Nonperturbative contributions of QCD condensates to $D^0\overline{D}^0$ mixing

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Croatian Science Foundation

This work was supported by the Croatian Science Foundation under the project number IP-2024-05-4427.

Brda 2025: Selected topics in BSM, *Ljubljana, Slovenia*
29 to 30 October 2025

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The formalism

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mass difference *decay difference* *possible CPV*

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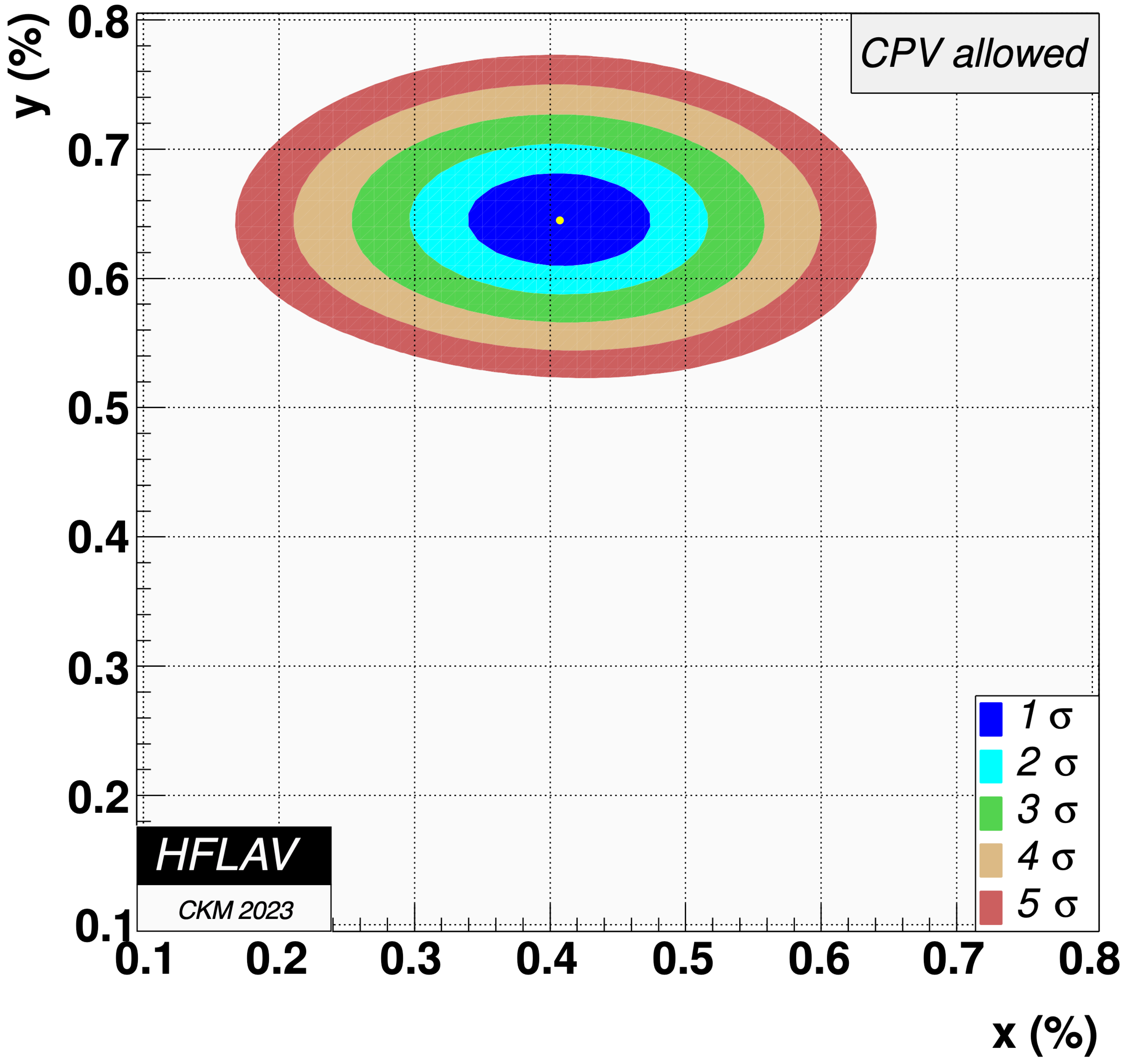

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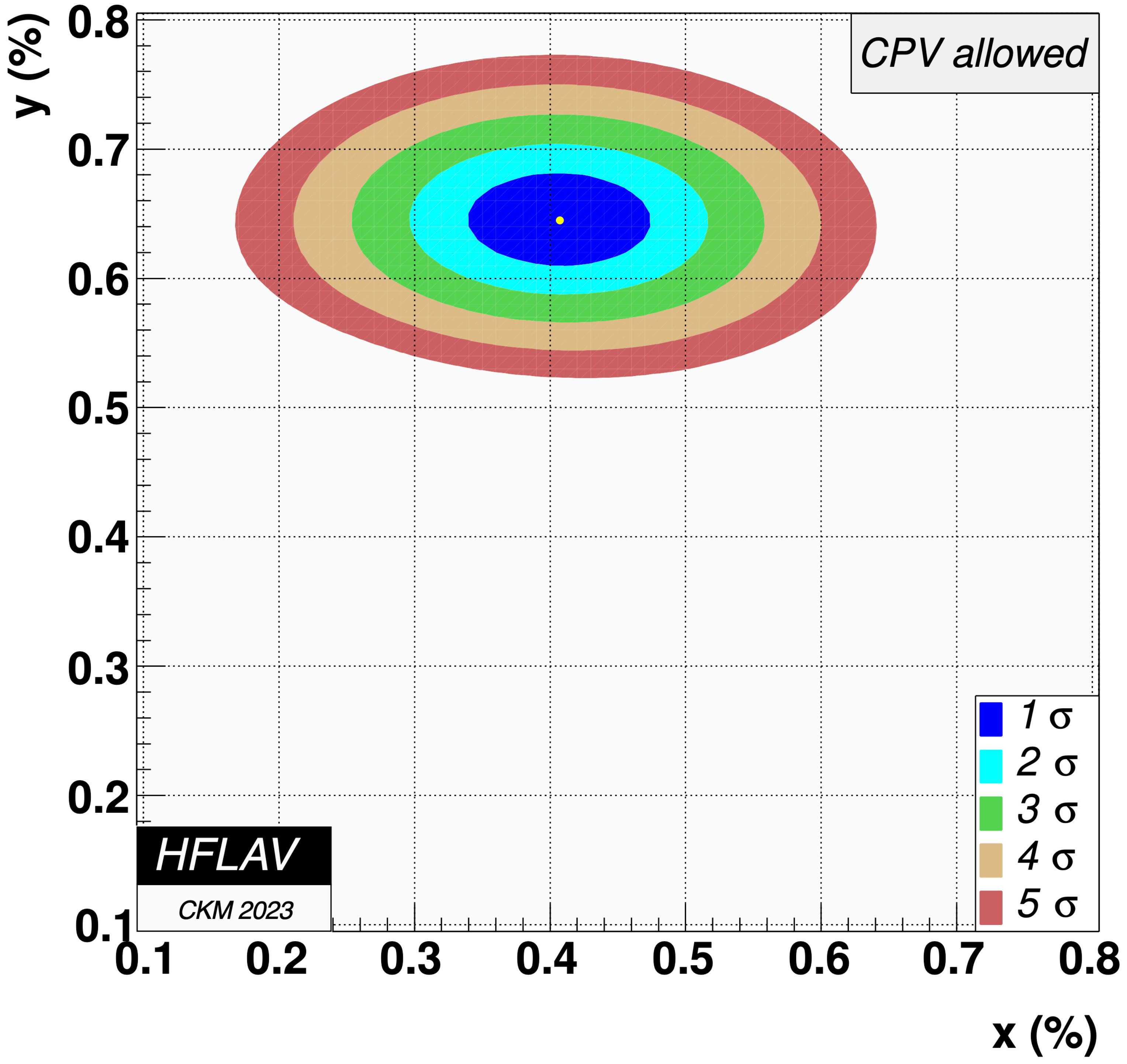

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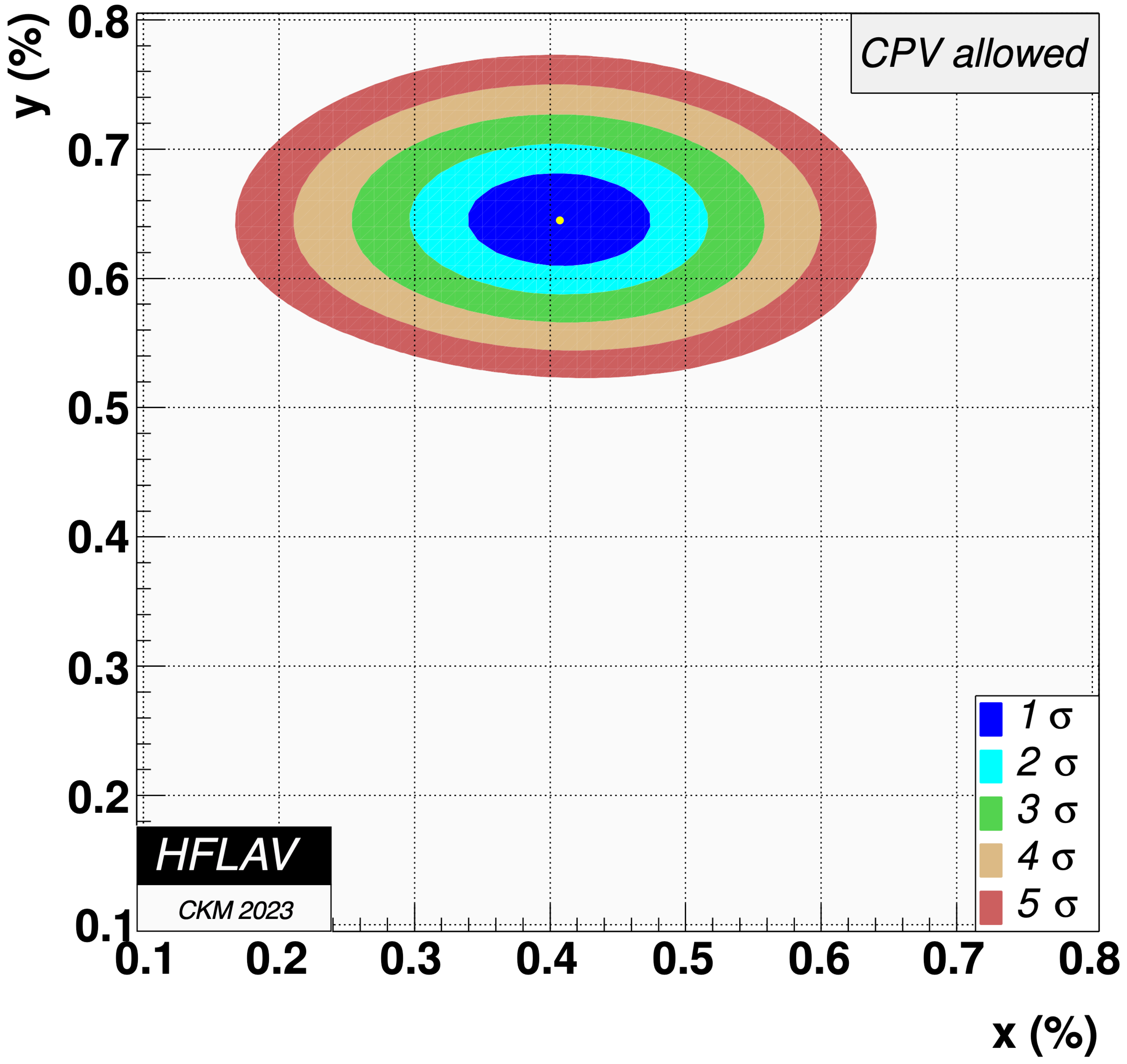

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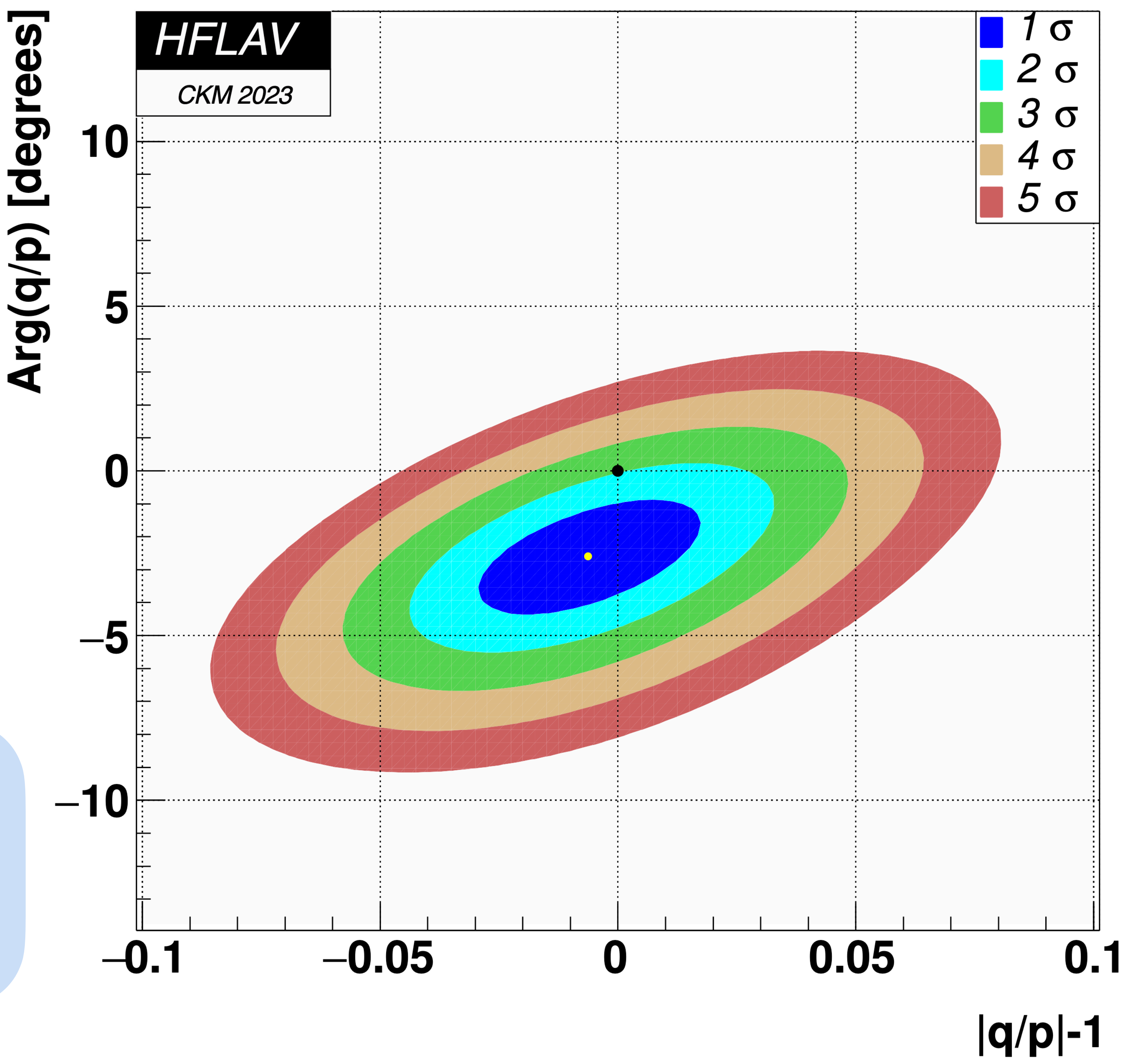

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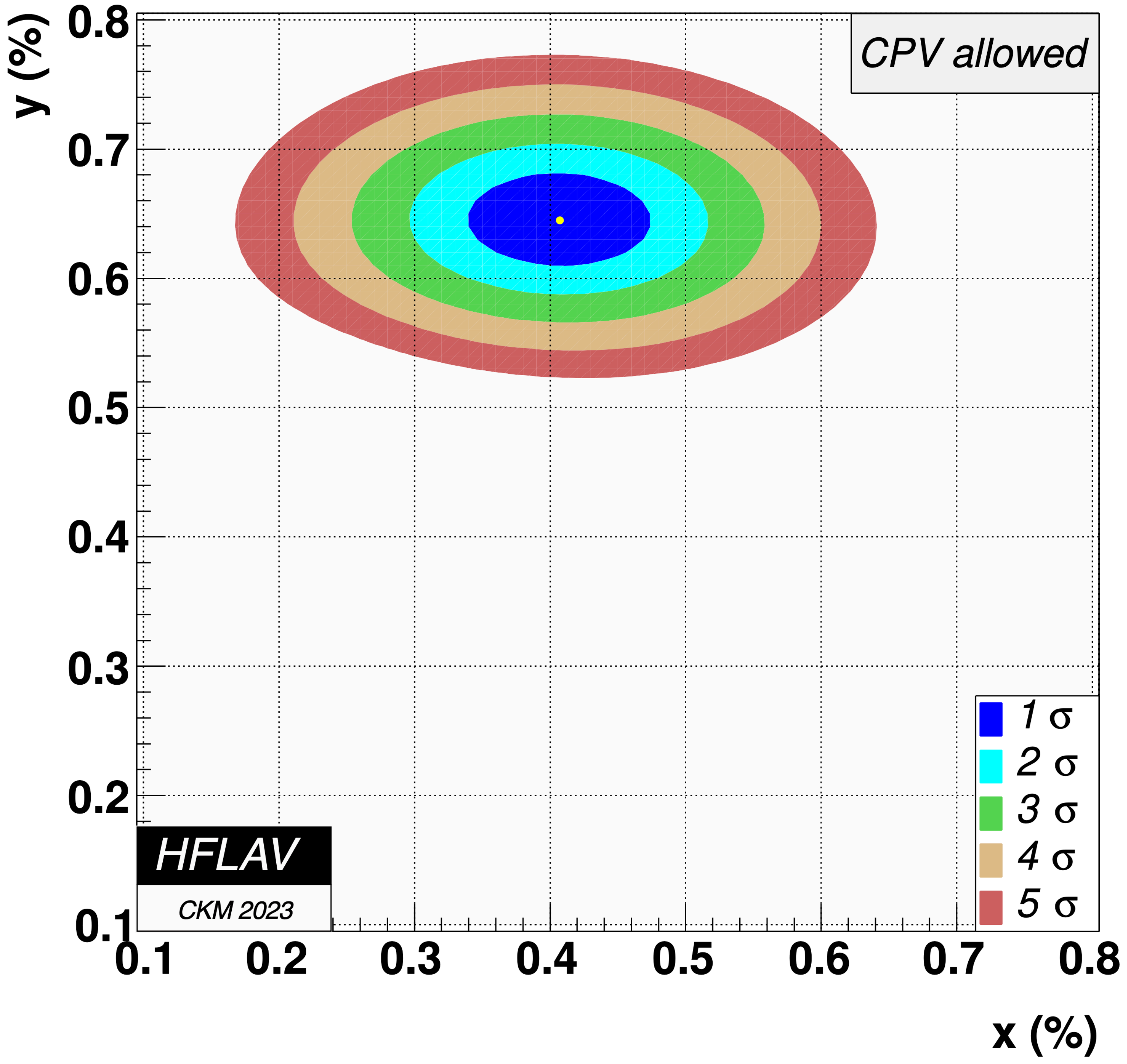

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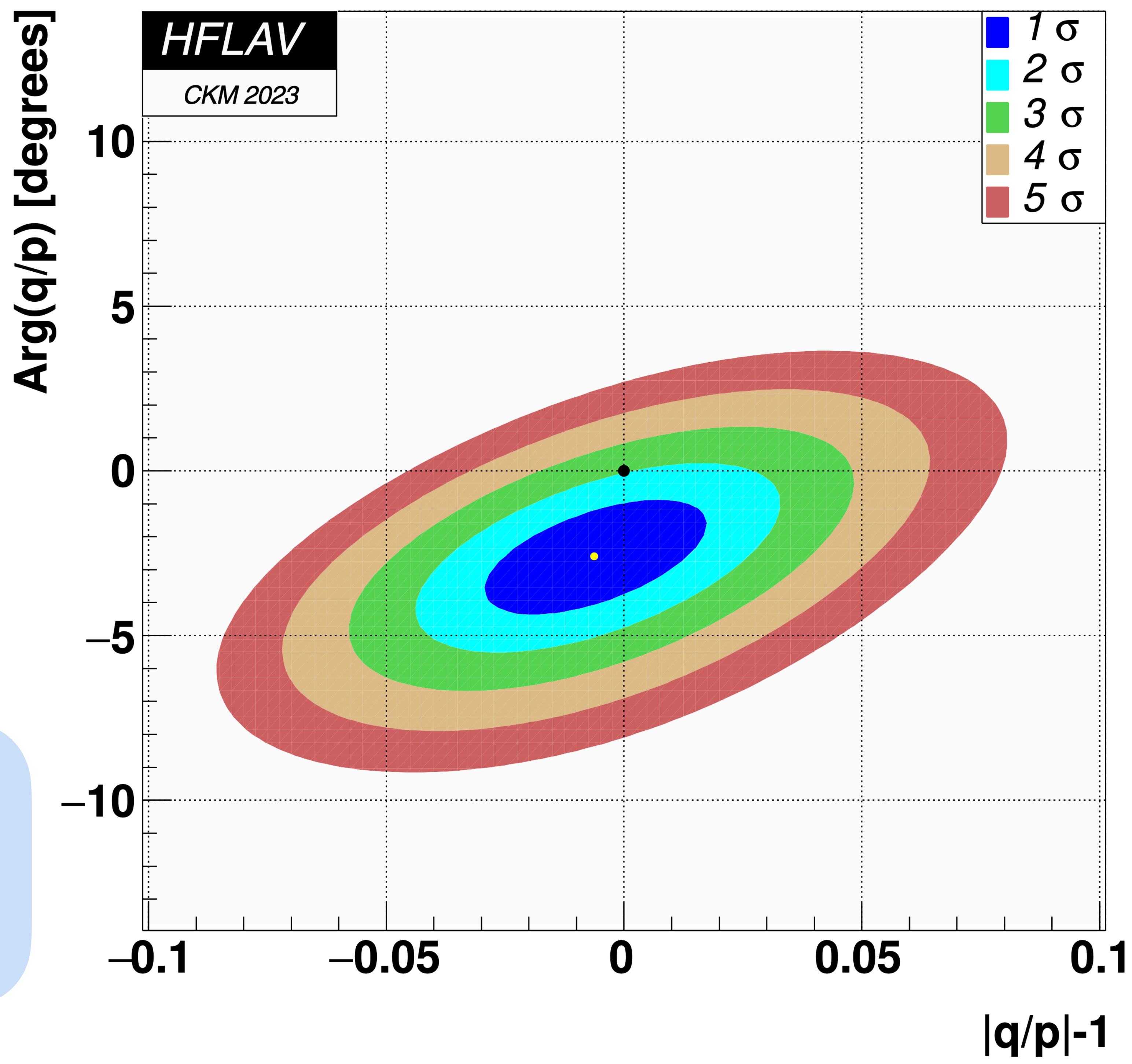

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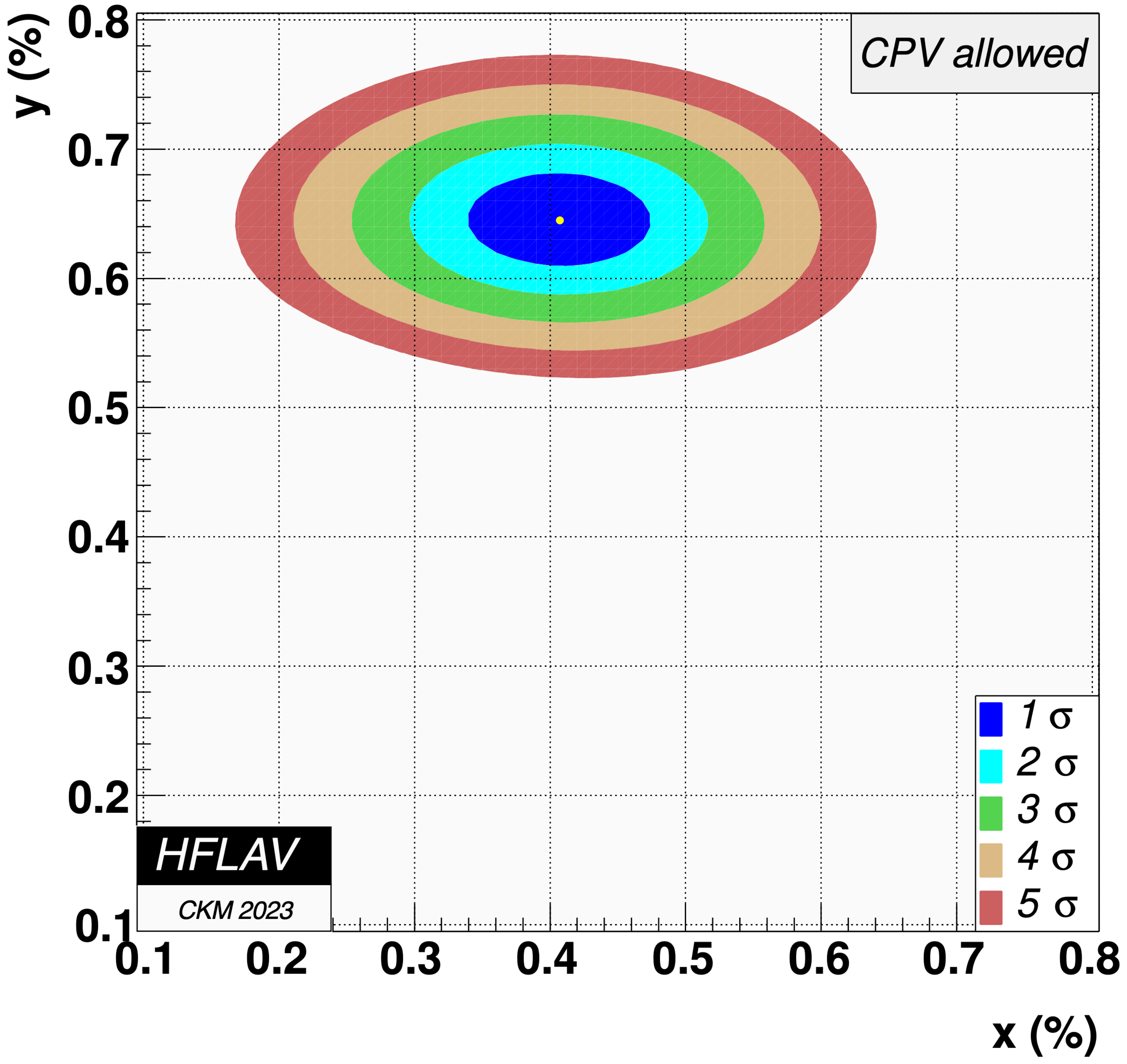

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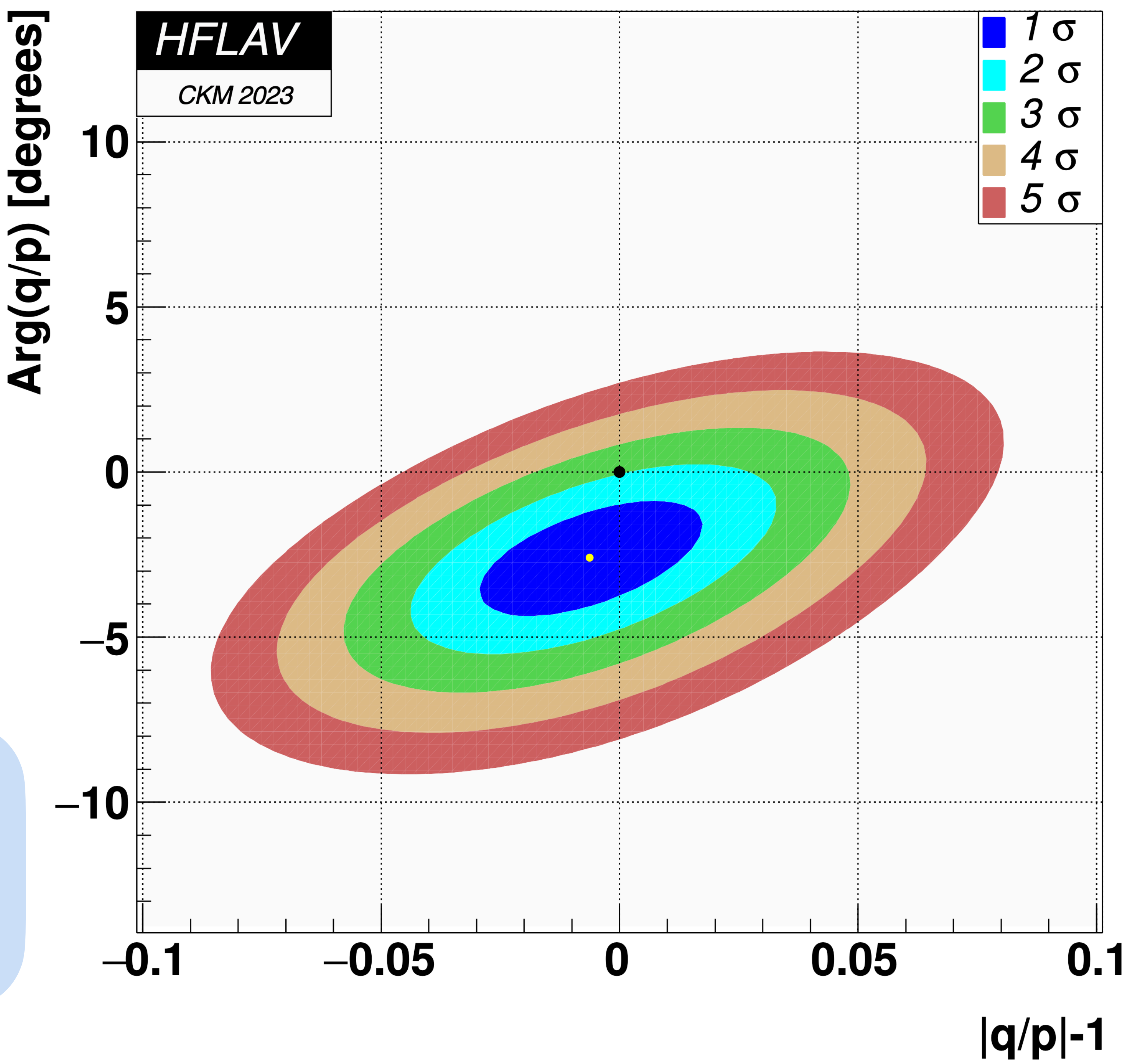

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In our calculations we assumed **no CPV**

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Theory - the perturbative contribution

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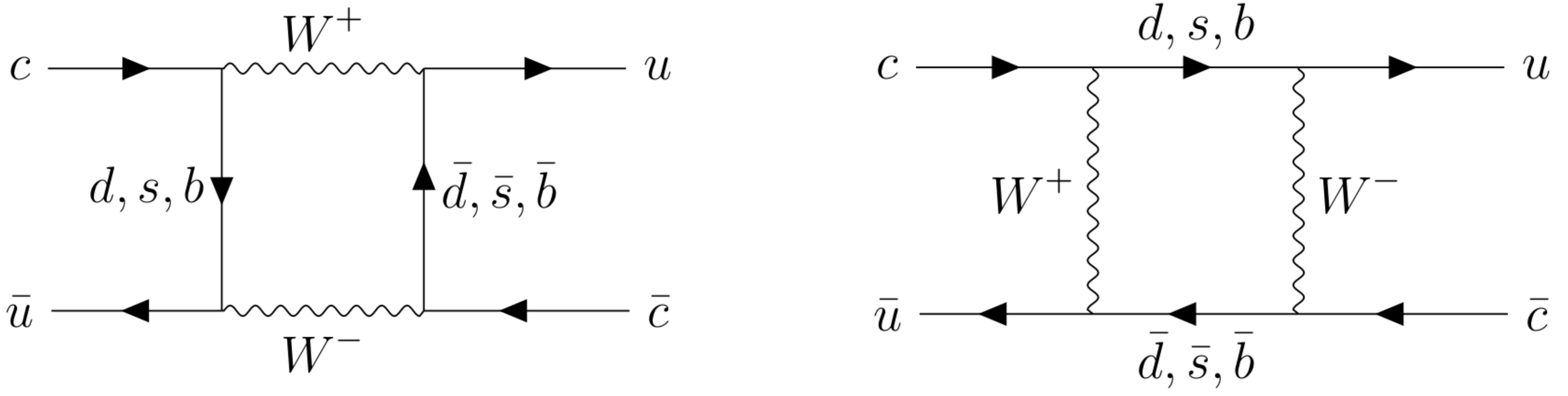

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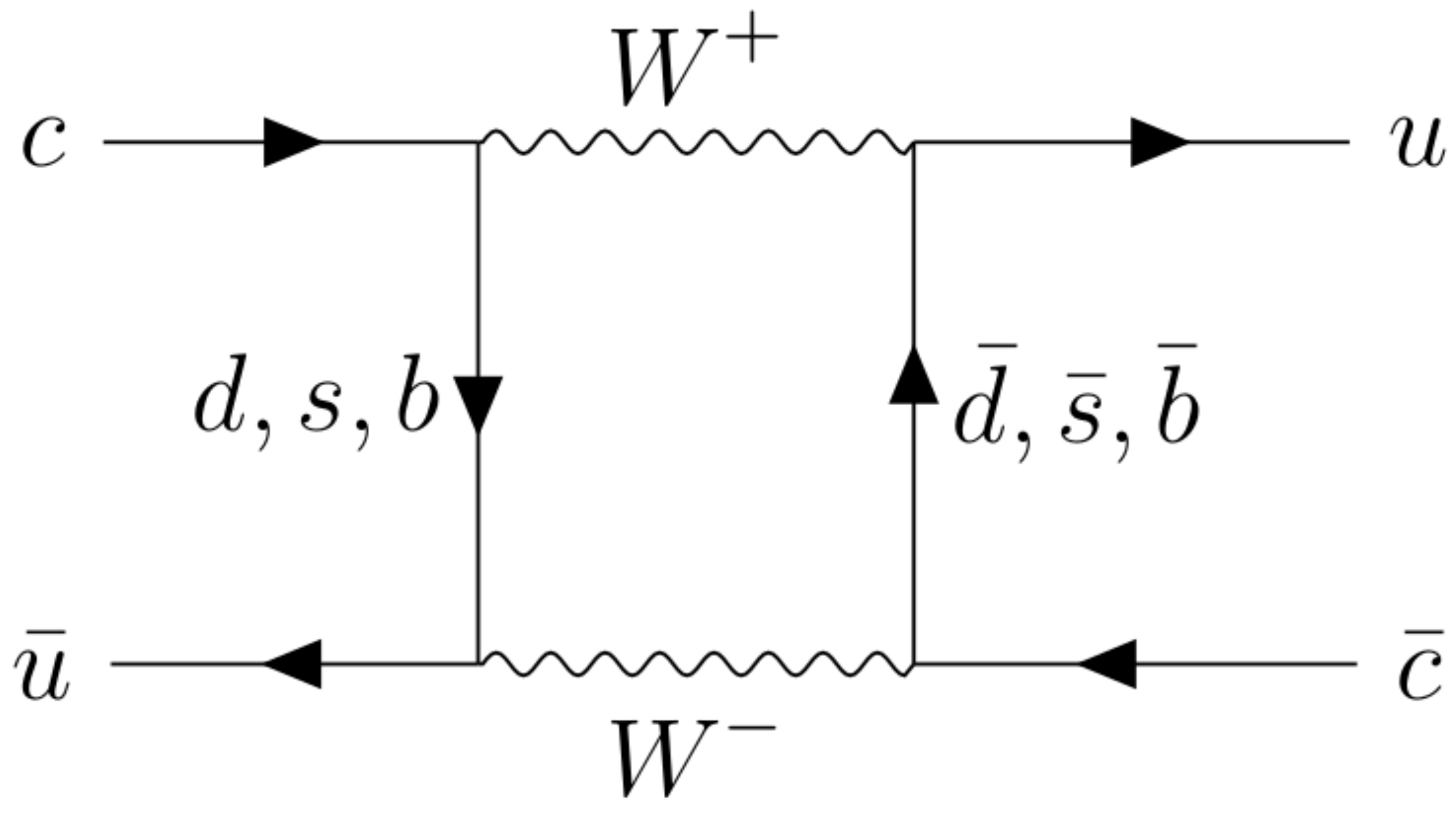

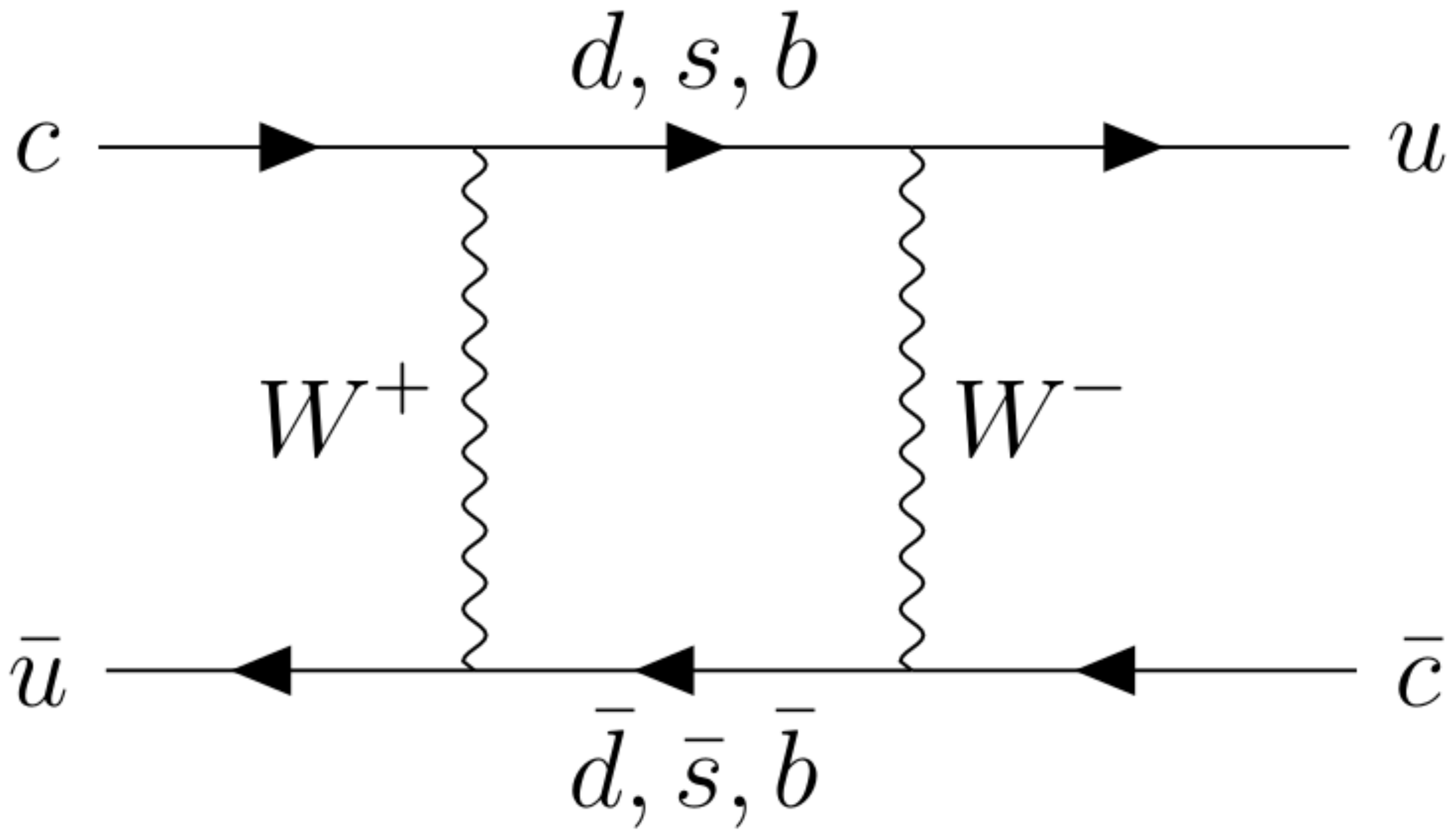

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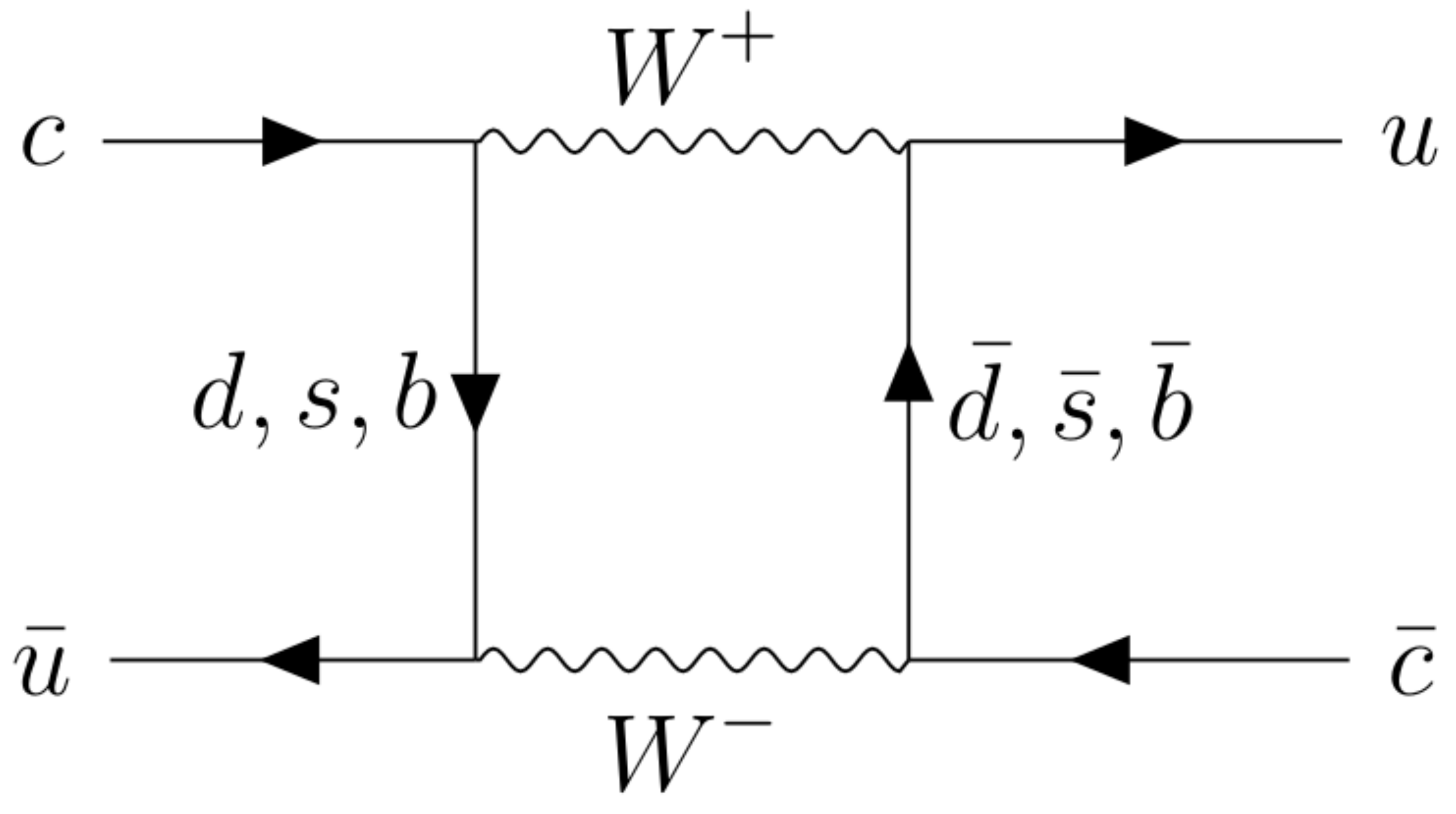

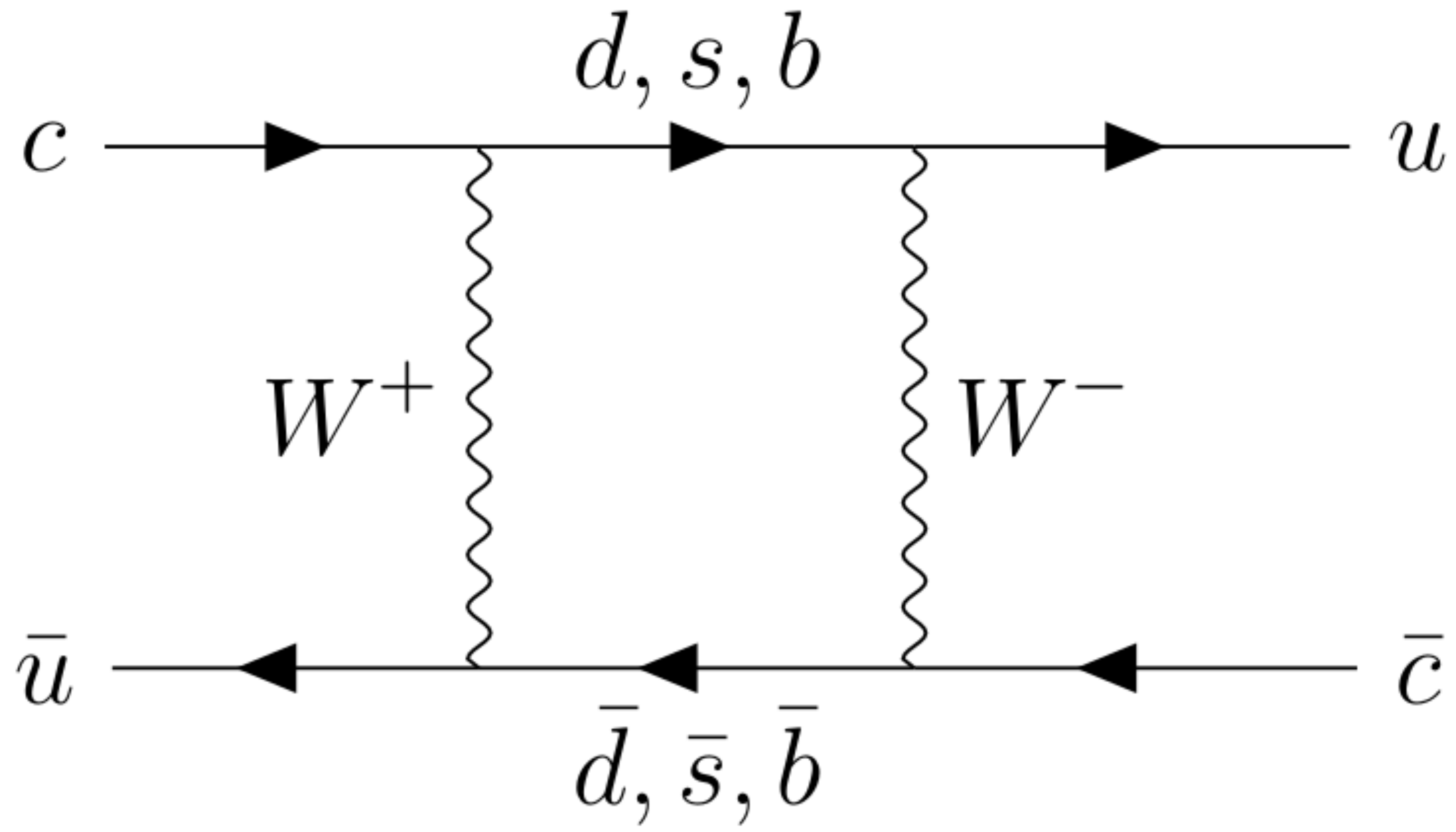

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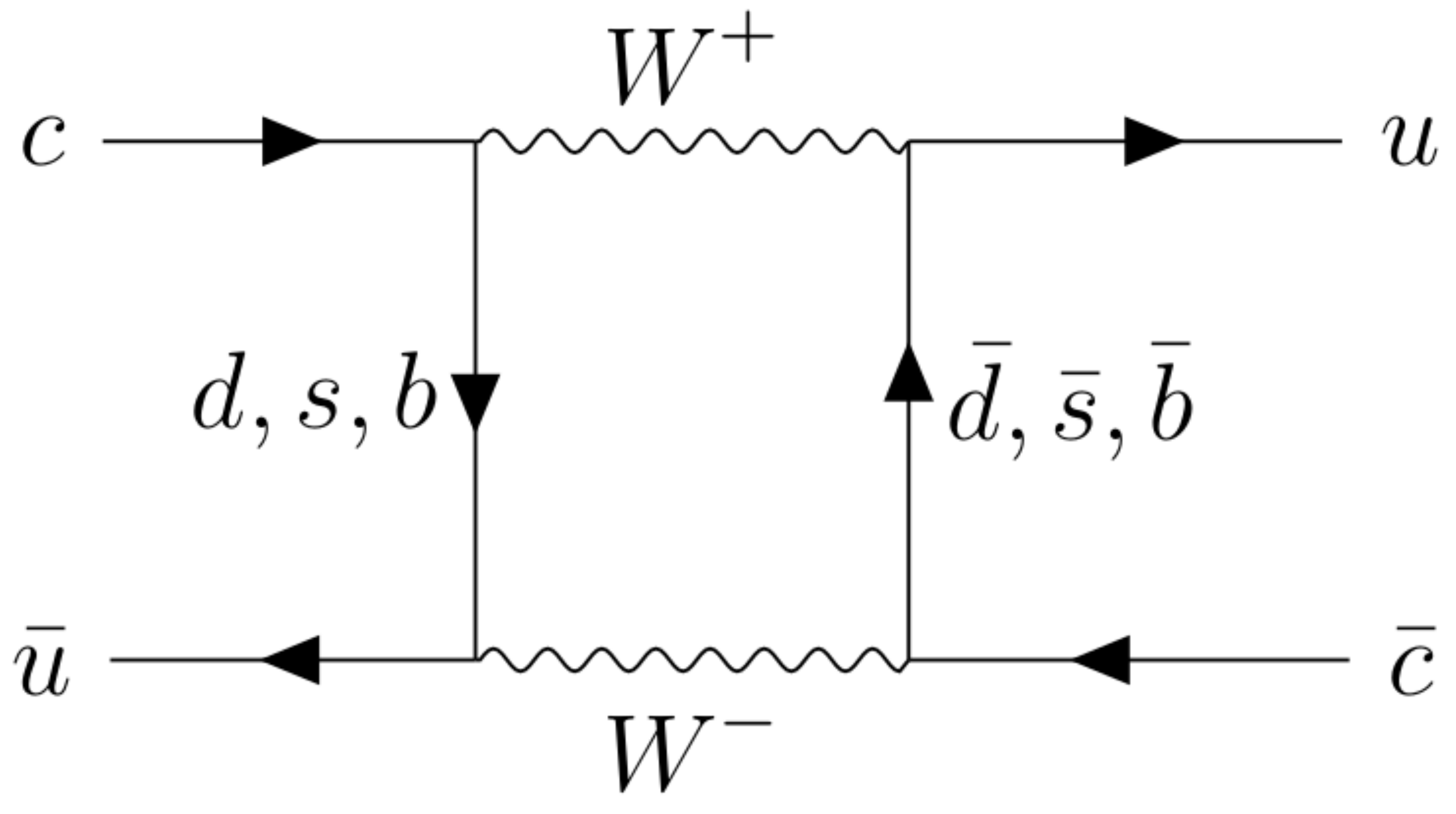

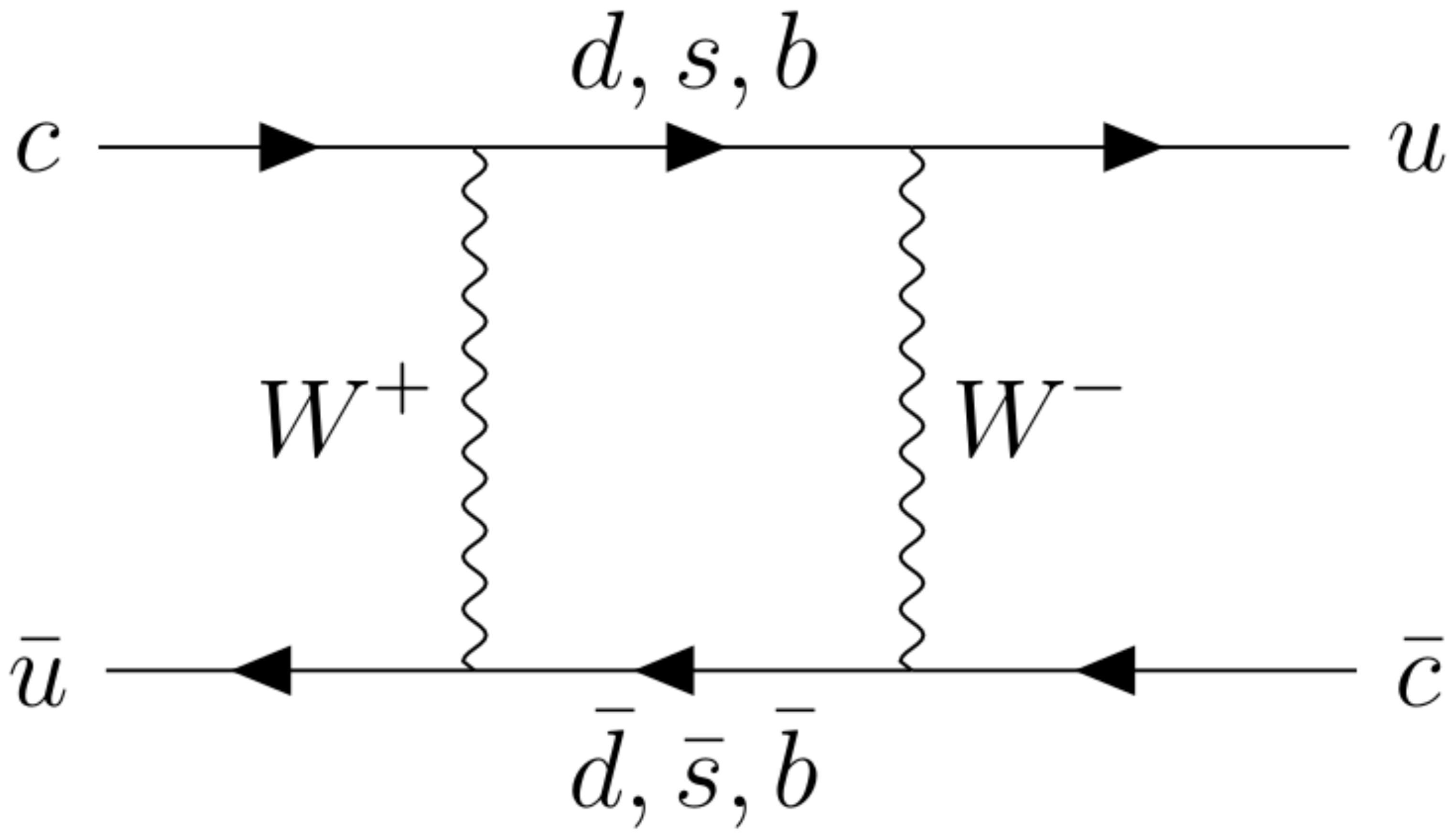

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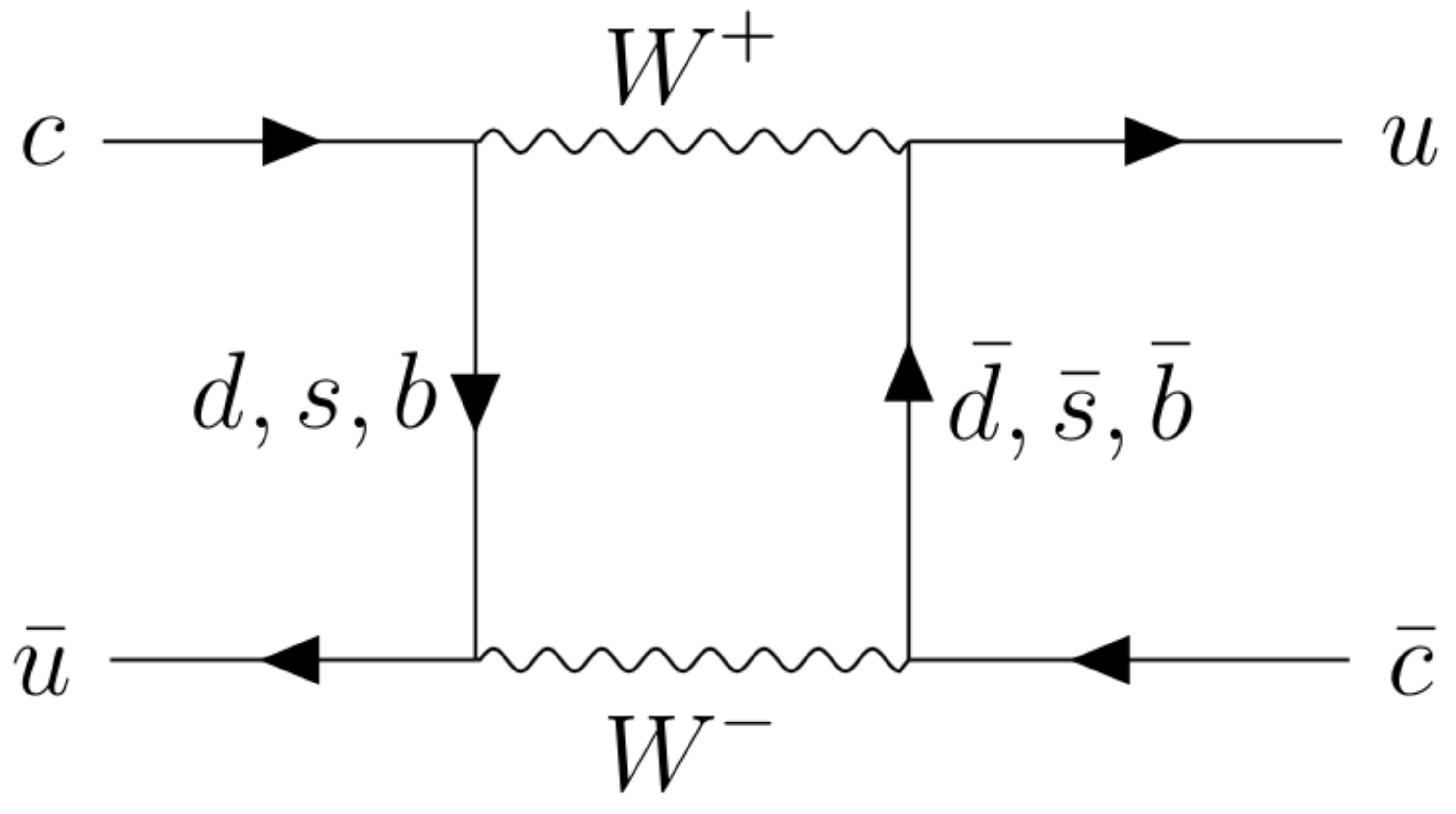

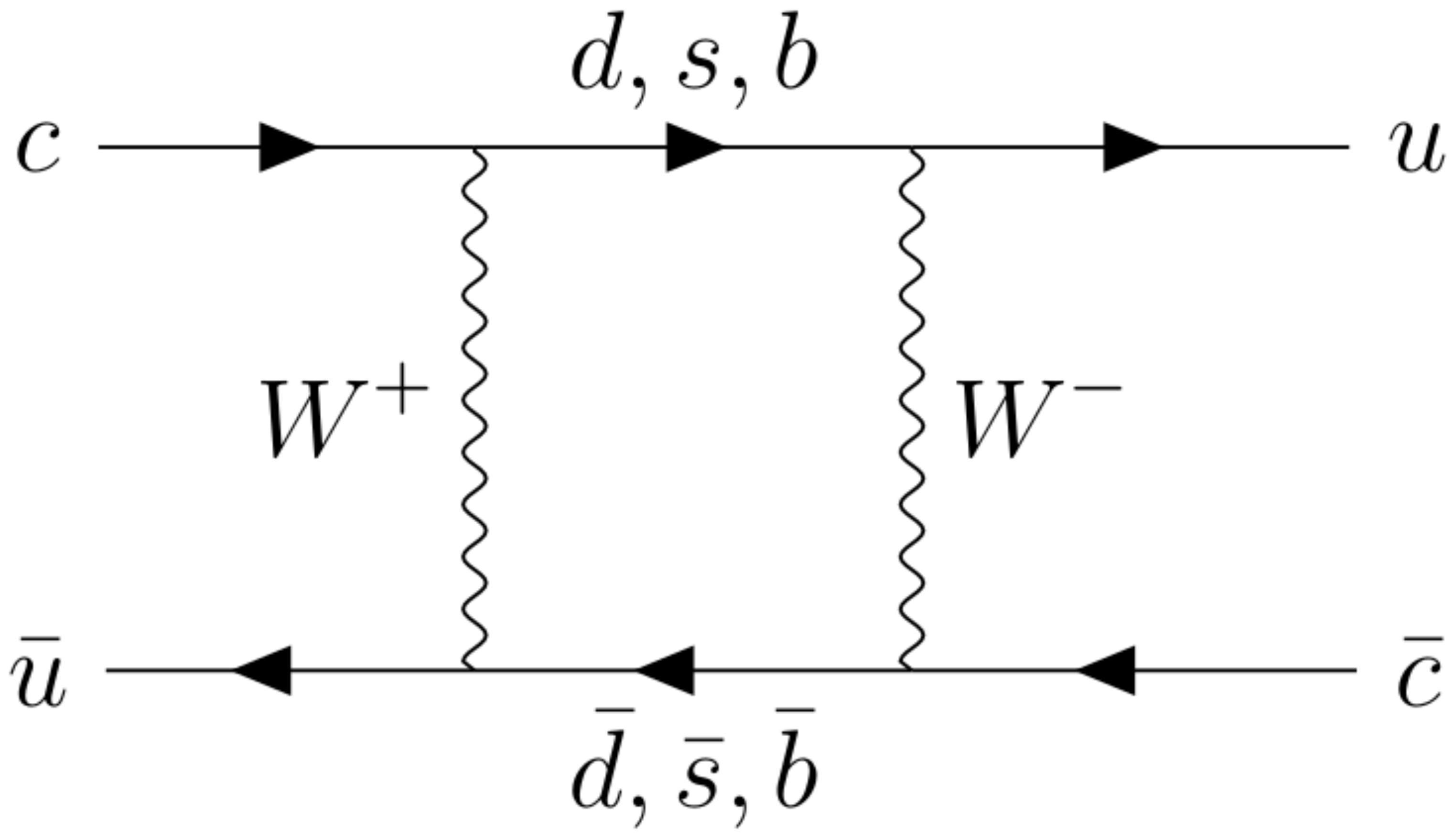

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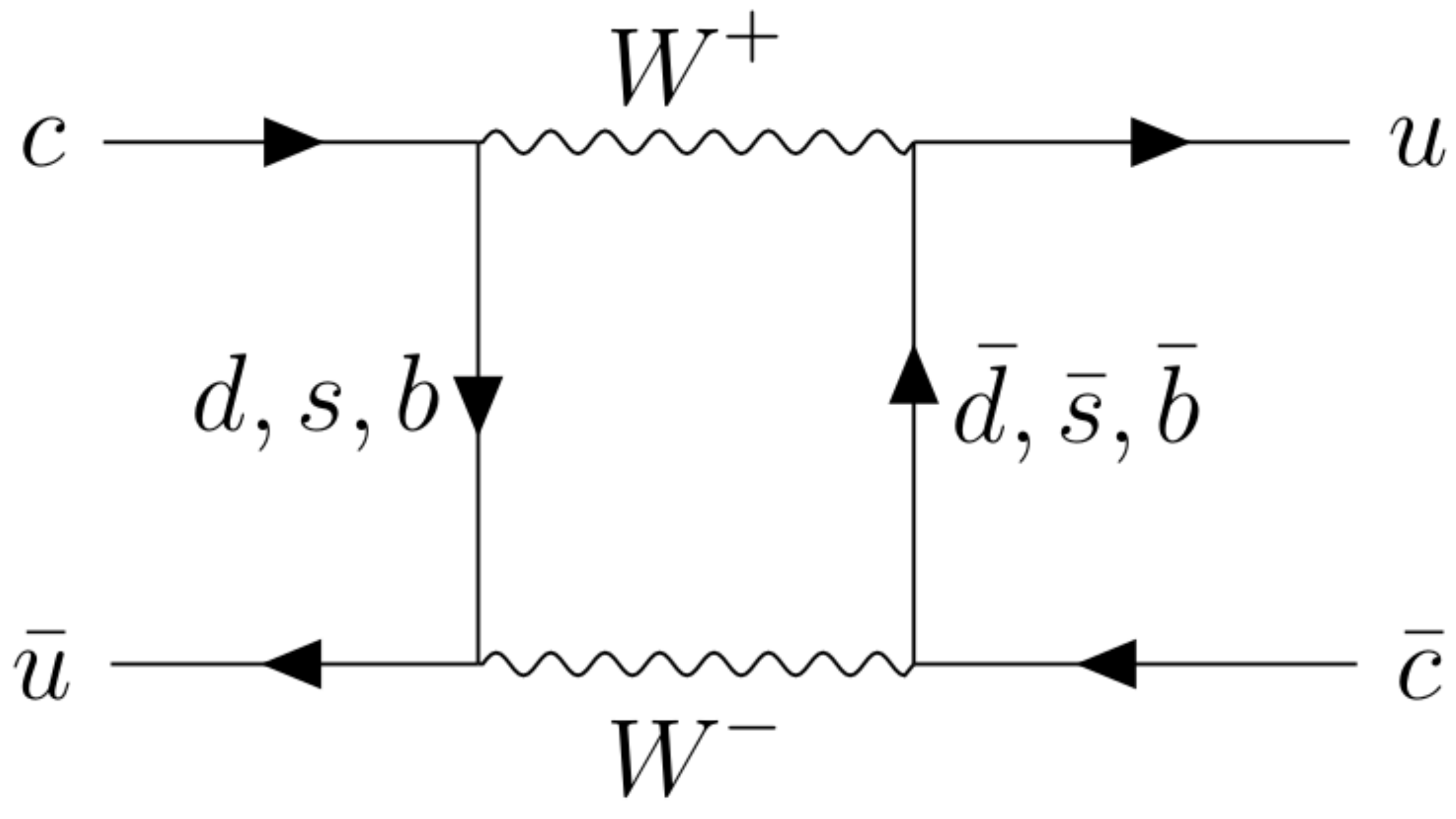

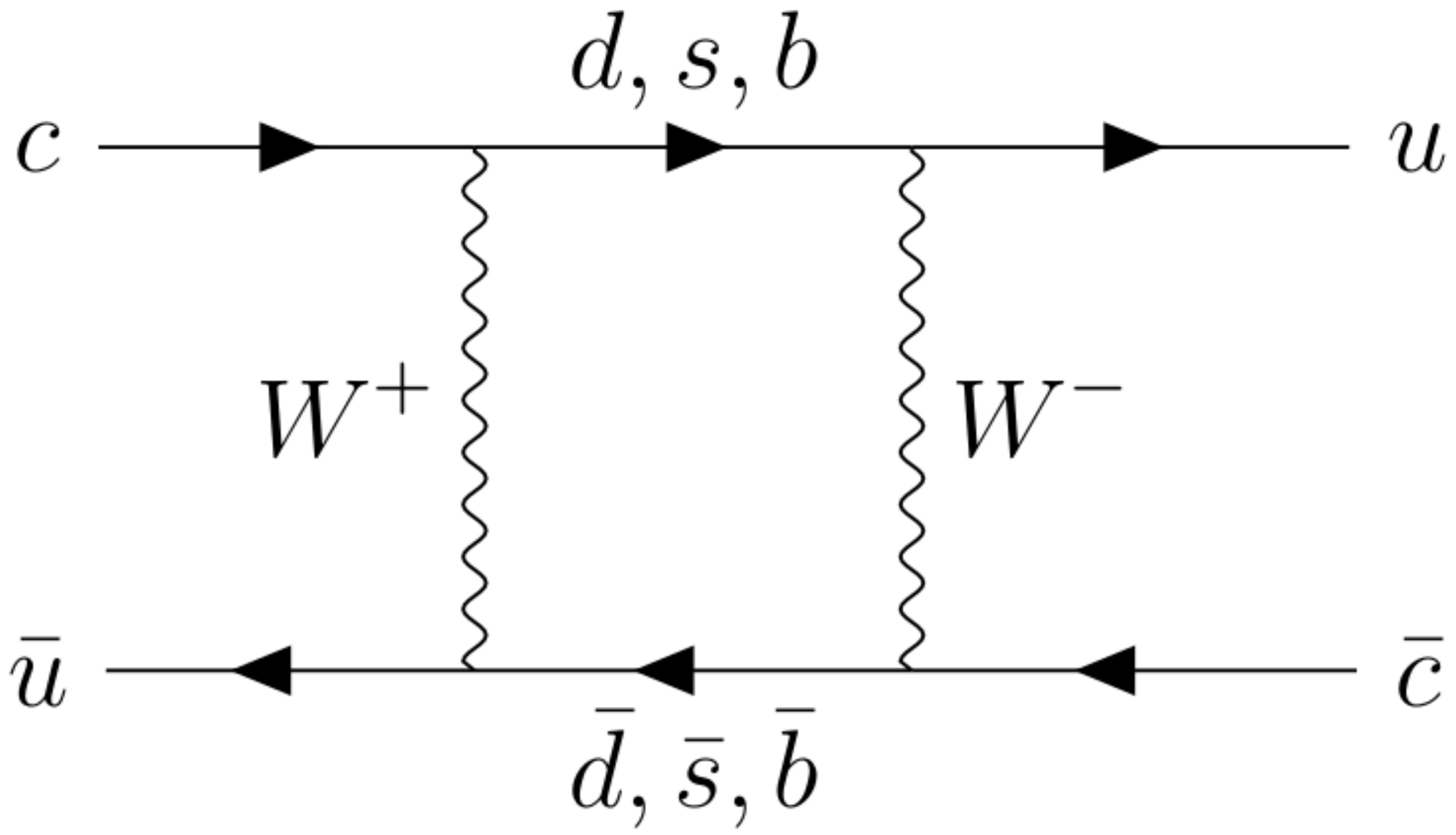

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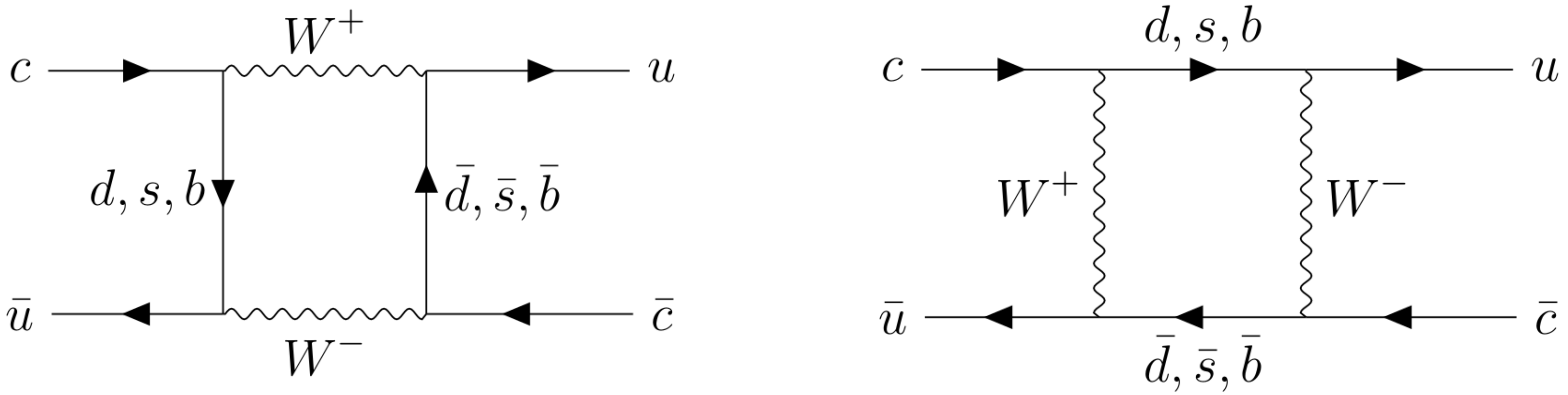

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- very effective cancellations happen leaving the result **far behind experimental value**; even though separate contributions are large

Different approaches

How to lift the GIM suppression?

Inclusive

- $SU(3)$ breaking effects from **higher-dimension operators**
- $SU(3)$ breaking effects from **alternative renormalisation scale setting**
- **NLO and mass corrections**

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- $SU(3)$ breaking effects from **final state phase space differences of intermediate bound states**
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QCD condensates

Motivation

☐ indicates weak effective vertex

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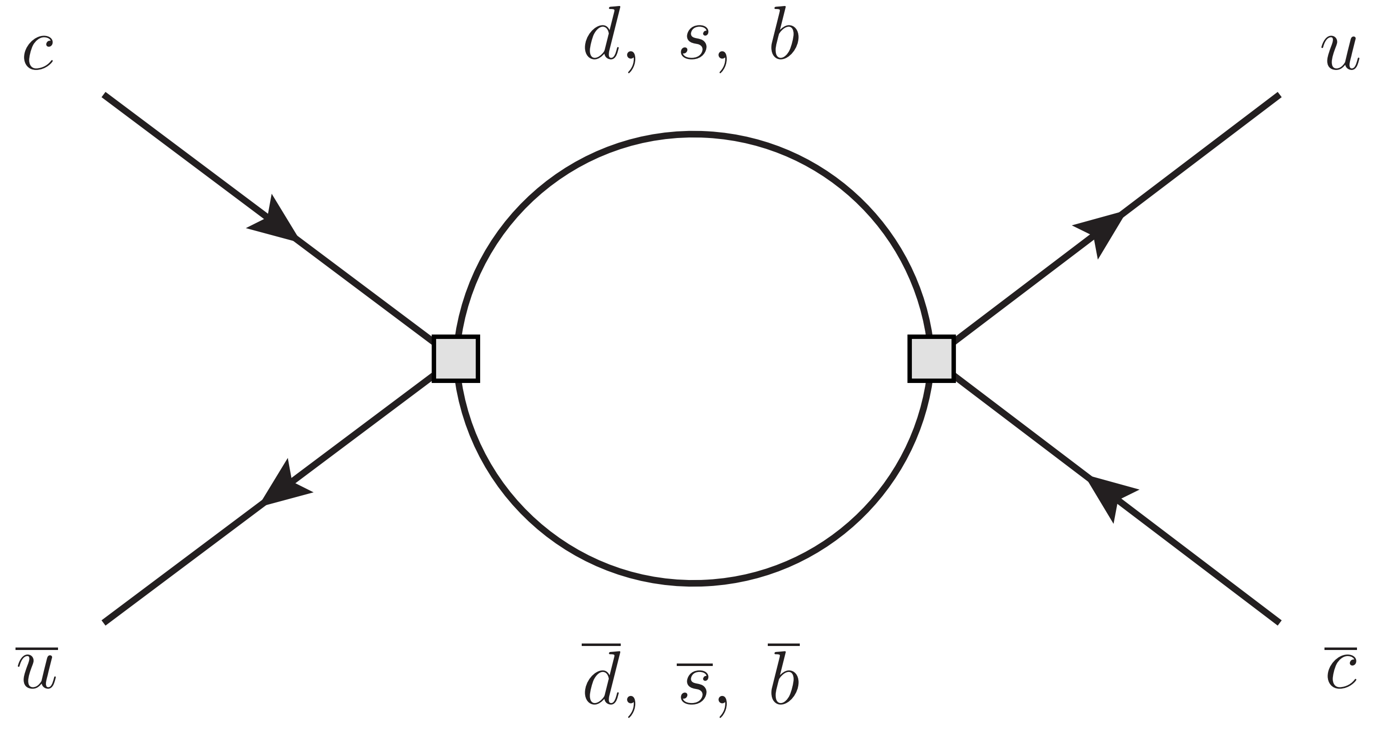
- leading order diagram $\propto \left(m_s/m_c\right)^4$

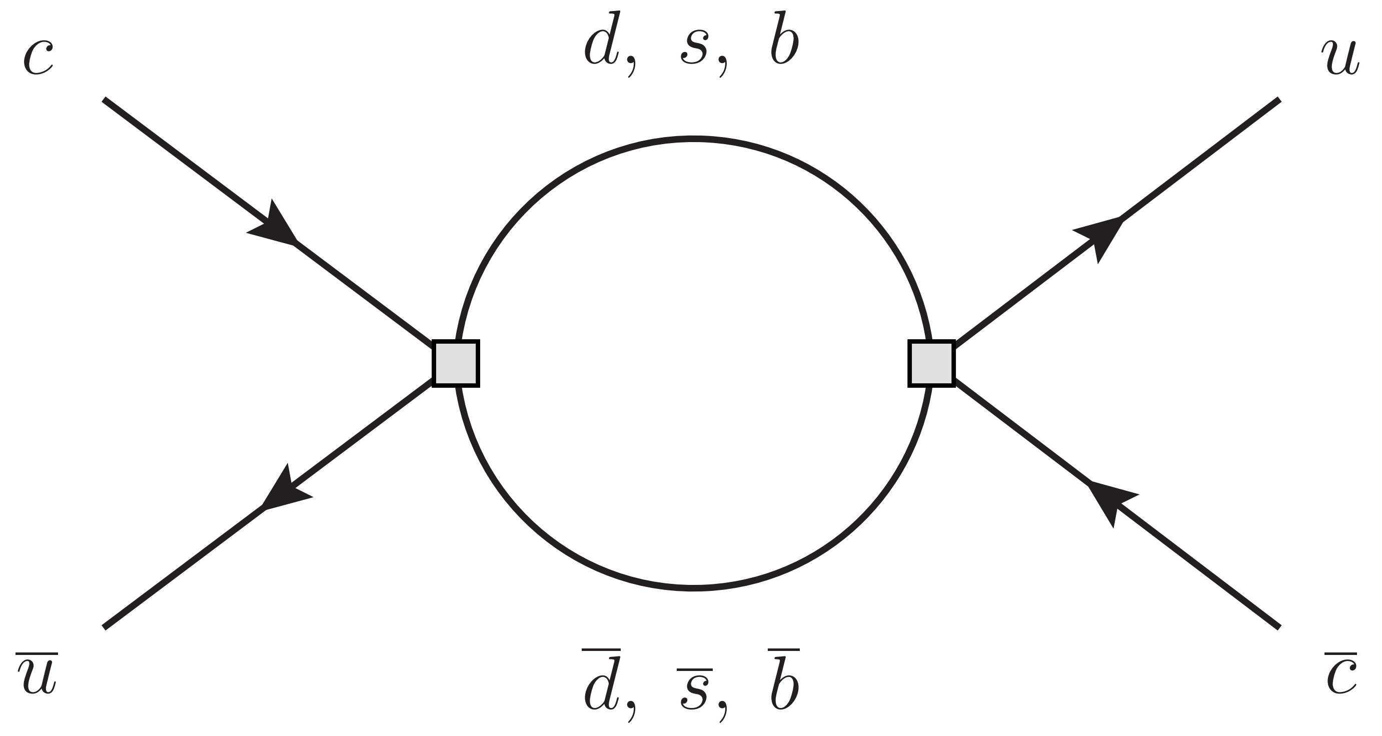


□ indicates weak effective vertex

QCD condensates

Motivation

- leading order diagram $\propto \left(m_s/m_c\right)^4$



$$\overbrace{q(0)\overline{q}}(x) \propto \frac{\not{p}+m_q}{p^2-m_q^2} = \frac{\not{p}+m_q}{p^2}\left(1+\frac{m_q^2}{p^2}+\ldots\right)$$

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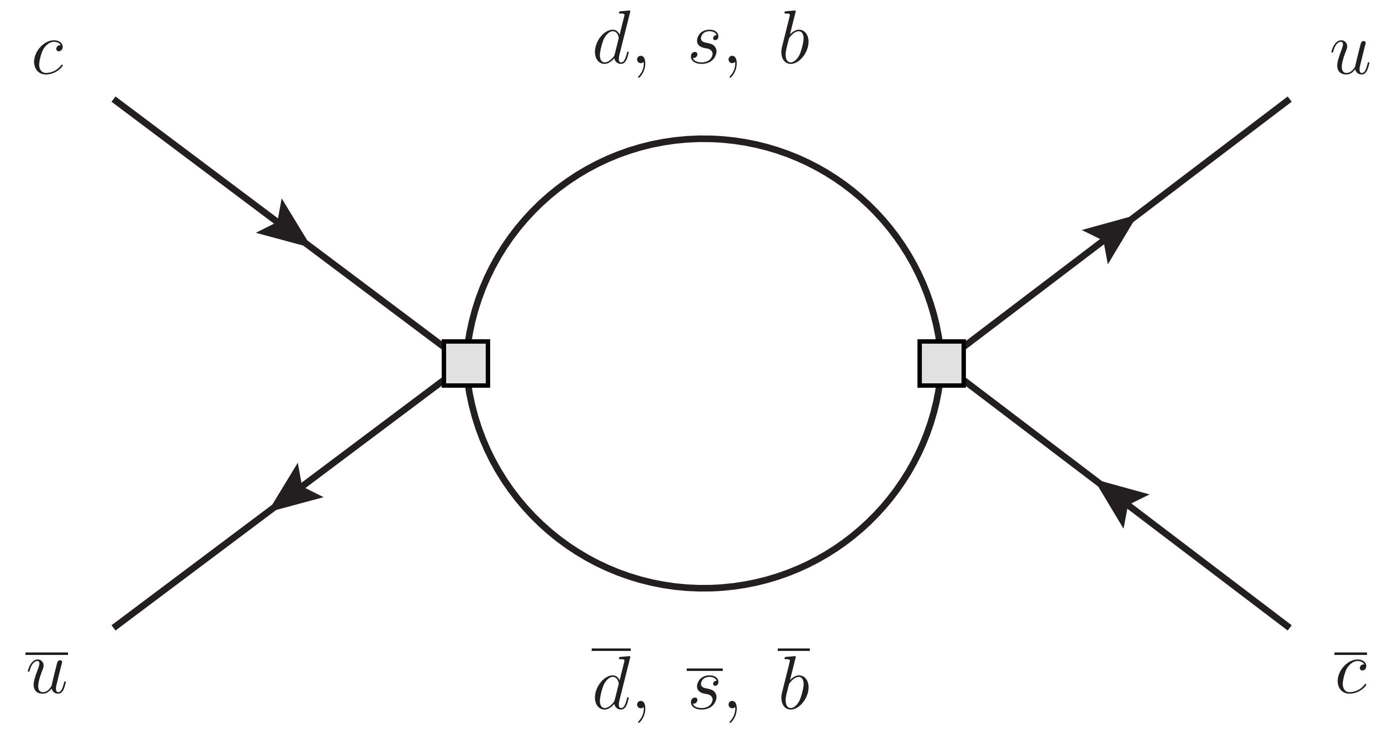

don't contract the quark fields

factorize them!

$$\contraction{}{q}{(0)}{\overline{q}}q(0)\overline{q}(x) \propto \frac{\not{p} + m_q}{p^2 - m_q^2} = \frac{\not{p} + m_q}{p^2}\left(1 + \frac{m_q^2}{p^2} + \dots\right)$$

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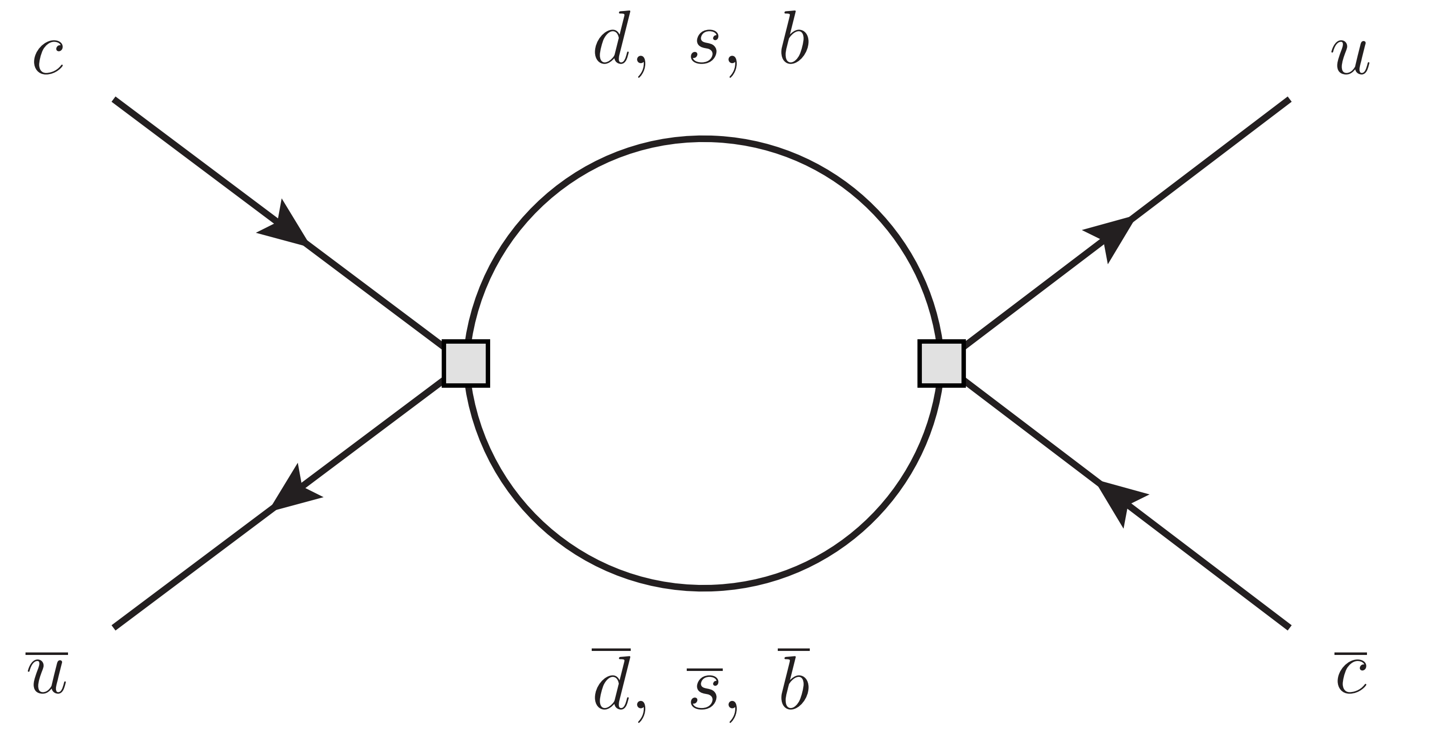

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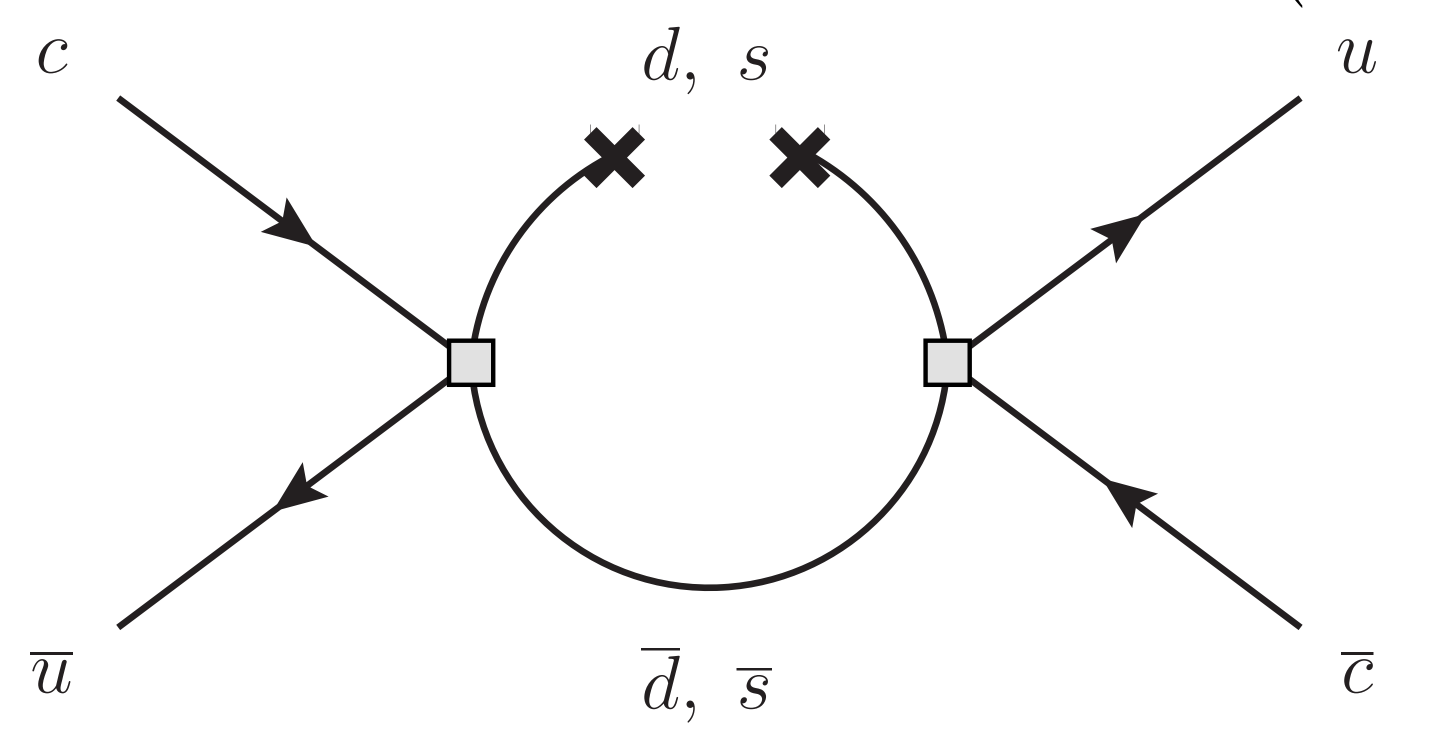
- **two-quark condensate** (dim-9 operator) $\propto \left(m_s/m_c\right)^3$



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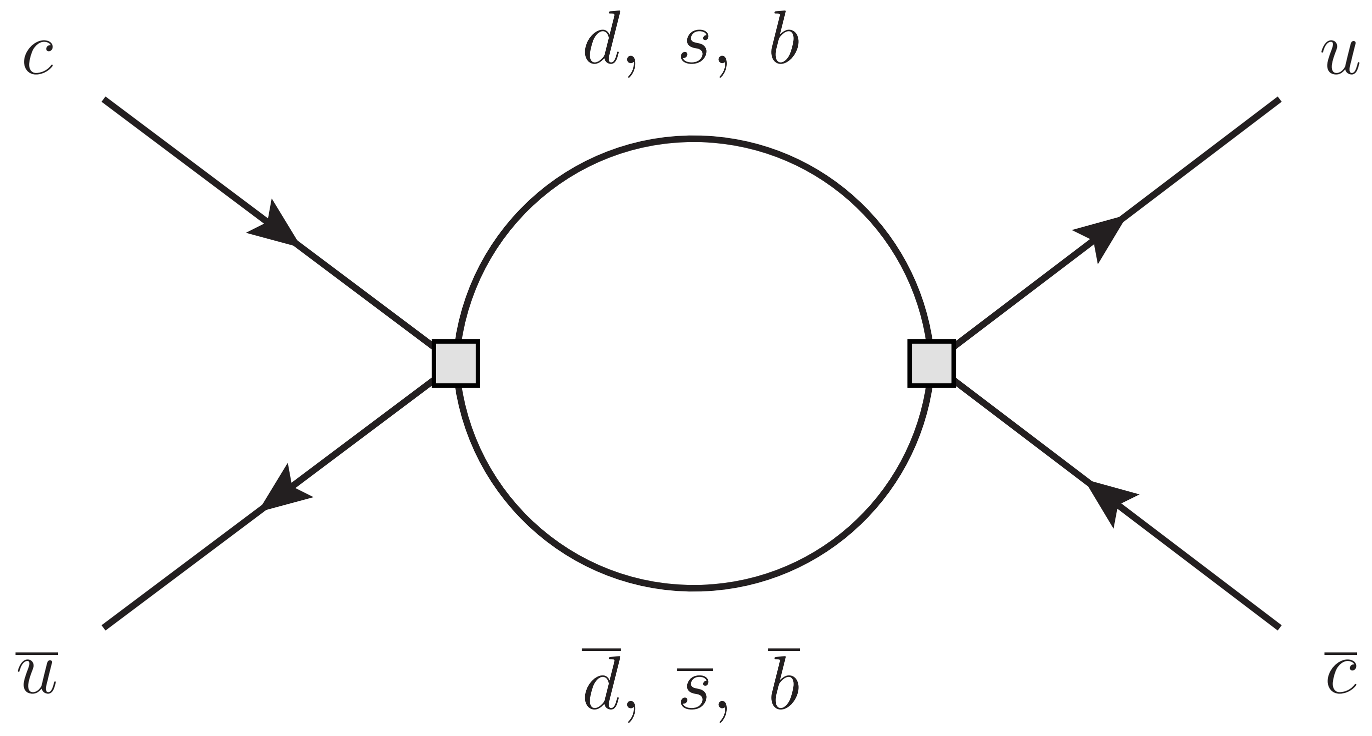

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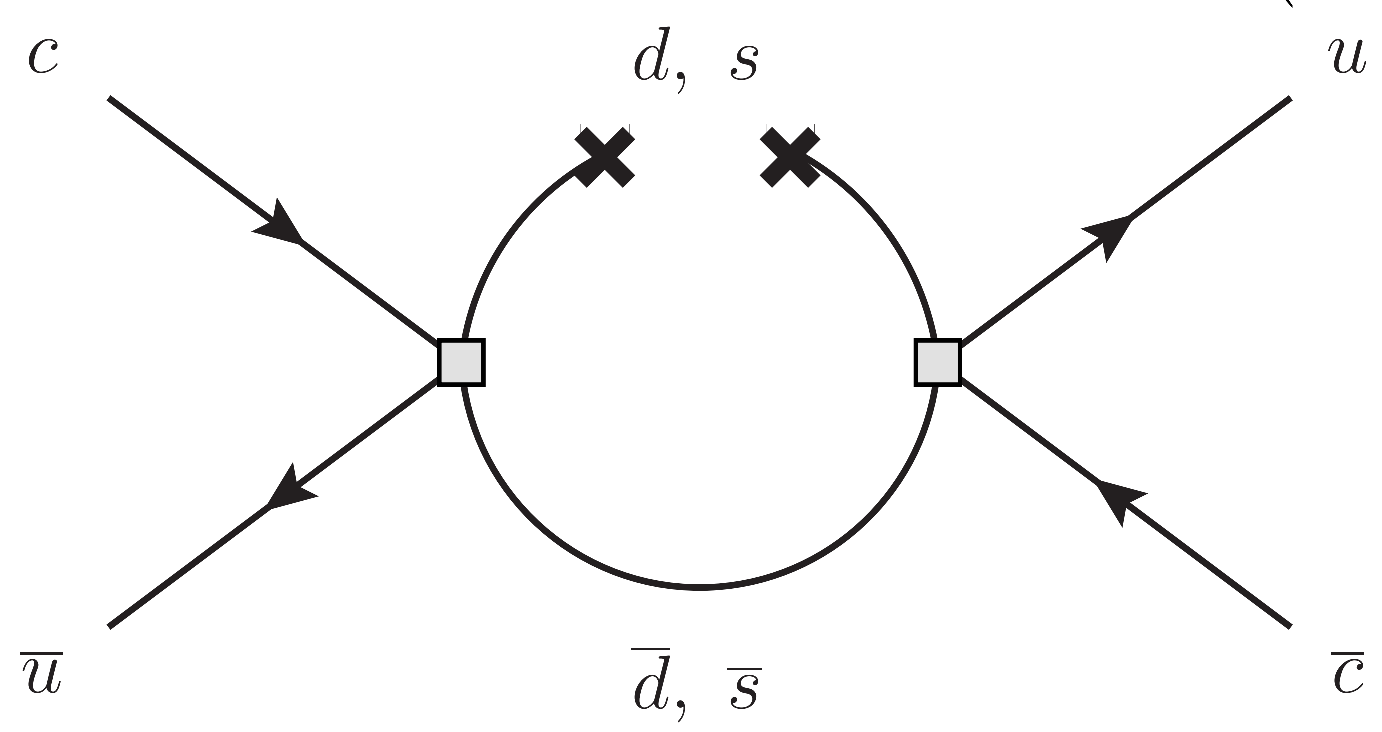

$$\langle \overline{q}(x) q(0) \rangle \propto 1 + i\not{x}\frac{m_q}{4} + \ldots$$

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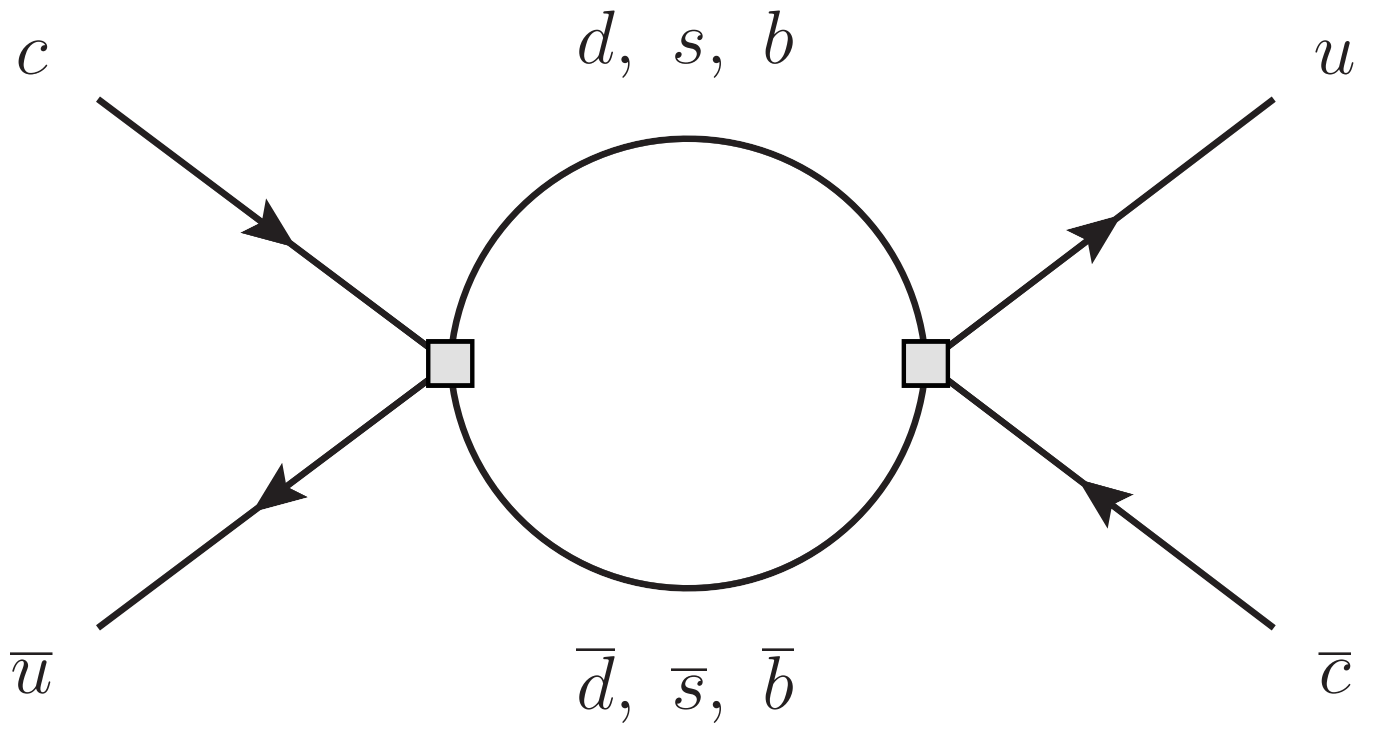

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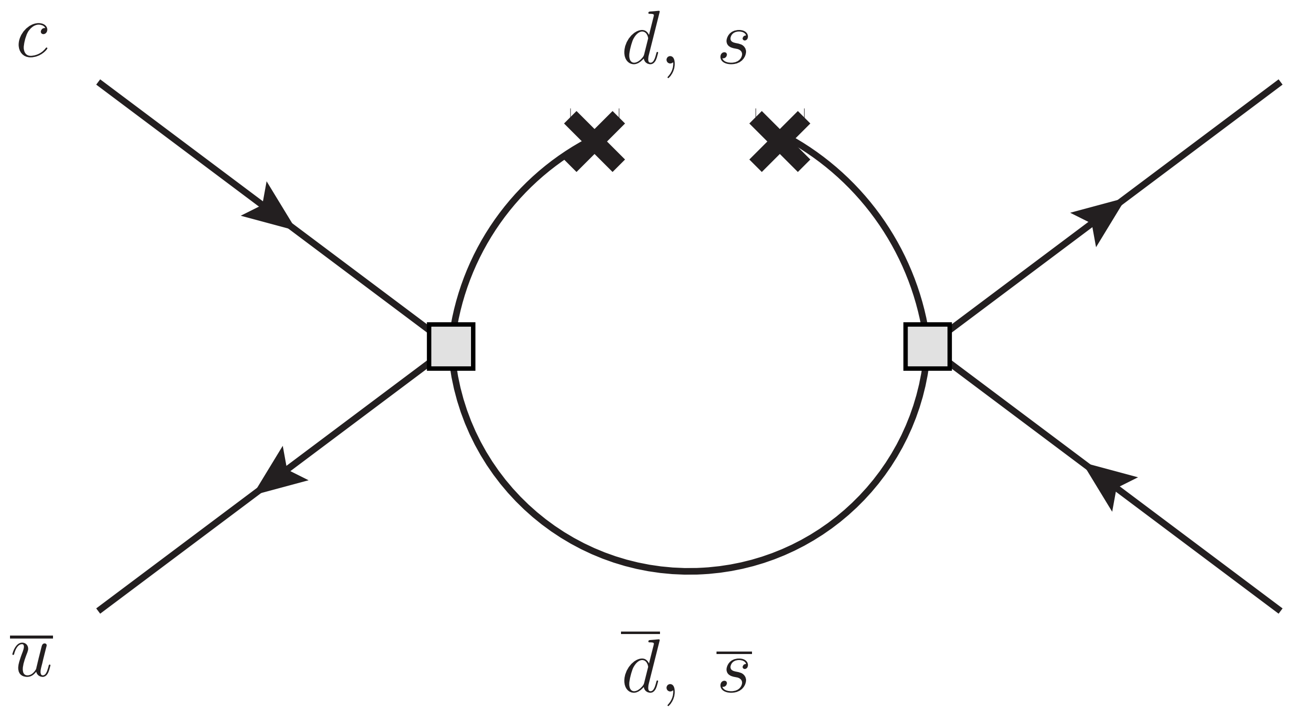

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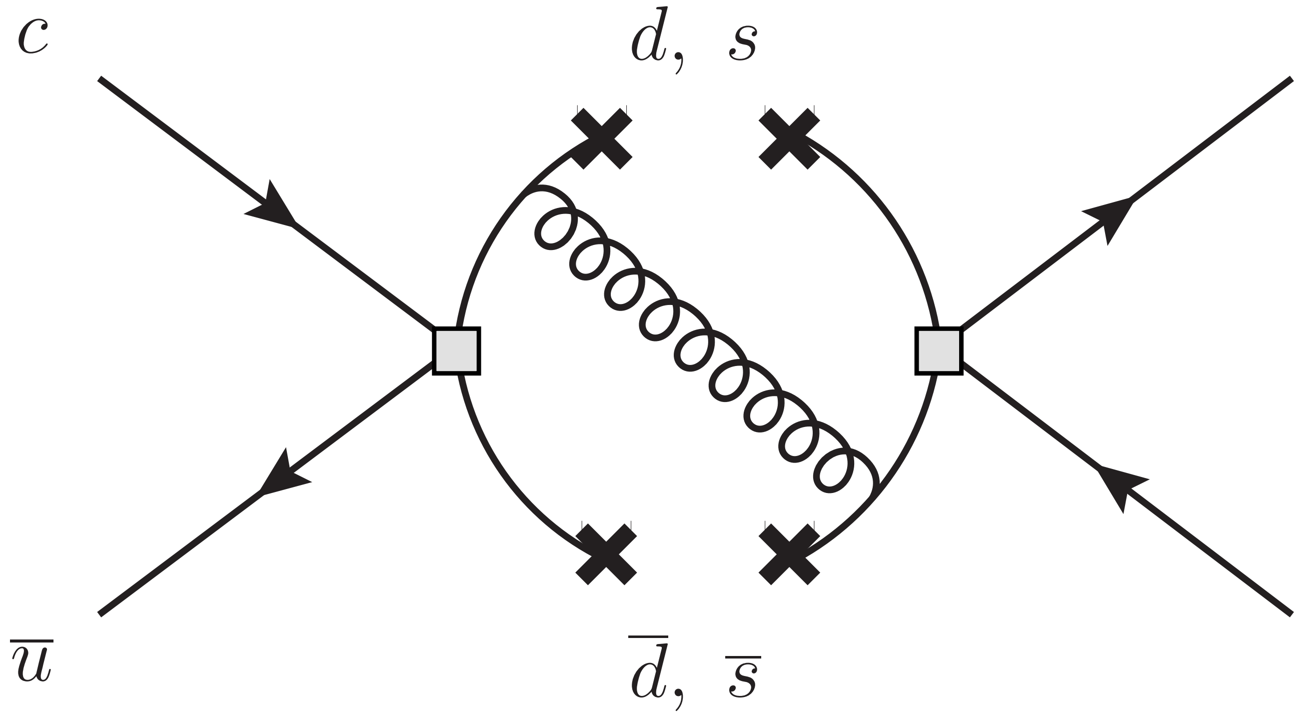

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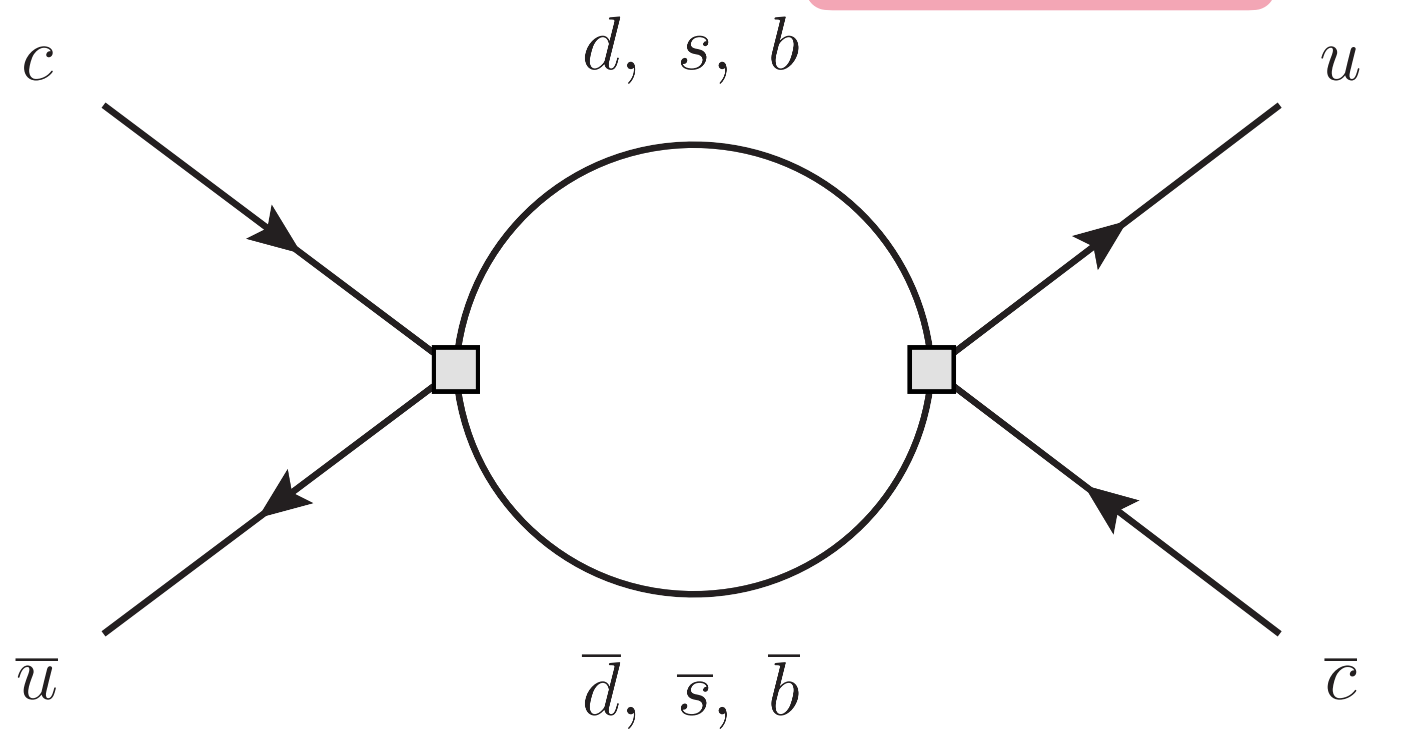

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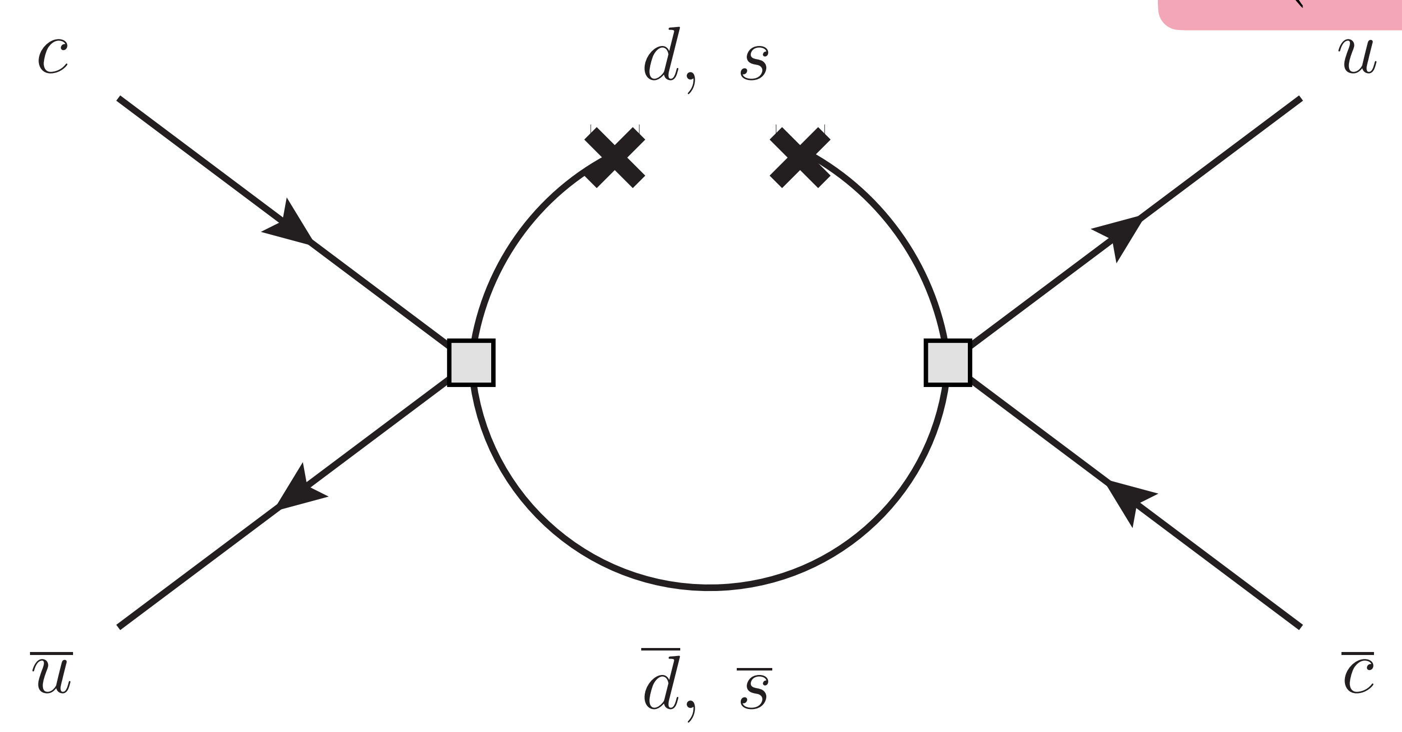

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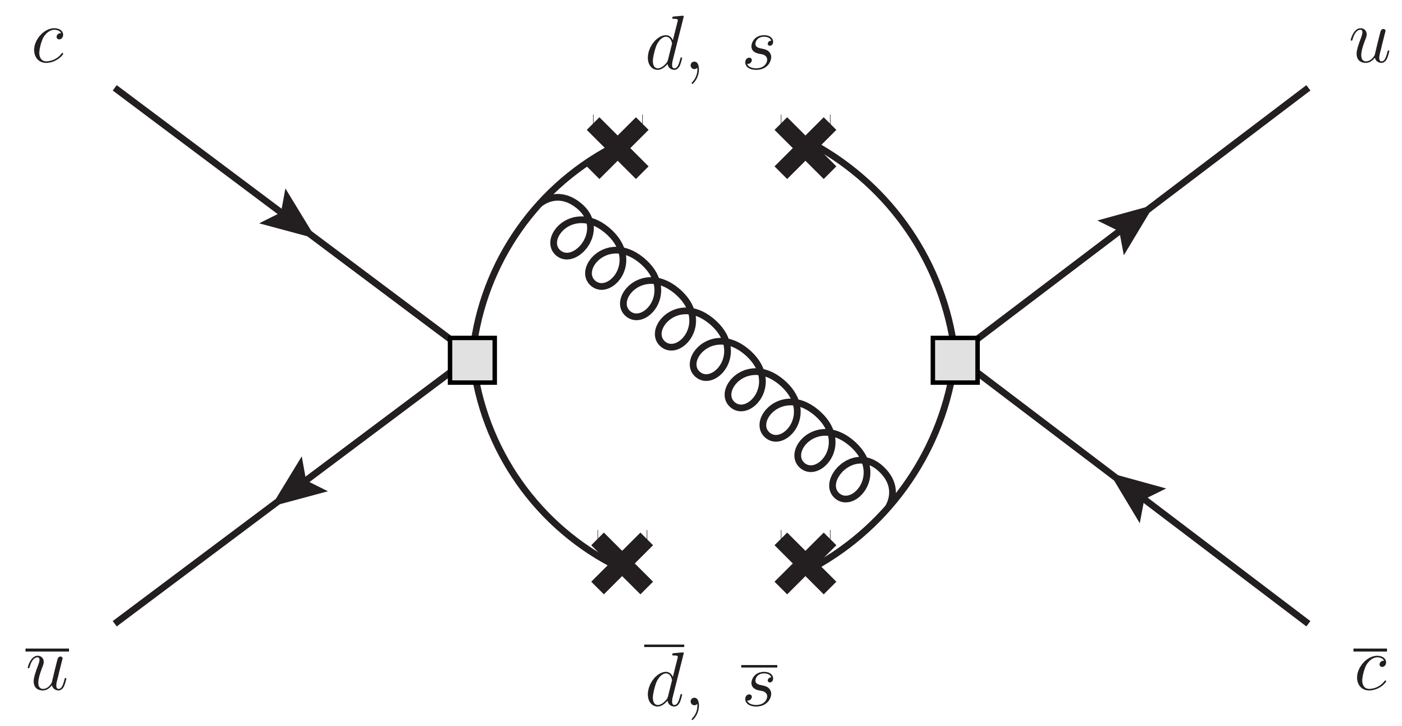

- we trade a power of m_s/m_c suppression for a suppression of the **higher dimensional operator**

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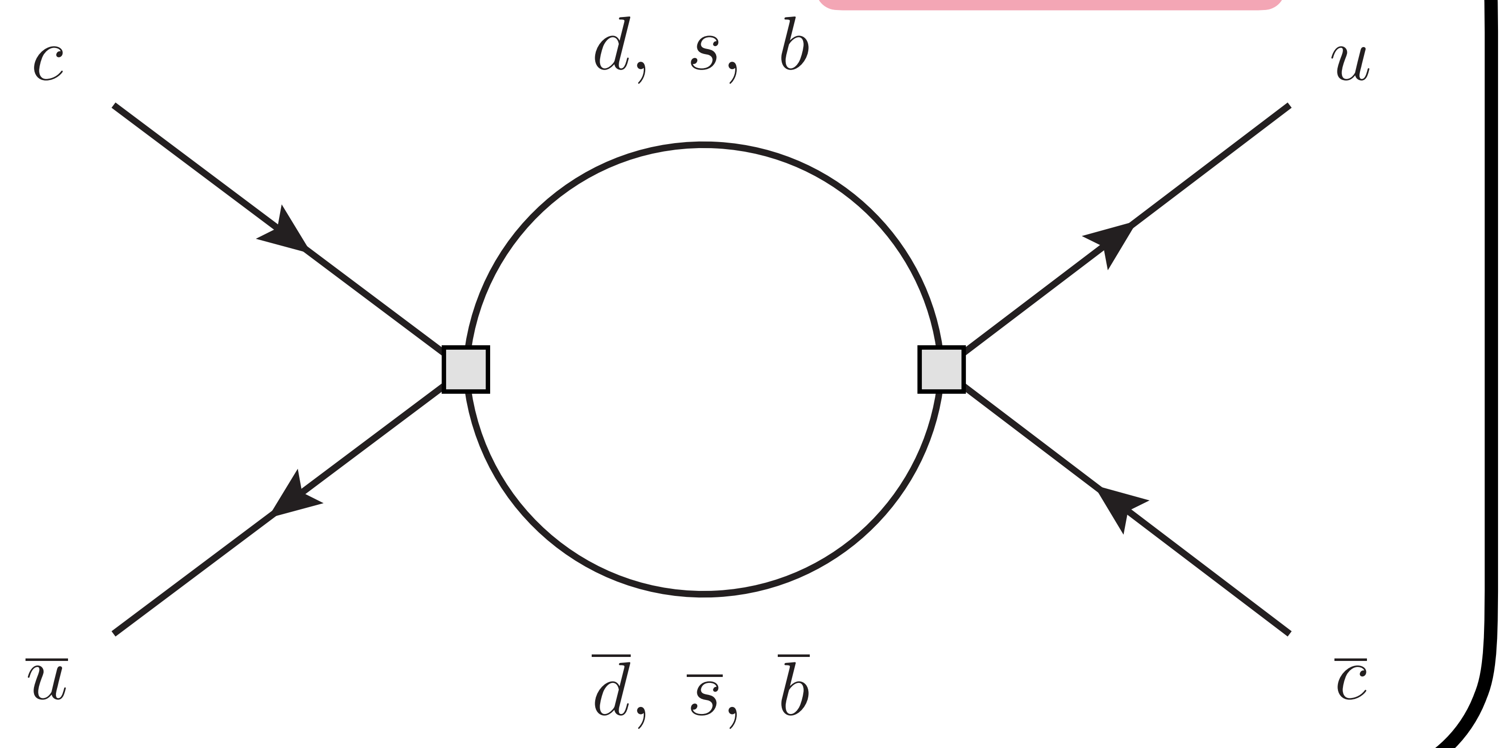

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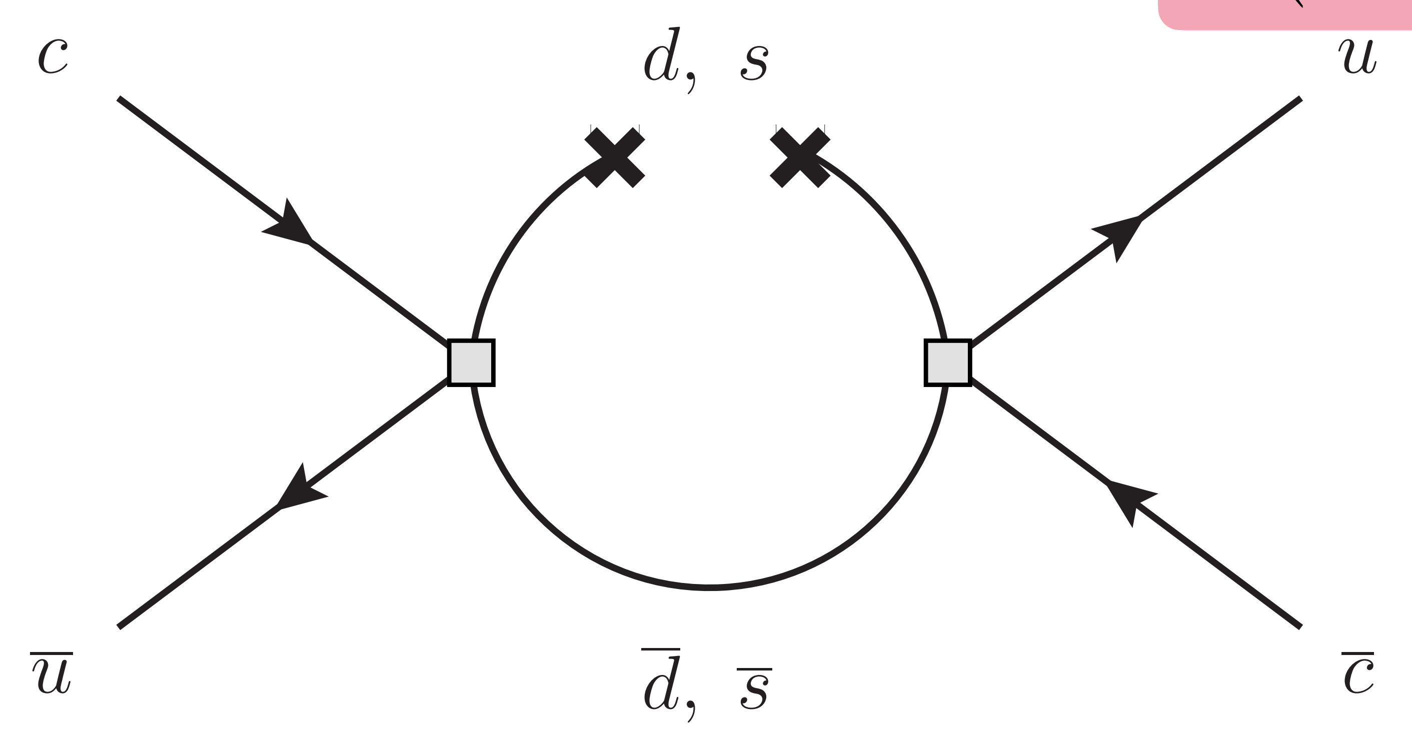

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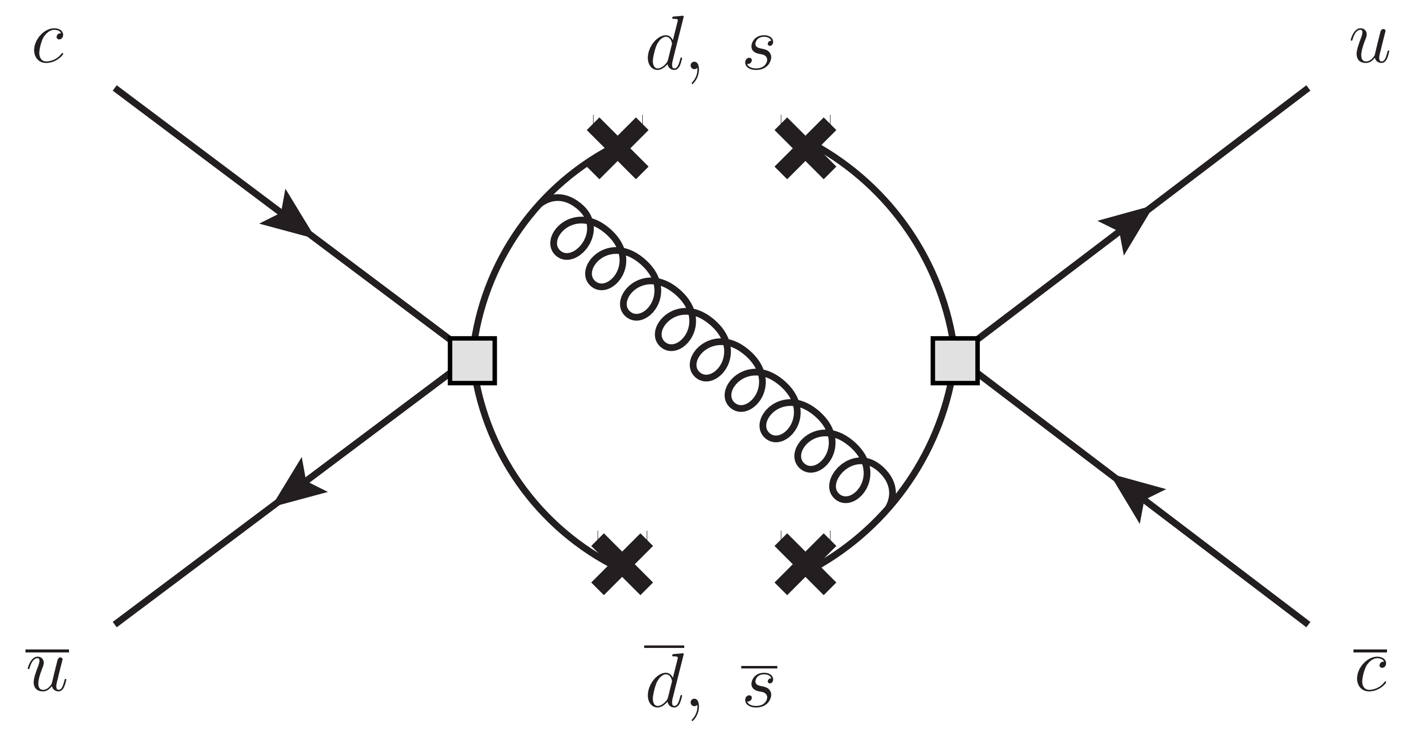

- we trade a power of m_s/m_c suppression for a suppression of the **higher dimensional operator**
- since the condensate diagram is not a loop diagram we expect a **relative $16\pi^2$ enhancement**

□ indicates weak effective vertex

QCD condensates

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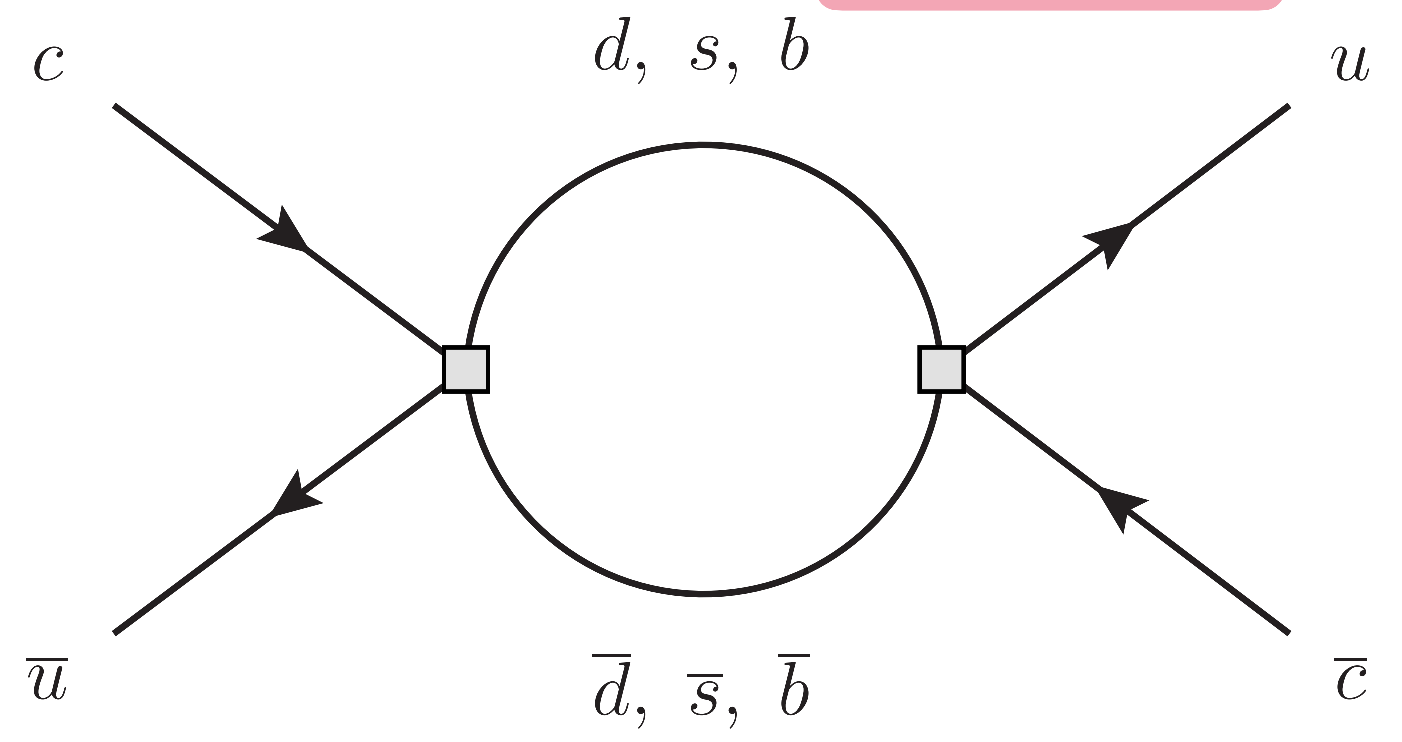

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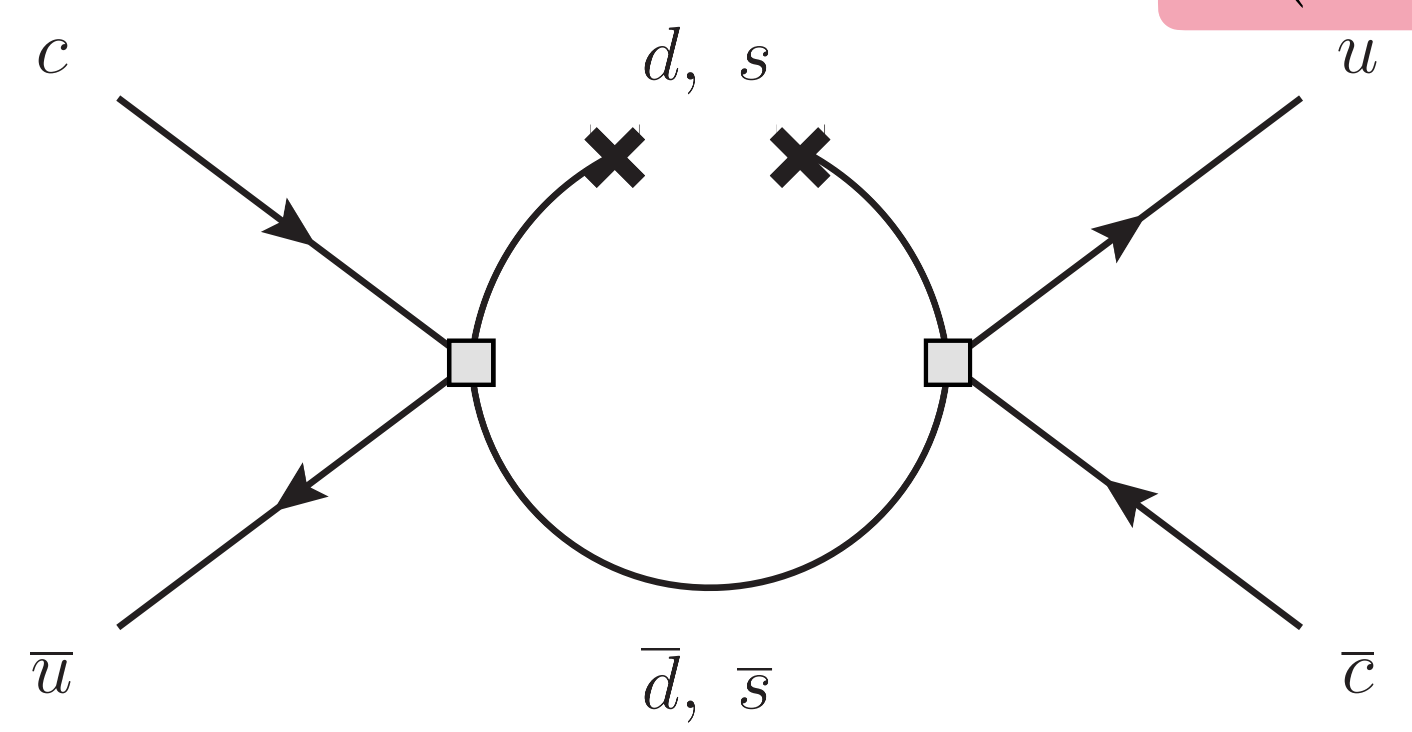

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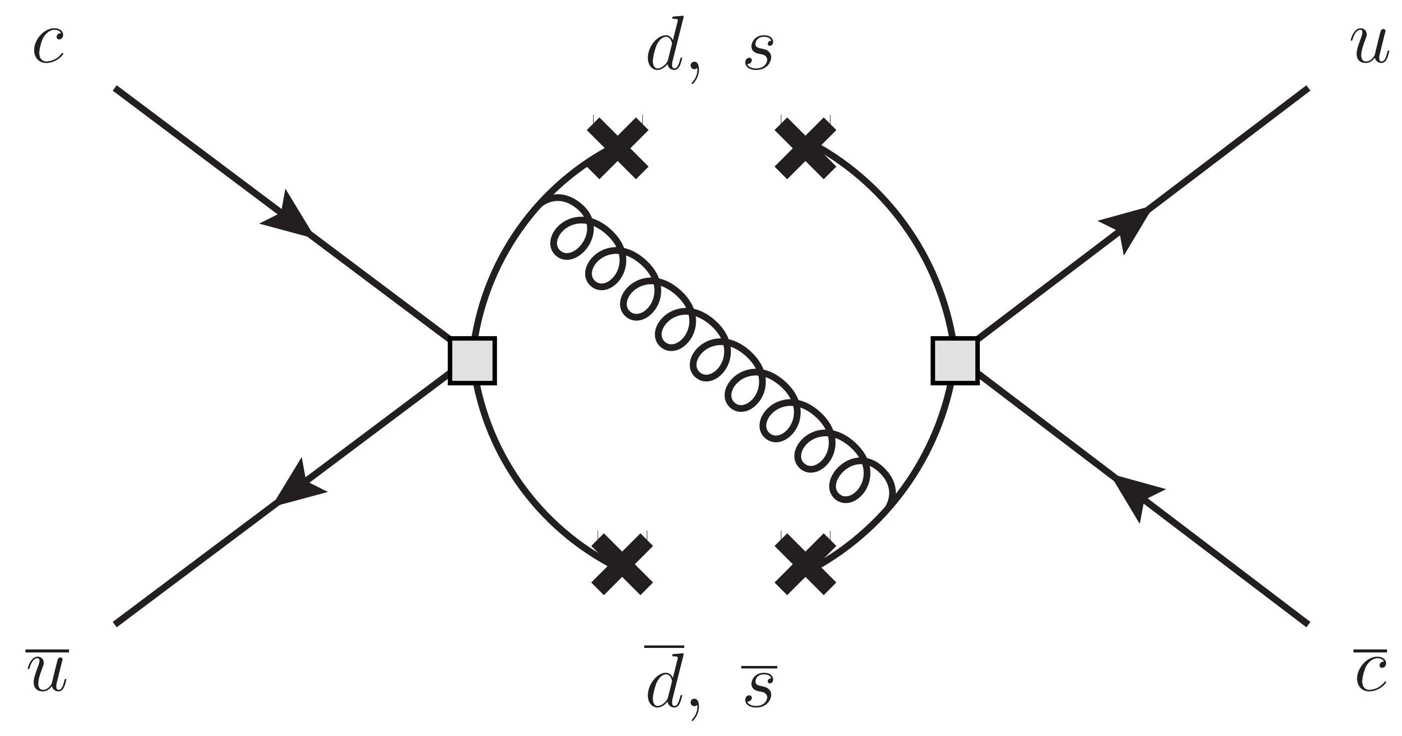

- we trade a power of m_s/m_c suppression for a suppression of the **higher dimensional operator**
- since the condensate diagram is not a loop diagram we expect a **relative $16\pi^2$ enhancement**
- suppression of the higher dimensional operators is in powers of $1/m_c$ which is **expected to be relatively weak**

□ indicates weak effective vertex

QCD condensates

Nonlocal condensates expanded in terms of local condensates

QCD condensates

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QCD condensates

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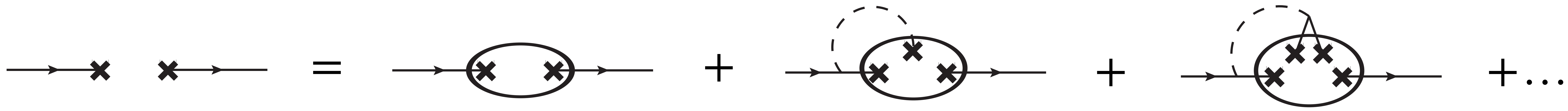

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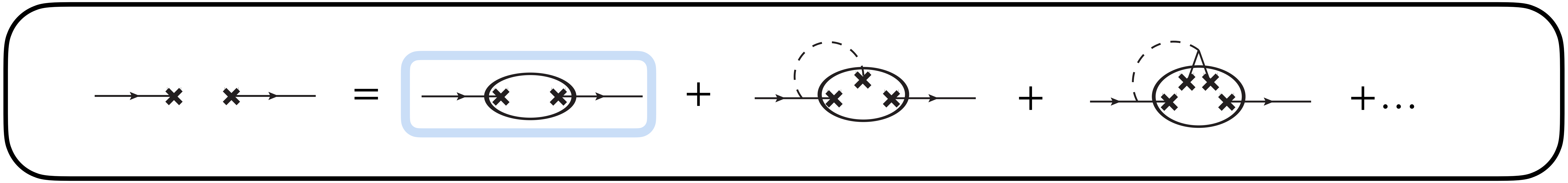

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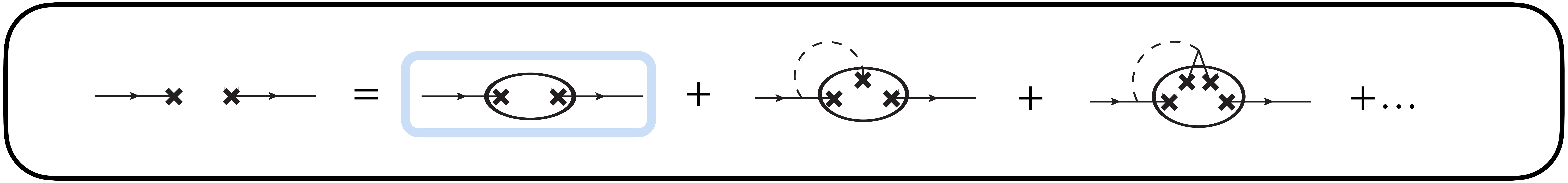

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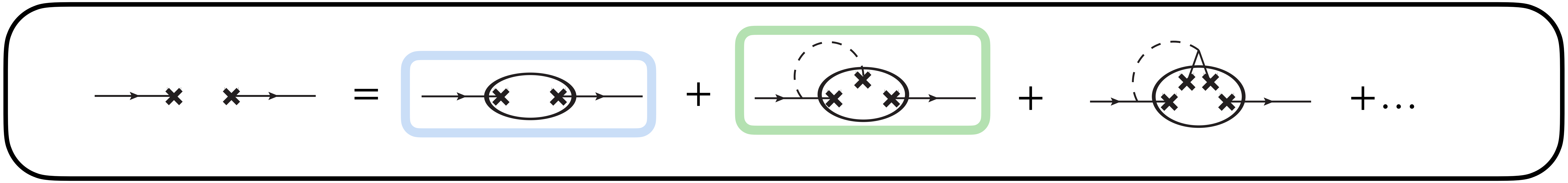

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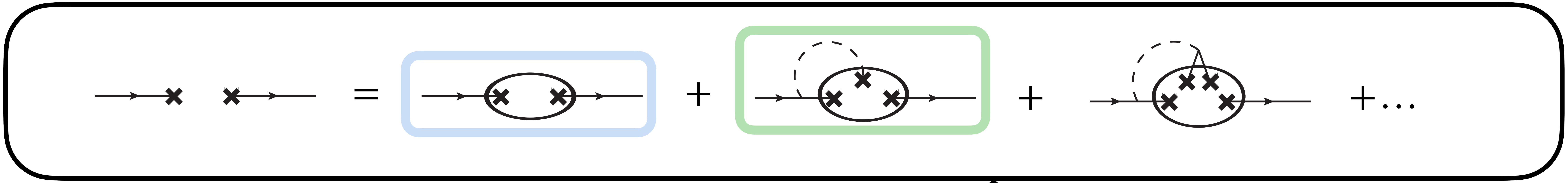

$\propto \langle \overline{q}q \rangle_0$ $\qquad$ $\propto \langle \overline{q}\sigma \cdot Gq \rangle_0 \sim \lambda_q^2$

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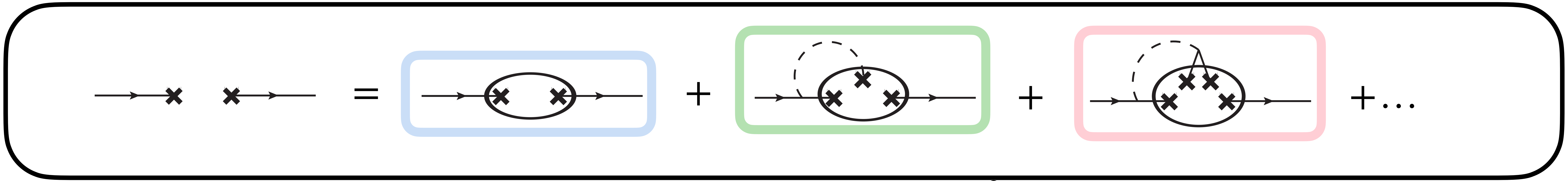

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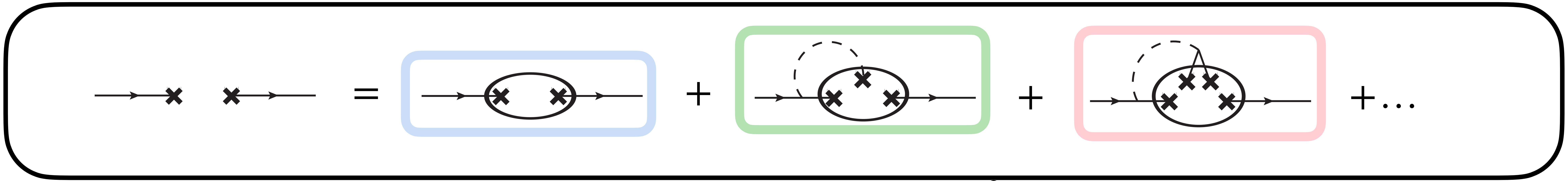

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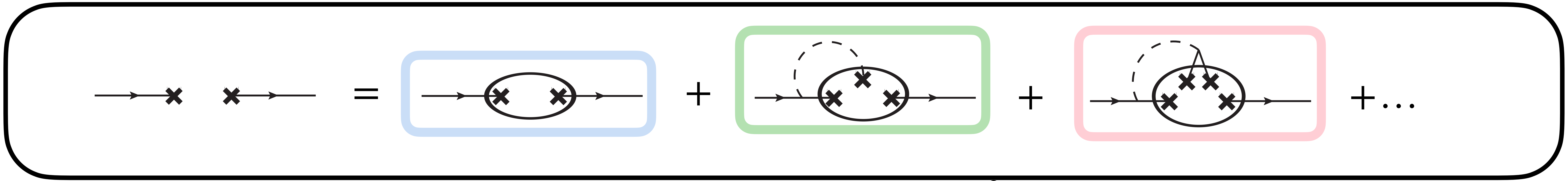

$$\propto \langle \bar{q}q\rangle_0 \qquad \propto \langle \bar{q}\sigma\cdot Gq\rangle_0 \sim \lambda_q^2 \qquad \propto \langle \bar{q}q\rangle_0^2$$

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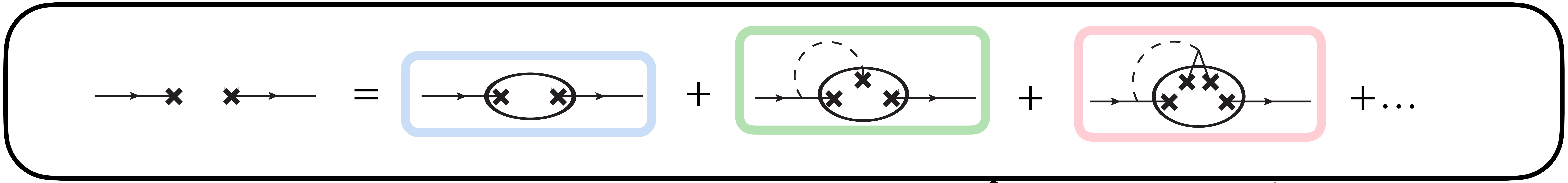

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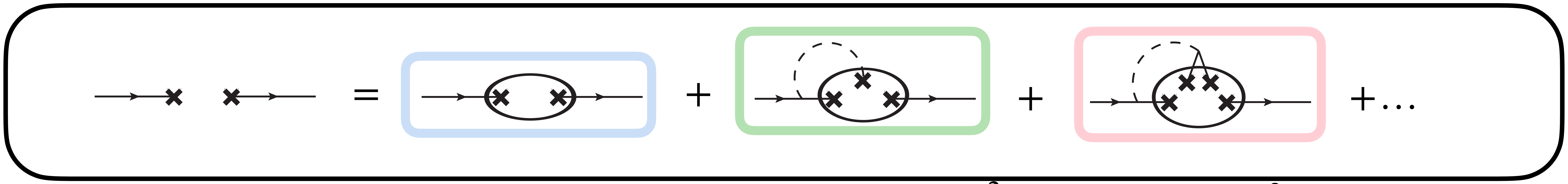

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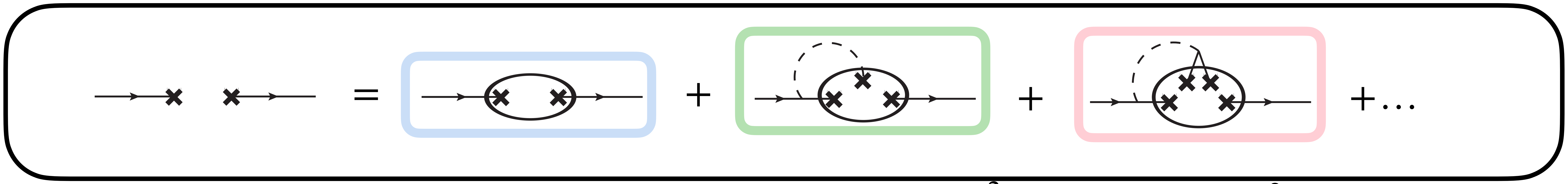

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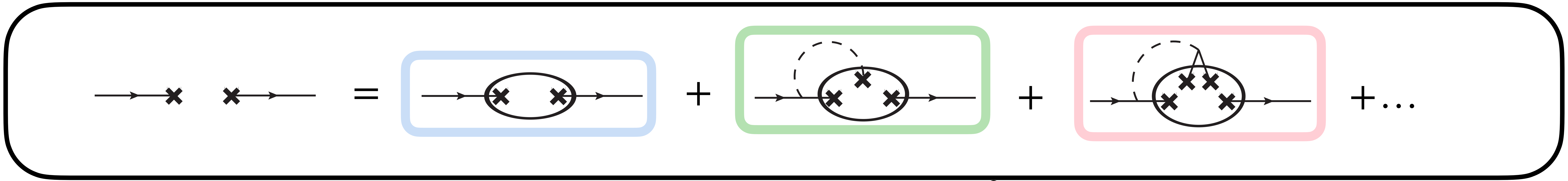

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due to Dirac structure, only the vector terms contribute

$\propto \gamma^\mu$

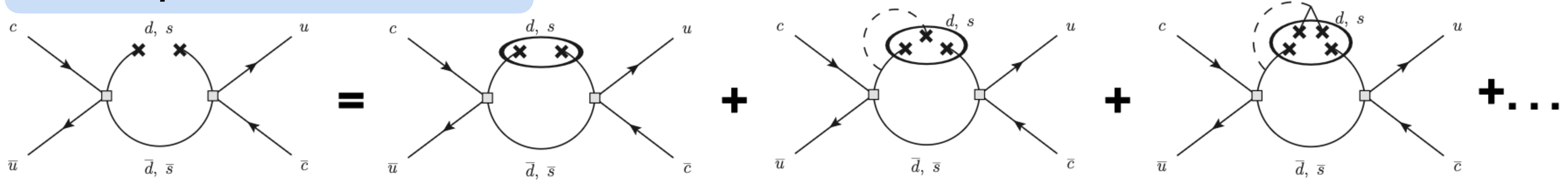
Contributions of QCD condensates to $D\overline{D}$ mixing

Nonlocal condensates expanded in terms of local condensates

Contributions of QCD condensates to $D\overline{D}$ mixing

Nonlocal condensates expanded in terms of local condensates

Dim-9 operator contribution



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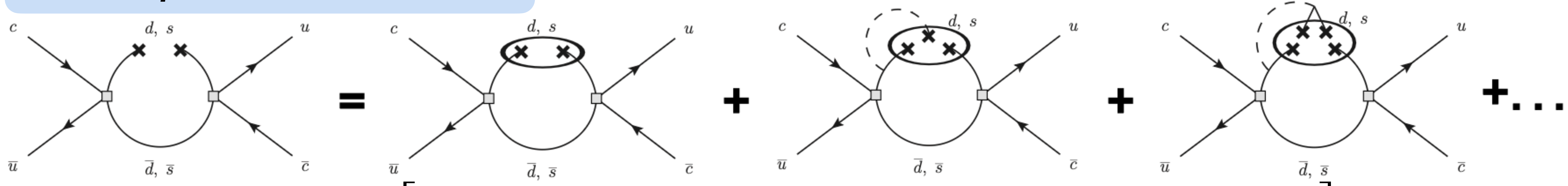
Nonlocal condensates expanded in terms of local condensates

Dim-9 operator contribution



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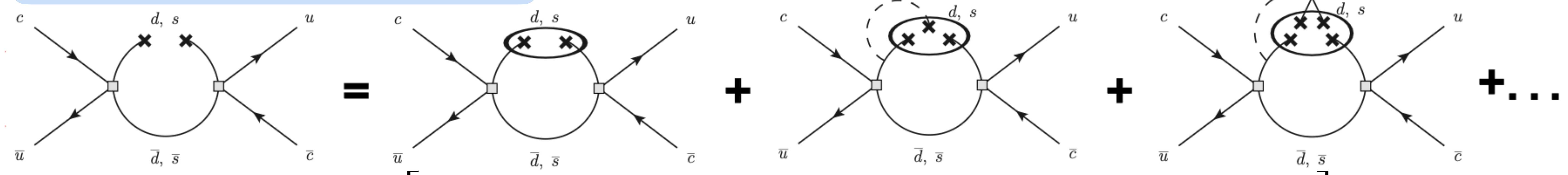
Contributions of QCD condensates to $D\overline{D}$ mixing

Nonlocal condensates expanded in terms of local condensates

Dim-9 operator contribution



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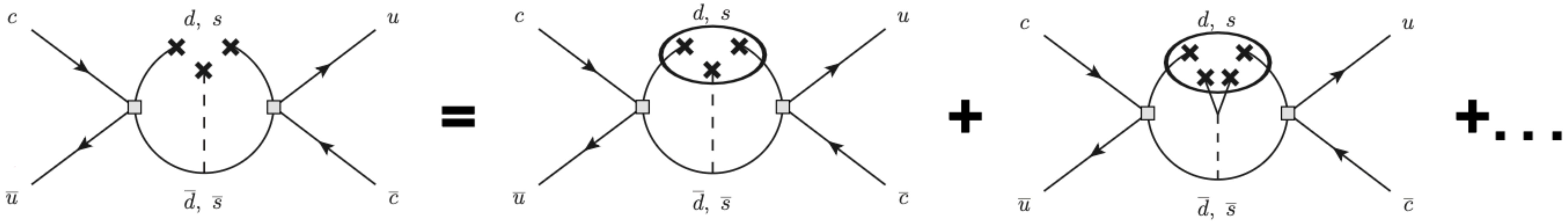
Dim-11 operator contribution



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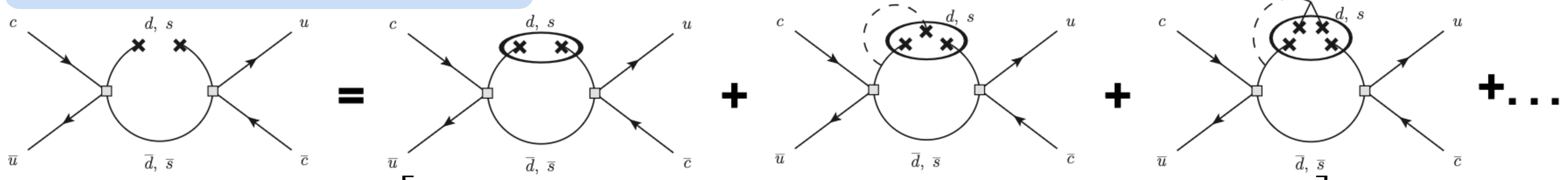
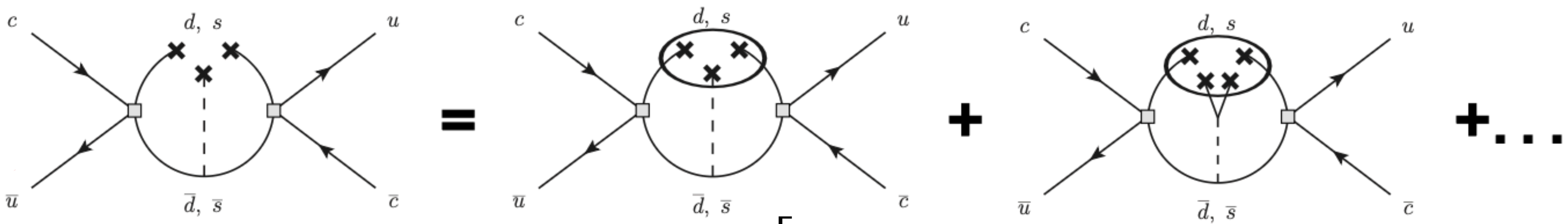
Nonlocal condensates expanded in terms of local condensates

Dim-9 operator contribution



$$\langle \bar{q}^a_\alpha(x) q^b_\beta(0)\rangle = \delta^{ab}\frac{\langle \bar{q}q\rangle_0}{12}\left[\delta_{\alpha\beta}\left(1+\frac{x^2}{8}\lambda_q^2+\dots\right)+i\not{x}_{\beta\alpha}\left(\frac{m}{4}+\frac{x^2}{4}\left(\frac{m\lambda_q^2}{12}-\frac{2}{81}\pi\alpha_s^{NP}\frac{\langle \bar{q}q\rangle_0^2}{\langle \bar{q}q\rangle_0}\right)+\dots\right)\right]$$

Dim-11 operator contribution



$$\langle \bar{q}^a_\alpha(x) G^{cd}_{\mu\nu}(0) q^b_\beta(0)\rangle = \frac{1}{384}\left(\delta^{bd}\delta^{ac}-\frac{1}{3}\delta^{cd}\delta^{ab}\right)\left[\left(\sigma_{\mu\nu}+m\left(i\sigma_{\mu\nu}\not{x}+\gamma_\mu x_\nu-\gamma_\nu x_\mu\right)\right)_{\beta\alpha}\lambda_q^2\langle \bar{q}q\rangle+\frac{i}{2}\left(\not{x}\sigma_{\mu\nu}\right)_{\beta\alpha}\frac{16}{9}\pi\alpha_s^{\mathrm{NP}}\langle \bar{q}q\rangle^2+\dots\right]$$

Contributions of QCD condensates to $D\overline{D}$ mixing

Nonlocal condensates expanded in terms of local condensates

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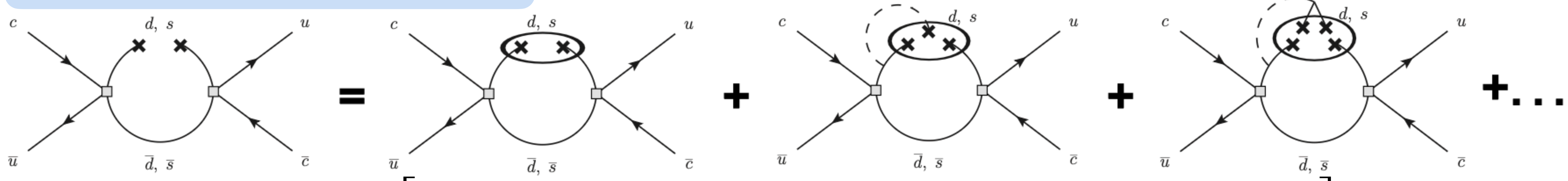

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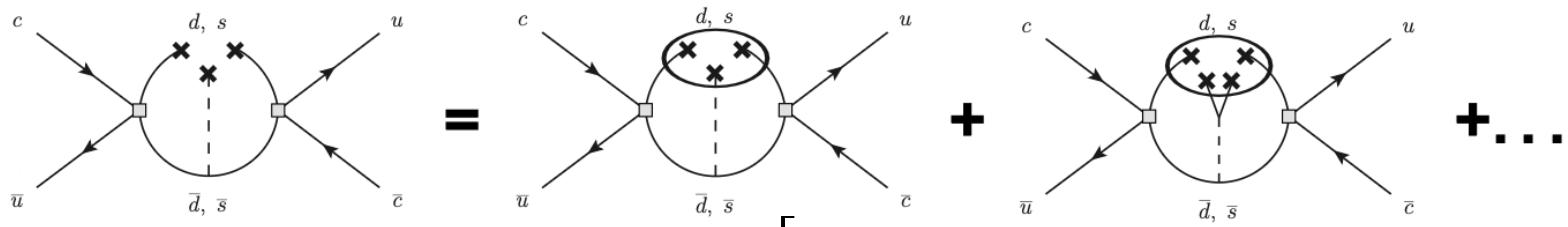

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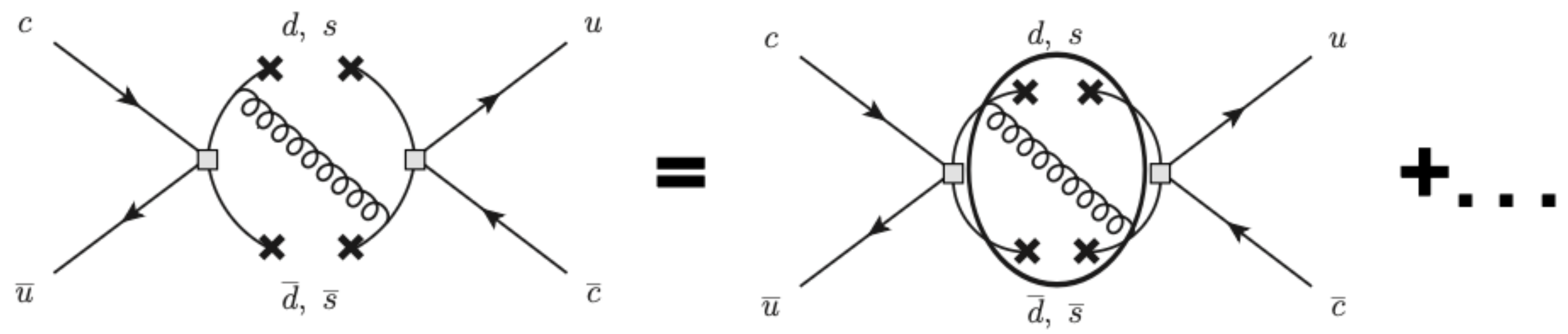
Dim-12 operator contribution



Contributions of QCD condensates to $D\overline{D}$ mixing

Nonlocal condensates expanded in terms of local condensates

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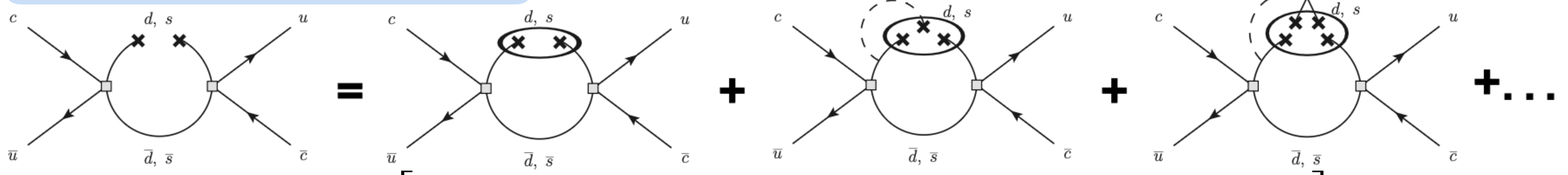

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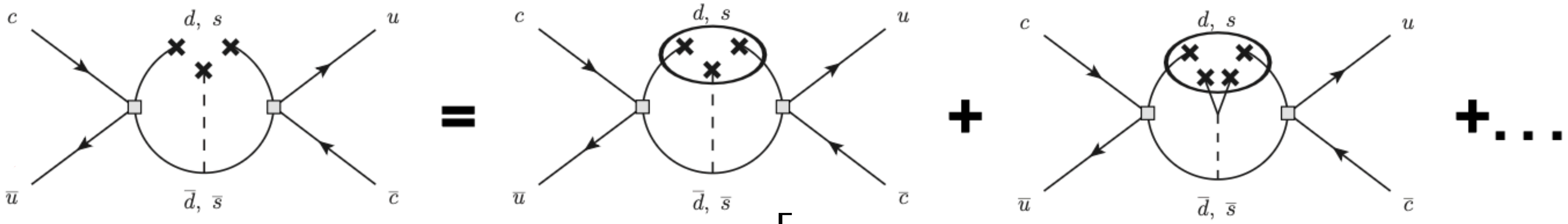

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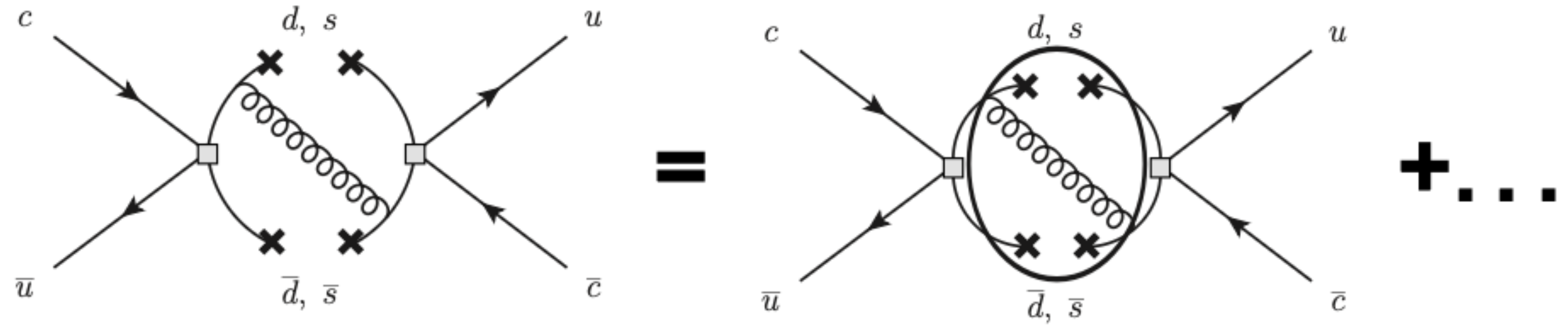

second factorization

$$\langle \bar{q}^a_1(x)_\alpha \bar{q}^d_4(y)_\delta q^c_3(z)_\gamma q^b_2(0)_\beta\rangle = \langle \bar{q}^a_1(x)_\alpha q^b_2(0)_\beta\rangle\langle \bar{q}^d_4(y)_\delta q^c_3(z)_\gamma\rangle - \langle \bar{q}^a_1(x)_\alpha q^c_3(z)_\gamma\rangle\langle \bar{q}^d_4(y)_\delta q^b_2(0)_\beta\rangle$$

Complexity of four-quark contribution

There is sixteen ways to connect the diagram

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- Note in weak vertices you find O_1 and O_2 operators which contract color in different ways
- All together this multitude of different contractions sums to an 'unpredictable result'

$$O_1 = \left(\overline{u}^i \gamma^\mu (1-\gamma^5) q^j\right)\left(\overline{q}^k \gamma_\mu (1-\gamma^5) c^l\right) \delta^{ij}\delta^{kl}$$

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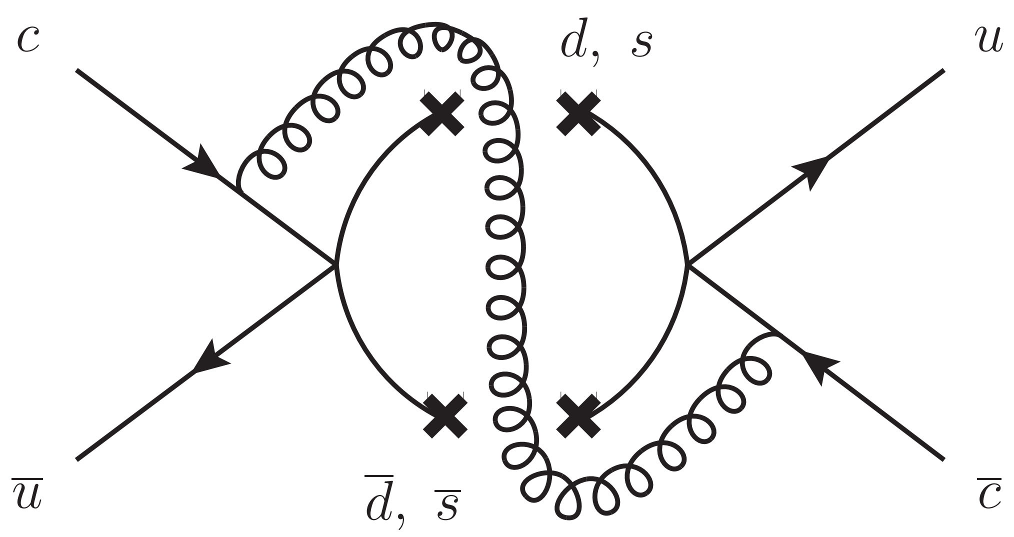

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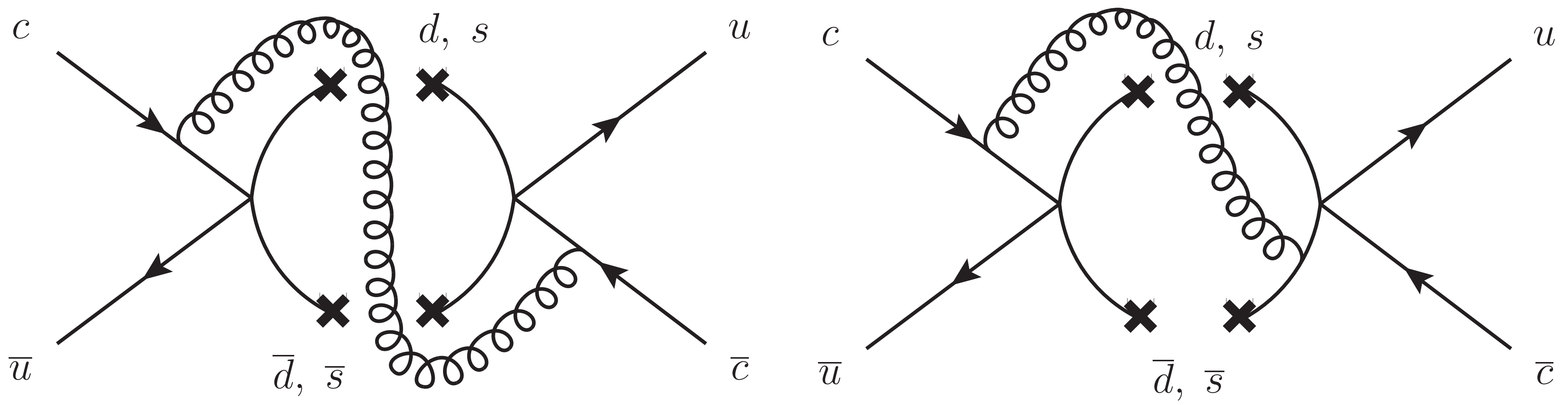

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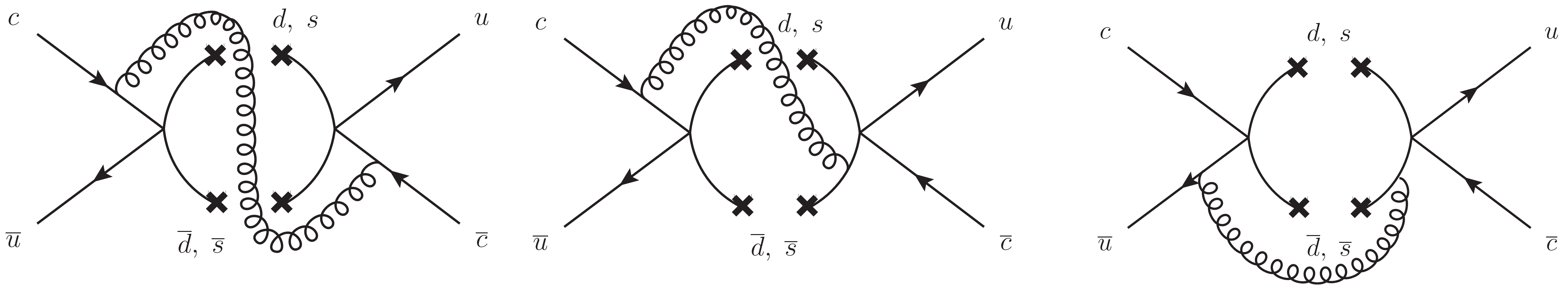

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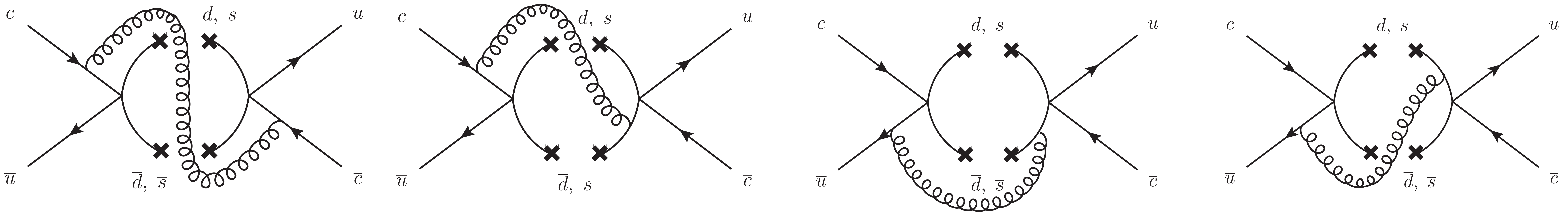

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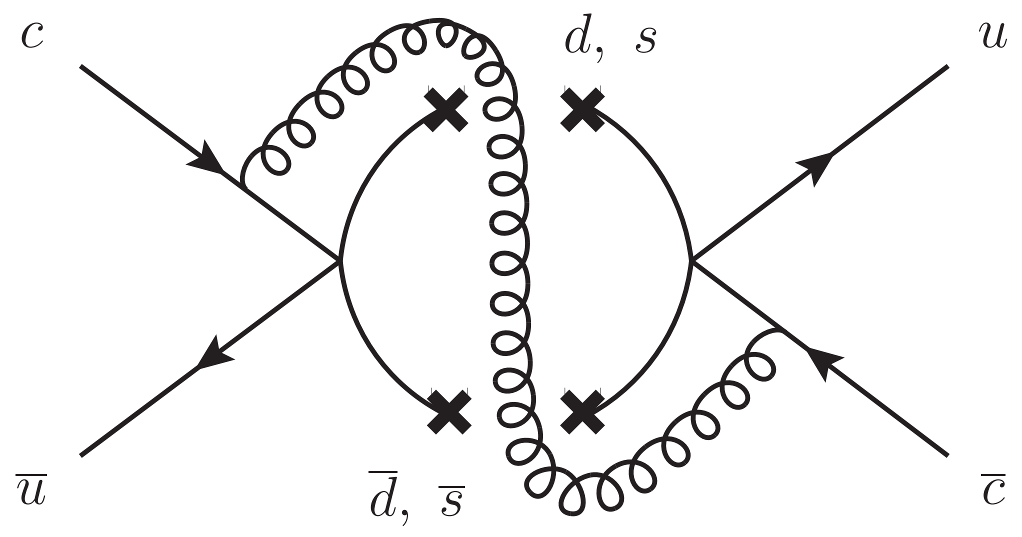

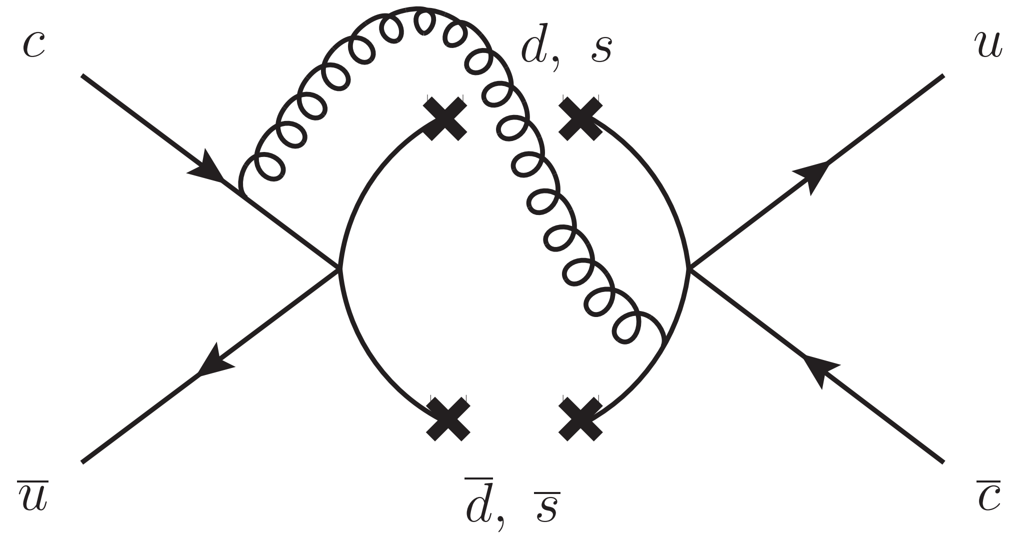

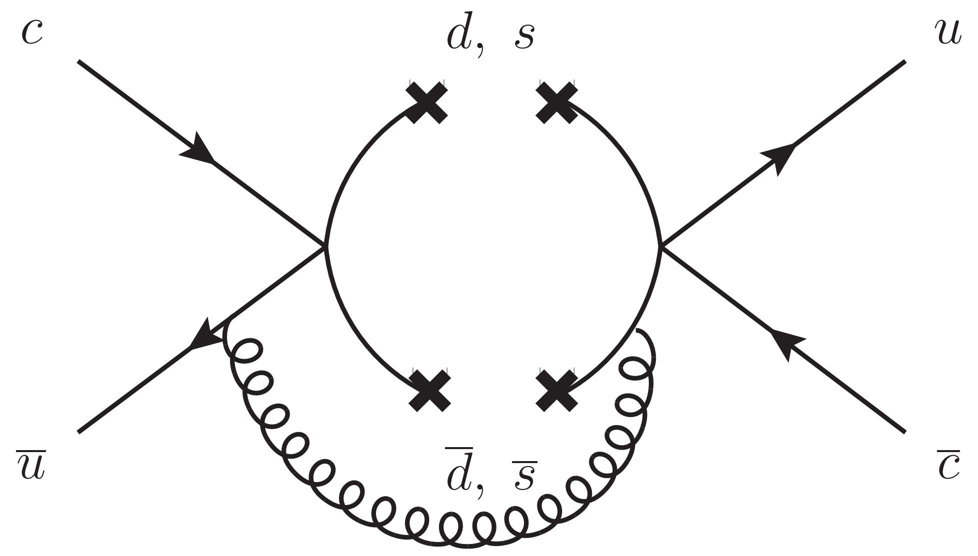

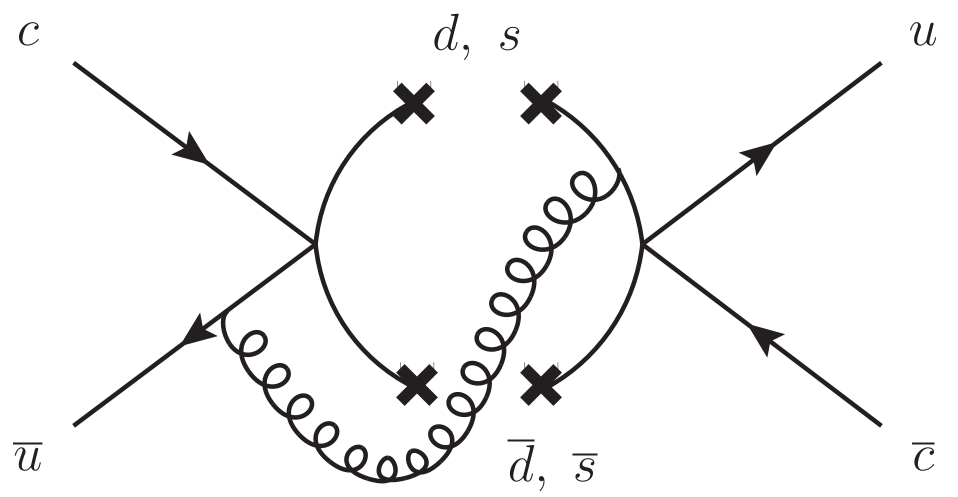

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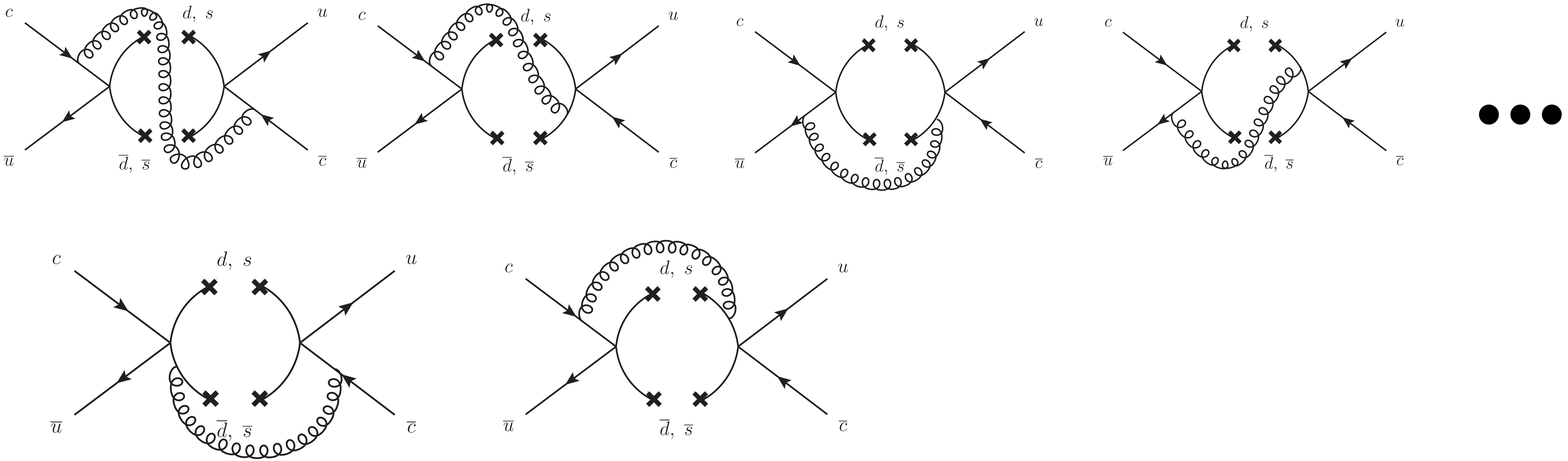

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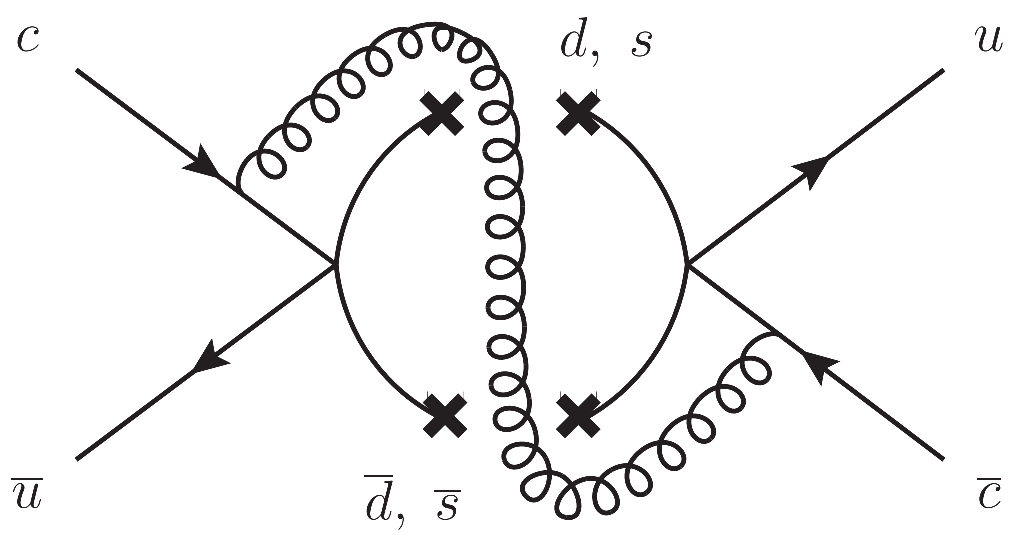

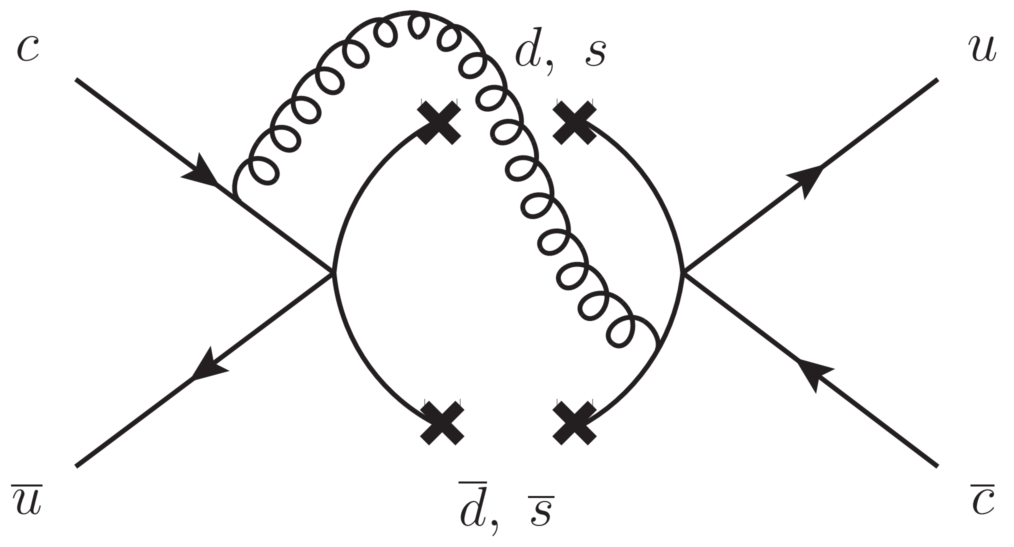

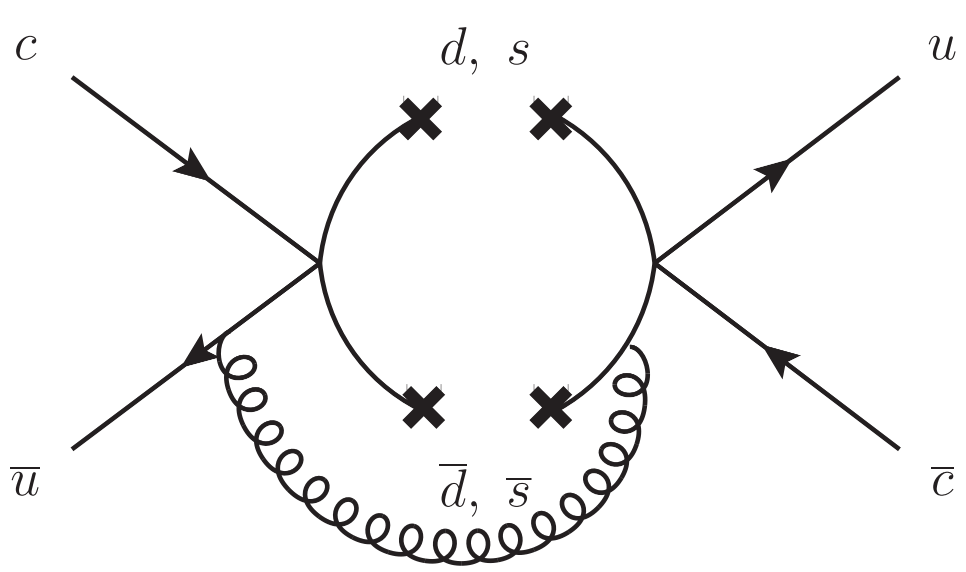

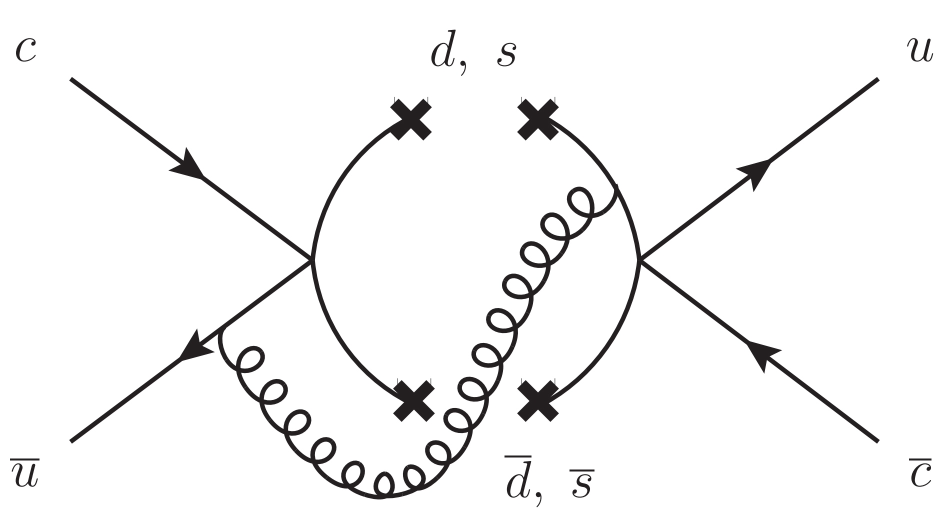

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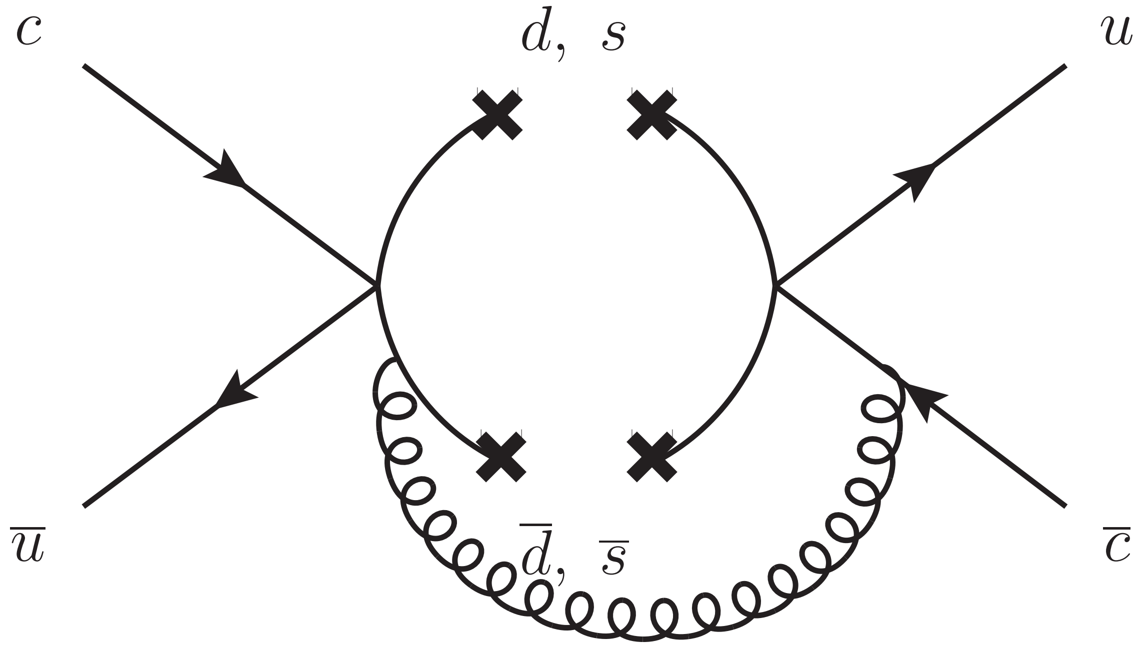

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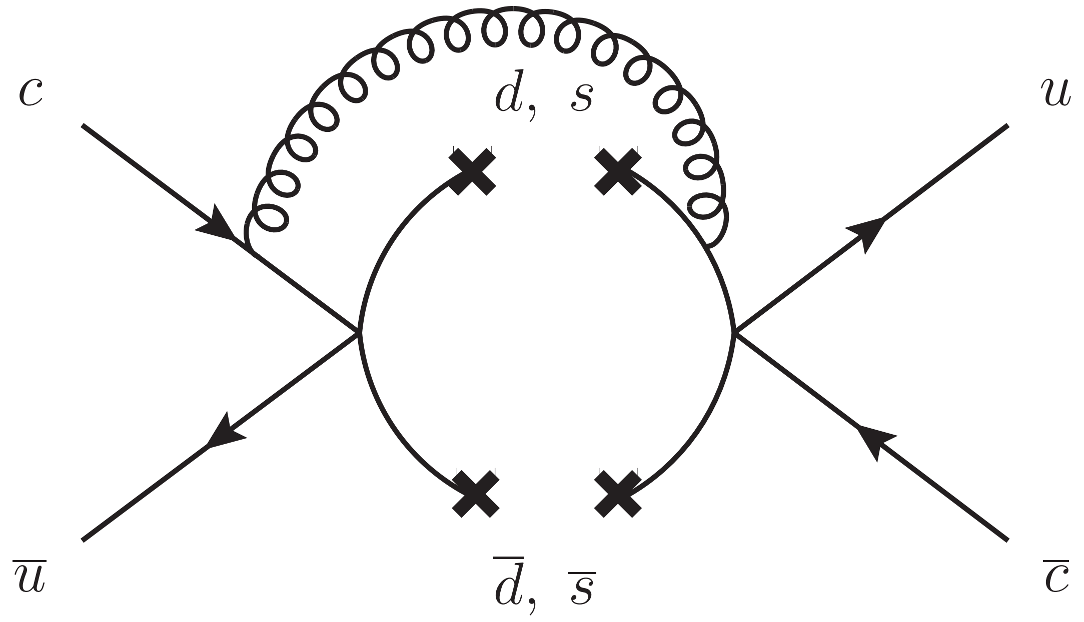

some simplifications are possible

CP symmetry

Final results

Analytic results (leading contributions)

Final results

Analytic results (leading contributions)

Perturbative contribution

$$x_D^{\mathrm{LO}} = -\frac{G_F^2}{2}\frac{m_c^2}{2M_D\Gamma_D}\xi_s^2 m_c^2 \frac{m_s^4}{m_c^4}\frac{1}{2\pi^2}\left(-C_1^2(2\langle O_{S-P}\rangle + \langle O_{V-A}\rangle) + 4C_1C_2\langle O_{S-P}\rangle + 6C_2^2\langle O_{S-P}\rangle\right)$$

Final results

Analytic results (leading contributions)

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Condensate contributions

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$$x_D^{\langle \bar{q}q\rangle} = -\frac{G_F^2}{2}\frac{m_c^2}{2M_D\Gamma_D}\xi_s^2 \frac{\langle \bar{s}s\rangle}{m_c^3}\frac{m_s^3}{m_c^3}\frac{2}{3}\left(C_1^2 - 2C_1C_2 - 3C_2^2\right)\left(4\langle O_{S-P}\rangle + \langle O_{V-A}\rangle\right)$$

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$$x_D^{\mathrm{LO}} = -\frac{G_F^2}{2}\frac{m_c^2}{2M_D\Gamma_D}\xi_s^2 m_c^2 \frac{m_s^4}{m_c^4}\frac{1}{2\pi^2}\left(-C_1^2(2\langle O_{S-P}\rangle + \langle O_{V-A}\rangle) + 4C_1C_2\langle O_{S-P}\rangle + 6C_2^2\langle O_{S-P}\rangle\right)$$

Condensate contributions

$$x_D^{\langle \bar{q}q\rangle} = -\frac{G_F^2}{2}\frac{m_c^2}{2M_D\Gamma_D}\xi_s^2 \frac{\langle \bar{s}s\rangle}{m_c^3}\frac{m_s^3}{m_c^3}\frac{2}{3}\left(C_1^2 - 2C_1C_2 - 3C_2^2\right)\left(4\langle O_{S-P}\rangle + \langle O_{V-A}\rangle\right)$$

$$x_D^{\langle \bar{q}Gq\rangle} = -\frac{G_F^2}{2}\frac{m_c^2}{2M_D\Gamma_D}\xi_s^2\left(\frac{\lambda_q^2}{m_c^2}\frac{\langle \bar{s}s\rangle}{m_c^3}\frac{m_s^3}{m_c^3} + \frac{8}{9}\pi\alpha_s^{NP}\frac{\langle \bar{s}s\rangle^2 - \langle \bar{d}d\rangle^2}{m_c^6}\frac{m_s^2}{m_c^2}\right)\frac{1}{6}\left(C_1^2\langle O_{V-A}\rangle + 8\langle O_{S-P}\rangle(C_1^2 + 4C_1C_2 + 6C_2^2)\right)$$

$$x_D^{\langle \bar{q}q\bar{q}q\rangle} = -\frac{G_F^2}{2}\frac{m_c^2}{2M_D\Gamma_D}\xi_s^2\frac{\langle \bar{s}s\rangle^2}{m_c^6}\frac{m_s^2}{m_c^2}\pi\alpha_s(\mu)\frac{1}{108}\left((104\langle O_{S-P}\rangle - 27\langle O_{V-A}\rangle)C_1^2 + 4C_1C_2(904\langle O_{S-P}\rangle + 129\langle O_{V-A}\rangle) + 9(440\langle O_{S-P}\rangle + 137\langle O_{V-A}\rangle)C_2^2\right)$$

Final results

Analytic results (leading contributions)

Perturbative contribution

$$x_D^{\mathrm{LO}} = -\frac{G_F^2}{2}\frac{m_c^2}{2M_D\Gamma_D}\xi_s^2 m_c^2 \frac{m_s^4}{m_c^4}\frac{1}{2\pi^2}\left(-C_1^2(2\langle O_{S-P}\rangle + \langle O_{V-A}\rangle) + 4C_1C_2\langle O_{S-P}\rangle + 6C_2^2\langle O_{S-P}\rangle\right)$$

matrix elements

$$\langle O_{V-A}\rangle = \frac{8}{3}f_D^2 B_D M_D^2 \quad \langle O_{S-P}\rangle = -\frac{5}{3}f_D^2 B_D^{(S)}\frac{M_D^2}{m_c^2}M_D^2$$

Condensate contributions

$$x_D^{\langle\bar{q}q\rangle} = -\frac{G_F^2}{2}\frac{m_c^2}{2M_D\Gamma_D}\xi_s^2\frac{\langle\bar{s}s\rangle}{m_c^3}\frac{m_s^3}{m_c^3}\frac{2}{3}\left(C_1^2 - 2C_1C_2 - 3C_2^2\right)\left(4\langle O_{S-P}\rangle + \langle O_{V-A}\rangle\right)$$

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Condensate contributions

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parametrically leading!

Final results

Analytic results (leading contributions)

Perturbative contribution

$$x_D^{\mathrm{LO}} = -\frac{G_F^2}{2}\frac{m_c^2}{2M_D\Gamma_D}\xi_s^2 m_c^2 \frac{m_s^4}{m_c^4}\frac{1}{2\pi^2}\left(-C_1^2(2\langle O_{S-P}\rangle + \langle O_{V-A}\rangle) + 4C_1C_2\langle O_{S-P}\rangle + 6C_2^2\langle O_{S-P}\rangle\right)$$

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Condensate contributions

$$x_D^{\langle \bar{q}q\rangle} = -\frac{G_F^2}{2}\frac{m_c^2}{2M_D\Gamma_D}\xi_s^2 \frac{\langle \bar{s}s\rangle}{m_c^3}\frac{m_s^3}{m_c^3}\frac{2}{3}\left(C_1^2 - 2C_1C_2 - 3C_2^2\right)\left(4\langle O_{S-P}\rangle + \langle O_{V-A}\rangle\right)$$

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parametrically leading!

Falk, Grossman, Ligeti, Petrov, *SU(3) breaking and D0 - anti-D0 mixing, Phys. Rev. D65 (2002) 054034,* **[hep-ph/0110317]**

Georgi, *D - anti-D mixing in heavy quark effective field theory, Phys. Lett. B 297 (1992) 353–357,* **[hep-ph/9209291]**

Final results

Parameters used in evaluation

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Bazavov et al., Short-distance matrix elements for D0-meson mixing for Nf = 2 + 1 lattice QCD, Phys. Rev. D 97 (2018) 034513, **[1706.04622]**.

$$\langle O_{V-A} \rangle = \left(\overline{u}^i \gamma^\mu (1-\gamma^5) q^i \right) \left(\overline{q}^j \gamma_\mu (1-\gamma^5) c^j \right)$$

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The condensate

Gell-Mann-Oakes-Renner relation

Reinders, Rubinstein, Yazaki, *Hadron Properties from QCD Sum Rules,* Phys. Rept., 127:1, 1985

$$\langle \overline{q}q\rangle_0 = -\frac{f_\pi M_\pi}{2}\frac{1}{m_d+m_u}$$

$$\frac{\langle \overline{s}s\rangle}{\langle \overline{d}d\rangle} = 0.8 \pm 0.1$$

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Other parameters

Particle Data Group Collaboration, *Review of particle physics,* Phys. Rev. D 110 (2024) 030001

parameter	value
$\xi_s = V_{us}^* V_{cs}$	$(0.22501 \pm 0.00068)(0.97349 \pm 0.00016)$
G_F	$1.1663788(6) \times 10^{-5}\,\mathrm{GeV}^{-2}$
$\overline{\mathrm{MS}}\, m_c(m_c)$	$1.2730 \pm 0.0046\,\mathrm{GeV}$
$\overline{\mathrm{MS}}\, m_s(2\,\mathrm{GeV})$	$93.5 \pm 0.8\,\mathrm{MeV}$
M_D	$1864.84 \pm 0.05\,\mathrm{GeV}$
Γ_D	$1.604 \times 10^{-12}\,\mathrm{GeV}$
m_d	$0.00470 \pm 0.00007\,\mathrm{GeV}$
$O_{V-A}(3\,\mathrm{GeV})$	0.322 ± 0.022
$O_{S-P}(3\,\mathrm{GeV})$	-0.624 ± 0.007
$\langle\overline{q}q\rangle(1\,\mathrm{GeV})$	$(-263.5\,\mathrm{MeV})^3$

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Chetyrkin, Kuhn, Steinhauser, *RunDec: A Mathematica package for running and decoupling of the strong coupling and quark masses,* Comput. Phys. Commun. 133 (2000) 43–65, **[hep-ph/0004189]**.

We run all parameters to renormalization scale $\mu = 1.3\,\mathrm{GeV}$ and evaluate our results.

Final results

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This is most widely used, but more recent lattice measurements give larger results which disagree with this one

- *Phys.Rev.D 87* ***(2013)*** *3, 034503*
- *Phys.Rev.D 100* ***(2019)*** *3, 034506*

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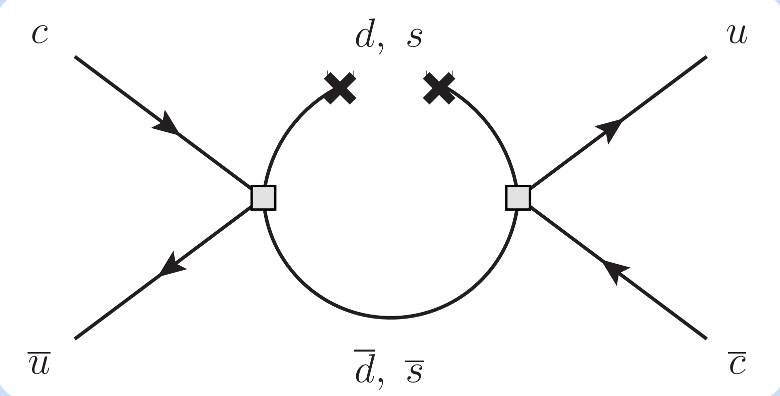
$$x_D^{\langle \overline{q}q \rangle} = (0.99 \pm 0.13) \times 10^{-5}$$



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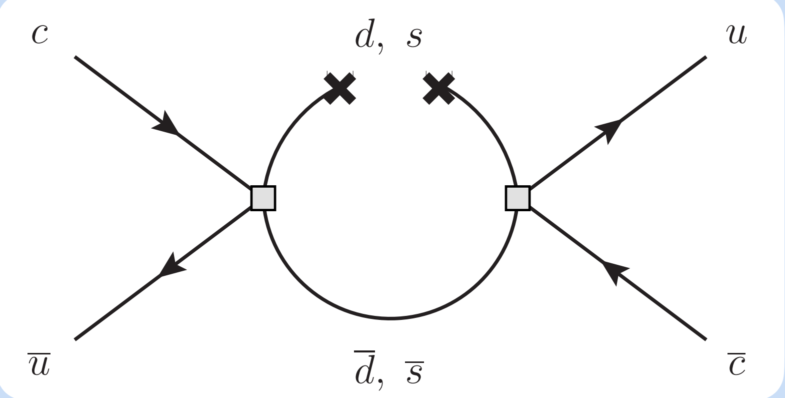
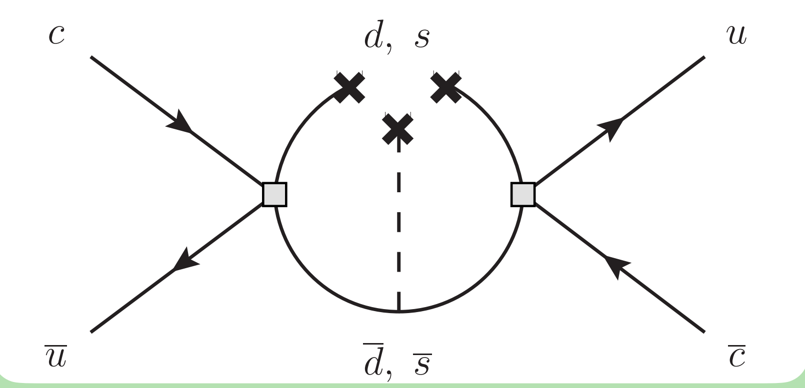


$$x_D^{\langle \overline{q}Gq \rangle} = (0.051 \pm 0.009) \times 10^{-5}$$



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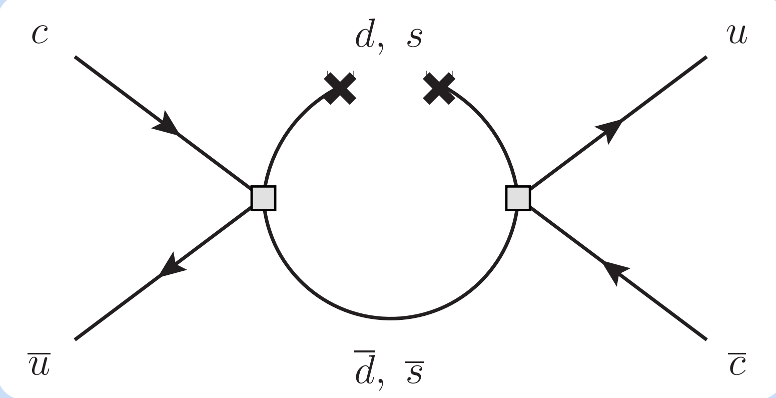
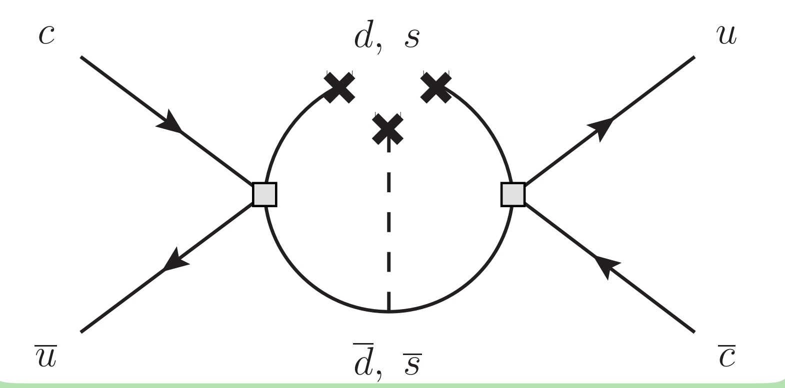
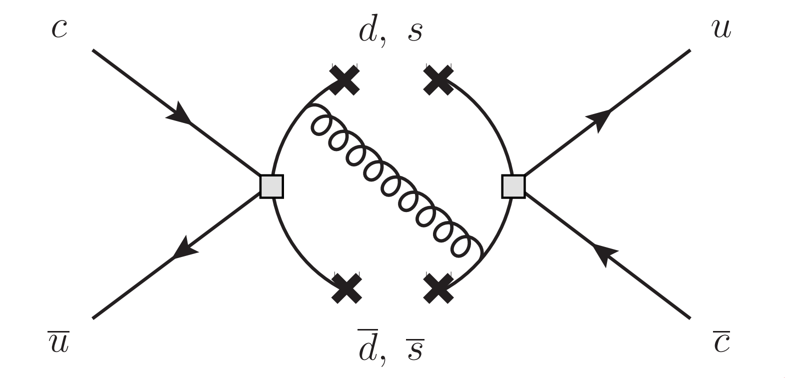
$$x_D^{\langle \bar{q}Gq \rangle} = (0.051 \pm 0.009) \times 10^{-5}$$

$$x_D^{\langle \bar{q}q\bar{q}q \rangle} = (0.24 \pm 0.06) \times 10^{-5}$$







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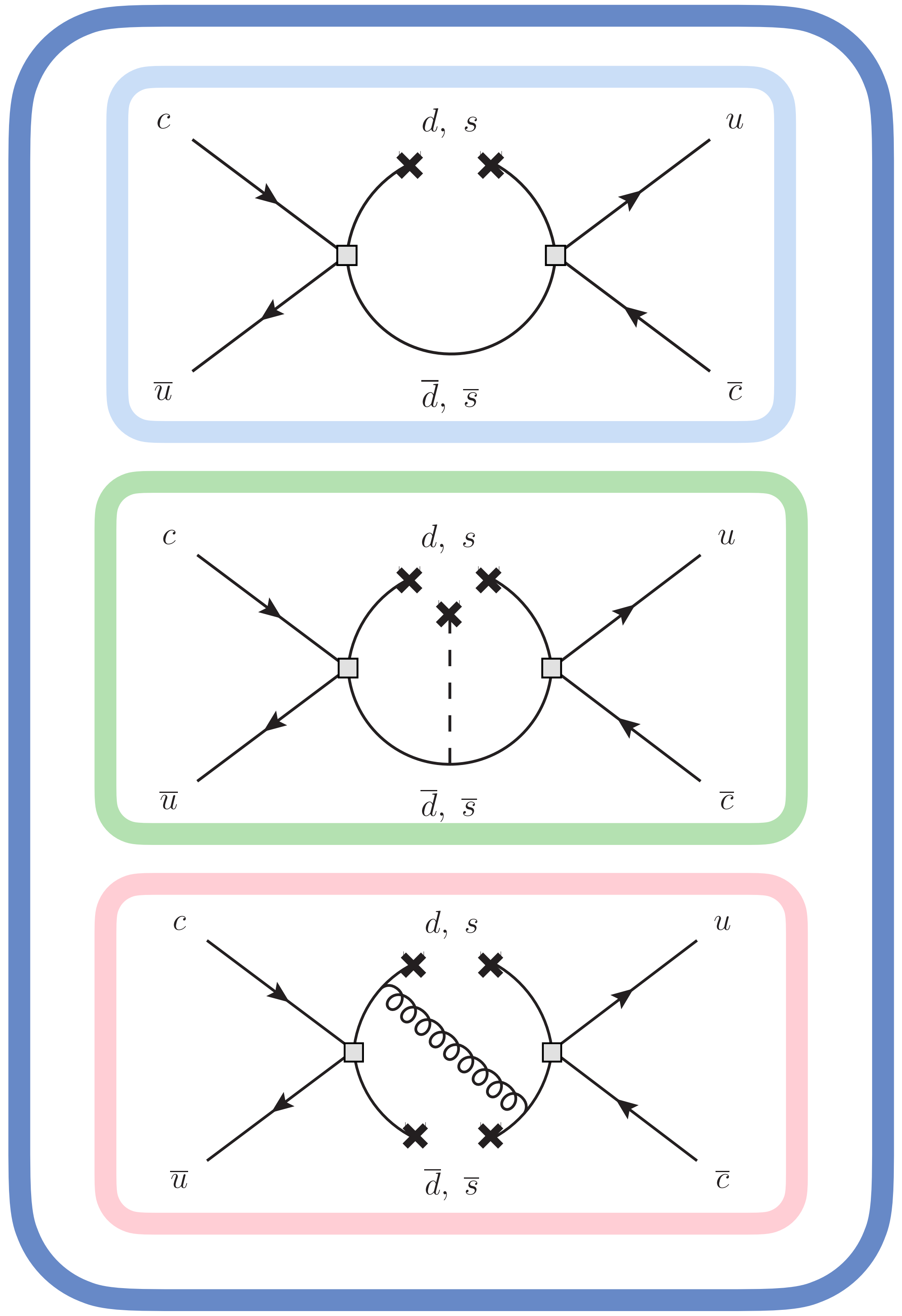
$$x_D^{\langle \bar{q}q\bar{q}q \rangle} = (0.24 \pm 0.06) \times 10^{-5}$$

Total nonperturbative

$$x_D^{NP} = (1.27 \pm 0.18) \times 10^{-5}$$



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$$x_D^{\langle \bar{q}q \rangle} = (0.99 \pm 0.13) \times 10^{-5}$$

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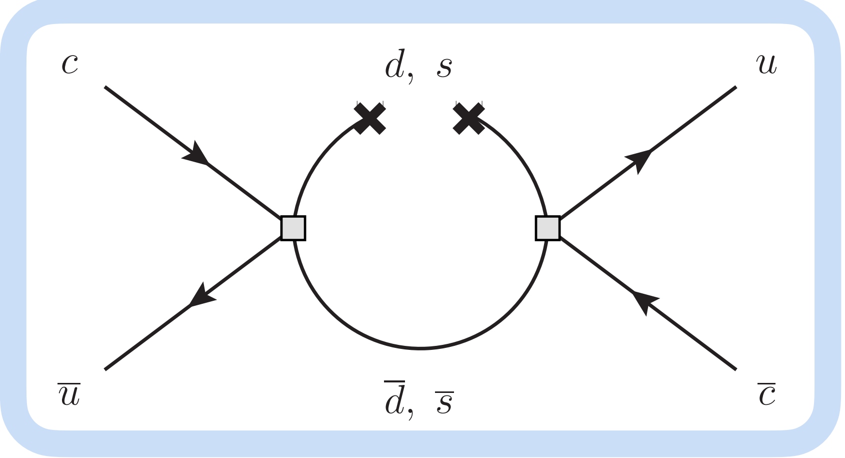
Total nonperturbative

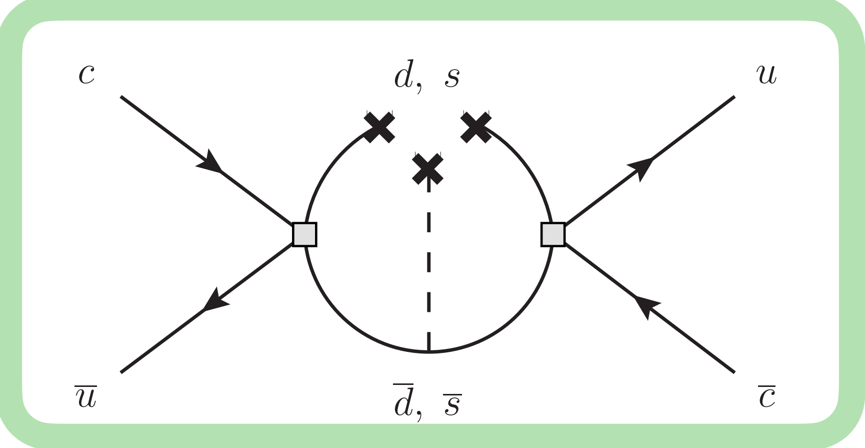
$$x_D^{NP} = (1.27 \pm 0.18) \times 10^{-5}$$

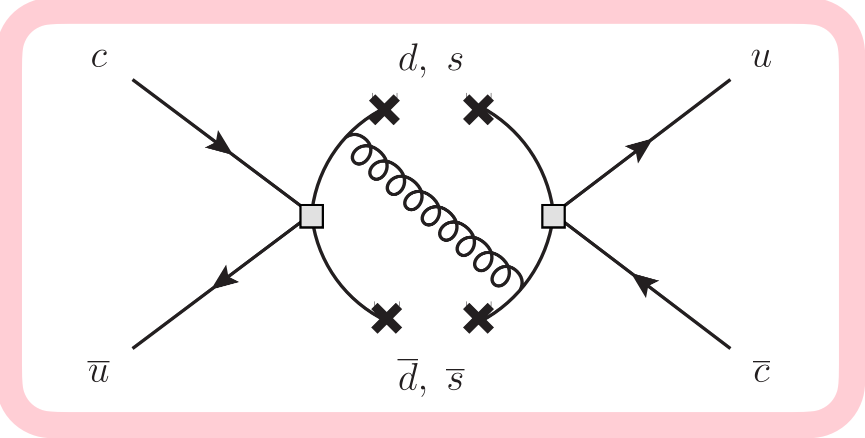
Total perturbative

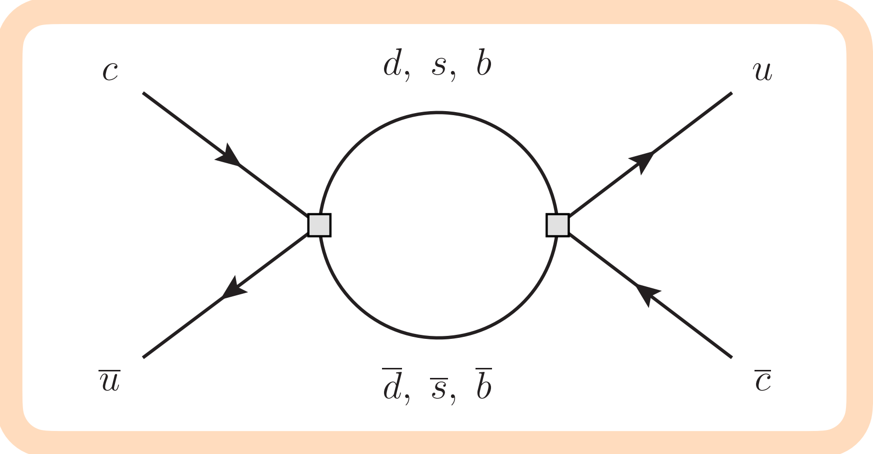
$$x_D^{LO+NLO} \approx 10^{-7}$$

Golowich, Petrov, *Short distance analysis of $D\overline{D}$ mixing*, **[hep-ph/0506185]**







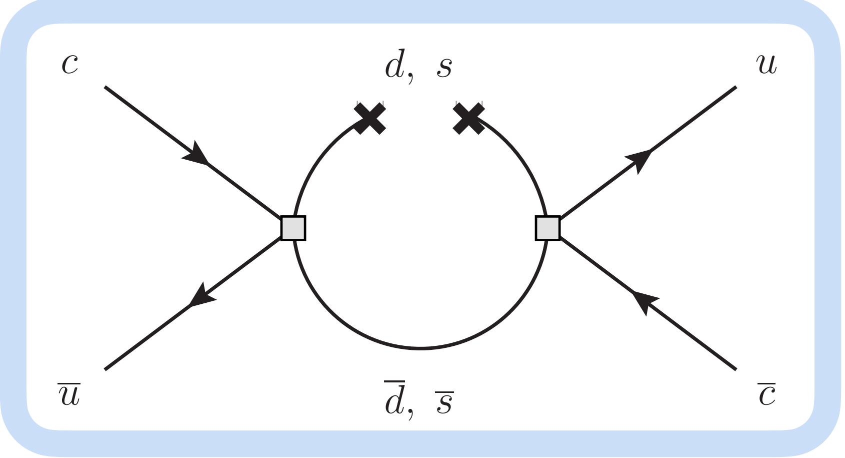


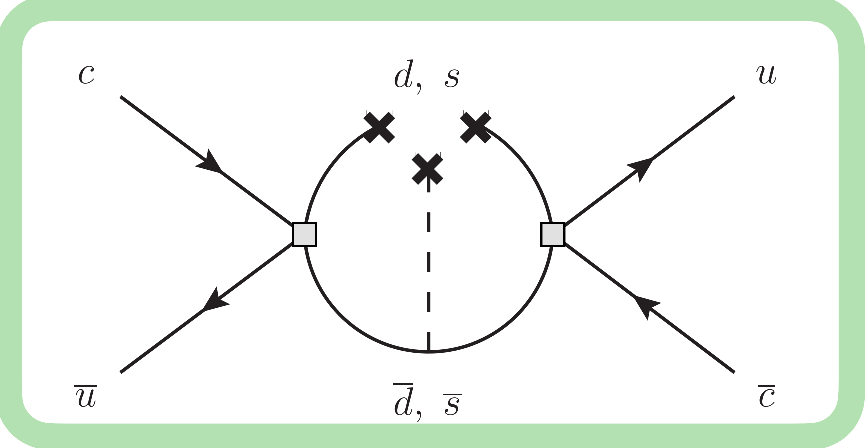
Final results

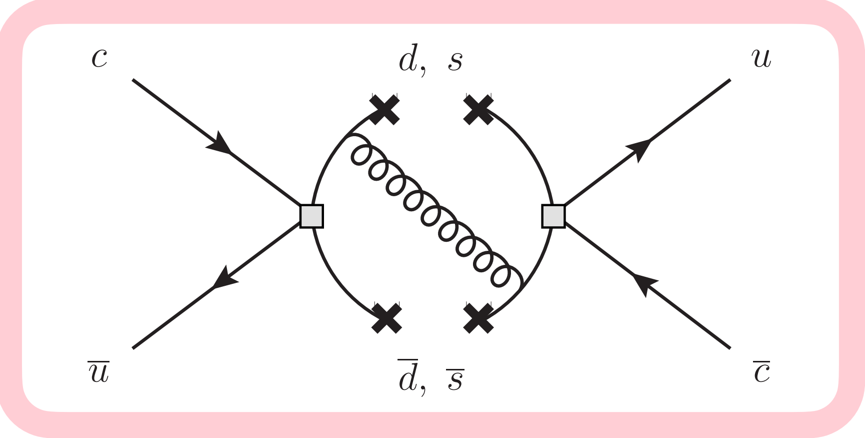
$$x_D^{\langle \bar{q}q \rangle} = (0.99 \pm 0.13) \times 10^{-5}$$

$$x_D^{\langle \bar{q}Gq \rangle} = (0.051 \pm 0.009) \times 10^{-5}$$

$$x_D^{\langle \bar{q}q\bar{q}q \rangle} = (0.24 \pm 0.06) \times 10^{-5}$$





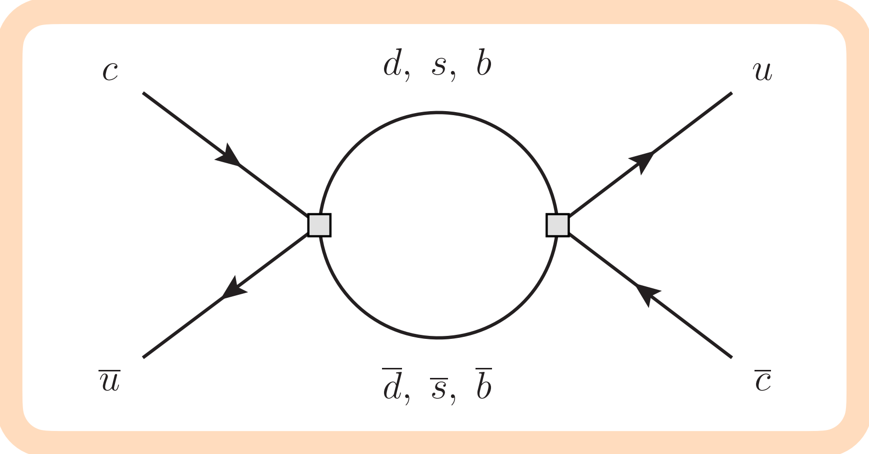


Total nonperturbative

$$x_D^{NP} = (1.27 \pm 0.18) \times 10^{-5}$$

Total perturbative

$$x_D^{LO+NLO} \approx 10^{-7}$$



Golowich, Petrov, *Short distance analysis of $D\bar{D}$ mixing*, **[hep-ph/0506185]**

HFLAV, *Averages of b-hadron, c-hadron, and τ-lepton properties as of 2023*, **[2411.18639]**

Experiment

$$x^{EXP} = (4.07 \pm 0.44) \times 10^{-3}$$

Final results

Dependencies on parameters

Final results

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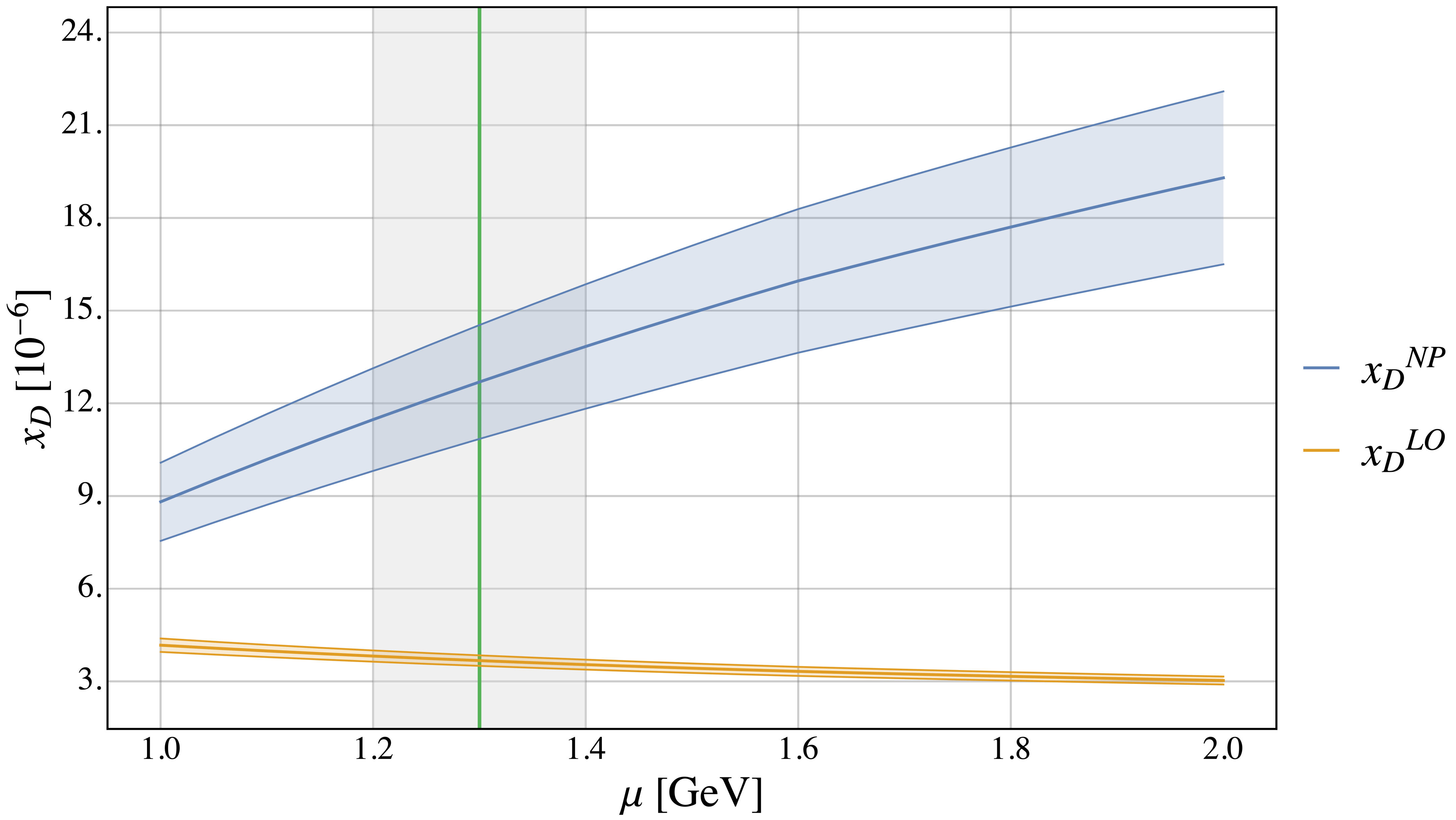

strong dependence on renormalization scale comes from the condensate

Final results

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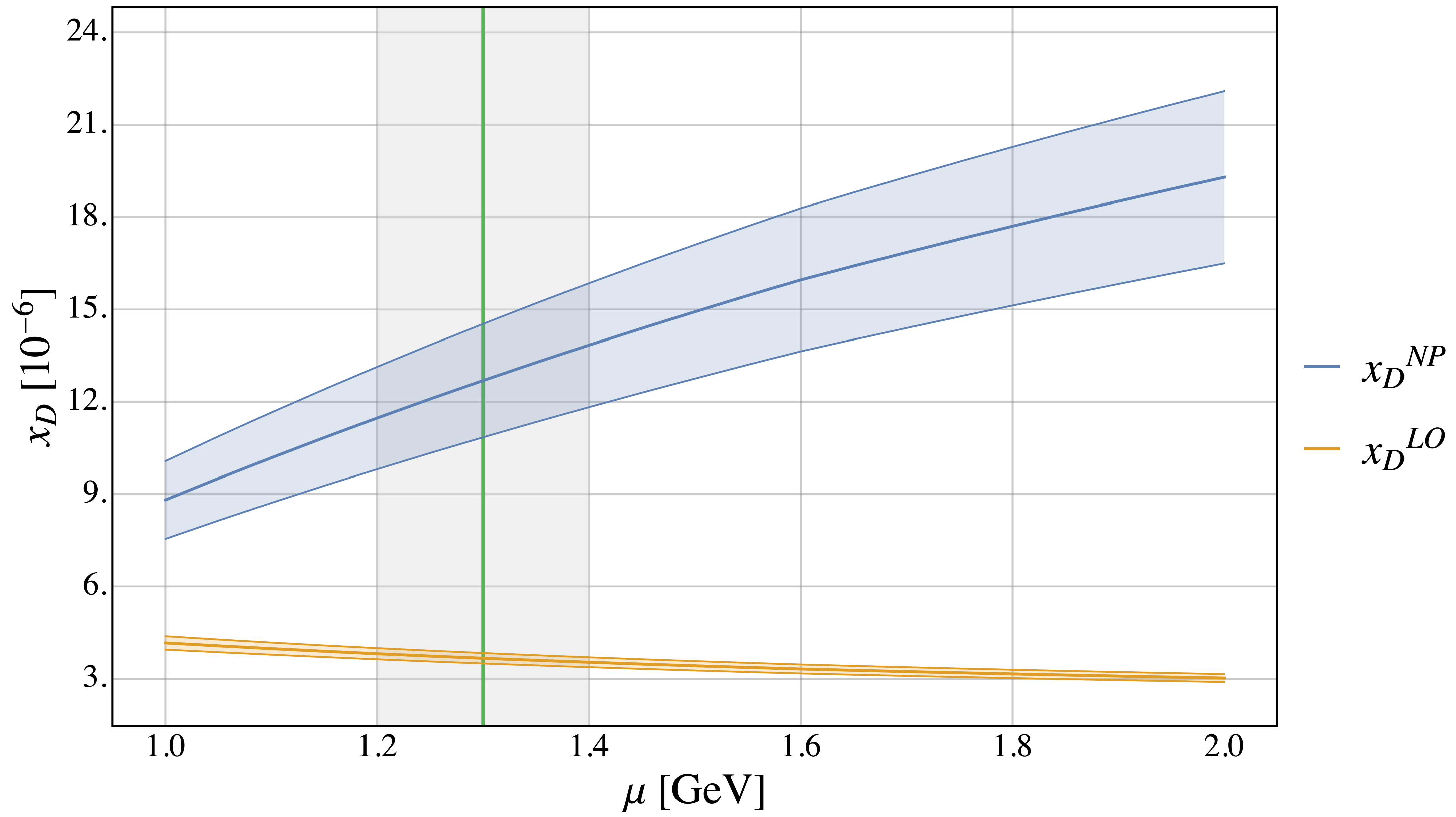

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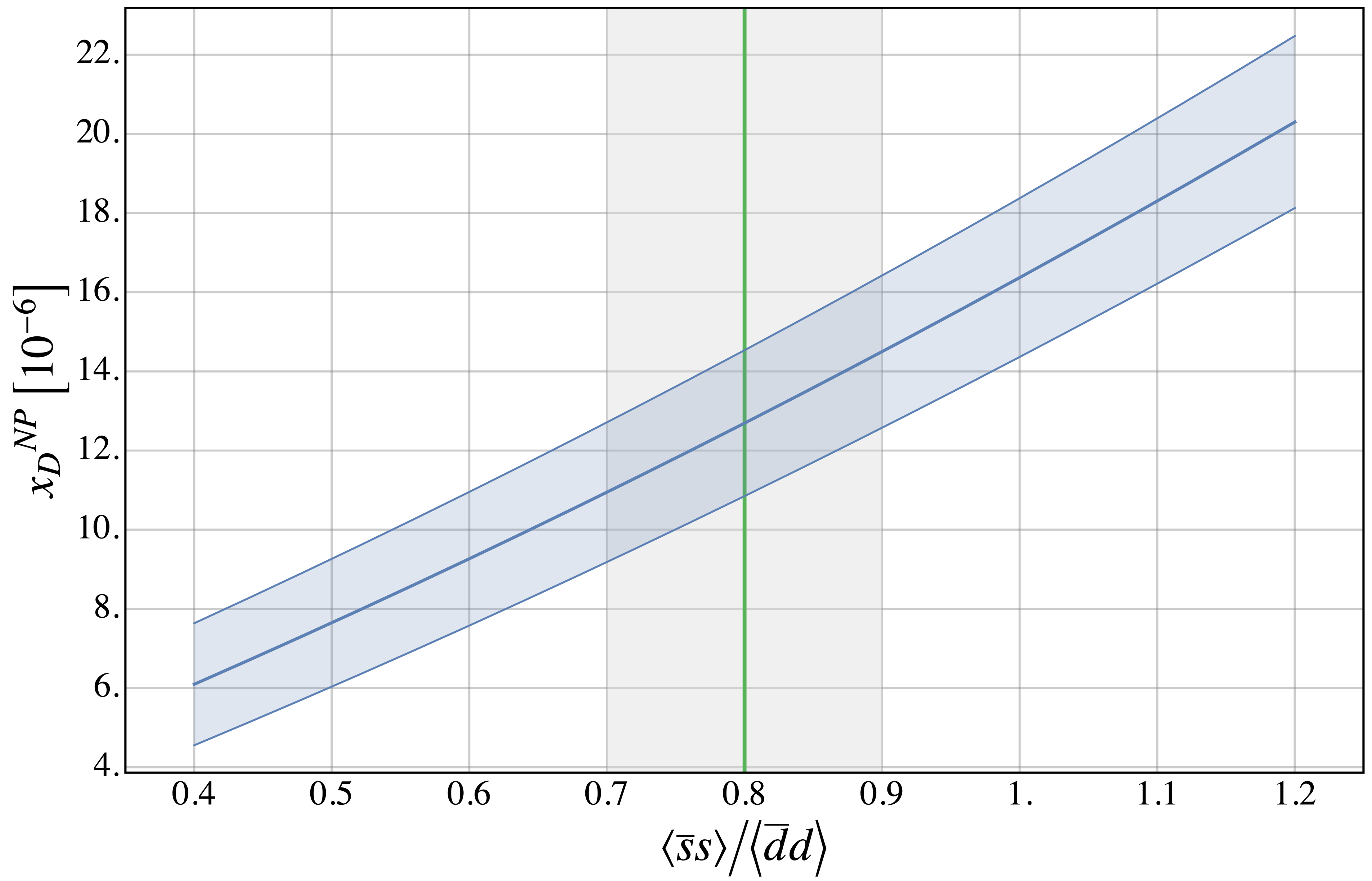


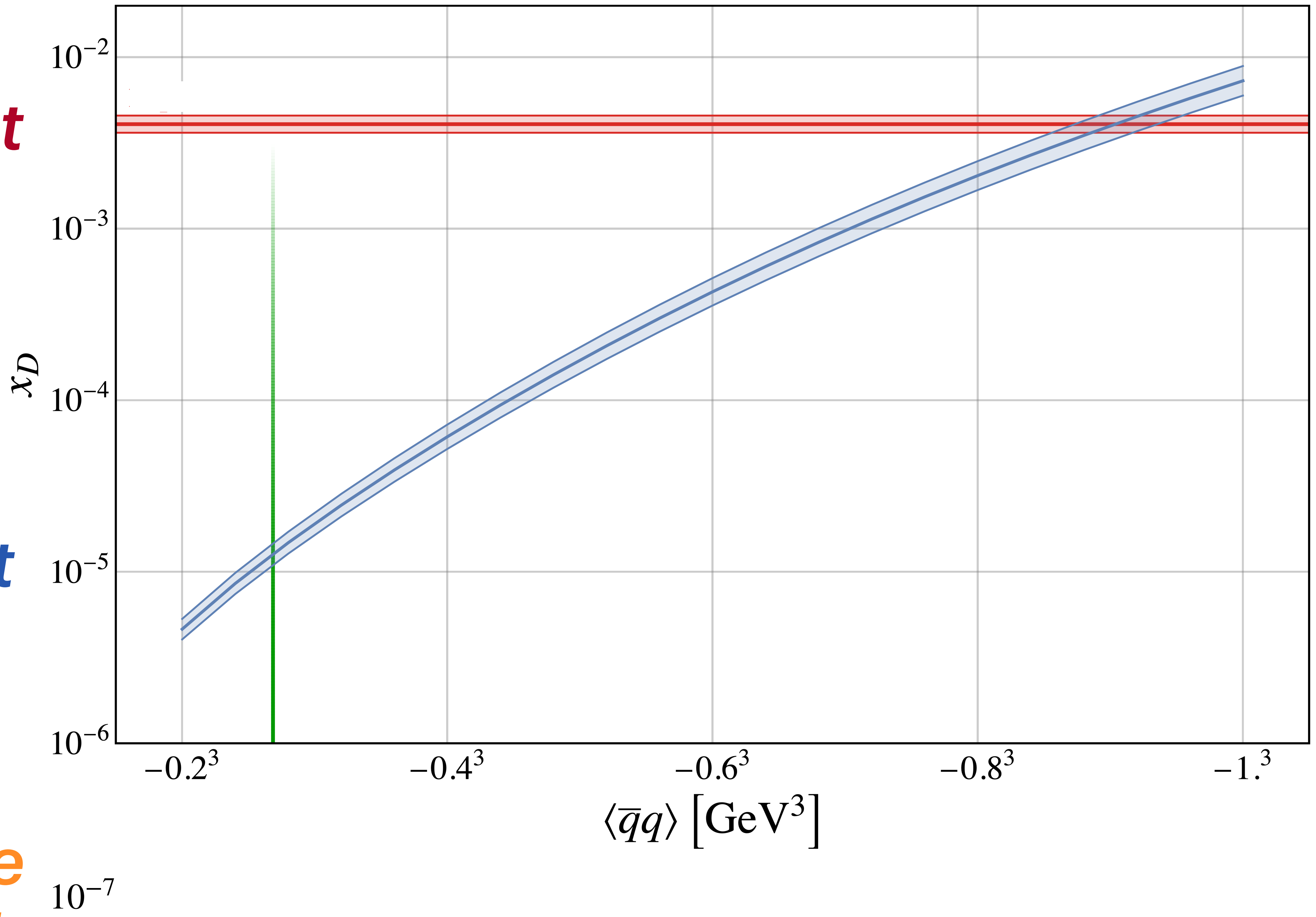
strong dependence on strange-to-light condensate ratio is explained by strong dependence on $\langle \bar{s}s \rangle$

Summary

Experiment

Our result

Perturbative result



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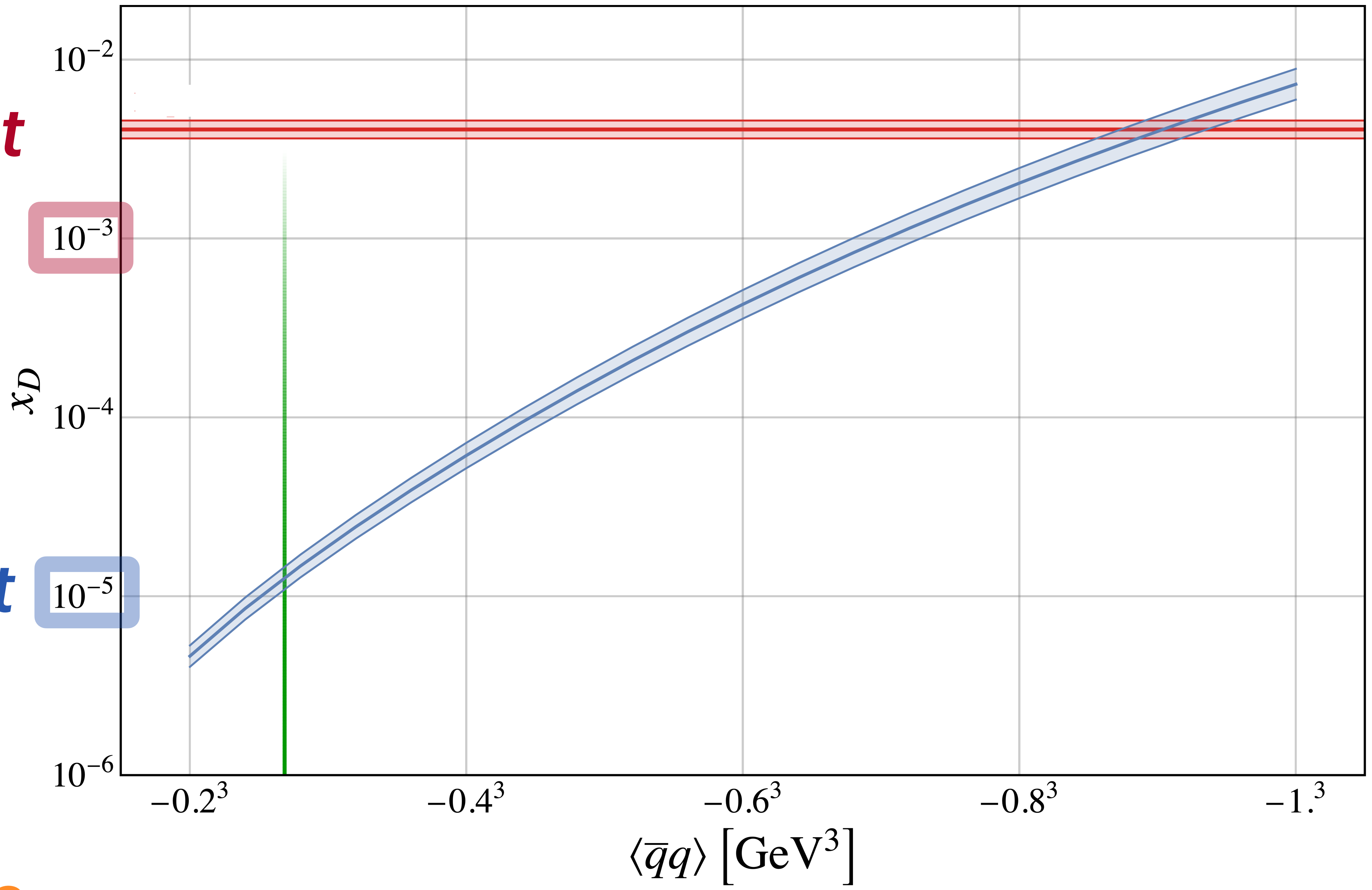

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10^{-7}

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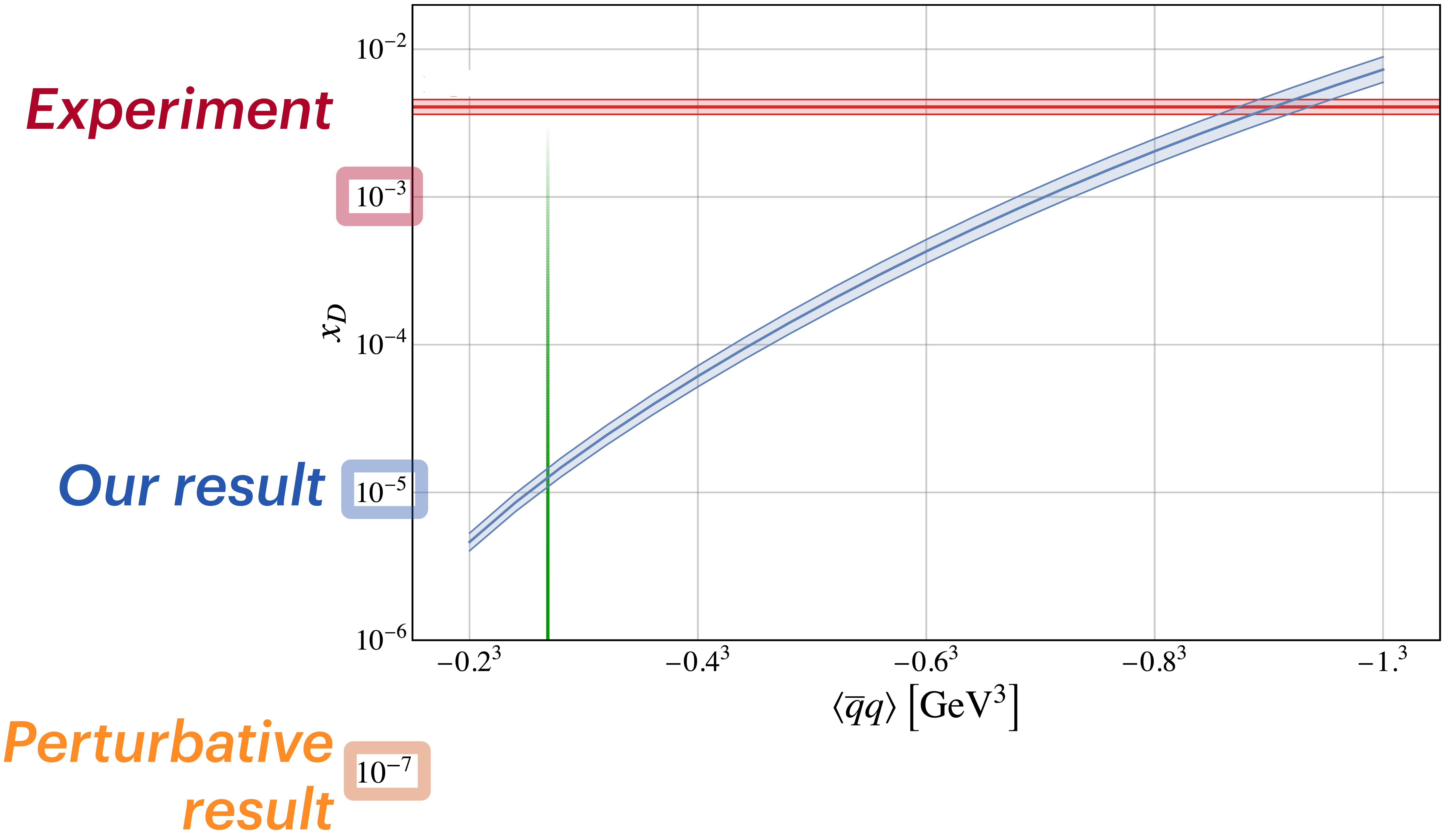

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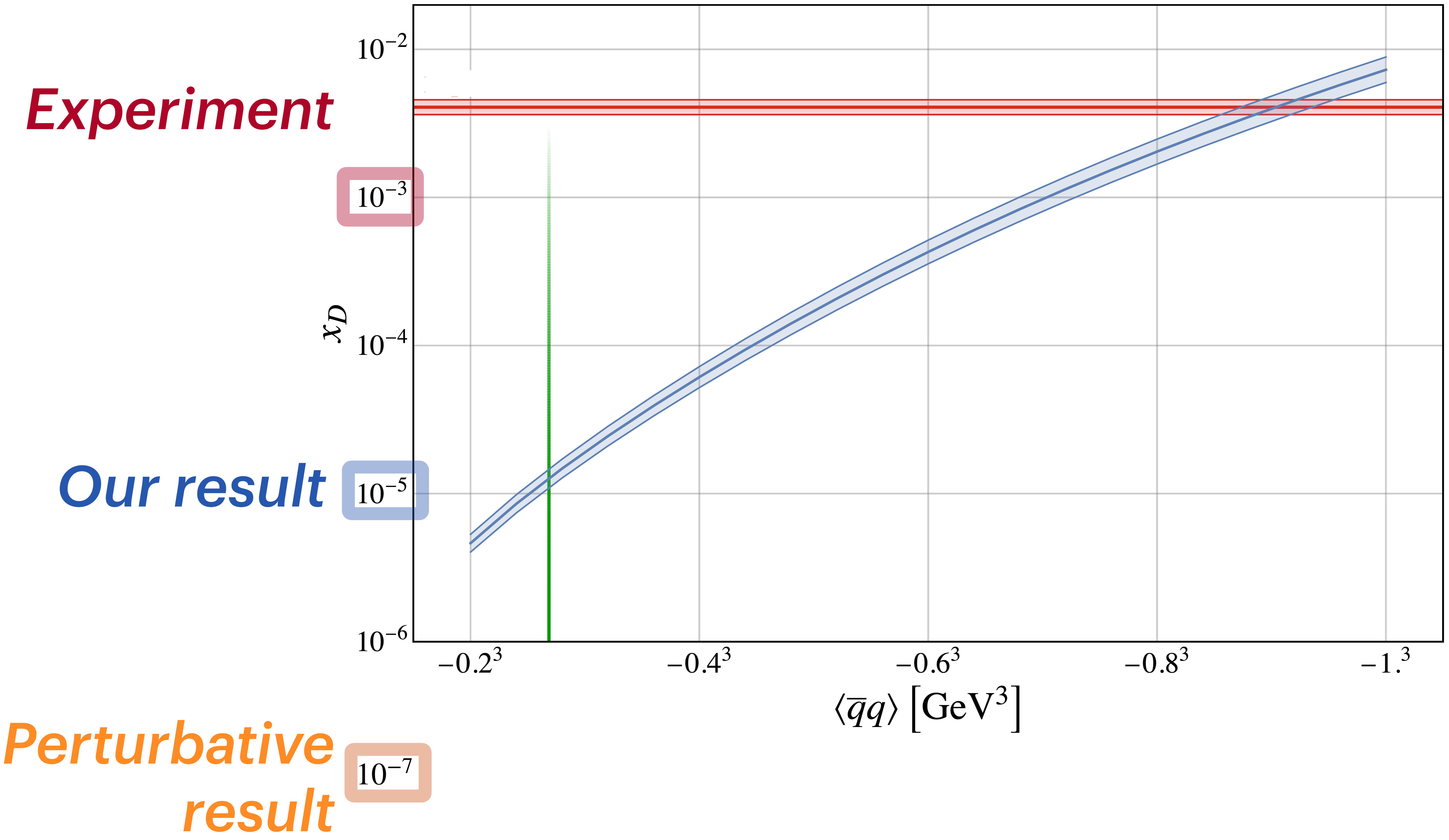

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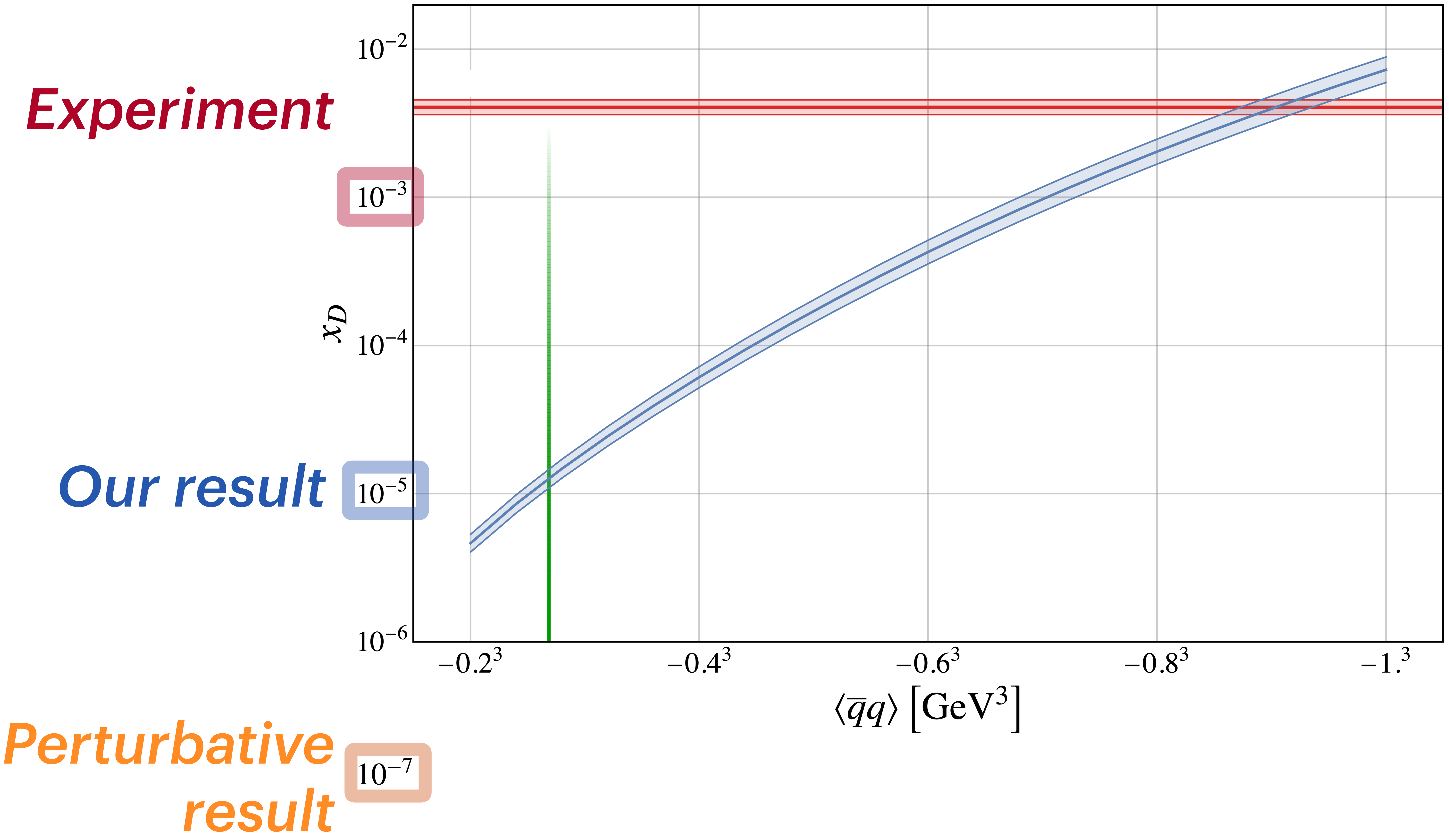

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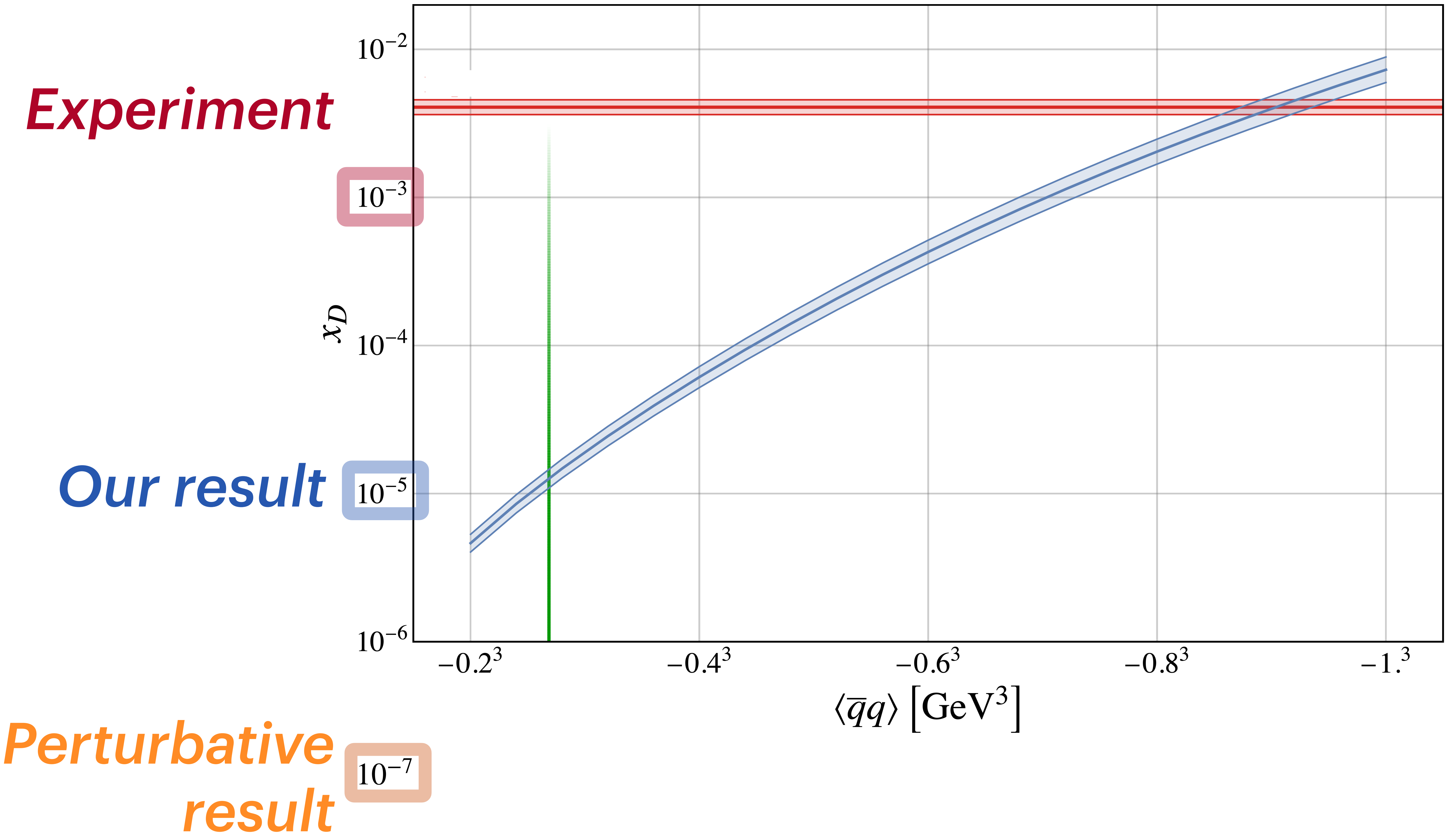

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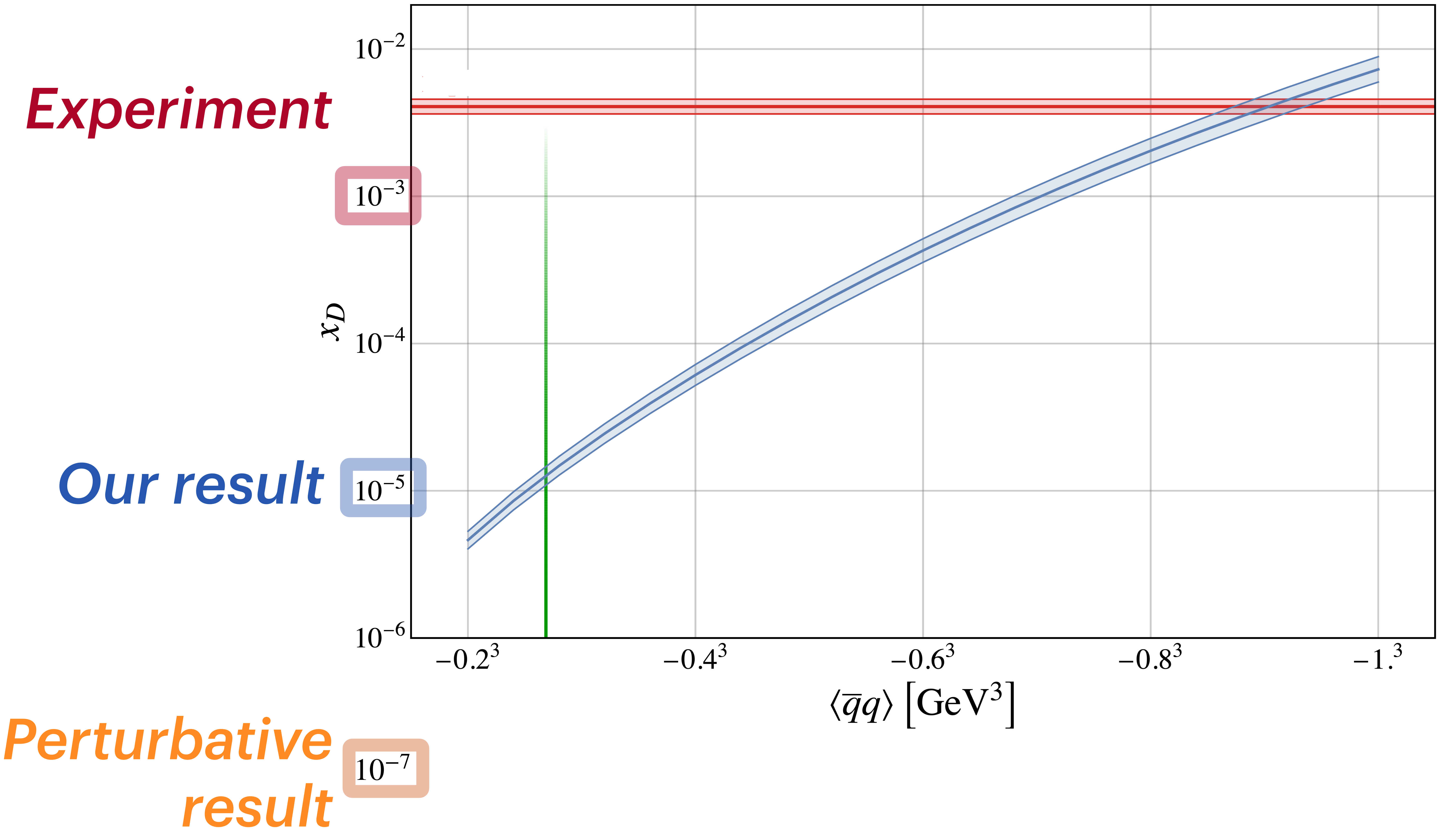

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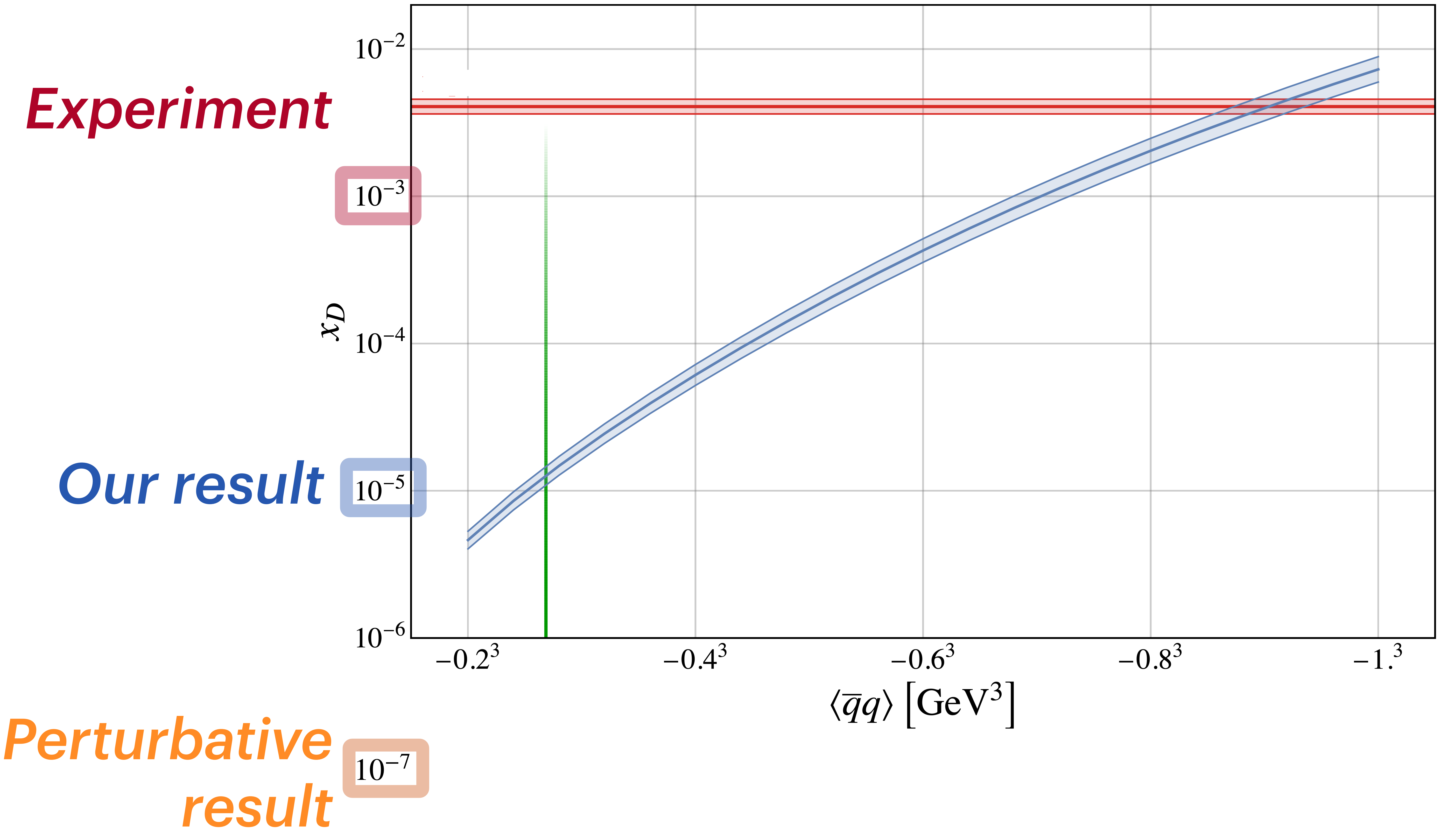

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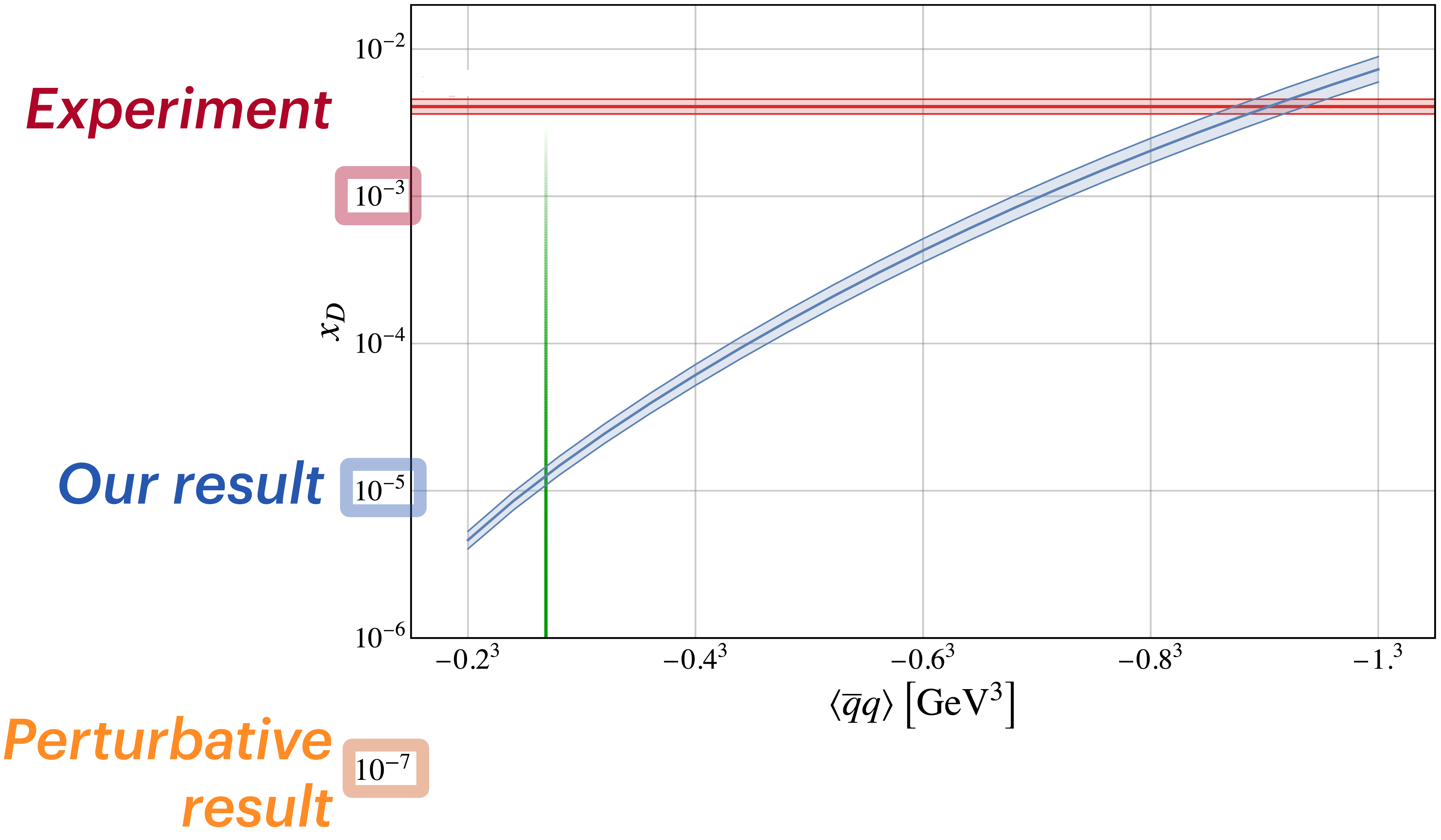

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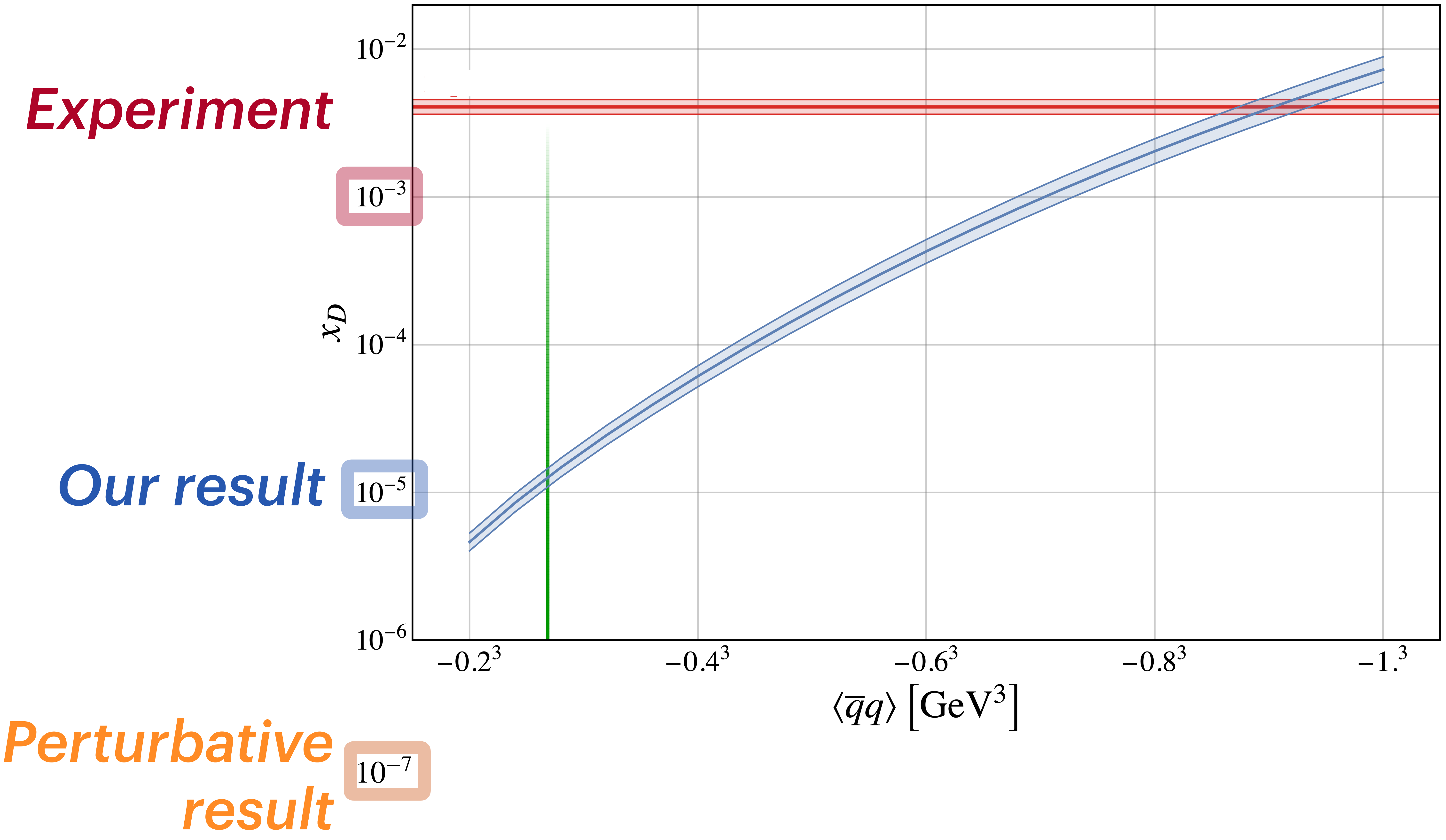

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  - our final result **improves on the perturbative LO+NLO by almost two orders of magnitude**, but still falls short of experimental value by another two orders

Thank you!

For more details feel free to take a look at the paper itself!

[2508.16337]

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[2]*Department of Physics, Faculty of Science, University of Zagreb, Croatia*

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This work was supported by the Croatian Science Foundation under the project number IP-2024-05-4427.

Brda 2025: Selected topics in BSM, *Ljubljana, Slovenia*

29 to 30 October 2025