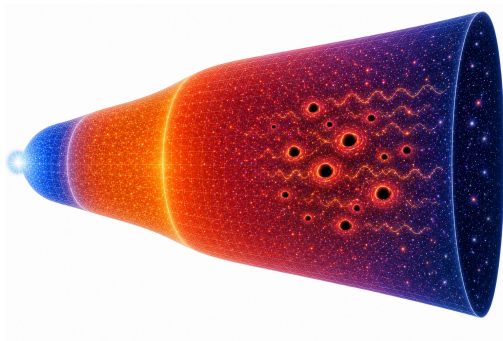


Lukewarm inflation

QCD axion dilution, primordial black holes
and gravitational waves



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- 2 A realization at the QCD axion scale & the baryon asymmetry
- 3 Generic signatures:
curvature perturbations, gravitational waves, PBHs
- 4 Summary & outlook

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Why (lukewarm) inflation?

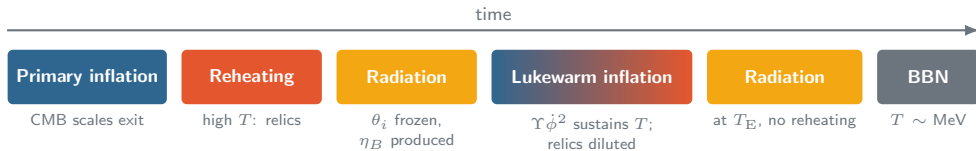
Relics after reheating, Thermal friction, the Warm Little Inflaton,
and strong dissipation at low T

Why a secondary inflationary stage?

CMB probes only
 $\lesssim 10$ e-folds

hot reheating
 $\Rightarrow$ unwanted relics

BSM: many scalars,
many possible stages



A short **warm** inflation stage at **low** T :
dilutes relics with entropy production and ends directly in radiation

Warm inflation in a nutshell

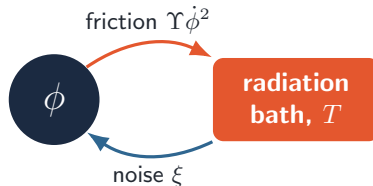
Slow roll sustained by **thermal friction**...

$$\ddot{\phi} + (3H + \Upsilon) \dot{\phi} + V'(\phi) = \xi$$

... that also **sustains a radiation bath**:

$$\dot{\rho}_R + 4H\rho_R = \Upsilon\dot{\phi}^2$$

$$Q \equiv \frac{\Upsilon}{3H}: \quad 3H(1 + Q) \dot{\phi} \simeq -V'(\phi), \quad \frac{\rho_R}{\rho_\phi} \simeq \frac{\epsilon_H}{2} \simeq \frac{1}{2} \frac{\epsilon_\phi}{1 + Q}$$



Slow roll by thermal friction; radiation takes over at $\epsilon_H = 1 \Rightarrow$ **no separate reheating stage.**

Realizing (luke)warm inflation in QFT

Challenges at finite temperature

- ▶ Keeping the fields light: $-\mathcal{L} \supset g \phi \bar{\psi} \psi \Rightarrow m_\psi = g\phi < T$
- ▶ Suppressing thermal backreaction: $\Delta V_T \sim m_\psi^2 T^2 \sim g^2 T^2 \phi^2 \Rightarrow \Delta\eta_\phi \sim g^2 \frac{T^2}{H^2} \gg 1$

Possible solution [Bastero-Gil, Berera, Ramos, Rosa 2016]

$$\begin{aligned} m_{\psi_1} &= g v_\Phi \cos(\phi/v_\Phi), & m_{\psi_2} &= g v_\Phi \sin(\phi/v_\Phi) \\ \Rightarrow \Delta V_T &\propto (m_{\psi_1}^2 + m_{\psi_2}^2) T^2 = g^2 v_\Phi^2 T^2 & (\phi\text{-independent}) \end{aligned}$$

Note: a single fermion is also viable (oscillatory thermal corrections) [Ferraz & Rosa 2023]

Fermion masses bounded by $g v_\Phi$, and the leading thermal correction is ϕ -independent.

The Warm Little Inflaton

- ▶ U(1) gauge symmetry spontaneously broken by two scalars

$$\Phi_1 = \frac{v_\Phi}{\sqrt{2}} e^{i(\alpha+\phi)/v_\Phi} \quad \Phi_2 = \frac{v_\Phi}{\sqrt{2}} e^{i(\alpha-\phi)/v_\Phi}$$

- ▶ inflaton ϕ = relative phase (gauge invariant $\Rightarrow$ any $V(\phi)$)
- ▶ discrete interchange symmetry: $\Phi_1 \leftrightarrow i \Phi_2, \psi_1 \leftrightarrow \psi_2$

$$\begin{aligned} -\mathcal{L}_{\phi\psi} &= \frac{g}{\sqrt{2}} (\Phi_1 + \Phi_2) \bar{\psi}_{1L} \psi_{1R} - i \frac{g}{\sqrt{2}} (\Phi_1 - \Phi_2) \bar{\psi}_{2L} \psi_{2R} + \text{h.c.} \\ &= gv_\Phi \cos\left(\frac{\phi}{v_\Phi}\right) \bar{\psi}_1 \psi_1 + gv_\Phi \sin\left(\frac{\phi}{v_\Phi}\right) \bar{\psi}_2 \psi_2 \end{aligned}$$

$$|m_{\psi_i}| \leq gv_\Phi: \text{ light fermions for } gv_\Phi \lesssim T \lesssim v_\Phi$$

Strong dissipation at low temperature

Fermion decays controlled by an additional Yukawa coupling y :

$$-\mathcal{L}_{\psi\sigma} = y \sigma \sum_{i=1,2} (\bar{\psi}_{iL} \psi_{\sigma R} + \bar{\psi}_{\sigma L} \psi_{iR}) + \text{h.c.} \quad \Rightarrow \quad \Upsilon \simeq C_T T, \quad C_T \propto \frac{g^2}{y^2} \lesssim 1$$

high- T regime $T \gtrsim m_{\psi_i}$; non-equilibrium QFT in the adiabatic regime

$$Q = \frac{C_T}{3} \frac{T}{H} \simeq \left(\frac{5}{\pi^2} \frac{\epsilon_H}{g_*} \right)^{1/2} C_T \frac{M_{\text{P}}}{T}$$

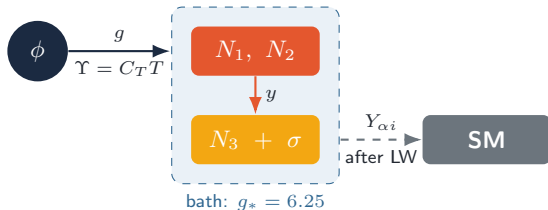
Lower $T \Rightarrow$ stronger dissipation: $Q \gg 1$ automatically for $T \ll C_T M_{\text{P}}$.

Lukewarm inflation with right-handed neutrinos

$$\psi_{1,2} \rightarrow N_{1,2}, \quad \psi_\sigma \rightarrow N_3 \text{ (lightest)}$$

$$-\mathcal{L} \supset g v_\Phi \left[\cos\left(\frac{\phi}{v_\Phi}\right) \bar{N}_1 N_1 + \sin\left(\frac{\phi}{v_\Phi}\right) \bar{N}_2 N_2 \right] + y \sigma \sum_{i=1,2} \bar{N}_i N_3 + Y_{\alpha i} \bar{N}_i \mathcal{H}^\dagger L_\alpha + \text{h.c.}$$

$\langle \sigma \rangle = 0$; $\mathcal{H}$: SM Higgs doublet



seesaw to generate $m_\nu \simeq 0.05 \text{ eV}$

$$\Gamma_{N \rightarrow \text{SM}} \ll H \text{ during LW}$$

$$\Gamma_{N_3 \rightarrow \text{SM}} > H(T_{BBN})$$

Only $N_{1,2,3}$ and σ in the bath;
The **SM redshifts away**; Repopulated by N_3 decays before BBN.

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Lukewarm inflation at the QCD axion scale

Right-handed neutrinos in the bath, $H \sim m_a$ at $T \lesssim \text{GeV}$, and an early matter era

The QCD axion with a GUT-scale decay constant

- ▶ QCD axion solves the **strong CP problem**: $\bar{\theta}$ becomes a field, a/f_a , that relaxes to zero
- ▶ Coherent oscillations from an initial misalignment $\theta_i \Rightarrow$ **dark matter** candidate

$$\Omega_a h^2 \simeq 0.12 \left(\frac{\theta_i}{0.86} \right)^2 \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{1.16}$$

misalignment mechanism, standard thermal history

$$\theta_i \sim 1 \Rightarrow f_a \lesssim 10^{12} \text{ GeV}$$

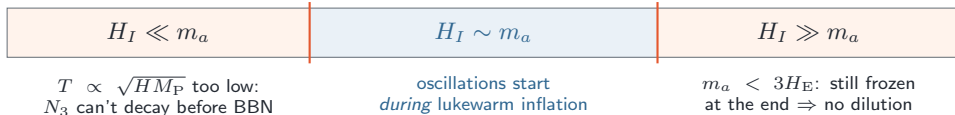
$$f_a = 10^{16} \text{ GeV} \Rightarrow \text{overabundant by} \\ \sim 10^4\text{--}10^5$$

Goal: $f_a \simeq 10^{15-16} \text{ GeV}$ with $\theta_i \sim 1$

Why $H_I \sim m_a$, and what it implies

Axion diluted only if it **oscillates while entropy is produced** ($s_{\text{osc}} \simeq s_E$):

$$\Omega_{a,0} = \Delta_s \frac{s_{\text{osc}}}{s_E} \left(\frac{a_{\text{osc}}}{a_E} \right)^3 \Omega_{a,\text{osc}}$$

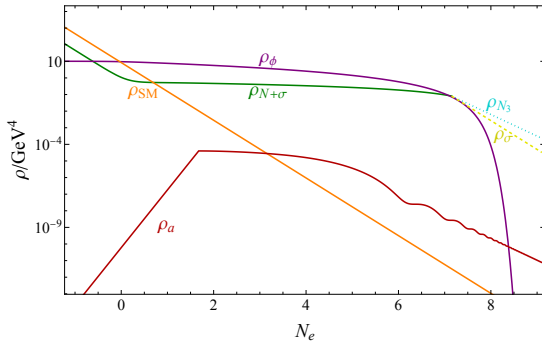


RH neutrinos: in the bath, $M_{N_3} \lesssim T_E$;
decay before BBN $\Rightarrow$ **a few hundred MeV**

Dissipation: $T/H \sim \text{GeV}/\text{neV}$
 $\Rightarrow Q = \frac{c_T}{3} \frac{T}{H} \sim 10^{12} (C_T/10^{-5})$

$H_I \sim m_a \Rightarrow$ lukewarm inflation at $T \sim \text{GeV}$, with $Q \gg 1$.

Background evolution



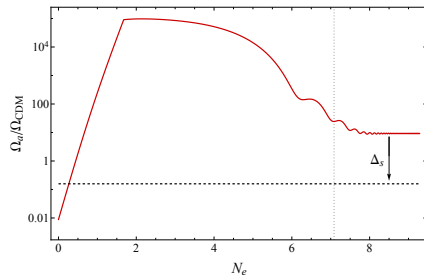
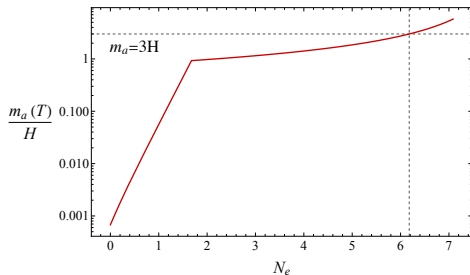
- ▶ SM: $\rho \propto a^{-4}$
- ▶ Bath in slow roll:
 $\rho_R/\rho_\phi \simeq \epsilon_H/2$
- ▶ Afterwards: N_3 matter era and σ dark radiation

Radiation smoothly takes over at the end, followed by an **early matter era** when $T < M_3$

Fig. $m_a/H_I \simeq 0.55$, $N_{\text{LW}} \simeq 7$, $C_T \simeq 10^{-3}$, $m_\phi \simeq 9$ meV, $f_a = 10^{16}$ GeV, $\theta_i = \pi/\sqrt{3}$

The QCD axion during lukewarm inflation

$$\ddot{\theta} + 3H\dot{\theta} + m_a^2(T_{\text{SM}}) \sin \theta = 0, \quad T_{\text{SM}} \propto a^{-1} \quad (\text{SM decoupled from the bath})$$



$m_a \sim H_I$: **critically damped**,
then oscillates while $s \simeq \text{const} \Rightarrow n_a/s$ strongly diluted.

Fig. $m_a/H_I \simeq 0.55$, $N_{\text{LW}} \simeq 7$, $C_T \simeq 10^{-3}$, $f_a = 10^{16}$ GeV, $\theta_i = \pi/\sqrt{3}$

After lukewarm inflation: entropy from N_3 decays

$$\Delta_s^{-1} \equiv \frac{(s a^3)_D}{(s a^3)_{N_3}} = \frac{7}{11} \left(1 + \frac{4}{7} \frac{T_D}{M_{N_3}} \right) \frac{M_{N_3}}{T_D}, \quad T_D = 5 \text{ MeV}$$

$\Omega_a/\Omega_{\text{CDM}} \simeq 27$ at the end of LW
 $\Rightarrow$ need $\Delta_s^{-1} \gtrsim 27$

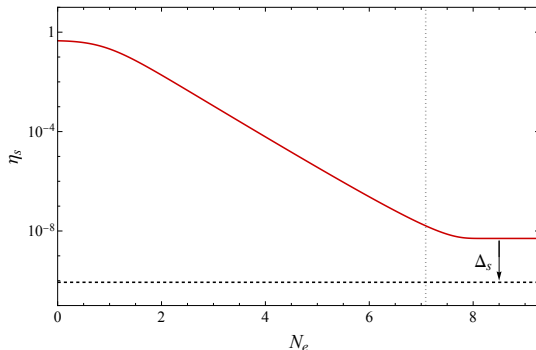
$$M_{N_3} \gtrsim 210 \text{ MeV}$$

N_3 in the bath during LW
 $\Rightarrow M_{N_3} \lesssim T_E$

$$M_{N_3} \lesssim 450 \text{ MeV}$$

N_3 decays give at most $\Delta_s^{-1} \simeq 58$: most of the dilution, a factor $\sim 10^4$, happens **during** lukewarm inflation.

Baryon asymmetry: diluted more than the axion



$$\eta_s \equiv n_B/s$$

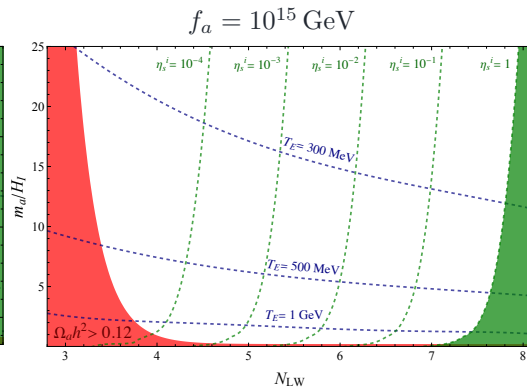
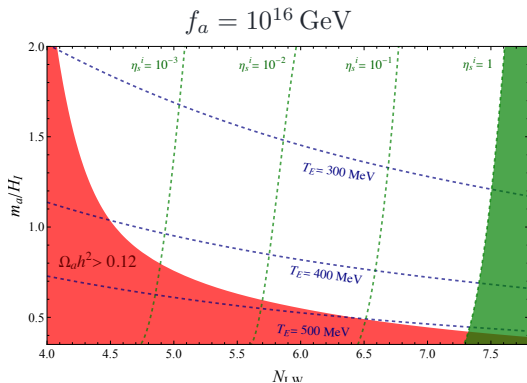
$$0.46 \longrightarrow 8.7 \times 10^{-11}$$

(Planck)

A large early asymmetry (e.g. Affleck–Dine) is brought down to the **observed value** by the same stage.

Fig. $m_a/H_I \simeq 0.55$, $N_{\text{LW}} \simeq 7$, $C_T \simeq 10^{-3}$

Viable parameter space: $\theta_i = \pi/\sqrt{3}$



Viable: $N_{LW} \approx 4-8$ e-folds, f_a up to 10^{16} GeV with $M_{N_i} \gtrsim 200$ MeV.

$$\Delta N_{\text{eff}} = \frac{4}{7} \left[\frac{11}{7} \frac{T_D}{M_{N_3}} g_{*s}^{\text{SM}} \right]^{4/3} \leq 0.047 \quad (T_D = 5 \text{ MeV}, M_{N_3} \geq 200 \text{ MeV})$$

The light scalar σ is dark radiation: $\Delta N_{\text{eff}} \lesssim 0.05$, within current bounds, near future CMB sensitivity.

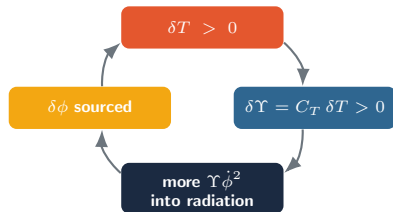
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Generic signatures of lukewarm inflation

Large, nearly scale-invariant curvature perturbations over $e^{N_{\text{LW}}}$ in $k \rightarrow$ SGWB & PBHs

Perturbations: thermal noise and a growing mode

$$\delta\ddot{\phi}_{\mathbf{k}} + (3H + \Upsilon) \delta\dot{\phi}_{\mathbf{k}} + \left(\frac{k^2}{a^2} + V''\right) \delta\phi_{\mathbf{k}} + \dot{\phi} \delta\Upsilon_{\mathbf{k}} = \xi_{\mathbf{k}} + \dots, \quad \delta\Upsilon_{\mathbf{k}} = C_T \delta T_{\mathbf{k}}$$
$$\langle \xi_{\mathbf{k}}(t) \xi_{\mathbf{k}'}(t') \rangle = 2\Upsilon T a^{-3} (2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}') \delta(t - t')$$



- ▶ $T \gtrsim H$: thermal noise $\gg$ quantum fluctuations
- ▶ $Q \gg 1$: modes freeze out *before* horizon crossing

Hotter regions dissipate more and heat up more: for $\Upsilon \propto T$, **curvature perturbations grow.**

Curvature power spectrum

Super-horizon curvature spectrum, $Q \gg 1$:

$$\Delta_{\mathcal{R}}^2 = \frac{\sqrt{3\pi}}{24\pi^2} \frac{V}{M_{\text{P}}^4} \frac{Q^{3/2}}{\epsilon_H} \frac{T}{H} G(Q), \quad G(Q) \simeq 5 \times 10^{-4} Q^3 \quad (\text{growing mode, Wl2easy})$$

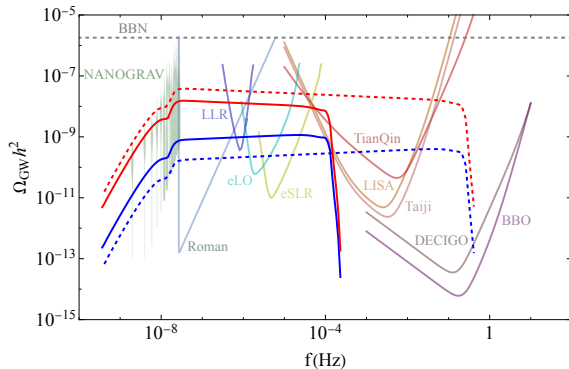
with the slow-roll relations and $\Upsilon = C_T T$:

$$\Delta_{\mathcal{R}}^2 \simeq 0.07 \epsilon_H^{3/4} \left(\frac{100}{g_*} \right)^{7/4} \left(\frac{C_T}{10^{-2}} \right)^{9/2} \left(\frac{10^7 \text{ GeV}}{T} \right)^{3/2}$$

$$\Delta_{\mathcal{R}}^2 = A_s (k/k_{\text{max}})^{n_s-1} \quad \text{for } k_{\text{min}} < k < k_{\text{max}}, \quad k_{\text{max}}/k_{\text{min}} = e^{N_{\text{LW}}} H_{\text{E}}/H_{\text{I}}$$

Large enhancement at low T , $\Delta_{\mathcal{R}}^2 \propto C_T^{9/2} T^{-3/2}$
nearly scale invariant over $e^{N_{\text{LW}}}$ in k .

Gravitational waves: a flat plateau



	T_E/TeV	N_{LW}	n_s
—	5	9	1.03
—	5	9	0.97
- - -	10^4	17	1.03
- - -	10^4	17	0.97

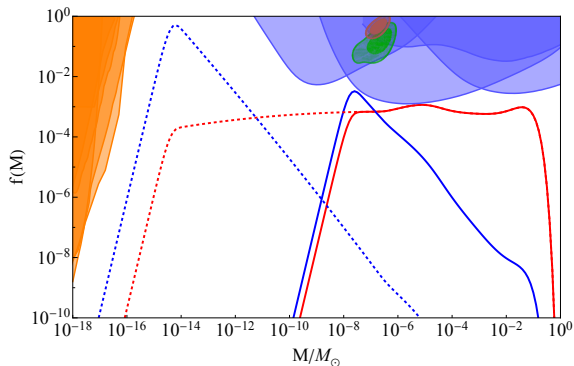
$$\Omega_{\text{GW}} \propto A_s^2 f^{2(n_s-1)}$$

$$f_{\text{max}} \simeq 2.6 \times 10^{-8} \text{ Hz} \frac{T_E}{\text{GeV}}$$

A flat SGWB over a large frequency range

$$f_{\text{max}}/f_{\text{min}} = e^{N_{LW}} H_E/H_I$$

Sub-solar mass primordial black holes



$$M_{\star} \simeq 3 \times 10^{-10} M_{\odot} \left(\frac{g_{\star}}{100} \right)^{-1/6} \left(\frac{\text{mHz}}{f_{\text{max}}} \right)^2$$

$$f_{\text{PBH}} \propto \begin{cases} M^{1+1/\gamma}, & M < M_{\star} \\ M^{-\frac{1}{2} - \frac{\kappa}{2}(n_s - 1)}, & M > M_{\star} \end{cases}$$

$$\kappa \equiv 81\delta_c^2/(16A_s)$$

$n_s > 1$: peak $n_s < 1$: plateau

All DM in **asteroid-mass PBHs** possible;
planetary masses could explain HSC-M31 events.

Fig. Orange: evaporation; blue: microlensing; red/green: HSC-M31 candidates, Sugiyama et al. arXiv:2602.05840

Early matter era after LW

- ▶ lower threshold for black hole formation δ_c
- ▶ spinning PBHs
- ▶ modified $\Omega_{\text{GW}}(f)$

order-of-magnitude estimate

$$f_{\text{max}} \sim 4.1 \times 10^{-9} \text{ Hz} \frac{T_{\text{E}}}{250 \text{ MeV}}$$

$$M_H(k_{\text{max}}) \sim M_{\odot}$$

The axion model points to the **PTA band and solar-mass PBHs** needs a dedicated analysis.

Lukewarm inflation

- ▶ secondary warm inflation at low T
- ▶ strong dissipation, $Q \propto C_T M_P / T$
- ▶ no reheating stage

QCD axion

- ▶ $f_a \simeq 10^{15-16}$ GeV with $\theta_i \sim 1$
- ▶ observed n_B/s from a large initial asymmetry
- ▶ RH neutrinos at a few hundred MeV

Signatures

- ▶ flat SGWB with a sharp cut-off
- ▶ sub-solar mass PBHs

Next: non-Gaussianity ($f_{\text{NL}} \simeq 8$), peak theory, early-matter-era effects, other lukewarm models.

Thank you!

Backup slides

Backup: axion cosmology and dilution mechanisms

- Mass (chiral perturbation theory, lattice):

$$m_a \simeq 5.7 \mu\text{eV} (10^{12} \text{ GeV}/f_a);$$

$$m_a^2(T) = m_a^2 (T/T_{\text{QCD}})^{-n} \text{ for } T > T_{\text{QCD}}, n \simeq 8$$

- Misalignment: $\theta_i \sim \mathcal{O}(1)$ (e.g. $\langle \theta_i^2 \rangle^{1/2} = \pi/\sqrt{3}$); frozen while $H \gg m_a(T)$
- Standard thermal history:

$$\Omega_a h^2 \simeq 0.12 \left(\frac{\theta_i}{0.86} \right)^2 \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{1.16}$$

$$f_a = 10^{16} \text{ GeV: overabundant by } \approx 4 \times 10^4 (\theta_i/0.86)^2$$

Existing dilution mechanisms

late entropy release (decays, moduli), thermal inflation, low-scale inflation, late accelerated phases.

Typical issues:

1. ad hoc fields with tuned decay rates
2. very low reheating temperatures
3. erase any pre-existing baryon asymmetry
4. tuning $m_\phi \sim H \sim m_a$

Goal: $f_a \simeq 10^{15-16} \text{ GeV}$, $\theta_i \sim 1$
and the observed n_B/s .

Backup: warm-inflation dynamics

Slow roll sustained by **thermal friction**...

$$\ddot{\phi} + (3H + \Upsilon) \dot{\phi} + V'(\phi) = \xi$$

... which also **sustains a radiation bath**:

$$\dot{\rho}_R + 4H\rho_R = \Upsilon\dot{\phi}^2$$

Slow-roll regime, with $Q \equiv \Upsilon/(3H)$:

$$3H(1 + Q) \dot{\phi} \simeq -V'(\phi), \quad 4H\rho_R \simeq \Upsilon\dot{\phi}^2$$

- Conditions: $\epsilon_\phi, |\eta_\phi| \ll 1 + Q$
 $\epsilon_\phi = \frac{M_{\text{P}}^2}{2} (V'/V)^2, \quad \eta_\phi = M_{\text{P}}^2 V''/V$
- For $Q \gg 1$: $\frac{\rho_R}{\rho_\phi} \simeq \frac{\epsilon_H}{2}, \quad \epsilon_H \simeq \frac{\epsilon_\phi}{1 + Q}$
radiation subdominant, yet $T > H$
- $\epsilon_H \rightarrow 1$: radiation takes over smoothly
 $\Rightarrow$ **no separate reheating**
- Strong dissipation: slow roll even for $m_\phi \gg H$

Backup: dissipation coefficient

$\psi_{1,2}$ decay through an additional Yukawa coupling y :

$$-\mathcal{L}_{\psi\sigma} = y \sigma \sum_{i=1,2} (\bar{\psi}_{iL} \psi_{\sigma R} + \bar{\psi}_{\sigma L} \psi_{iR}) + \text{h.c.}$$

Adiabatic, high- T regime ($T \gtrsim m_{\psi_i}$), from non-equilibrium QFT:

$$\Upsilon \simeq C_T T, \quad C_T \simeq \frac{3g^2}{y^2 (1 - 0.34 \log y)}$$

([A] Eq. (11); $C_T \lesssim 1$ means no large hierarchy $g \gg y$)

Using $\rho_R = \frac{\pi^2}{30} g_* T^4 \simeq \frac{\epsilon_H}{2} 3H^2 M_P^2$:

$$Q = \frac{C_T}{3} \frac{T}{H} \simeq \left(\frac{5}{\pi^2} \frac{\epsilon_H}{g_*} \right)^{1/2} C_T \frac{M_P}{T}$$

Lower $T \Rightarrow$ **stronger dissipation.**
 $T \ll C_T M_P \Rightarrow Q \gg 1$
automatically.

Lukewarm inflation is *easier* to realize than GUT-scale warm inflation (weak/mild dissipation there).

Backup: slow-roll solution, quadratic potential

$V = \frac{1}{2}m_\phi^2\phi^2$, $\Upsilon = C_T T$, $Q \gg 1$; N_e : e-folds since the onset (I);

$x \equiv 1 - N_e/(N_{\text{LW}} + \frac{5}{6})$:

$$\phi = \phi_I x^{5/6}, \quad T = T_I x^{1/6}, \quad H = H_I x^{5/6}$$

$\epsilon_H = -d \ln H / dN_e = 1$ at $N_e = N_{\text{LW}}$ (end, E; slow-roll estimate)

$$T_I^4 \simeq \frac{75}{2\pi^2 g_*} \frac{H_I^2 M_{\text{P}}^2}{N_{\text{LW}} + 5/6}, \quad m_\phi^2 \simeq \frac{5}{6} \frac{C_T H_I T_I}{N_{\text{LW}} + 5/6}$$

- ▶ Three parameters: $(H_I, N_{\text{LW}}, C_T)$
- ▶ Background $H(N_e), T(N_e)$ is **independent of C_T** for $Q \gg 1$: C_T only fixes m_ϕ (and the perturbation amplitude)
- ▶ T_I fixed by H_I and N_{LW} alone
- ▶ Other potentials allowed (only $\mathbb{Z}_2$ -even required), e.g. hybrid-like $V_0 + \frac{1}{2}m_\phi^2\phi^2$

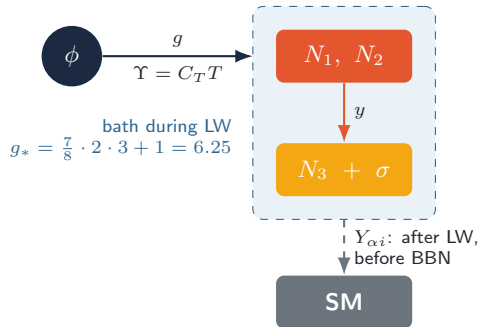
Backup: right-handed-neutrino sector

- ▶ WLI with $\psi_{1,2} \rightarrow N_1, N_2$, $\psi_\sigma \rightarrow N_3$ (lightest), σ a light scalar, $\langle \sigma \rangle = 0$; U(1) may be identified with lepton number; Majorana-type couplings ($\mathcal{O}(1)$ factors absorbed in g, y)
- ▶ Type-I seesaw links to the SM ($\mathcal{H}$: Higgs doublet, $\langle \mathcal{H} \rangle = v_H \simeq 174 \text{ GeV}$):

$$\mathcal{L}_{\text{Yuk}} = -Y_{\alpha i} \bar{N}_i \mathcal{H}^\dagger L_\alpha + \text{h.c.}$$

$$m_\nu \simeq 0.05 \text{ eV} \Rightarrow \sum_\alpha |Y_{\alpha i}|^2 \simeq 1.65 \times 10^{-15} M_{N_i} / \text{GeV}$$

- ▶ $\Gamma_{N_i} \simeq \frac{G_F^2 M_{N_i}^5}{96\pi^3} \sum_\alpha |U_{\alpha i}|^2$, $|U_{\alpha i}|^2 = \frac{v_H^2 |Y_{\alpha i}|^2}{M_{N_i}^2}$: too slow to keep the SM in the bath $\Rightarrow$ **SM redshifts away** during LW



BBN: $\Gamma_{N_3} > H(T_D \simeq 5 \text{ MeV}) \Rightarrow M_{N_i} \gtrsim 0.1\text{--}1 \text{ GeV}$, depending on the flavour structure of the mixing

Backup: scales and inflaton mass

Require $H_I \sim m_a \simeq 0.57 \text{ neV} (10^{16} \text{ GeV}/f_a)$:

$$T_I \simeq \frac{1.6 \text{ GeV}}{g_*^{1/4}} \left(\frac{H_I}{m_a} \right)^{\frac{1}{2}} \left(\frac{10^{16} \text{ GeV}}{f_a} \right)^{\frac{1}{2}} \left(N_{\text{LW}} + \frac{5}{6} \right)^{-\frac{1}{4}}$$

$$m_\phi \simeq 0.8 \text{ eV} \frac{C_T^{1/2}}{g_*^{1/8}} \left(\frac{H_I}{m_a} \right)^{\frac{3}{4}} \left(\frac{10^{16} \text{ GeV}}{f_a} \right)^{\frac{3}{4}} \left(N_{\text{LW}} + \frac{5}{6} \right)^{-\frac{5}{8}}$$

Benchmark ($g_* = 6.25$, $N_{\text{LW}} = 7$, $m_a/H_I \simeq 0.55$, $f_a = 10^{16} \text{ GeV}$): $T_I \simeq 0.8 \text{ GeV}$; numerically $T_E \simeq 450 \text{ MeV}$.

- ▶ $T \lesssim \text{GeV} \Rightarrow$ upper bound
 $M_{N_3} \lesssim T_E$; with the BBN bound:
 $M_{N_i} \sim \text{few} \times 100 \text{ MeV}$
- ▶ Sub-eV inflaton, yet $m_\phi \gg H_I$:
possible only because $Q \gg 1$
($Q \sim \frac{C_T}{3} \frac{T}{H} \sim 10^{12} C_T / 10^{-5}$ here)
- ▶ Sub-Planckian field:
 $\phi \sim (H/m_\phi) M_P \ll M_P$
- ▶ $T/H \gtrsim \Lambda_{\text{QCD}}/m_a \sim f_a/\Lambda_{\text{QCD}} \gg 1$:
very strong dissipation

Backup: scalar-induced gravitational waves

Second-order tensor modes sourced by $\Delta_{\mathcal{R}}^2$ re-entering in the radiation era:

$$\Omega_{\text{GW}}^{(c)}(k) = \frac{1}{6} \int_0^\infty dv \int_{|1-v|}^{1+v} du \left[\frac{4v^2 - (1 + v^2 - u^2)^2}{4uv} \right]^2 \overline{\mathcal{I}_{\text{RD}}^2}(u, v) \Delta_{\mathcal{R}}^2(kv) \Delta_{\mathcal{R}}^2(ku)$$

$\overline{\mathcal{I}_{\text{RD}}^2} \equiv \lim_{x \rightarrow \infty} x^2 \overline{I_{\text{RD}}^2}(u, v, x)$, $x = k\tau$: oscillation-averaged kernel in radiation domination

Main features

$$\Omega_{\text{GW}} h^2|_0 \simeq 0.83 \left(\frac{g_{*,c}}{10.75} \right)^{-1/3} \Omega_{r,0} h^2 \Omega_{\text{GW}}^{(c)}$$

$$\Omega_{r,0} h^2 \simeq 4.2 \times 10^{-5}$$

$f_{\text{max}} \equiv k_{\text{max}}/(2\pi a_0)$, re-entry at $T \simeq T_{\text{E}}$ in a radiation era:

$$f_{\text{max}} \simeq 2.6 \times 10^{-8} \text{ Hz} \left(\frac{T_{\text{E}}}{\text{GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6}$$

$$\Omega_{\text{GW}} h^2 \propto \begin{cases} f^3, & f < f_{\text{min}} \\ A_s^2 (f/f_{\text{max}})^{2(n_s-1)}, & f_{\text{min}} < f < f_{\text{max}} \\ 0, & f > 2f_{\text{max}} \end{cases}$$

$$\frac{f_{\text{max}}}{f_{\text{min}}} = \frac{k_{\text{max}}}{k_{\text{min}}} = e^{N_{\text{LW}}} \frac{H_{\text{E}}}{H_{\text{I}}}$$

$f > 2f_{\text{max}}$: forbidden by momentum conservation

Kohri & Terada, PRD 97 (2018) 123532; Espinosa, Racco, Riotto, JCAP 09 (2018) 012; [B] Eq. (8)

Backup: PBH formation (Press–Schechter)

$\delta \simeq -\frac{4}{9}(aH)^{-2}\nabla^2\mathcal{R} > \delta_c$ collapses into $M = \mathcal{K}M_H(\delta - \delta_c)^\gamma$; $\mathcal{K} \simeq 4$, $\gamma \simeq 0.36$, $\delta_c = 0.49$ for $w = \frac{1}{3}$, realistic $w(T)$ through EW, QCD, $e^\pm$

$$f_{\text{PBH}}(M) = \frac{2\mathcal{K}/\gamma}{\Omega_{\text{CDM}}} \int d\ln M_H \sqrt{\frac{M_{\text{eq}}}{M_H}} \frac{\mu^{1+\frac{1}{\gamma}}}{\sqrt{2\pi\Sigma^2}} \exp\left[-\frac{(\mu^{1/\gamma} + \delta_c)^2}{2\Sigma^2}\right], \quad \mu \equiv \frac{M}{\mathcal{K}M_H}$$

$$\Sigma^2(M_H) = \int d\ln k \frac{16}{81} \left(\frac{k}{aH}\right)^4 \Delta_{\mathcal{R}}^2(k) e^{-k^2/(aH)^2}, \quad (aH)^{-1} \text{ at re-entry of } M_H$$

Main features

$$f_{\text{PBH}} \propto \begin{cases} M^{1+1/\gamma}, & M < M_\star \\ M^{-\frac{1}{2}-\frac{\kappa}{2}(n_s-1)}, & M_\star < M < M_{\text{max}} \end{cases} \quad M_\star \simeq 5 \times 10^{-7} M_\odot \left(\frac{100}{g_\star}\right)^{\frac{1}{2}} \left(\frac{T_E}{\text{TeV}}\right)^{-2}$$

$$\kappa \equiv 81 \delta_c^2 / (16 A_s) \approx 30\text{--}45 \text{ (benchmarks)}$$

$$M_{\text{max}}/M_\star = (k_{\text{max}}/k_{\text{min}})^2; \\ n_s > 1 - \kappa^{-1}: \text{ peak at } M_\star; \quad n_s \lesssim 1 - \kappa^{-1}: \text{ plateau}$$

Byrnes, Hindmarsh, Young, Hawkins, JCAP 08 (2018) 041; Carr, Clesse, García-Bellido, Kühnel, PDU 31 (2021) 100755; Musco et al. PRD 103 (2021) 063538; Domènech, Pi, Wang, arXiv:2602.24061; [B] Eq. (9) & suppl.