



Timing & Charge Analysis for TI-LGADs

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Testbeam analysis meeting

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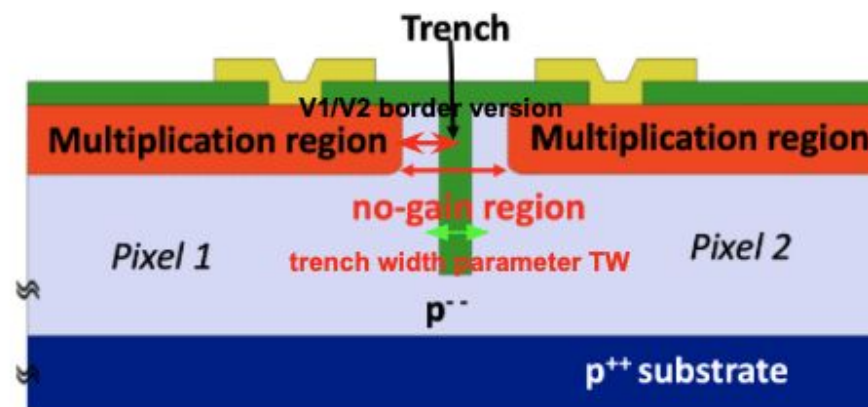


Motivation for timing/charge analysis of an TI-LGAD

- Per-DUT timing resolution (σ_t) from constant-fraction-discriminated pulse timing, measured against a reference signal (TREF: 1.2 x 1.2 mm² HPK LGAD sensor).
- How that resolution degrades as a track approaches the inter-pixel distance -> the key feature studied here
- A single “bulk” resolution number per pad, so the trench-approach curves can be read as departures from a clean baseline
- Charge response of each pad vs. track position -> in the bulk, and approaching the inter-pad gap.
- How charge is shared between two neighboring pads near that gap
- An absolute charge scale in fC, plus normalized observables

Charge analysis built on the timing macro

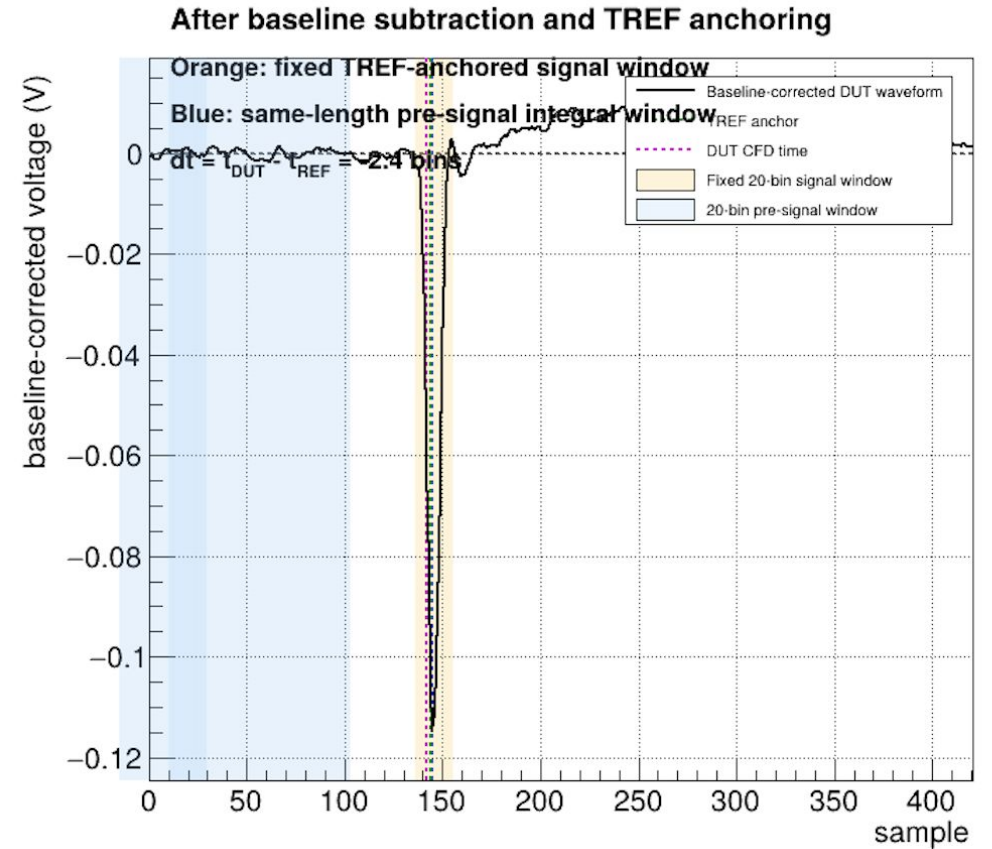
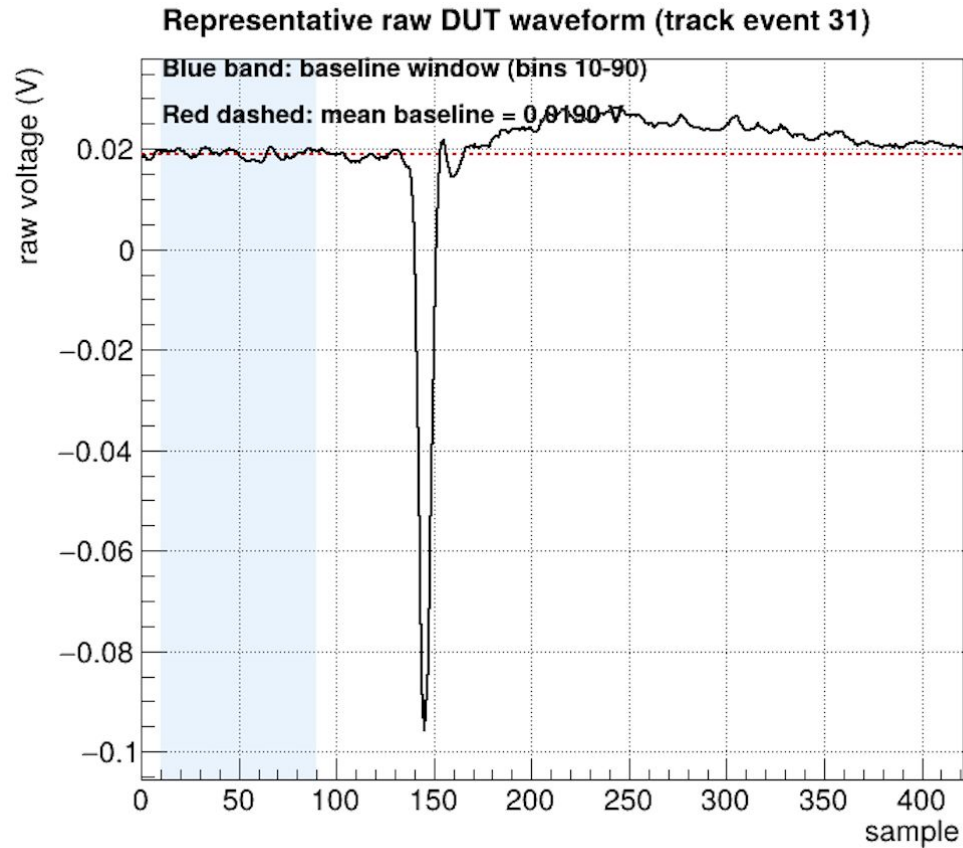
- Every operating point (which CFD fraction to use, per channel) is scanned and fixed, on bulk data
- charge macro based on timing
- Same event / track → waveform matching
- Same “bulk region, safely away from the gap” method already used for the timing bulk-ROI



notice: bulk here is the part of the pad that is far enough from the trench

Example event showing the fixed TREF-anchored signal definition

- TREF waveform defines the time anchor.
- DUT signal is integrated in a fixed 20-bin window relative to that anchor.
- A same-length pre-signal window is used for the baseline reference.
- dt is the time difference between DUT and reference.





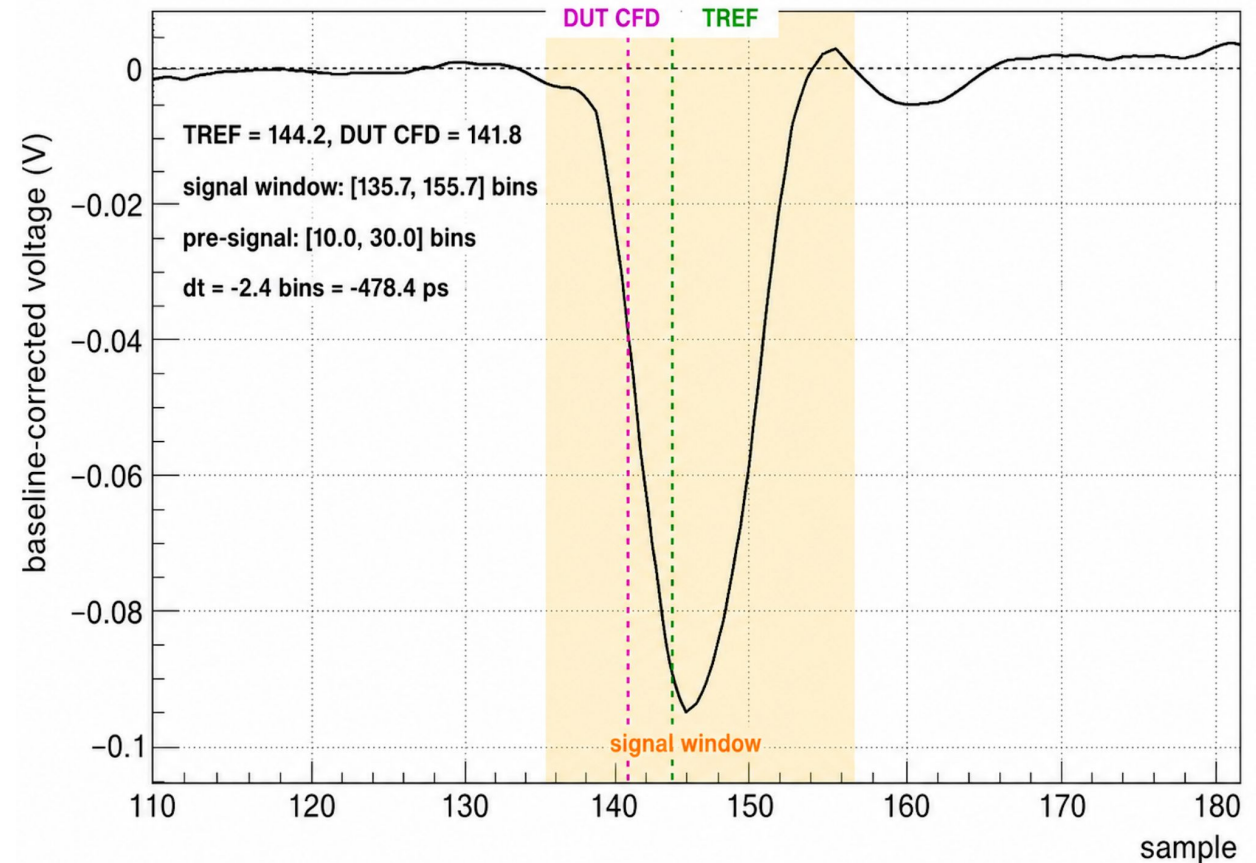
From Waveform to Charge

- Baseline -> mean of the first 100 samples (also gives the noise RMS).
- Charge = signed sum of (baseline - V) over a fixed window, converted to fC via the 0.2 ns/sample period at 5 GS/s.
- The window is not gated on the DUT's own CFD success -> every track gets a charge, including faint inter-pad events.
- Nominal window: 5 bins before + 15 after center = 4.2 ns. Short (4+11) / long (7+18) variants are recorded only as a stability cross-check, never for event selection.

In general:

- TREF CFD defines the event time anchor
- Same-length pre-signal window used for baseline reference
- $dt = t_{\text{DUT}} - t_{\text{REF}}$
- The DUT charge is integrated in a fixed 20-bin window placed relative to the TREF CFD anchor, to avoid the pedestal bias of peak-centering while retaining the main part of the DUT pulse for the selected events in this run

Fixed-anchor signal extraction and timing definition



$$\text{threshold} = k_{\text{CFD}} \times A_{\text{peak}}$$

timestamp = linear interpolation of the crossing between the two samples straddling threshold



Calibrating TREF: the Triplet Method

$$\sigma_{\text{TREF}}^2 = \frac{1}{2}(\sigma_{1R}^2 + \sigma_{2R}^2 - \sigma_{12}^2)$$

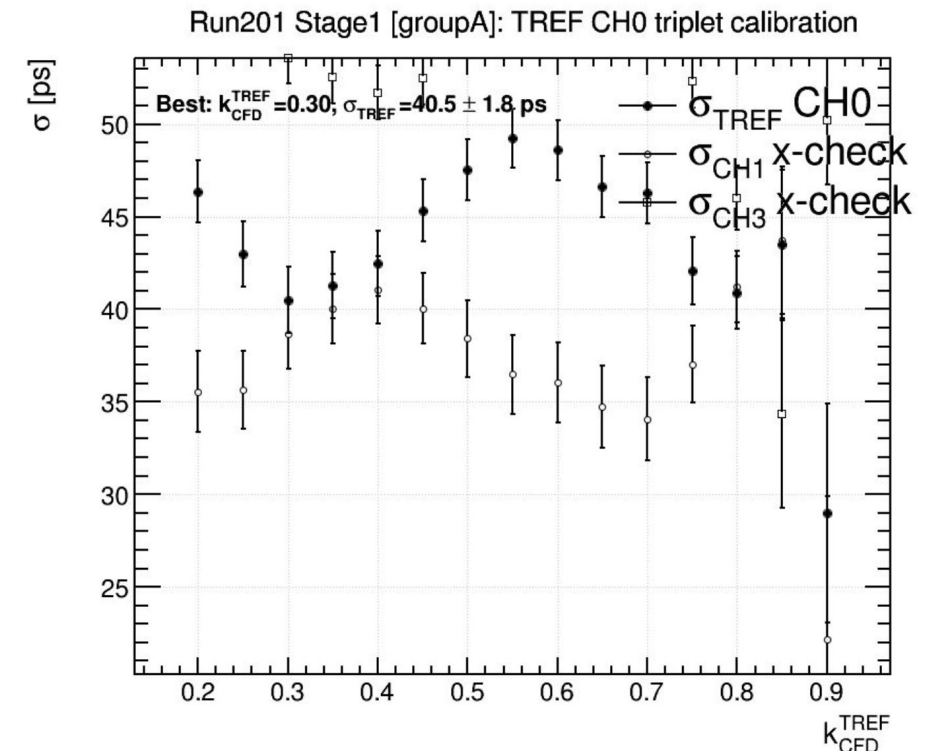
three pairwise timing widths (TREF-DUT1, TREF-DUT2, DUT1-DUT2) solve algebraically for each individual channel's own resolution

- TREF's own resolution is extracted from three pairwise widths among TREF and two calibration DUTs, using the same three-channel variance decomposition throughout this analysis.
- k_{TREF} is scanned, both calibration DUTs are held at a fixed, standard fraction so only TREF's own operating point moves.
- A scan point is only eligible as “best” if it clears BOTH a relative-error floor and a minimum-statistics floor
- Result: one k_{TREF} and one σ_{REF} , per group, and used as the fixed reference for every subtraction in every later stage.

with tracking included $\sigma_{\text{TREF}} = 40.5 \pm 1.8 \text{ ps}$

Frozen once, reused everywhere

- This is the ONE place σ_{TREF} is measured in the entire macro.
- Later resolves an independent “anchor triplet” using the frozen k's, purely as a cross-check
- Every quoted σ_{DUT} (the bulk ROI, the 1D profile, the 2D map) subtracts the same σ_{TREF} , in quadrature



Per-Channel Scan, and Fixing How “Best k” Is Chosen

$$\sigma_{\text{DUT}}(k) = \sqrt{(\sigma_{\text{pair}}(k)^2 - \sigma_{\text{TREF}}^2)} \quad \epsilon_{\text{CFD}}(k) = N_{\text{timestamped}}(k) / N_{\text{preselected}}$$

$N_{\text{preselected}}$ is fixed ONCE per channel — independent of which k is being scanned; tracks landing on the pad with good S/N ~**1000 events**

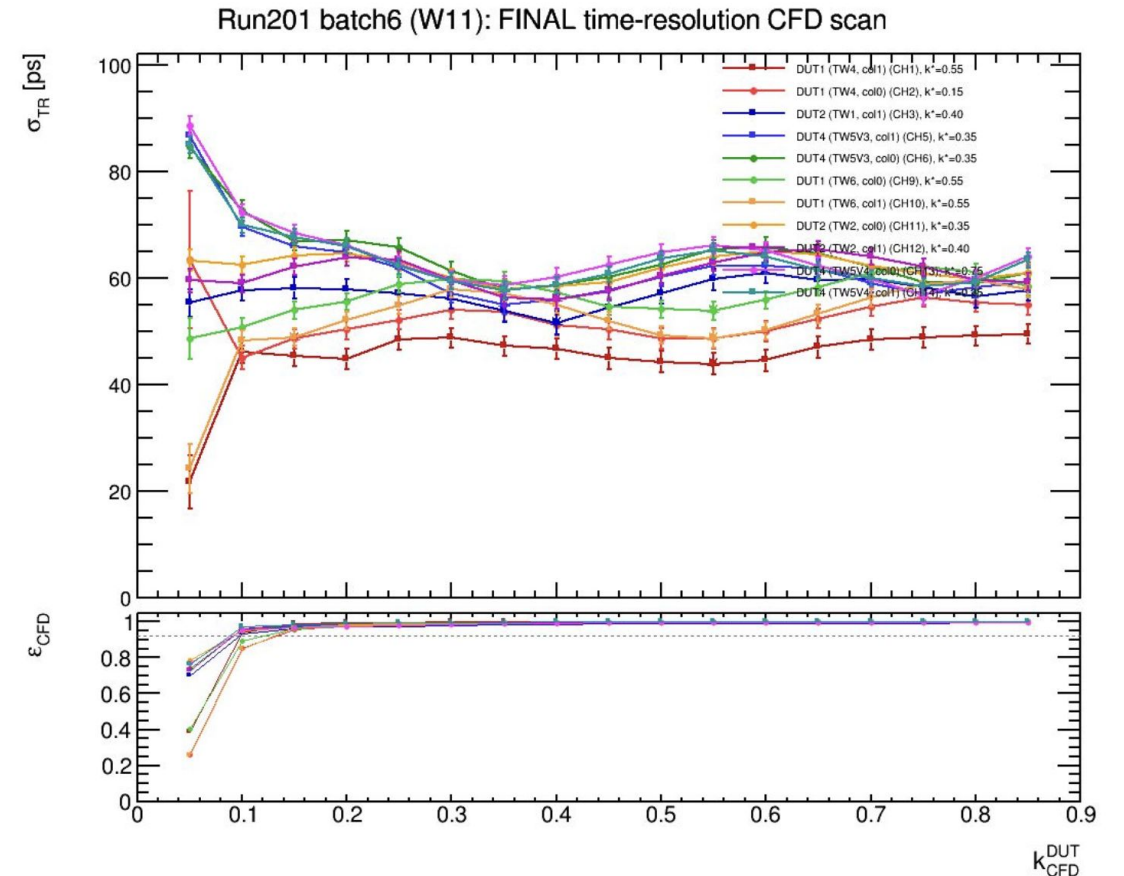
- For each of the 12 DUT channels, k_{DUT} is scanned against that group's now-fixed TREF; each point's σ_{pair} has σ_{TREF} quadrature-subtracted to give $\sigma_{\text{DUT}}(k)$.
- Best-k is only eligible if its CFD efficiency clears 92 % of a k-independent preselected population (scanning with different k)

For the later spatial analysis (after k_{CFD} is frozen): statistics per spatial bin are much smaller >>

$$\sigma_{68} = \frac{1}{2} (Q_{84} - Q_{16})$$

the central-68% half-width of the Δt distribution — a percentile, not a fitted parameter

- A spatial bin in the 1D profile or 2D map typically has 60-200 tracks — enough for a percentile, but a known-fragile regime for a binned χ^2 Gaussian fit (most of a 120-bin histogram sits empty).
- very bin still carries a Gaussian-fit cross-check column alongside σ_{68} in the numeric output tables.
- If the two disagree by more than 20% of σ_{68} in a given bin, that bin is explicitly flagged





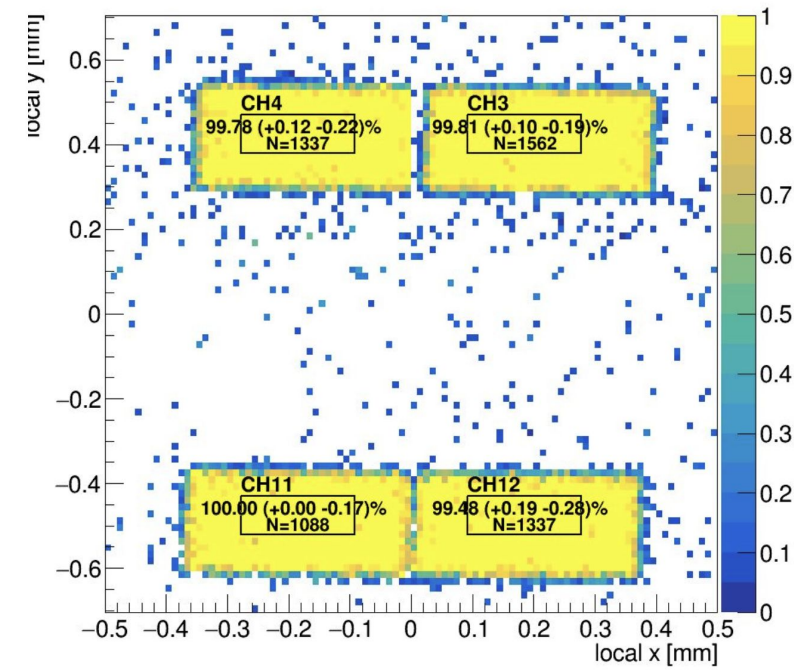
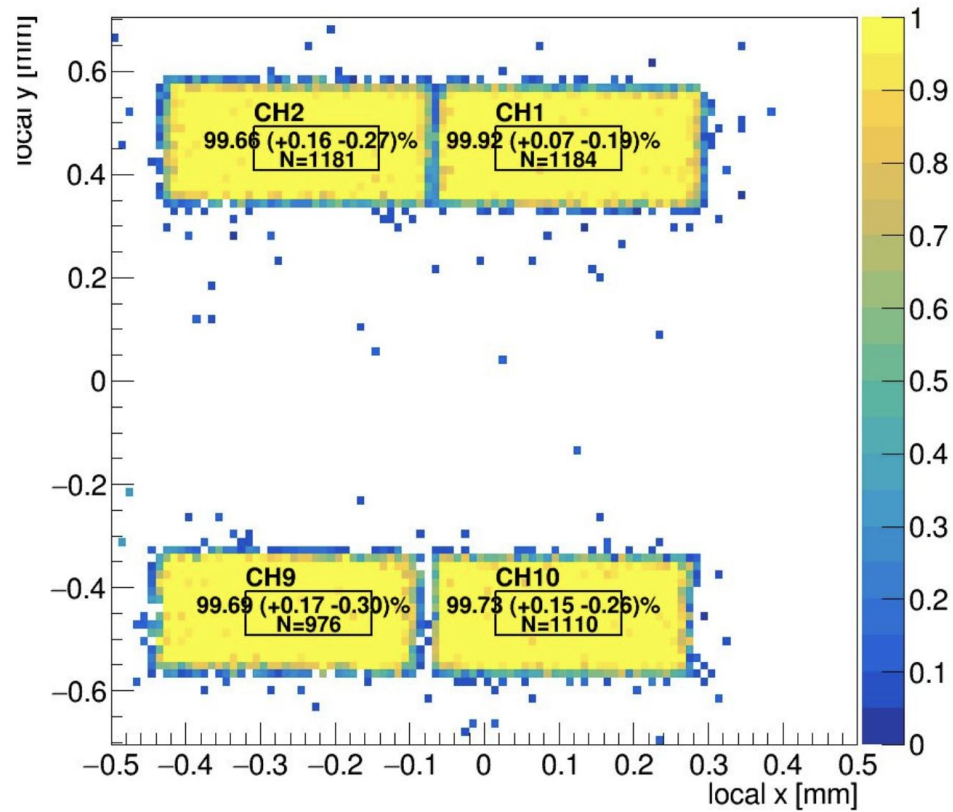
Timing resolution evaluation for W11 V2–TW4, unirradiated TI-LGAD biased @ 150 V

- Added at the per-event level: a signed waveform area (V·ns), an anchored peak amplitude, and their ratio (area/peak, in ns -> an effective local pulse width), all binned and plotted vs. distance from the trench next to the timing panels.
- The center it integrates around is not the DUT's own CFD-found peak -> it's a TREF-predicted position: a channel-specific DUT-minus-TREF peak-bin offset, calibrated once on clean bulk tracks (SNR ≥ 8 , robust median/MAD outlier rejection, ≥ 120 clean differences required) and then held fixed.

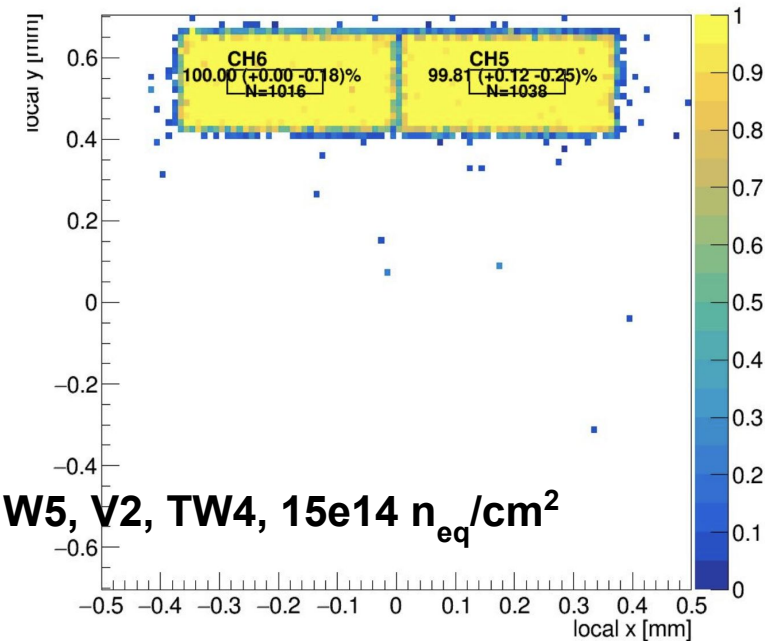
Pad	NCFD	kCFD	σ_t [ps]	Gaussian cross-check:
Pad 0	978	0.15	42.3 ± 3.0	Pad 0: 40.6 ± 2.7 ps Pad 1: 38.8 ± 2.9 ps
Pad 1	897	0.55	40.6 ± 3.6	

- CFD timing efficiency $\epsilon_{\text{CFD}}(x,y)$ — is this bin's population actually being timestamped, or mostly failing?
- Selected-track occupancy $N(x,y)$ — is a grey/degraded region actually sampled by real tracks, or just empty?
- Every quantitative bin requires ≥ 120 CFD-valid tracks
- A separate, finer binning ($5 \times 10 \mu\text{m}$) grid, built for visualization: σ_{68} is computed from the 120 nearest CFD-valid hits on the SAME physical side of the trench, within an $80 \mu\text{m}$ cap.

W5, V2, TW1-2, $0e14$ n_{eq}/cm^2 , angled at 10°

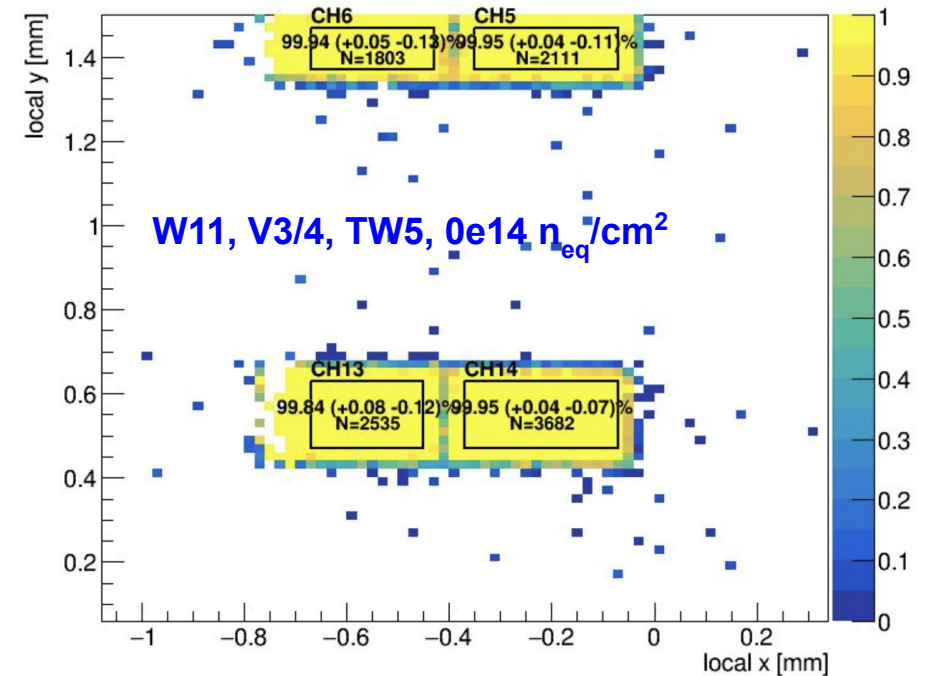
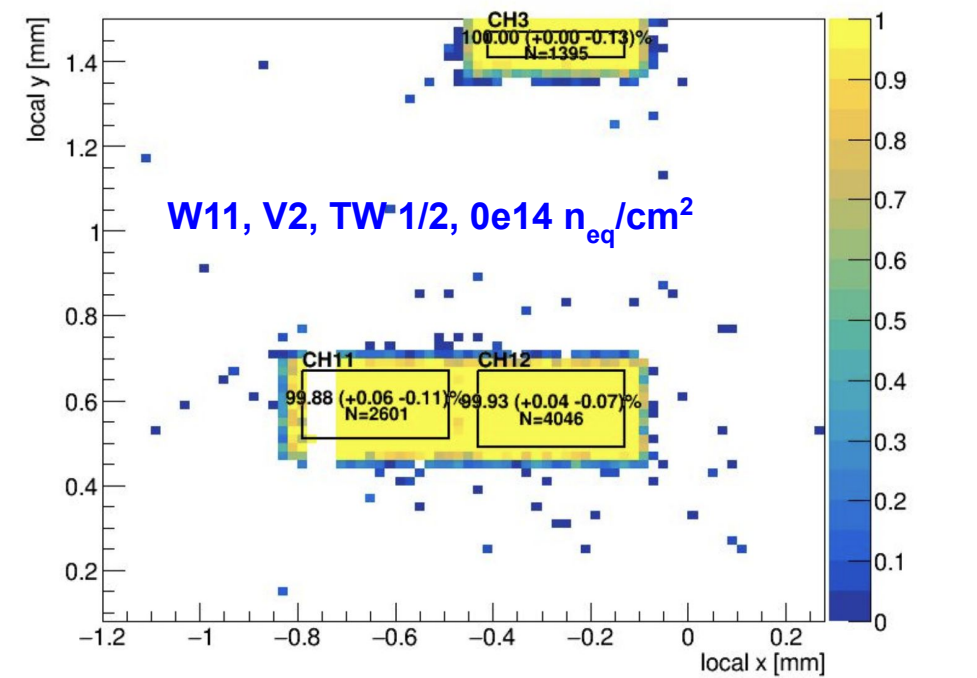
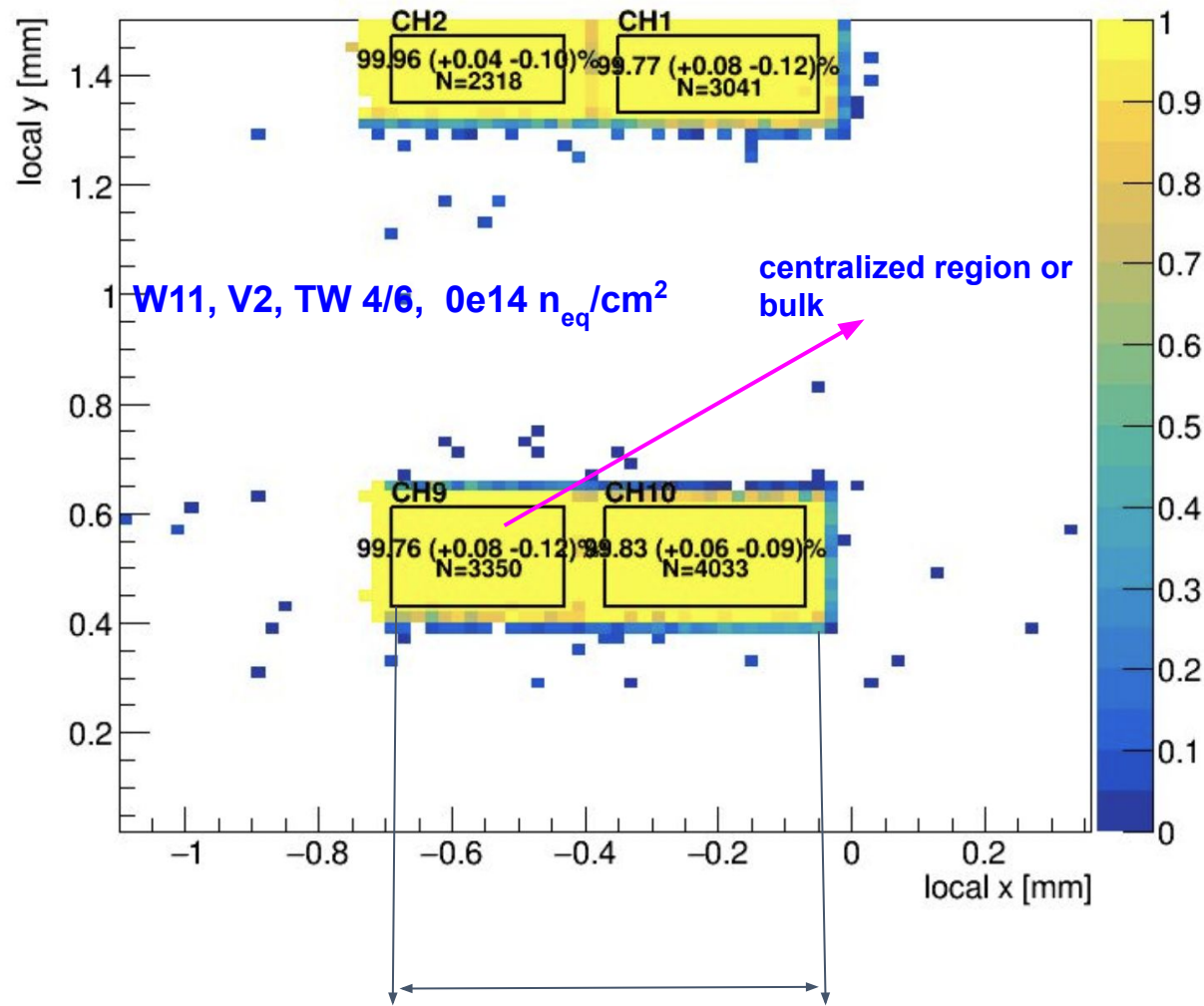


W5, V2, TW2-3, $25e14$ n_{eq}/cm^2

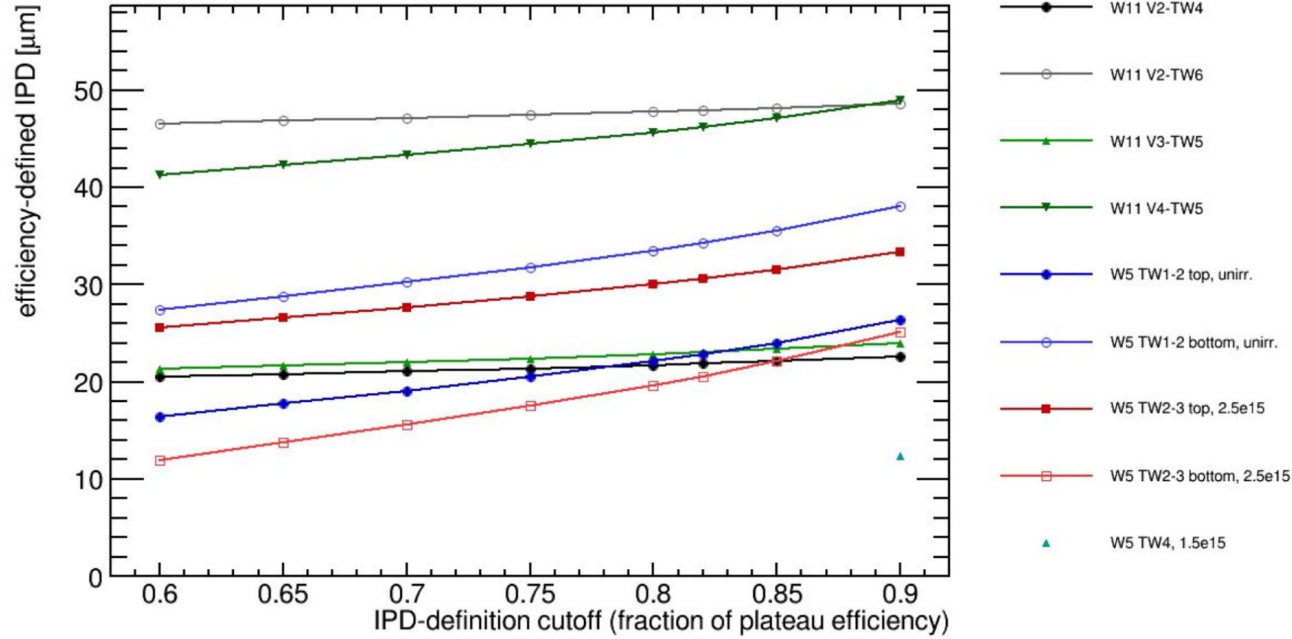
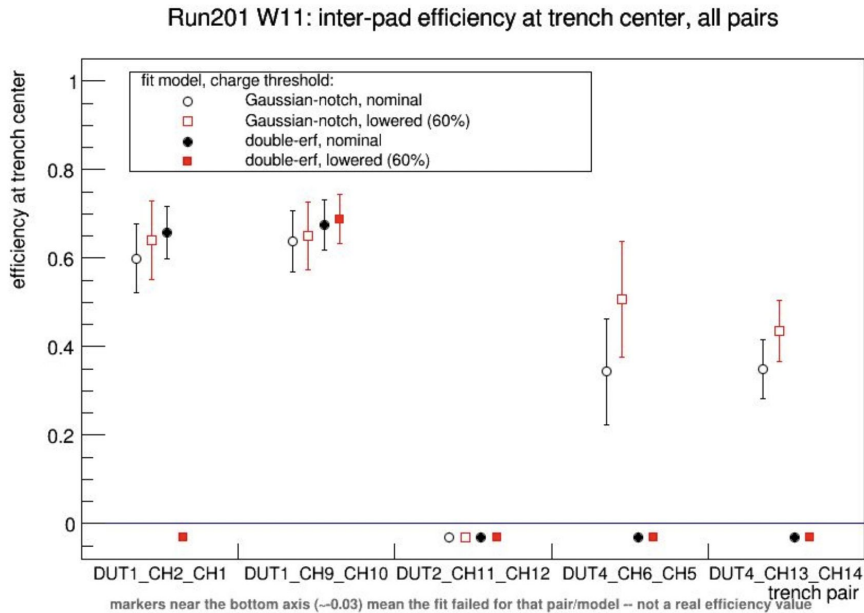
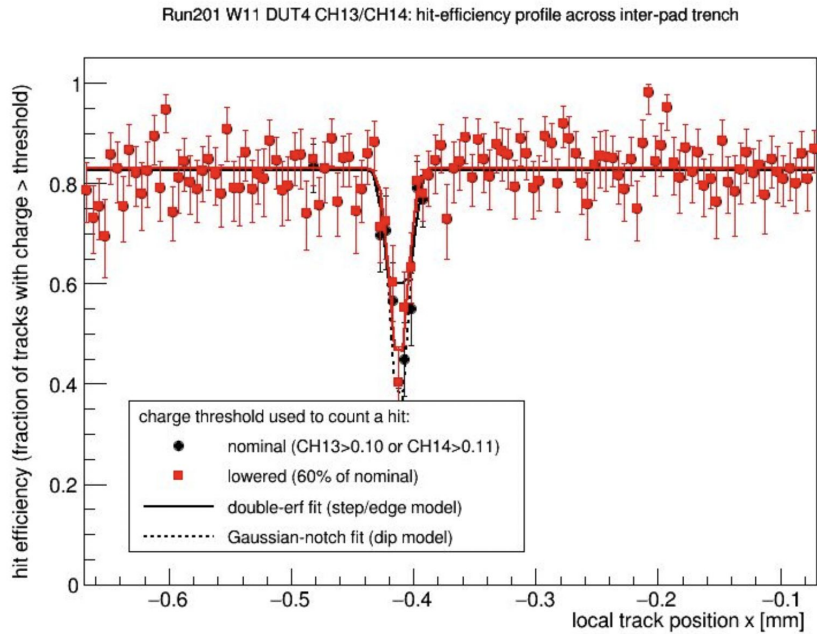
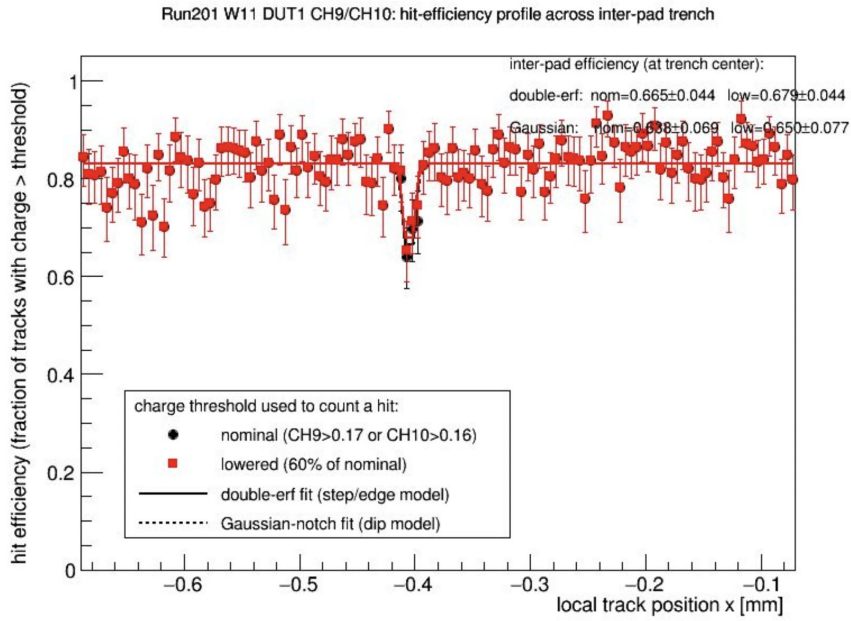


W5, V2, TW4, $15e14$ n_{eq}/cm^2

Wafer 11 designs, unirradiated;

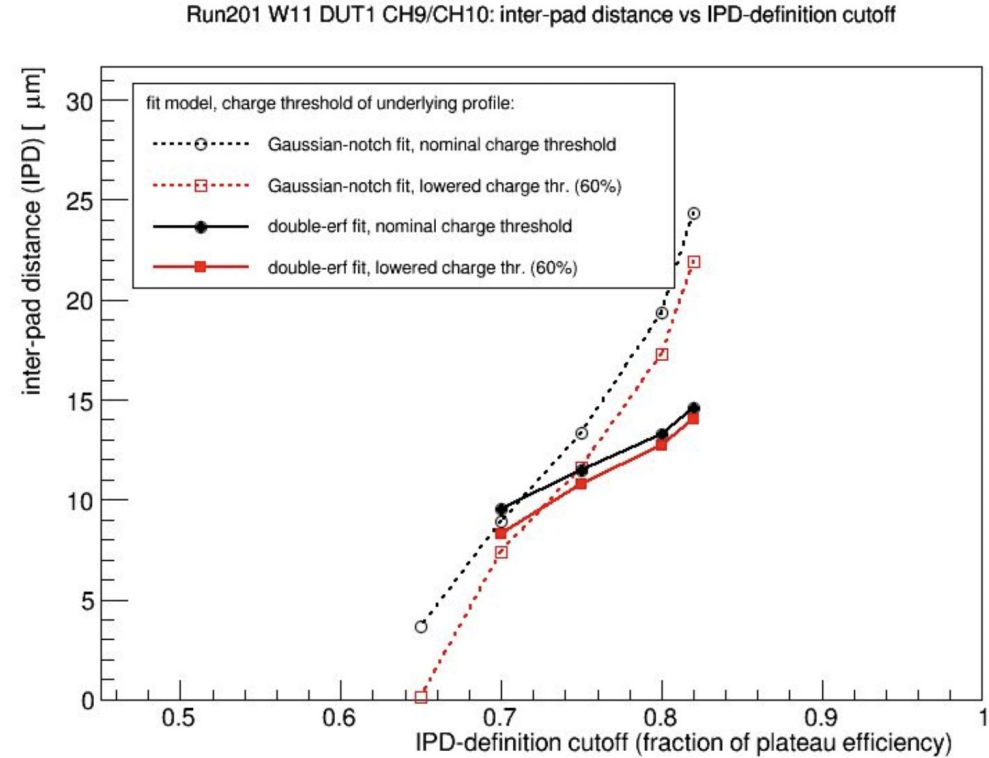
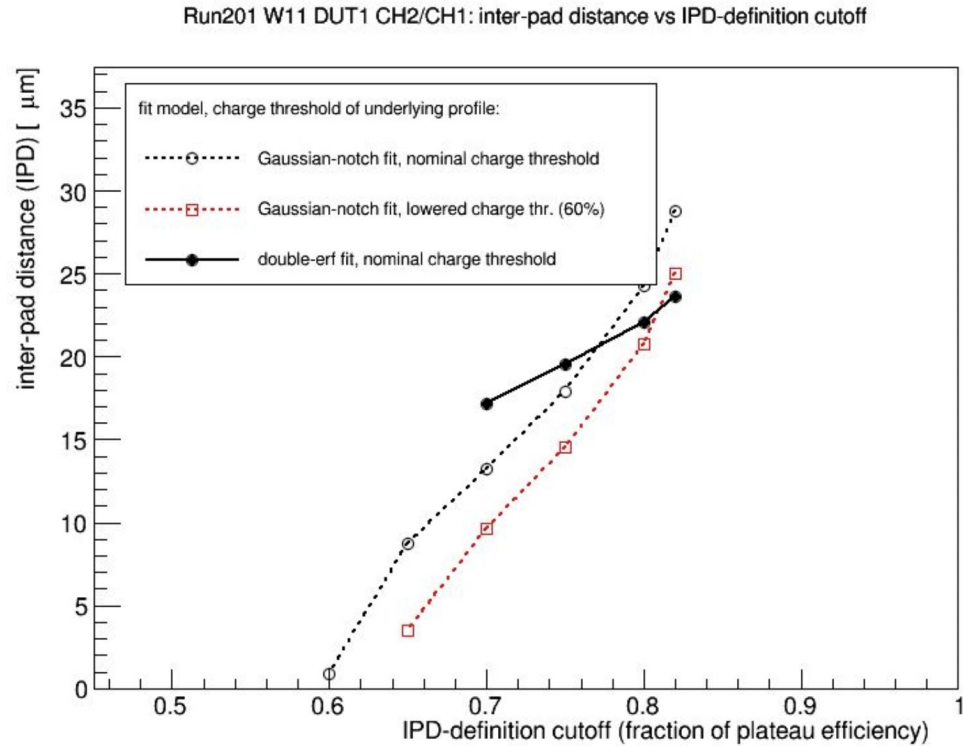


IPD results for wafer 11 and 5



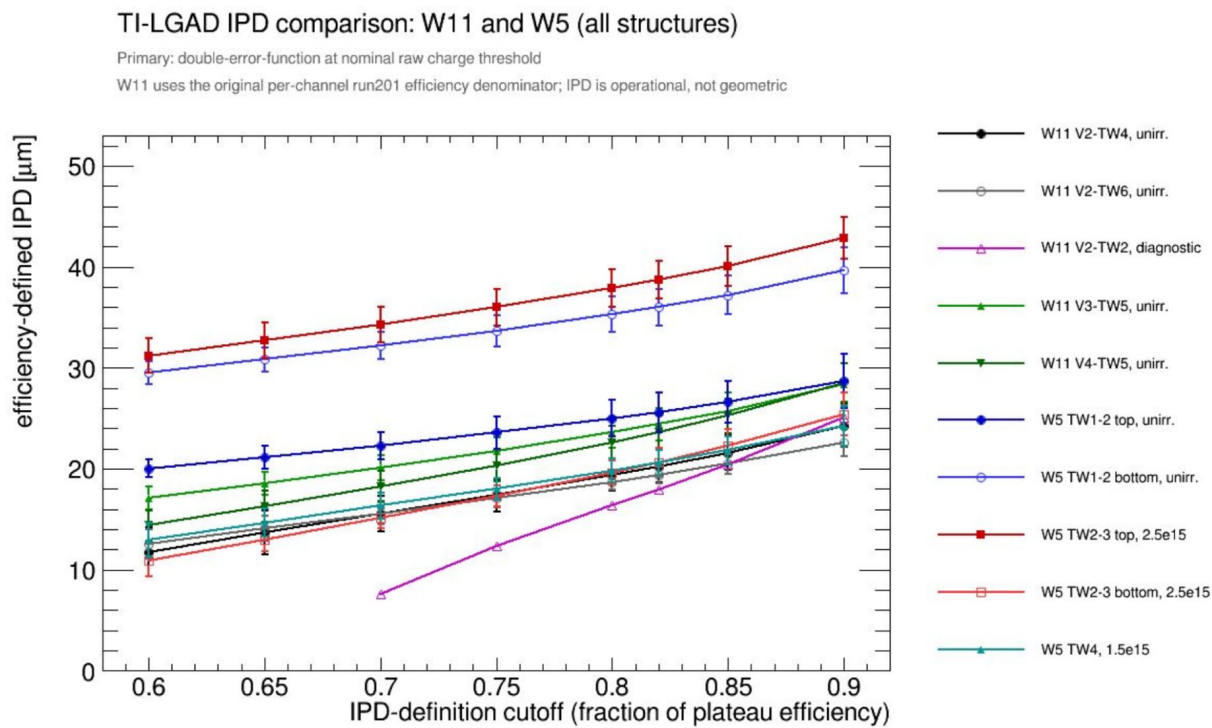
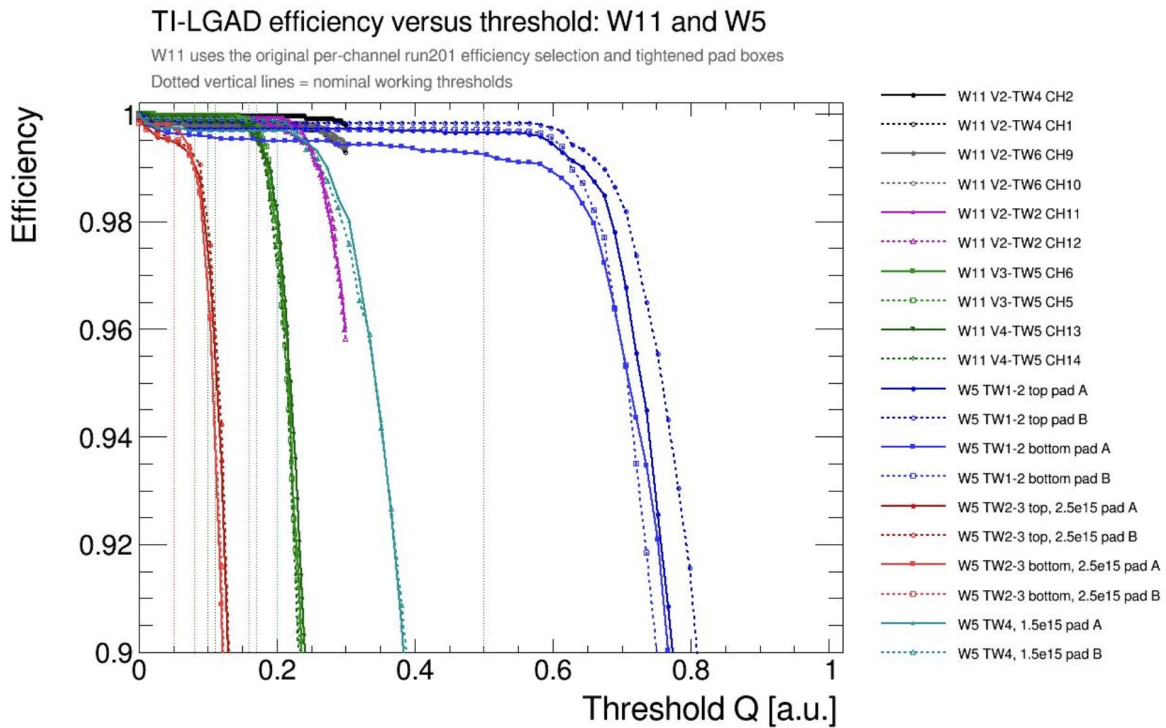
IPD is defined as the distance between the two double-erf crossings at a chosen fraction of the plateau efficiency

IPD for TI-LGAD from wafer 11 versions V2 and V3/4

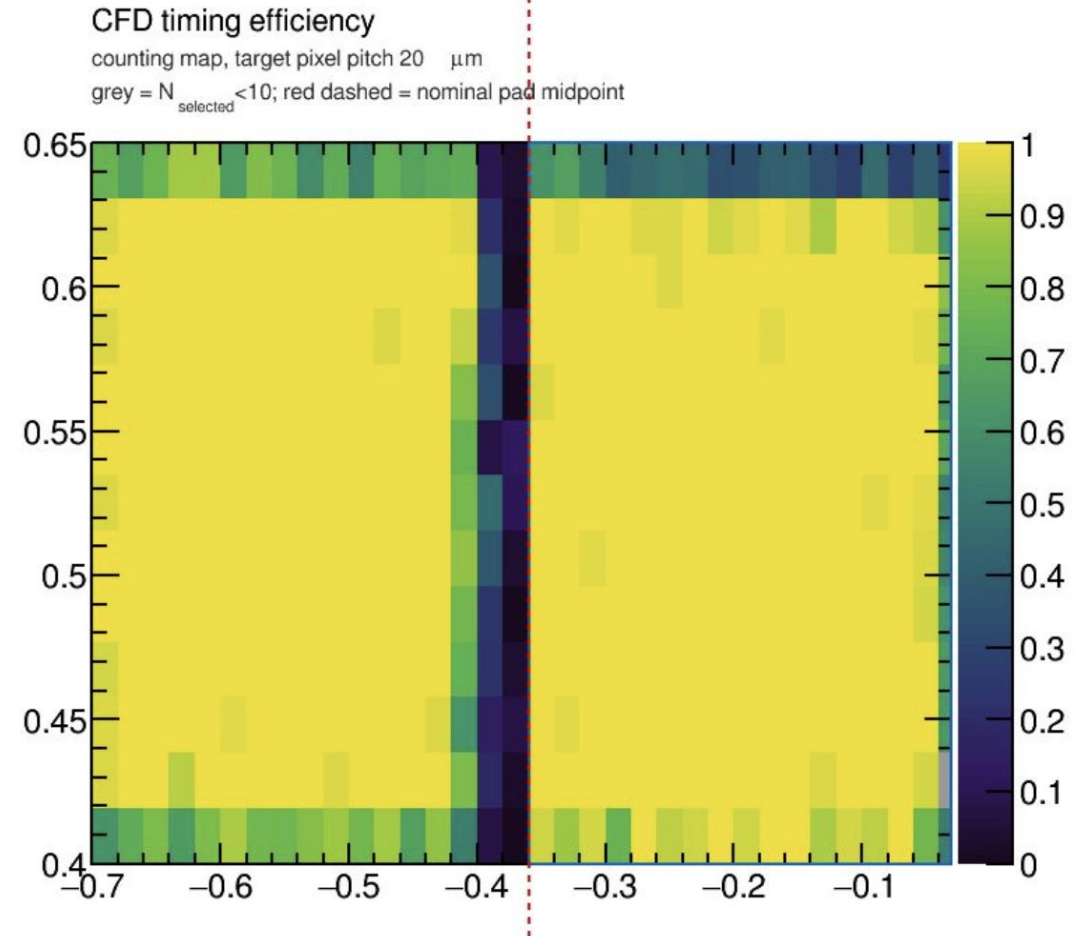
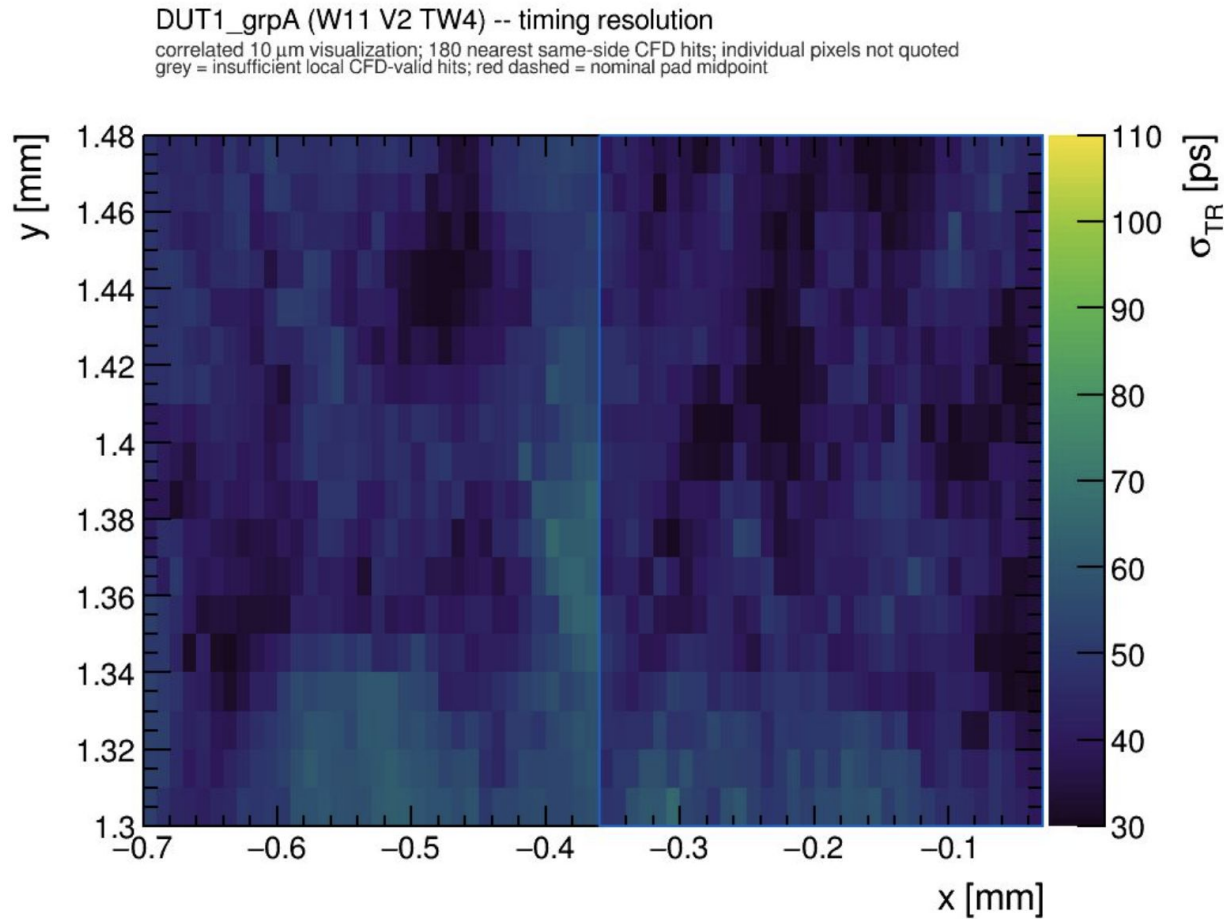


The lowered-threshold curve ($0.6 Q_{\text{thr}}$) is used to assess the sensitivity of the extracted IPD to the hit-threshold definition.

Efficiency vs threshold and IPD for TI-LGAD from wafer 5 and 11



Spatial timing performance — W11 V2-TW4, unirradiated, 150 V



- Bulk timing resolution: ~40–42 ps.
- CFD efficiency decreases locally in the inter-pixel region, accompanied by a degradation of σ_t .
- Away from the inter-pixel region, timing recovers to the clean-bulk level. **Results for wafer 5 in the backup slides**



Charge Spectrum

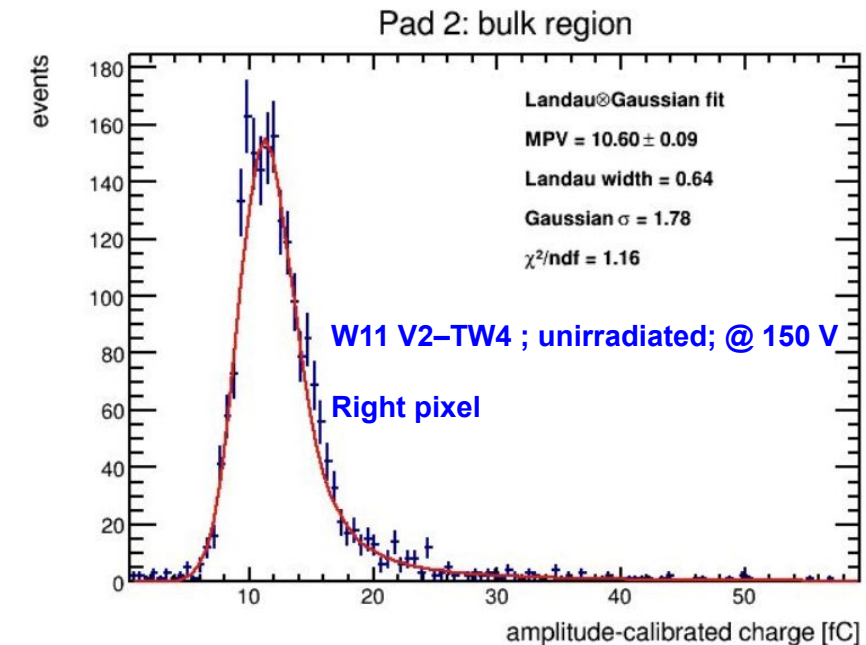
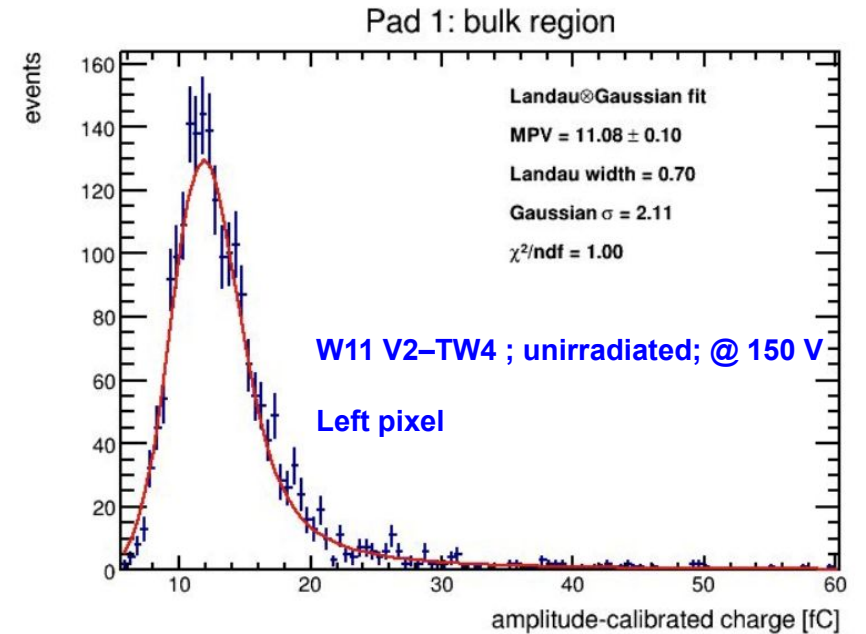
Kernel Density Estimate approach (Close to the inter-pixel region)

- Gaussian kernel density estimate over the charge sample.
- Bandwidth estimator: Silverman's rule $\times n^{-0.2}$, with a grid search + parabolic refinement for the peak.
- resampling gives the uncertainty.
- Used for: the fine $5\text{ }\mu\text{m}$ spatial profile and the 2D local maps, where a bin's population is often modest.

Landau $\otimes$ Gaussian fit

- Explicit physical model: Landau energy-loss fluctuation convolved with Gaussian electronic noise.
- Returns MPV, Landau width, Gaussian σ , and χ^2/ndf — with real error bars.
- Accepted only if the fit converges with good covariance and $\chi^2/\text{ndf} < 10$; otherwise the KDE mode is reported instead
- Used for: the bulk spectra — this is the primary quoted number for each pad's undisturbed response.

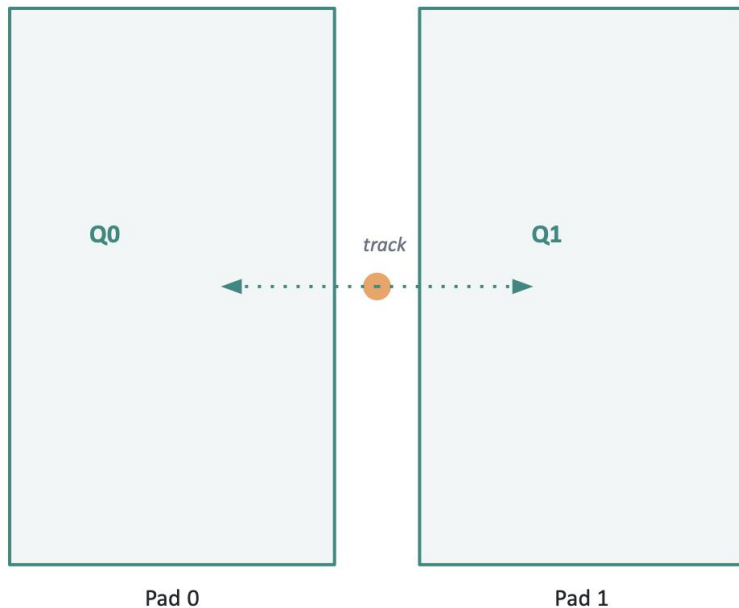
$$Q[\text{fC}] = \frac{V_{\text{peak}}[\text{mV}]}{5.67},$$



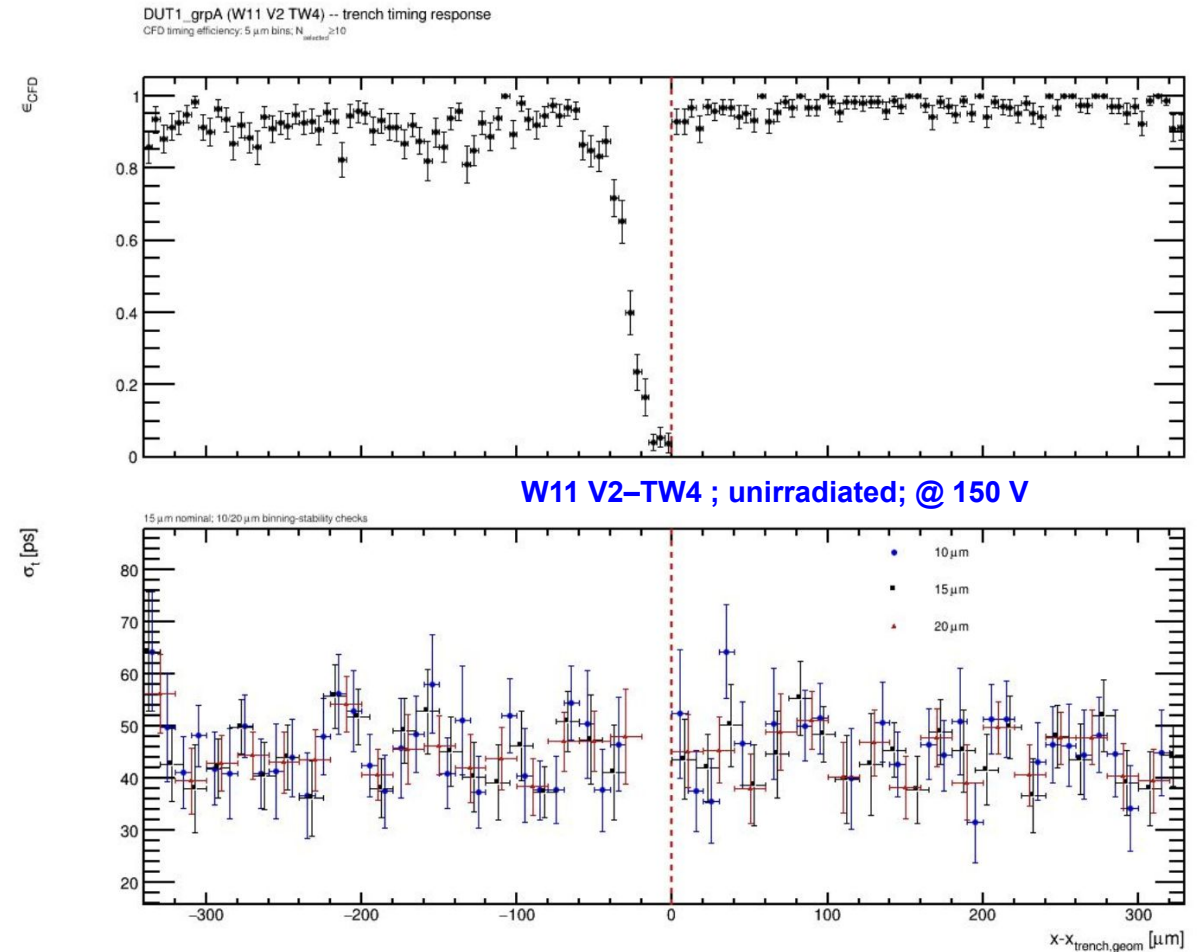


One Charge Sample, Both Pads

- For every accepted track, BOTH neighboring pads are integrated — not just the one geometrically “hit.”
- This is what makes charge sharing measurable: an event near the gap leaves a real, comparable signal on each side, not a single ambiguous number.
- Each sample stores: q_0 / q_1 in fC, the raw noise-only integral (a floor estimate), peak amplitude, and the short/long-window variants.



both integrated for every track, regardless of which pad it geometrically hit



Bulk Reference & Normalized Observables

- “Bulk” = a fixed band, 0.10–0.23 mm from the pad's own inner edge, central-y only — safely away from both the gap and the outer edge, the same logic as the timing bulk-ROI.
- Each pad's own bulk Landau⊗Gaussian MPV becomes that pad's normalization scale.
- Every event's charge is then expressed as a fraction of its own pad's bulk MPV: $a = Q_0 / \text{MPV}_0$, $c = Q_1 / \text{MPV}_1$.

$$Q_{\text{sum,norm}} = a + c$$

≈1 in the bulk; drops if charge is truly lost crossing the gap

$$\eta = (c - a) / (a + c)$$

−1 = all charge in pad 0, +1 = all charge in pad 1

Normalization per pad

- Removes pad-to-pad gain/response differences before comparing charge sharing
- Makes $Q_{\text{sum,norm}}$ and η insensitive to the absolute-fC calibration
- Every downstream spatial product (profile, 2D maps) is built on TOP of these two normalized numbers, plus the plain assigned charge in fC.

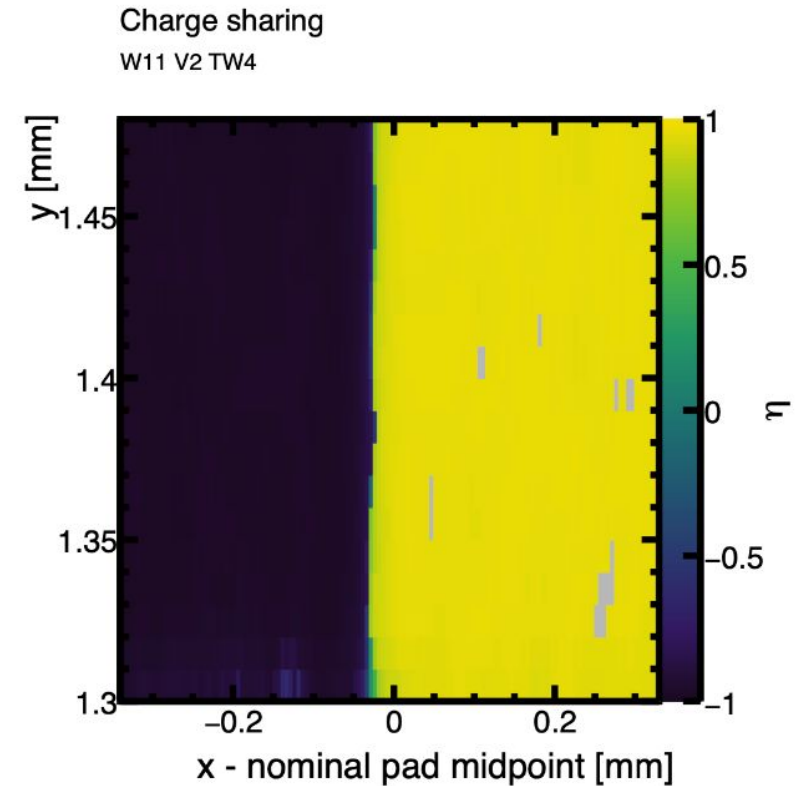
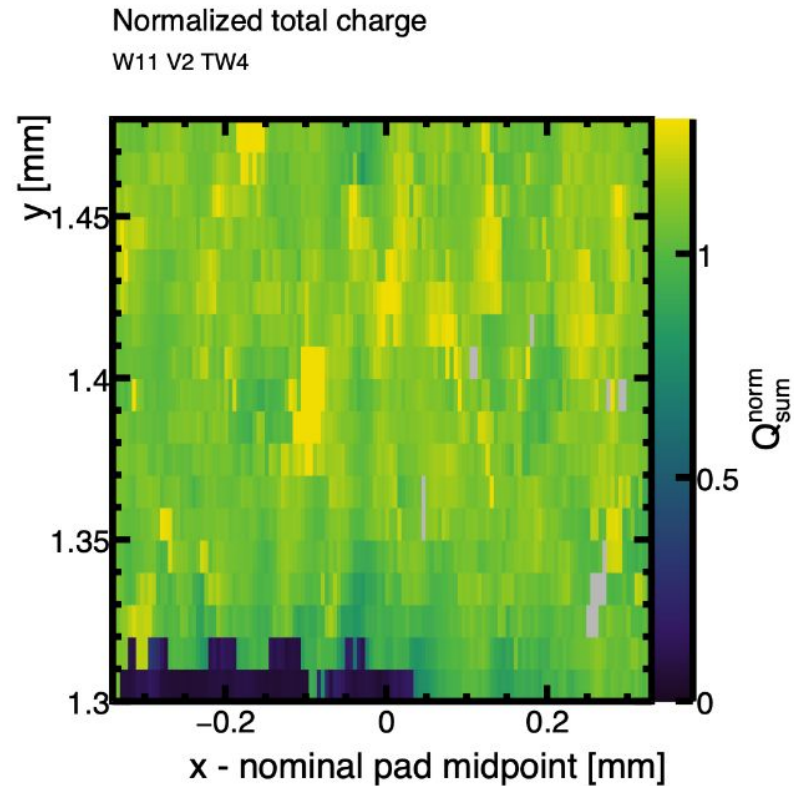
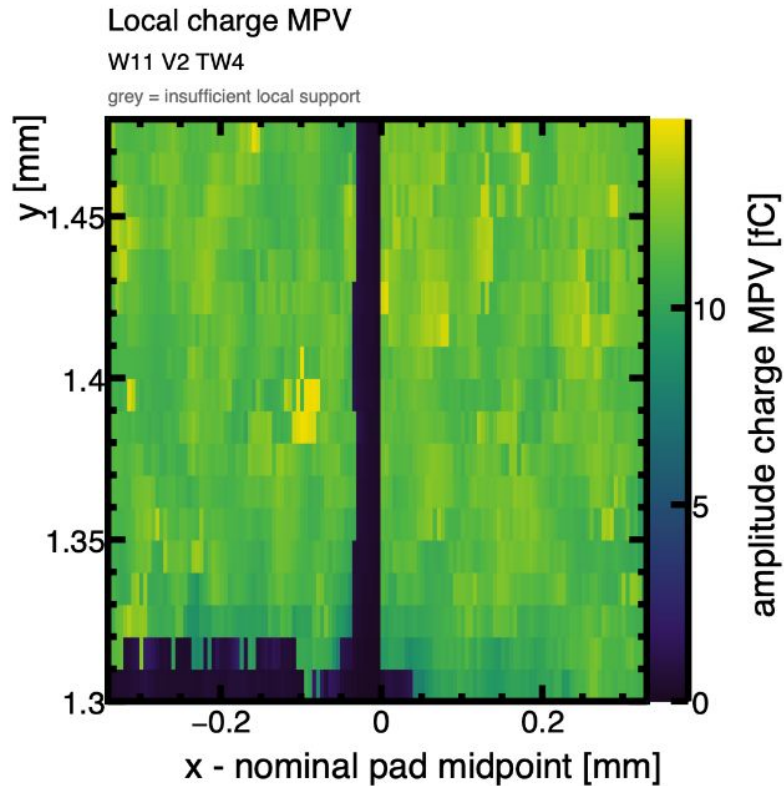


Fine 2D Maps: Local Neighborhoods

- Query grid: $5\text{ }\mu\text{m (x)} \times 10\text{ }\mu\text{m (y)}$
- Each query point's value = KDE mode of its 60 nearest same-side charge samples — not a fixed spatial bin.
- Two safeguards against over-interpreting sparse regions: the 60th-nearest neighbor must be within $50\text{ }\mu\text{m}$, AND the closest real track within $12\text{ }\mu\text{m}$.
- Every physical map (local MPV, $Q_{\text{sum,norm}}$, η) ships with its own explicit validity mask, plus a separate “effective radius” map — so a genuinely low charge near zero is never confused with “no data here.”

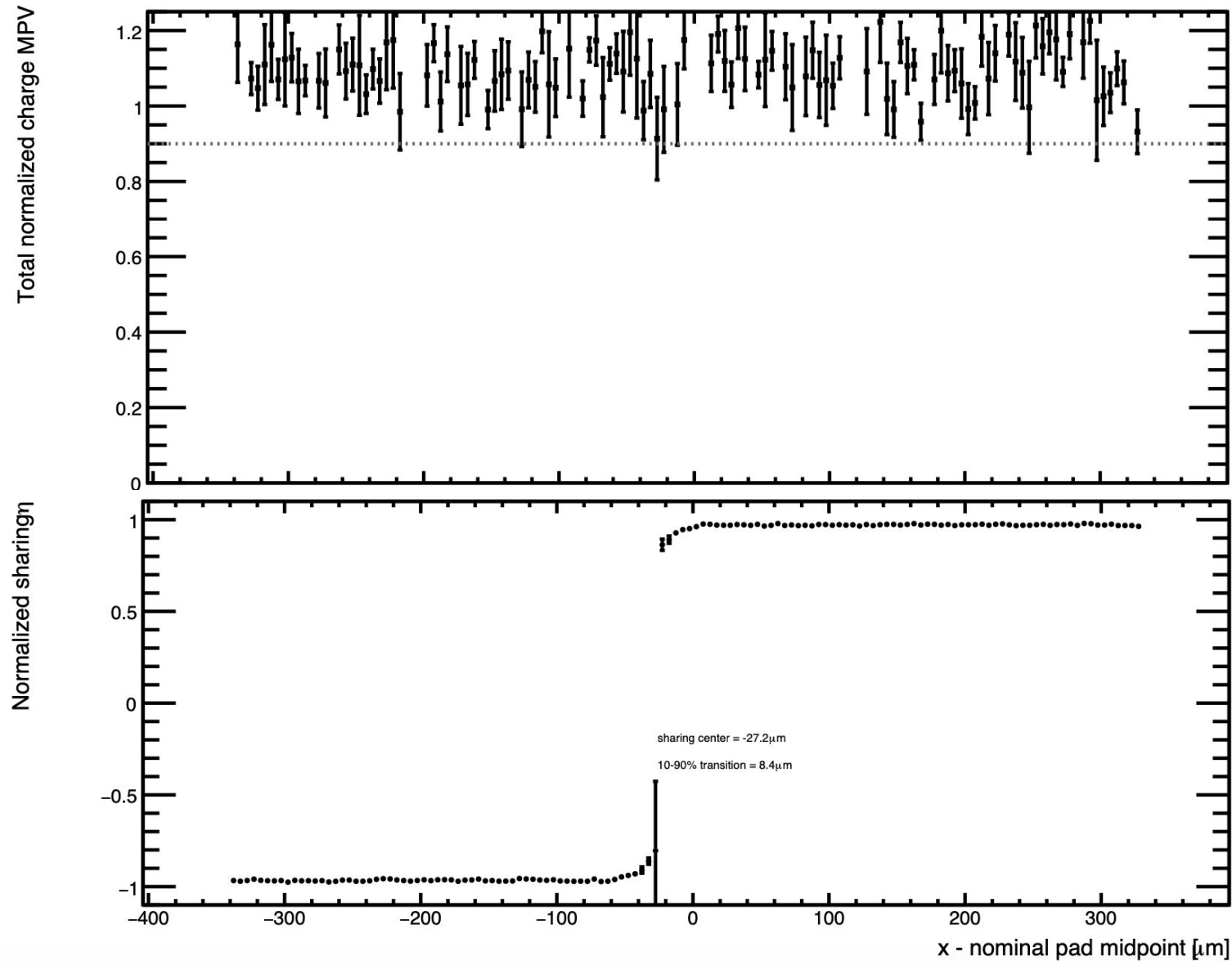


2D charge response and charge sharing — W11 V2–TW4, 150 V



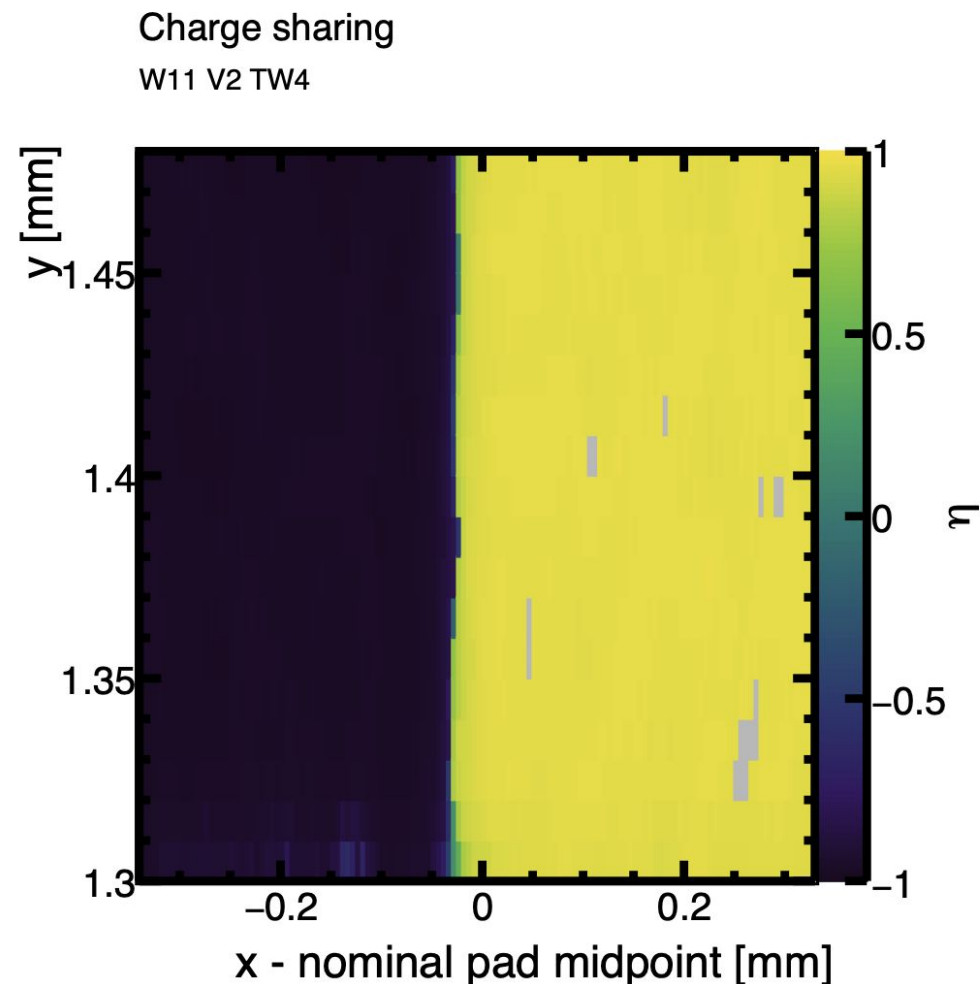
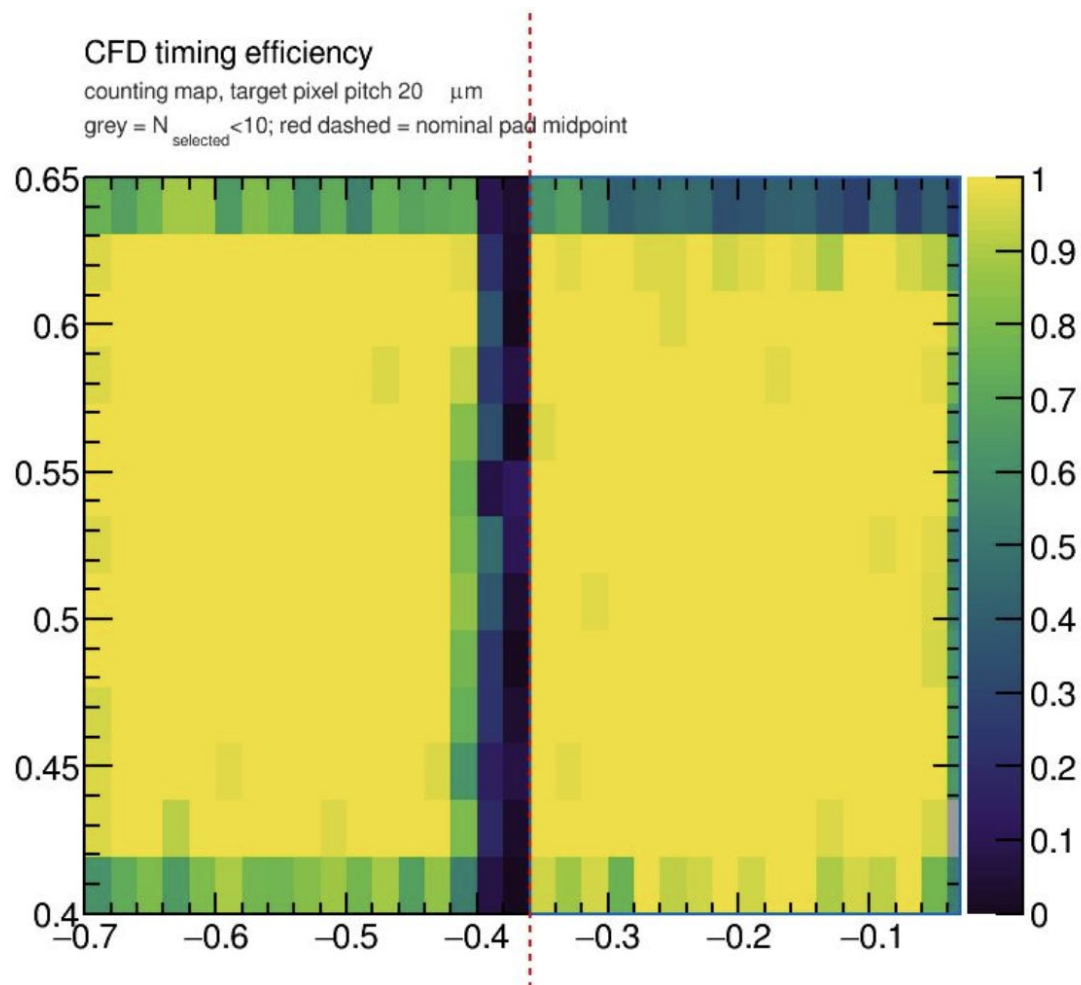
- The inter-pixel structure is localized in 2D.
- The summed response and sharing observable show the same spatial transition.
- The 2D result is consistent with the 1D charge-sharing profile.

Charge sharing across the inter-pixel region W11 V2-TW4, 150 V



- The summed two-pad response is approximately preserved across most of the transition region.
- The signal transfers sharply from one neighboring pixel to the other.
- Charge-sharing transition width (10–90%): ~8.4 μm.

Timing and charge resolve the same inter-pixel structure — W11 V2-TW4, 150 V



- Bulk timing remains at ~40–42 ps away from the inter-pixel region.
- The probability of obtaining a CFD timestamp drops locally where the signal is shared between neighboring pixels.
- Timing and charge therefore provide complementary views of the same inter-pixel response.

Spatial timing response — DUT2 W5 V2 TW 2/3, $2.5 \times 10^{15} n_{eq}/cm^2$

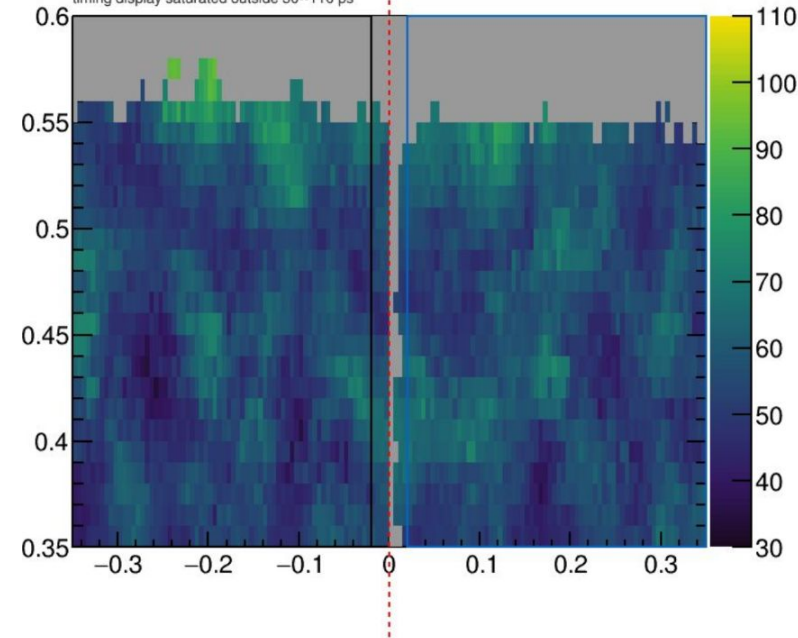
DUT2_grpA (run13231 DUT2 top (CH4-CH3))

correlated local timing-resolution estimate

5x10 μm query grid; 120 nearest same-side CFD hits; median $r_{all} = 23 \mu m$

correlated estimates; grey = insufficient local CFD support; red dashed = nominal pad midpoint

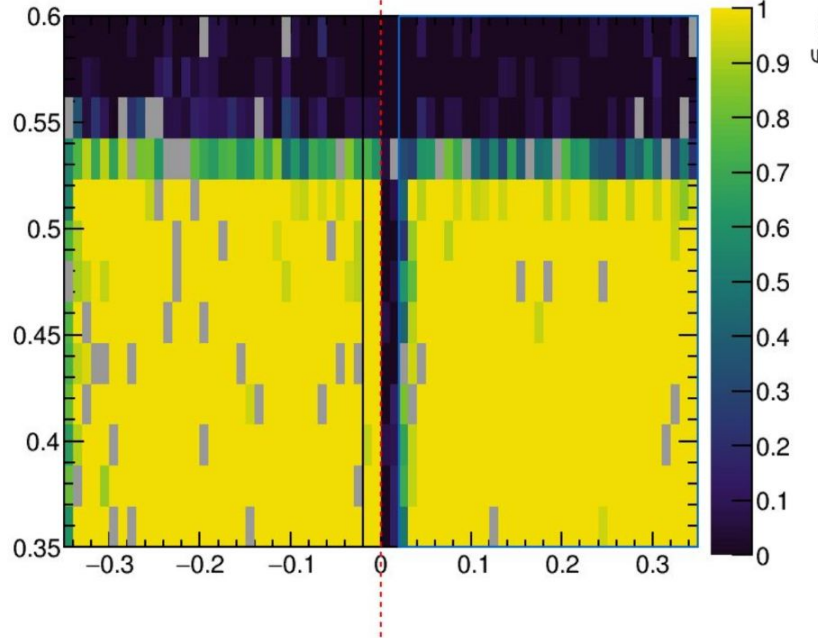
timing display saturated outside 30--110 ps



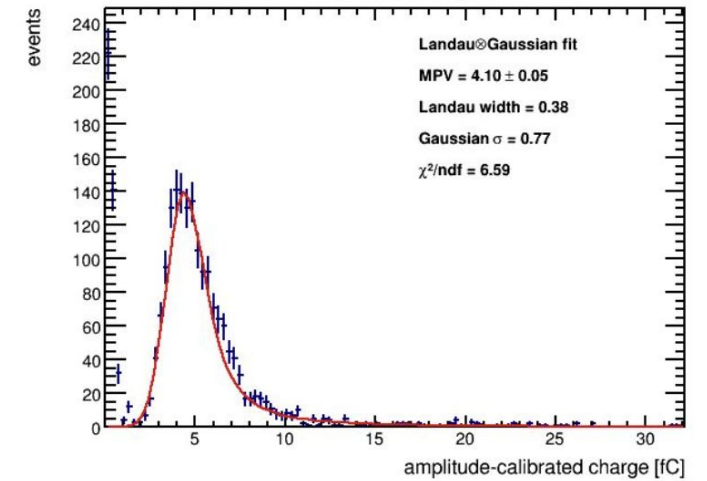
CFD timing efficiency

10x20 μm counting bins; grey = $N_{selected} < 10$

micron-scale inter-pad response is quantified with the 1D 10/15/20 μm profiles

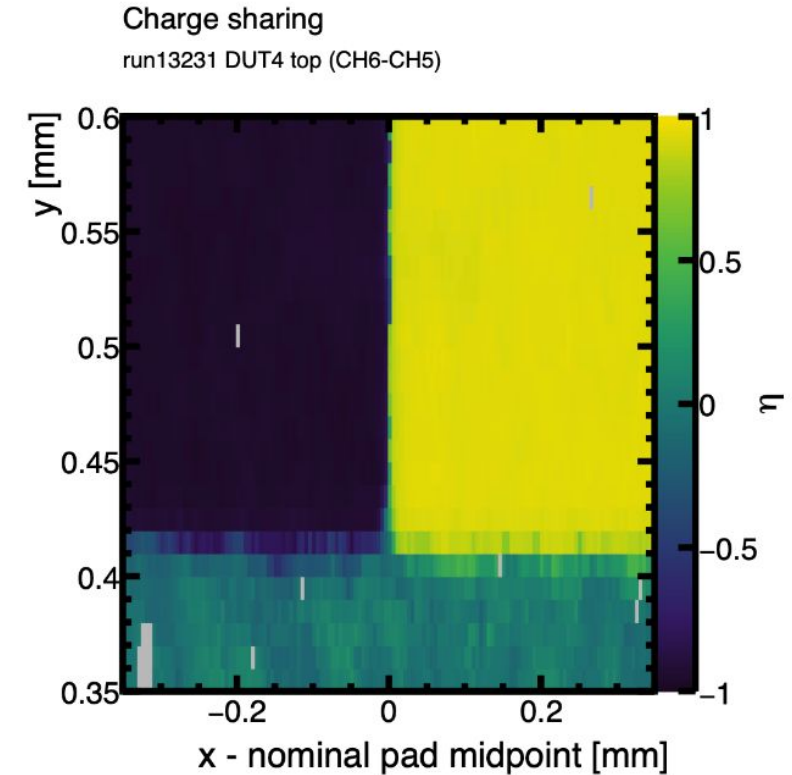
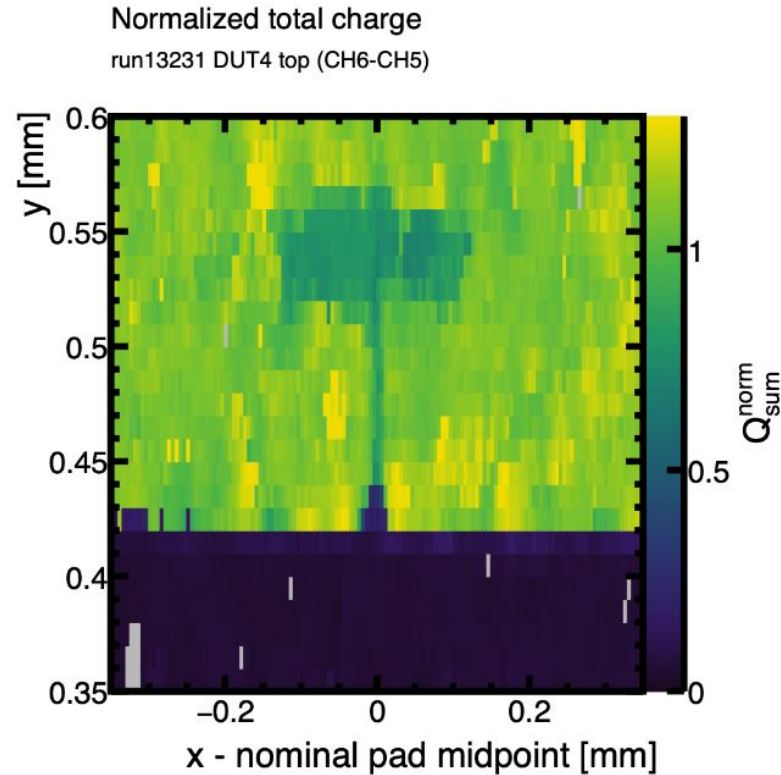
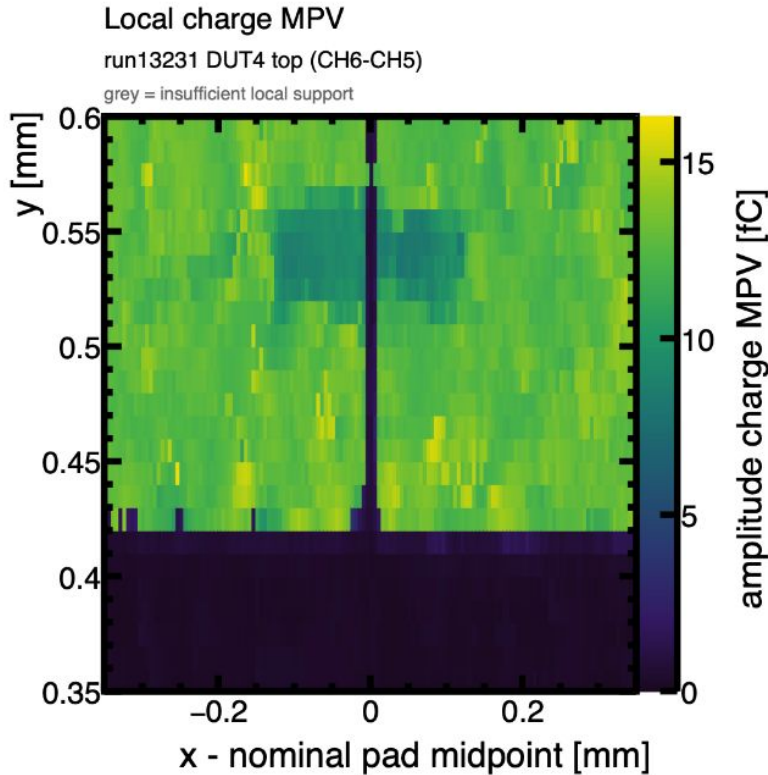


Pad 1: bulk region



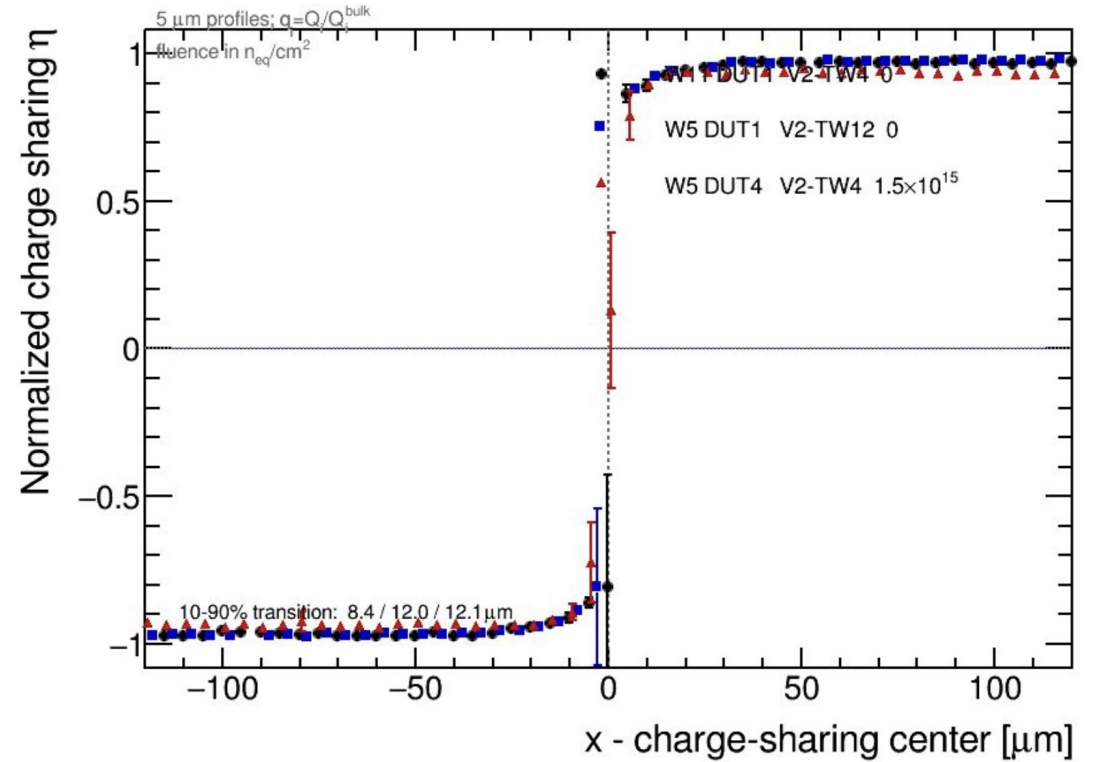
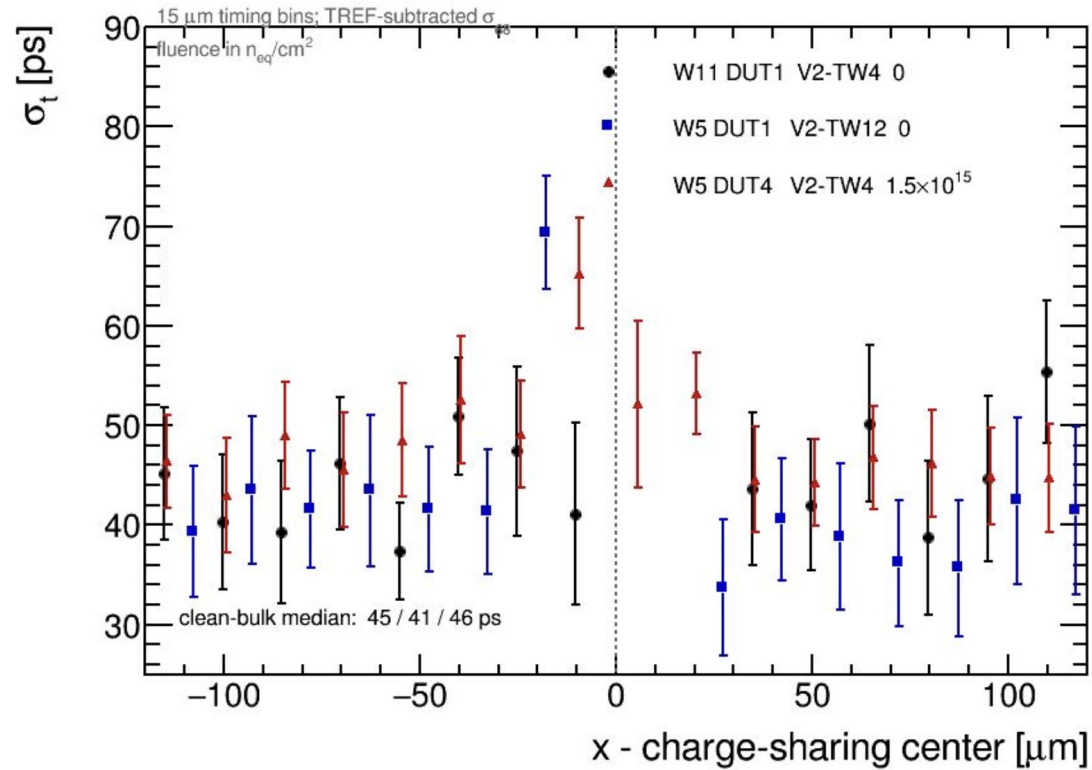
- Bulk timing: $\sim 55\text{--}60$ ps at $2.5 \times 10^{15} n_{eq}/cm^2$.
- Timing resolution degrades locally at the inter-pixel region.
- The degradation coincides with a localized loss of CFD timing efficiency.

Spatial charge response after irradiation — W5 V2 TW4, $1.5 \times 10^{15} n_{eq}/cm^2$



- A clear charge-sharing transition remains visible after irradiation.
- The signal transfers between the two neighboring pads across the inter-pixel region.
- The normalized total response and sharing observable remain spatially localized around the transition.

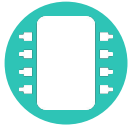
Inter-pixel timing and charge sharing in W11 and W5 TI-LGADs



- A narrow charge-sharing transition is observed in all three devices: 8.4 μm for W11 and $\sim 12 \mu\text{m}$ for both W5 devices.
- The timing degradation is localized around the charge-sharing region; away from it, the bulk timing is recovered.
- The irradiated W5 DUT4 retains a sharp inter-pixel transition and ~ 46 ps bulk timing at $1.5 \times 10^{15} n_{\text{eq}}/\text{cm}^2$.



BACKUP SLIDES

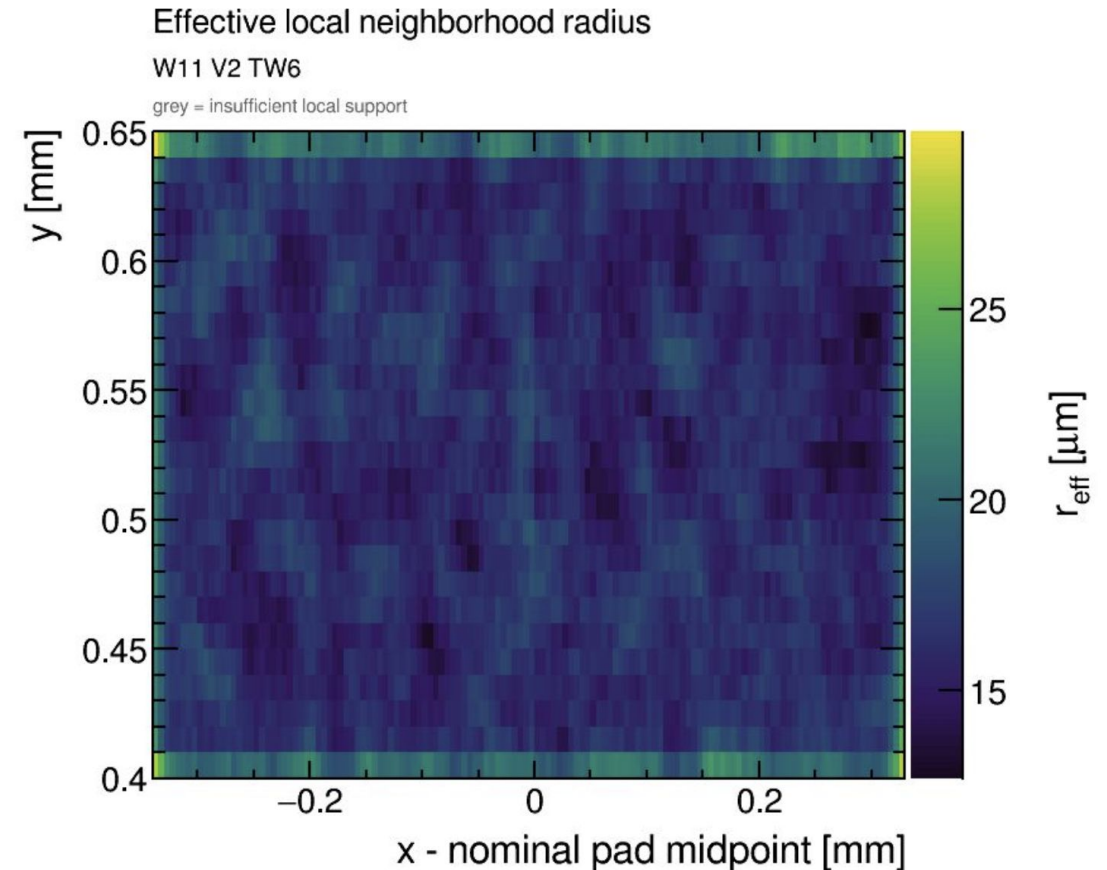


Converting to Absolute Charge (fC)

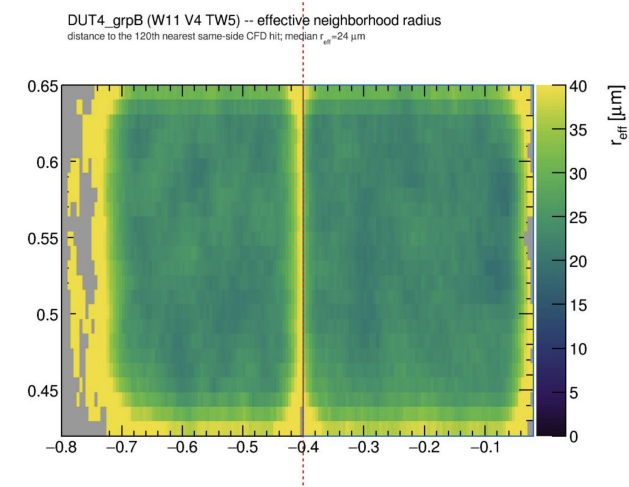
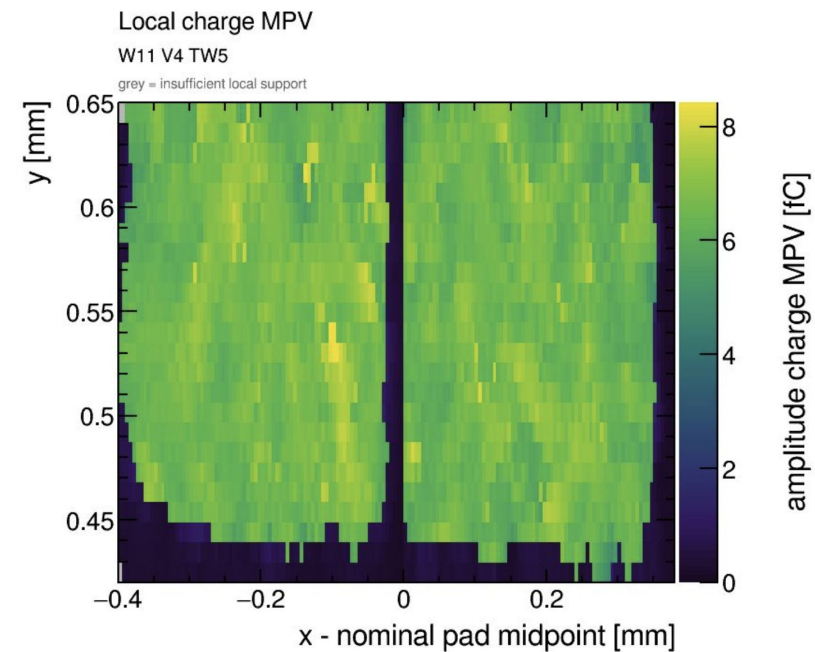
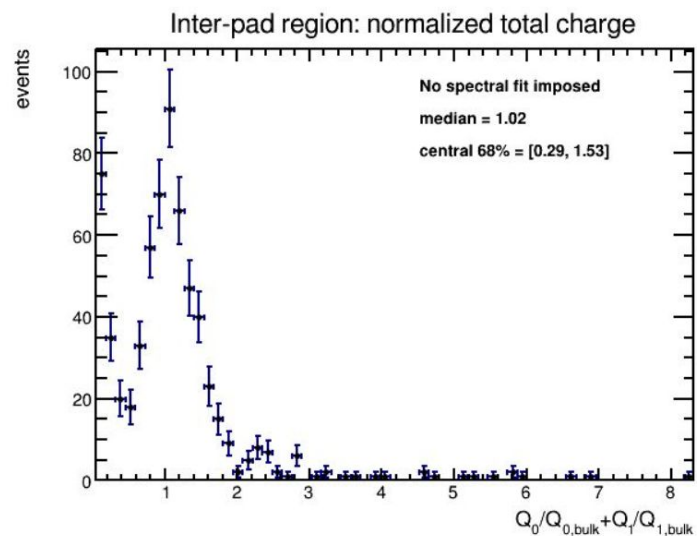
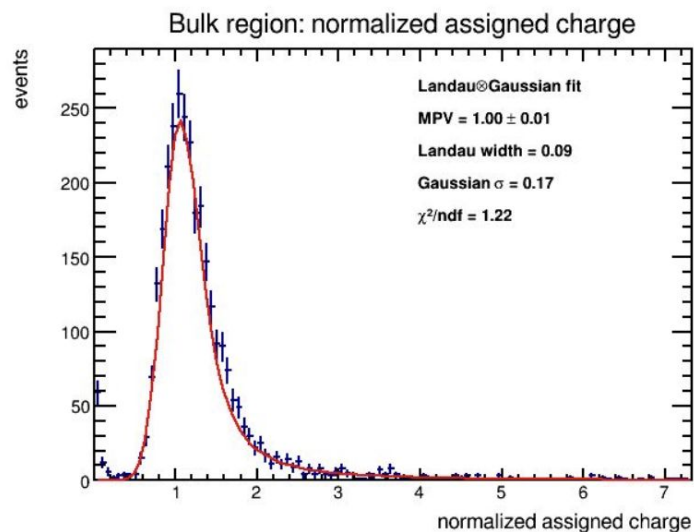
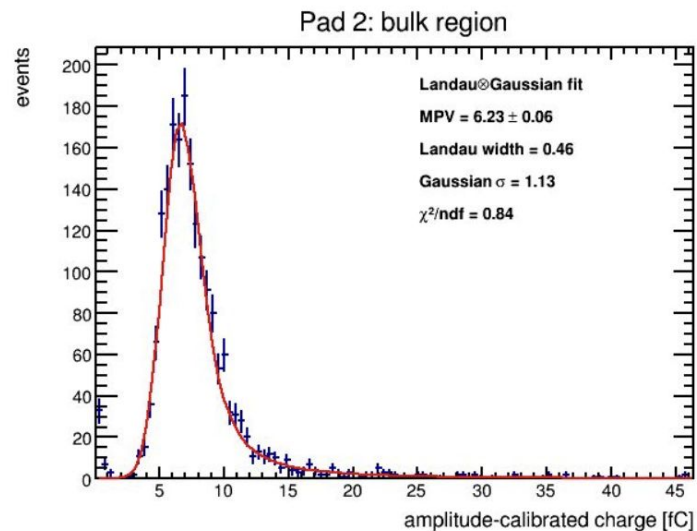
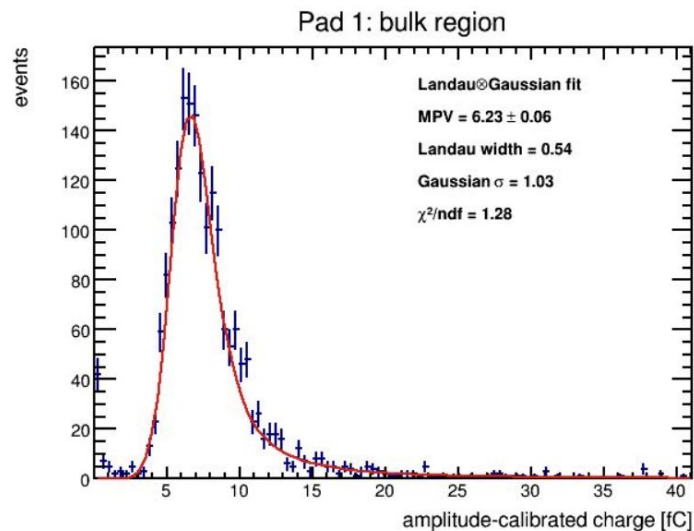
$$Q \text{ [fC]} = V_{\text{peak}} \text{ [mV]} / 5.67$$

5.67 ± 0.25 mV/fC; peak found in a ±0.6 ns window anchored to TREF's predicted pulse center

- Charge is read from the pulse's peak amplitude, not the integrated area — searched only in a narrow ±3-sample (±0.6 ns) window centered on the position TREF predicts, so a noise spike elsewhere on the trace can't be mistaken for the pulse.
- The 5.67 ± 0.25 mV/fC scale is the same amplitude-based transimpedance constant independently published for this exact Chubut board (5.67×10¹² V/C)
- This is still the only place an external, absolute calibration enters — and it now carries a quantified ±4.4% (0.25/5.67) common systematic on every absolute fC number, not folded into the per-fit statistical errors.



W11 V 3/4 TW5, unirradiated @120 V



W11 V 3/4 TW5, unirradiated @120 V

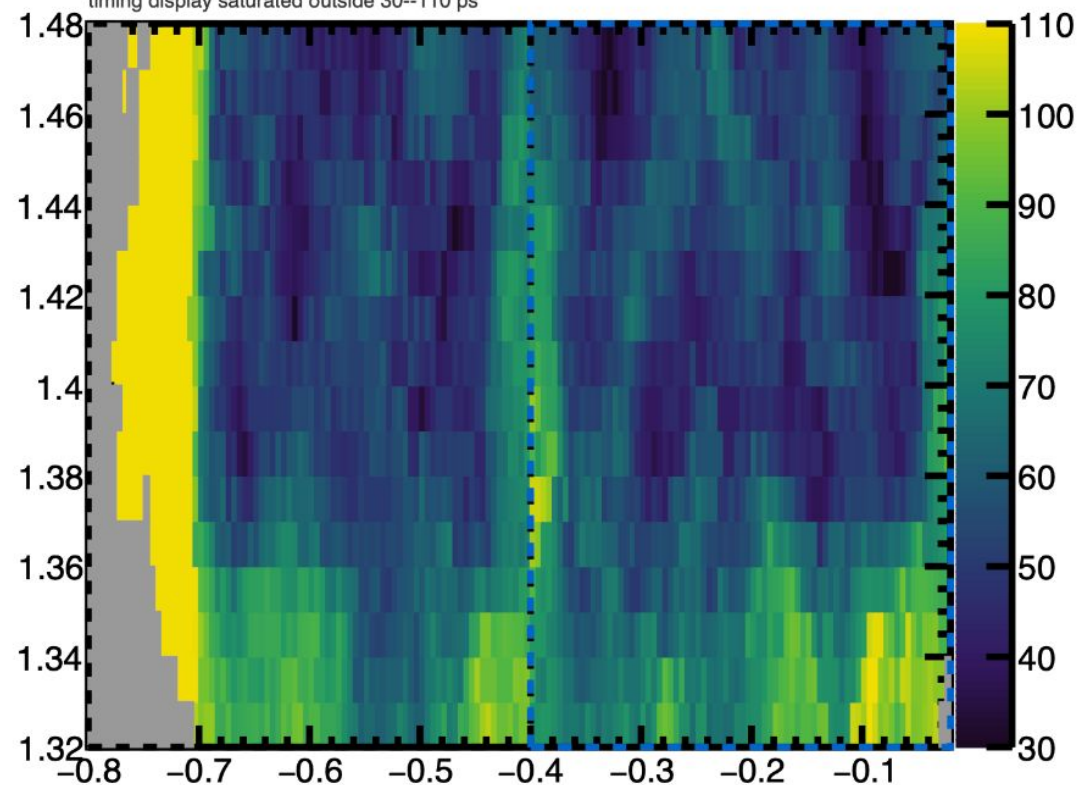
DUT4_grpA (W11 V3 TW5)

correlated local timing-resolution estimate

5x10 μm query grid; 120 nearest same-side CFD hits; median $r_{\text{eff}}=25 \mu\text{m}$

correlated estimates; grey = insufficient local CFD support; red dashed = nominal pad midpoint

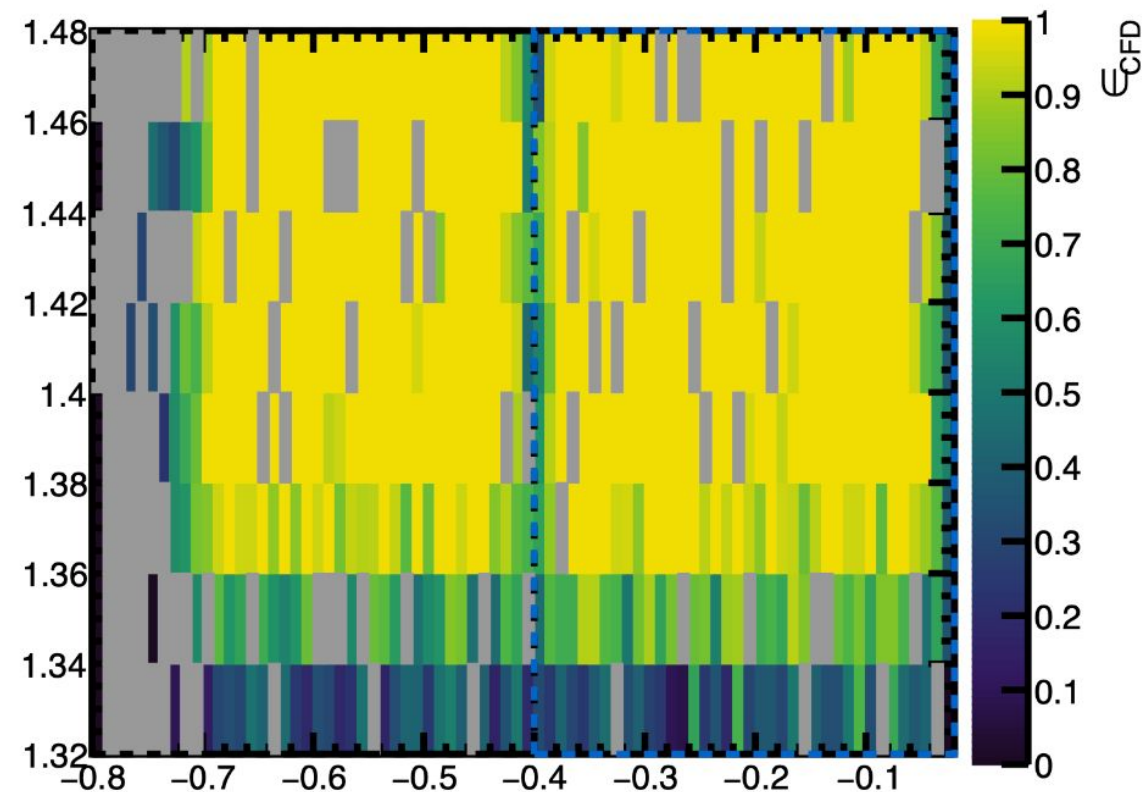
timing display saturated outside 30--110 ps



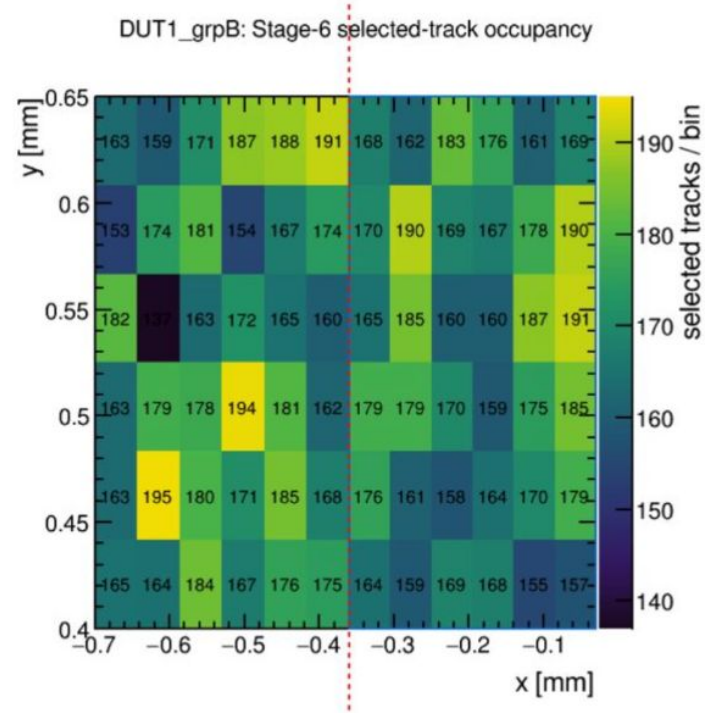
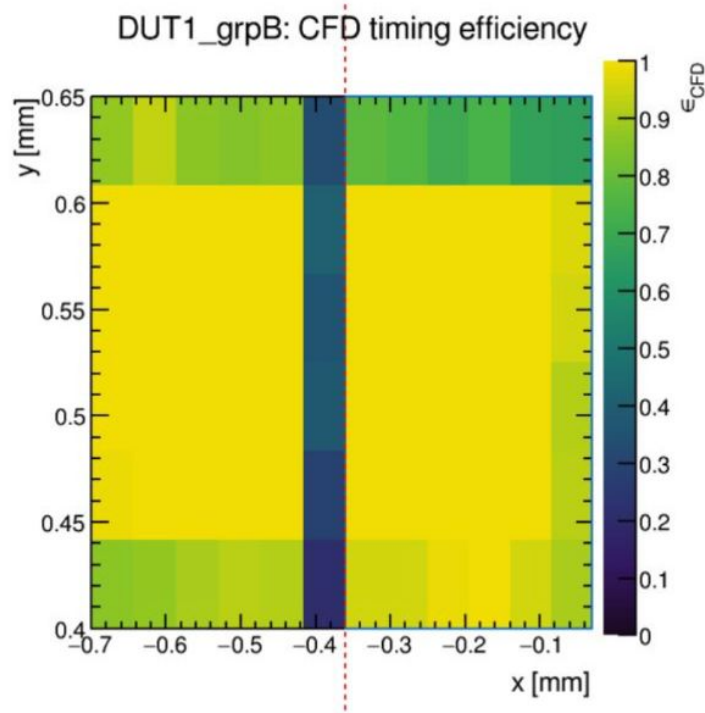
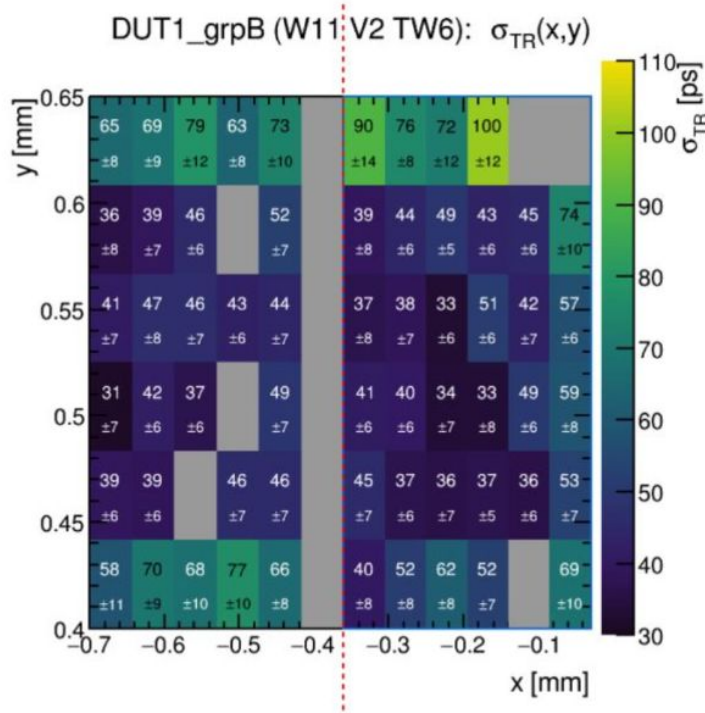
CFD timing efficiency

10x20 μm counting bins; grey = $N_{\text{selected}} < 10$

micron-scale inter-pad response is quantified with the 1D 10/15/20 μm profiles



Spatial timing response and track coverage — W11 V2-TW6, 150 V

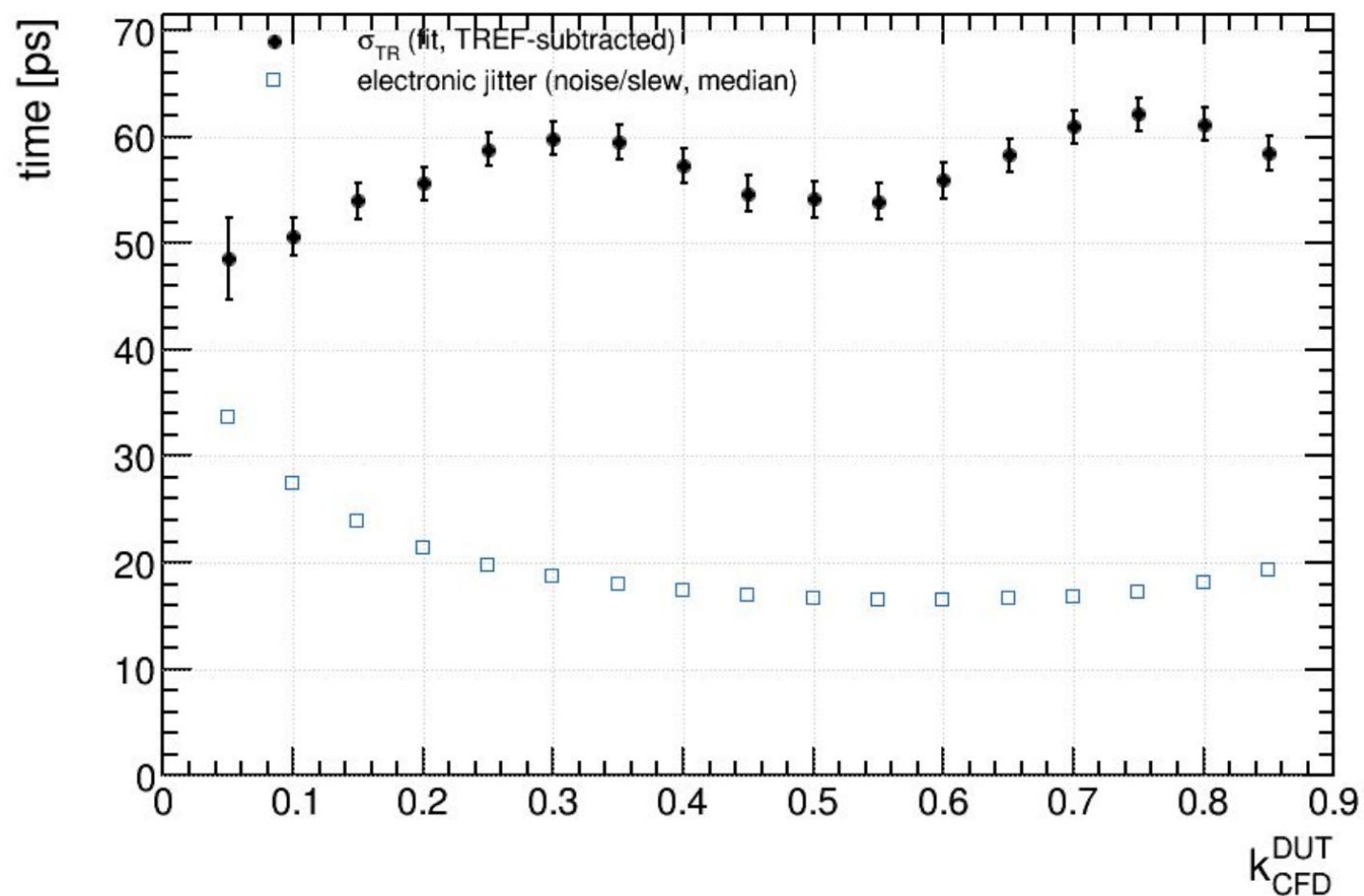


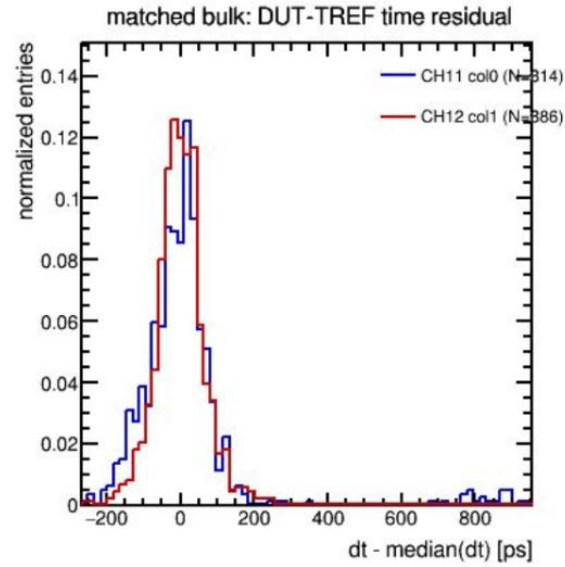
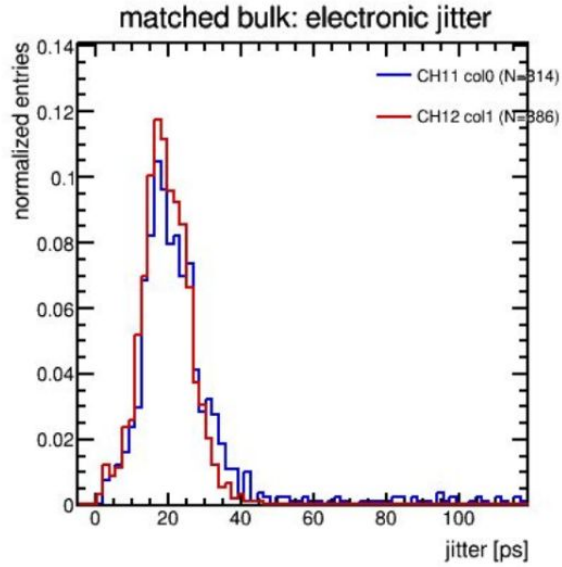
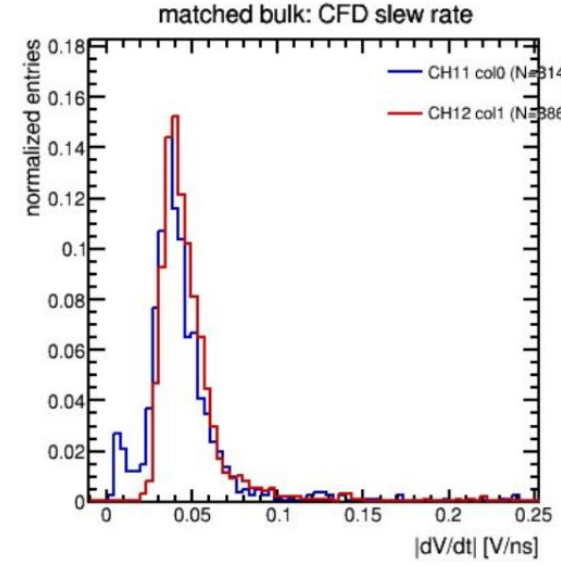
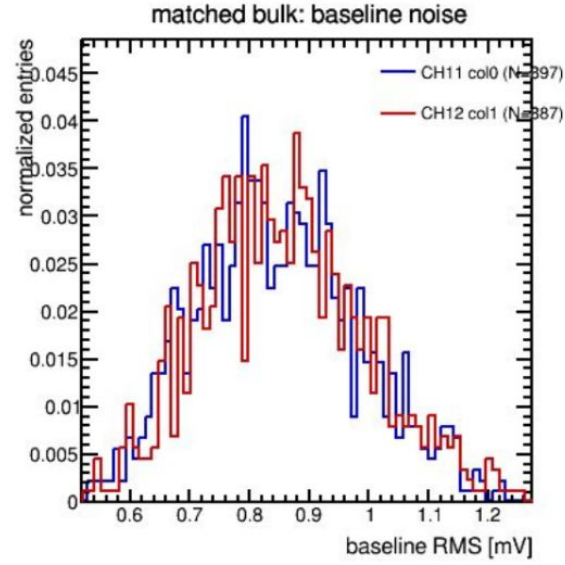
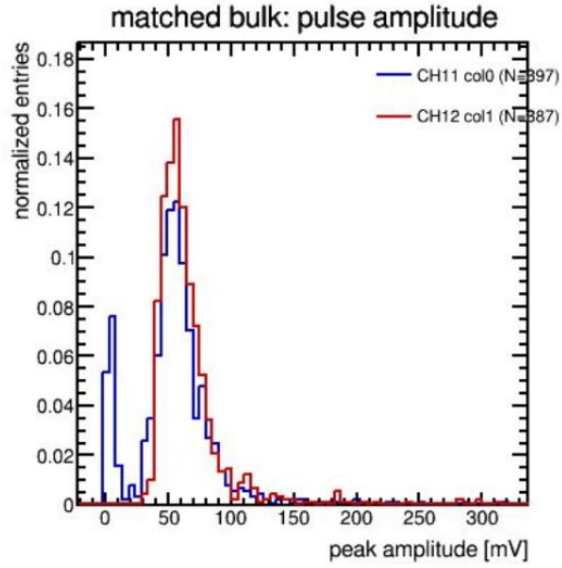
Electronic-jitter diagnostic vs CFD fraction — W11 V2-TW6

DUT1 (TW6, col0) -- CH9

σ_{TR} (fit, TREF-subtracted) and electronic jitter vs k_{CFD}

Best: $k_{\text{CFD}}=0.55$, $\sigma_{\text{TR}}=53.9 \pm 1.7$ ps





DUT2 group B = W11 TW2_V2

matched $|x-x_{\text{bench}}| = 100\text{-}170\ \mu\text{m}$

common $y = 0.490\text{-}0.660\ \text{mm}$

CH11 col0: Nsel=897, NCFD=814, $\epsilon=0.907$

median A/noise/slew/jitter = 54.28 mV / 0.845 mV / 0.040 V/ns / 21.0 ps

$\sigma_{\text{DUT}}^{\text{ss}} = 64.57 \pm 4.01\ \text{ps}$

CH12 col1: Nsel=887, NCFD=886, $\epsilon=0.999$

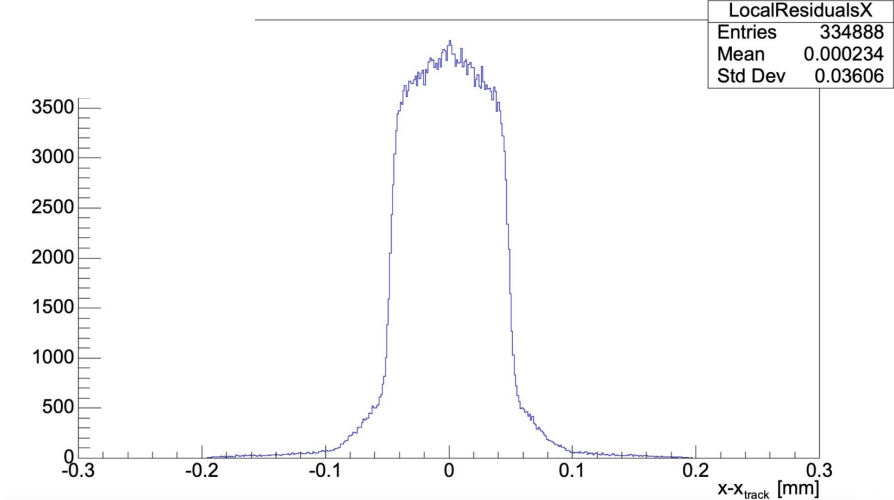
median A/noise/slew/jitter = 59.03 mV / 0.854 mV / 0.044 V/ns / 19.1 ps

$\sigma_{\text{DUT}}^{\text{ss}} = 39.14 \pm 3.27\ \text{ps}$

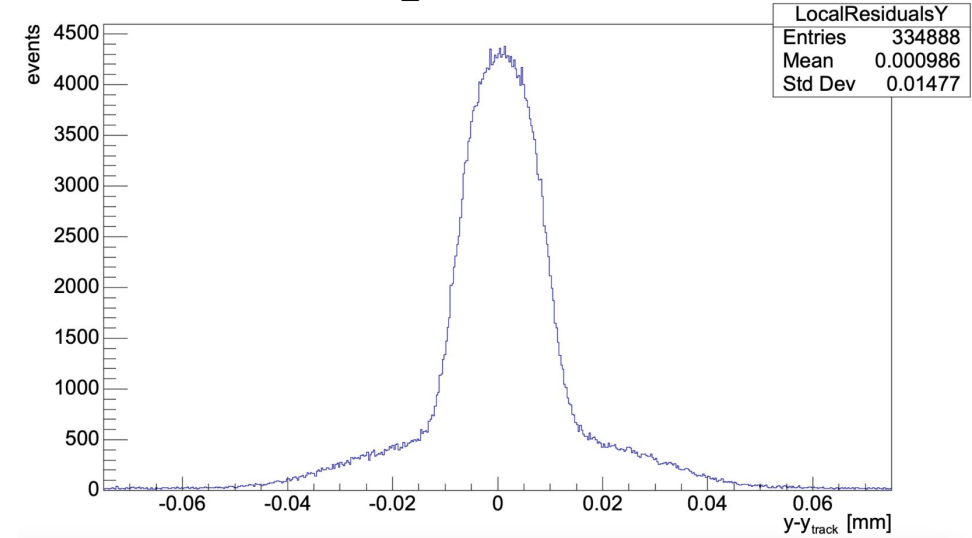
DIAGNOSTIC ONLY: same frozen k_{CFD} , same Stage-1 TREF, same matched geometry

Use this to identify whether the CH11/CH12 timing asymmetry is pulse/electronics driver

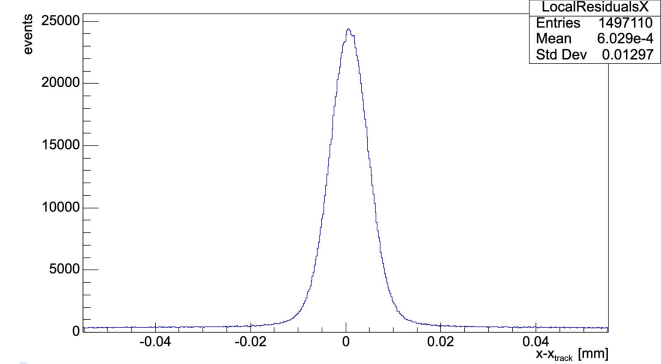
RD53B_115Local Residual X



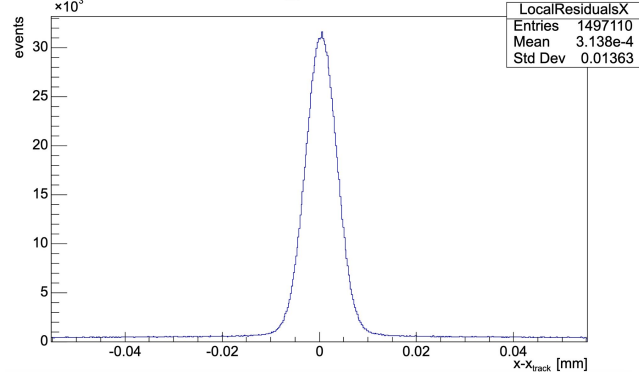
RD53B_115Local Residual Y



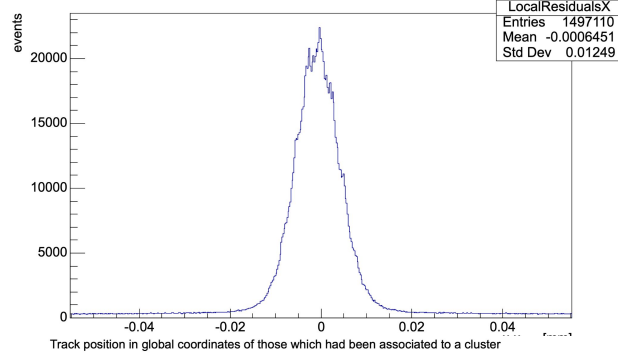
MIMOSA26_0Local Residual X



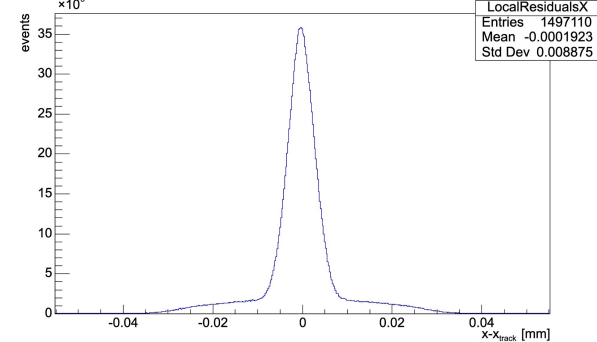
MIMOSA26_3Local Residual X



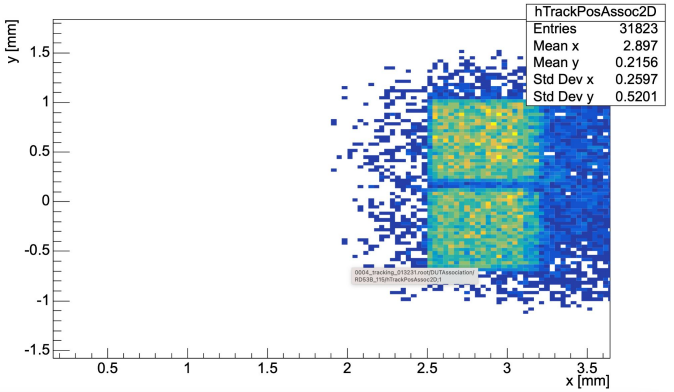
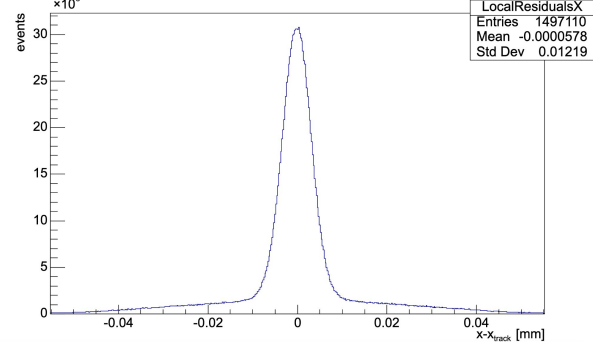
MIMOSA26_2Local Residual X



MIMOSA26_1Local Residual X



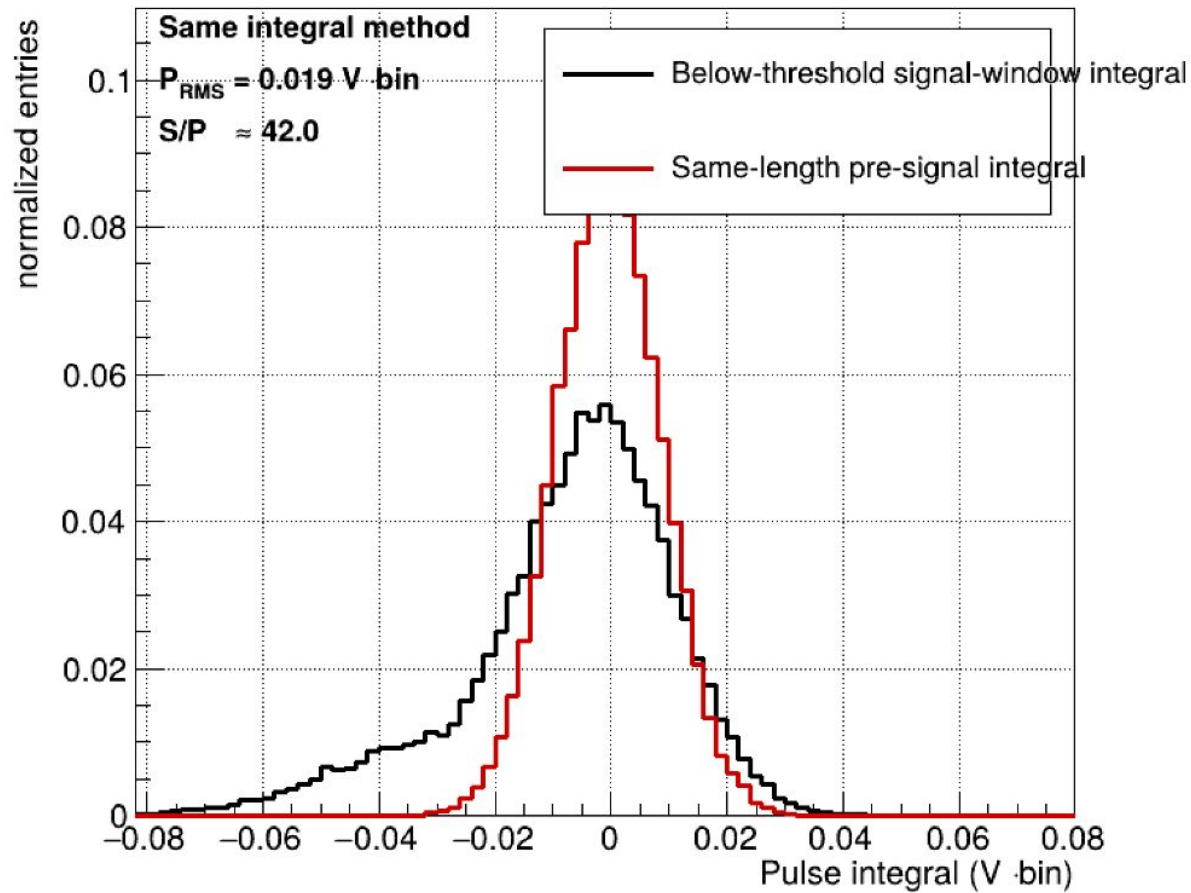
MIMOSA26_4Local Residual X



Pedestal Noise Comparison Before and After Irradiation

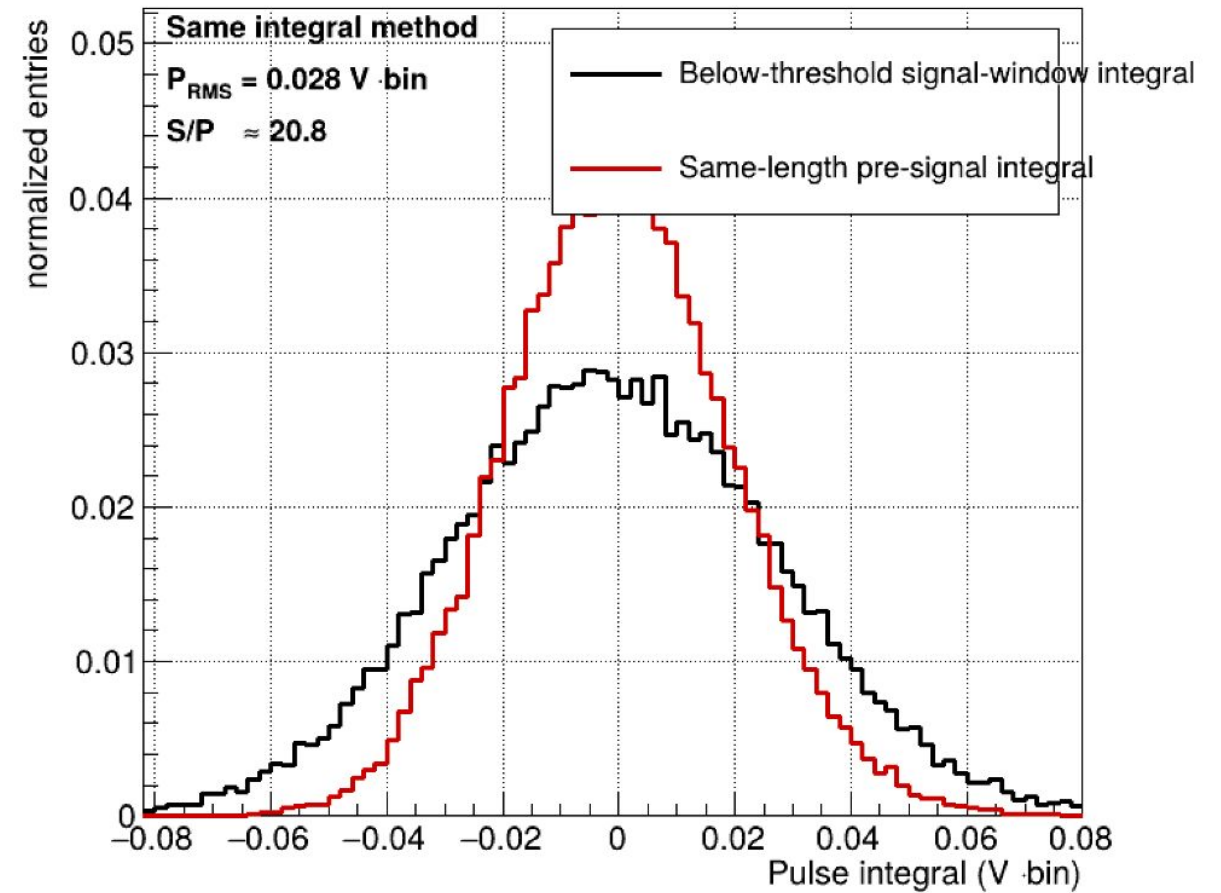
DUT1_0 non-irrad pixel 1

Pedestal-focused comparison



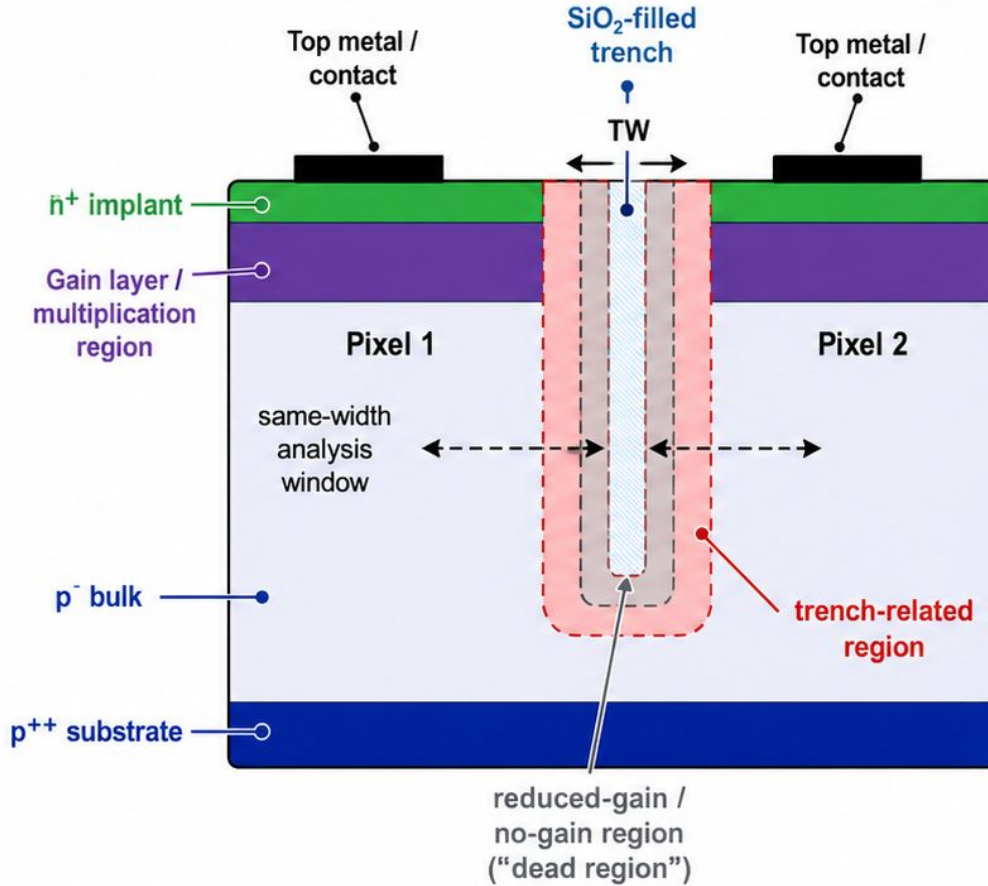
DUT3 1.5e15 pixel 1

Pedestal-focused comparison

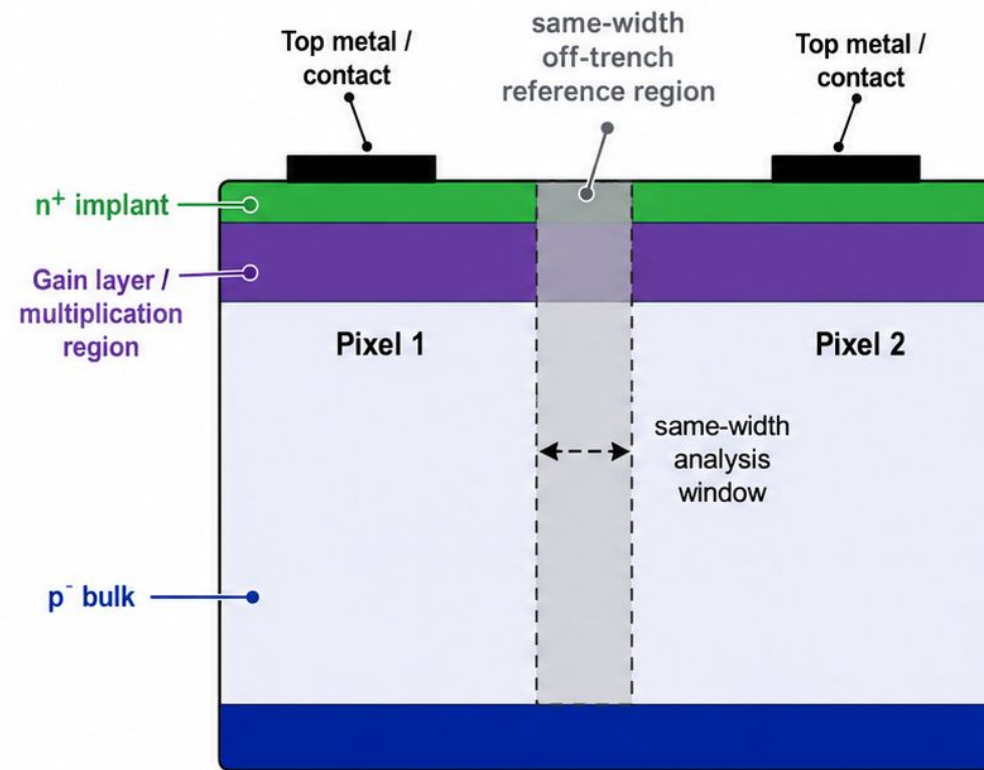


Analysis of trench and off-trench regions with tracks required

Trench region



Off-trench reference region



Red shading:
trench-related region

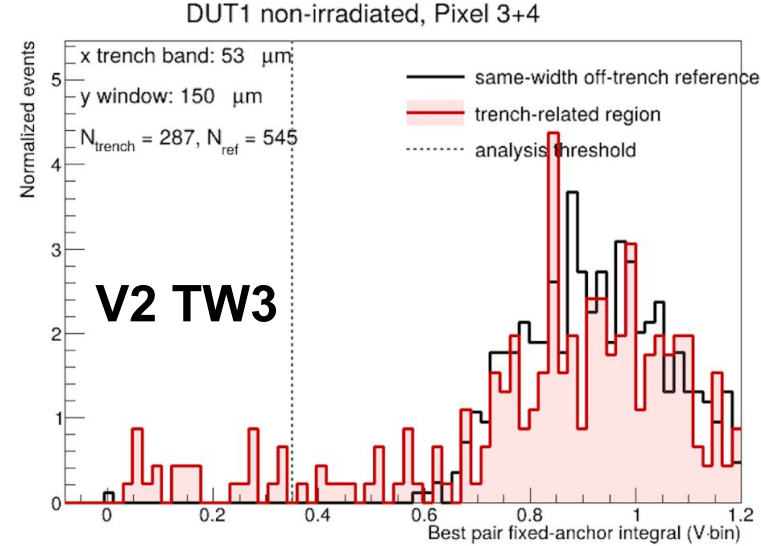
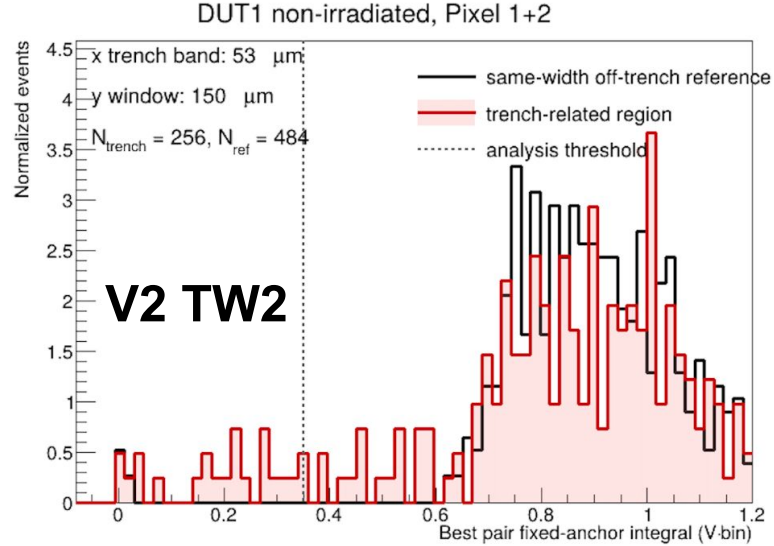


Gray shading:
same-width off-trench
reference region

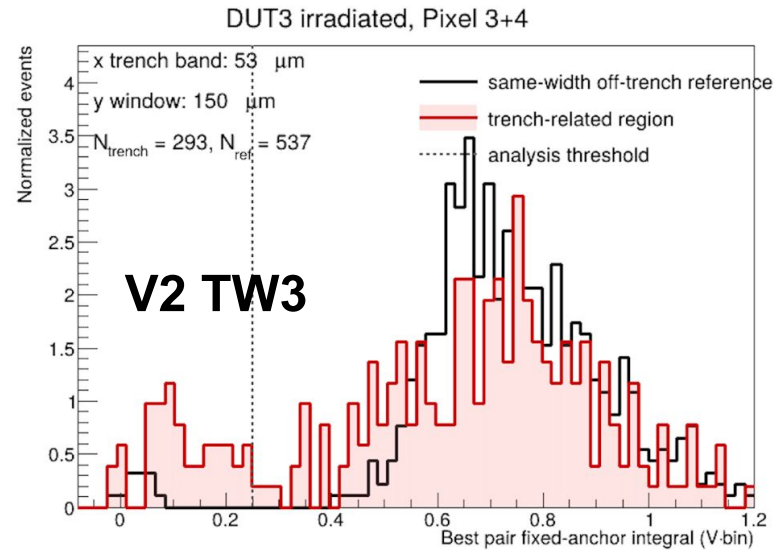
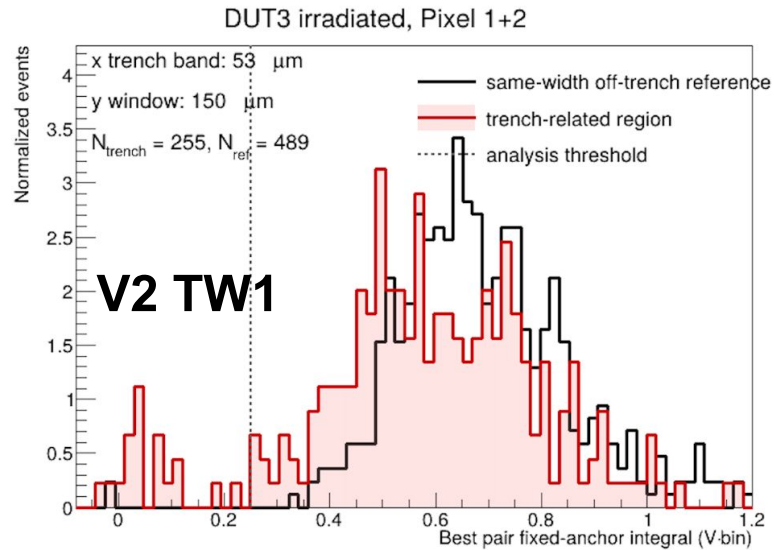


Blue hatched fill:
SiO₂-filled trench

How does the trench-region signal spectrum compare to a nearby non-trench region?



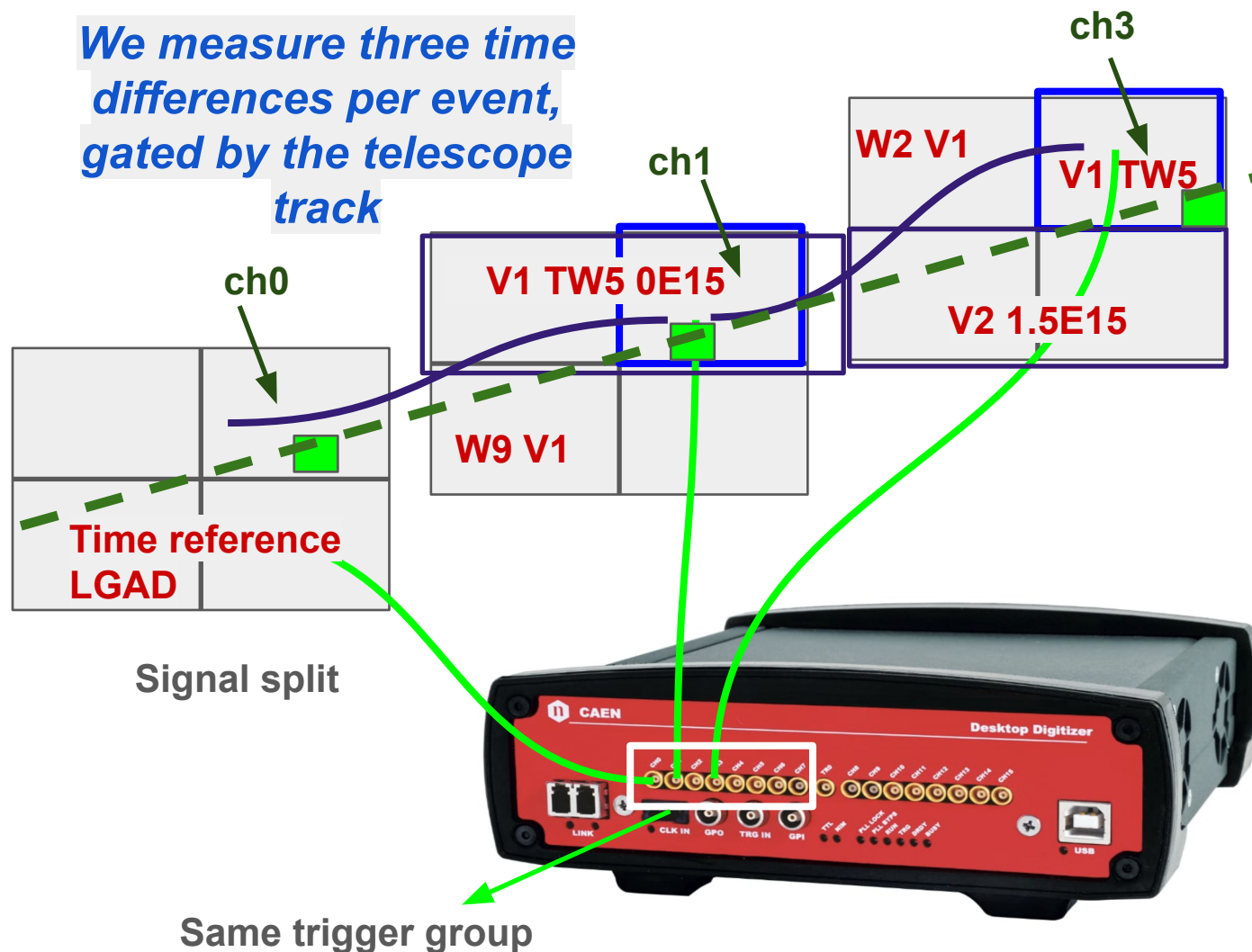
The trench-related region shows a lower-charge population than the off-trench reference region, which is consistent with the local efficiency loss.



x trench band: 53 μm

y trench band: 150 μm

Tracking-Gated¹ Timing Results Explanation



For each event, the track is interpolated across the three sensors, and the time differences between their signals are calculated.

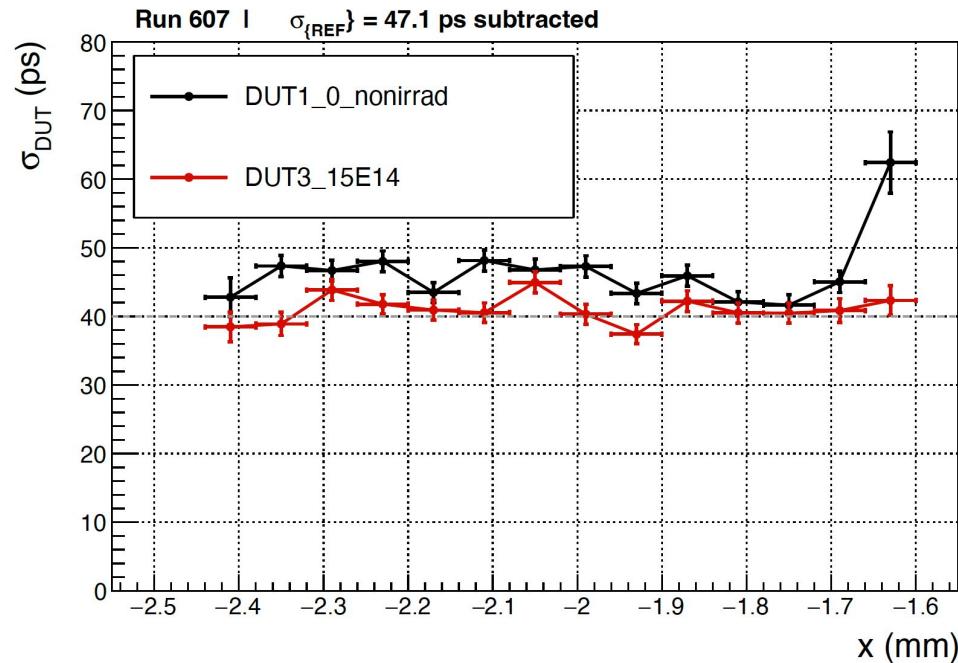
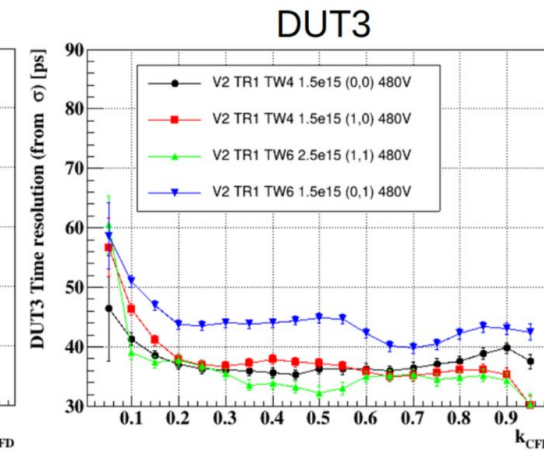
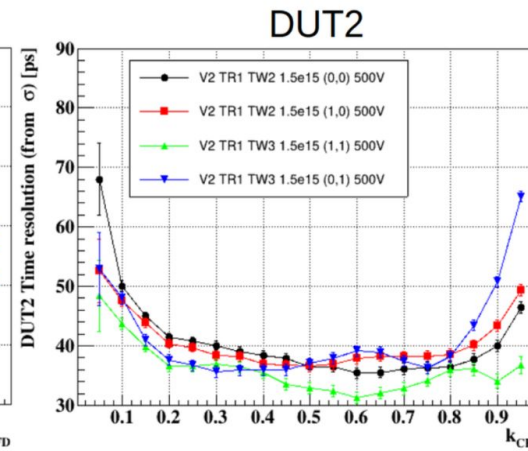
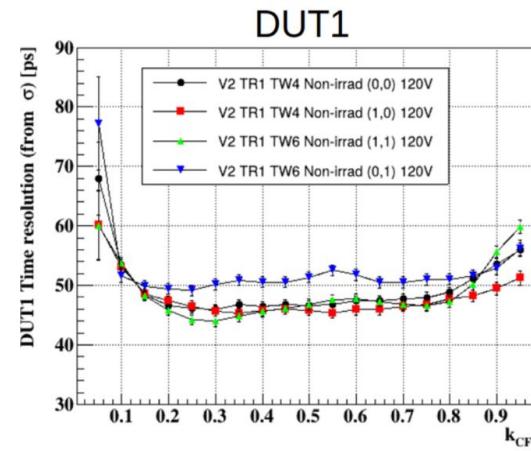
- $A = t(\text{ch1}) - t(\text{ch0})$ (non-irradiated DUT - REF)
- $B = t(\text{ch3}) - t(\text{ch0})$ (irradiated DUT - REF)
- $C = t(\text{ch1}) - t(\text{ch3})$ (DUT - DUT, REF-independent)

Triplet system equations:

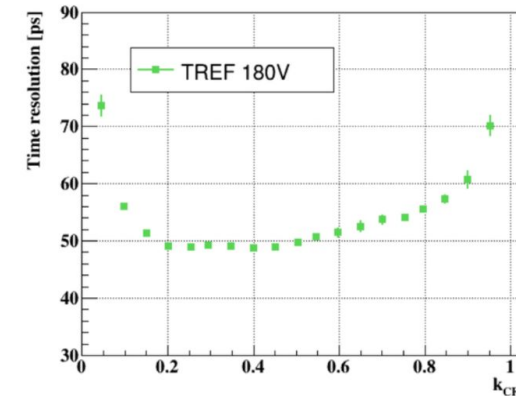
$$A^2 = \sigma_1^2 + \sigma_R^2, \quad B^2 = \sigma_3^2 + \sigma_R^2, \quad C^2 = \sigma_1^2 + \sigma_3^2$$

¹ Track gating is the process of keeping only data associated with valid particle tracks and rejecting unrelated or noisy events

Timing tracking-anchored results



Cuts:
NPeaks=1
RiseTime>0.6
(to get rid of digitizer fast noise)



0,1	1,1
0,0	1,0

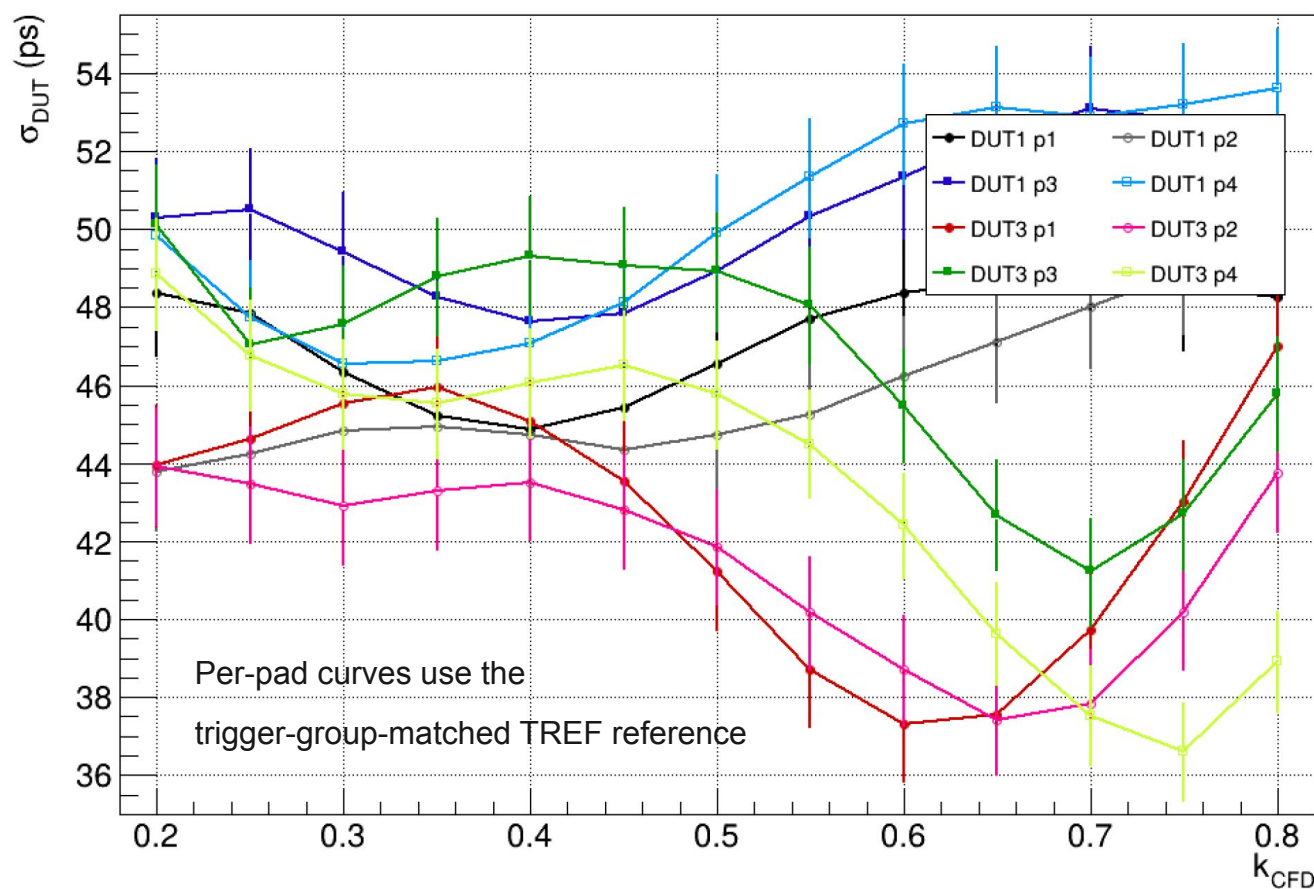
Comparison with Marcos' results 33

Batch 3: $\sigma_{\text{REF}} = 47.1$ ps (triplet method)

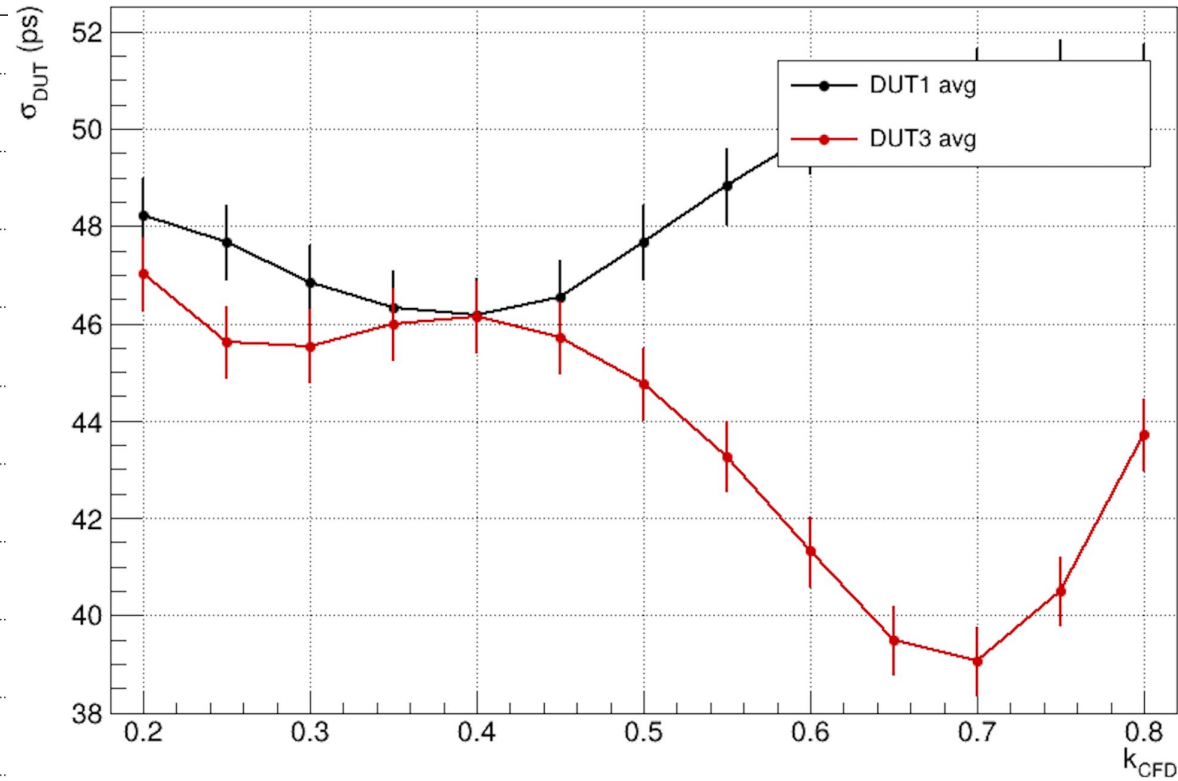
	Batch 2 run509	Batch 3 run607
DUT1 non-irrad	~47 ps @ 120V	43 ps @ 165V
DUT3 1.5×10^{14}	~38 ps @ 500V	38 ps @ 495V
TREF 180V	~49 ps	47 ps ✓

Threshold and CFD scans with track-based TREF-anchor selection

Per-pad timing vs CFD | $\sigma_{\text{ref}} = 47.1$ ps



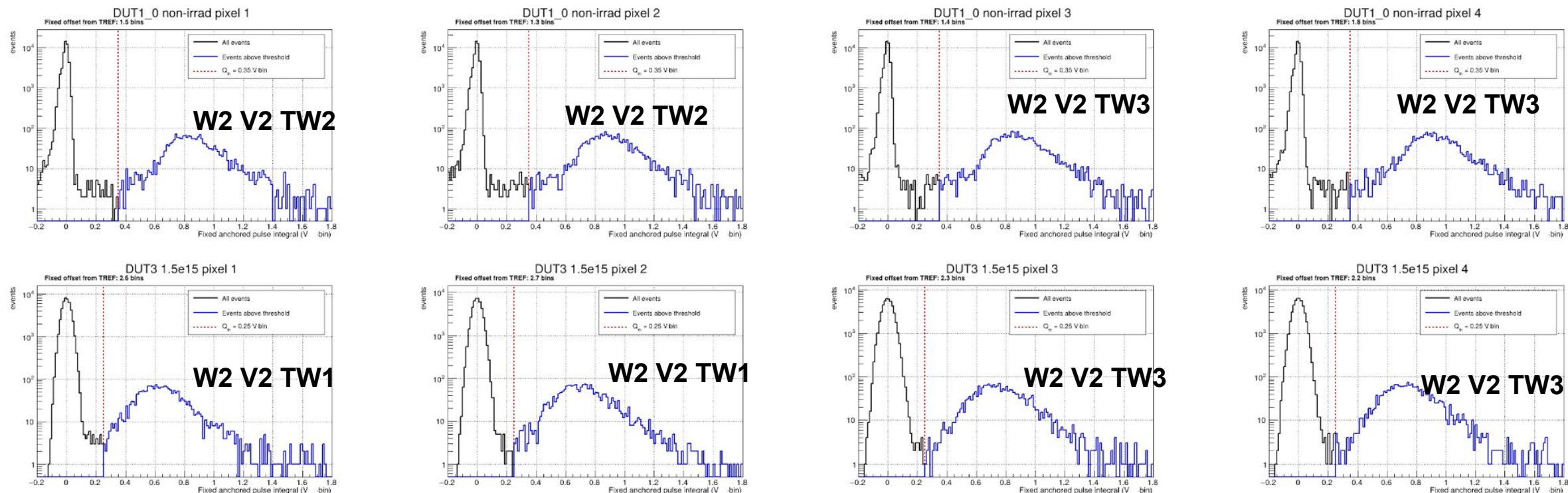
Detector-averaged timing shown as a function of CFD fraction



DUT 1: W2 V2 TW2/3 pixel 1/2 (p1/2) @ 165 V

DUT3: W2 V2 TW1/3 pixel 1/2 (p1/2) @ 550 V

Pulse-integral distributions for TI-LGAD pixels before and after irradiation

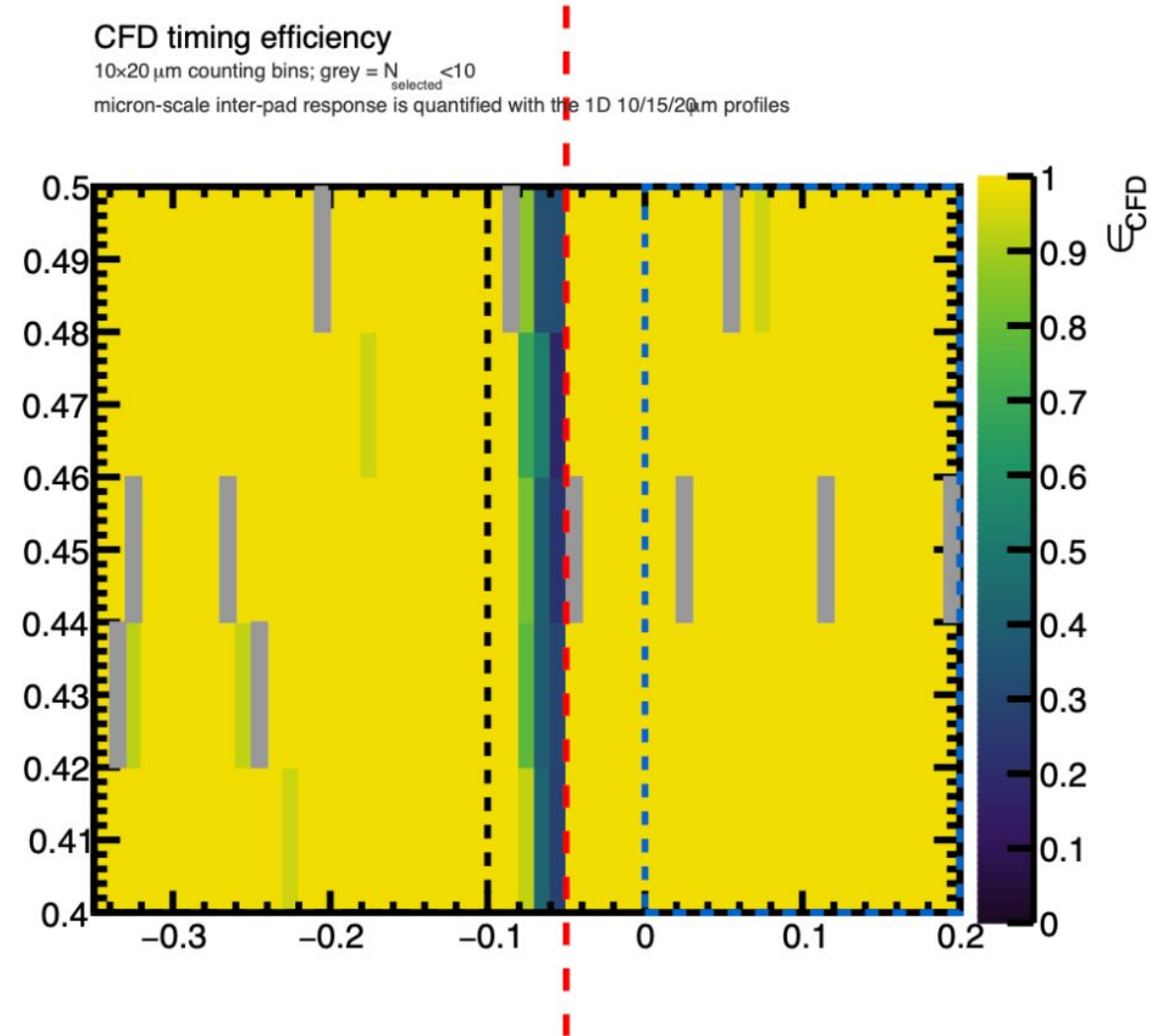
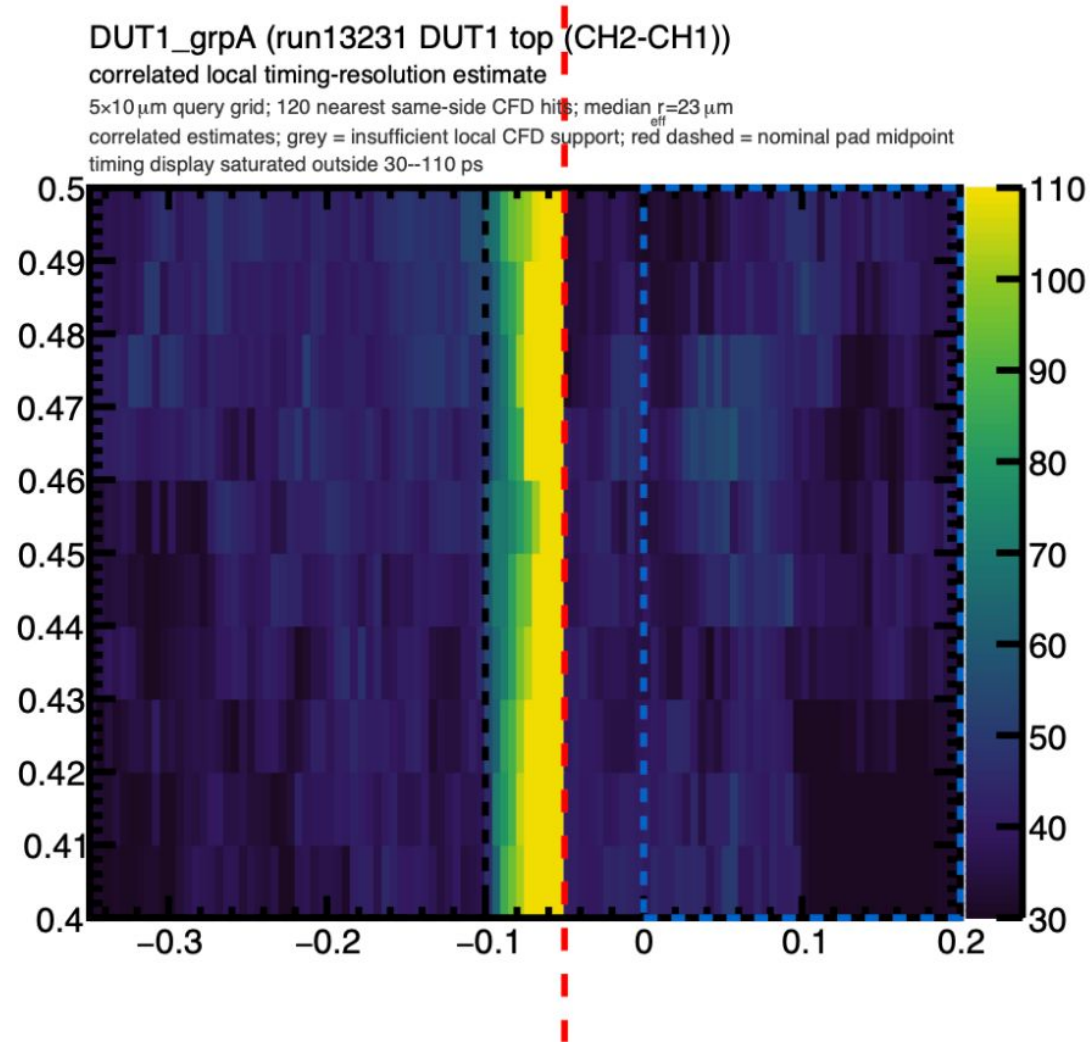


Top: non-irradiated DUT

Bottom: DUT irradiated to $1.5 \times 10^{15} \text{ n}_{\text{eq}}/\text{cm}^2$

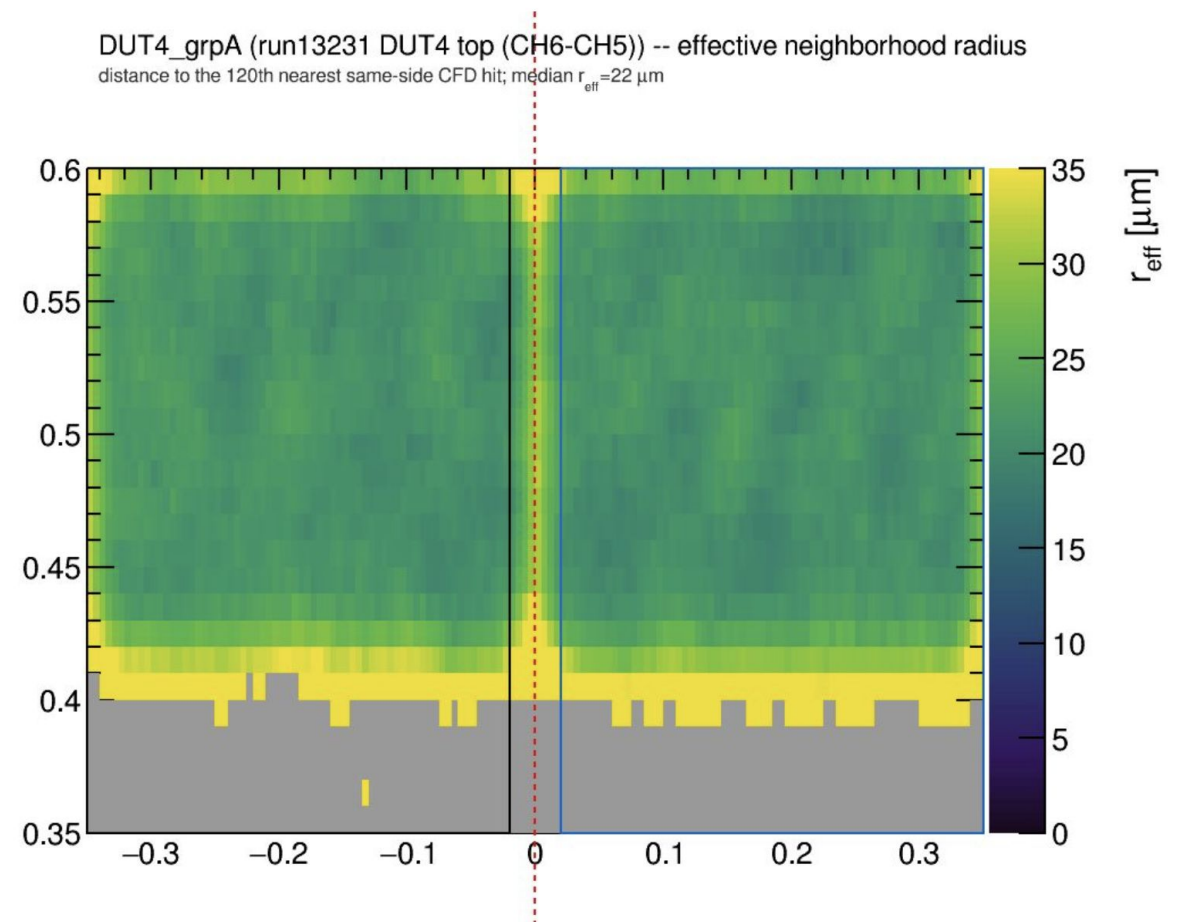
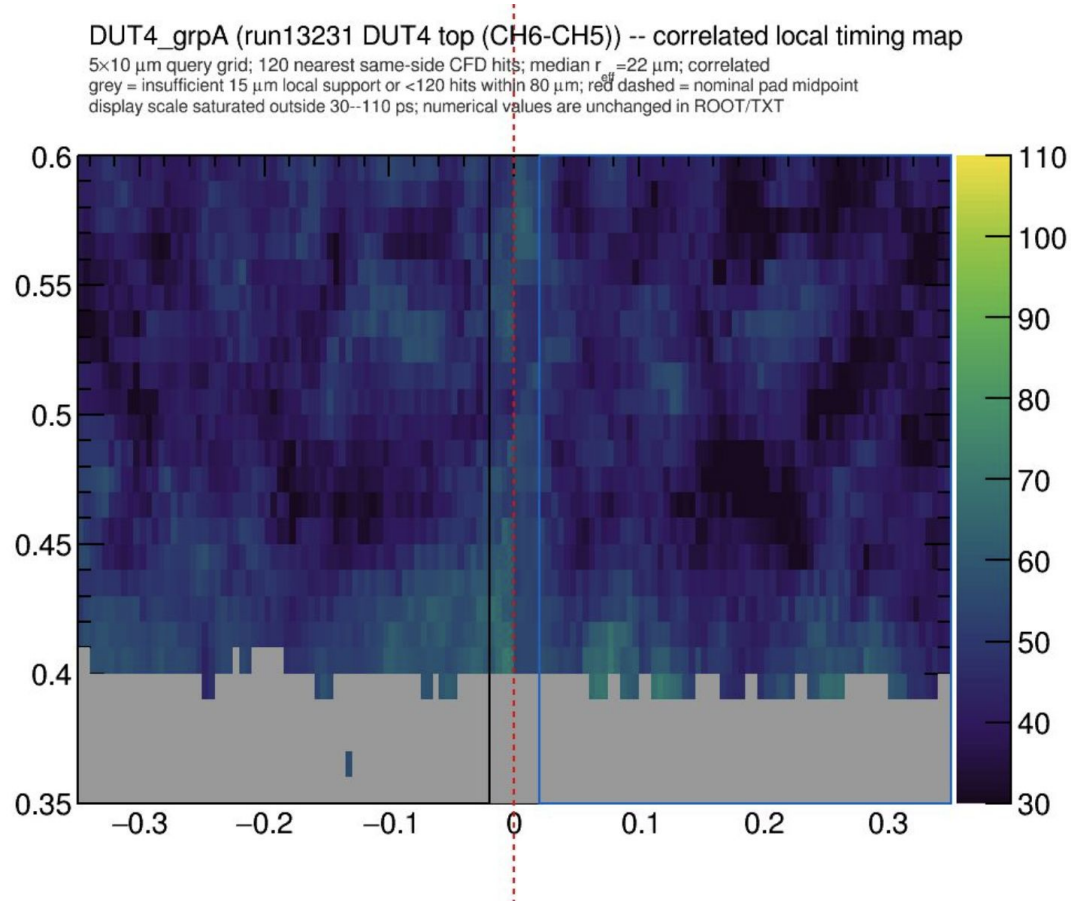
Black distributions contain all events, blue distributions show events passing the pulse-integral threshold

W5 V2 TW12 0e15 inclined at 10°



Clean-bulk timing resolution: ~38 ps

Spatial timing for DUT4 W5 V2 TW4, $1.5 \times 10^{15} \text{ n}_{\text{eq}}/\text{cm}^2$

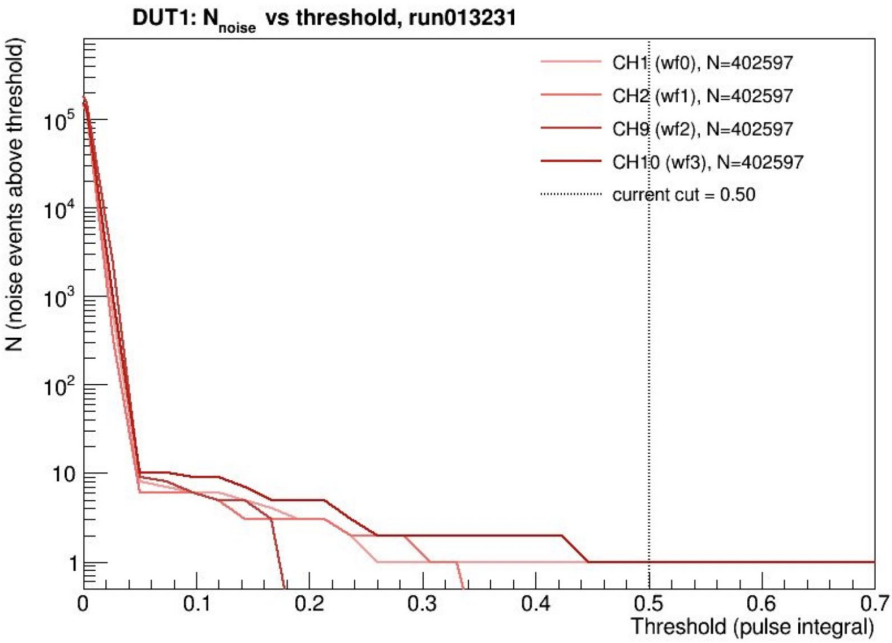


Bulk timing: ~42–46 ps.

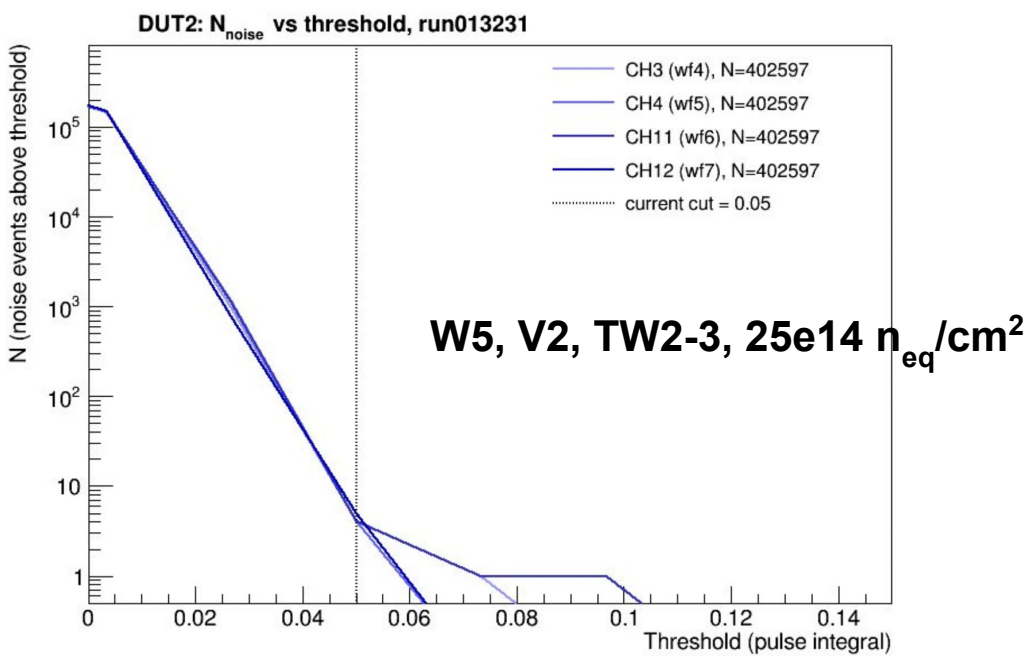
Local estimates use 120 same-side CFD hits, with median $r_{\text{eff}} \approx 22 \mu\text{m}$.

The relatively uniform r_{eff} confirms that the timing map is not driven by strongly varying local statistics.

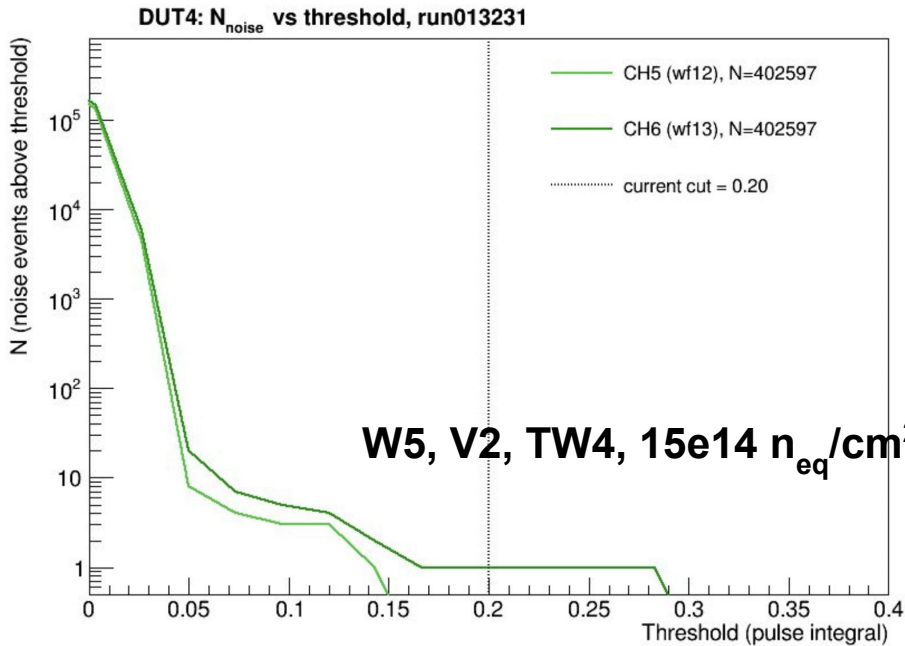
Wafer 5 analysis



W5, V2, TW1-2, 0e14 n_{eq} /cm², angled at 10°



W5, V2, TW2-3, 25e14 n_{eq} /cm²



W5, V2, TW4, 15e14 n_{eq} /cm²



Anchoring the Window: DUT-TREF Peak-Offset Calibration

- The window's center can't depend on the DUT's own CFD edge -> so it's anchored to TREF's peak, plus a fixed per-channel time offset.
- That offset is measured once per channel, using only “clean bulk” tracks (inset from the pad edges), with $S/N > 8$ required on both TREF and the DUT.
- Raw peak-bin difference (DUT – TREF) is collected for up to 2000 such events per channel.
- Offset = robust median of that sample; spread = $1.4826 \times \text{MAD}$; a 5σ clip removes outliers before finalizing.
- A channel is calibrated only if ≥ 120 events survive — otherwise its charge samples are skipped entirely, never guessed.

Calibration gates, at a glance

Target sample	2000 events / channel
Minimum sample	120 events / channel
SNR gate	amplitude $> 8 \times$ noise RMS
Outlier clip	5σ (robust MAD-based)
Bulk region	inset from pad edges



Bulk ROI: One Timing Resolution Per Pad

- Scanning in the bulk is done for every k_{CFD} (TREF and DUT) and fixed before any position-resolved analysis.
- The bulk ROI pools all clean-bulk tracks from a fixed box each pad's own geometric box, shrunk inward away from both the trench and the outer sensor edge — into one sample, and computes σ_{DUT} once with the same σ_{68} /Gaussian-cross-check machinery used everywhere else.
- The 1D trench profile uses two bin widths (10 μm for pure counting quantities like amplitude/efficiency/jitter, a coarser 15 μm for σ_t 's fit stability) with edges anchored at $x=0$ so no bin ever straddles and mixes both sides of the trench, and that $x=0$ itself is refined from a validity-gated erf fit to the data's own efficiency.. An analytical per-event jitter curve is then quadrature-subtracted from σ_t to expose a non-jitter residual



This number is jitter-INCLUSIVE

- The bulk ROI's σ_{DUT} is total detector resolution — electronic jitter is NOT subtracted here.
- A separate, analytical per-event jitter estimate (noise/slew at the CFD crossing) exists and IS quadrature-subtracted