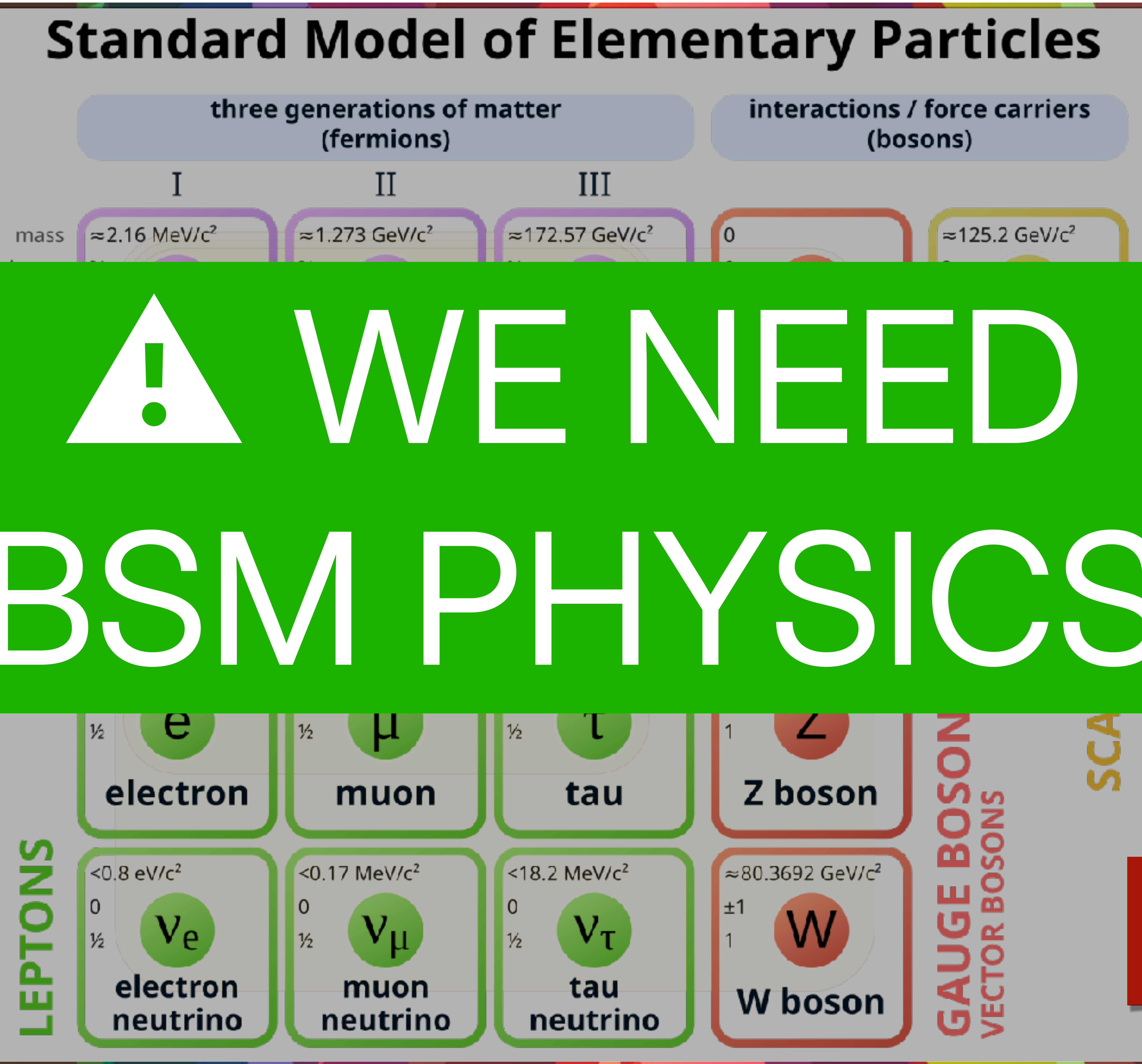
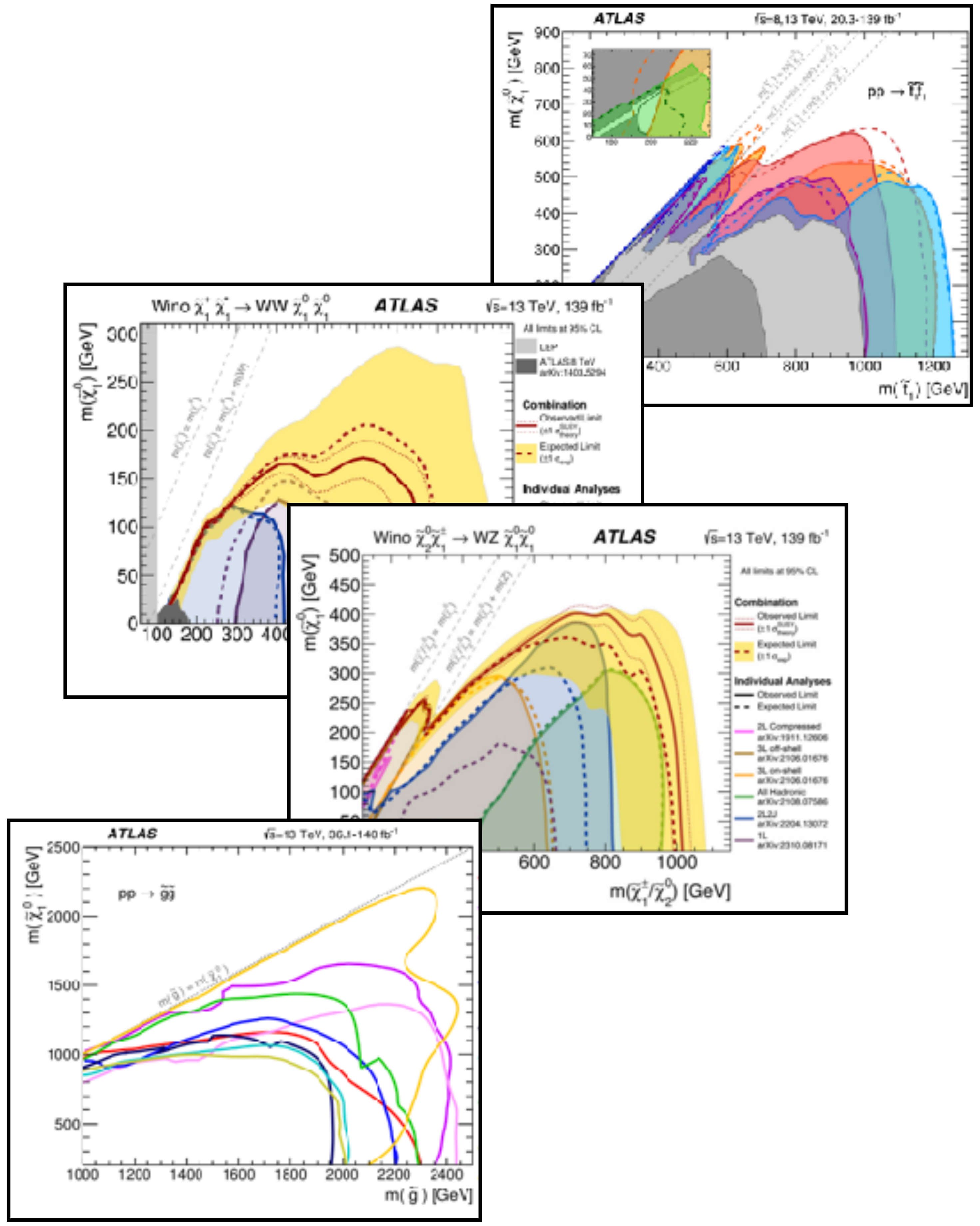
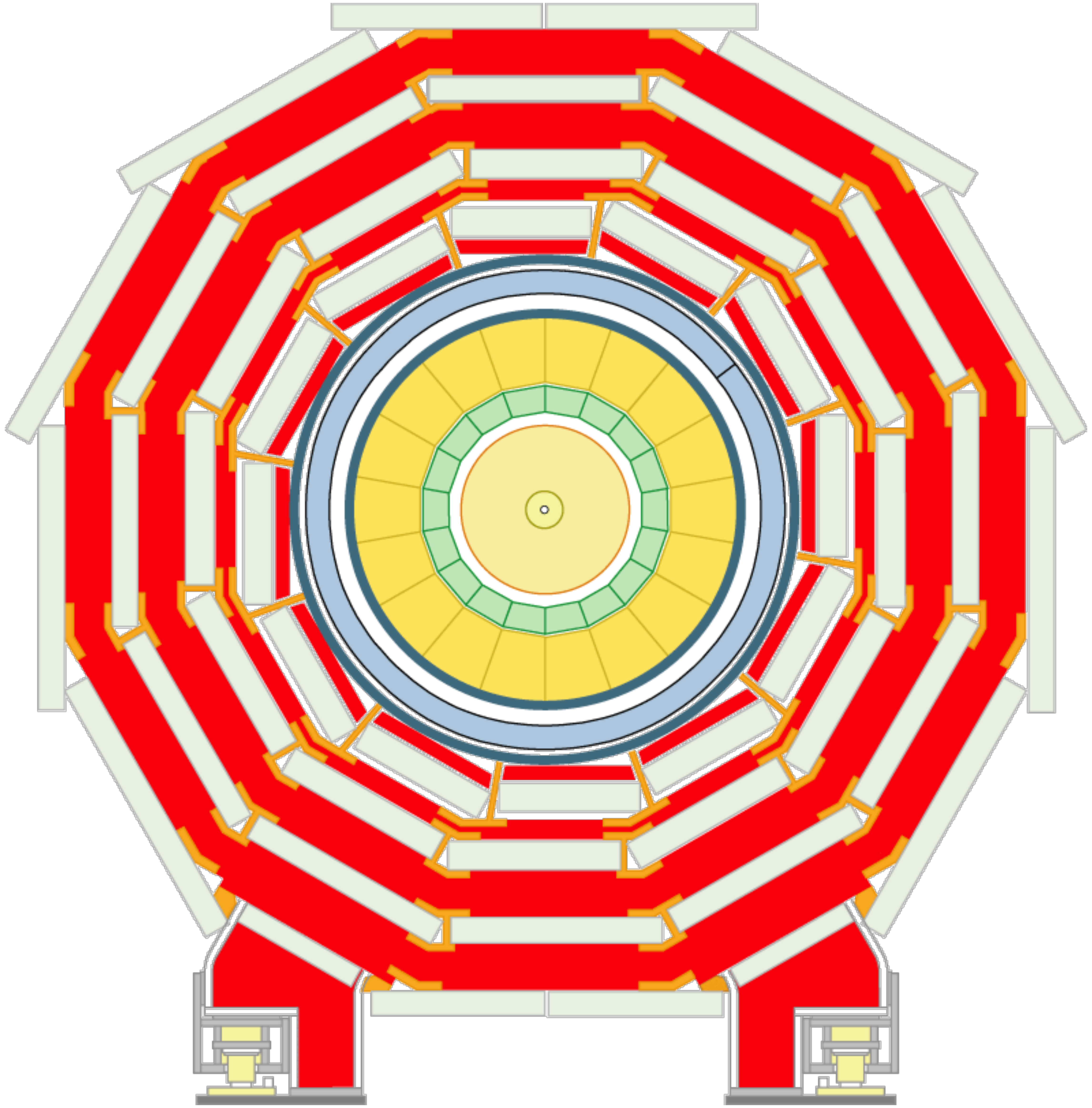


⚠ WE NEED BSM PHYSICS

IMG source: https://en.wikipedia.org/wiki/Standard_Model

Searches for BSM at the LHC

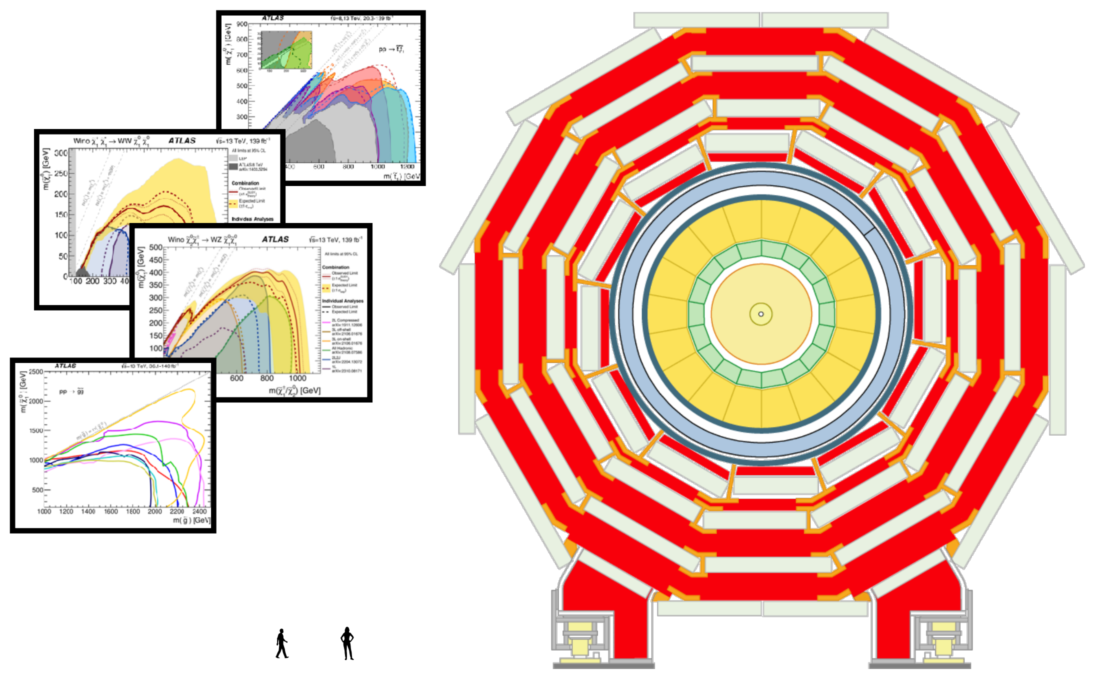




plots from: https://doi.org/10.1016/j.physrep.2024.09.010

Searches for BSM at the LHC

reinterpretation



WIMPs

AXIONS

LEPTOQUARK

DARK SECTOR

LR-SYMMETRY

COMPOSITENESS

SUPERSYMMETRY

NEUTRINO MODELS

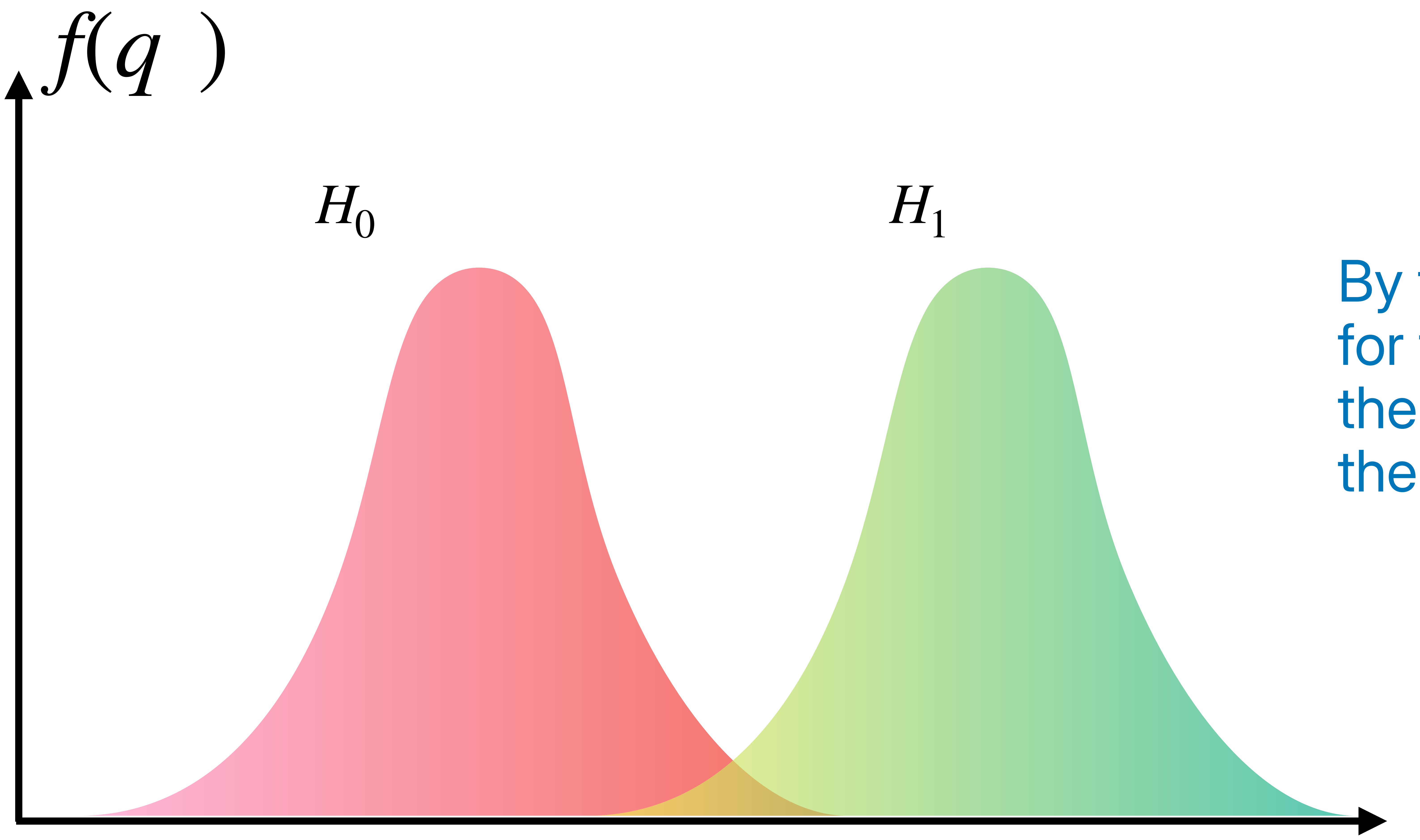
EXTRA DIMENSIONS

TWO HIGGS DOUBLETS

MILICHARGED PARTICLES

Reinterpretation

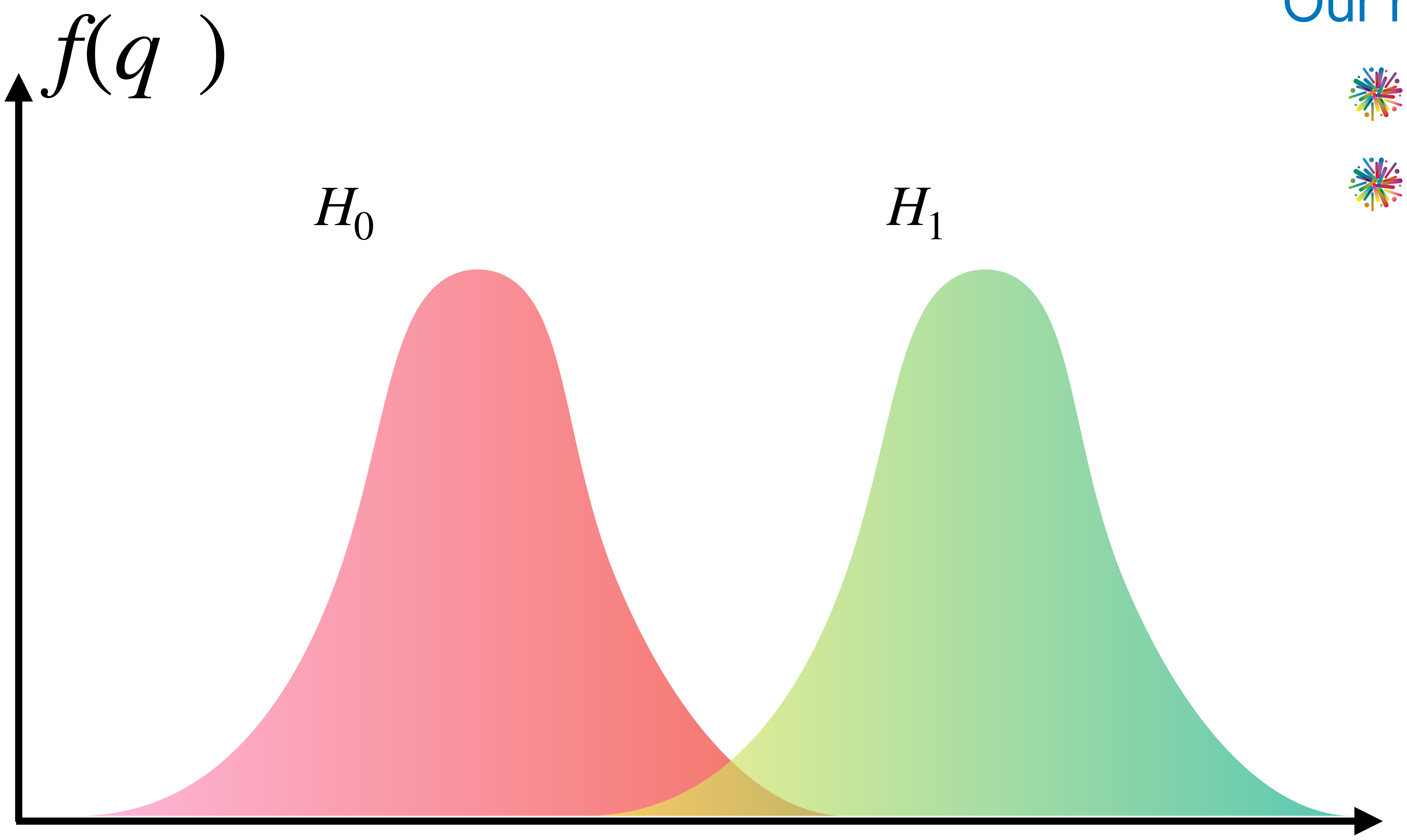
Hypothesis testing



By the Neyman-Pearson lemma, for testing simple hypotheses the most powerful test statistic is the likelihood ratio

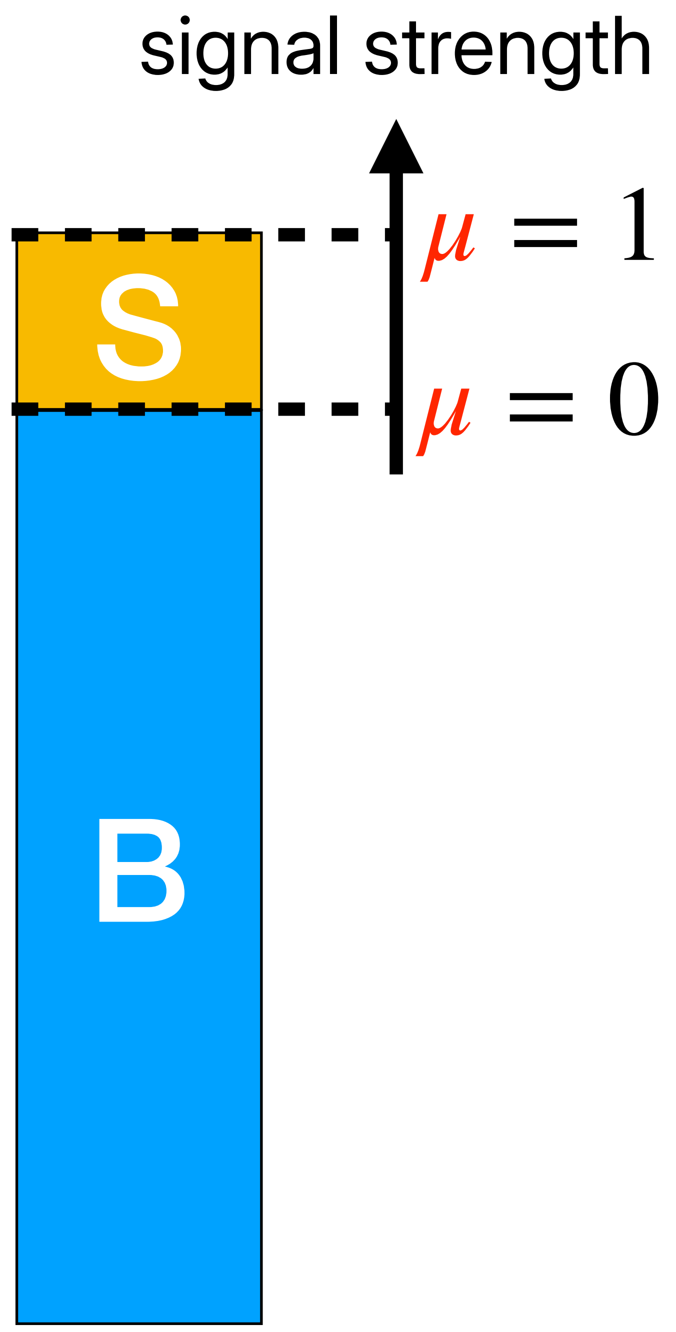
$$q = -2\ln \frac{L\left(\text{data}\,|\,H_0\right)}{L\left(\text{data}\,|\,H_1\right)}$$

Hypothesis testing

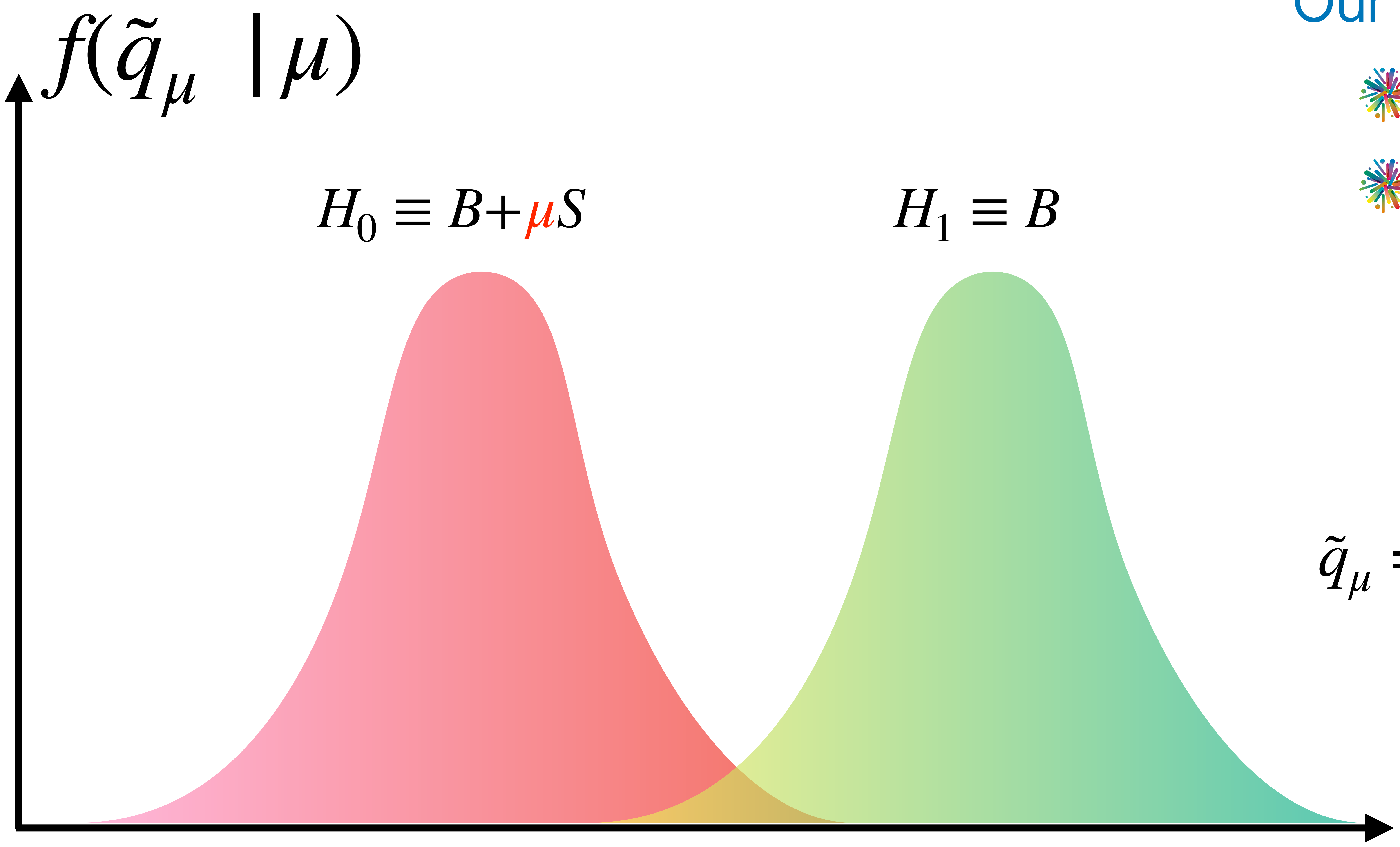


Our hypotheses are composite:

- We have niussance parameters θ
- We have a parameter of interest μ



Hypothesis testing



Our hypotheses are composite:

- We have niussance parameters θ
- We have a parameter of interest μ

$$\tilde{q}_\mu = \begin{cases} -2\ln \dfrac{L\left(\mu, \hat{\hat{\theta}}(\mu)\right)}{L\left(0, \hat{\hat{\theta}}(0)\right)} & \hat{\mu} < 0, \\ -2\ln \dfrac{L\left(\mu, \hat{\hat{\theta}}(\mu)\right)}{L\left(\hat{\mu}, \hat{\theta}\right)} & 0 \le \hat{\mu} \le \mu, \\ 0 & \hat{\mu} > \mu \end{cases}$$

$\hat{\square}$ unconditional MLE

$\hat{\hat{\square}}(\mu)$ conditional MLE

Full likelihood models by ATLAS (Hist Factory)

$$L\left(n, a \,|\, \mu, \theta\right) = \prod_{c}^{\text{channels}} \prod_{b}^{\text{bins}_c} \text{Pois}\left(\boxed{n_{cb}} \,|\, \nu_{cb}(\boxed{\mu}, \boxed{\theta})\right) \prod_{\theta} p_\theta\left(\boxed{a_\theta} \,|\, \boxed{\theta}\right)$$

Full likelihood models by ATLAS (Hist Factory)

channel data

auxiliary data

$$L\left(n, a \,|\, \mu, \theta\right) = \underbrace{\prod_{c}^{\text{channels}} \prod_{b}^{\text{bins}_c} \mathrm{Pois}\left(\boxed{n_{cb}} \,|\, \nu_{cb}(\boxed{\mu}, \boxed{\theta})\right)}_{\text{simultaneous measurement of multiple channels}} \underbrace{\prod_{\theta} p_{\theta}\left(\boxed{a_{\theta}} \,|\, \boxed{\theta}\right)}_{\text{constraint terms for "auxiliary measurements"}}$$

free parameters

constrained parameters

simultaneous measurement
of multiple channels

constraint terms for
"auxiliary measurements"

Full likelihood models by ATLAS (Hist Factory)

auxiliary data

channel data

$$L\left(n, a \,|\, \mu, \theta\right) = \underbrace{\prod_{c}^{\text{channels}} \prod_{b}^{\text{bins}_c} \mathrm{Pois}\left(\boxed{n_{cb}} \,|\, \nu_{cb}(\boxed{\mu}, \boxed{\theta})\right)}_{\text{simultaneous measurement of multiple channels}} \underbrace{\prod_{\theta} p_\theta\left(\boxed{a_\theta} \,|\, \boxed{\theta}\right)}_{\text{constraint terms for "auxiliary measurements"}}$$

free parameters

constrained parameters

simultaneous measurement of multiple channels

constraint terms for "auxiliary measurements"

$$\nu_{cb}(\mu,\theta) = \sum_{s}^{\text{samples}} \nu_{scb}(\mu,\theta) = \sum_{s}^{\text{samples}} \underbrace{\left(\prod_{\kappa} \kappa_{scb}(\mu,\theta)\right)}_{\textbf{multiplicative modifiers}} \left(\underbrace{\nu^0_{scb}(\mu,\theta)}_{\textbf{const. nominal rate}} + \underbrace{\sum_{\Delta} \Delta_{scb}(\mu,\theta)}_{\textbf{additive modifiers}}\right)$$

multiplicative modifiers **const. nominal rate** **additive modifiers**

Why bother?

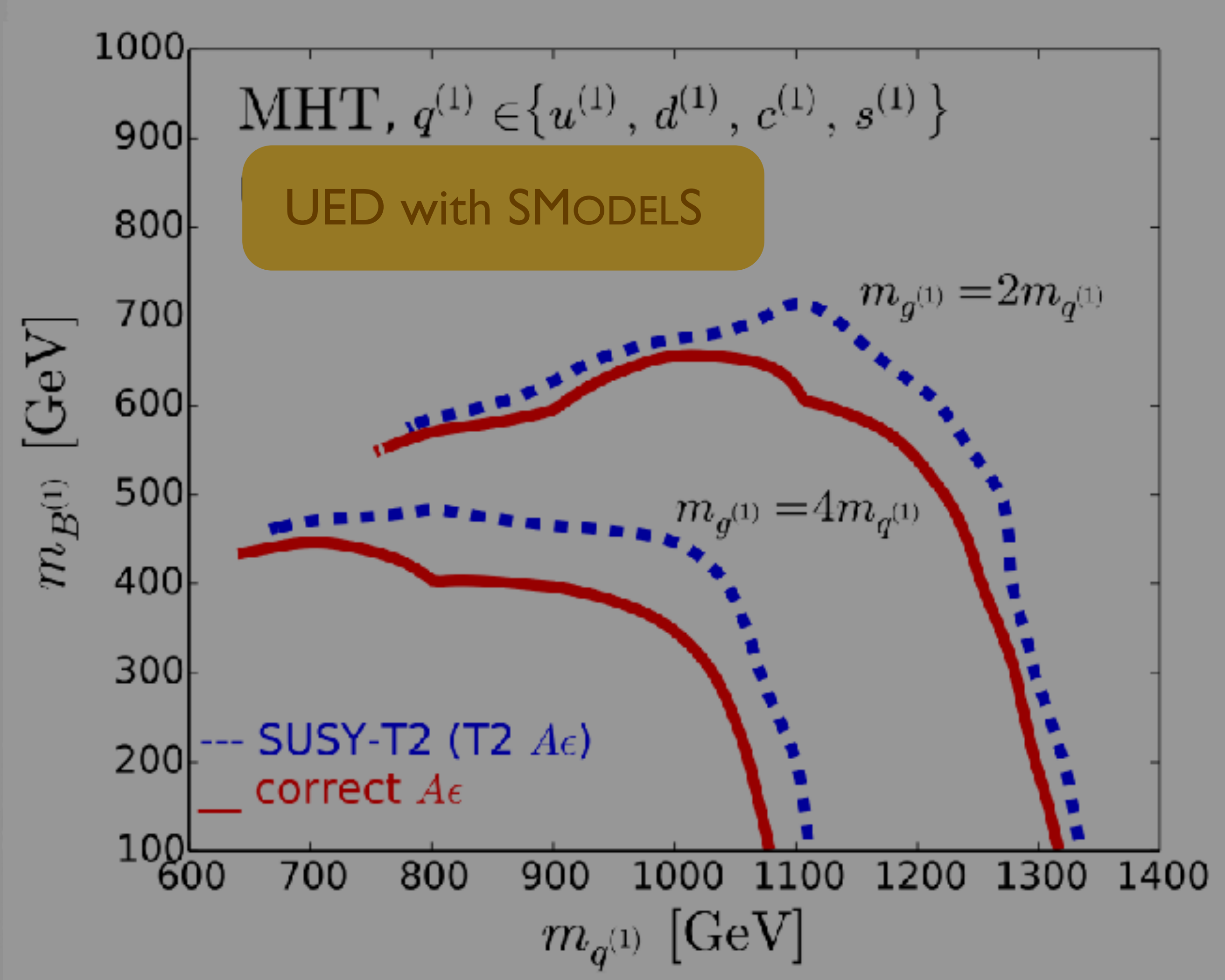
SMS-based *public* tools

SMS-based

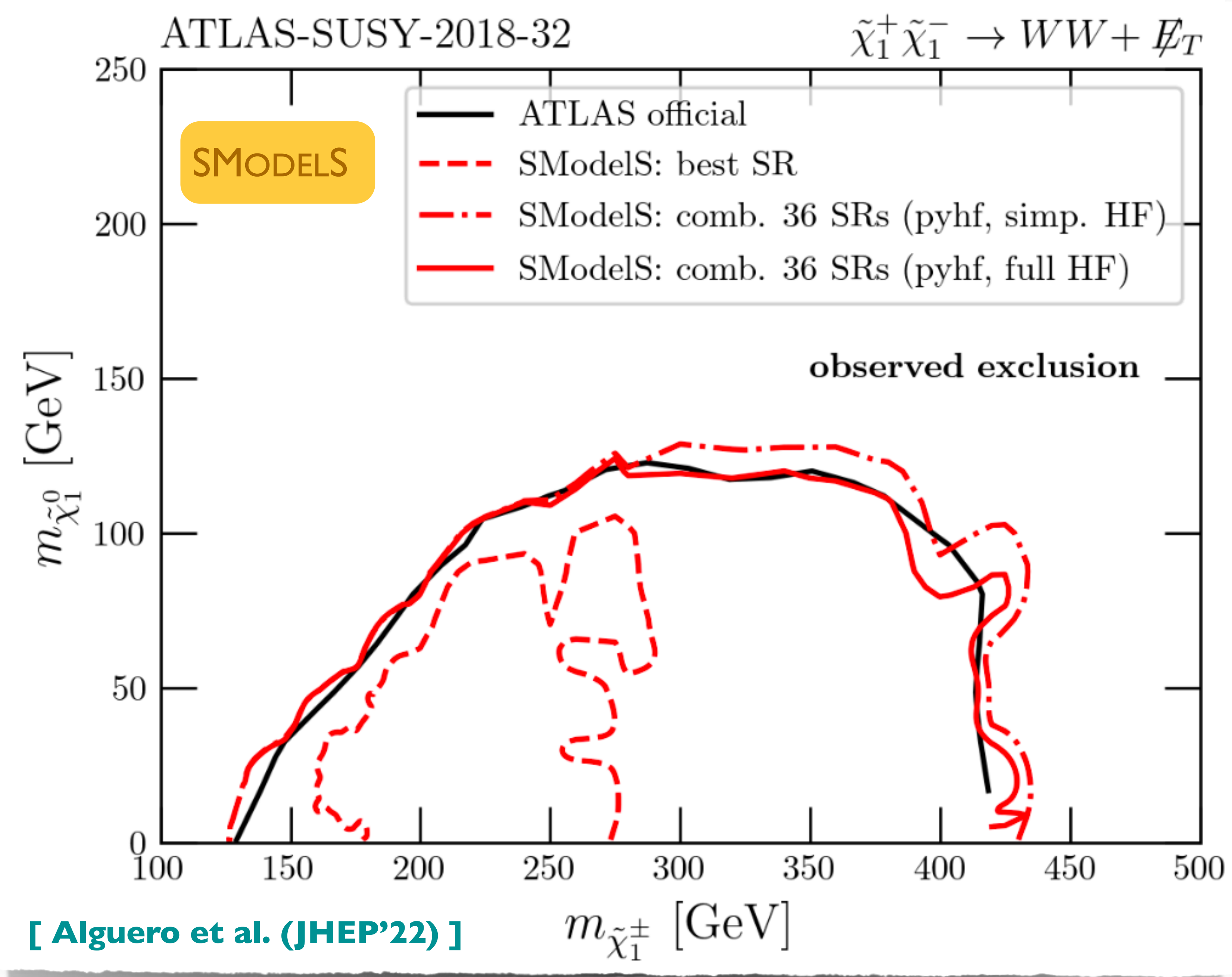
- SMODELS [$O(100)$ analyses, from GITHUB]
  - ⋆ Validation example (ATLAS-SUSY-2018-32)
    - ➙ $\tilde{W}/\tilde{B}$ scenario with 2 leptons and $\not{E}_T$
    - ➙ PYHF model crucial

[Kraml et al. (EPJC'14); Alguero et al. (JHEP'22)]
[Altakach et al. (SciPost'24)]

- Dark photons: DARKCAST [from GITLAB] [Ilten et al. (JHEP'18)]

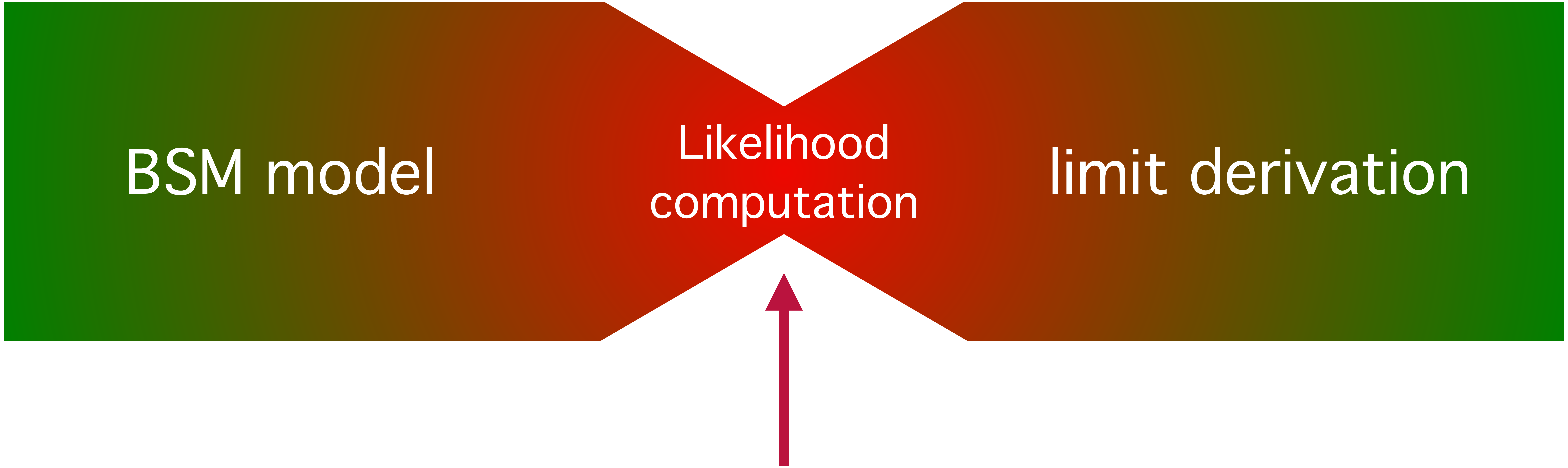


[Edelhäuser, Krämer, Sonneveld (JHEP'15)]



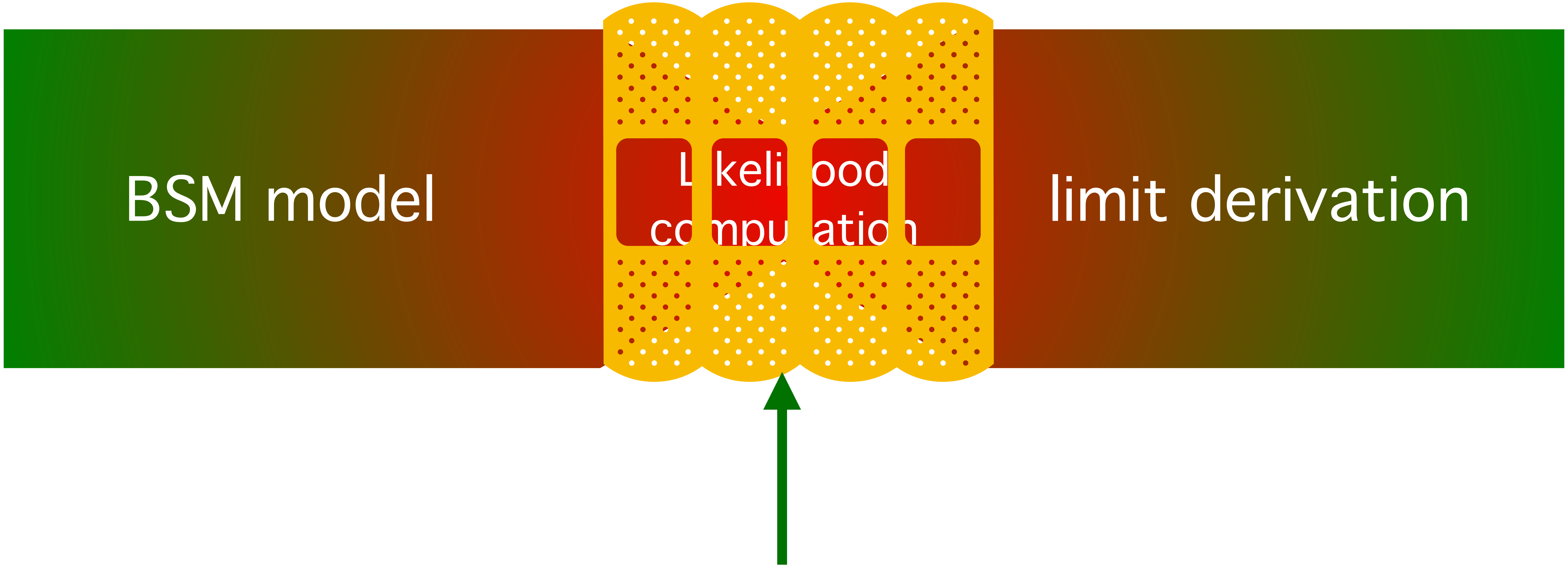
[Alguero et al. (JHEP'22)]

Computational bottleneck



Full statistical model calculations enter here

Computational bottleneck



Machine learning enters here

The goal

Use Machine Learning to create surrogates to Full Statistical Models that would be **accurate** but **much faster** to use.

The approach

If we have a regressor for profiled likelihoods, we can calculate limits.

$$\Delta_{\text{data}}^{(\mu_1,\ldots\mu_N)} \equiv \ln \frac{L\left(\text{data}\,\middle|\,\mu_1,\ldots,\mu_N;\ \hat{\hat{\theta}}(\mu_1,\ldots,\mu_N)\right)}{L\left(\text{data}\,\middle|\,0,\ldots,0;\ \hat{\hat{\theta}}(0,\ldots,0)\right)}$$

The approach

varying signal
in each bin independently

profiling

$$\Delta_{\mathrm{data}}^{(\mu_1,\ldots\mu_N)} \equiv \ln \frac{L\left(\mathrm{data}\,\middle|\,\mu_1,\ldots,\mu_N;\ \hat{\hat{\theta}}(\mu_1,\ldots,\mu_N)\right)}{L\left(\mathrm{data}\,\middle|\,0,\ldots,0;\ \hat{\hat{\theta}}(0,\ldots,0)\right)}$$

Four data sets are used:
1) expected data
2) expected Asimov data
3) observed data
4) observed Asimov data

The approach

regression

yields → likelihoods → limits

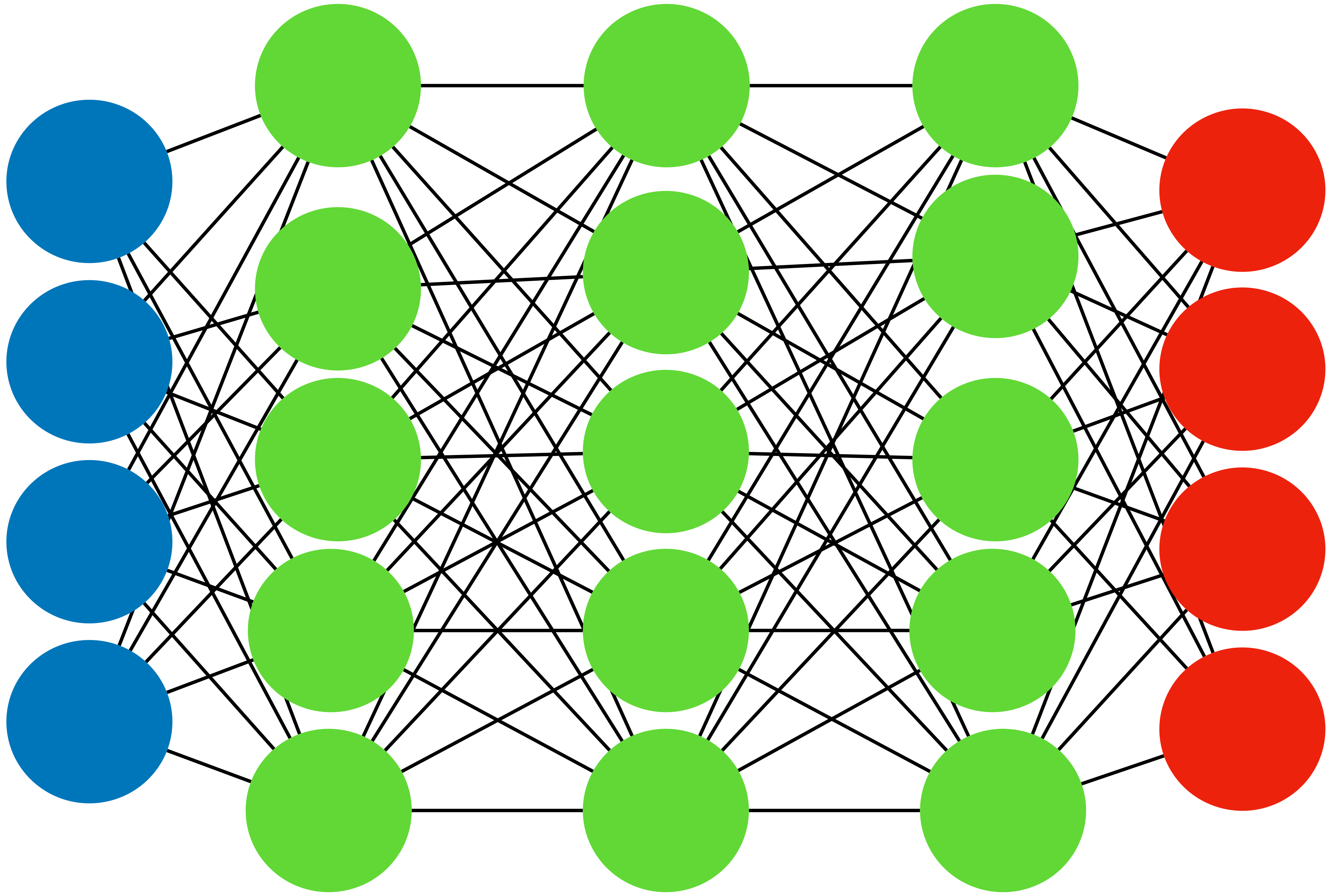
n_1

n_2

n_3

$\vdots$

n_N



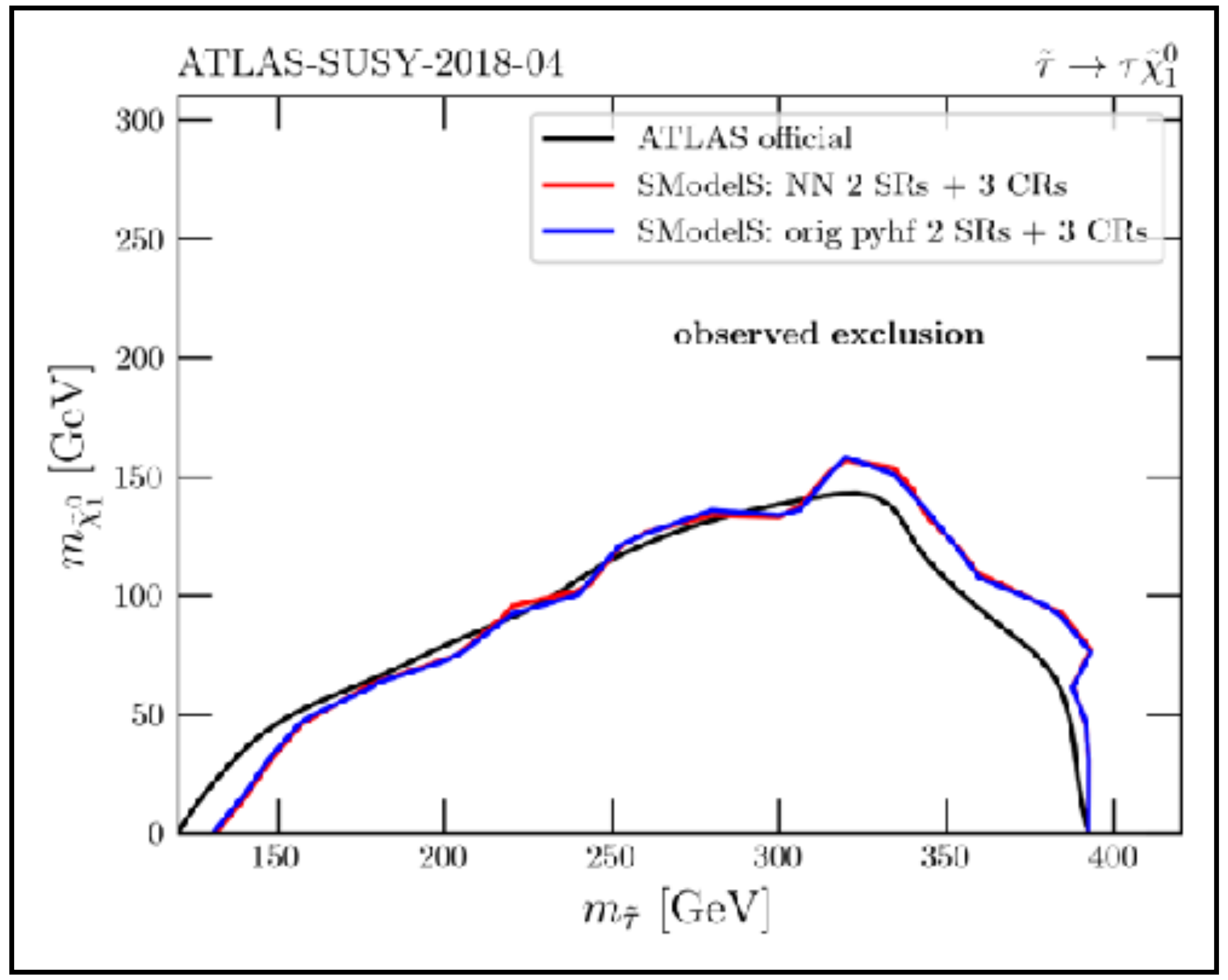
Fully connected network

$\Delta_{\text{exp}}^{(\mu_1,\ldots\mu_N)}$

$\Delta_{\text{exp Asimov}}^{(\mu_1,\ldots\mu_N)}$

$\Delta_{\text{obs}}^{(\mu_1,\ldots\mu_N)}$

$\Delta_{\text{obs Asimov}}^{(\mu_1,\ldots\mu_N)}$



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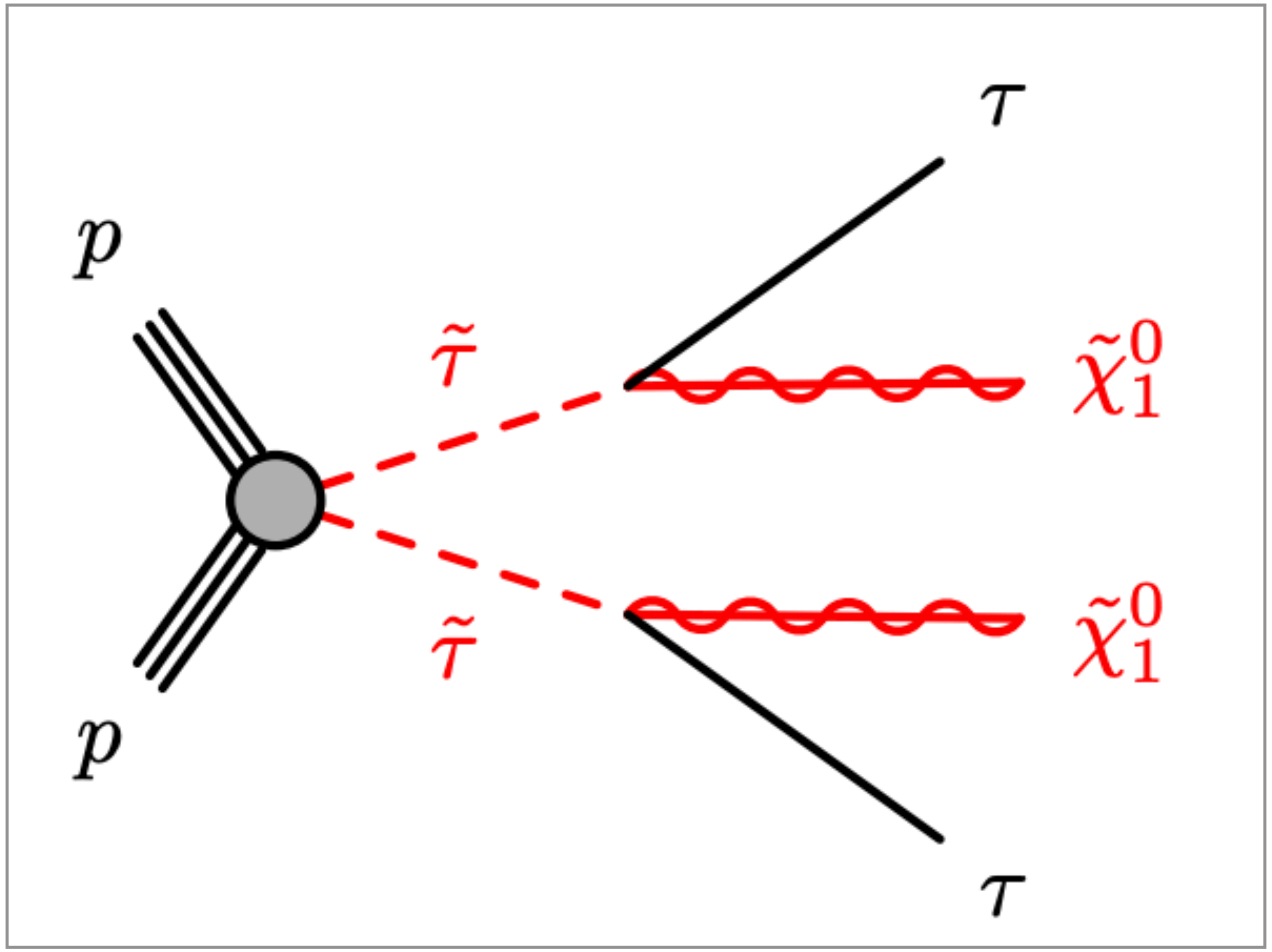
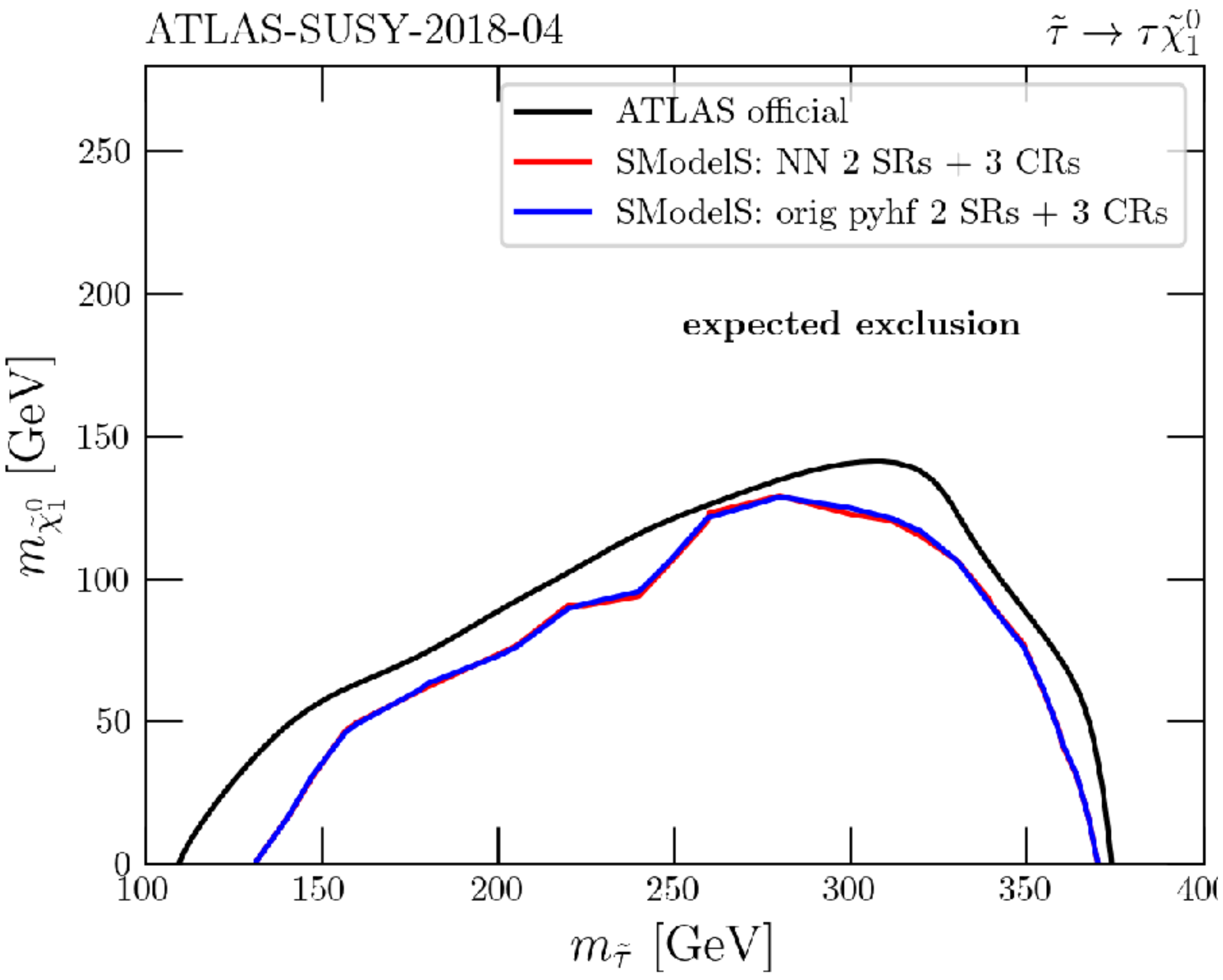
ATLAS-SUSY-2018-04 (1911.0660)

- Search for direct stau production in events with two hadronic τ-leptons in $\sqrt{s}$=13 TeV pp collisions with the ATLAS detector
- 2 signal bins, 3 control bins
- fit **without** the CRs

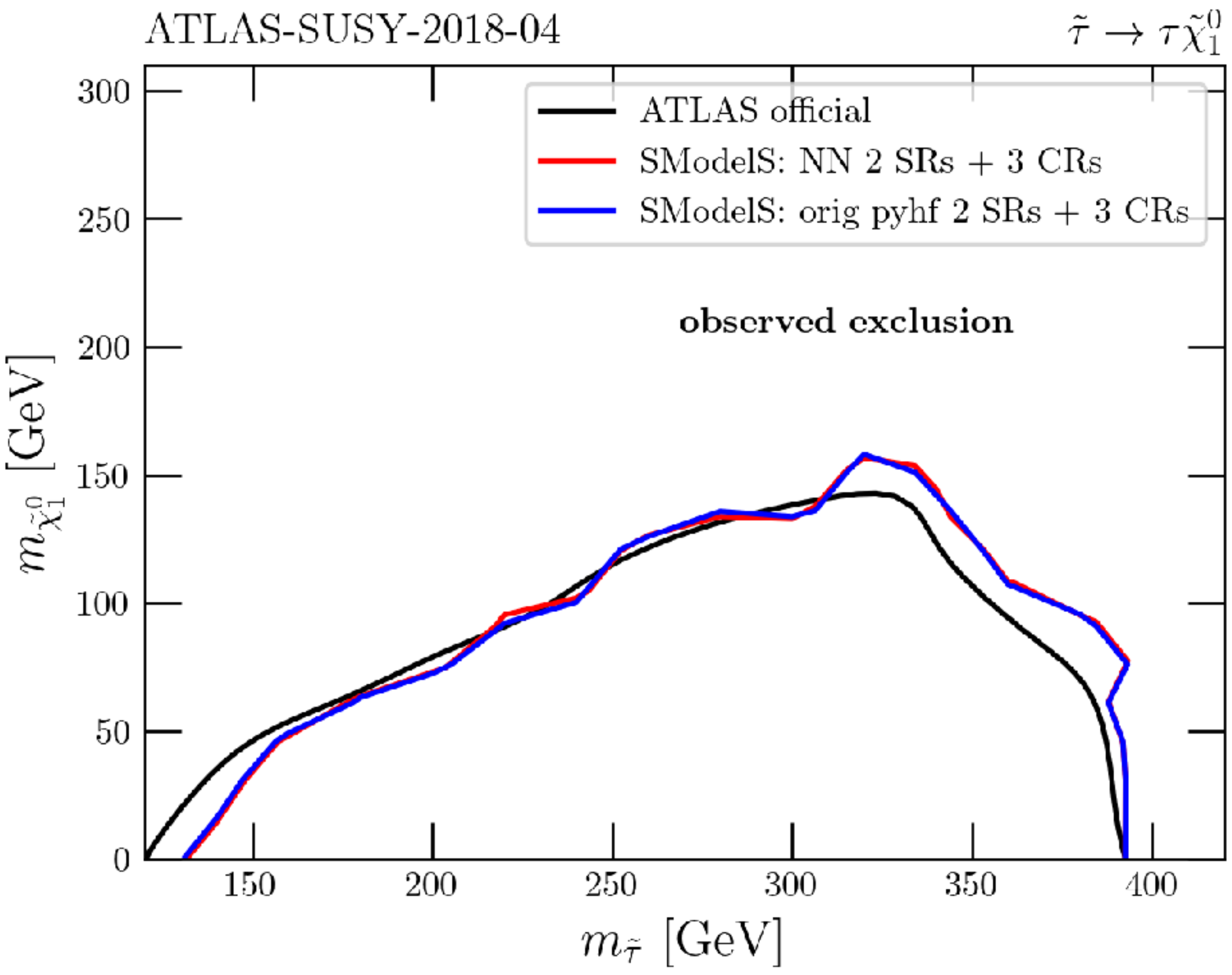


ATLAS-SUSY-2018-04 (1911.0660)

expected

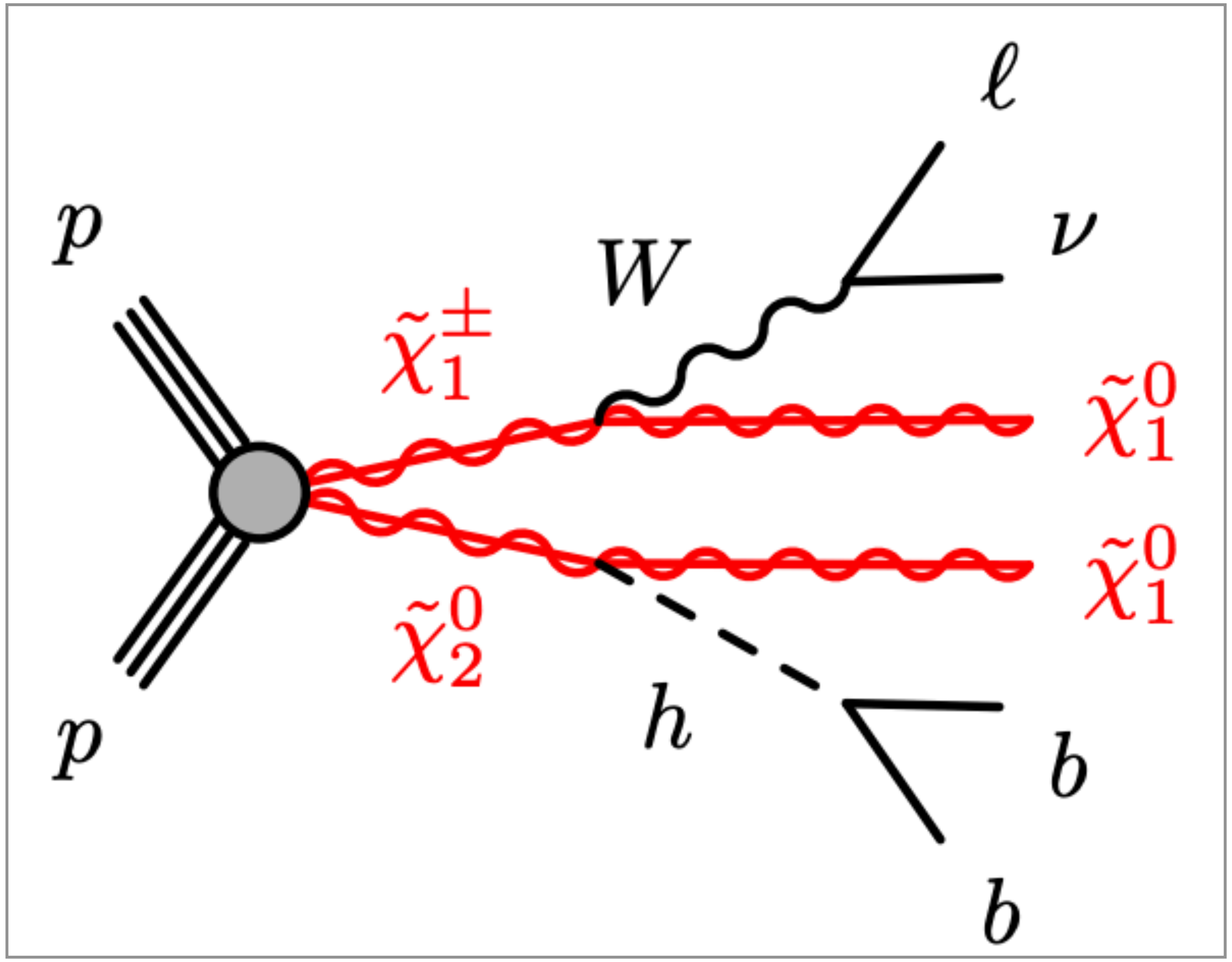


observed



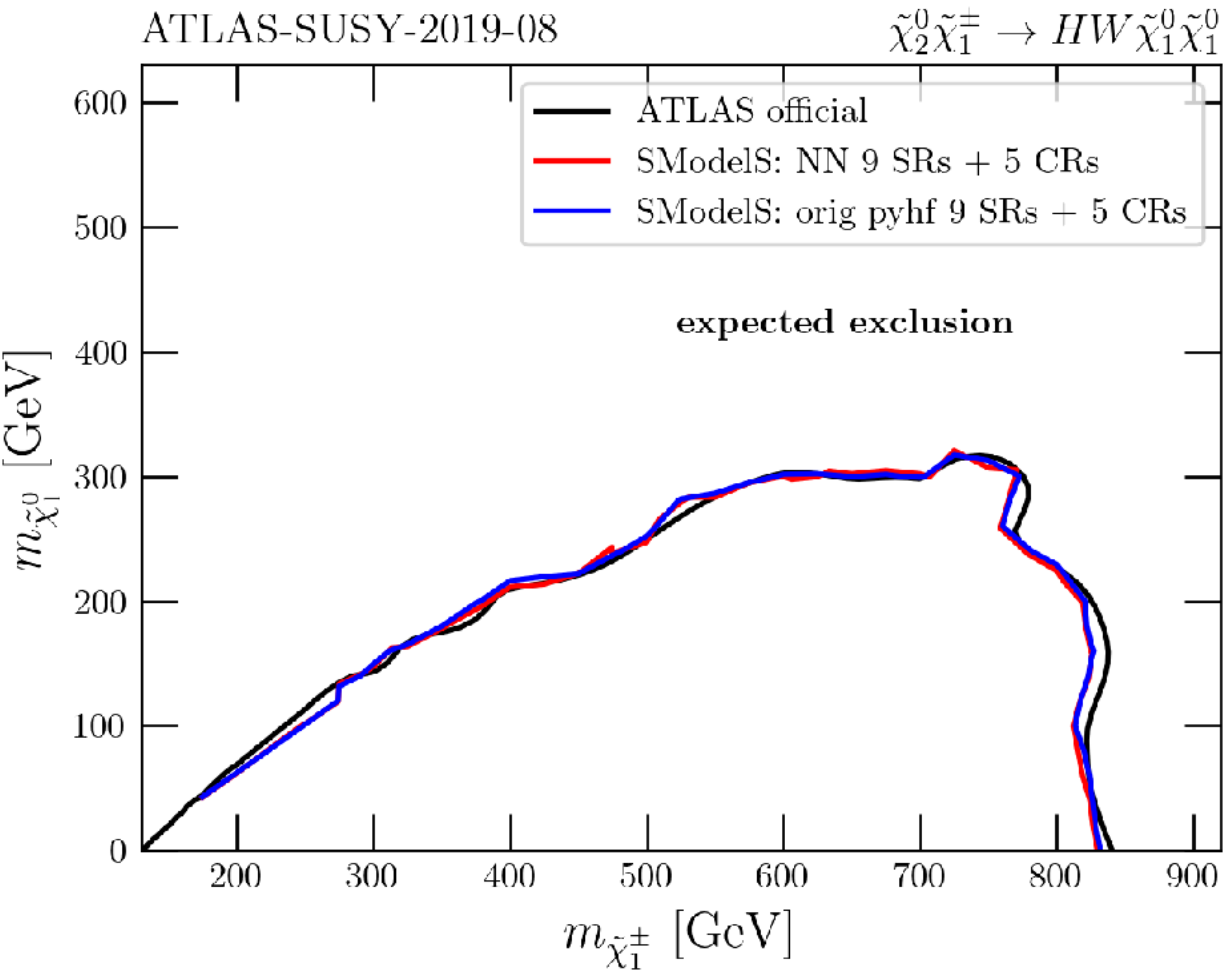
ATLAS-SUSY-2019-08 (1909.09926)

- Search for direct production of electroweakinos in final states with one lepton, missing transverse momentum and a Higgs boson decaying into two b-jets in pp collisions at $\sqrt{s}$=13 TeV with the ATLAS detector
- 9 signal bins, 5 control bins

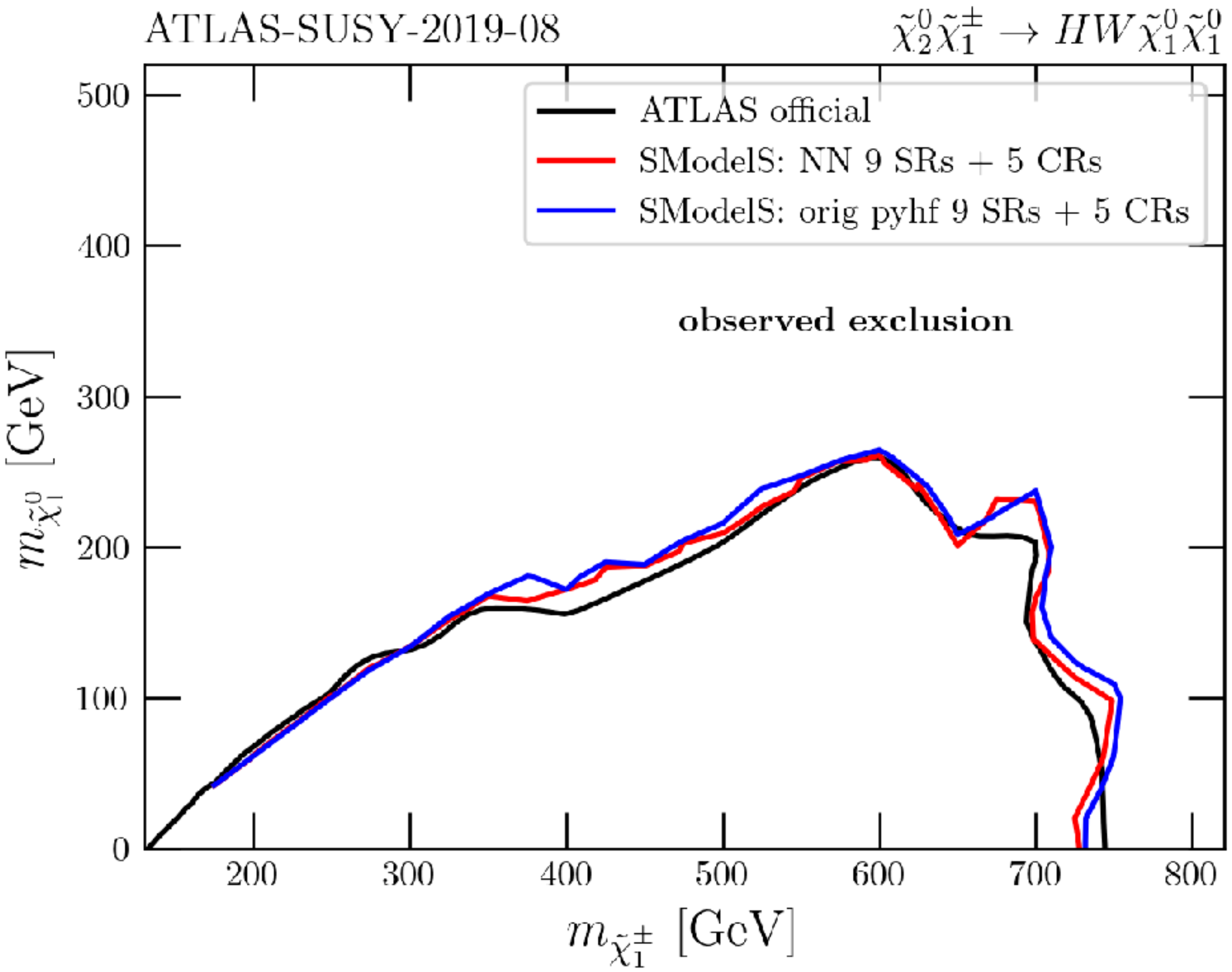


ATLAS-SUSY-2019-08 (1909.09926)

expected



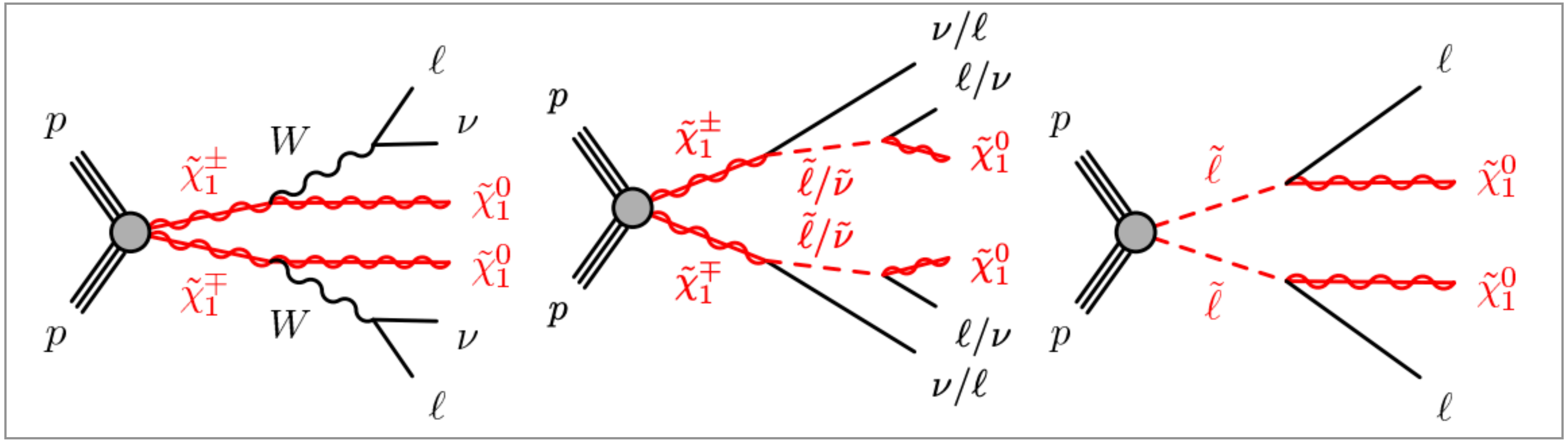
observed



ATLAS-SUSY-2018-32 (1908.08215)

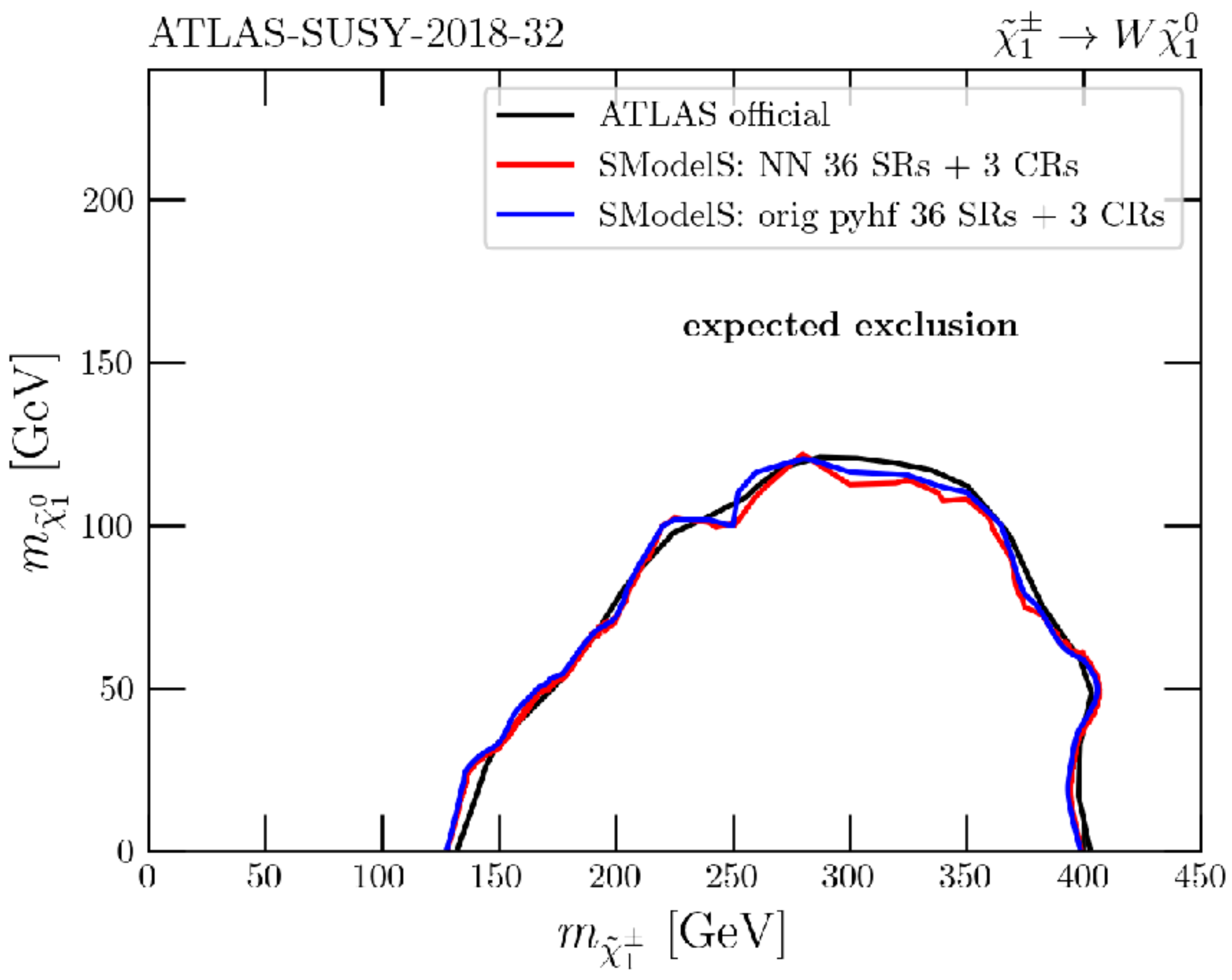
- Search for electroweak production of charginos and sleptons decaying into final states with two leptons and missing transverse momentum in s=√13 TeV pp collisions using the ATLAS detector

- 36 SRs and 3 CRs

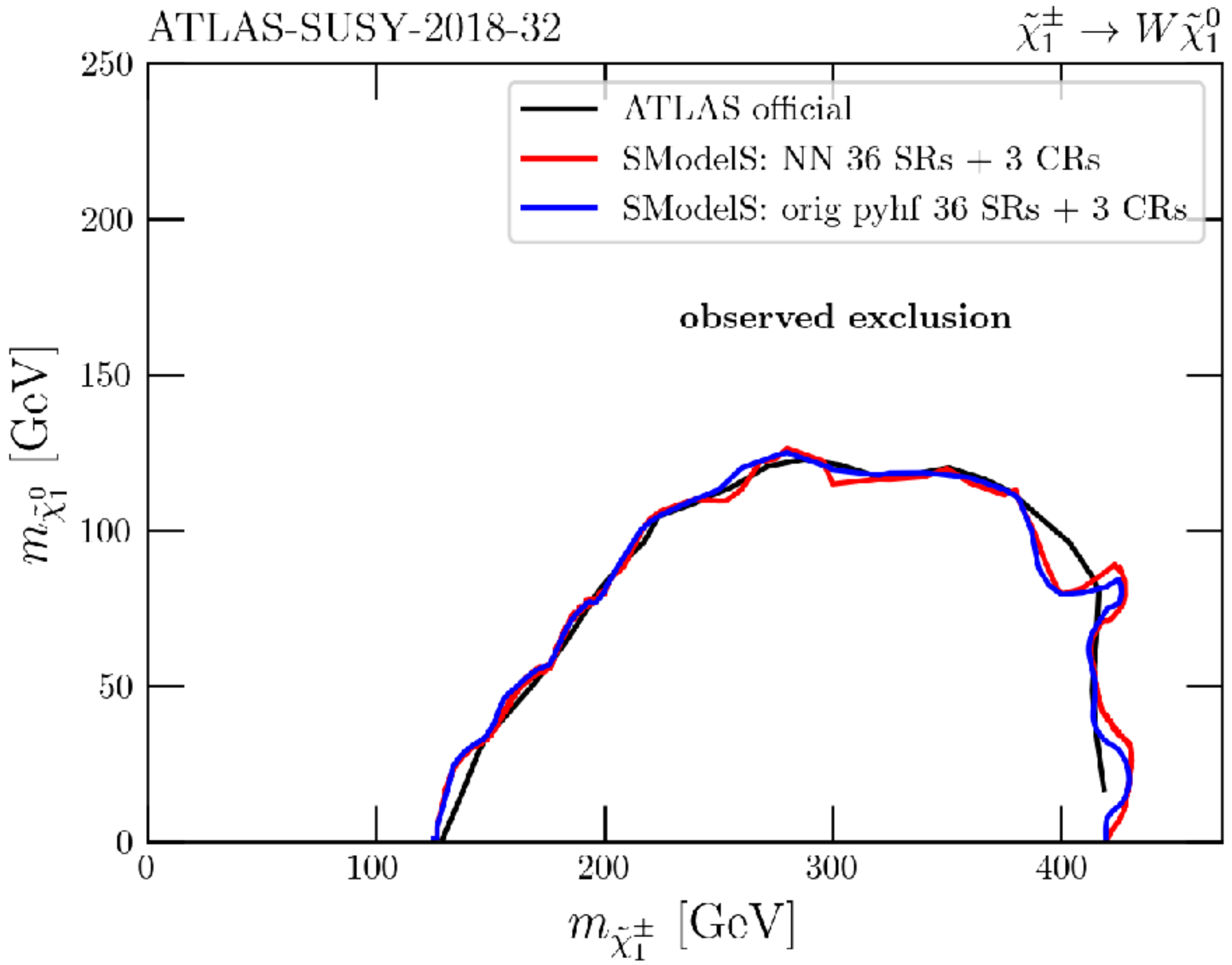


ATLAS-SUSY-2018-32 (1908.08215)

expected

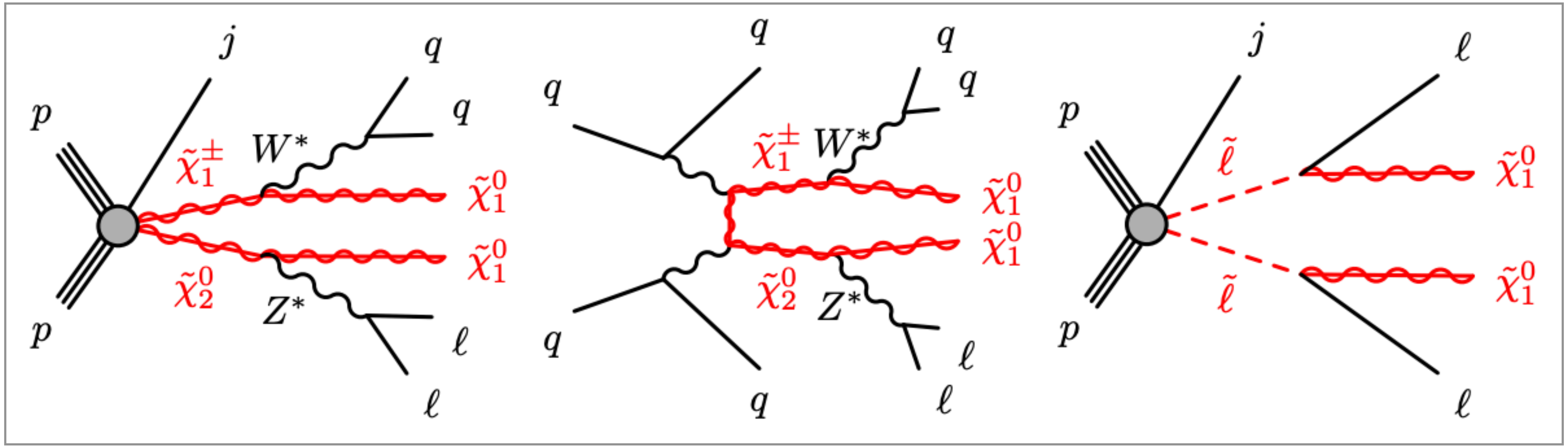


observed



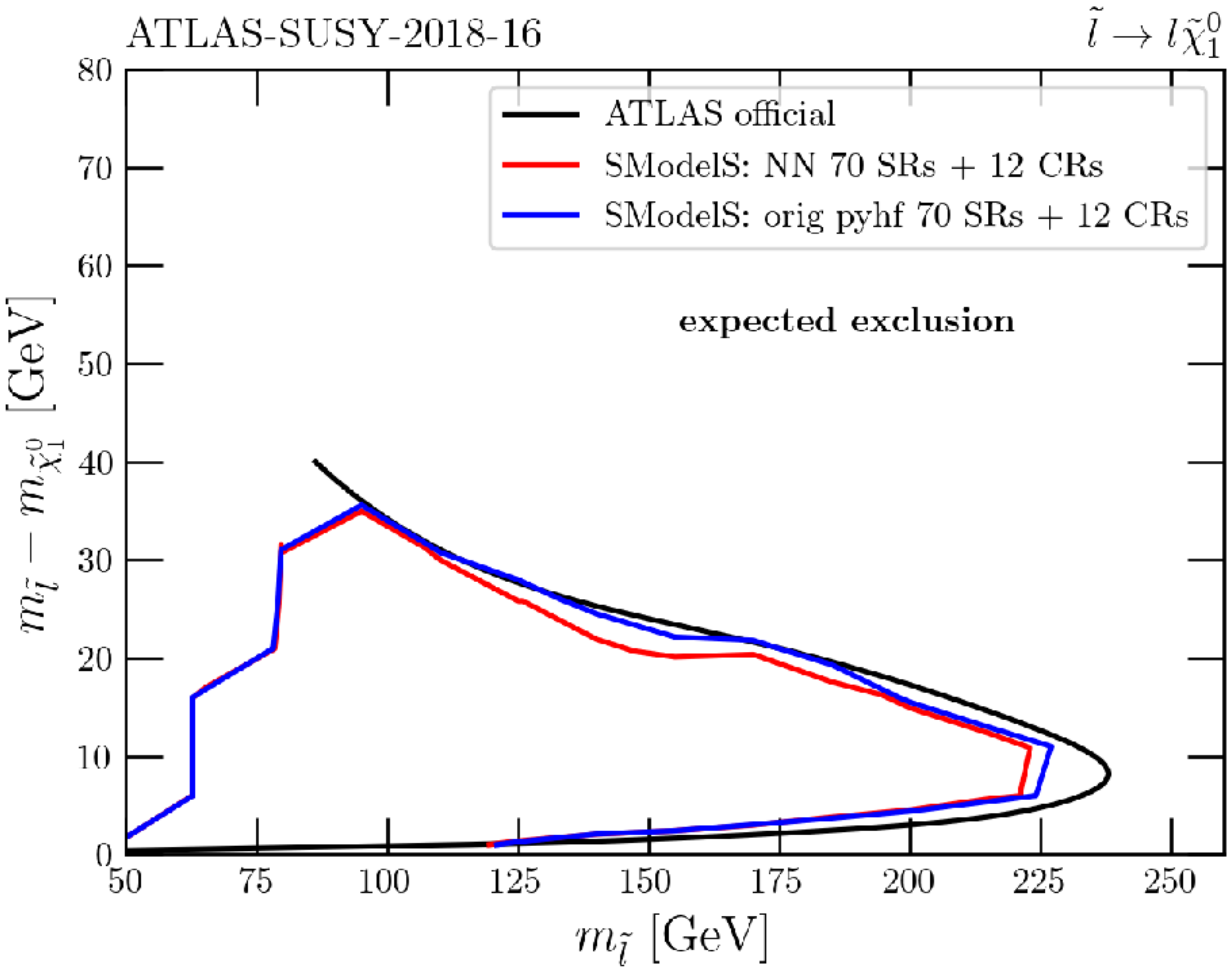
ATLAS-SUSY-2018-16 (1911.12606)

- Searches for electroweak production of supersymmetric particles with compressed mass spectra in $\sqrt{s}$=13 TeV pp collisions with the ATLAS detector
- 2 subanalyses: sleptons (32+6) and EWKinos (38+6)
- Subanalyses combination: 70 SRs + 12 CRs (disjoint in this particular case)

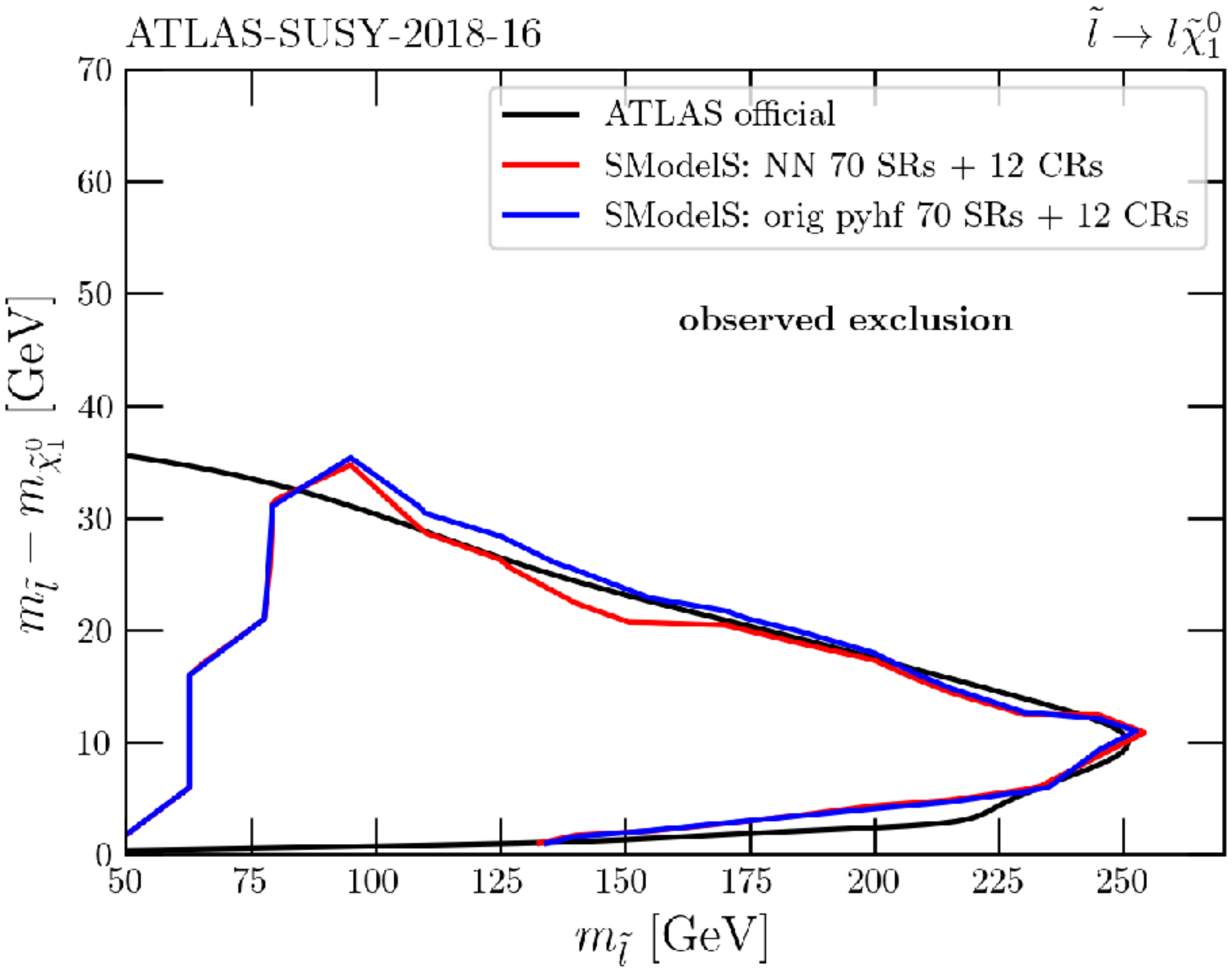


ATLAS-SUSY-2018-16 (1911.12606)

expected (sleptons)



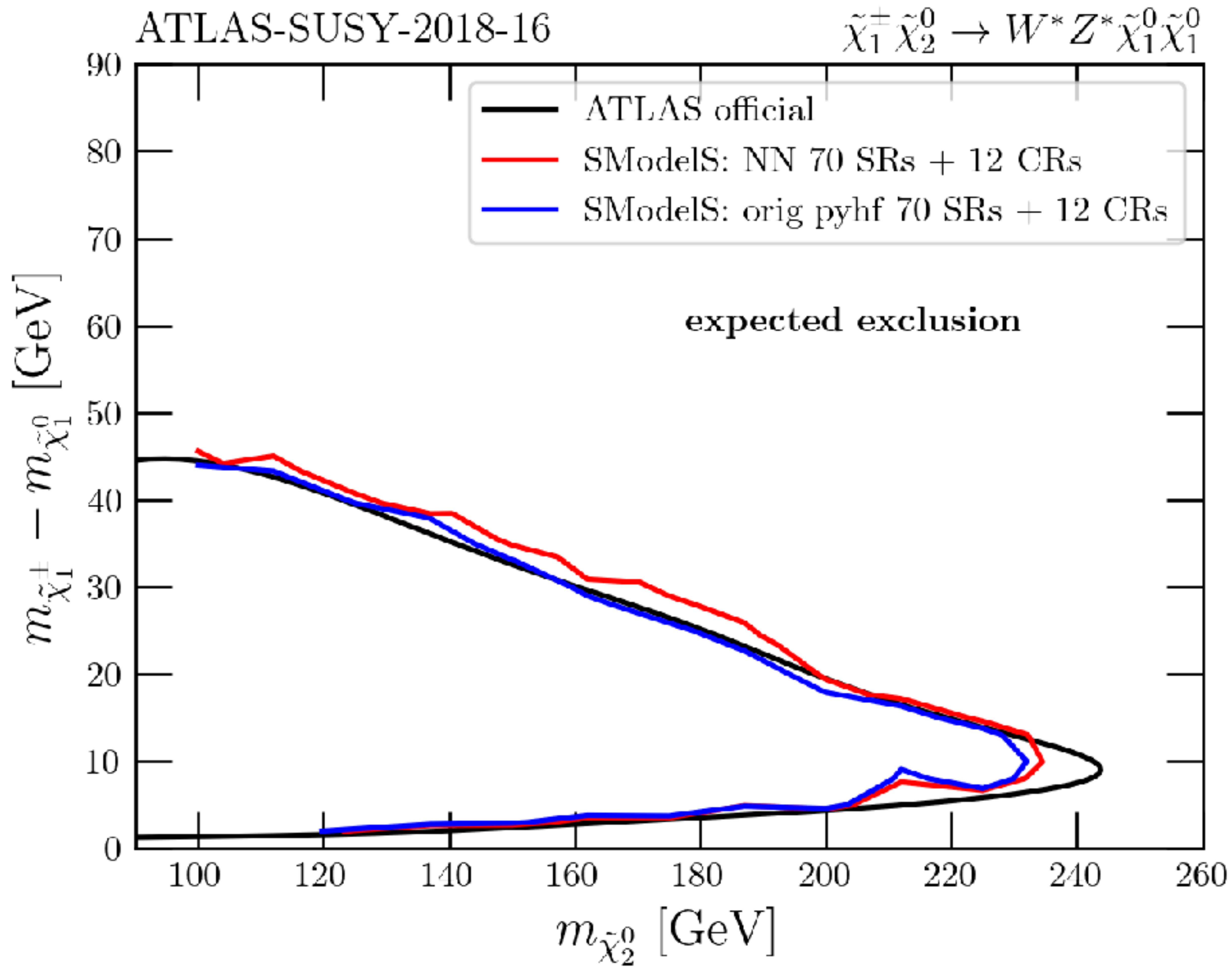
observed (sleptons)

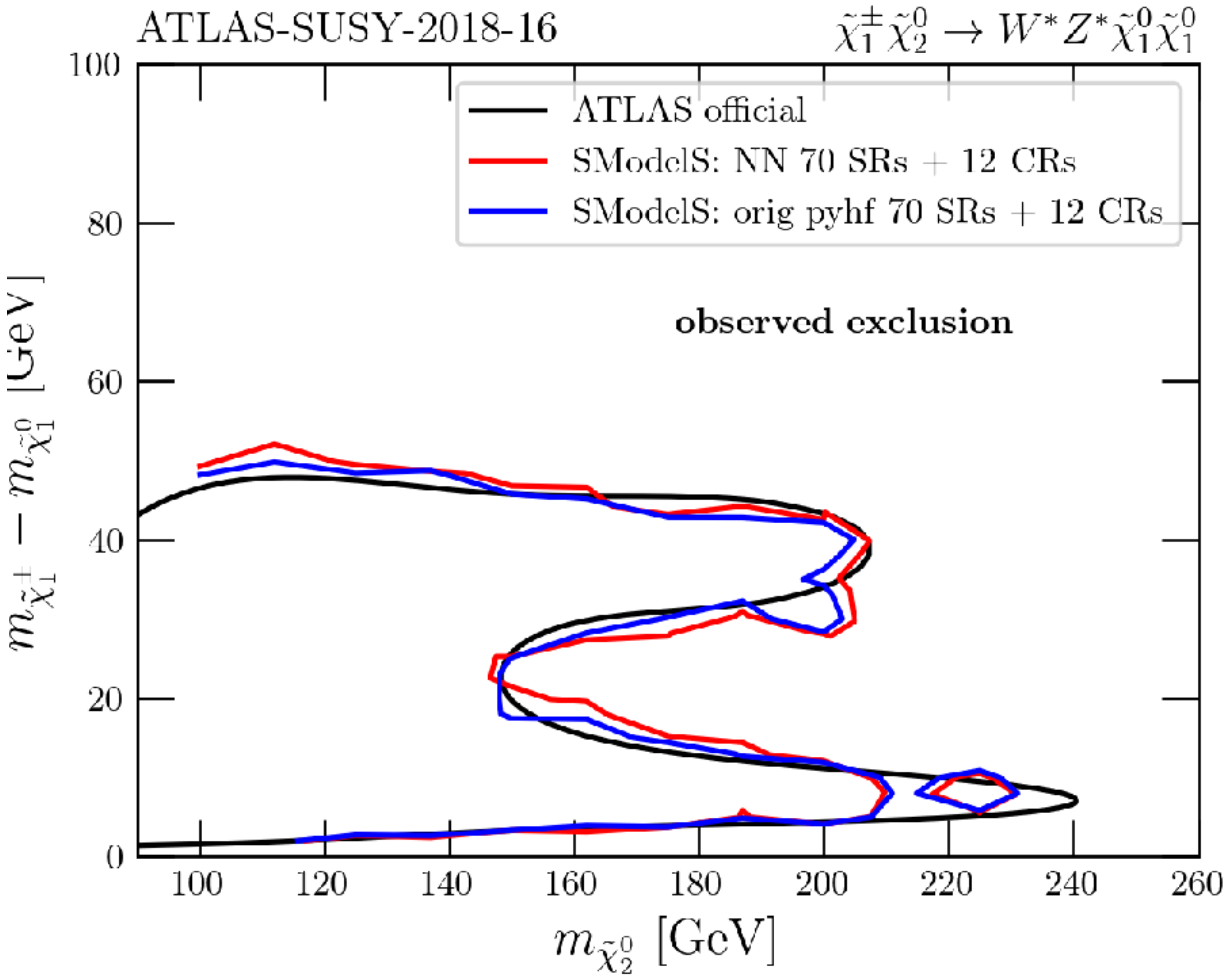


ATLAS-SUSY-2018-16 (1911.12606)

expected (EWKinos)

observed (EWKinos)





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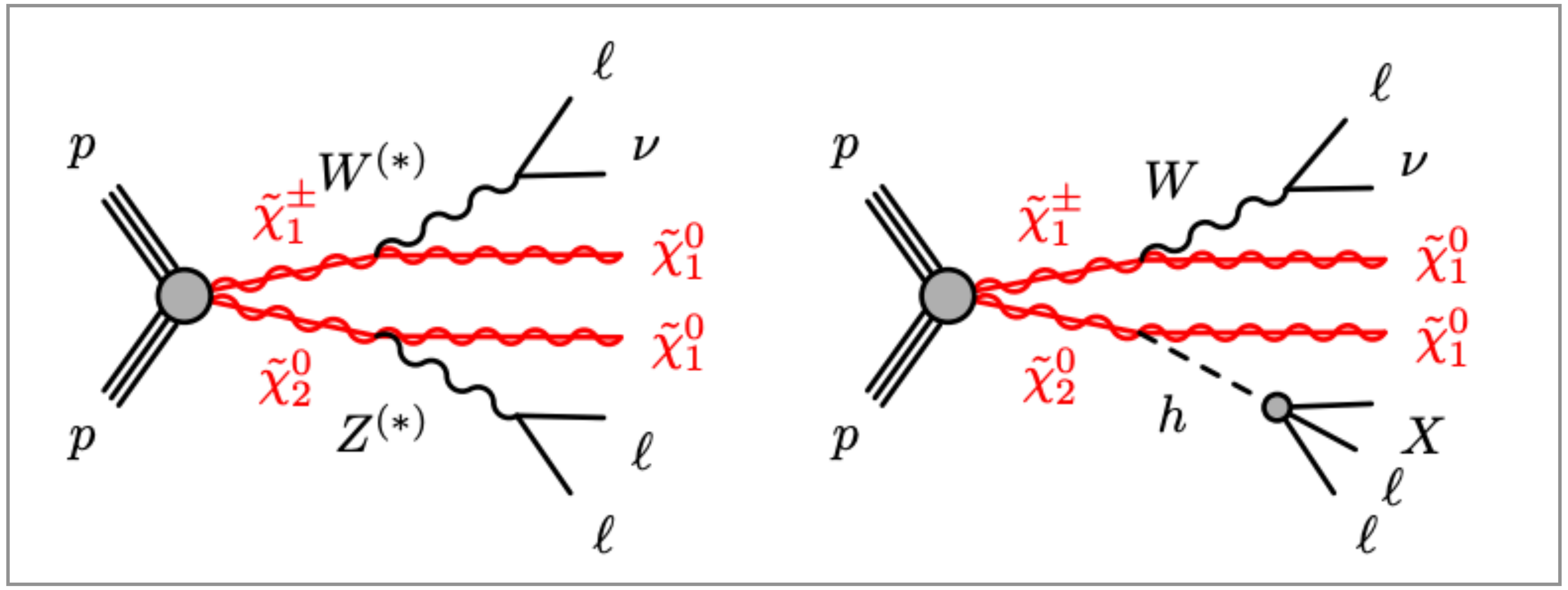
ATLAS-SUSY-2019-09 (2106.01676)

- Search for chargino-neutralino pair production in final states with three leptons and missing transverse momentum in √s=13 TeV pp collisions with the ATLAS detector

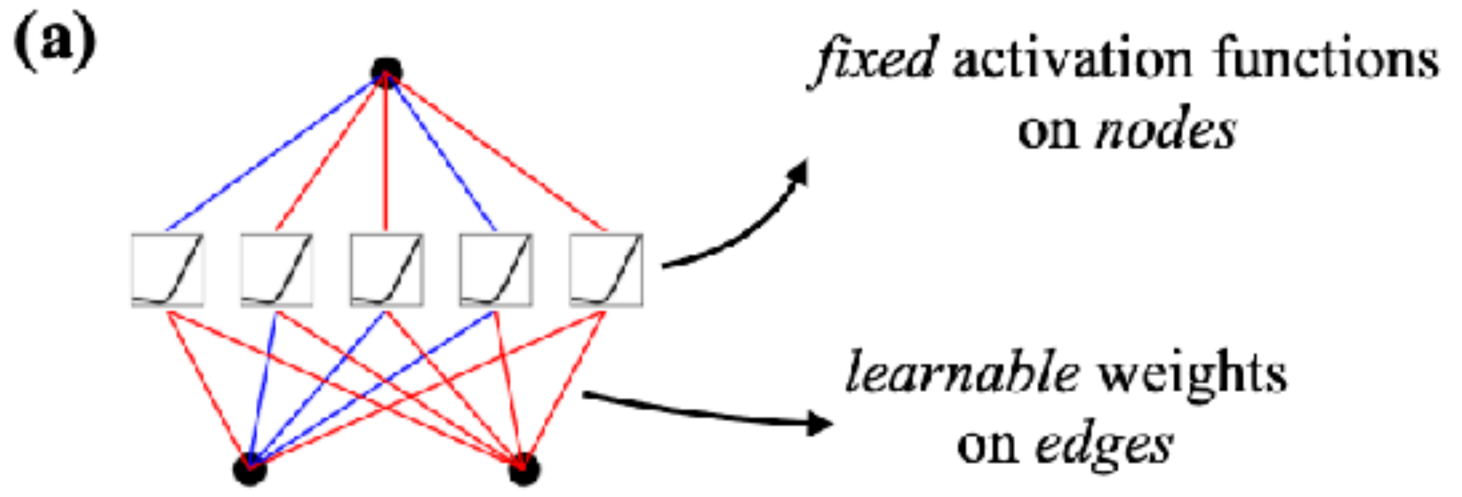
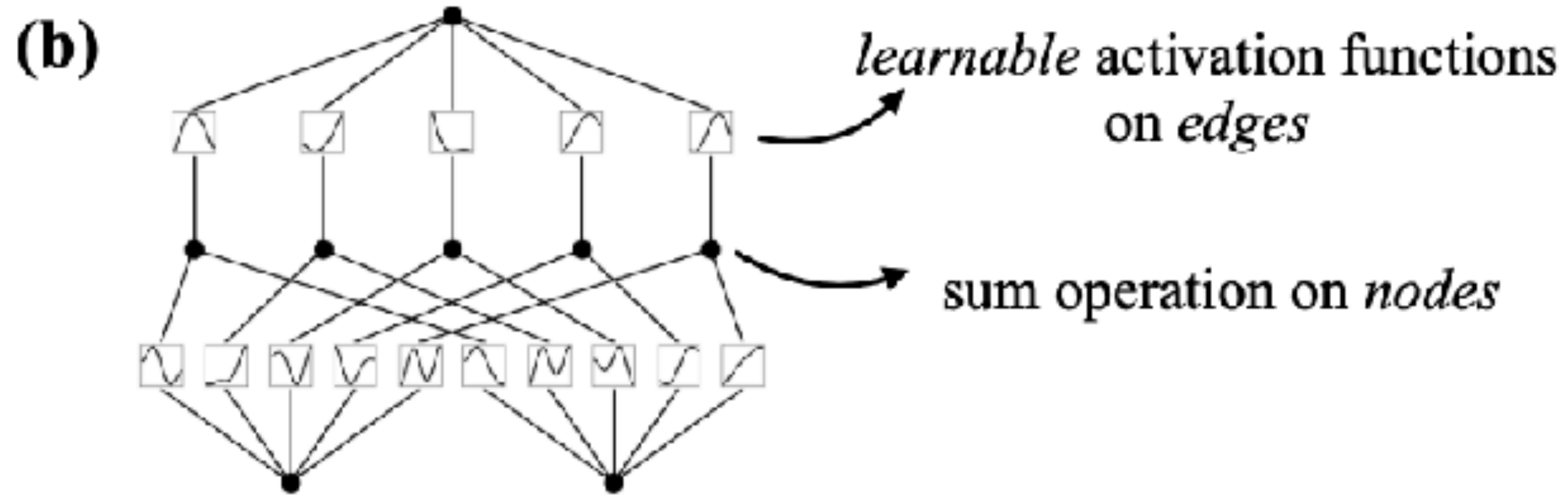
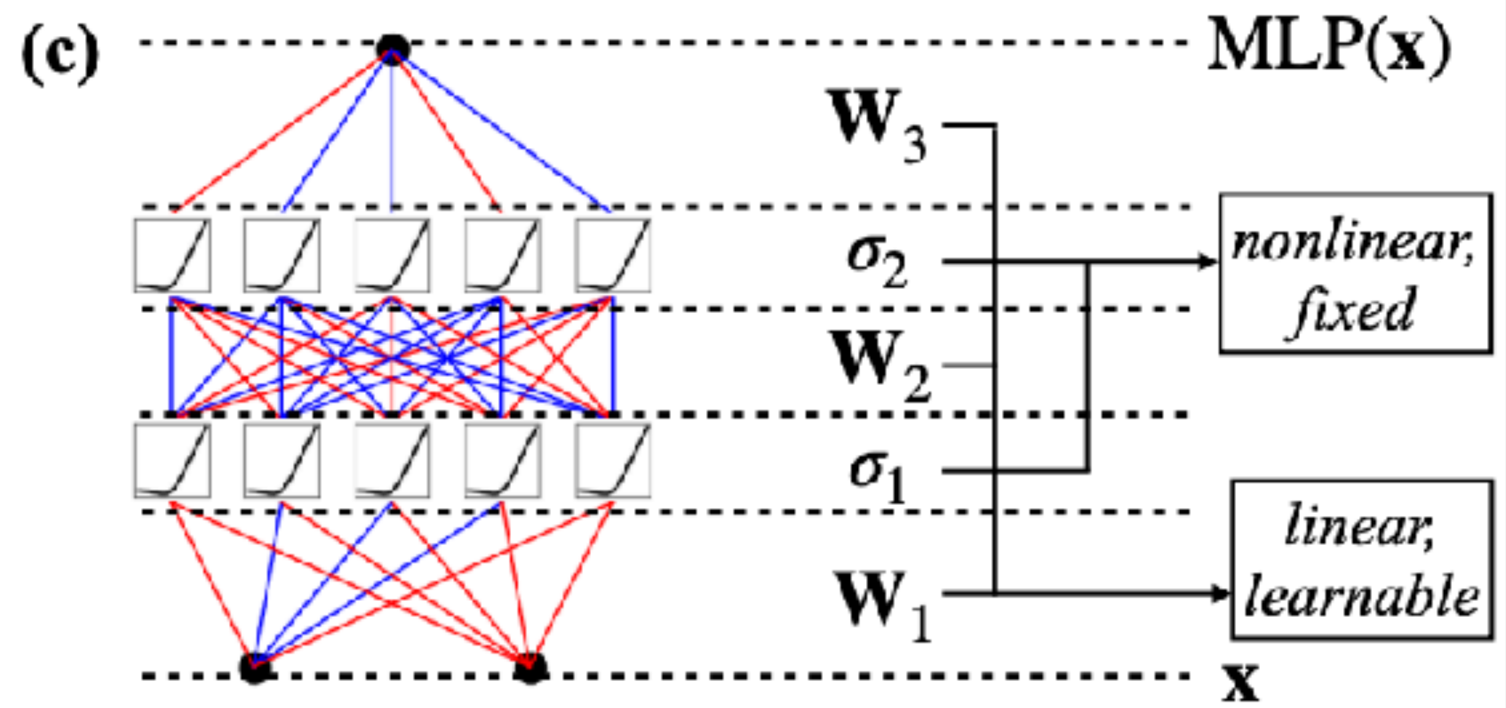
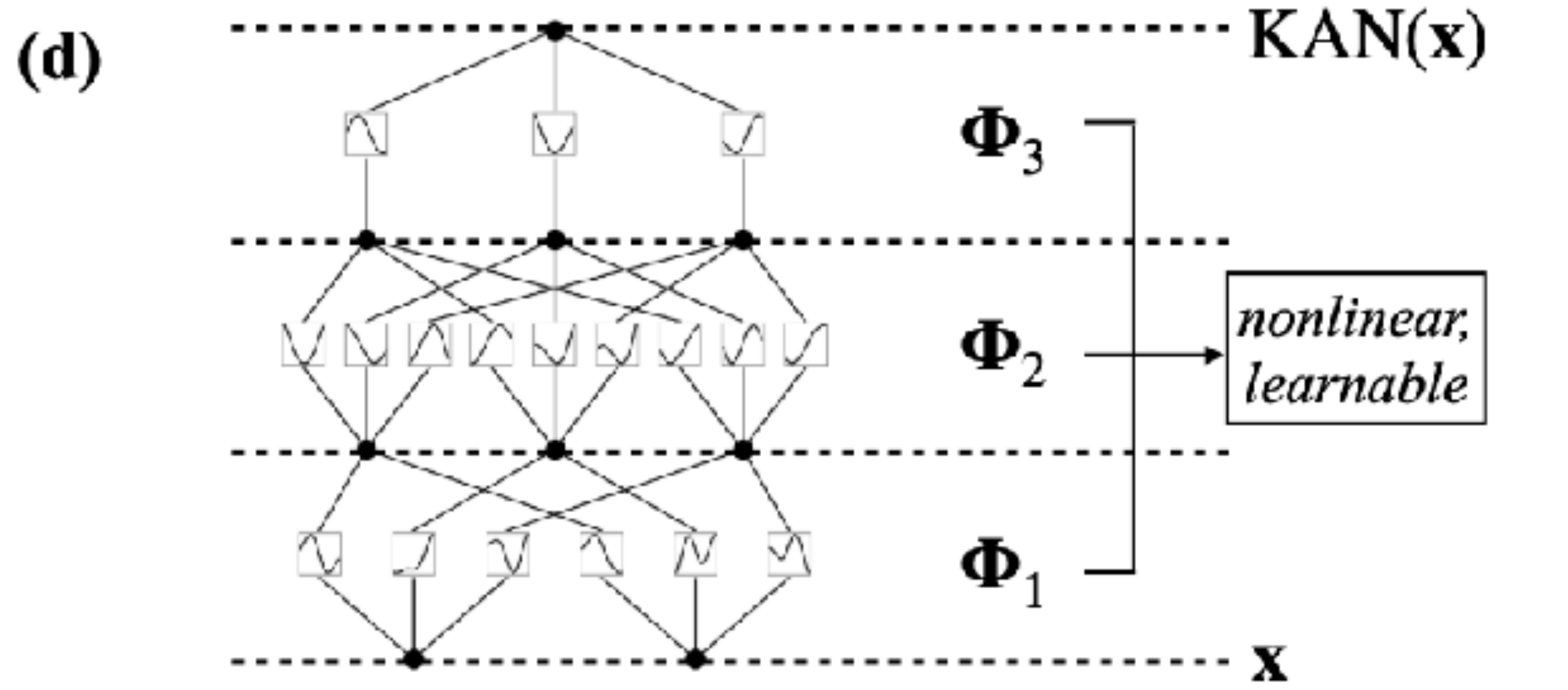
- 4 subanalyses:
  - **onshell** (W/Z when decaying) **wino/bino** (22+2)
  - **offshell wino/bino minus** (sign of the product of two neutralino mass eigenstates) (32+2)
  - **offshell wino/bino plus** (32+2)
  - **offshell higgsino** (32+2)

- Subanalyses combination: 54 SRs + 4 CRs (full overlap for offshell subanalyses)



Kolmogorov-Arnold networks

from Liu et al (2404.19756)

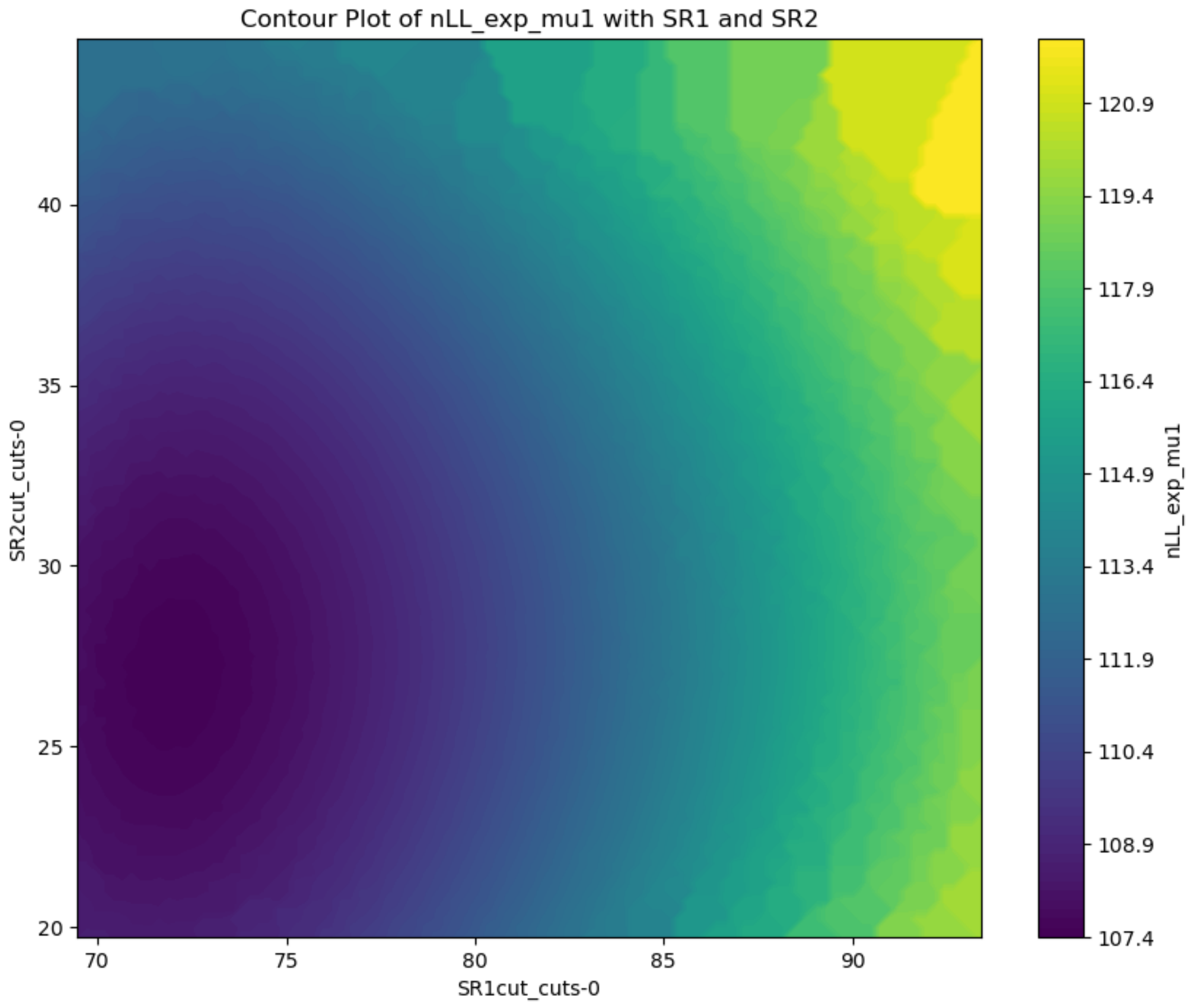
Model	Multi-Layer Perceptron (MLP)	Kolmogorov-Arnold Network (KAN)
Theorem	Universal Approximation Theorem	Kolmogorov-Arnold Representation Theorem
Formula (Shallow)	$f(\mathbf{x}) \approx \sum_{i=1}^{N(\epsilon)} a_i \sigma(\mathbf{w}_i \cdot \mathbf{x} + b_i)$	$f(\mathbf{x}) = \sum_{q=1}^{2n+1} \Phi_q \left(\sum_{p=1}^{n} \phi_{q,p}(x_p) \right)$
Model (Shallow)		
Formula (Deep)	$\mathrm{MLP}(\mathbf{x}) = (\mathbf{W}_3 \circ \sigma_2 \circ \mathbf{W}_2 \circ \sigma_1 \circ \mathbf{W}_1)(\mathbf{x})$	$\mathrm{KAN}(\mathbf{x}) = (\mathbf{\Phi}_3 \circ \mathbf{\Phi}_2 \circ \mathbf{\Phi}_1)(\mathbf{x})$
Model (Deep)		

Kolmogorov-Arnold networks

- KANs allow to find analytical formulas describing data

- It might be possible, in some cases, to find analytical approximations to likelihood and speed up computations even further

ATLAS-SUSY-2018-04 (1911.0660)



Summary

- Selecting the right BSM model enhances chances of new physics discovery
- Reinterpretation allows to constrain untested models
- Proper hypotheses testing is possible with published Full Statistical Models
- However, using these models constitute a **computational bottleneck**
- Machine Learning surrogates offer **good accuracy with much** (orders of magnitude) **faster computations**
- Preliminary results with MLPs are promising
- The next step will be to try KANs
- Trained models will be published to be used by the community

Tatra mountains, Poland