

Nonperturbative contributions of QCD condensates to $D^0\bar{D}^0$ mixing

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Introduction

The formalism

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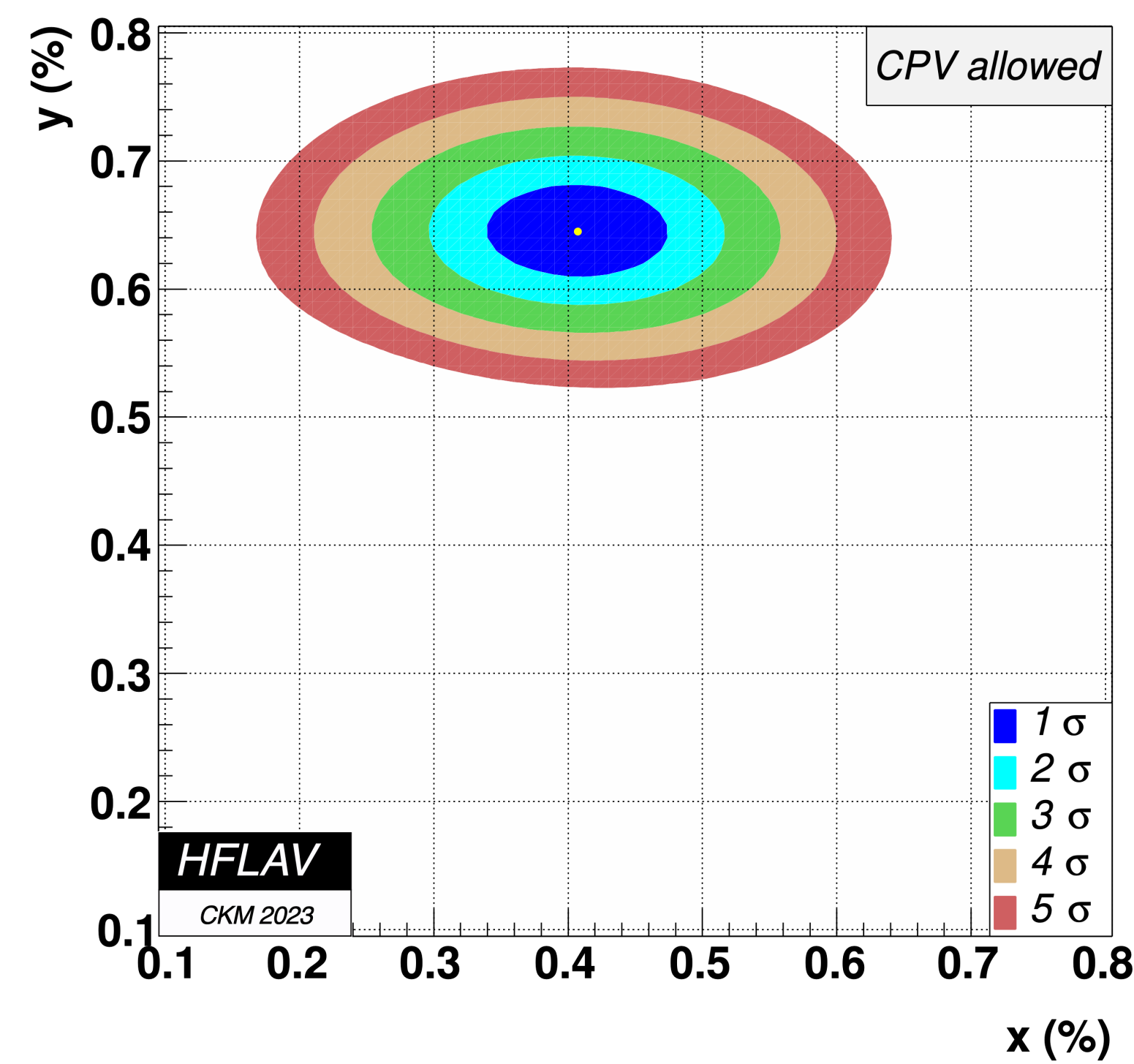
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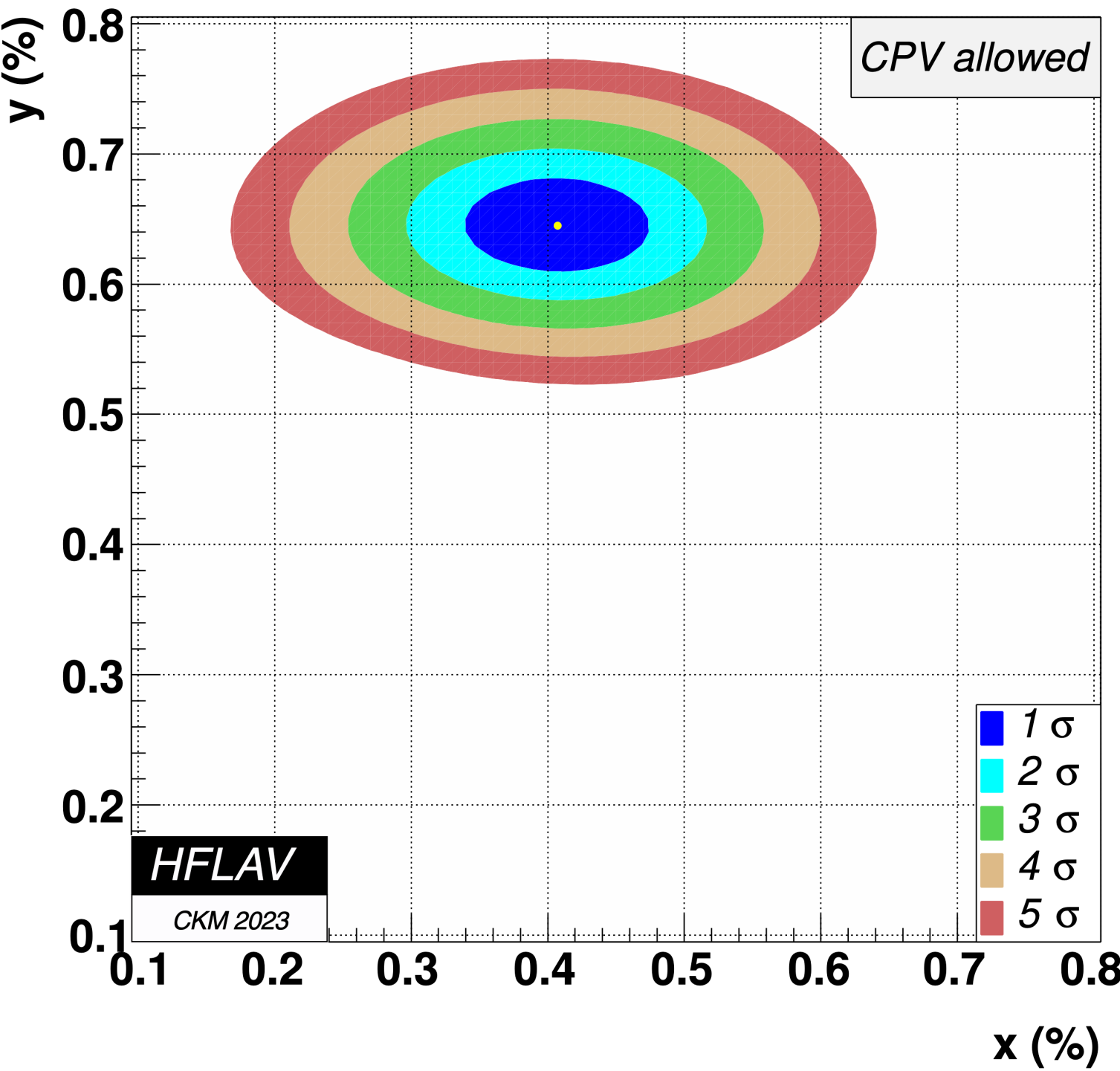
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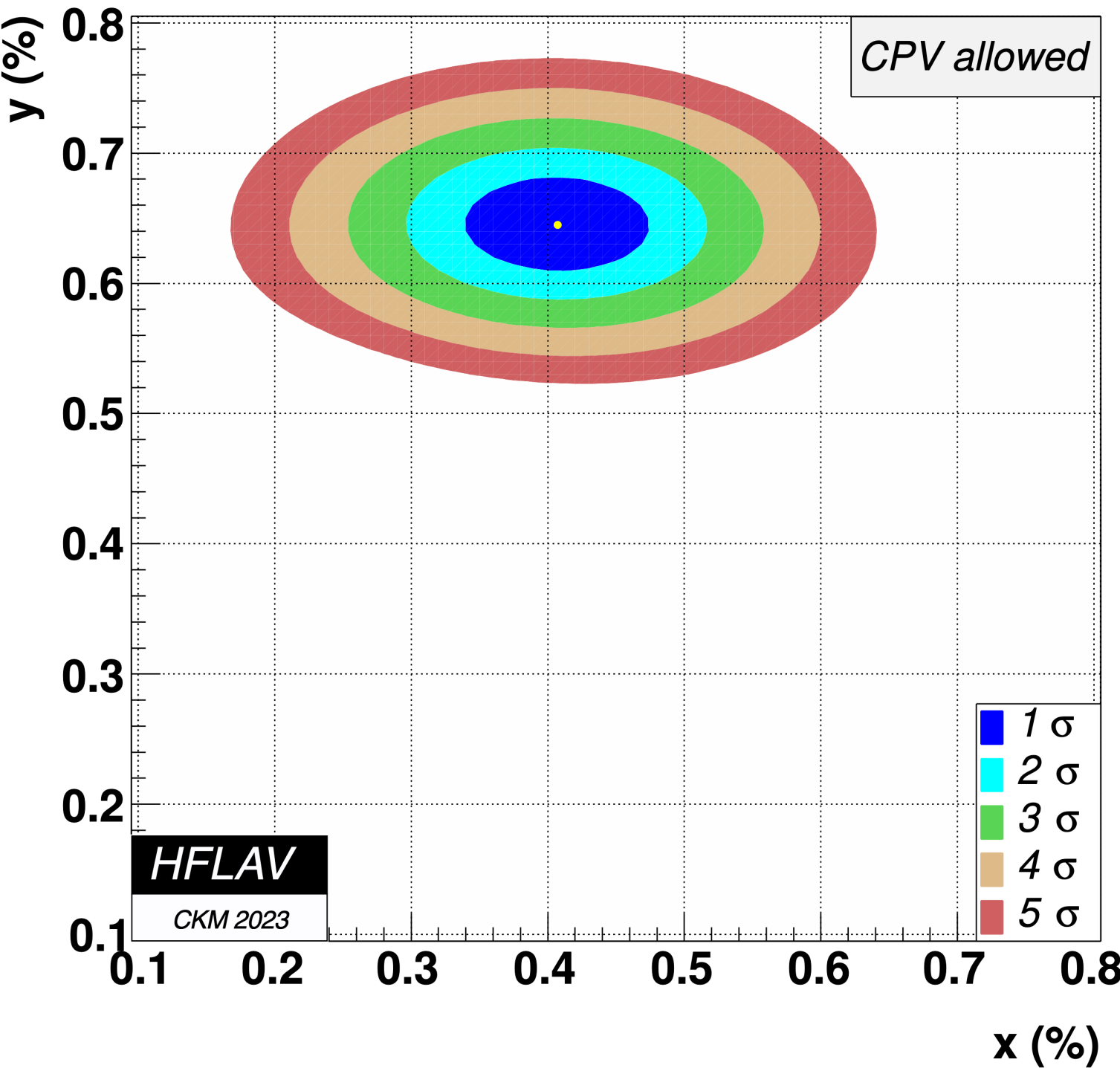
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Clear evidence for mixing!

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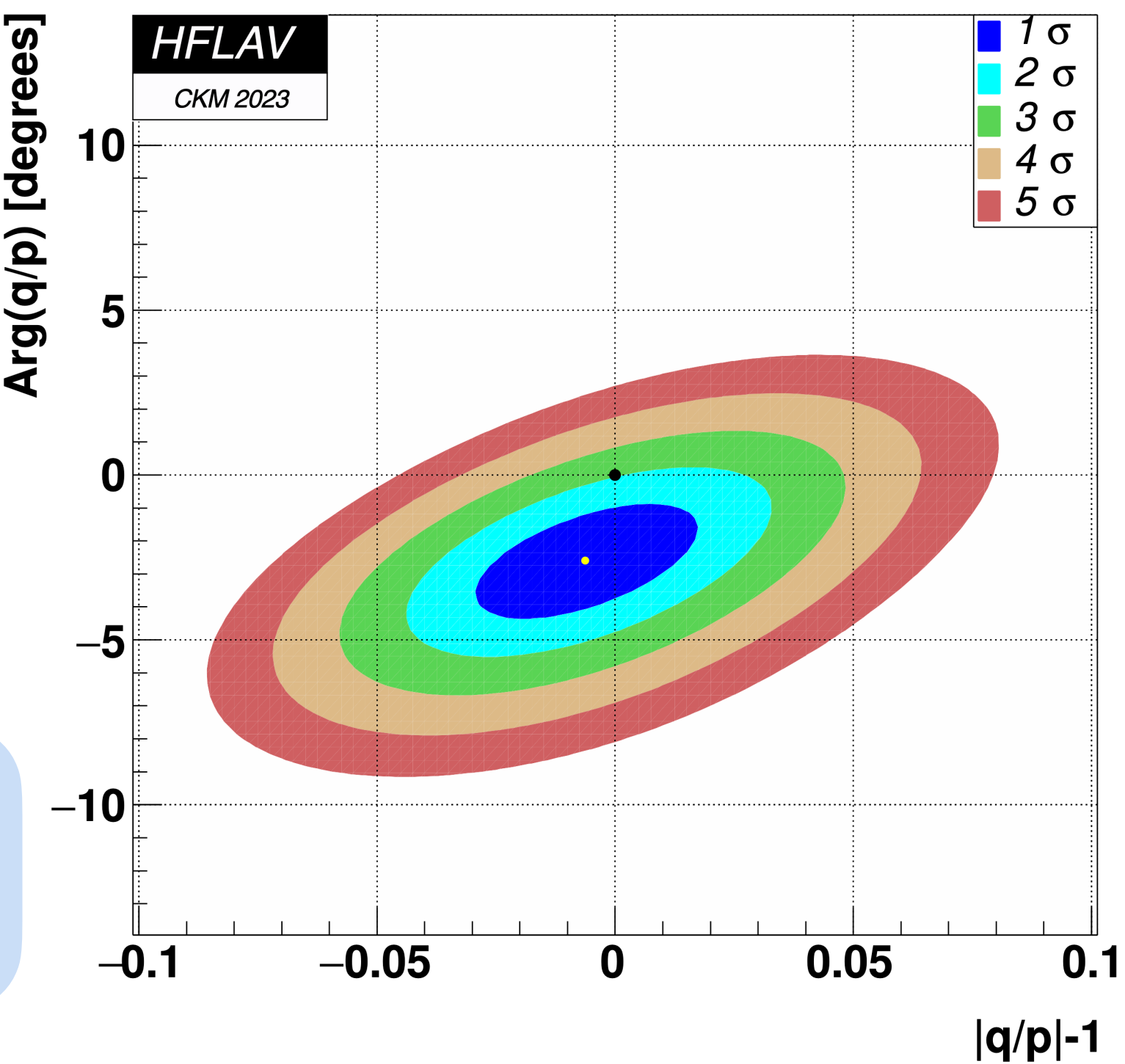
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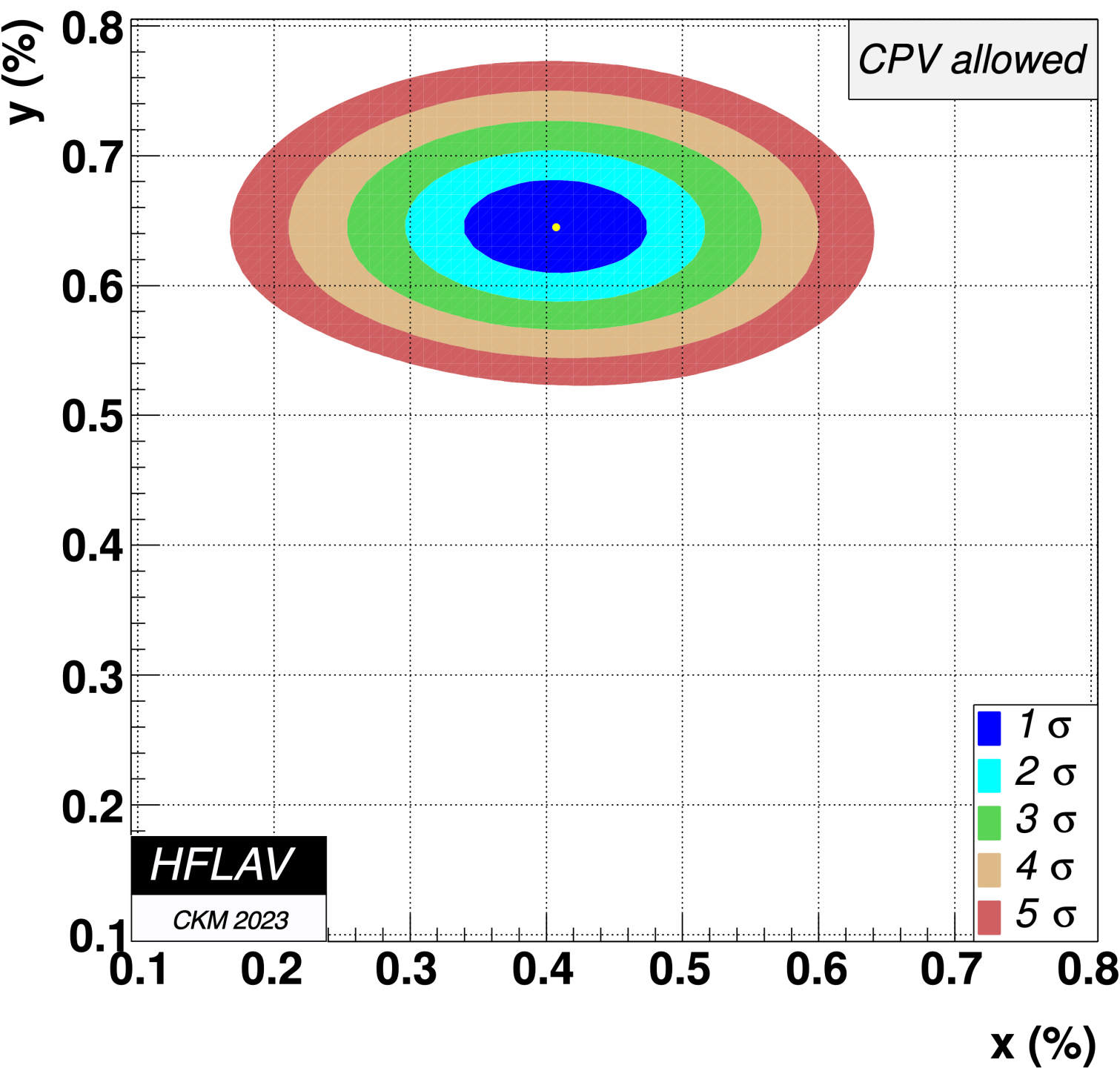


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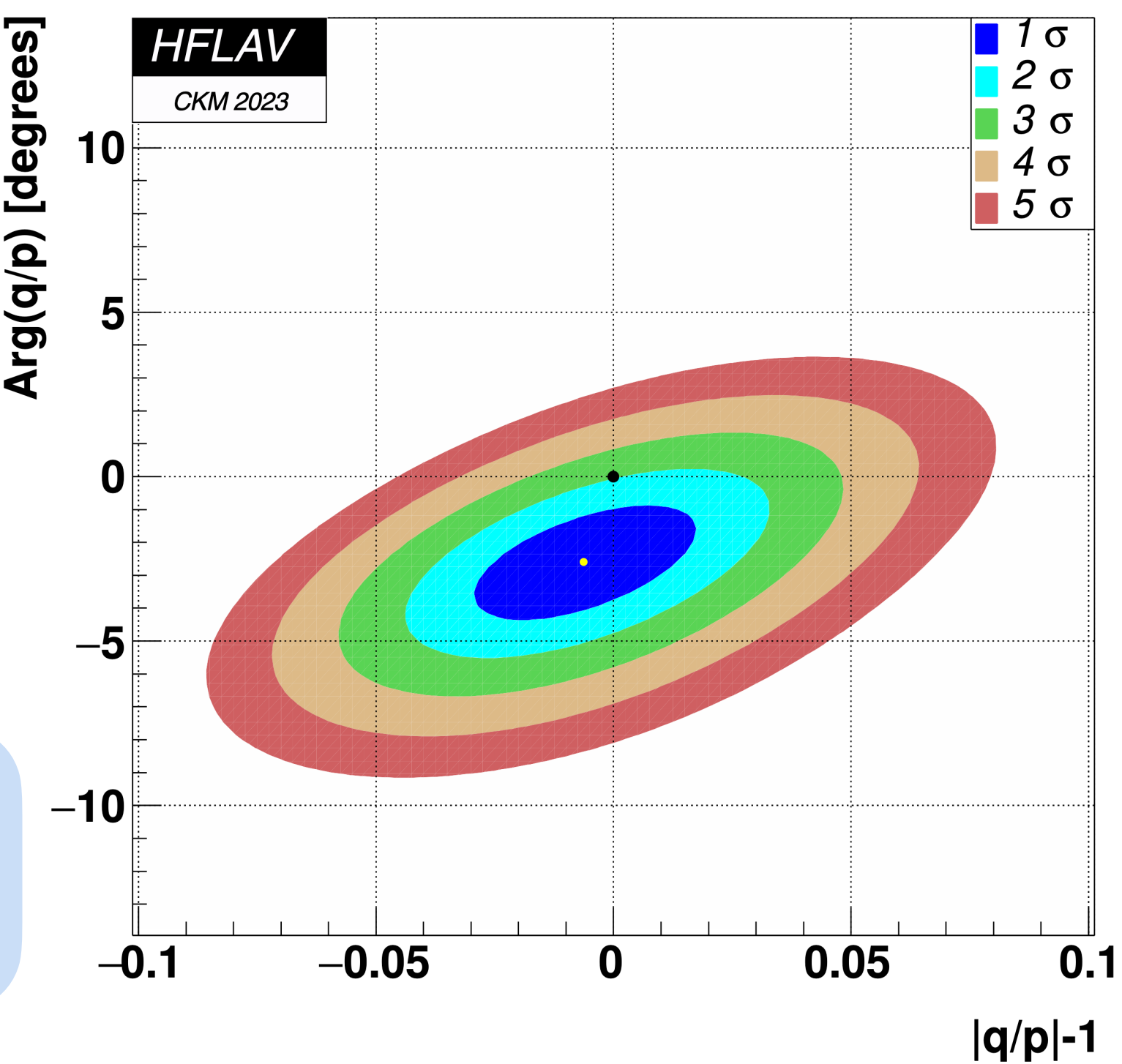
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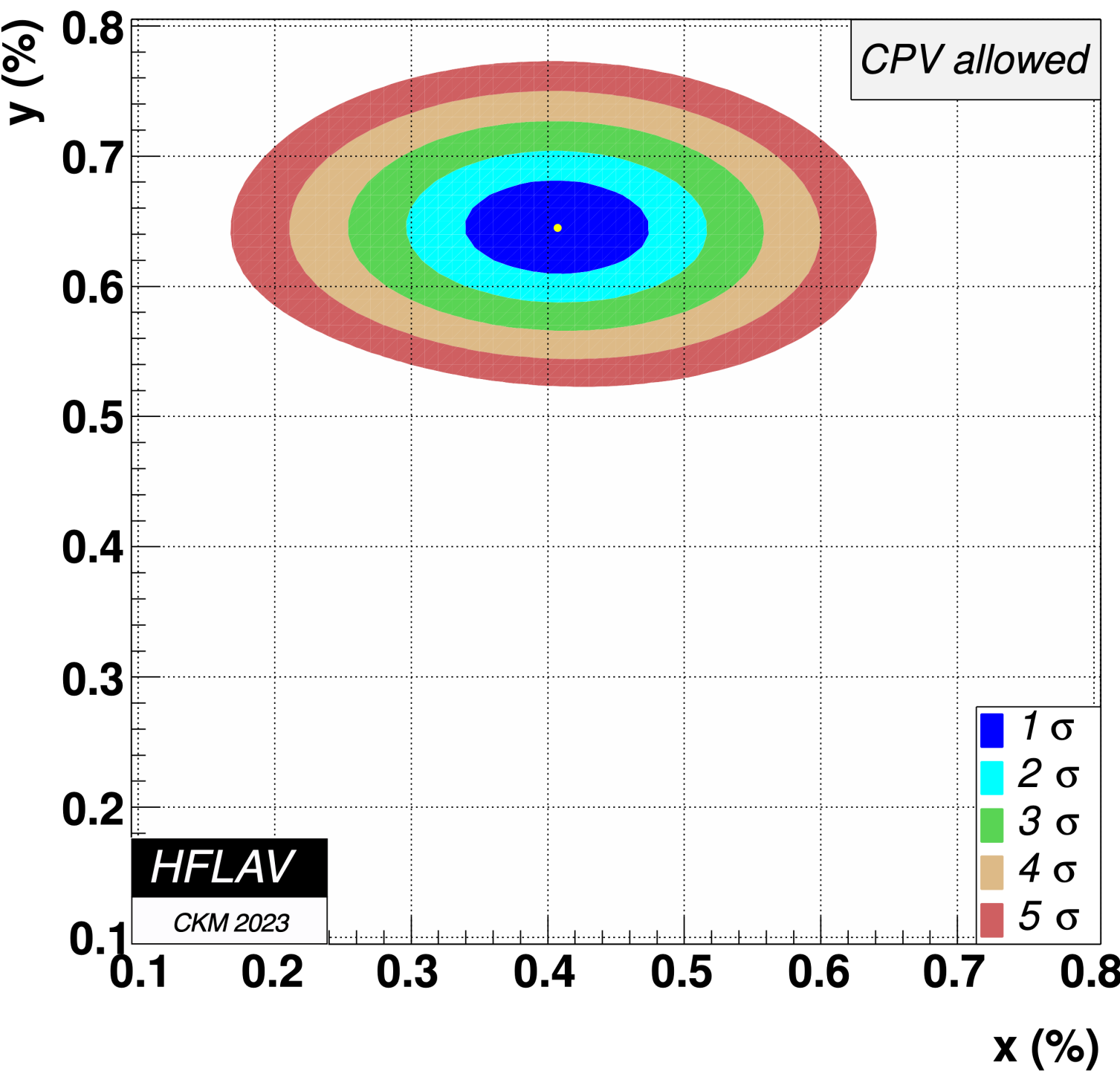
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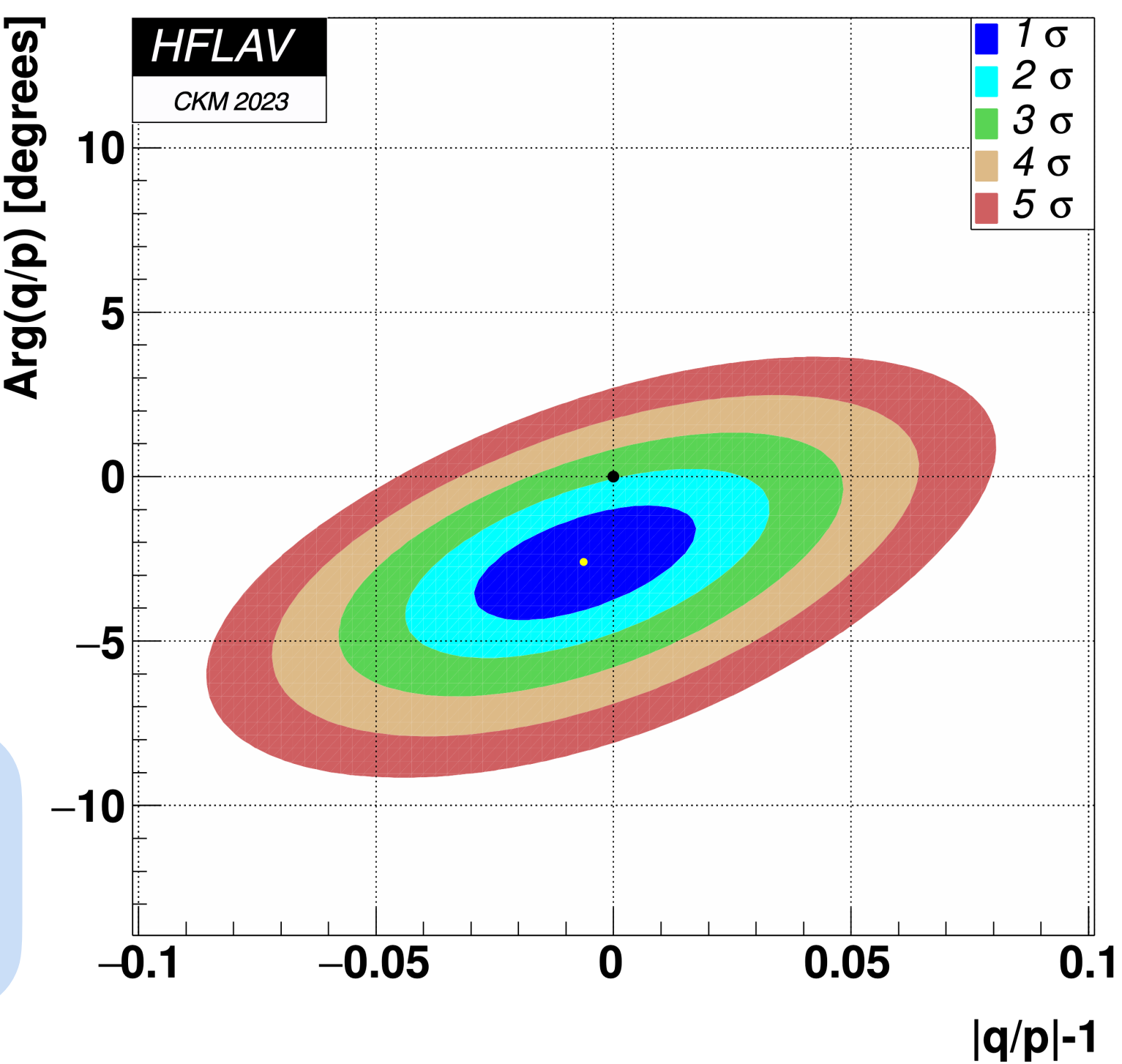
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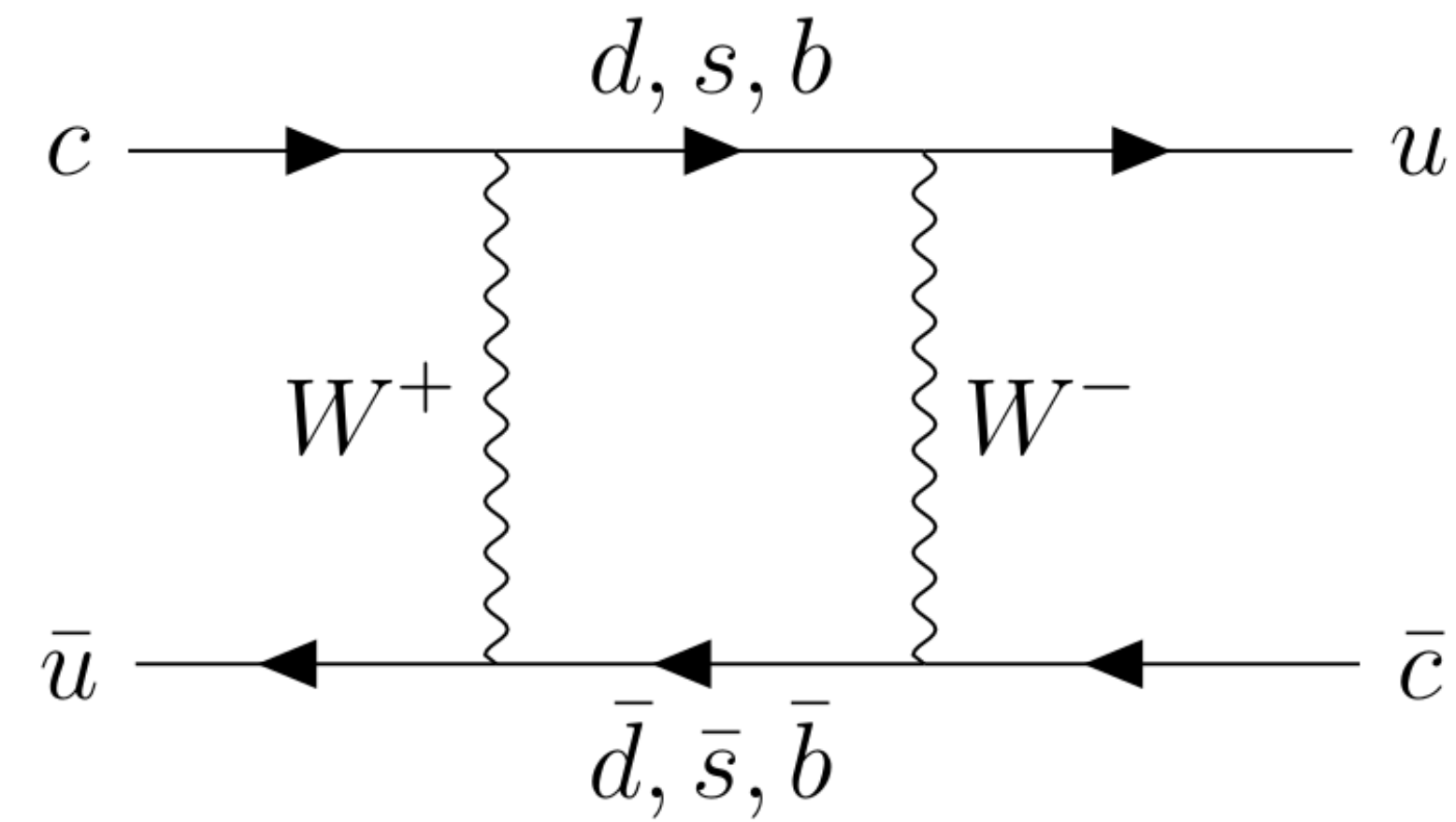
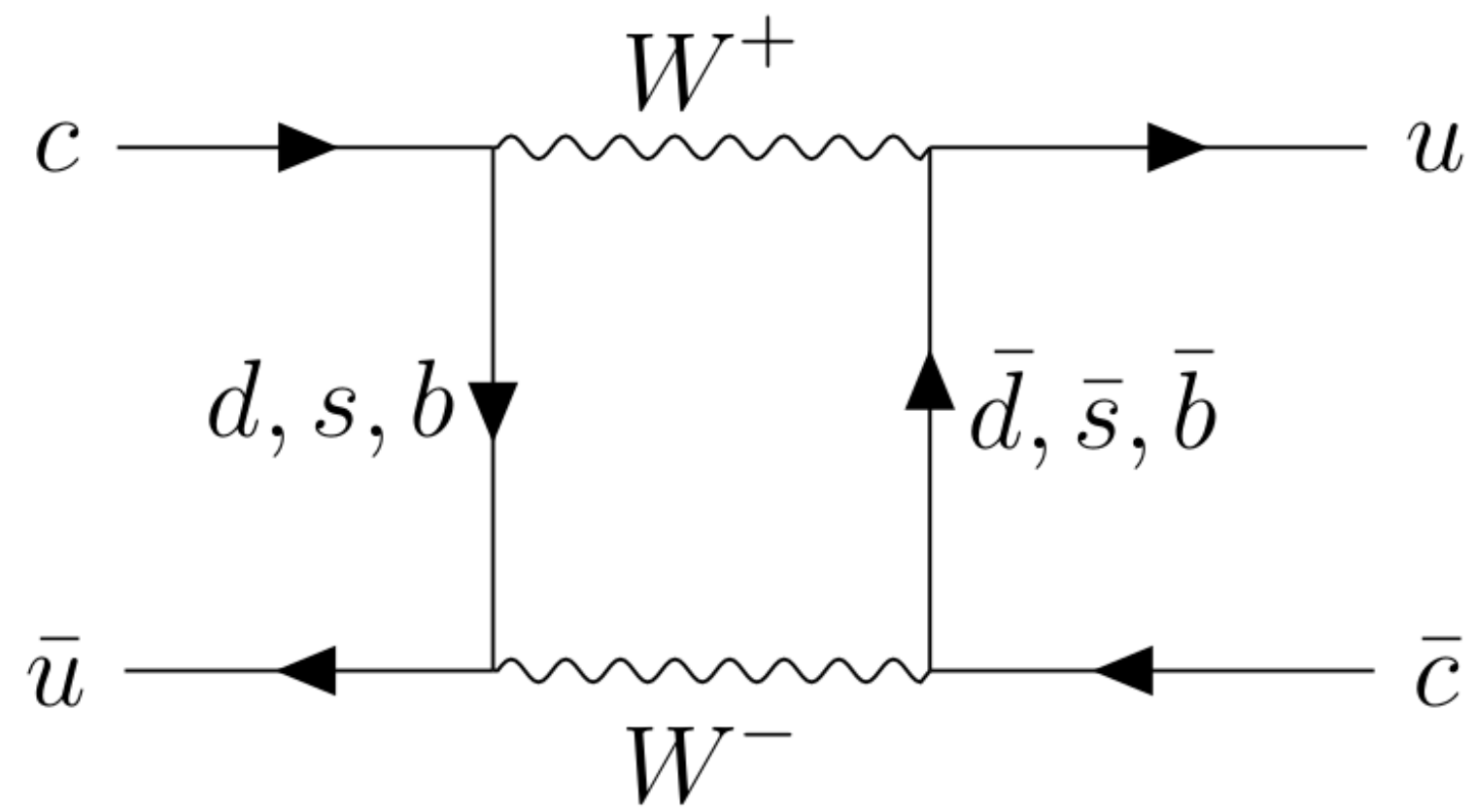
In our calculations we assumed no CPV

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Theory - the perturbative contribution

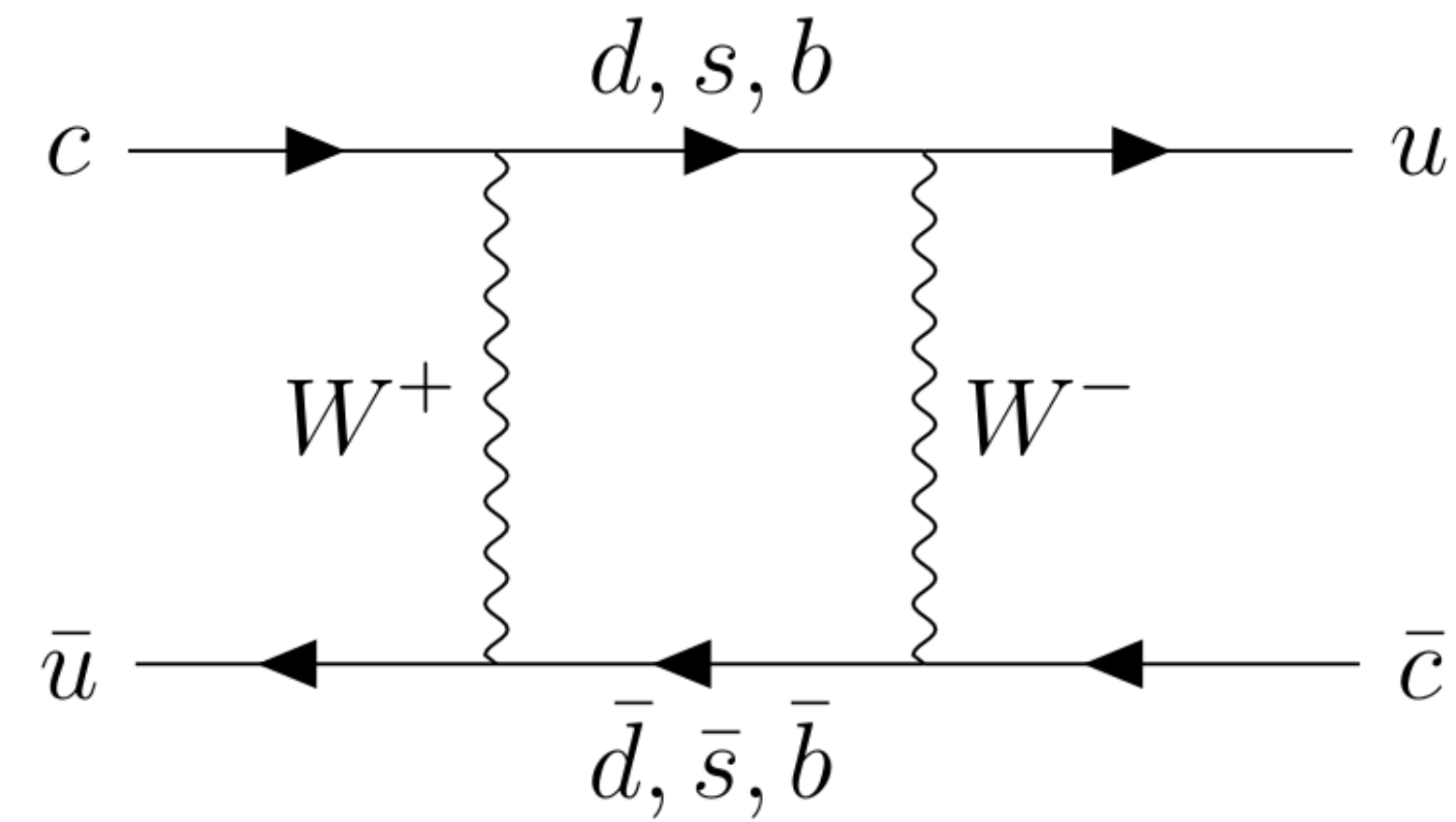
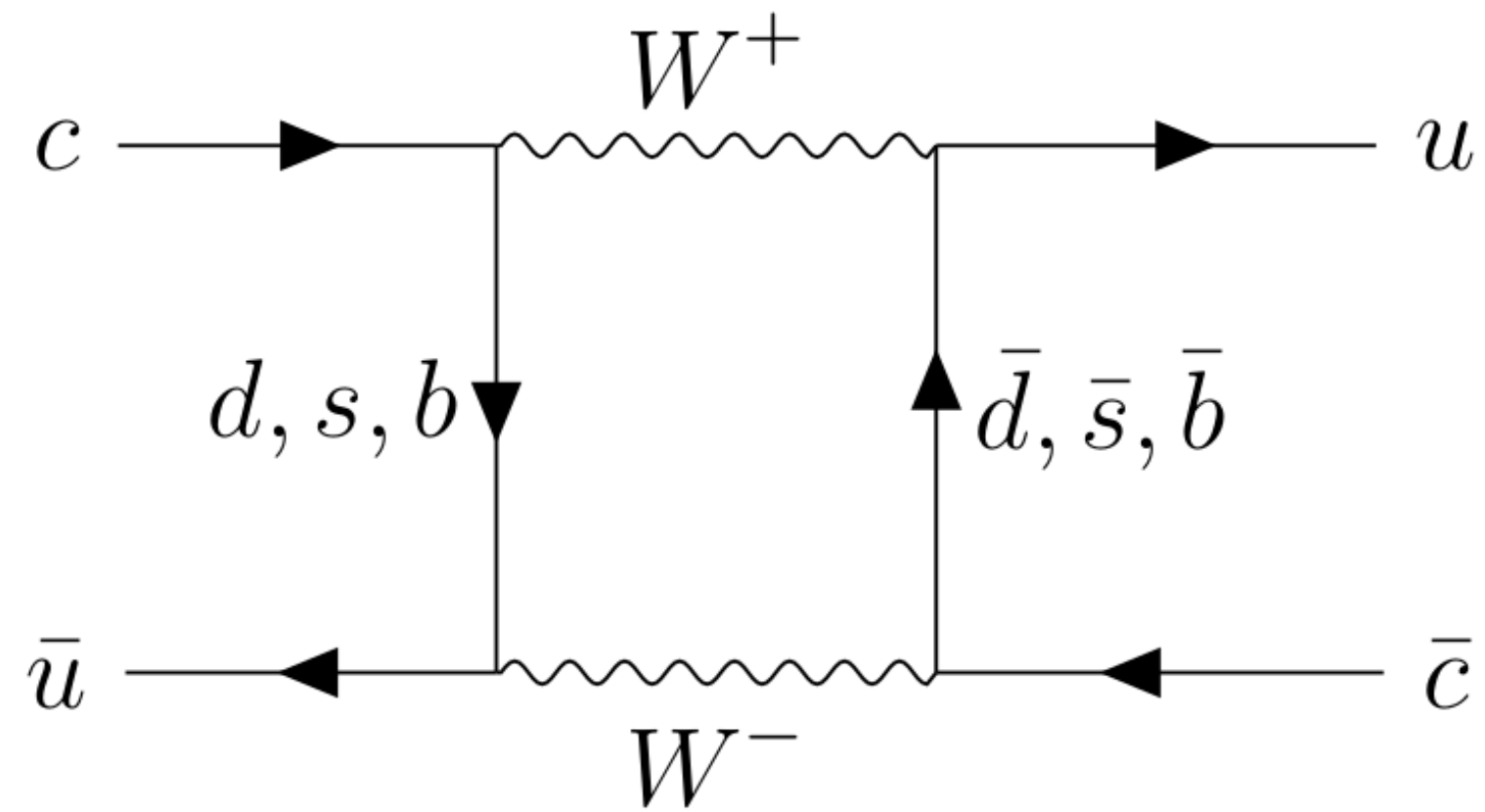
Introduction

Theory - the perturbative contribution



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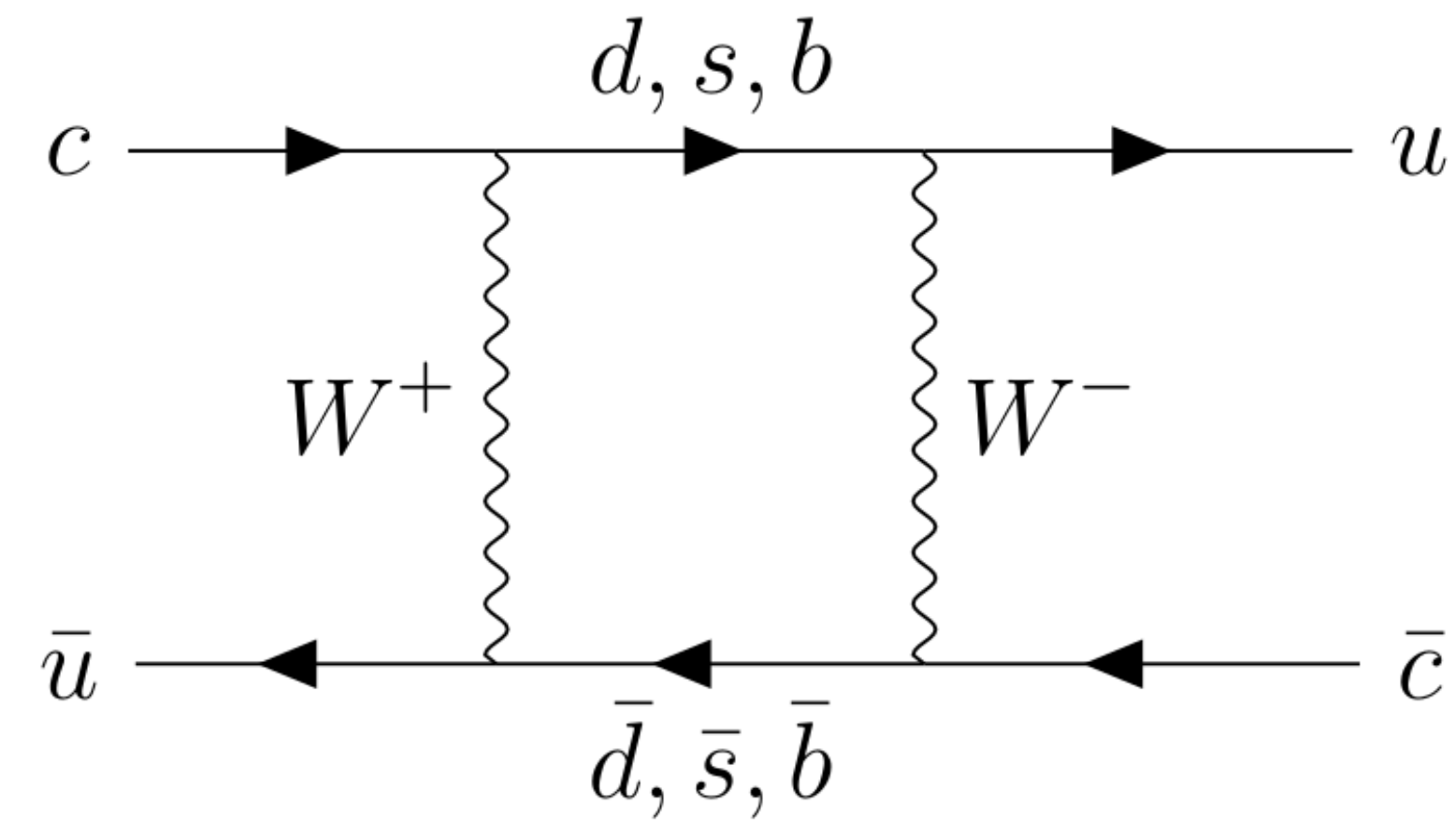
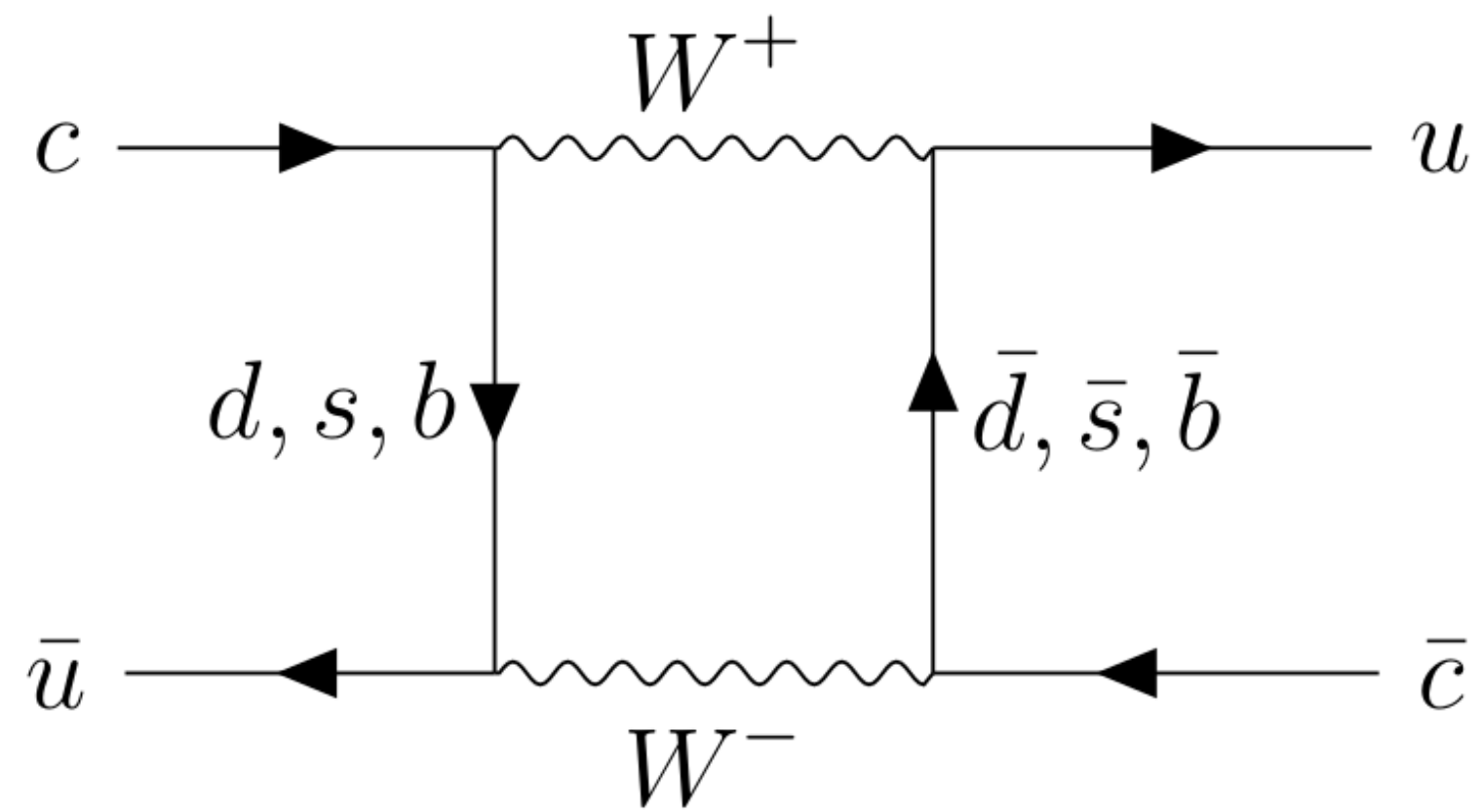
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$$\mathcal{A} = \xi_s^2(A_{ss} - 2A_{ds} + A_{dd}) + 2\xi_s\xi_b(A_{dd} - A_{ds}) + \xi_b^2A_{dd}$$

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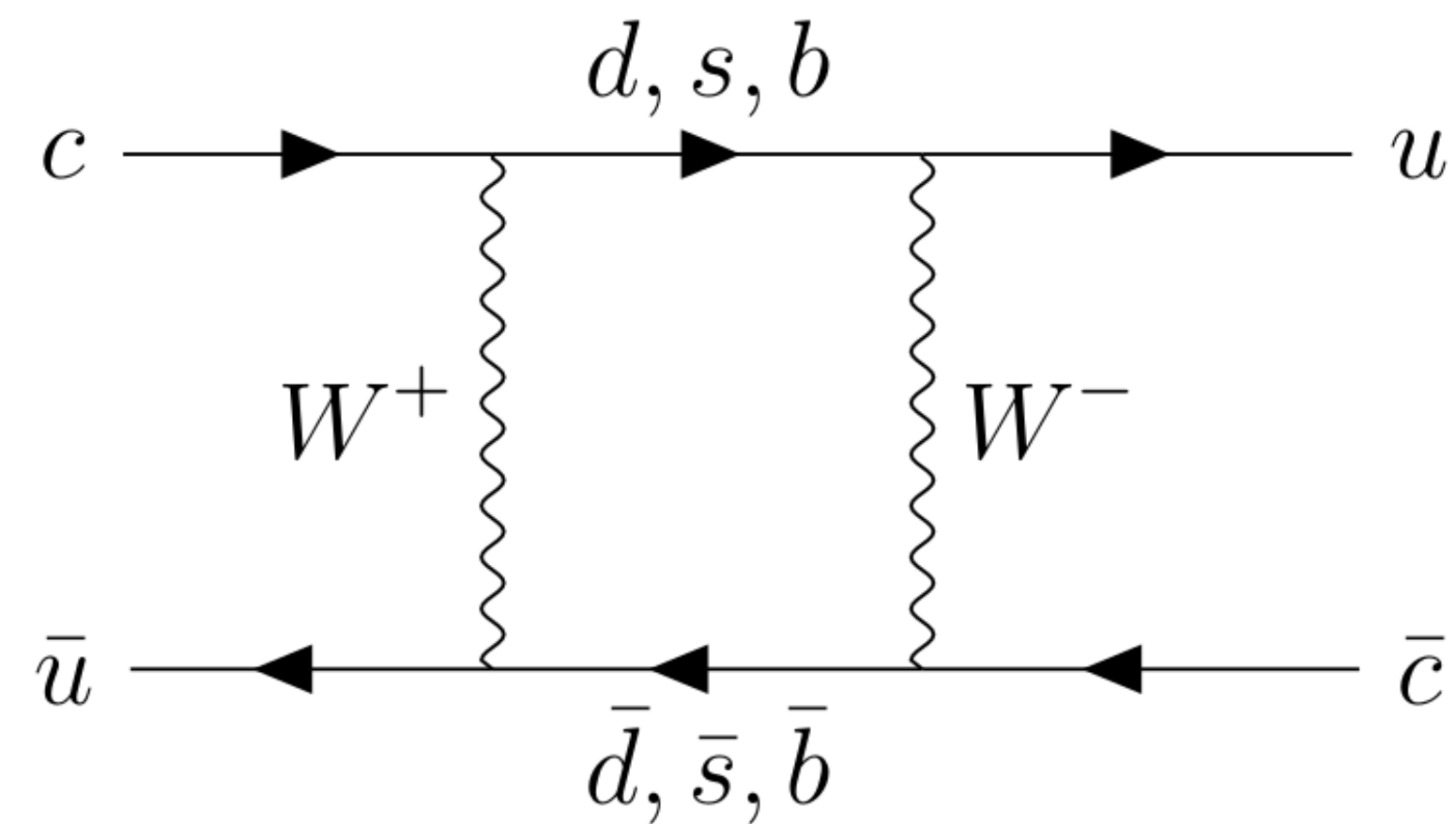
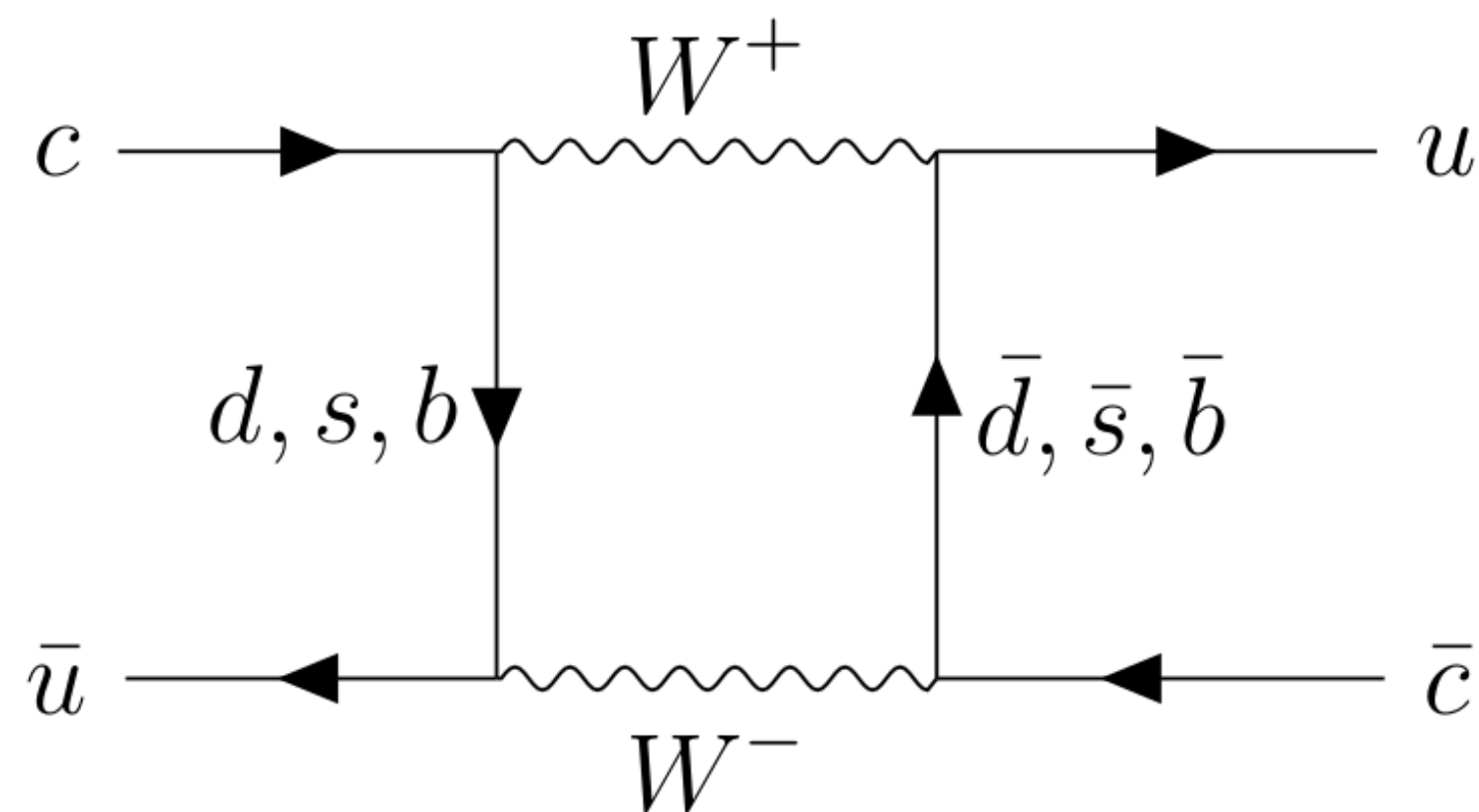
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CKM leading

'doubly' GIM suppressed

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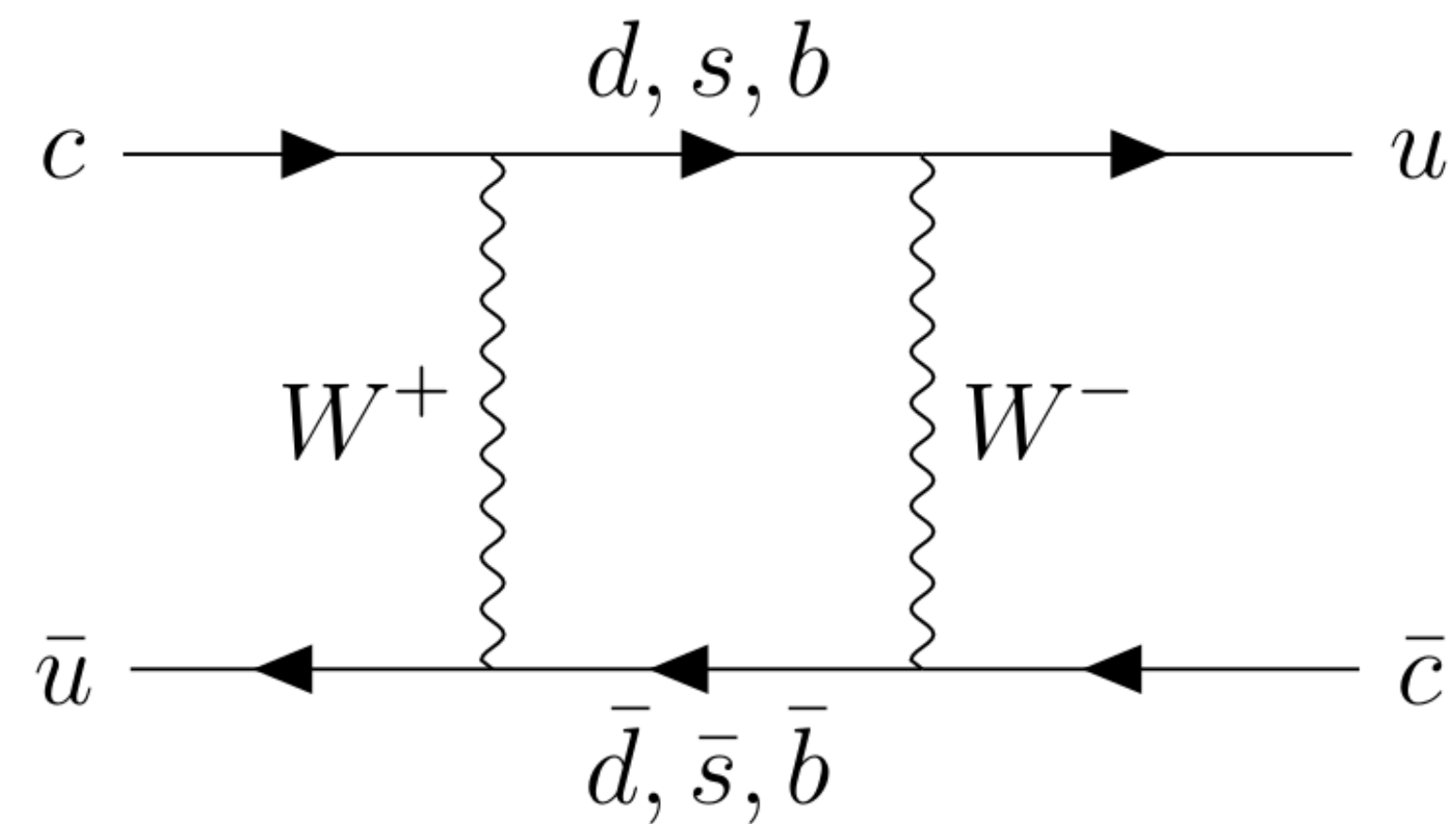
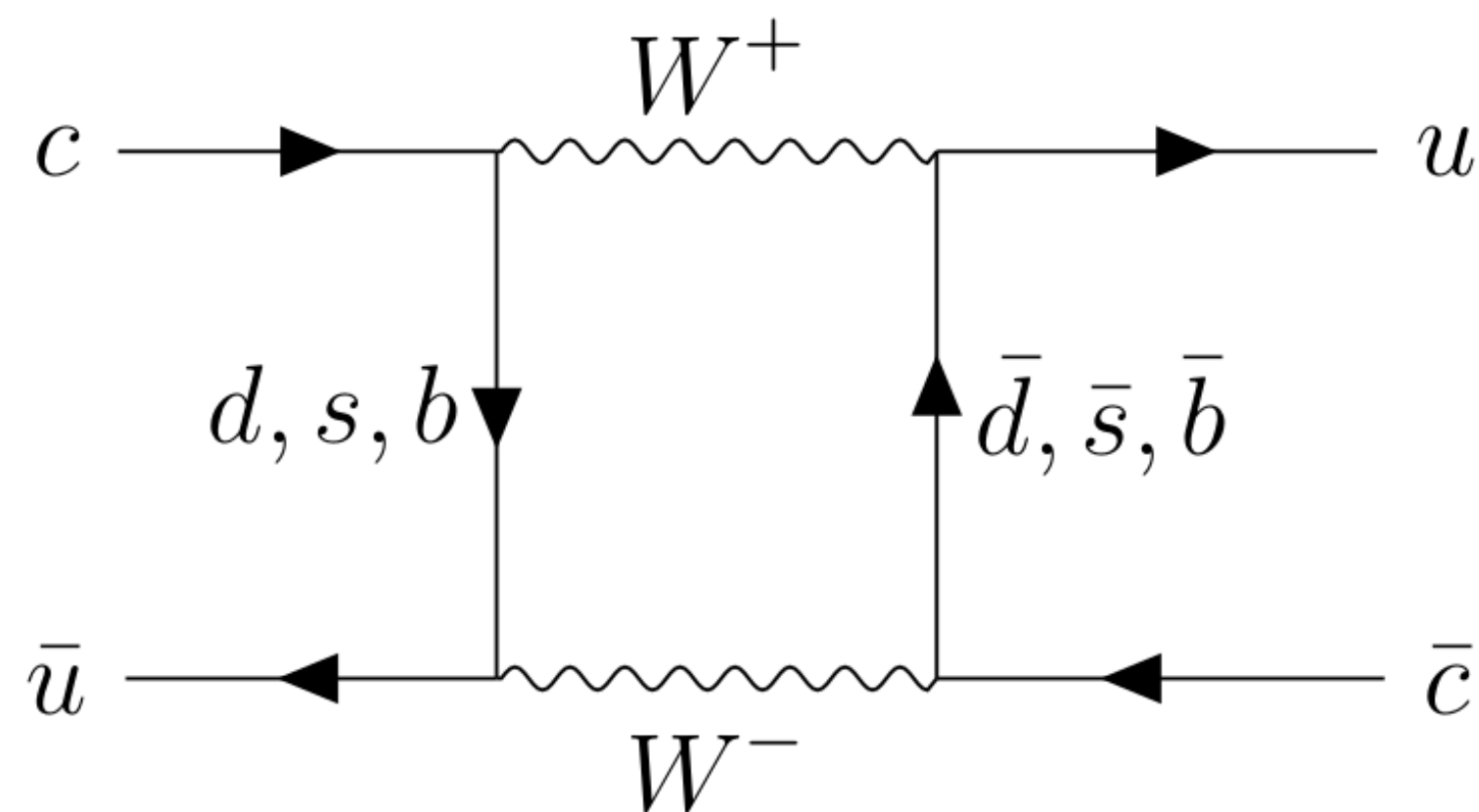
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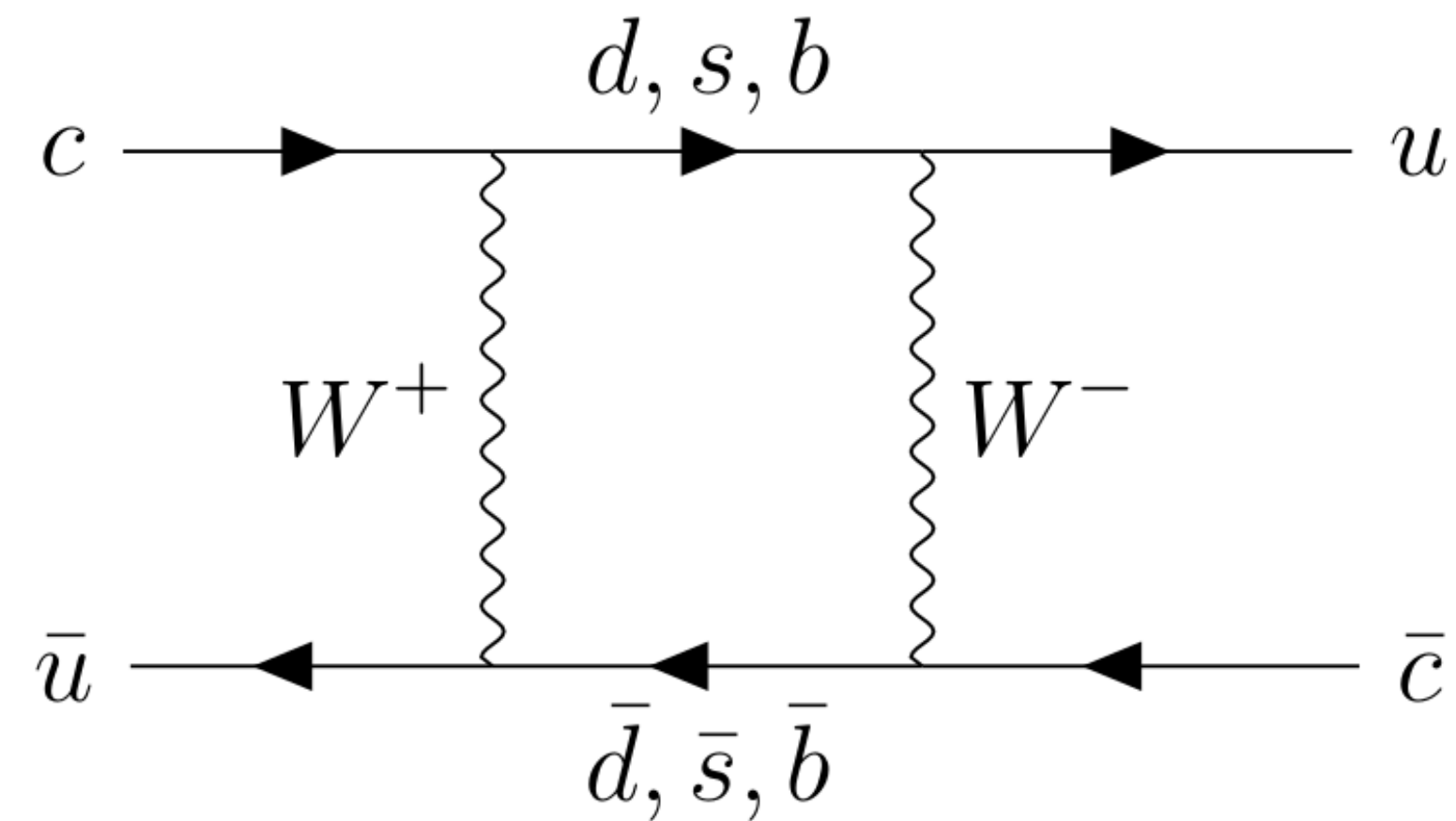
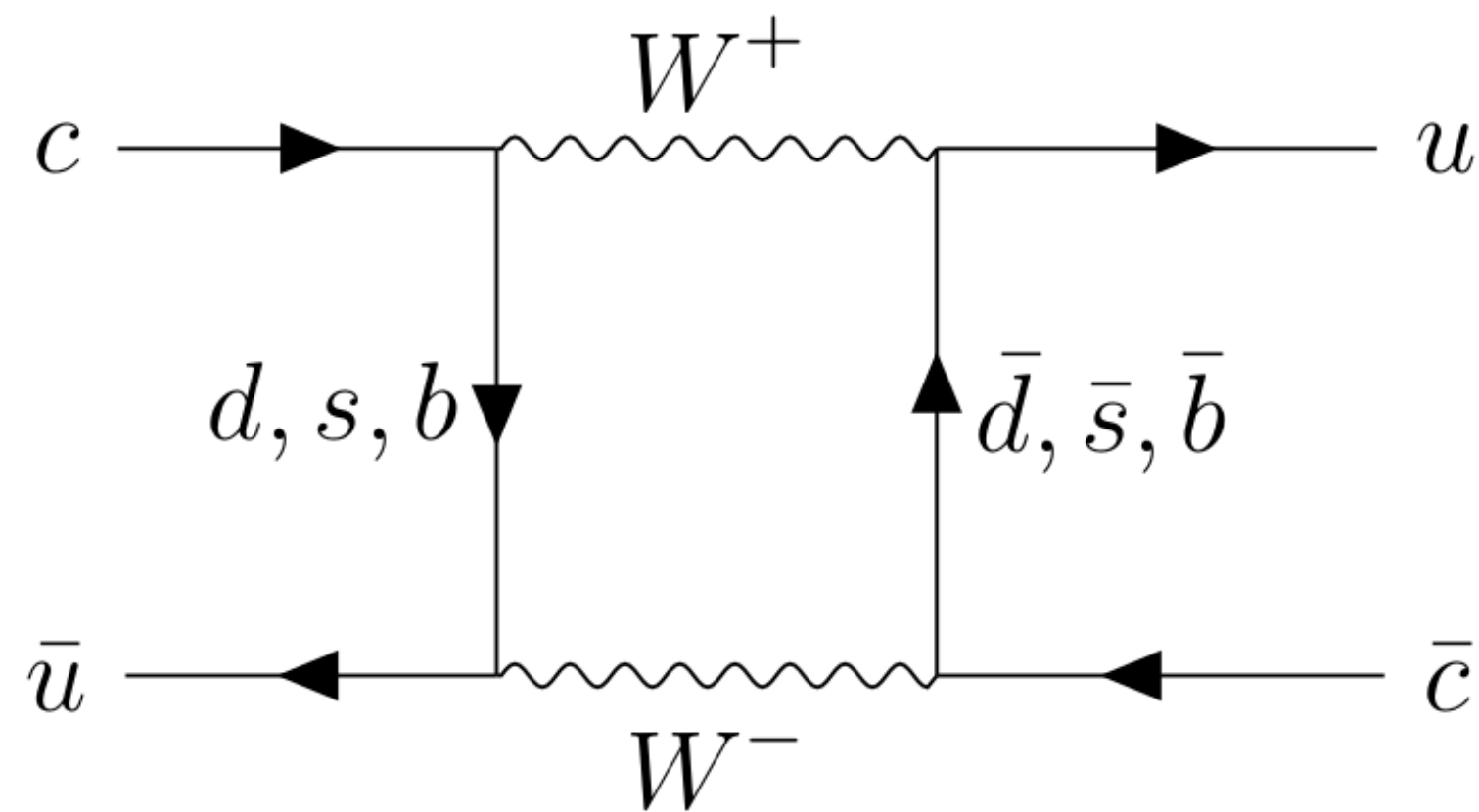
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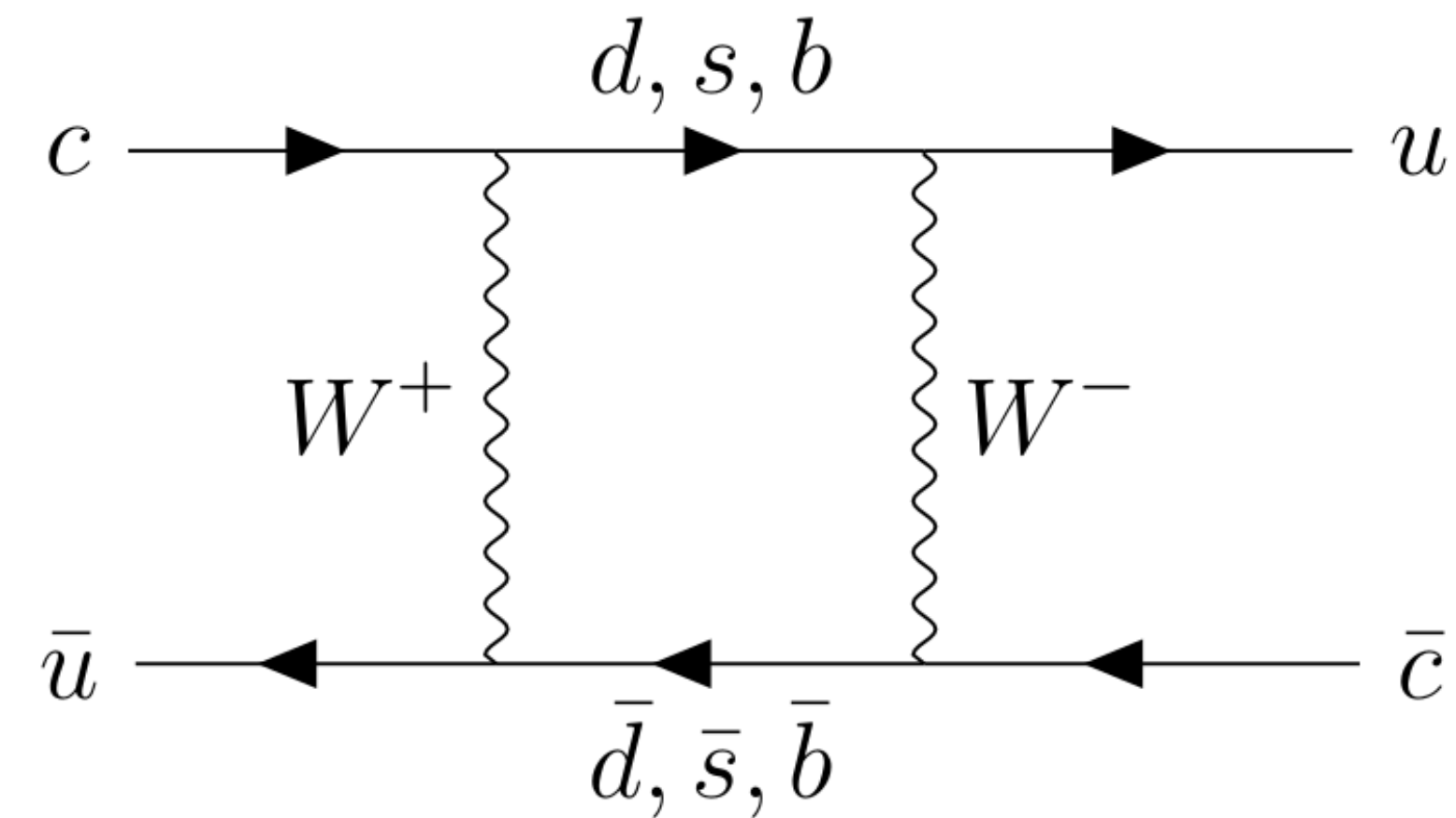
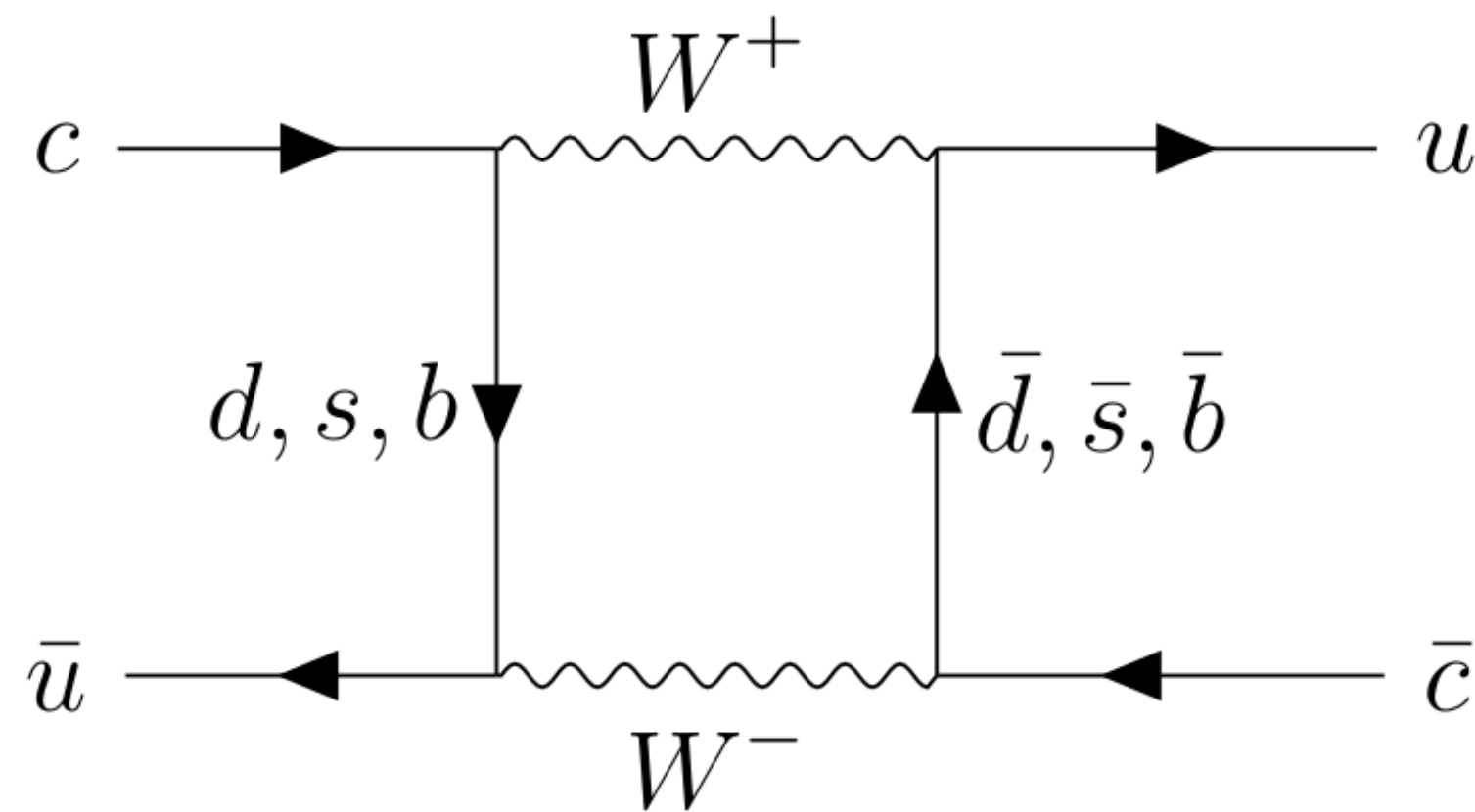
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• very effective cancellations happen leaving the result **far behind experimental value**; even though separate contributions are large

Different approaches

How to lift the GIM suppression?

Inclusive

- $SU(3)$ breaking effects from **higher-dimension operators**
- $SU(3)$ breaking effects from **alternative renormalisation scale setting**
- **NLO and mass corrections**

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- $SU(3)$ breaking effects from **final state phase space differences of intermediate bound states**
- Inclusion of **multi-body states**
- Topological amplitude approach

Dispersive

- Mixing parameters x and y connected through a dispersion relation
- Inclusion of $SU(3)$ breaking effects from **thresholds of different D meson decay channels**

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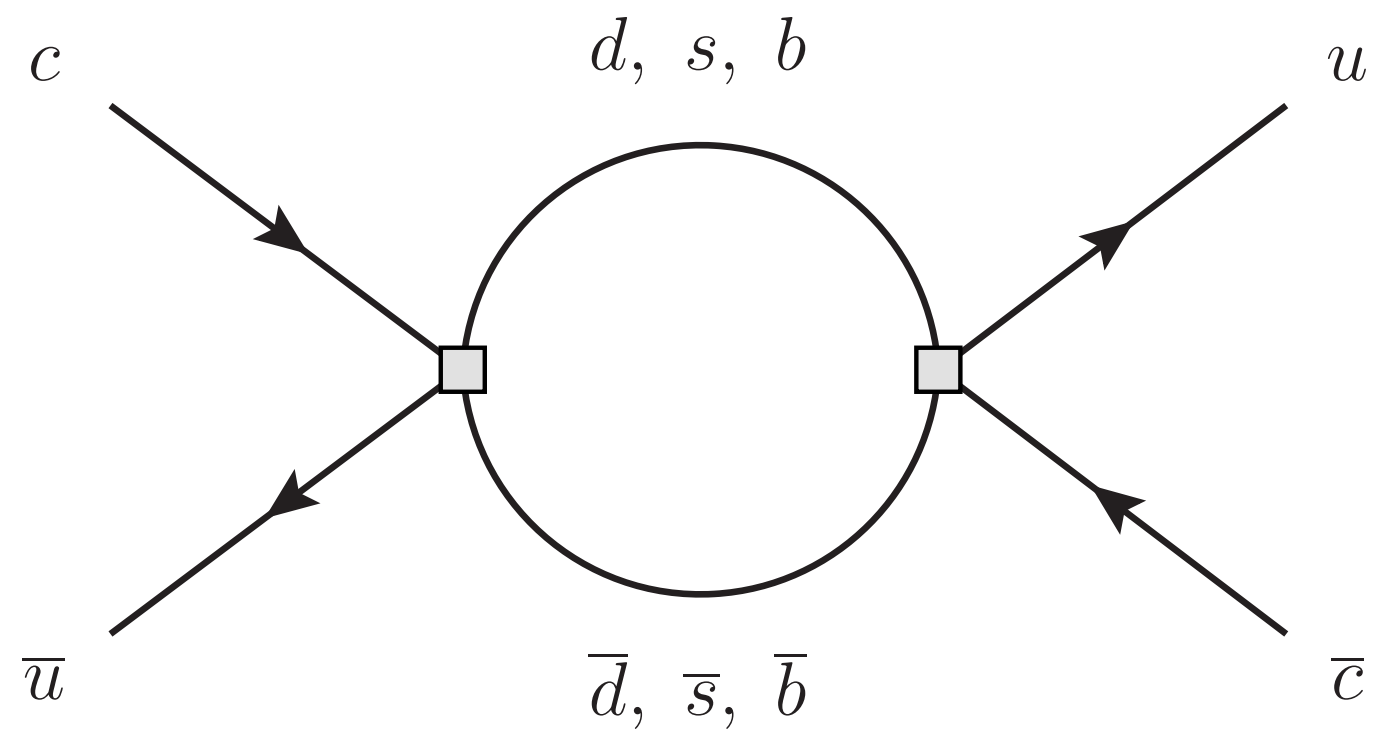
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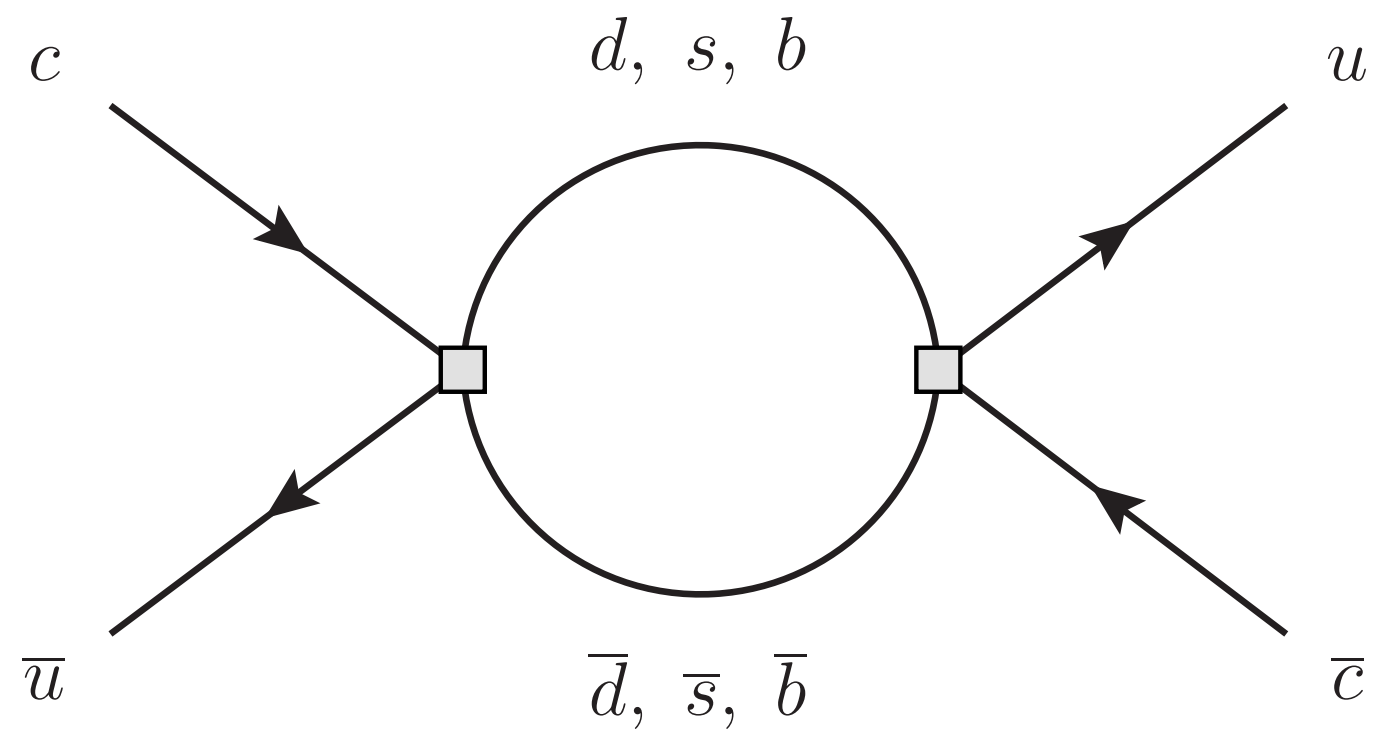
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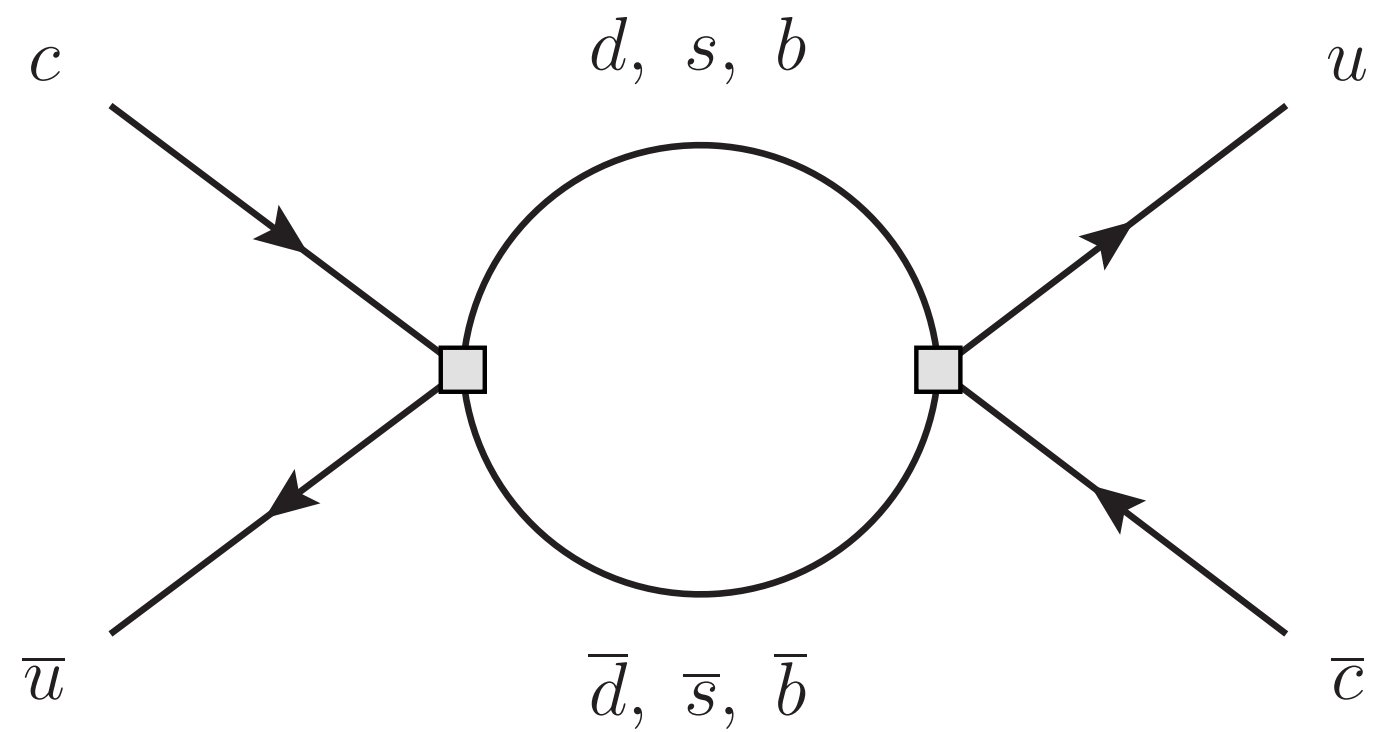


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QCD condensates

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don't contract the quark fields



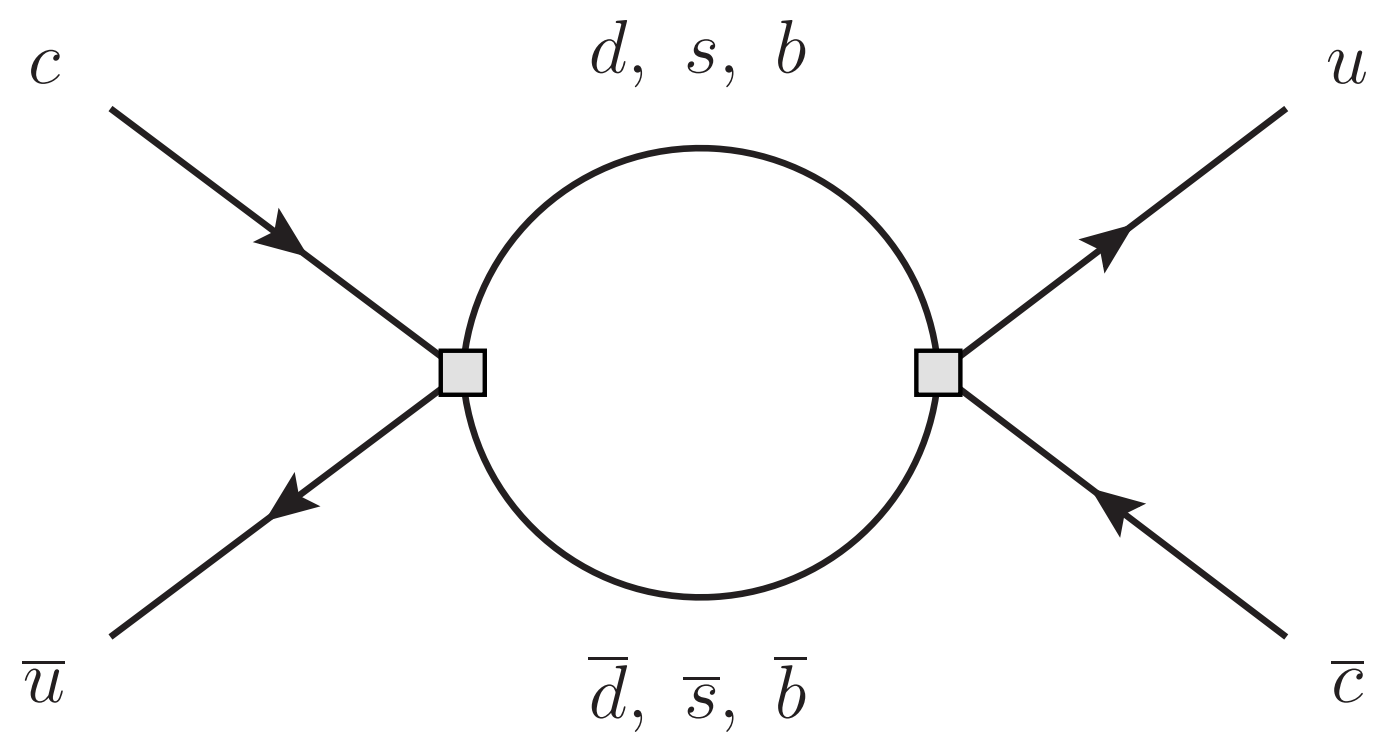
factorize them!

$$\overline{q(0)}q(x) \propto \frac{\not{p} + m_q}{p^2 - m_q^2} = \frac{\not{p} + m_q}{p^2} \left(1 + \frac{m_q^2}{p^2} + \dots \right)$$

QCD condensates

Motivation

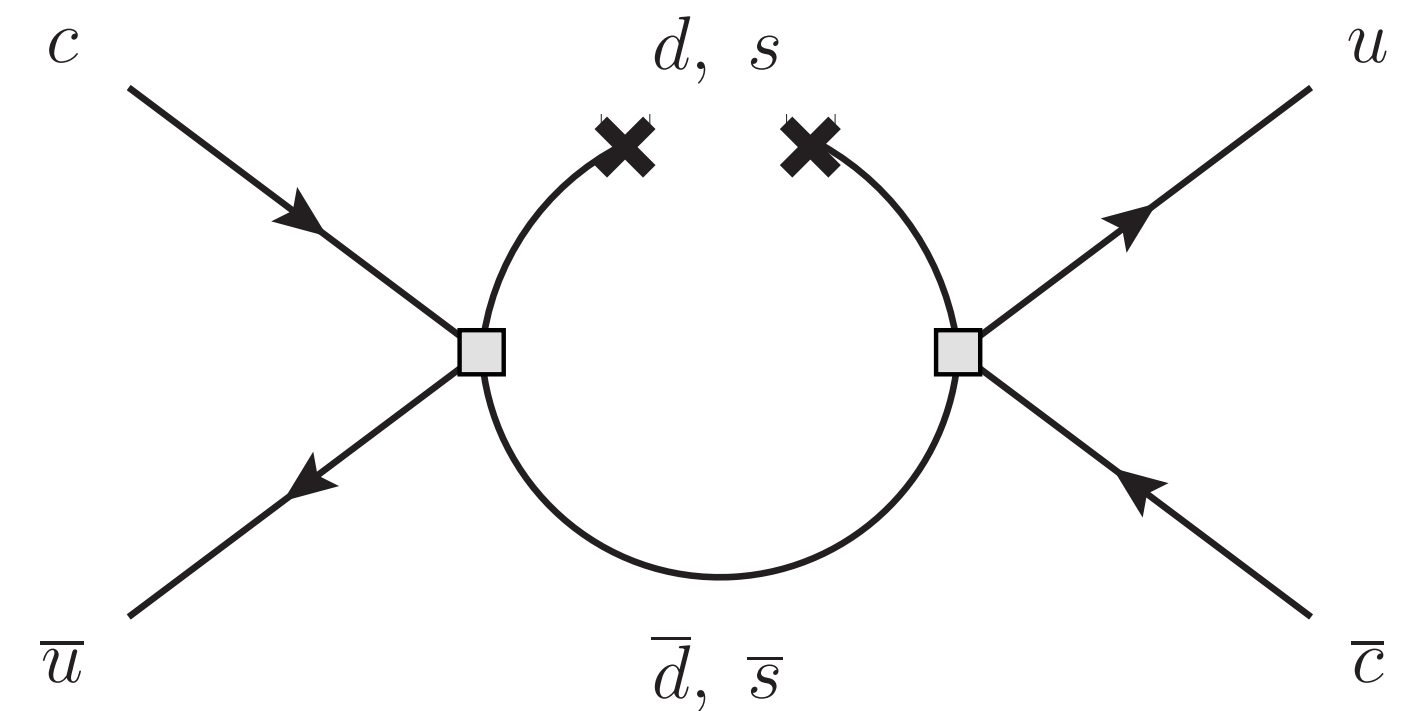
- leading order diagram $\propto (m_s/m_c)^4$



don't contract the quark fields



- two-quark condensate (dim-9 operator) $\propto (m_s/m_c)^3$

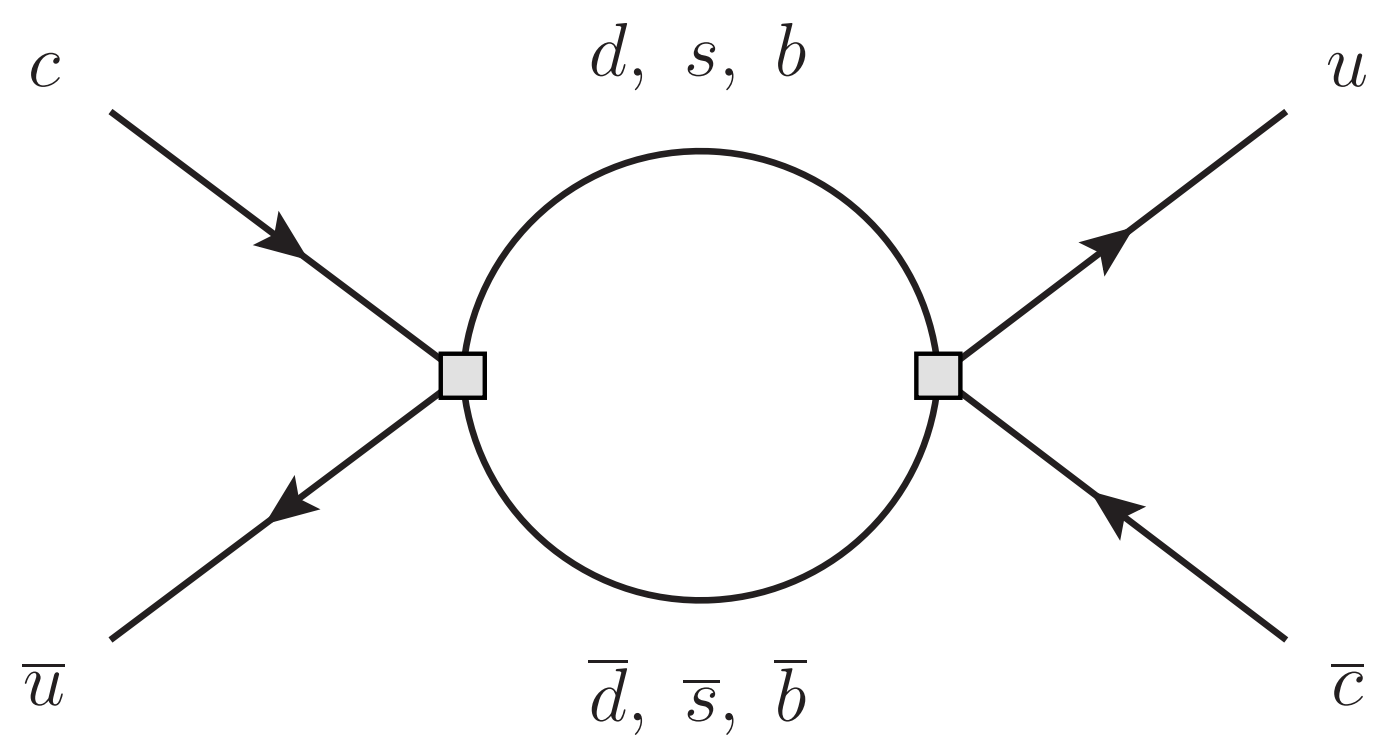


$$\overline{q(0)}q(x) \propto \frac{\not{p} + m_q}{p^2 - m_q^2} = \frac{\not{p} + m_q}{p^2} \left(1 + \frac{m_q^2}{p^2} + \dots \right)$$

QCD condensates

Motivation

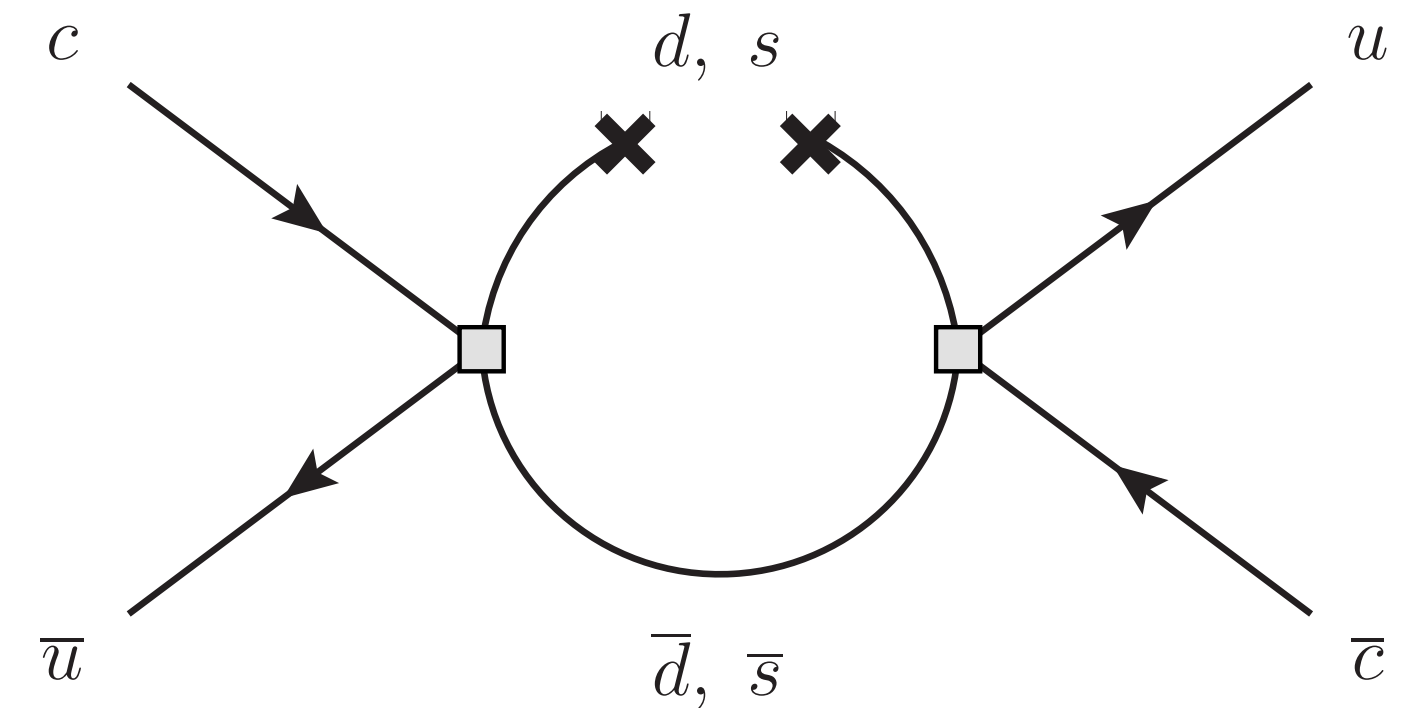
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don't contract the quark fields



- two-quark condensate (dim-9 operator) $\propto (m_s/m_c)^3$



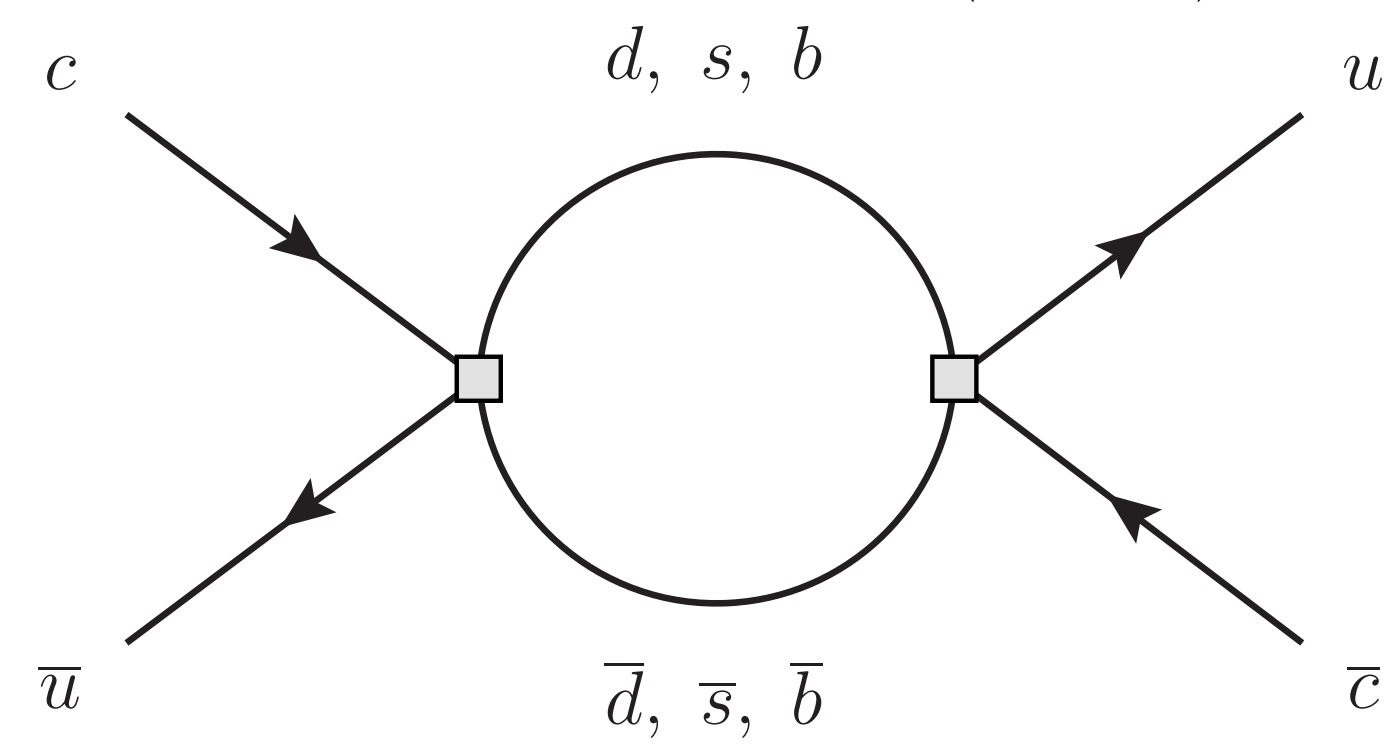
$$\langle \bar{q}(x)q(0) \rangle \propto 1 + i\gamma_5 \frac{m_q}{4} + \dots$$

$$\overline{q(0)}q(x) \propto \frac{\not{p} + m_q}{p^2 - m_q^2} = \frac{\not{p} + m_q}{p^2} \left(1 + \frac{m_q^2}{p^2} + \dots \right)$$

QCD condensates

Motivation

- leading order diagram $\propto (m_s/m_c)^4$



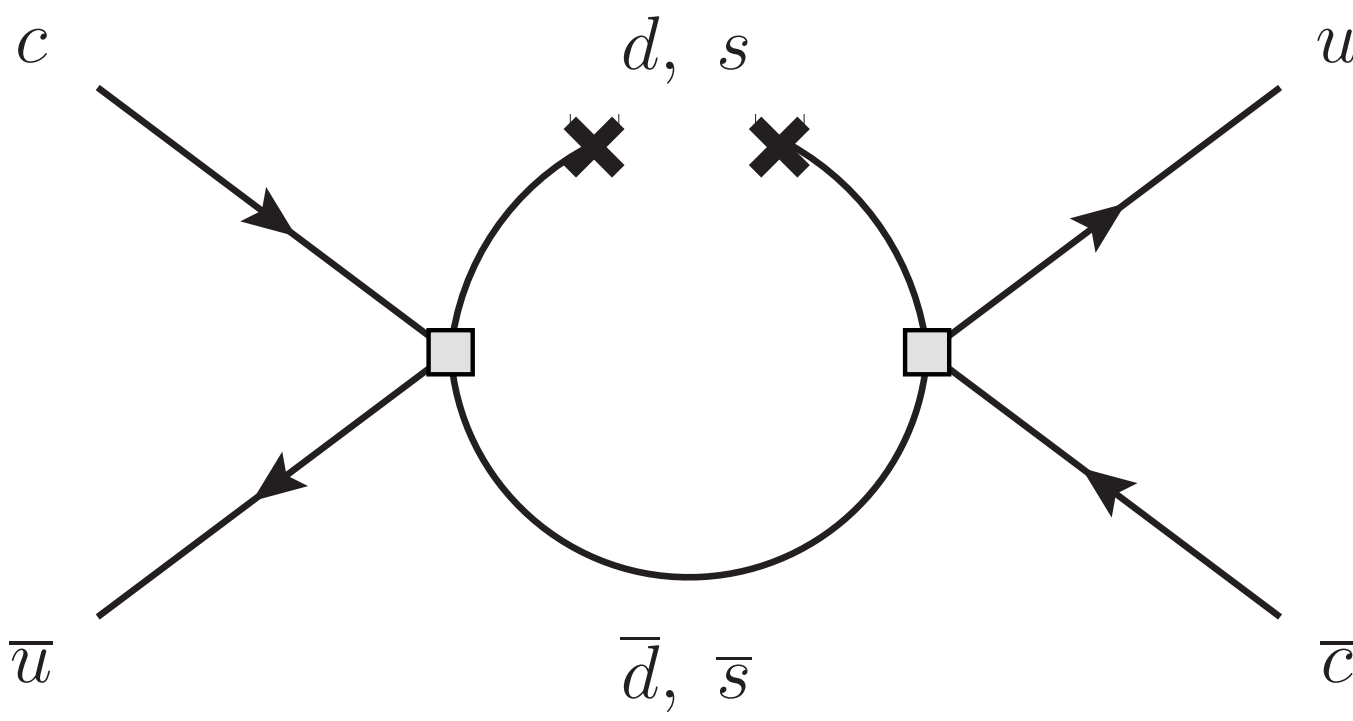
don't contract the quark fields



factorize them!

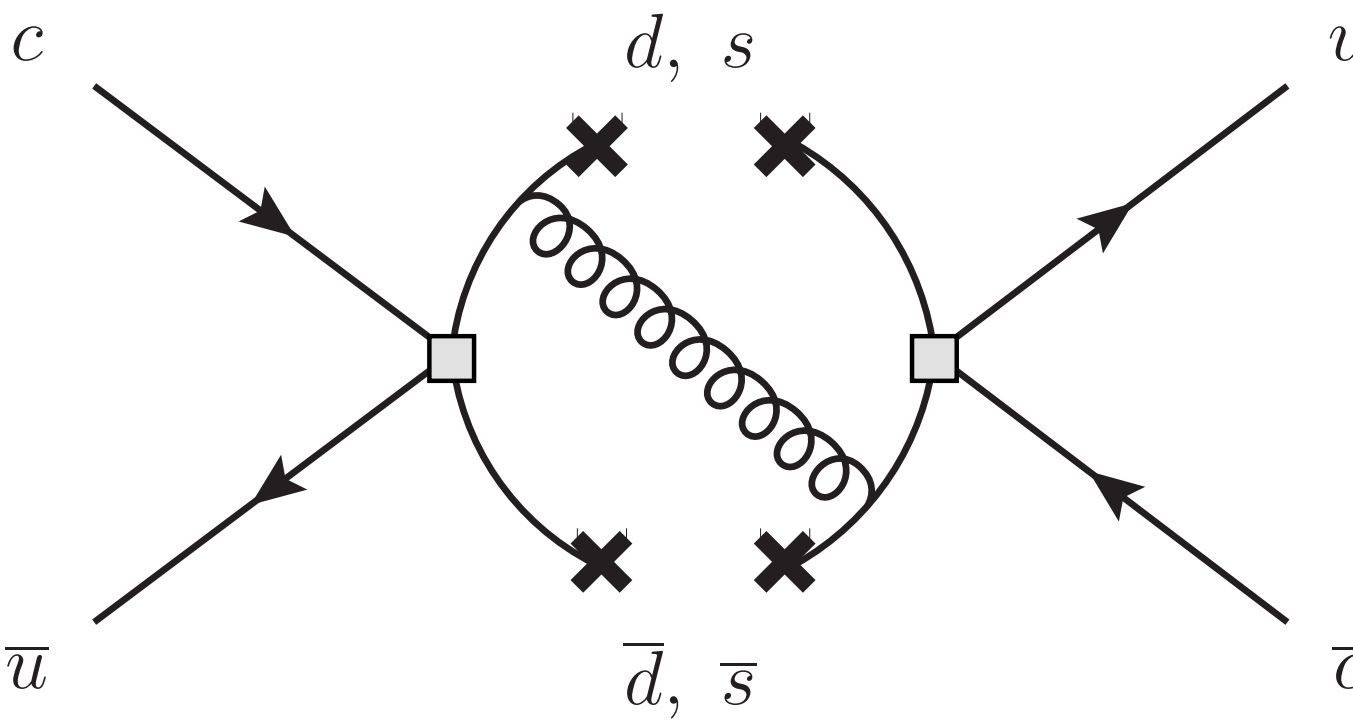
$$\overline{q(0)}q(x) \propto \frac{\not{p} + m_q}{p^2 - m_q^2} = \frac{\not{p} + m_q}{p^2} \left(1 + \frac{m_q^2}{p^2} + \dots \right)$$

- two-quark condensate (dim-9 operator) $\propto (m_s/m_c)^3$



$$\langle \bar{q}(x)q(0) \rangle \propto 1 + i\chi \frac{m_q}{4} + \dots$$

- four-quark condensate (dim-12 operator) $\propto (m_s/m_c)^2$

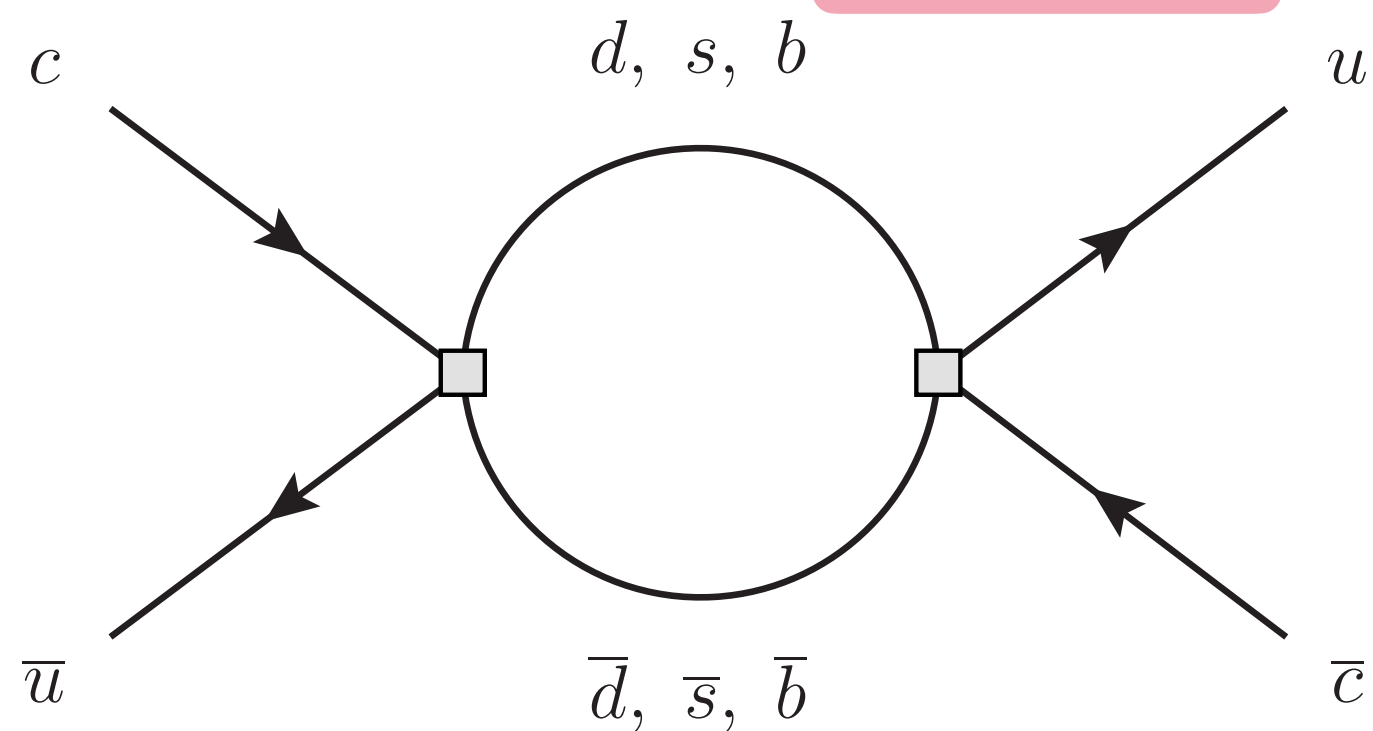


□ indicates weak effective vertex

QCD condensates

Motivation

- leading order diagram $\propto (m_s/m_c)^4$



don't contract the quark fields

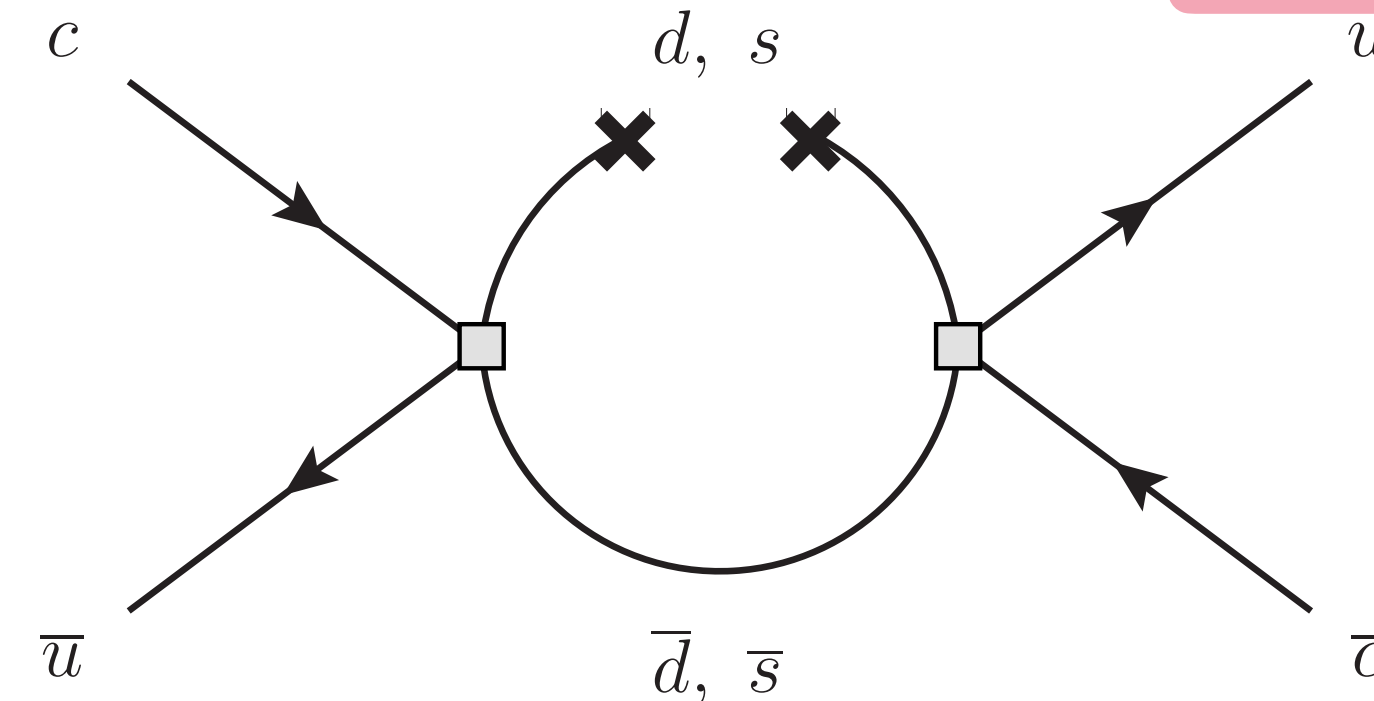


$$\overline{q(0)}q(x) \propto \frac{\not{p} + m_q}{p^2 - m_q^2} = \frac{\not{p} + m_q}{p^2} \left(1 + \frac{m_q^2}{p^2} + \dots \right)$$

- we trade a power of m_s/m_c suppression for a suppression of the **higher dimensional operator**

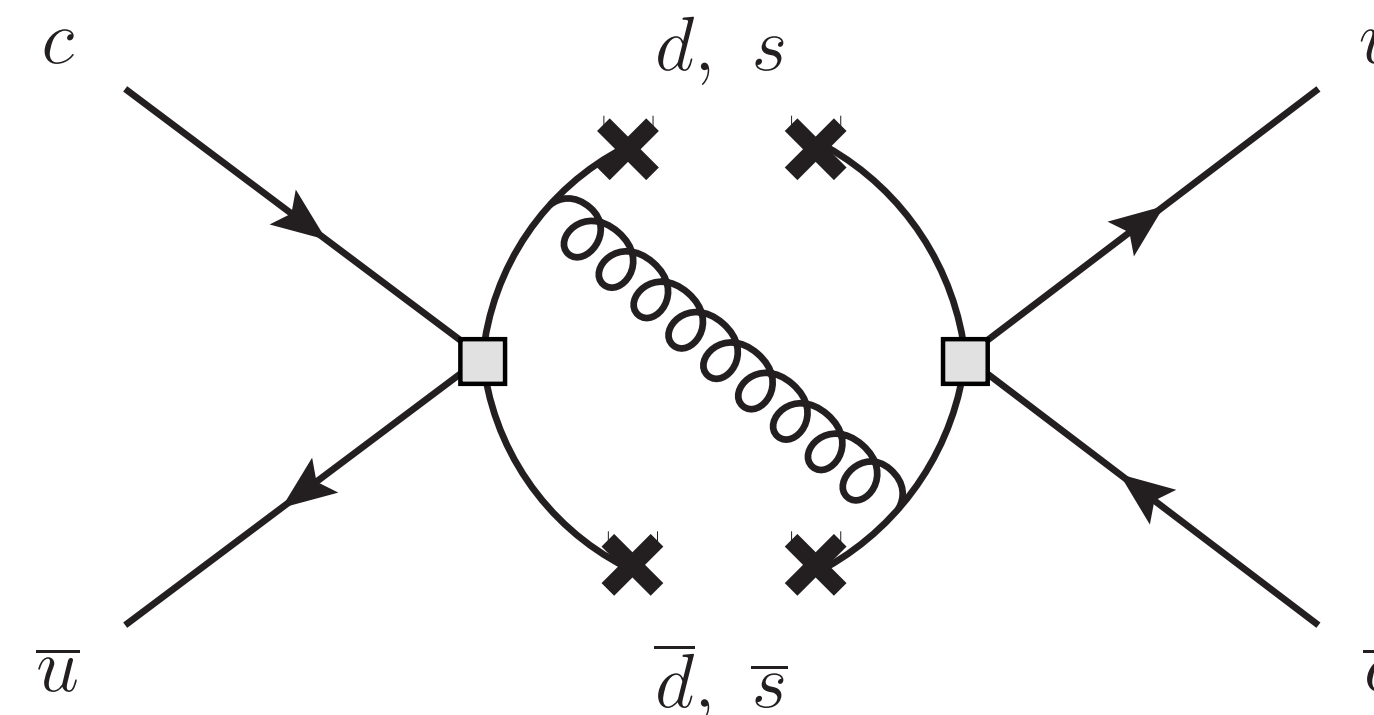
□ indicates weak effective vertex

- two-quark condensate (dim-9 operator) $\propto (m_s/m_c)^3$



$$\langle \bar{q}(x)q(0) \rangle \propto 1 + i\gamma_5 \frac{m_q}{4} + \dots$$

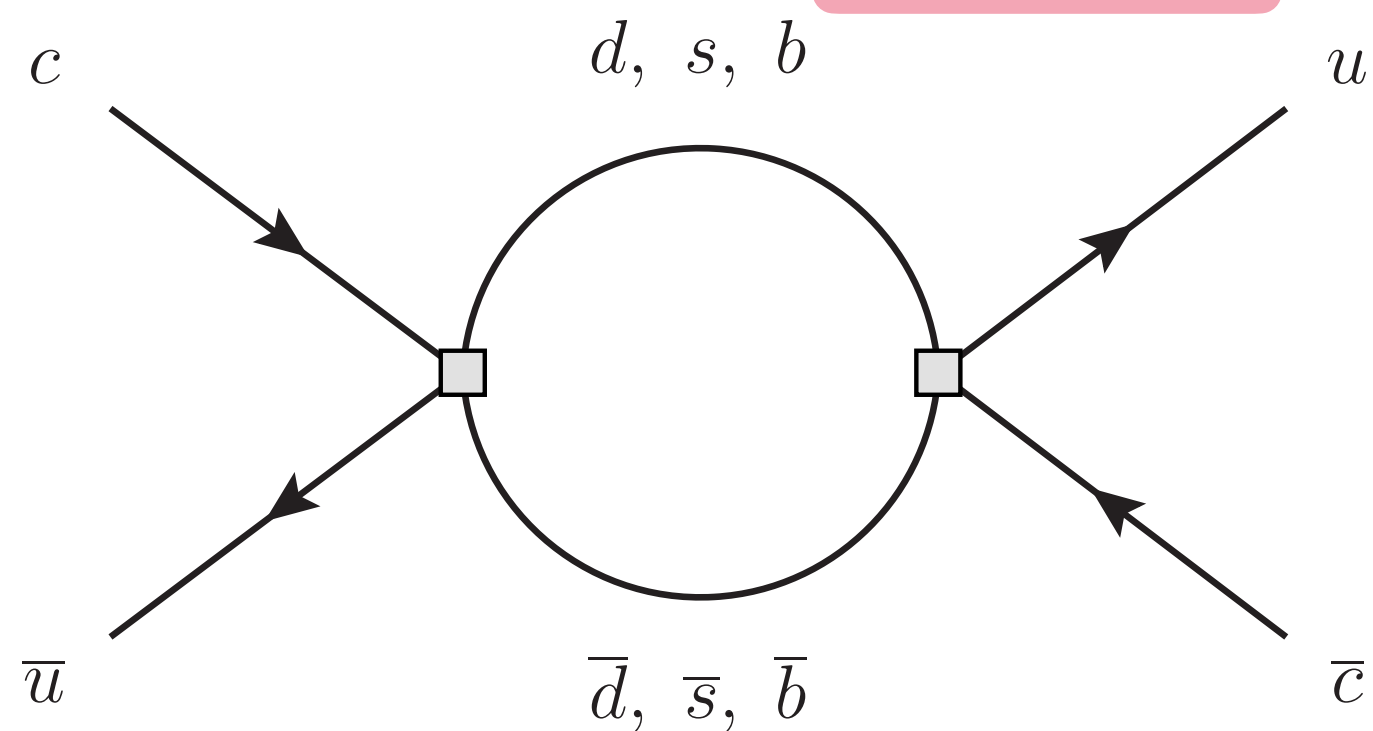
- four-quark condensate (dim-12 operator) $\propto (m_s/m_c)^2$



QCD condensates

Motivation

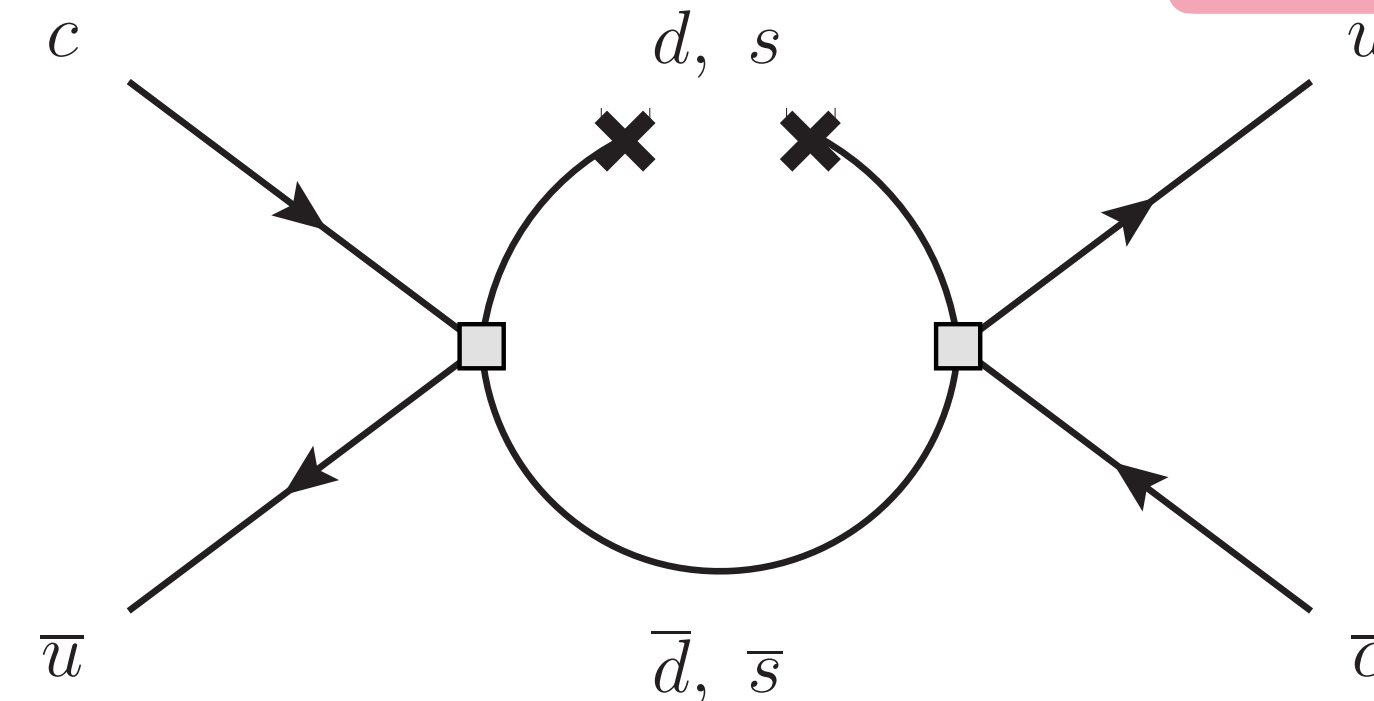
- leading order diagram $\propto (m_s/m_c)^4$



don't contract the quark fields



- two-quark condensate (dim-9 operator) $\propto (m_s/m_c)^3$

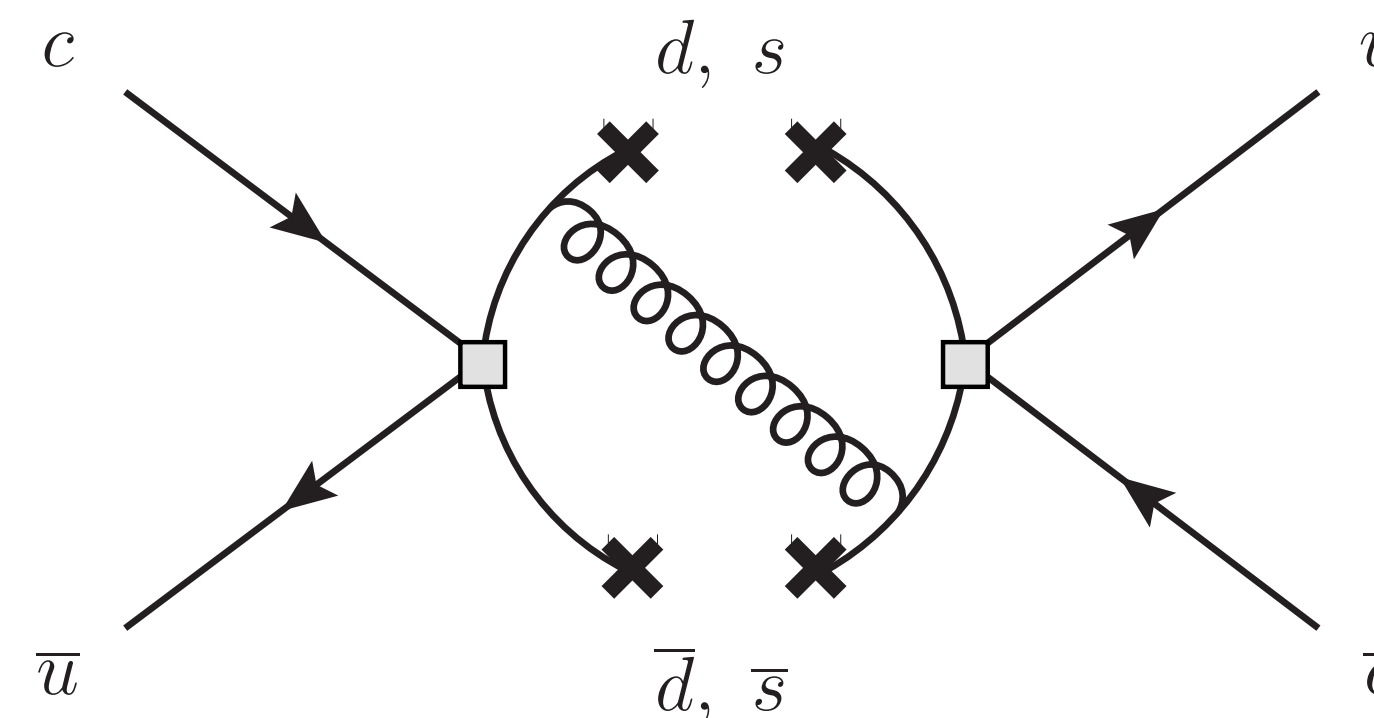


$$\langle \bar{q}(x)q(0) \rangle \propto 1 + i\chi \frac{m_q}{4} + \dots$$

$$\overline{q(0)}q(x) \propto \frac{\not{p} + m_q}{p^2 - m_q^2} = \frac{\not{p} + m_q}{p^2} \left(1 + \frac{m_q^2}{p^2} + \dots \right)$$

- we trade a power of m_s/m_c suppression for a suppression of the **higher dimensional operator**
- since the condensate diagram is not a loop diagram we expect a **relative $16\pi^2$ enhancement**

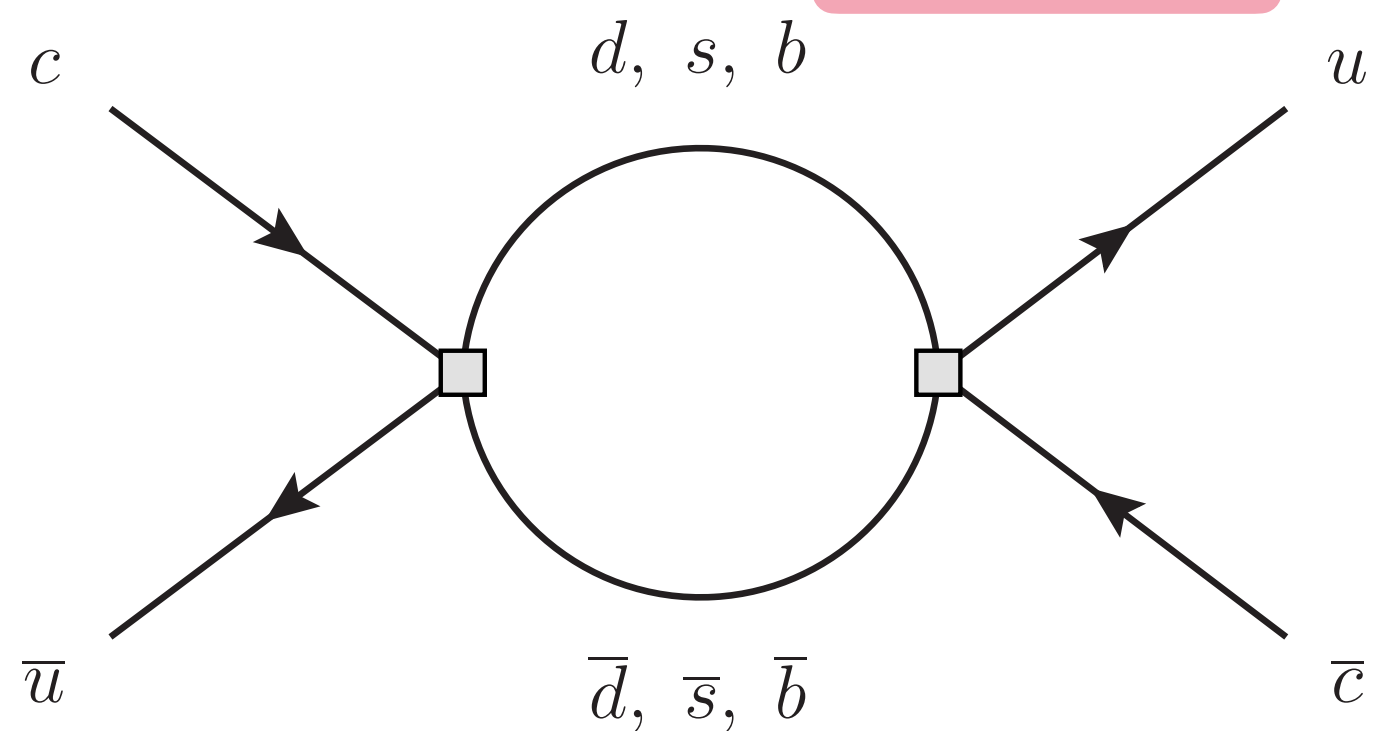
- four-quark condensate (dim-12 operator) $\propto (m_s/m_c)^2$



QCD condensates

Motivation

- leading order diagram $\propto (m_s/m_c)^4$



don't contract the quark fields

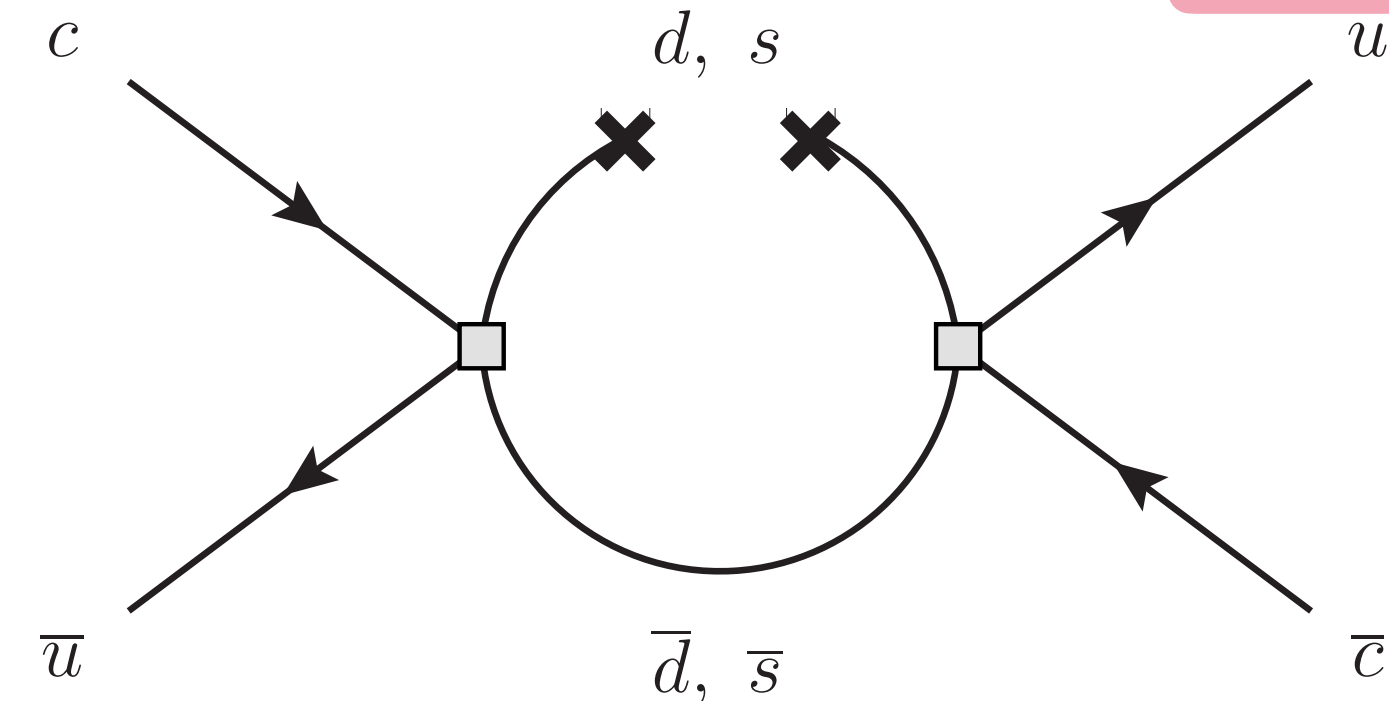


$$\overline{q(0)}q(x) \propto \frac{\not{p} + m_q}{p^2 - m_q^2} = \frac{\not{p} + m_q}{p^2} \left(1 + \frac{m_q^2}{p^2} + \dots \right)$$

- we trade a power of m_s/m_c suppression for a suppression of the **higher dimensional operator**
- since the condensate diagram is not a loop diagram we expect a **relative $16\pi^2$ enhancement**
- suppression of the higher dimensional operators is in powers of $1/m_c$ which is **expected to be relatively weak**

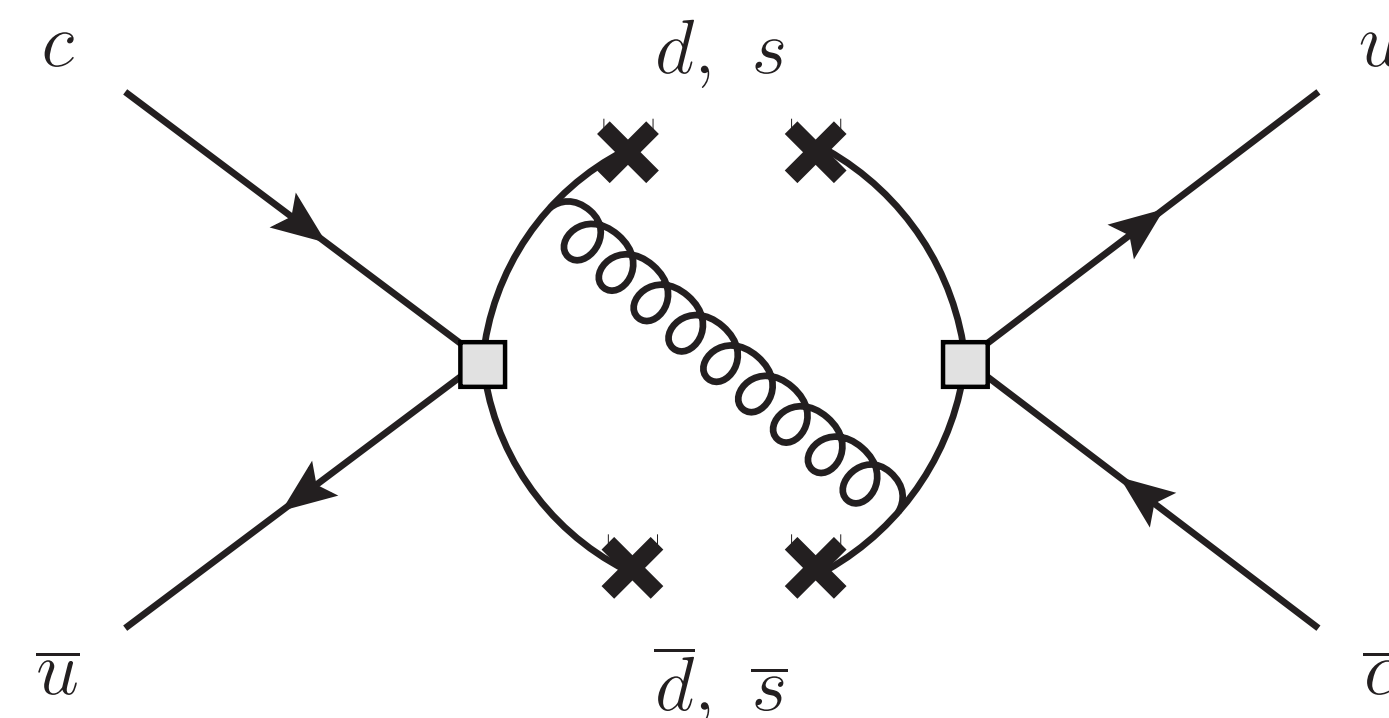
□ indicates weak effective vertex

- two-quark condensate (dim-9 operator) $\propto (m_s/m_c)^3$



$$\langle \bar{q}(x)q(0) \rangle \propto 1 + i\gamma_5 \frac{m_q}{4} + \dots$$

- four-quark condensate (dim-12 operator) $\propto (m_s/m_c)^2$



QCD condensates

Nonlocal condensates expanded in terms of local condensates

QCD condensates

Nonlocal condensates expanded in terms of local condensates

- Shifman, Vainshtein, Zakharov, QCD and Resonance Physics. Theoretical Foundations, Nucl. Phys. B147, Issue 5, 1979, 385-447

QCD condensates

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QCD condensates

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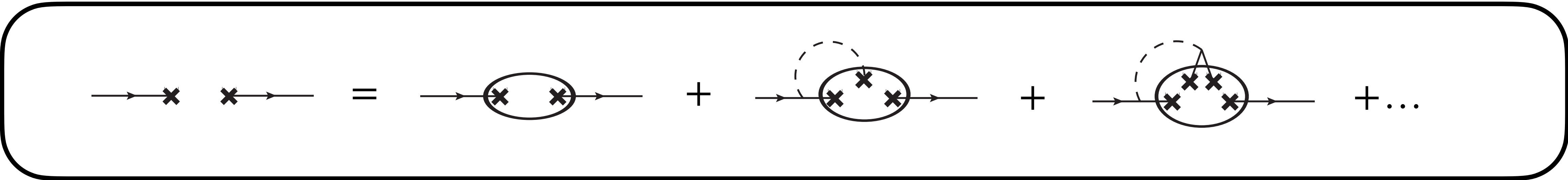
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$$\longrightarrow \times \quad \times \longrightarrow =$$

QCD condensates

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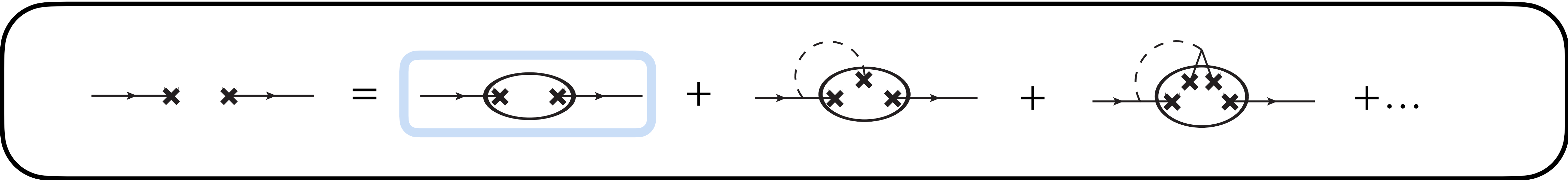
Nonlocal condensates expanded in terms of local condensates



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Nonlocal condensates expanded in terms of local condensates



QCD condensates

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Nonlocal condensates expanded in terms of local condensates

The diagram shows the expansion of a nonlocal condensate into a series of local condensates. On the left, a horizontal line with two vertices marked by 'x' is shown. This is followed by an equals sign. The first term on the right is a horizontal line with two vertices marked by 'x', where the two vertices are enclosed in an oval. This term is highlighted with a light blue rectangular box. This is followed by a plus sign, then a horizontal line with two vertices marked by 'x', where the two vertices are enclosed in an oval and a dashed arc connects the two vertices. This is followed by a plus sign, then a horizontal line with two vertices marked by 'x', where the two vertices are enclosed in an oval and a dashed arc connects the two vertices, with a triangle connecting the two vertices. This is followed by a plus sign and an ellipsis '...'. Below the first term, the expression $\propto \langle \bar{q}q \rangle_0$ is written.

$$\propto \langle \bar{q}q \rangle_0$$

QCD condensates

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Nonlocal condensates expanded in terms of local condensates

The diagram illustrates the expansion of a nonlocal condensate into a series of local condensates. On the left, a horizontal line with two vertices marked by 'x' is shown. This is followed by an equals sign and a sum of terms. The first term is enclosed in a light blue rounded rectangle and shows a loop with two 'x' vertices. The second term is enclosed in a light green rounded rectangle and shows a loop with three 'x' vertices and a dashed line connecting two of them. The third term shows a loop with four 'x' vertices and a dashed line connecting two of them. The series continues with an ellipsis. Below the first term, the expression $\propto \langle \bar{q}q \rangle_0$ is written.

$$\rightarrow \times \quad \times \rightarrow = \boxed{\rightarrow \times \quad \times \rightarrow} + \boxed{\rightarrow \times \quad \times \quad \times \rightarrow} + \rightarrow \times \quad \times \quad \times \quad \times \rightarrow + \dots$$

$\propto \langle \bar{q}q \rangle_0$

QCD condensates

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Nonlocal condensates expanded in terms of local condensates

The diagram illustrates the expansion of a nonlocal condensate into local condensates. On the left, a horizontal line with two vertices marked by 'x' is shown. This is equated to a sum of terms. The first term is enclosed in a blue box and shows a loop with two vertices, representing the quark condensate $\propto \langle \bar{q}q \rangle_0$. The second term is enclosed in a green box and shows a loop with three vertices, representing the gluon condensate $\propto \langle \bar{q}\sigma \cdot Gq \rangle_0 \sim \lambda_q^2$. The third term shows a loop with four vertices, and the sequence continues with an ellipsis.

$$\rightarrow \times \quad \times \rightarrow = \quad \propto \langle \bar{q}q \rangle_0 \quad + \quad \propto \langle \bar{q}\sigma \cdot Gq \rangle_0 \sim \lambda_q^2 \quad + \quad \dots$$

QCD condensates

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Nonlocal condensates expanded in terms of local condensates

The diagram illustrates the expansion of a nonlocal condensate into local condensates. On the left, a nonlocal condensate is represented by two vertices (marked with 'x') connected by a horizontal line. This is set equal to a sum of three terms, each enclosed in a colored box (blue, green, and pink), followed by an ellipsis. The first term (blue box) shows a local condensate with two vertices inside an oval, connected by a horizontal line. The second term (green box) shows a local condensate with three vertices inside an oval, connected by a horizontal line, with a dashed arc above it. The third term (pink box) shows a local condensate with four vertices inside an oval, connected by a horizontal line, with a dashed arc above it. Below the first term is the expression $\propto \langle \bar{q}q \rangle_0$. Below the second and third terms is the expression $\propto \langle \bar{q}\sigma \cdot Gq \rangle_0 \sim \lambda_q^2$.

$$\rightarrow \times \quad \times \rightarrow = \boxed{\rightarrow \times \quad \times \rightarrow} + \boxed{\rightarrow \times \quad \times \quad \times \rightarrow} + \boxed{\rightarrow \times \quad \times \quad \times \quad \times \rightarrow} + \dots$$

$$\propto \langle \bar{q}q \rangle_0 \qquad \propto \langle \bar{q}\sigma \cdot Gq \rangle_0 \sim \lambda_q^2$$

QCD condensates

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Nonlocal condensates expanded in terms of local condensates

The diagram illustrates the expansion of a nonlocal condensate into local condensates. On the left, a nonlocal condensate is represented by two vertices (marked with 'x') connected by a horizontal line. This is set equal to a sum of three terms, each enclosed in a colored box (blue, green, and pink), followed by an ellipsis. Each term shows a different internal structure for the condensate, with vertices and internal lines. Below each term is a corresponding mathematical expression for the condensate's value.

$$\begin{aligned}
 & \text{Nonlocal condensate} = \text{Blue box} + \text{Green box} + \text{Pink box} + \dots \\
 & \propto \langle \bar{q}q \rangle_0 \qquad \propto \langle \bar{q} \sigma \cdot G q \rangle_0 \sim \lambda_q^2 \qquad \propto \langle \bar{q}q \rangle_0^2
 \end{aligned}$$

QCD condensates

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Nonlocal condensates expanded in terms of local condensates

$$\begin{aligned}
 & \text{---} \times \times \text{---} = \text{---} \times \times \text{---} + \text{---} \times \times \times \text{---} + \text{---} \times \times \times \times \text{---} + \dots \\
 & \propto \langle \bar{q}q \rangle_0 \qquad \propto \langle \bar{q} \sigma \cdot G q \rangle_0 \sim \lambda_q^2 \qquad \propto \langle \bar{q}q \rangle_0^2
 \end{aligned}$$

$$\langle \bar{q}_\alpha^a(x) q_\beta^b(0) \rangle = \delta^{ab} \frac{\langle \bar{q}q \rangle_0}{12} \left[\delta_{\alpha\beta} \left(1 + \frac{x^2}{8} \lambda_q^2 + \dots \right) + i x_{\beta\alpha} \left(\frac{m}{4} + \frac{x^2}{4} \left(\frac{m \lambda_q^2}{12} - \frac{2}{81} \pi \alpha_s^{NP} \frac{\langle \bar{q}q \rangle_0^2}{\langle \bar{q}q \rangle_0} \right) + \dots \right) \right]$$

QCD condensates

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Nonlocal condensates expanded in terms of local condensates

$$\begin{aligned}
 \text{---} \times \times \text{---} &= \text{---} \times \times \text{---} + \text{---} \times \times \text{---} + \text{---} \times \times \text{---} + \dots \\
 &\propto \langle \bar{q}q \rangle_0 \qquad \propto \langle \bar{q} \sigma \cdot G q \rangle_0 \sim \lambda_q^2 \qquad \propto \langle \bar{q}q \rangle_0^2
 \end{aligned}$$

$$\langle \bar{q}_\alpha^a(x) q_\beta^b(0) \rangle = \delta^{ab} \frac{\langle \bar{q}q \rangle_0}{12} \left[\delta_{\alpha\beta} \left(1 + \frac{x^2}{8} \lambda_q^2 + \dots \right) + i x_{\beta\alpha} \left(\frac{m}{4} + \frac{x^2}{4} \left(\frac{m \lambda_q^2}{12} - \frac{2}{81} \pi \alpha_s^{NP} \frac{\langle \bar{q}q \rangle_0^2}{\langle \bar{q}q \rangle_0} \right) + \dots \right) \right]$$

QCD condensates

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Nonlocal condensates expanded in terms of local condensates

$$\begin{aligned}
 & \longrightarrow \times \quad \times \longrightarrow = \boxed{\longrightarrow \times \times \longrightarrow} + \boxed{\longrightarrow \times \times \times \longrightarrow} + \boxed{\longrightarrow \times \times \times \times \longrightarrow} + \dots \\
 & \qquad \qquad \qquad \propto \langle \bar{q}q \rangle_0 \qquad \qquad \propto \langle \bar{q} \sigma \cdot G q \rangle_0 \sim \lambda_q^2 \qquad \qquad \propto \langle \bar{q}q \rangle_0^2
 \end{aligned}$$

$$\langle \bar{q}_\alpha^a(x) q_\beta^b(0) \rangle = \delta^{ab} \frac{\langle \bar{q}q \rangle_0}{12} \left[\delta_{\alpha\beta} \left(1 + \frac{x^2}{8} \lambda_q^2 + \dots \right) + i x_{\beta\alpha} \left(\frac{m}{4} + \frac{x^2}{4} \left(\frac{m \lambda_q^2}{12} - \frac{2}{81} \pi \alpha_s^{NP} \frac{\langle \bar{q}q \rangle_0^2}{\langle \bar{q}q \rangle_0} \right) + \dots \right) \right]$$

QCD condensates

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Nonlocal condensates expanded in terms of local condensates

$$\begin{aligned}
 & \longrightarrow \times \quad \times \longrightarrow = \boxed{\longrightarrow \times \times \longrightarrow} + \boxed{\longrightarrow \times \times \times \longrightarrow} + \boxed{\longrightarrow \times \times \times \times \longrightarrow} + \dots \\
 & \qquad \qquad \qquad \propto \langle \bar{q}q \rangle_0 \qquad \qquad \propto \langle \bar{q} \sigma \cdot G q \rangle_0 \sim \lambda_q^2 \qquad \qquad \propto \langle \bar{q}q \rangle_0^2
 \end{aligned}$$

$$\langle \bar{q}_\alpha^a(x) q_\beta^b(0) \rangle = \delta^{ab} \frac{\langle \bar{q}q \rangle_0}{12} \left[\delta_{\alpha\beta} \left(1 + \frac{x^2}{8} \lambda_q^2 + \dots \right) + i x_{\beta\alpha} \left(\frac{m}{4} + \frac{x^2}{4} \left(\frac{m \lambda_q^2}{12} - \frac{2}{81} \pi \alpha_s^{NP} \frac{\langle \bar{q}q \rangle_0^2}{\langle \bar{q}q \rangle_0} \right) + \dots \right) \right]$$

QCD condensates

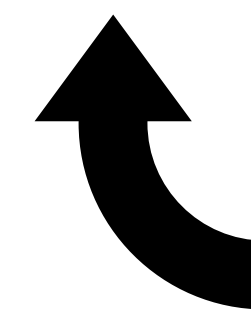
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Nonlocal condensates expanded in terms of local condensates

$$\rightarrow \times \times \rightarrow = \boxed{\rightarrow \times \times \rightarrow} + \boxed{\rightarrow \times \times \rightarrow} + \boxed{\rightarrow \times \times \rightarrow} + \dots$$

$$\propto \langle \bar{q}q \rangle_0 \quad \propto \langle \bar{q} \sigma \cdot G q \rangle_0 \sim \lambda_q^2 \quad \propto \langle \bar{q}q \rangle_0^2$$

$$\langle \bar{q}_\alpha^a(x) q_\beta^b(0) \rangle = \delta^{ab} \frac{\langle \bar{q}q \rangle_0}{12} \left[\delta_{\alpha\beta} \left(1 + \frac{x^2}{8} \lambda_q^2 + \dots \right) + i x_{\beta\alpha} \left(\frac{m}{4} + \frac{x^2}{4} \left(\frac{m \lambda_q^2}{12} - \frac{2}{81} \pi \alpha_s^{NP} \frac{\langle \bar{q}q \rangle_0^2}{\langle \bar{q}q \rangle_0} \right) + \dots \right) \right]$$



due to Dirac structure, only the vector terms contribute
 $\propto \gamma^\mu$

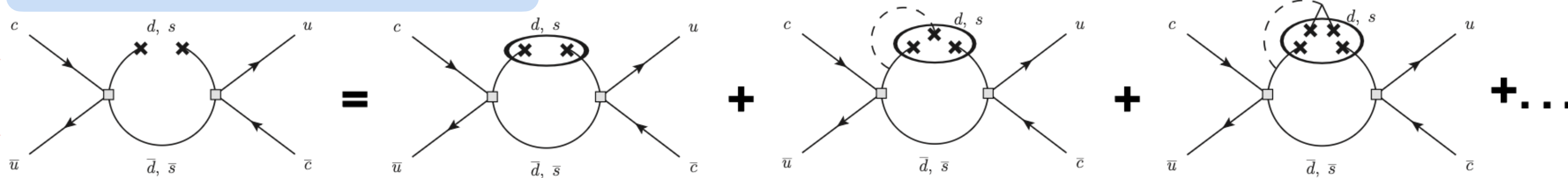
Contributions of QCD condensates to $D\bar{D}$ mixing

Nonlocal condensates expanded in terms of local condensates

Contributions of QCD condensates to $D\bar{D}$ mixing

Nonlocal condensates expanded in terms of local condensates

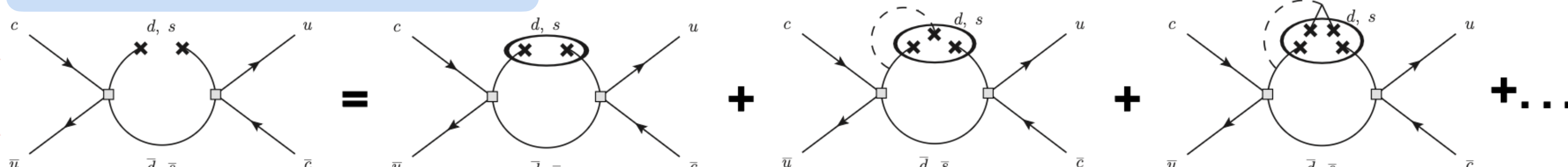
Dim-9 operator contribution



Contributions of QCD condensates to $D\bar{D}$ mixing

Nonlocal condensates expanded in terms of local condensates

Dim-9 operator contribution

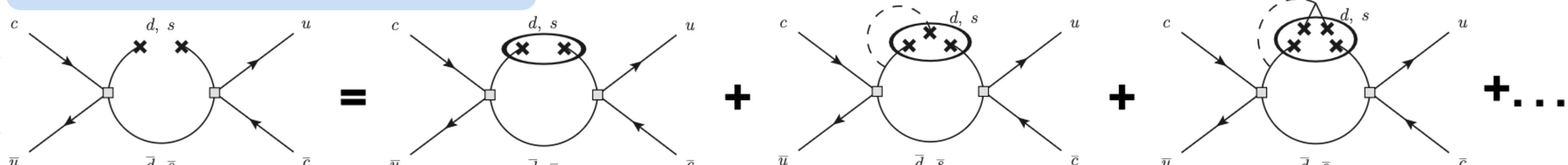


$$\langle \bar{q}_\alpha^a(x) q_\beta^b(0) \rangle = \delta^{ab} \frac{\langle \bar{q}q \rangle_0}{12} \left[\delta_{\alpha\beta} \left(1 + \frac{x^2}{8} \lambda_q^2 + \dots \right) + i \chi_{\beta\alpha} \left(\frac{m}{4} + \frac{x^2}{4} \left(\frac{m \lambda_q^2}{12} - \frac{2}{81} \pi \alpha_s^{NP} \frac{\langle \bar{q}q \rangle_0^2}{\langle \bar{q}q \rangle_0} \right) + \dots \right) \right]$$

Contributions of QCD condensates to $D\bar{D}$ mixing

Nonlocal condensates expanded in terms of local condensates

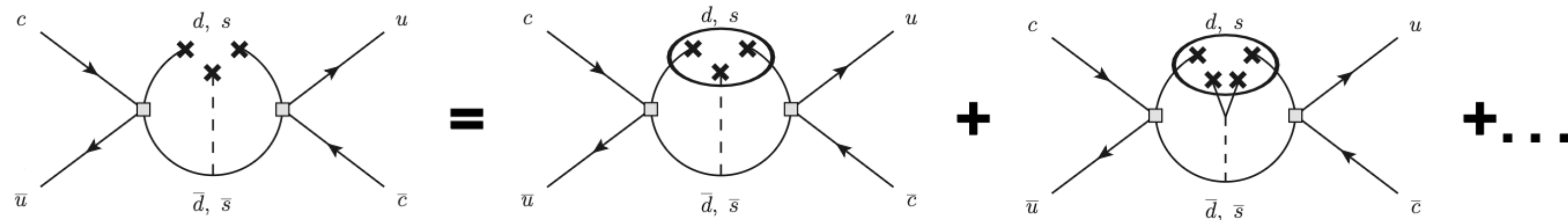
Dim-9 operator contribution



The diagram shows the expansion of a nonlocal condensate into a series of local operators. On the left, a circle with two vertices (squares) and four external lines (c, u, u-bar, c-bar) is shown. The top arc is labeled 'd, s' and the bottom arc is labeled 'd-bar, s-bar'. Two 'x' marks are on the top arc. This is equal to a sum of terms: the first term has two 'x' marks on the top arc; the second term has three 'x' marks on the top arc and a dashed line above it; the third term has four 'x' marks on the top arc and a dashed line above it; followed by an ellipsis.

$$\langle \bar{q}_\alpha^a(x) q_\beta^b(0) \rangle = \delta^{ab} \frac{\langle \bar{q}q \rangle_0}{12} \left[\delta_{\alpha\beta} \left(1 + \frac{x^2}{8} \lambda_q^2 + \dots \right) + i \chi_{\beta\alpha} \left(\frac{m}{4} + \frac{x^2}{4} \left(\frac{m \lambda_q^2}{12} - \frac{2}{81} \pi \alpha_s^{NP} \frac{\langle \bar{q}q \rangle_0^2}{\langle \bar{q}q \rangle_0} \right) + \dots \right) \right]$$

Dim-11 operator contribution

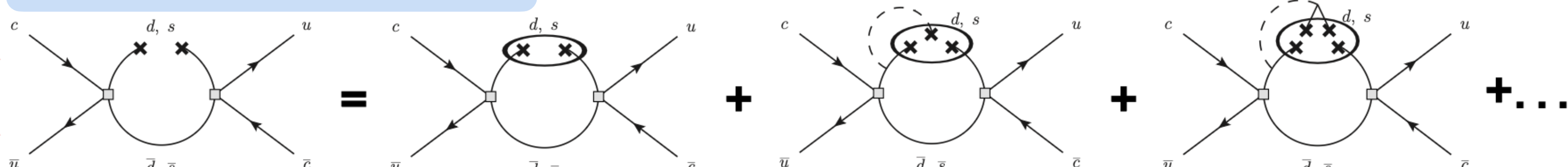


The diagram shows the expansion of a nonlocal condensate into a series of local operators. On the left, a circle with two vertices (squares) and four external lines (c, u, u-bar, c-bar) is shown. The top arc is labeled 'd, s' and the bottom arc is labeled 'd-bar, s-bar'. A dashed vertical line connects the two vertices. This is equal to a sum of terms: the first term has two 'x' marks on the top arc and a dashed vertical line; the second term has three 'x' marks on the top arc and a dashed vertical line; followed by an ellipsis.

Contributions of QCD condensates to $D\bar{D}$ mixing

Nonlocal condensates expanded in terms of local condensates

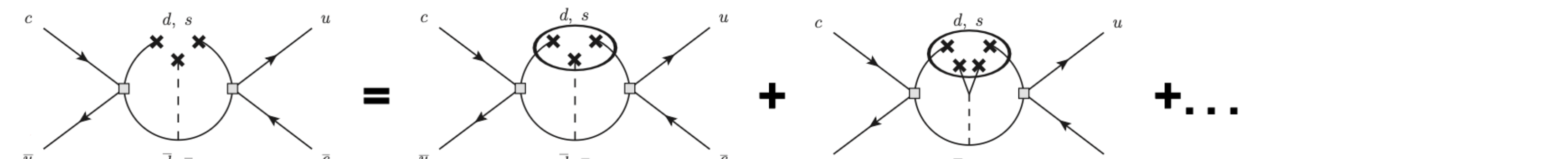
Dim-9 operator contribution



The diagram shows the expansion of a nonlocal condensate into a series of local condensates. On the left, a quark loop with external lines labeled $c, \bar{c}, u, \bar{u}$ and internal lines labeled $d, \bar{d}, s, \bar{s}$ is shown. This is equal to a sum of terms: the first term is the same loop with two 'x' marks on the top line; the second term is the same loop with four 'x' marks on the top line; the third term is the same loop with six 'x' marks on the top line; and so on, indicated by '+...'. Below the diagrams is the mathematical expression for the expansion:

$$\langle \bar{q}_\alpha^a(x) q_\beta^b(0) \rangle = \delta^{ab} \frac{\langle \bar{q}q \rangle_0}{12} \left[\delta_{\alpha\beta} \left(1 + \frac{x^2}{8} \lambda_q^2 + \dots \right) + i \chi_{\beta\alpha} \left(\frac{m}{4} + \frac{x^2}{4} \left(\frac{m \lambda_q^2}{12} - \frac{2}{81} \pi \alpha_s^{NP} \frac{\langle \bar{q}q \rangle_0^2}{\langle \bar{q}q \rangle_0} \right) + \dots \right) \right]$$

Dim-11 operator contribution



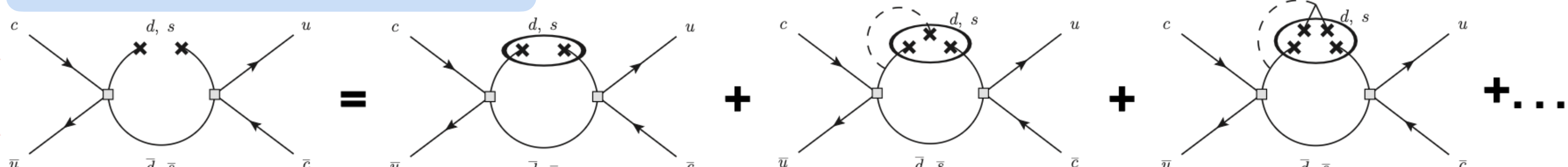
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$$\langle \bar{q}_\alpha^a(x) G_{\mu\nu}^{cd}(0) q_\beta^b(0) \rangle = \frac{1}{384} \left(\delta^{bd} \delta^{ac} - \frac{1}{3} \delta^{cd} \delta^{ab} \right) \left[\left(\sigma_{\mu\nu} + m \left(i \sigma_{\mu\nu} \not{x} + \gamma_\mu x_\nu - \gamma_\nu x_\mu \right) \right)_{\beta\alpha} \lambda_q^2 \langle \bar{q}q \rangle + \frac{i}{2} \left(x \sigma_{\mu\nu} \right)_{\beta\alpha} \frac{16}{9} \pi \alpha_s^{NP} \langle \bar{q}q \rangle^2 + \dots \right]$$

Contributions of QCD condensates to $D\bar{D}$ mixing

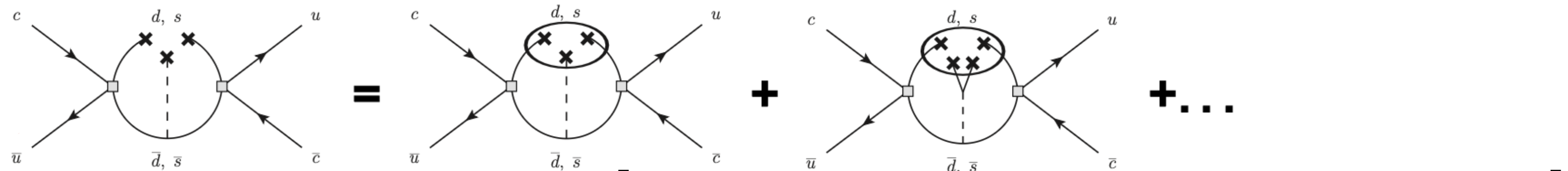
Nonlocal condensates expanded in terms of local condensates

Dim-9 operator contribution



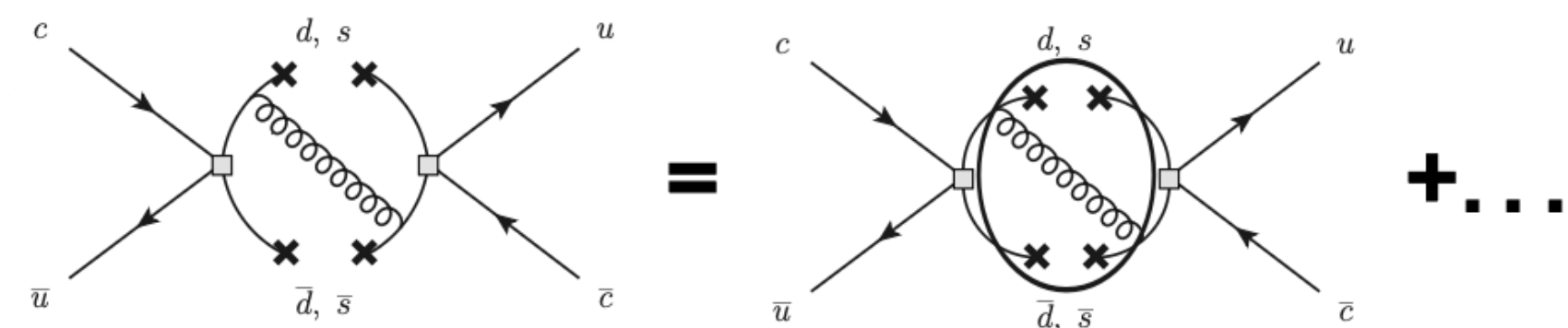
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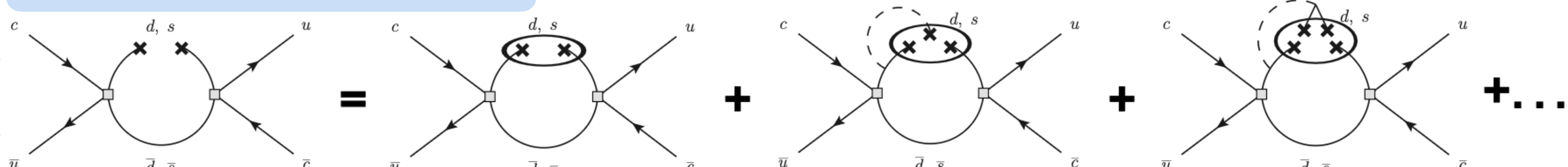
Dim-12 operator contribution



Contributions of QCD condensates to $D\bar{D}$ mixing

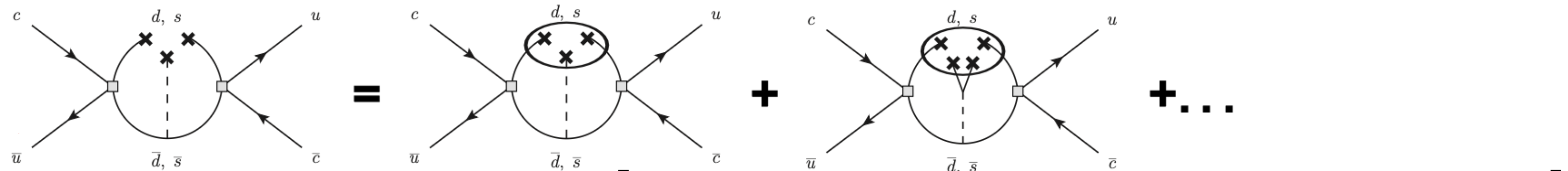
Nonlocal condensates expanded in terms of local condensates

Dim-9 operator contribution



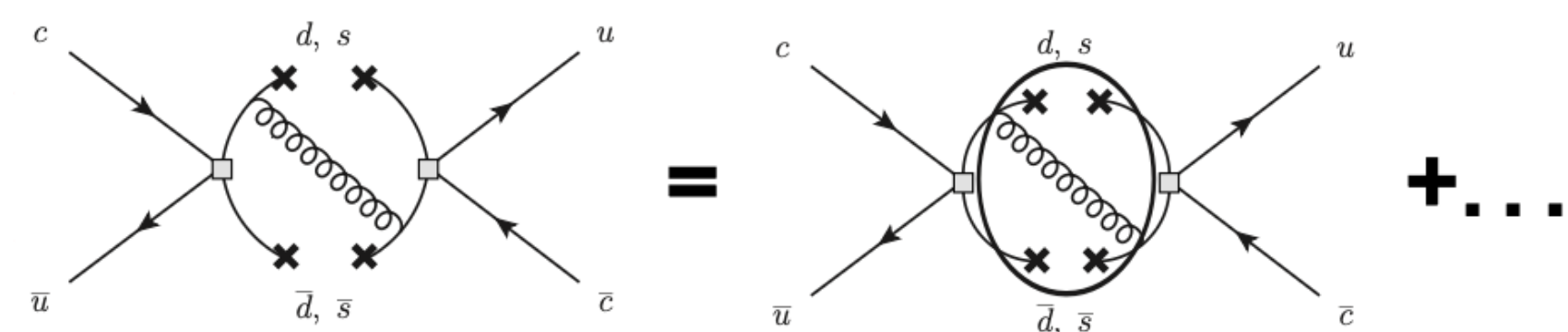
$$\langle \bar{q}_\alpha^a(x) q_\beta^b(0) \rangle = \delta^{ab} \frac{\langle \bar{q}q \rangle_0}{12} \left[\delta_{\alpha\beta} \left(1 + \frac{x^2}{8} \lambda_q^2 + \dots \right) + i \chi_{\beta\alpha} \left(\frac{m}{4} + \frac{x^2}{4} \left(\frac{m \lambda_q^2}{12} - \frac{2}{81} \pi \alpha_s^{\text{NP}} \frac{\langle \bar{q}q \rangle_0^2}{\langle \bar{q}q \rangle_0} \right) + \dots \right) \right]$$

Dim-11 operator contribution



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Dim-12 operator contribution



second factorization

$$\langle \bar{q}_1^a(x) \bar{q}_4^d(y) q_3^c(z) q_2^b(0) \rangle = \langle \bar{q}_1^a(x) q_2^b(0) \rangle_\beta \langle \bar{q}_4^d(y) q_3^c(z) \rangle_\gamma - \langle \bar{q}_1^a(x) q_3^c(z) \rangle_\gamma \langle \bar{q}_4^d(y) q_2^b(0) \rangle_\beta$$

Complexity of four-quark contribution

There is sixteen ways to connect the diagram

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- Note in weak vertices you find O_1 and O_2 operators which contract color in different ways
- All together this multitude of different contractions sums to an 'unpredictable result'

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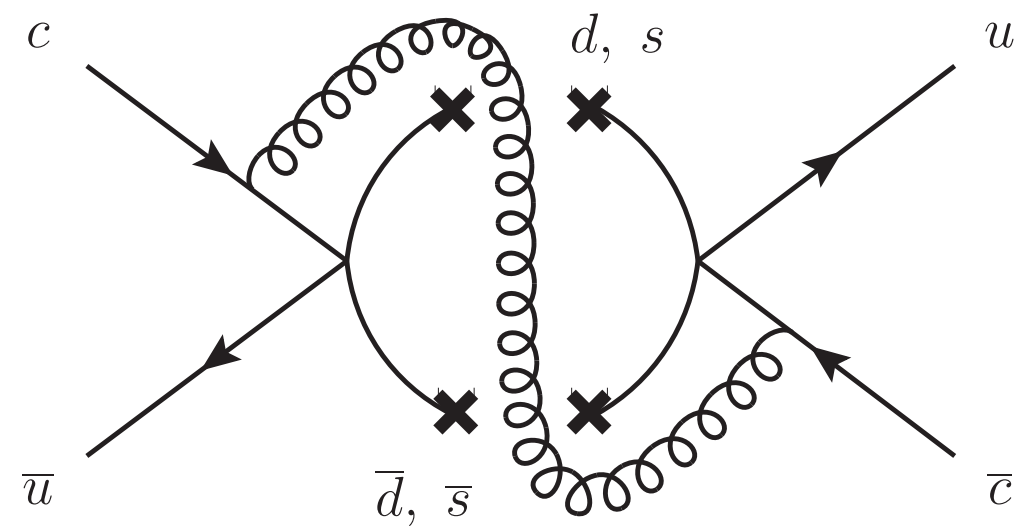
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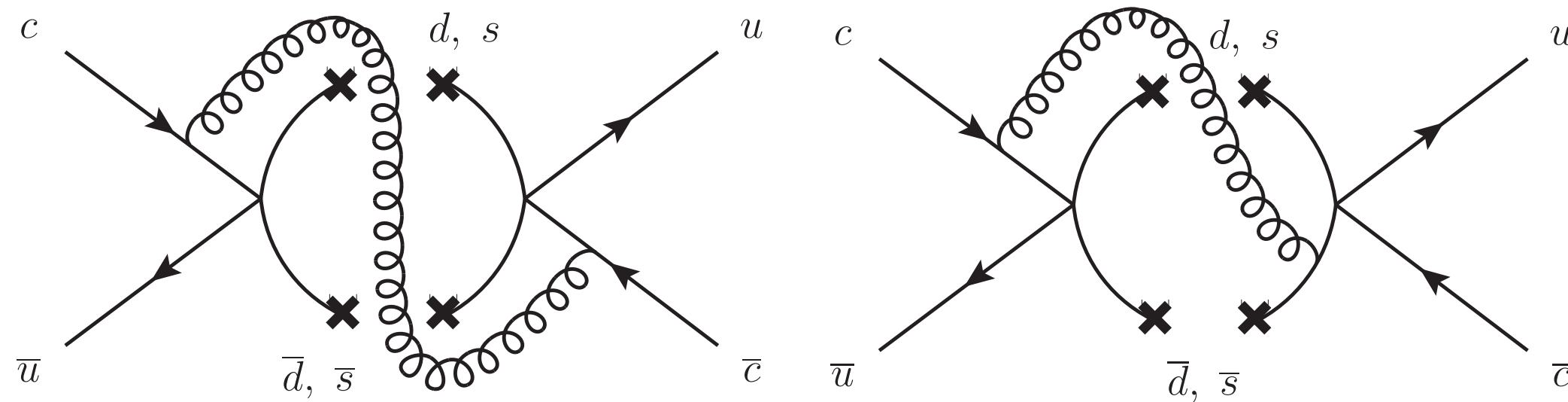
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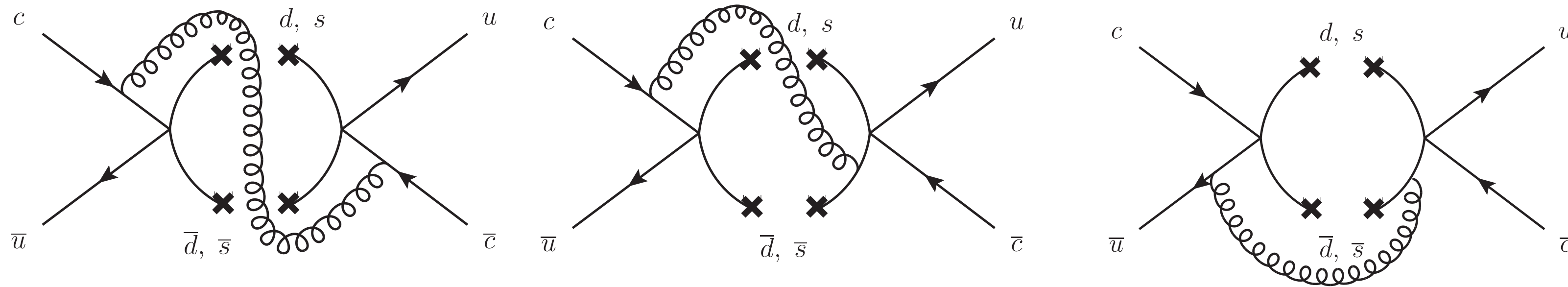
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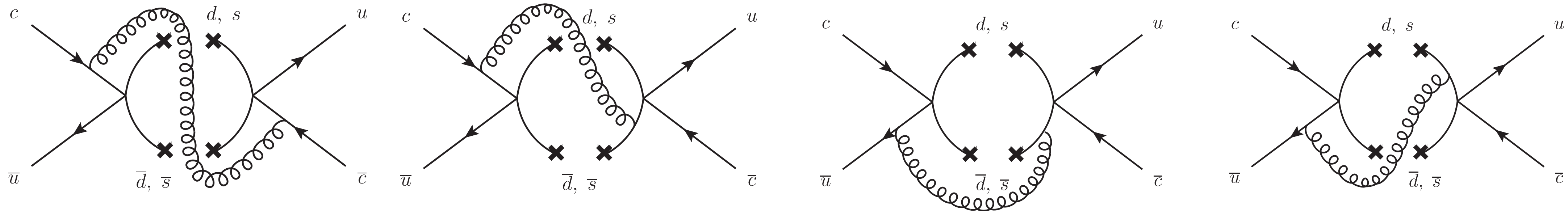
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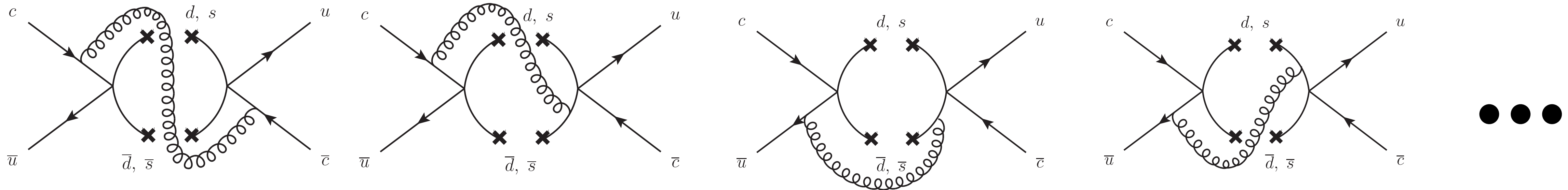
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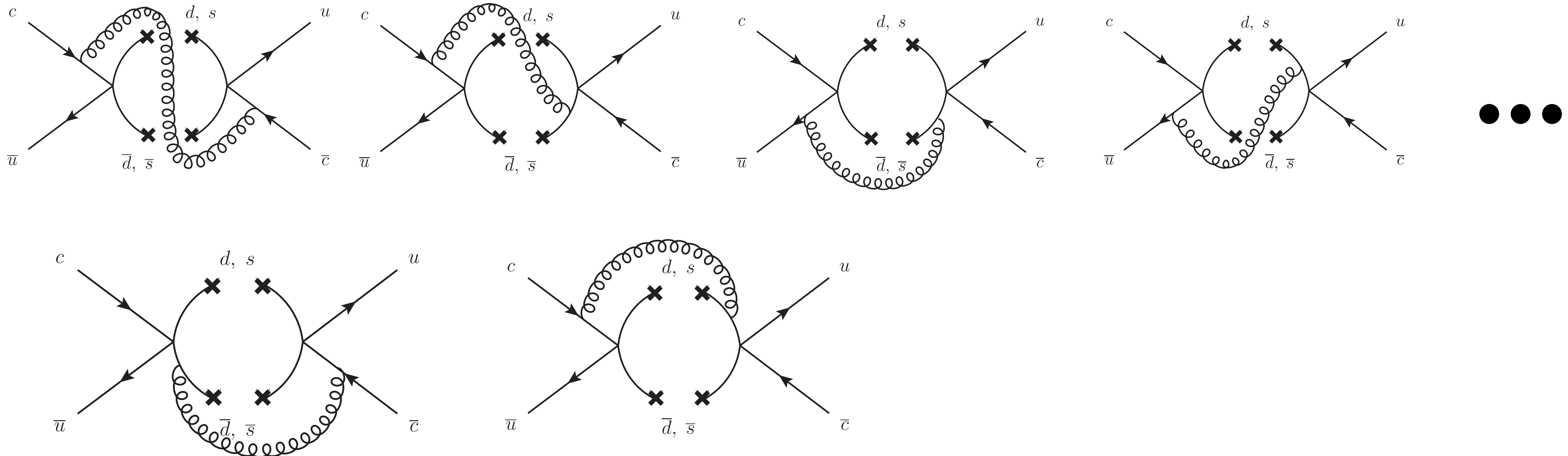
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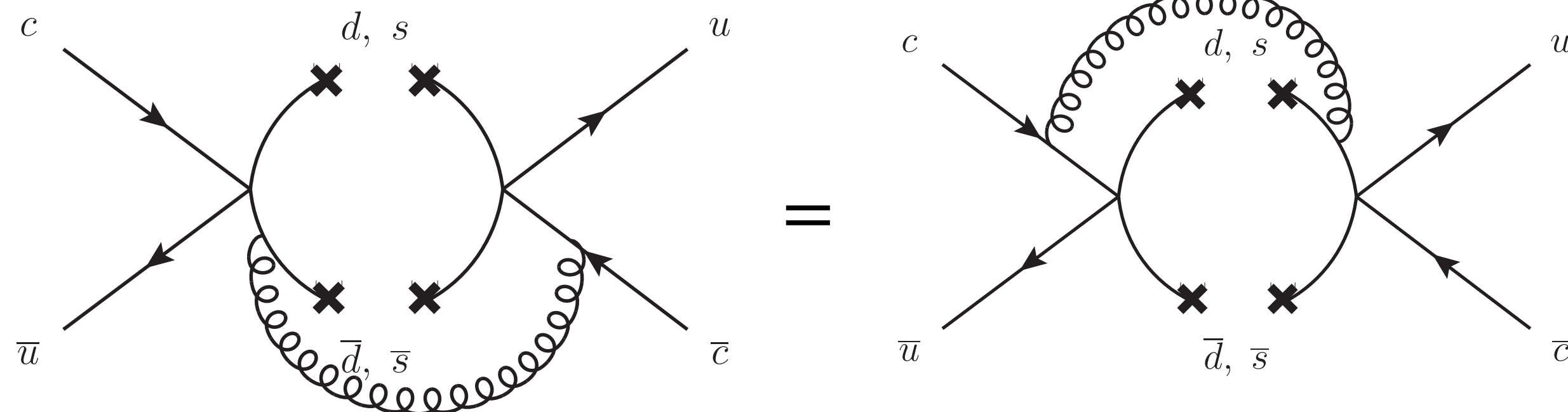
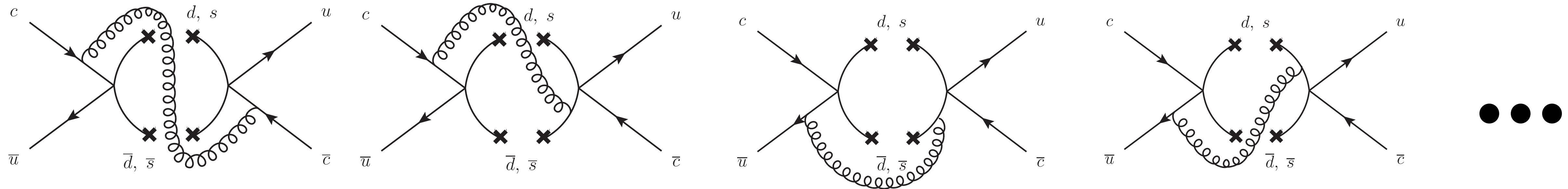
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some simplifications are possible
CP symmetry

Final results

Analytic results (leading contributions)

Final results

Analytic results (leading contributions)

Perturbative contribution

$$x_D^{\text{LO}} = -\frac{G_F^2}{2} \frac{m_c^2}{2M_D \Gamma_D} \xi_s^2 m_c^2 \frac{m_s^4}{m_c^4} \frac{1}{2\pi^2} \left(-C_1^2 (2\langle O_{S-P} \rangle + \langle O_{V-A} \rangle) + 4C_1 C_2 \langle O_{S-P} \rangle + 6C_2^2 \langle O_{S-P} \rangle \right)$$

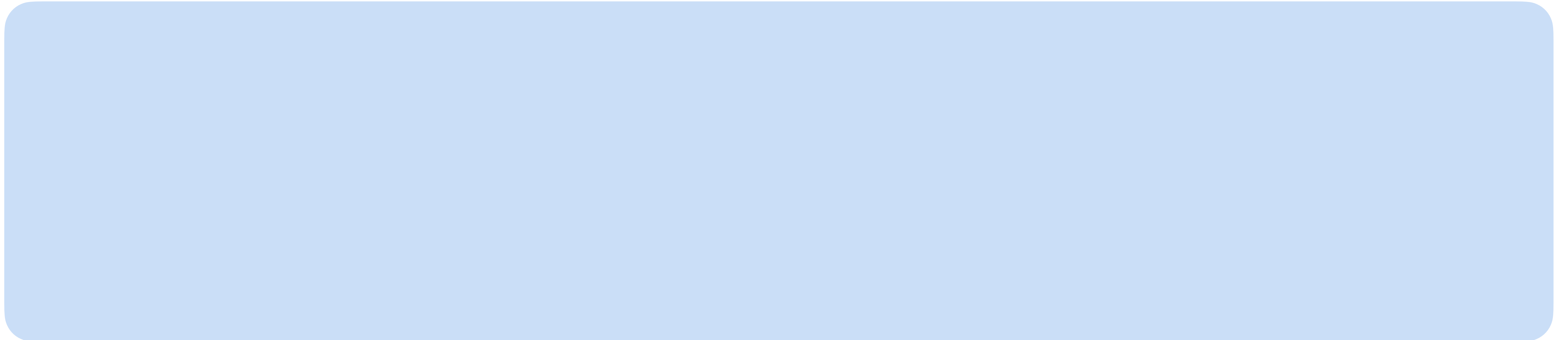
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Condensate contributions



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$$x_D^{\langle \bar{q}q \rangle} = -\frac{G_F^2}{2} \frac{m_c^2}{2M_D \Gamma_D} \xi_s^2 \frac{\langle \bar{s}s \rangle}{m_c^3} \frac{m_s^3}{m_c^3} \frac{2}{3} (C_1^2 - 2C_1 C_2 - 3C_2^2) (4\langle O_{S-P} \rangle + \langle O_{V-A} \rangle)$$

Final results

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$$x_D^{\langle \bar{q}q \rangle} = -\frac{G_F^2}{2} \frac{m_c^2}{2M_D \Gamma_D} \xi_s^2 \frac{\langle \bar{s}s \rangle}{m_c^3} \frac{m_s^3}{m_c^3} \frac{2}{3} (C_1^2 - 2C_1 C_2 - 3C_2^2) (4\langle O_{S-P} \rangle + \langle O_{V-A} \rangle)$$

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$$x_D^{\langle \bar{q}q\bar{q}q \rangle} = -\frac{G_F^2}{2} \frac{m_c^2}{2M_D \Gamma_D} \xi_s^2 \frac{\langle \bar{s}s \rangle^2}{m_c^6} \frac{m_s^2}{m_c^2} \pi \alpha_s(\mu) \frac{1}{108} \left((104\langle O_{S-P} \rangle - 27\langle O_{V-A} \rangle) C_1^2 + 4C_1 C_2 (904\langle O_{S-P} \rangle + 129\langle O_{V-A} \rangle) + 9(440\langle O_{S-P} \rangle + 137\langle O_{V-A} \rangle) C_2^2 \right)$$

Final results

Analytic results (leading contributions)

Perturbative contribution

$$x_D^{\text{LO}} = -\frac{G_F^2}{2} \frac{m_c^2}{2M_D \Gamma_D} \xi_s^2 m_c^2 \frac{m_s^4}{m_c^4} \frac{1}{2\pi^2} \left(-C_1^2 (2\langle O_{S-P} \rangle + \langle O_{V-A} \rangle) + 4C_1 C_2 \langle O_{S-P} \rangle + 6C_2^2 \langle O_{S-P} \rangle \right)$$

matrix elements

$$\langle O_{V-A} \rangle = \frac{8}{3} f_D^2 B_D M_D^2 \quad \langle O_{S-P} \rangle = -\frac{5}{3} f_D^2 B_D^{(S)} \frac{M_D^2}{m_c^2} M_D^2$$

Condensate contributions

$$x_D^{\langle \bar{q}q \rangle} = -\frac{G_F^2}{2} \frac{m_c^2}{2M_D \Gamma_D} \xi_s^2 \frac{\langle \bar{s}s \rangle}{m_c^3} \frac{m_s^3}{m_c^3} \frac{2}{3} (C_1^2 - 2C_1 C_2 - 3C_2^2) (4\langle O_{S-P} \rangle + \langle O_{V-A} \rangle)$$

$$x_D^{\langle \bar{q}Gq \rangle} = -\frac{G_F^2}{2} \frac{m_c^2}{2M_D \Gamma_D} \xi_s^2 \left(\frac{\lambda_q^2}{m_c^2} \frac{\langle \bar{s}s \rangle}{m_c^3} \frac{m_s^3}{m_c^3} + \frac{8}{9} \pi \alpha_s^{NP} \frac{\langle \bar{s}s \rangle^2 - \langle \bar{d}d \rangle^2}{m_c^6} \frac{m_s^2}{m_c^2} \right) \frac{1}{6} (C_1^2 \langle O_{V-A} \rangle + 8\langle O_{S-P} \rangle (C_1^2 + 4C_1 C_2 + 6C_2^2))$$

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Final results

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matrix elements

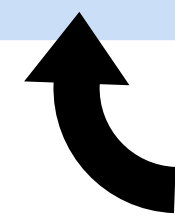
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Condensate contributions

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$$x_D^{\langle \bar{q}q\bar{q}q \rangle} = -\frac{G_F^2}{2} \frac{m_c^2}{2M_D \Gamma_D} \xi_s^2 \frac{\langle \bar{s}s \rangle^2}{m_c^6} \frac{m_s^2}{m_c^2} \pi \alpha_s(\mu) \frac{1}{108} ((104\langle O_{S-P} \rangle - 27\langle O_{V-A} \rangle) C_1^2 + 4C_1 C_2 (904\langle O_{S-P} \rangle + 129\langle O_{V-A} \rangle) + 9(440\langle O_{S-P} \rangle + 137\langle O_{V-A} \rangle) C_2^2)$$



parametrically leading!

Final results

Analytic results (leading contributions)

Perturbative contribution

$$x_D^{\text{LO}} = -\frac{G_F^2}{2} \frac{m_c^2}{2M_D \Gamma_D} \xi_s^2 m_c^2 \frac{m_s^4}{m_c^4} \frac{1}{2\pi^2} \left(-C_1^2 (2\langle O_{S-P} \rangle + \langle O_{V-A} \rangle) + 4C_1 C_2 \langle O_{S-P} \rangle + 6C_2^2 \langle O_{S-P} \rangle \right)$$

matrix elements

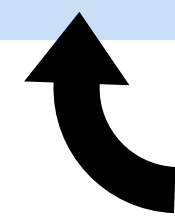
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Condensate contributions

$$x_D^{\langle \bar{q}q \rangle} = -\frac{G_F^2}{2} \frac{m_c^2}{2M_D \Gamma_D} \xi_s^2 \frac{\langle \bar{s}s \rangle}{m_c^3} \frac{m_s^3}{m_c^3} \frac{2}{3} (C_1^2 - 2C_1 C_2 - 3C_2^2) (4\langle O_{S-P} \rangle + \langle O_{V-A} \rangle)$$

$$x_D^{\langle \bar{q}Gq \rangle} = -\frac{G_F^2}{2} \frac{m_c^2}{2M_D \Gamma_D} \xi_s^2 \left(\frac{\lambda_q^2}{m_c^2} \frac{\langle \bar{s}s \rangle}{m_c^3} \frac{m_s^3}{m_c^3} + \frac{8}{9} \pi \alpha_s^{NP} \frac{\langle \bar{s}s \rangle^2 - \langle \bar{d}d \rangle^2}{m_c^6} \frac{m_s^2}{m_c^2} \right) \frac{1}{6} (C_1^2 \langle O_{V-A} \rangle + 8\langle O_{S-P} \rangle (C_1^2 + 4C_1 C_2 + 6C_2^2))$$

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parametrically leading!

Falk, Grossman, Ligeti, Petrov, *SU(3) breaking and D0 - anti-D0 mixing*, Phys. Rev. D65 (2002) 054034, [hep-ph/0110317]

Georgi, *D - anti-D mixing in heavy quark effective field theory*, Phys. Lett. B 297 (1992) 353–357, [hep-ph/9209291]

Final results

Parameters used in evaluation

Final results

Parameters used in evaluation

Matrix elements	Bazavov et al. , Short-distance matrix elements for D0-meson mixing for Nf = 2 + 1 lattice QCD, Phys. Rev. D 97 (2018) 034513, [1706.04622] .
$\langle O_{V-A} \rangle = \left(\bar{u}^i \gamma^\mu (1 - \gamma^5) q^i \right) \left(\bar{q}^j \gamma_\mu (1 - \gamma^5) c^j \right)$	
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The condensate	Gell-Mann-Oakes-Renner relation Reinders, Rubinstein, Yazaki , <i>Hadron Properties from QCD Sum Rules</i> , Phys. Rept., 127:1, 1985
$\langle \bar{q}q \rangle_0 = -\frac{f_\pi M_\pi}{2} \frac{1}{m_d + m_u}$	$\frac{\langle \bar{s}s \rangle}{\langle \bar{d}d \rangle} = 0.8 \pm 0.1$

Final results

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Other parameters	Particle Data Group Collaboration, Review of particle physics, Phys. Rev. D 110 (2024) 030001	

parameter	value
$\xi_s = V_{us}^* V_{cs}$	$(0.22501 \pm 0.00068)(0.97349 \pm 0.00016)$
G_F	$1.1663788(6) \times 10^{-5} \text{ GeV}^{-2}$
$\overline{\text{MS}} m_c(m_c)$	$1.2730 \pm 0.0046 \text{ GeV}$
$\overline{\text{MS}} m_s(2 \text{ GeV})$	$93.5 \pm 0.8 \text{ MeV}$
M_D	$1864.84 \pm 0.05 \text{ GeV}$
Γ_D	$1.604 \times 10^{-12} \text{ GeV}$
m_d	$0.00470 \pm 0.00007 \text{ GeV}$
$O_{V-A}(3 \text{ GeV})$	0.322 ± 0.022
$O_{S-P}(3 \text{ GeV})$	-0.624 ± 0.007
$\langle \bar{q}q \rangle(1 \text{ GeV})$	$(-263.5 \text{ MeV})^3$

Final results

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We run all parameters to renormalization scale $\mu = 1.3 \text{ GeV}$ and evaluate our results.

Final results

Parameters used in evaluation

Matrix elements

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This is most widely used, but more recent lattice measurements give larger results which disagree with this one

- Phys.Rev.D 87 (2013) 3, 034503
- Phys.Rev.D 100 (2019) 3, 034506

The condensate

Gell-Mann-Oakes-Renner relation
Reinders, Rubinstein, Yazaki, Hadron Properties from QCD Sum Rules, Phys. Rept., 127:1, 1985

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Other parameters

Particle Data Group Collaboration, Review of particle physics, Phys. Rev. D 110 (2024) 030001

Running

Chetyrkin, Kuhn, Steinhauser, RunDec: A Mathematica package for running and decoupling of the strong coupling and quark masses, Comput. Phys. Commun. 133 (2000) 43–65, [hep-ph/0004189].

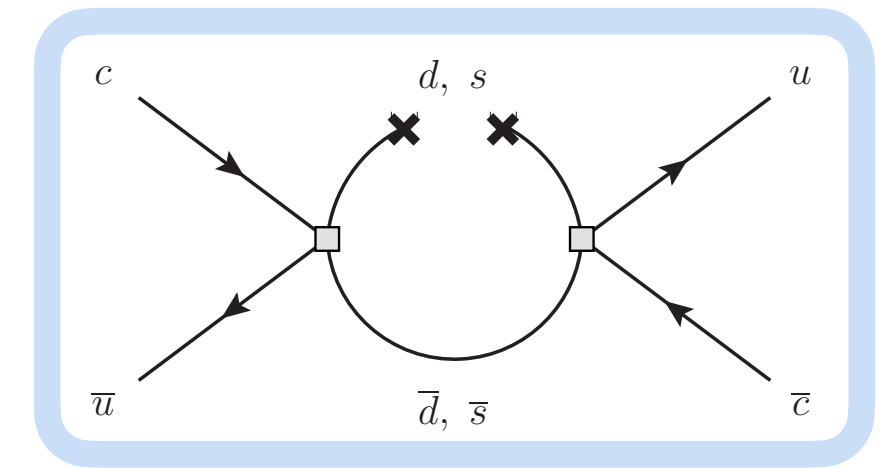
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Final results

Final results

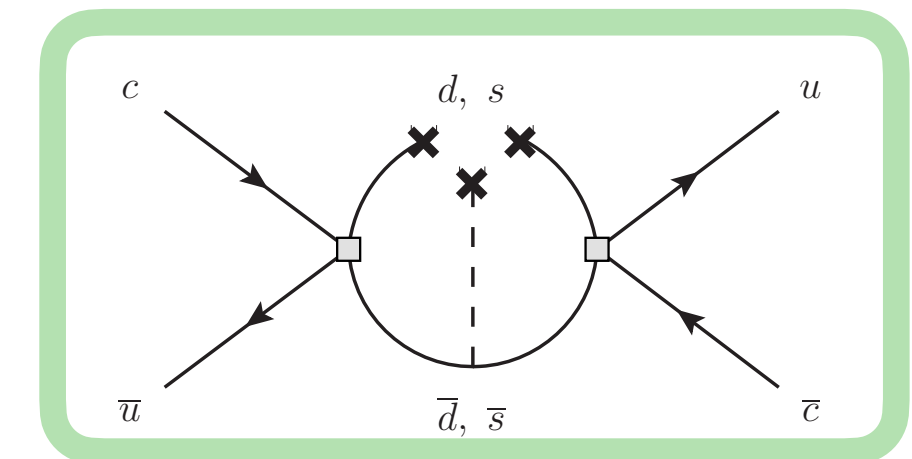
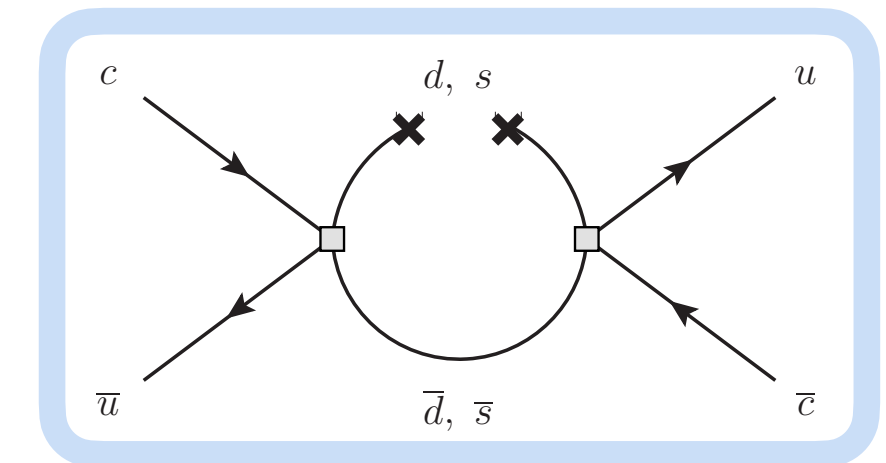
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Final results

$$x_D^{\langle \bar{q}q \rangle} = (0.99 \pm 0.13) \times 10^{-5}$$

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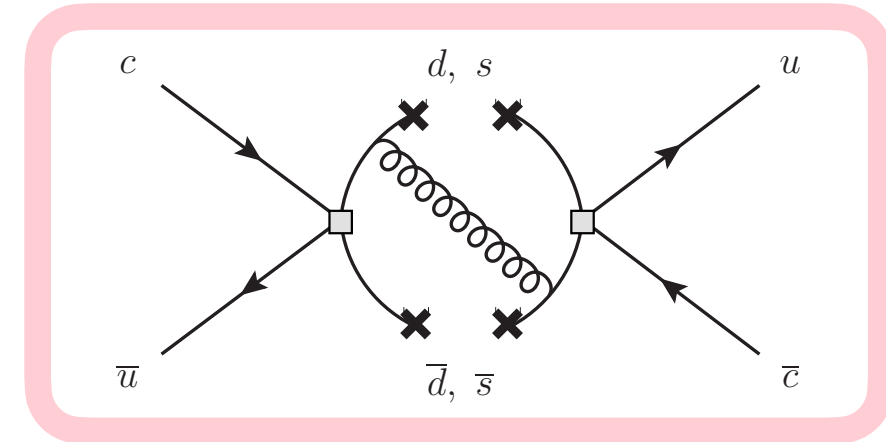
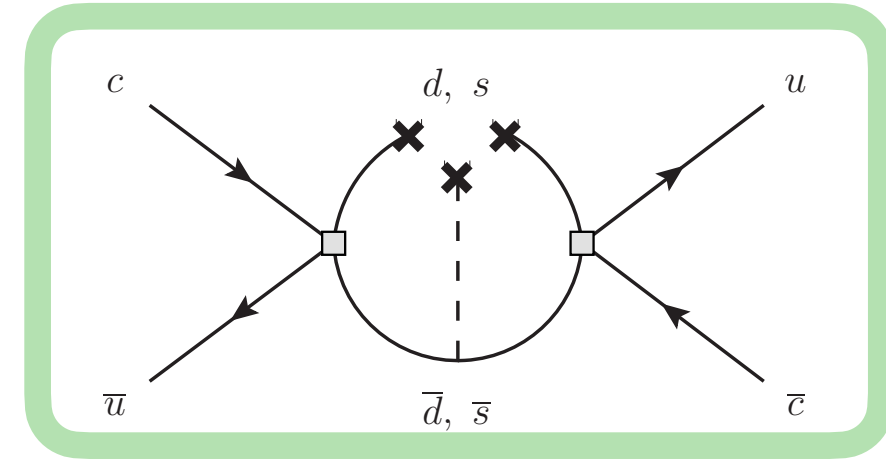
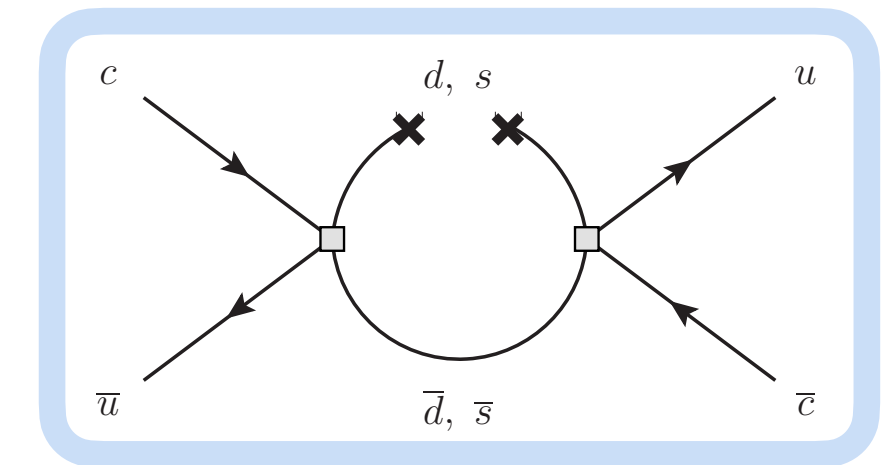


Final results

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$$x_D^{\langle \bar{q}Gq \rangle} = (0.051 \pm 0.009) \times 10^{-5}$$

$$x_D^{\langle \bar{q}q\bar{q}q \rangle} = (0.24 \pm 0.06) \times 10^{-5}$$



Final results

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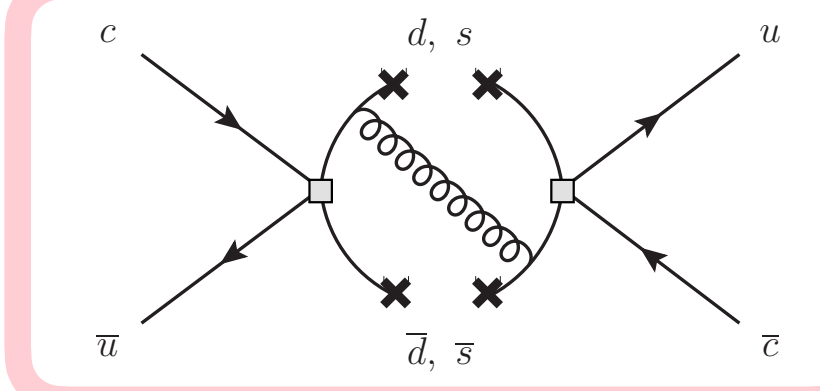
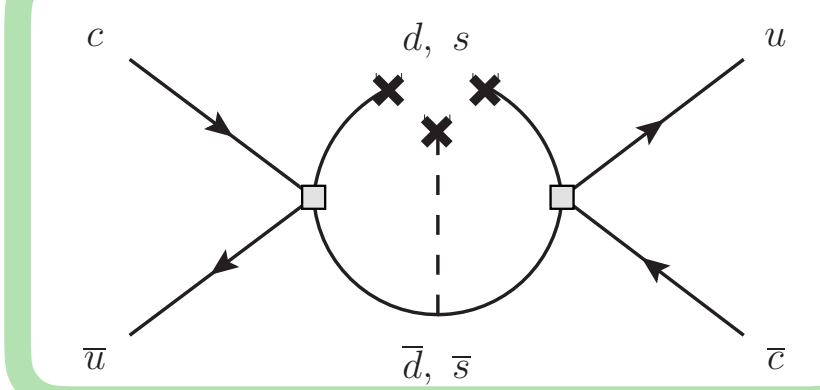
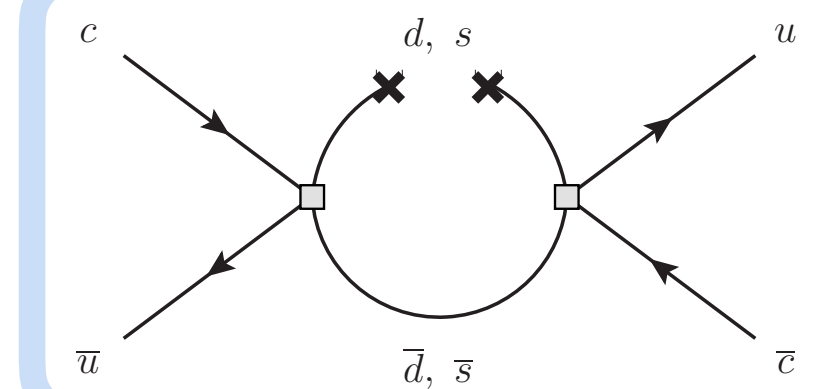
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=====

$$x_D^{NP} = (1.27 \pm 0.18) \times 10^{-5}$$

Total nonperturbative



Final results

$$x_D^{\langle \bar{q}q \rangle} = (0.99 \pm 0.13) \times 10^{-5}$$

$$x_D^{\langle \bar{q}Gq \rangle} = (0.051 \pm 0.009) \times 10^{-5}$$

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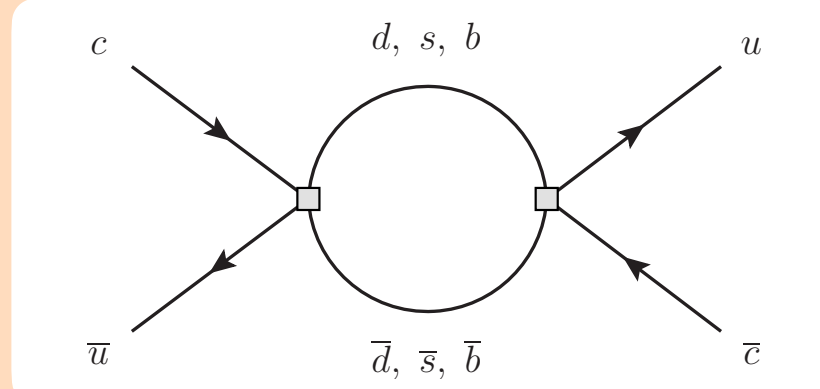
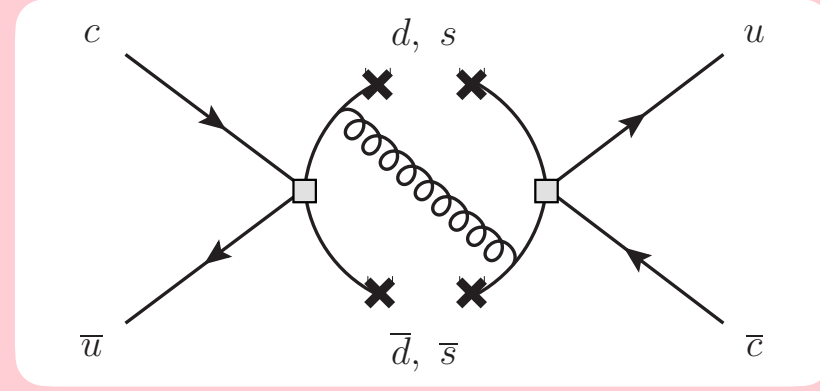
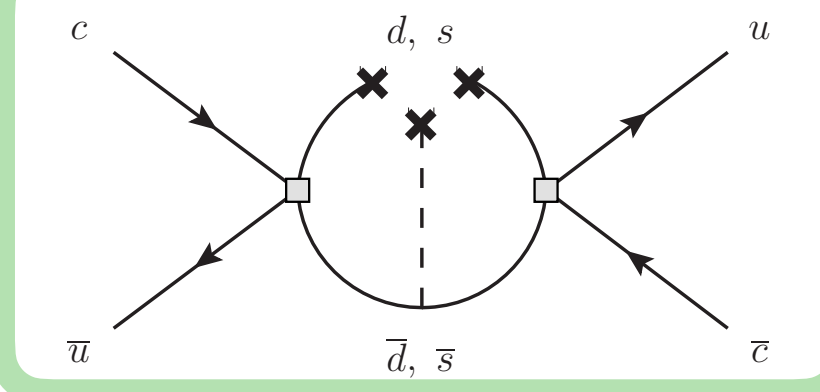
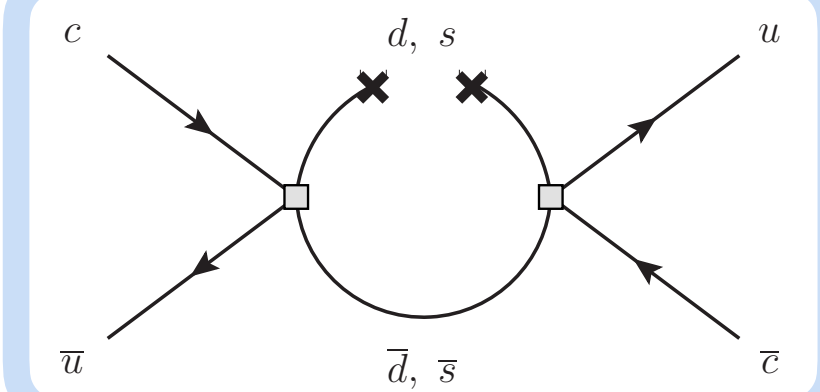
$$x_D^{NP} = (1.27 \pm 0.18) \times 10^{-5}$$

Total nonperturbative

Total perturbative

$$x_D^{LO+NLO} \approx 10^{-7}$$

Golowich, Petrov, Short distance analysis of $D\bar{D}$ mixing, [hep-ph/0506185]



Final results

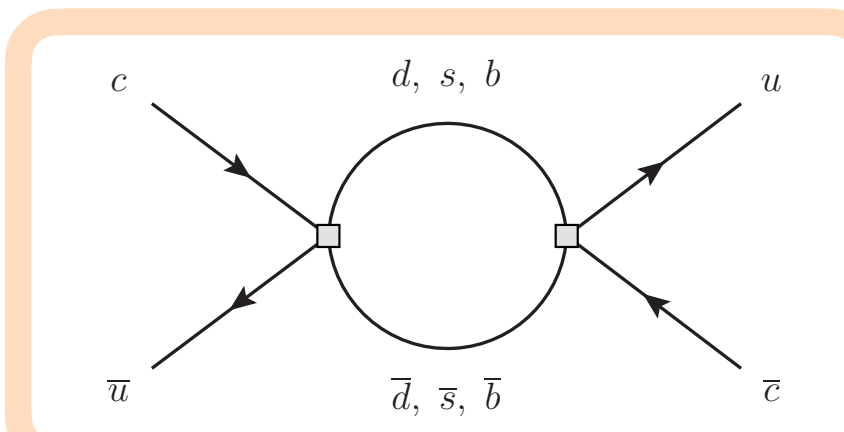
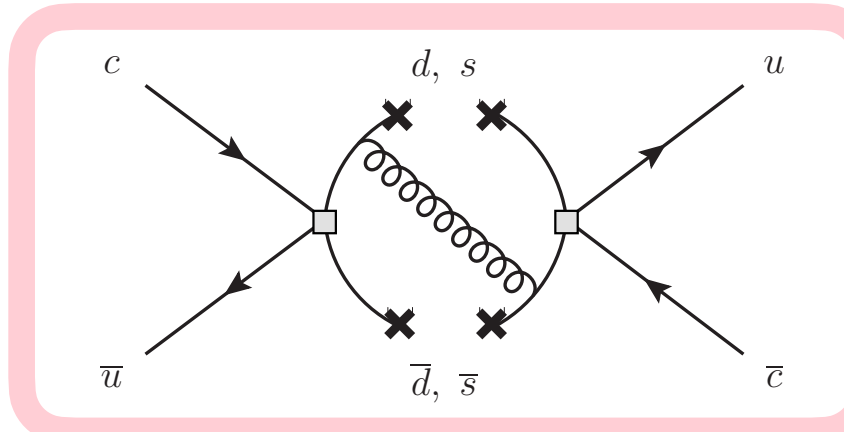
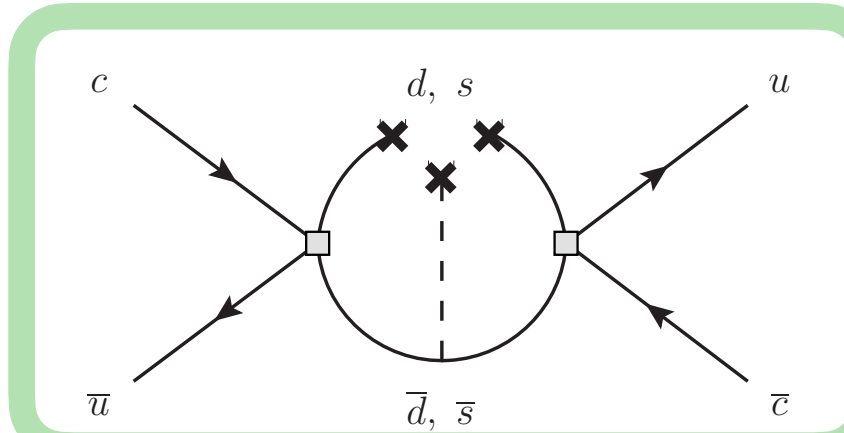
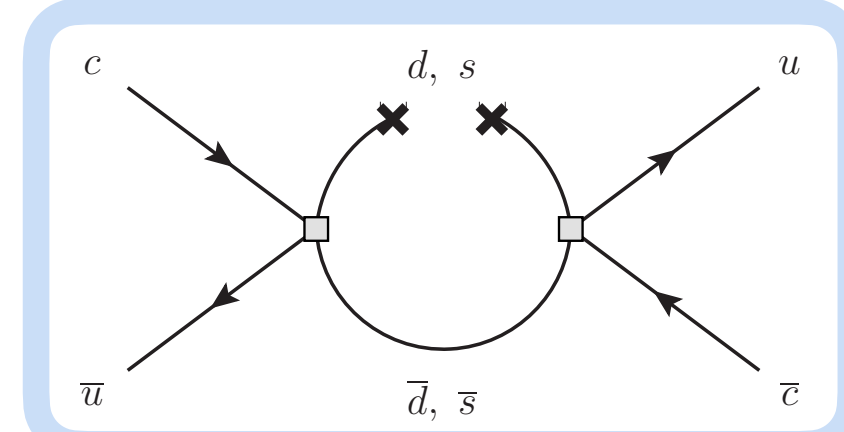
$$x_D^{\langle \bar{q}q \rangle} = (0.99 \pm 0.13) \times 10^{-5}$$

$$x_D^{\langle \bar{q}Gq \rangle} = (0.051 \pm 0.009) \times 10^{-5}$$

$$x_D^{\langle \bar{q}q\bar{q}q \rangle} = (0.24 \pm 0.06) \times 10^{-5}$$

$$x_D^{NP} = (1.27 \pm 0.18) \times 10^{-5}$$

Total nonperturbative



Total perturbative

$$x_D^{LO+NLO} \approx 10^{-7}$$

Golowich, Petrov, Short distance analysis of $D\bar{D}$ mixing, [hep-ph/0506185]

HFLAV, Averages of b -hadron, c -hadron, and τ -lepton properties as of 2023, [2411.18639]

Experiment

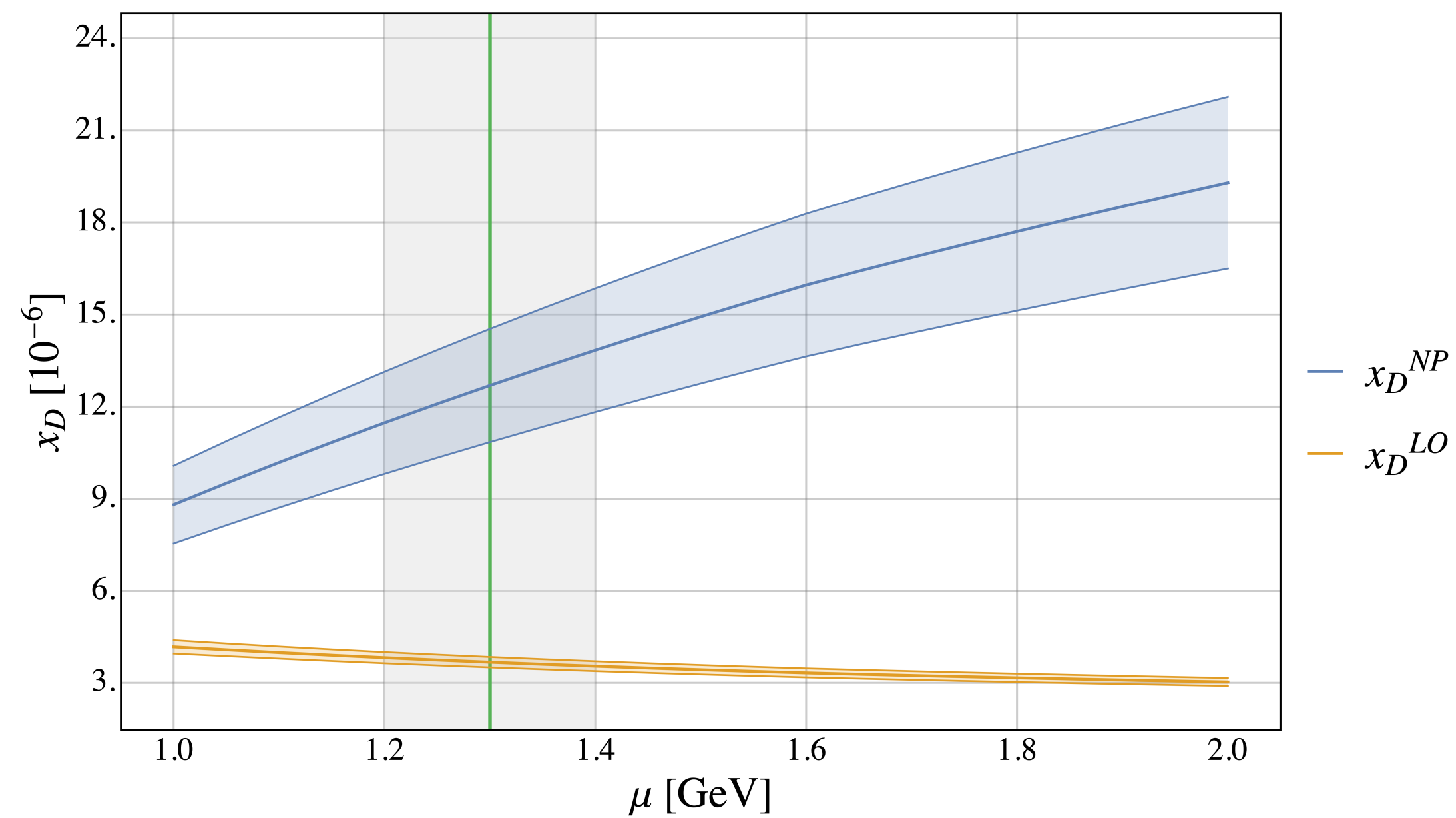
$$x^{EXP} = (4.07 \pm 0.44) \times 10^{-3}$$

Final results

Dependencies on parameters

Final results

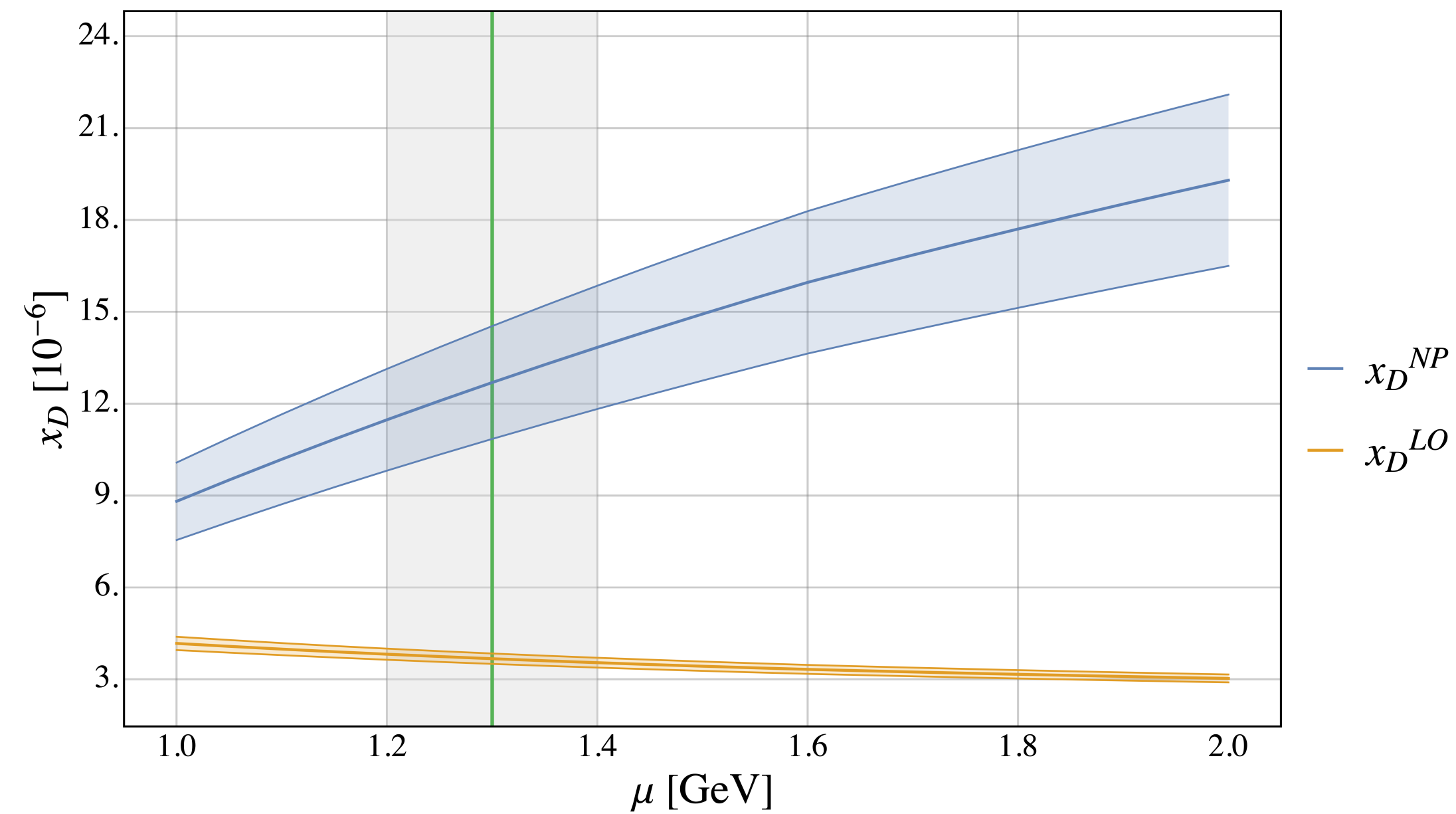
Dependencies on parameters



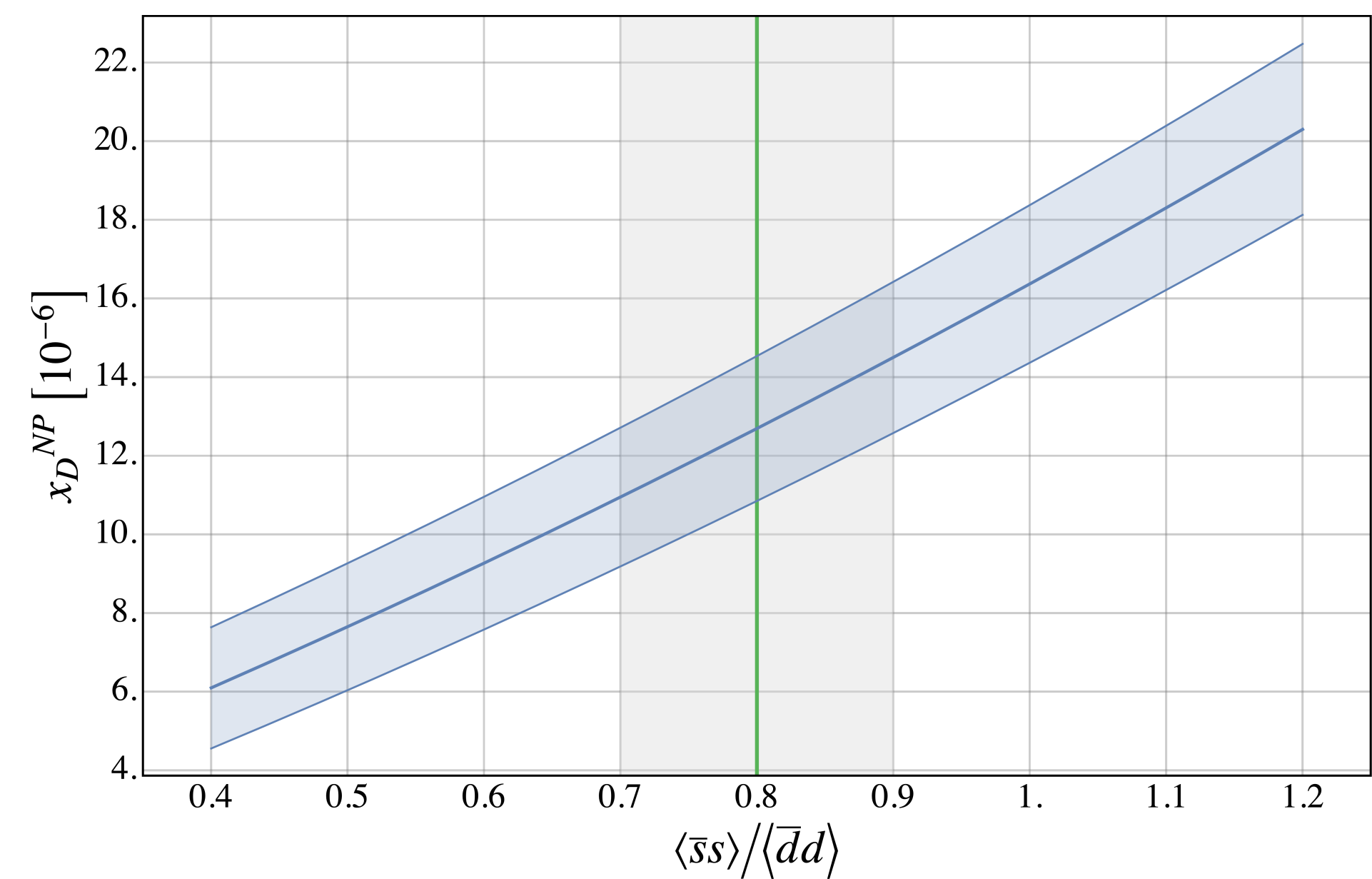
*strong dependence on renormalization scale
comes from the condensate*

Final results

Dependencies on parameters



*strong dependence on renormalization scale
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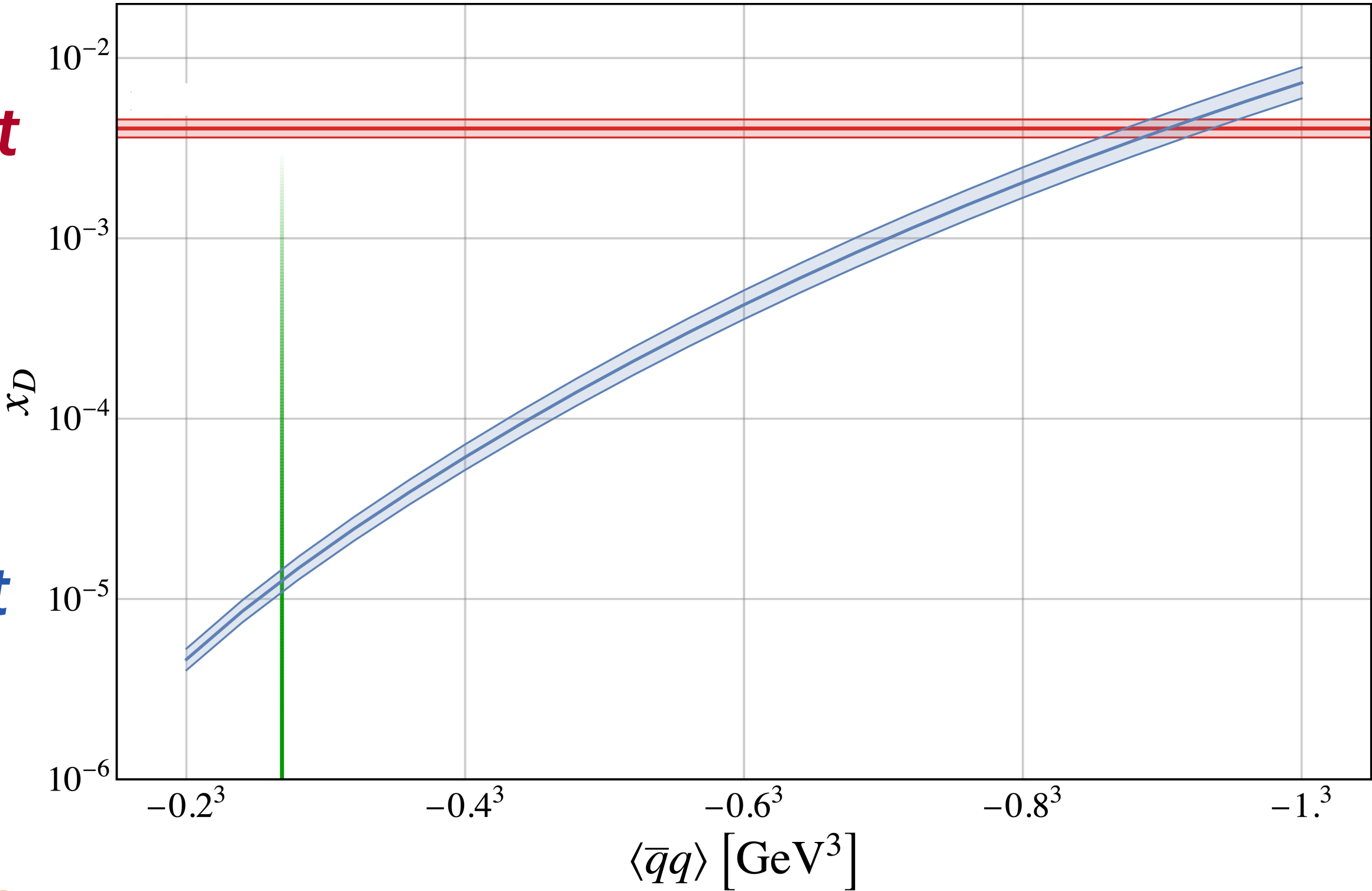
*strong dependence on strange-to-light condensate
ratio is explained by strong dependence on $\langle \bar{s}s \rangle$*

Summary

Experiment

Our result

*Perturbative
result*

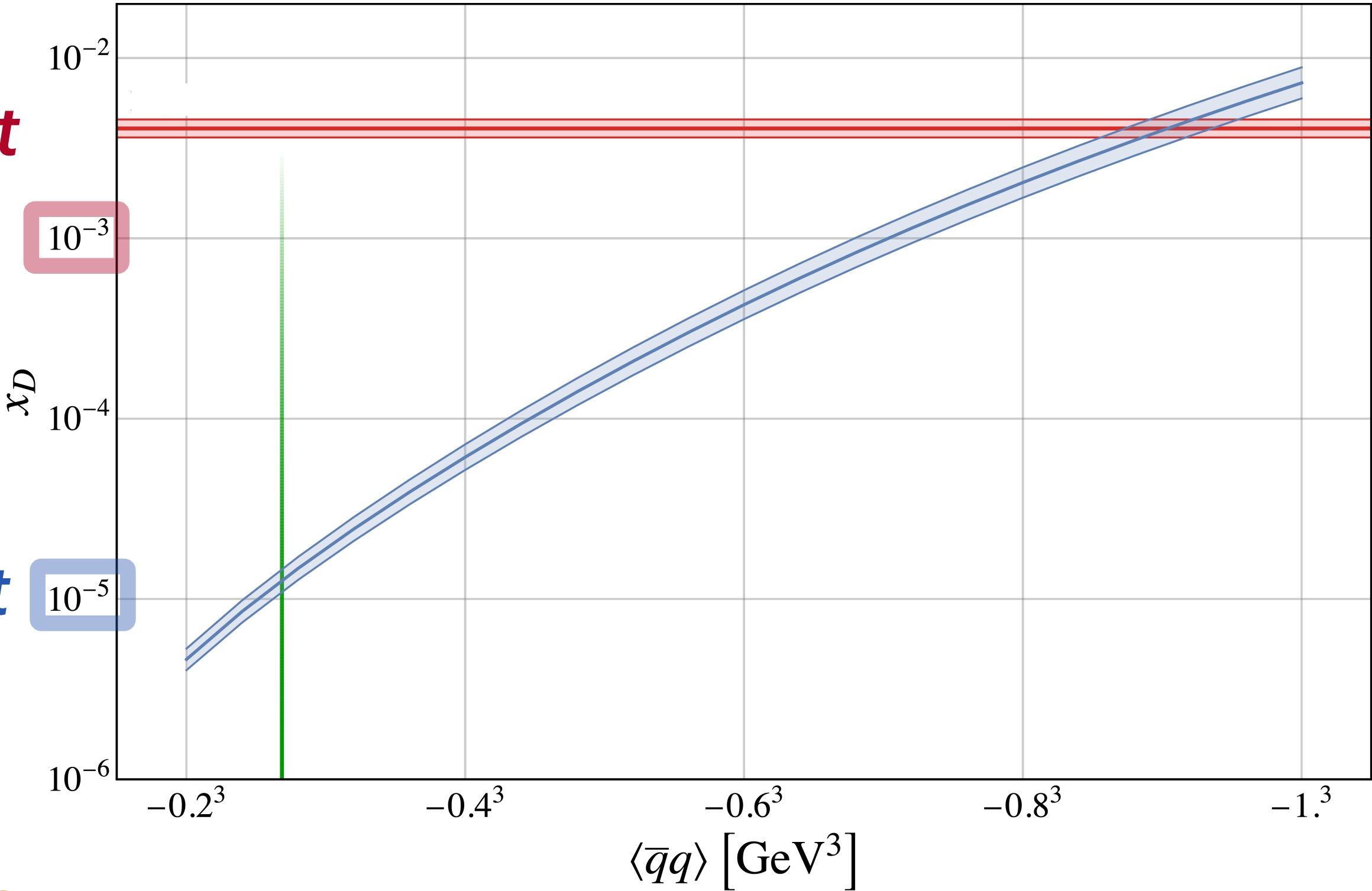


Summary

Experiment

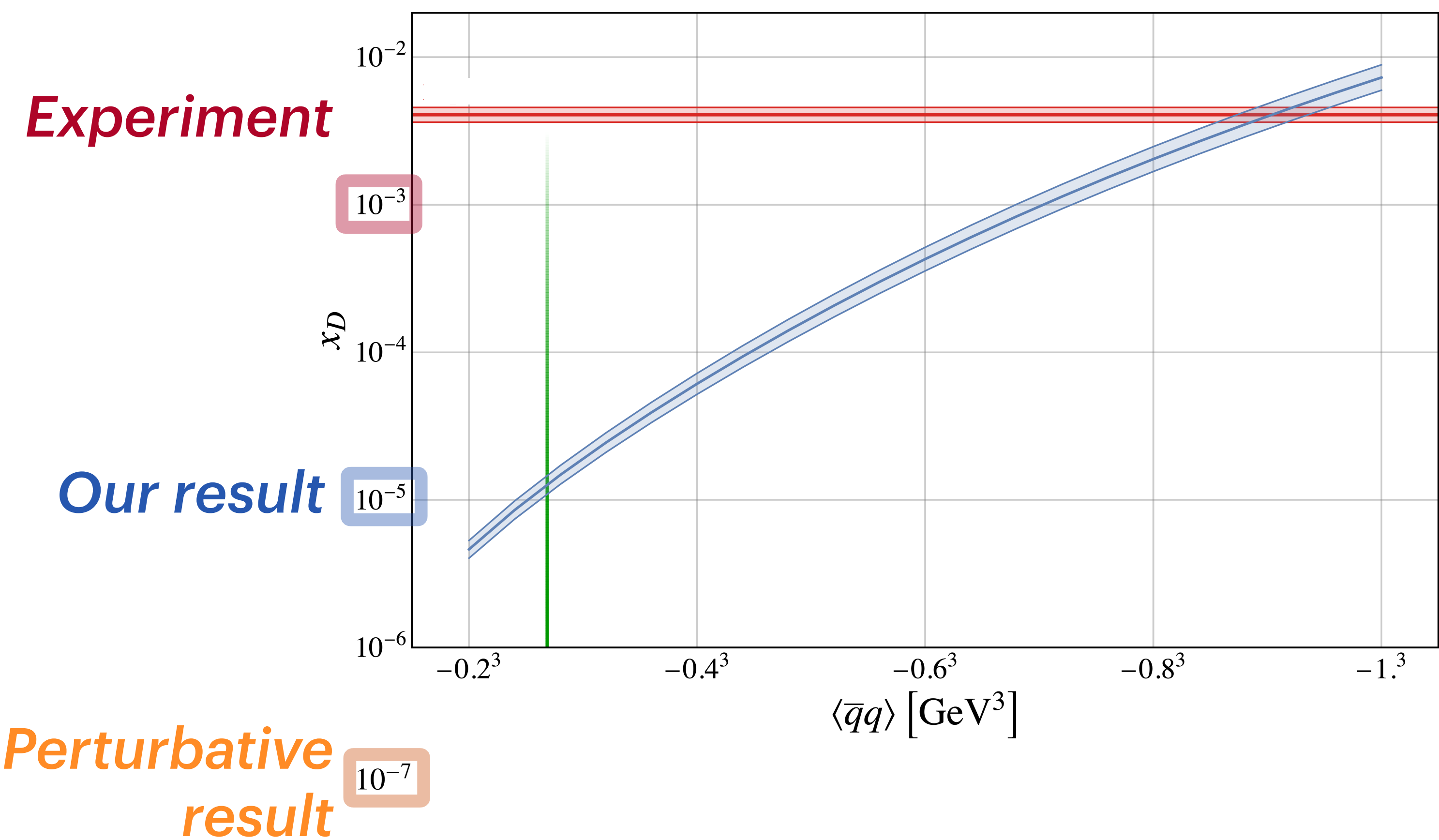
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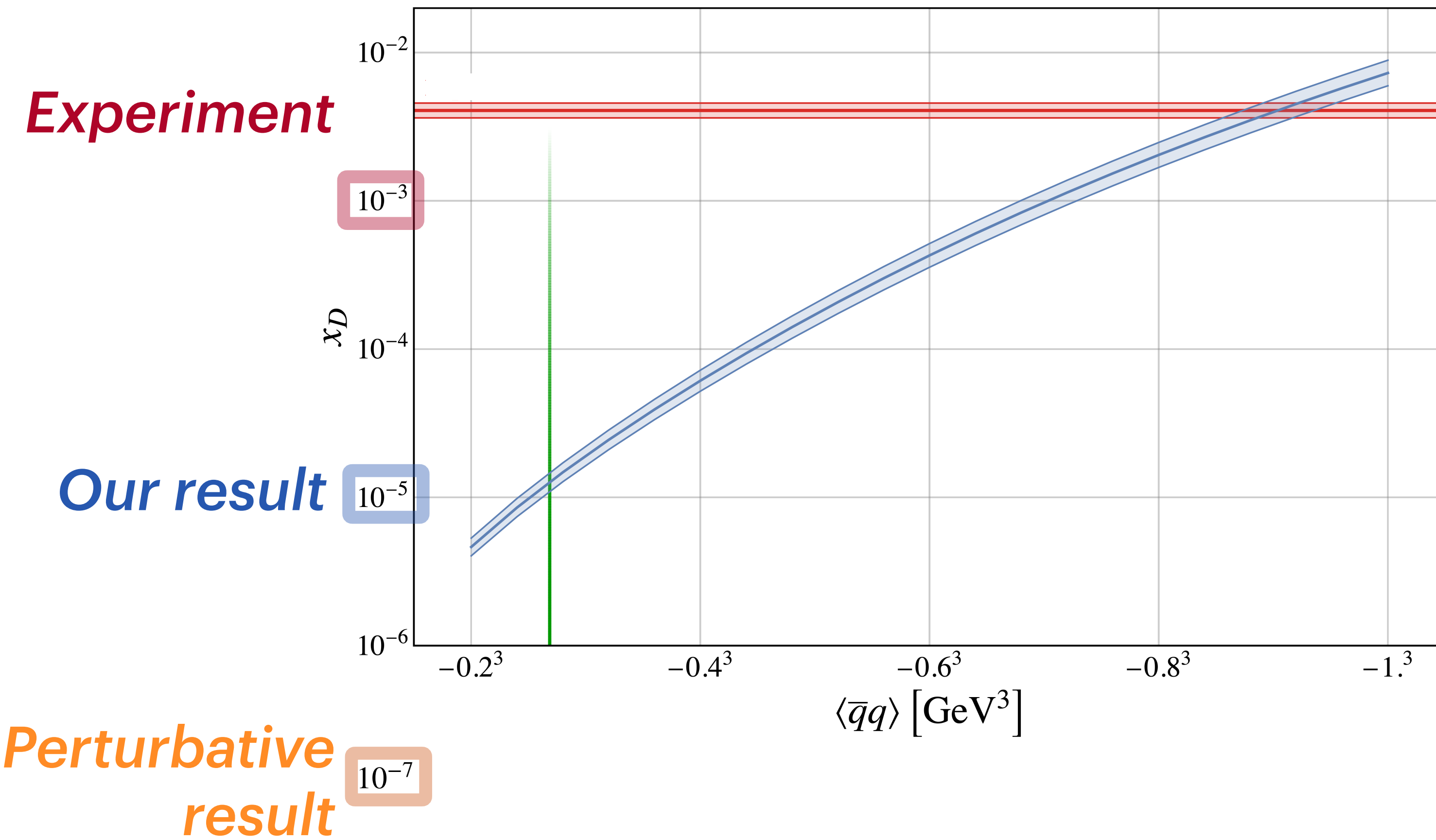


Summary

- Previous analyses point to $D\bar{D}$ mixing being within the Standard Model

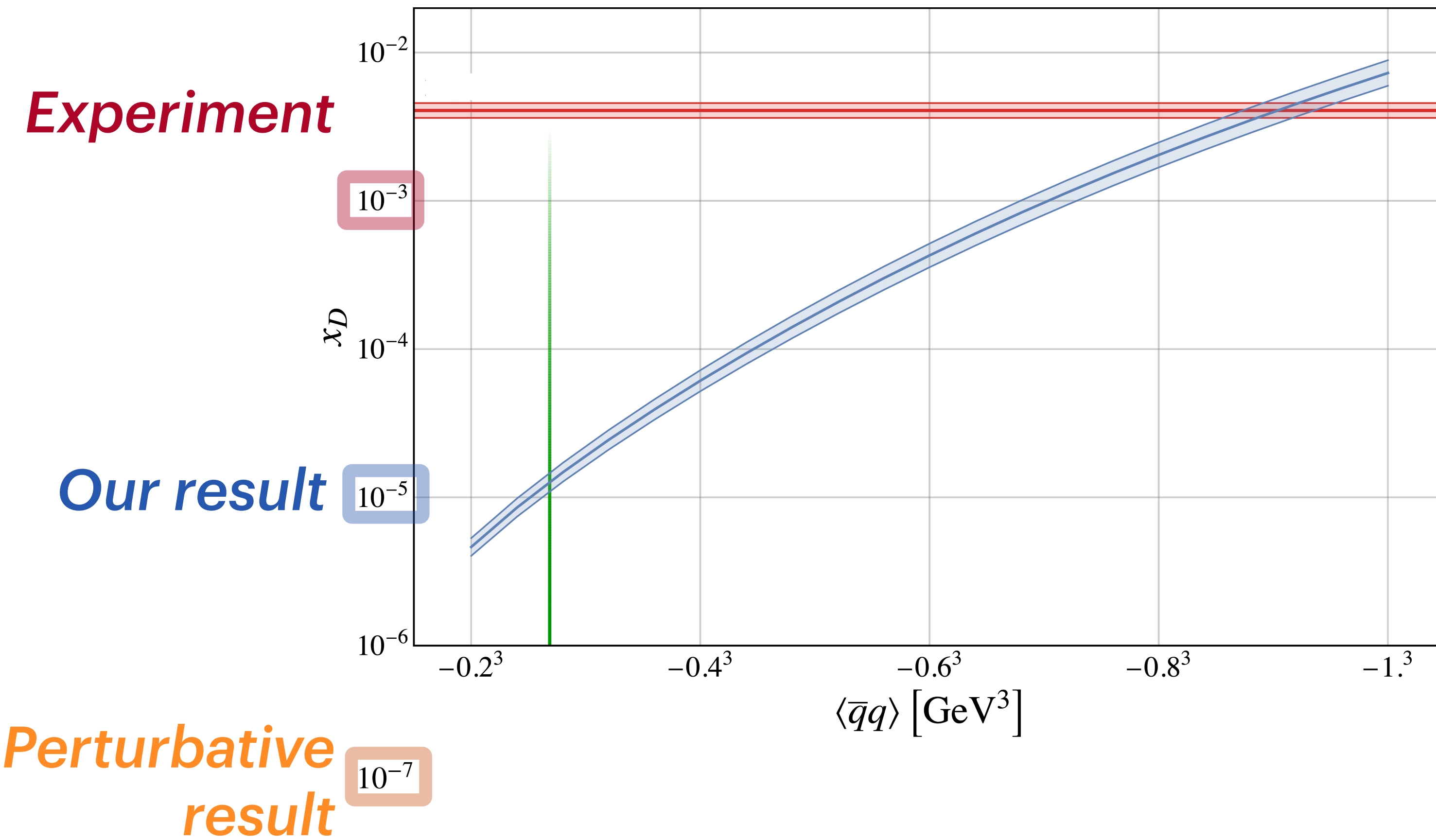


Summary



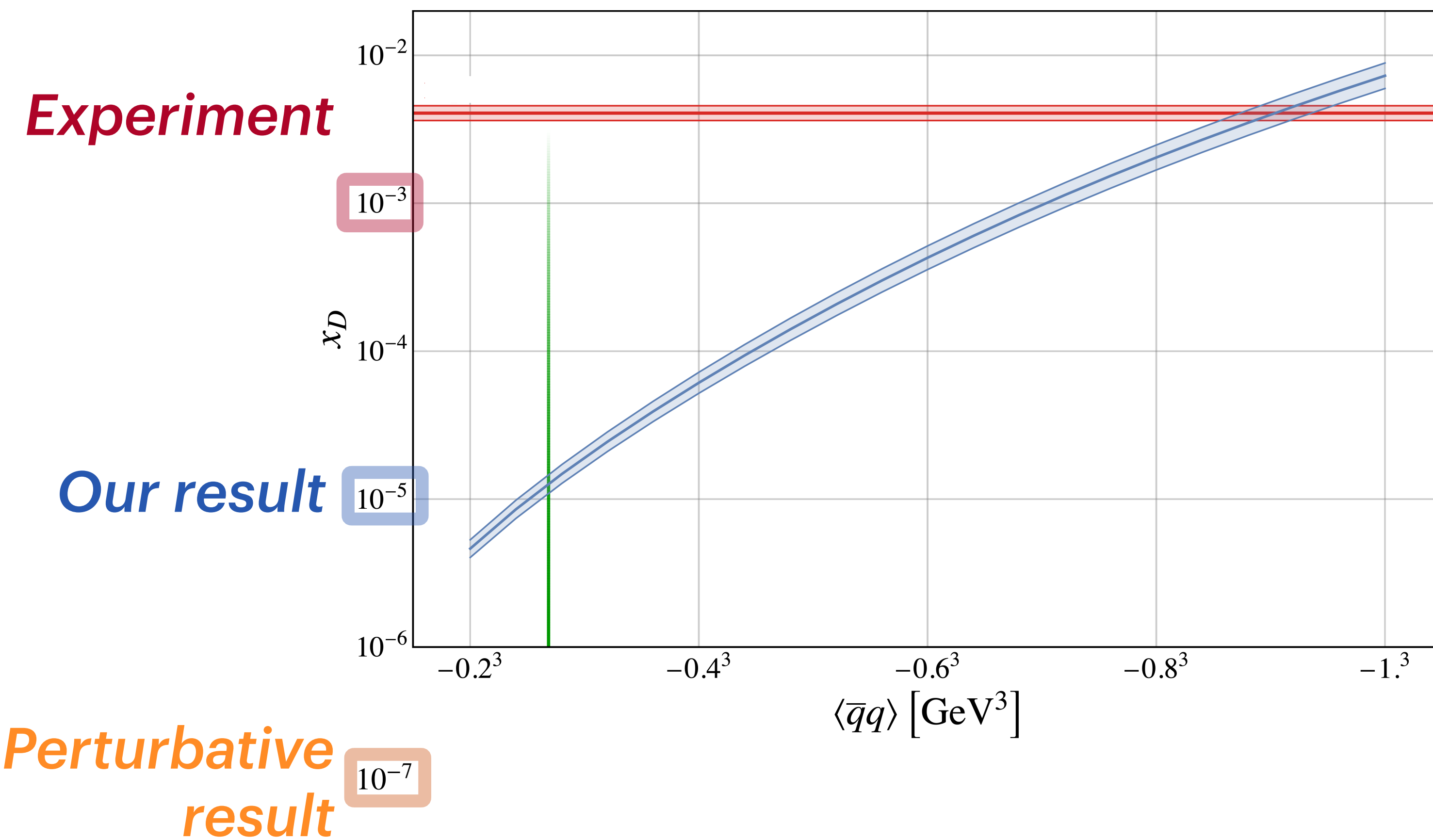
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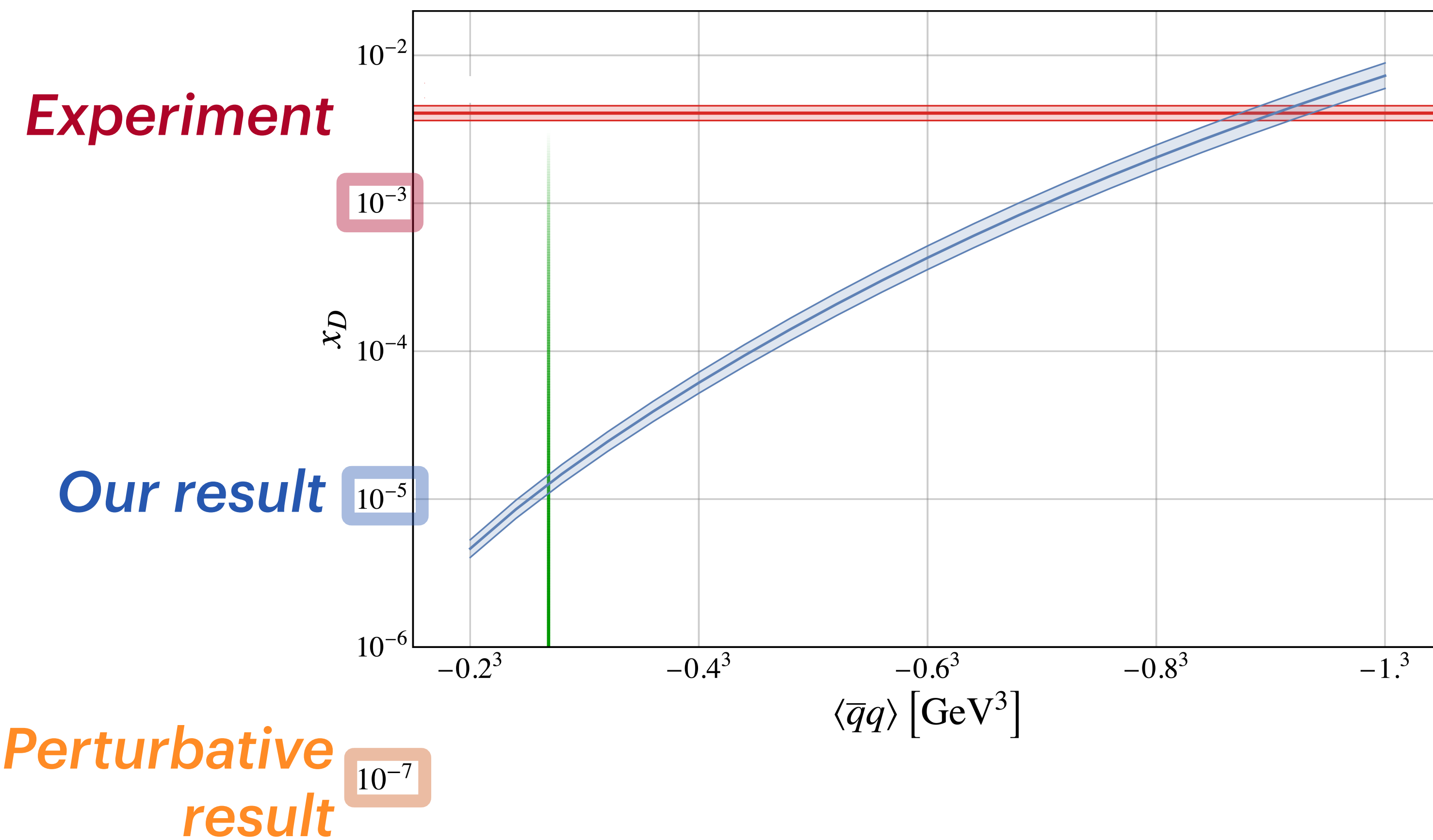
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- We introduce nonperturbative dynamics through nonlocal QCD condensates and

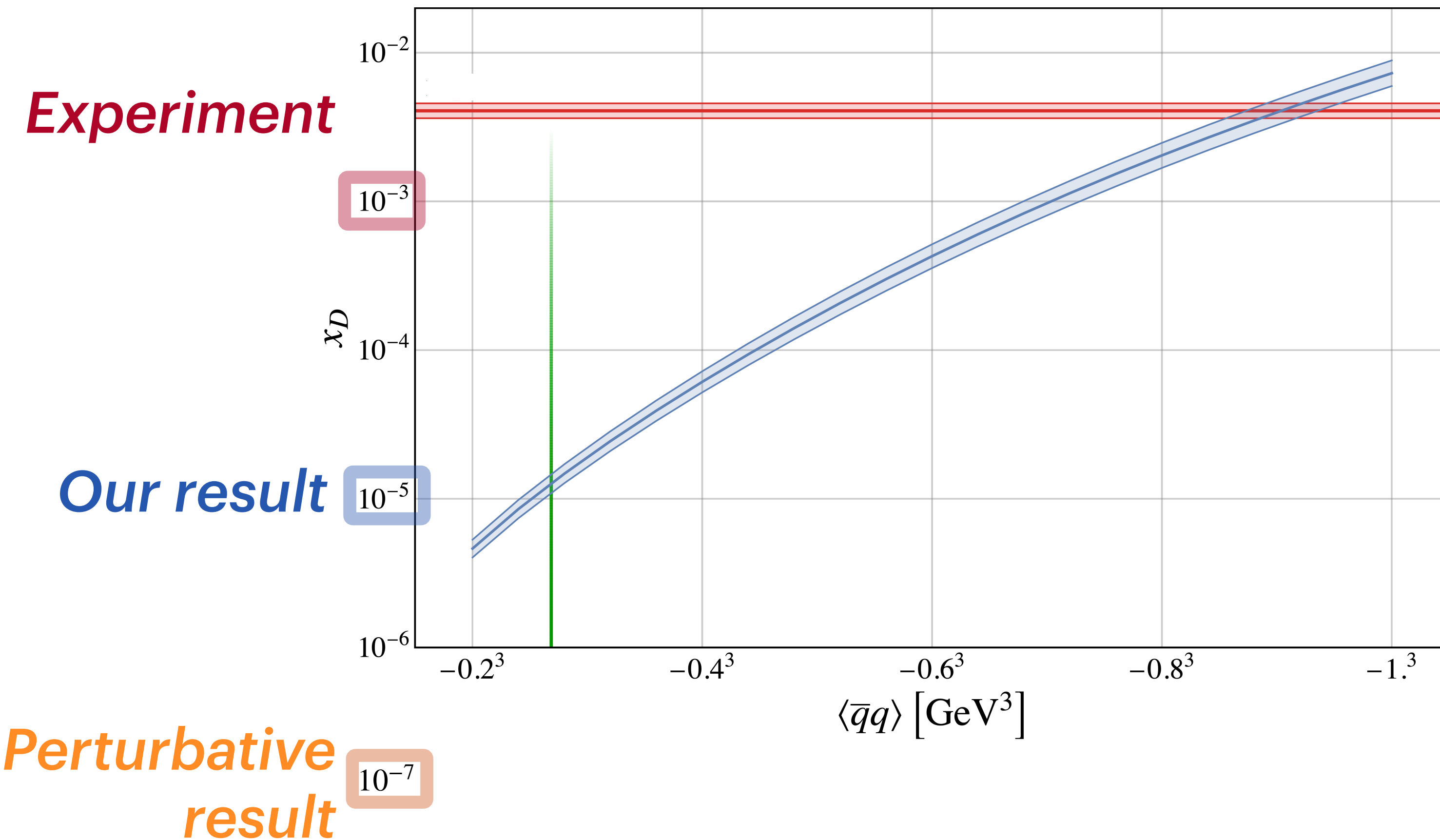
Summary



- Previous analyses point to $D\bar{D}$ mixing being within the Standard Model
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- We introduce nonperturbative dynamics through nonlocal QCD condensates and
 - provide a comprehensive and detailed general analysis

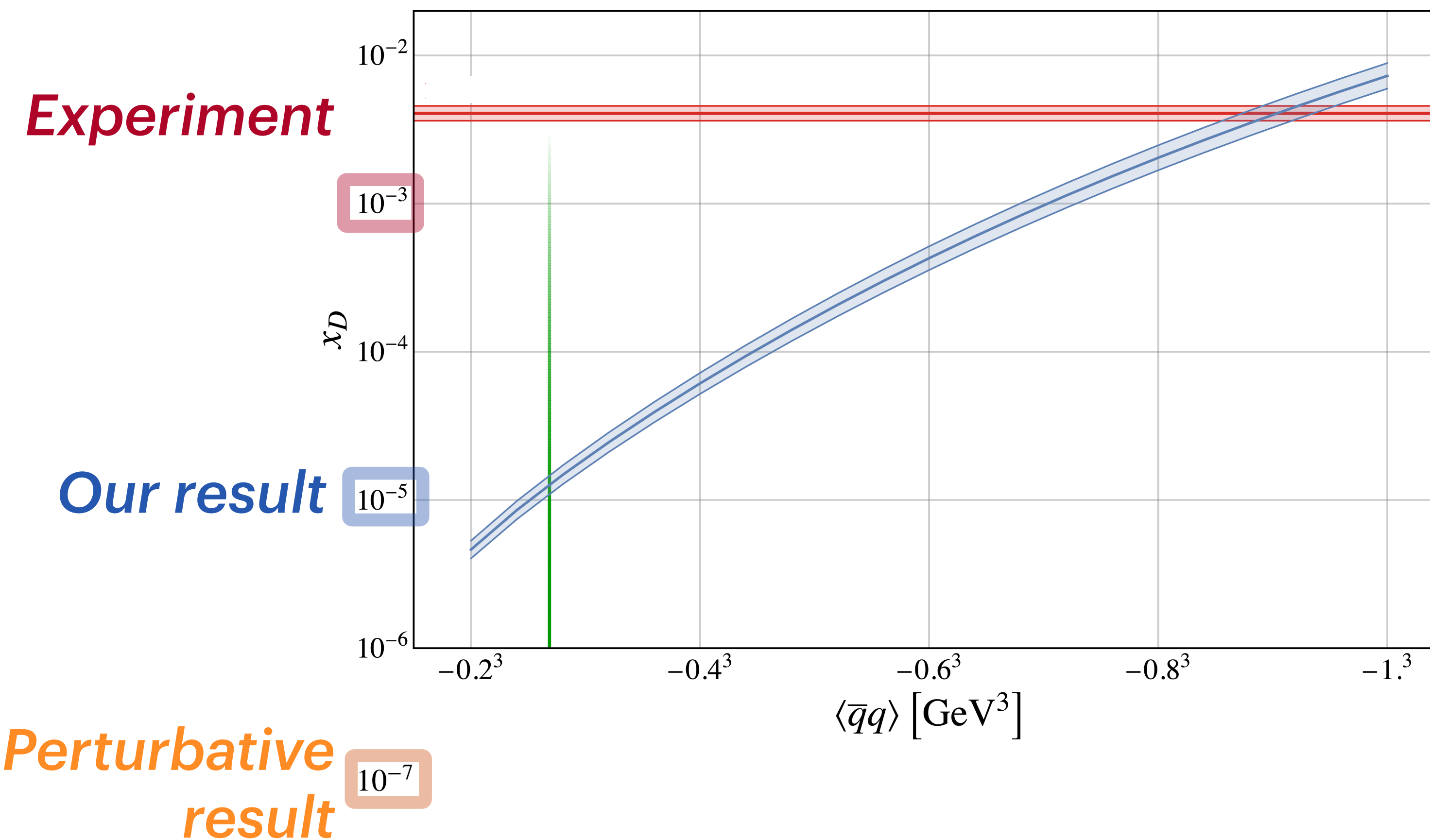
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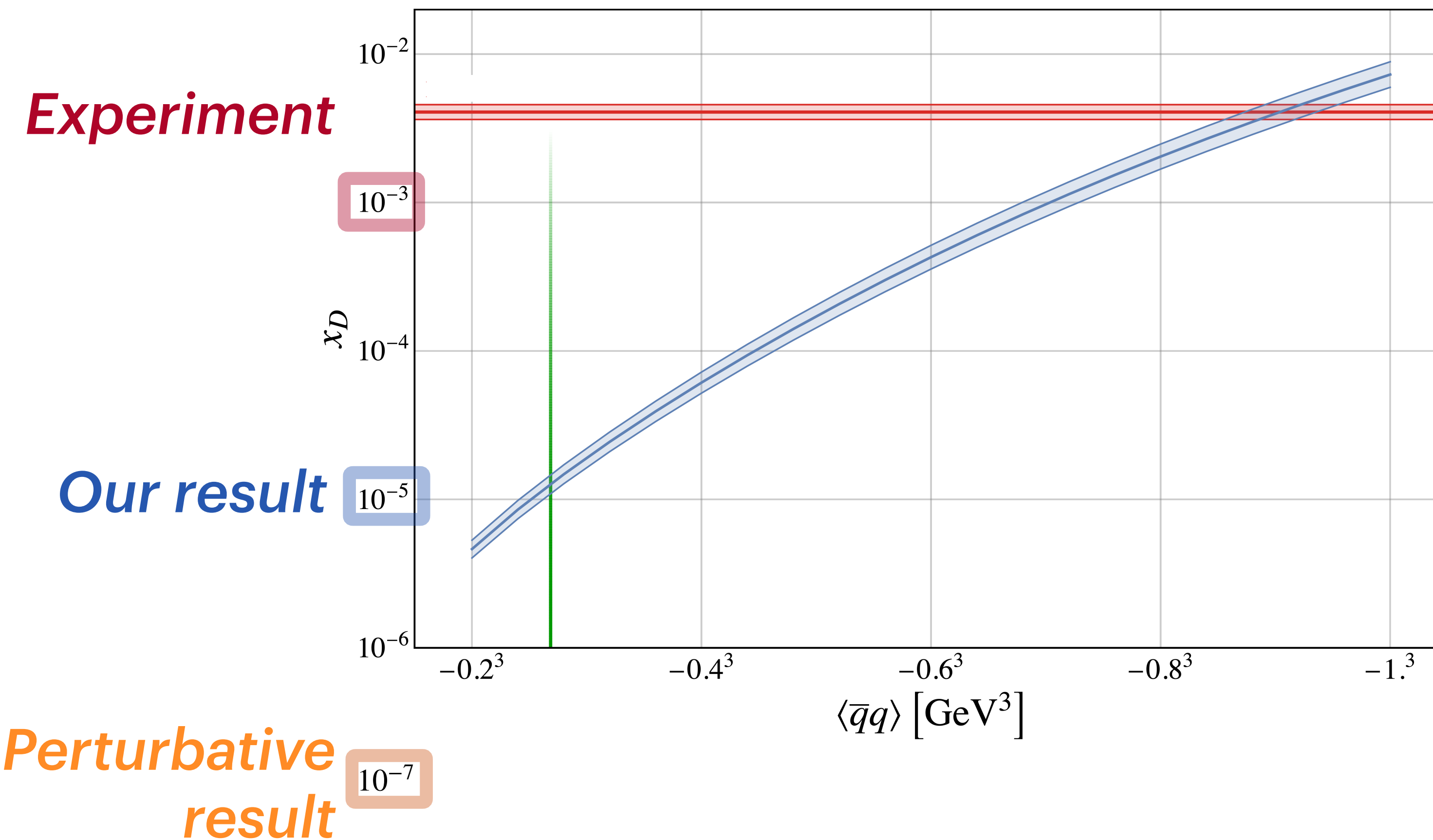
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- We introduce nonperturbative dynamics through nonlocal QCD condensates and
 - provide a comprehensive and detailed general analysis
 - for the first time, we find the **mixed condensate and four-quark condensate** contributions
 - we confirm the **four-quark contribution to be parametrically leading** as was predicted
 - our final result **improves on the perturbative LO+NLO by almost two orders of magnitude**, but still falls short of experimental value by another two orders

Thank you!

*For more details feel free to take a look at the
paper itself!*

[2508.16337]

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29 to 30 October 2025***