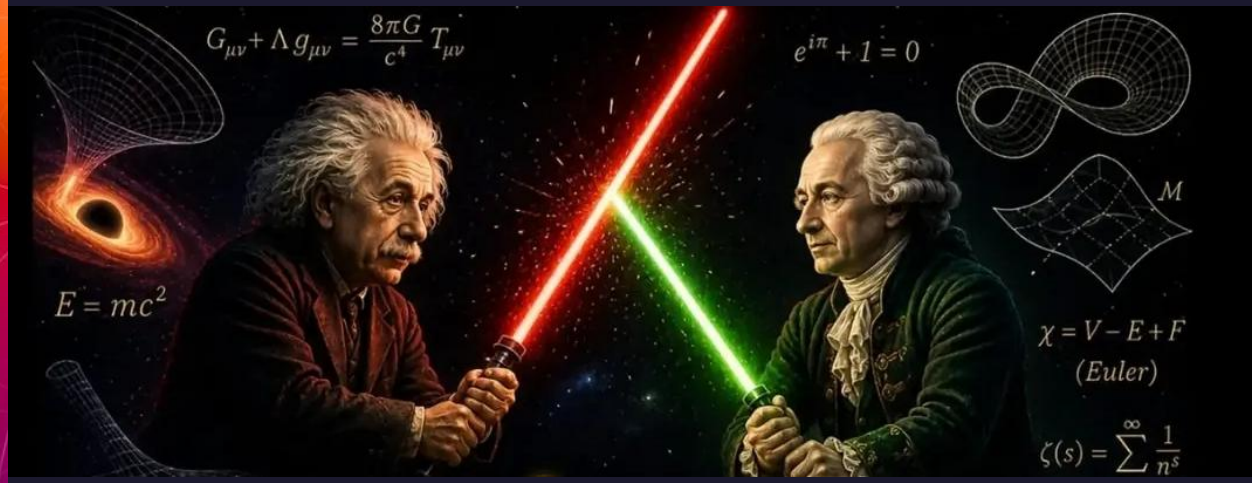


Exploring higher-derivative terms in gravity (in the era of AI)

Dr. Eric Lescano

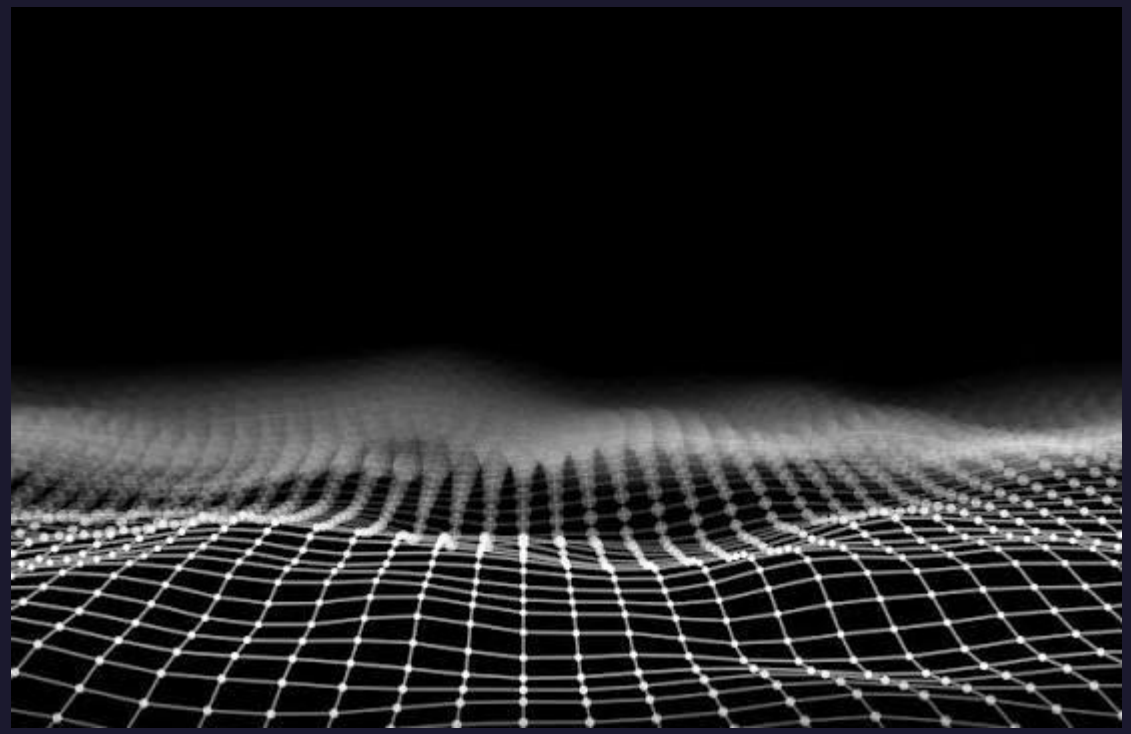
member of the glorious SQUASH team,
Teslova headquarters

BRDA 2026: Challenges in particle physics and cosmology

Outline of the talk

1. Quick introduction to quantum gravity and hep-th
2. The open problem: alpha'-corrections and beyond
3. Manifest T-duality approach.
4. The obstruction and the two branches
5. New idea in 2025/2026: the off-shell double-copy
5. AI assistance: what, how, why?
6. Final messages

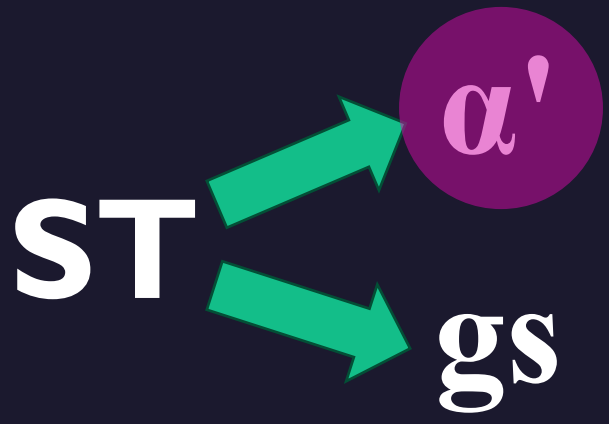
Why quantum gravity?



Investing in the mathematics of quantum gravity provides an indirect way to validate and refine the foundational principles of quantum mechanics (example: holography)

- Beyond Einstein's theory: The unification between GR and QM remains a fundamental open problem. Quantum effects become essential when spacetime curvature and gravitational dynamics reach extreme regimes.
- The early universe: Quantum-gravity corrections may change the picture of the initial cosmological evolution, including scenarios involving a pre-Big-Bang phase or a resolution of classical singularities.
- Holography and new mathematical structures: Gauge/gravity duality connects gravitational physics to quantum field theory, revealing unexpected links between spacetime, quantum information, geometry and strongly coupled systems.

The open problem: alpha'-corrections and beyond



Quantum gravity is not just about quantizing Einstein's equations. It is also about understanding what new structures emerge when spacetime itself is treated as part of a quantum system.

String theory provides a particularly rich framework for exploring this question. Its low-energy limit reproduces gravitational dynamics, but it also predicts an infinite tower of corrections controlled by the string length scale, conventionally organized in powers of the parameter so-called alpha'.

Can we understand these corrections systematically?



From a quantum gravity model, we expect corrections to the Einstein field equations

Why alpha'-corrections?



The opportunity: beyond a single effective action

Can similar structures reveal new gravitational effective theories, potentially with improved ultraviolet properties?

The ultraviolet problem of gravity:

General relativity is an effective field theory, but is non-renormalizable: increasingly higher-curvature counterterms are needed at higher loop orders. At sufficiently high energies, its low-energy expansion is no longer enough to describe the underlying physics.

String theory changes the picture

String theory replaces point particles with extended objects and introduces a fundamental length scale, ℓ_s, with $\alpha' \sim \ell_s^2$.

Its low-energy gravitational action contains an infinite series of higher-derivative corrections, schematically:

$$S_{\text{eff}} = S_{\text{EH}} + \alpha' S_1 + (\alpha')^2 S_2 + \cdots$$

Example: bosonic (super)gravity+alpha'-corrections

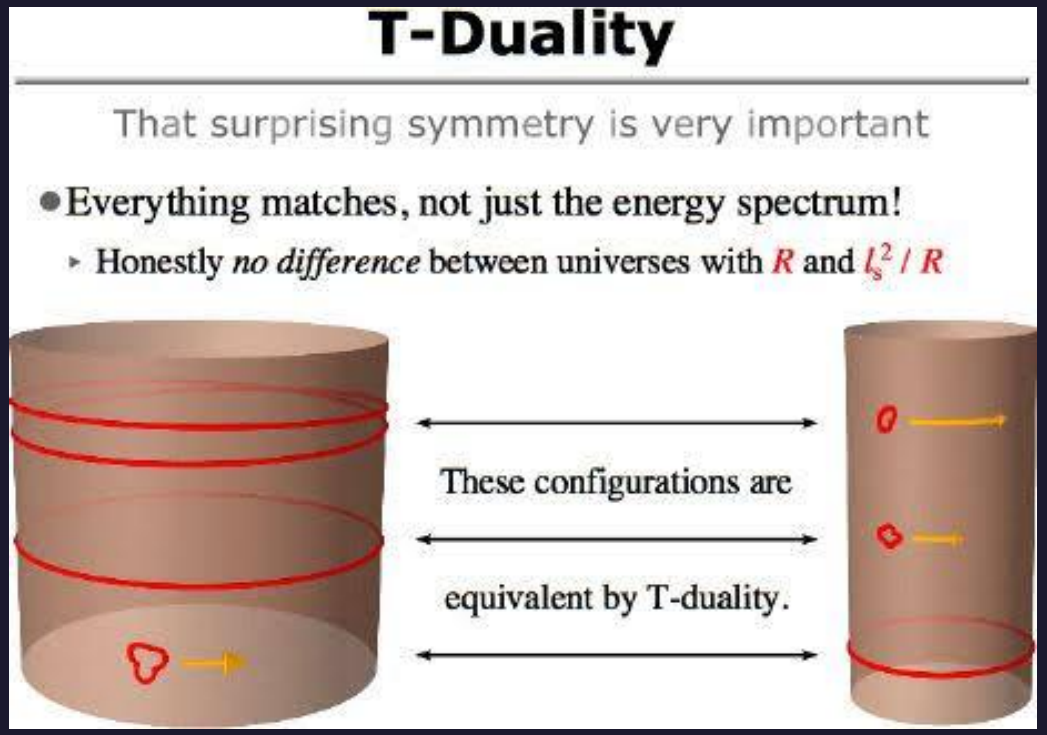
The supergravity Lagrangian is the effective Lagrangian that emerges from the low energy limit of string theory. This expansion is valid precisely because the string length scale is much smaller than the characteristic curvature scale of the spacetime. If we just focus on the metric, the effective Lagrangian up to six-derivative terms is…

$$S = \int d^D x \sqrt{-g} \left(R + \frac{\alpha'}{4} R_{\mu\nu\rho\lambda} R^{\mu\nu\rho\lambda} - \frac{\alpha'^2}{12} R^{\mu}{}_{\nu}{}^{\rho\lambda} R^{\nu}{}_{\sigma\lambda}{}^{\tau} R^{\sigma}{}_{\rho\tau\mu} - \frac{\alpha'^2}{12} R^{\mu\nu}{}_{\rho\lambda} R^{\sigma}{}_{\tau}{}^{\rho\lambda} R^{\tau}{}_{\mu\sigma\nu} + \cdots \right)$$

Which are the symmetries fixing the coefficients?

Old ideas and obstacles: manifest T-duality approach

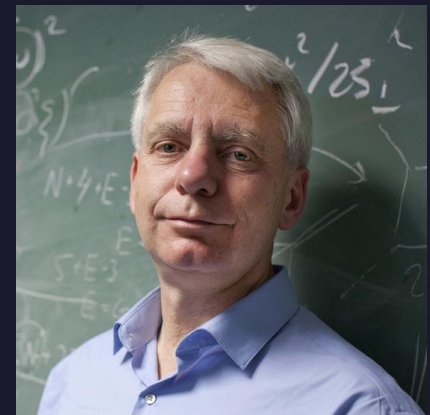
Initially, people tried to construct alpha'-corrections from scattering amplitudes and beta functions computations. But these methods were extremely complicated.



However, a new idea appeared!

In string theory, compactifying a spatial direction can lead to equivalent physics at different radii. T-duality exchanges momentum and winding modes and, in suitable settings, relates large and small compactification radii.

This suggested that duality should constrain the structure of the effective higher-derivative action.



The manifest T-duality approach (Double Field Theory)

The idea of this approach is to construct a new geometry, compatible with the T-duality symmetry: O(D,D)

	$O(D,D)$ invariant metric η_{MN}
Lie derivative $\mathcal{L}_\xi$	Generalized Lie derivative $\widehat{\mathcal{L}}_\xi$
Lie bracket $[\xi,\eta]$	C-bracket $[\xi,\eta]_C$
Metric $g_{\mu\nu}$, b-field $b_{\mu\nu}$, and dilaton ϕ	Generalized metric $\mathcal{H}_{MN}$ and generalized dilaton d
Levi-Civita connection ∇	Undetermined DFT connection ∇_M
Riemann tensor $R_{\mu\nu\rho}{}^\sigma$	DFT Riemann tensor $\mathcal{R}_{MNKL}$
Ricci tensor $R_{\mu\nu}$	DFT Ricci tensor $\mathcal{R}_{MN}$
Ricci scalar R	Generalized Ricci scalar $\mathcal{R}$

The manifest T-duality approach (Double Field Theory)

While the two-derivative supergravity action is captured by the generalized Ricci scalar, there exist T-duality invariant deformations giving rise the six-derivative bosonic Lagrangian

- $O(D,D)$ invariant metric η_{MN}
- Generalized Lie derivative $\widehat{\mathcal{L}}_\xi$
- C-bracket $[\xi,\eta]_C$
- Generalized metric $\mathcal{H}_{MN}$ and generalized dilaton d
- Undetermined DFT connection ∇_M
- DFT Riemann tensor $\mathcal{R}_{MNKL}$
- DFT Ricci tensor $\mathcal{R}_{MN}$
- Generalized Ricci scalar $\mathcal{R}$

T-duality can fix the bosonic Lagrangian,
and manifest T-duality is good up to the six-derivative case…

$$\sqrt{-g}\left(R+\frac{\alpha'}{4}R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda}-\frac{\alpha'^2}{12}R^{\mu}{}_{\nu}{}^{\rho\lambda}R^{\nu}{}_{\sigma\lambda}{}^{\tau}R^{\sigma}{}_{\rho\tau\mu}-\frac{\alpha'^2}{12}R^{\mu\nu}{}_{\rho\lambda}R^{\sigma}{}_{\tau}{}^{\rho\lambda}R^{\tau}{}_{\mu\sigma\nu}+\cdots\right)$$

An obstacle in the manifest duality approach

If one attemps the strategy of rewrite the effective theory and combine the field content into generalized deformations up to 8-derivative terms…



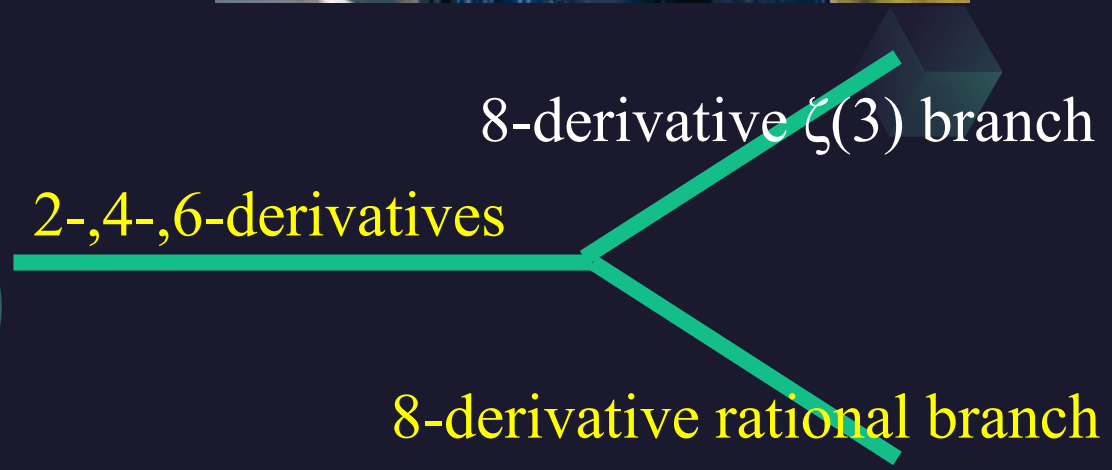
There are terms proportional to $\zeta(3)$!!!

For example, Riem^4 terms…

This means that manifest T-duality

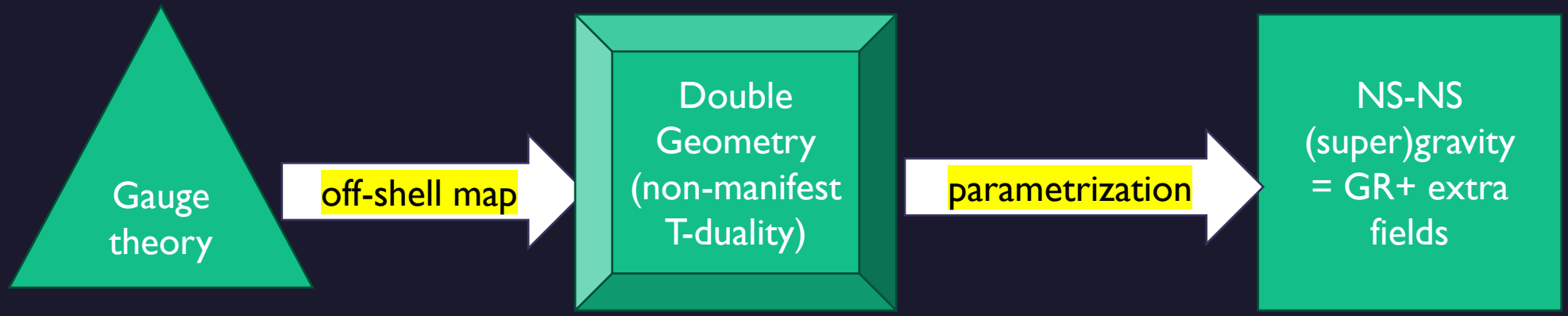
only can capture one branch of the theory



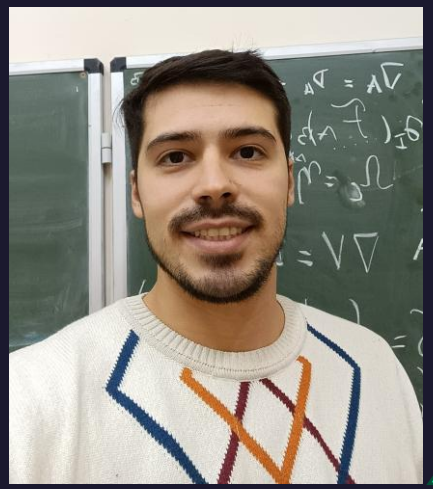


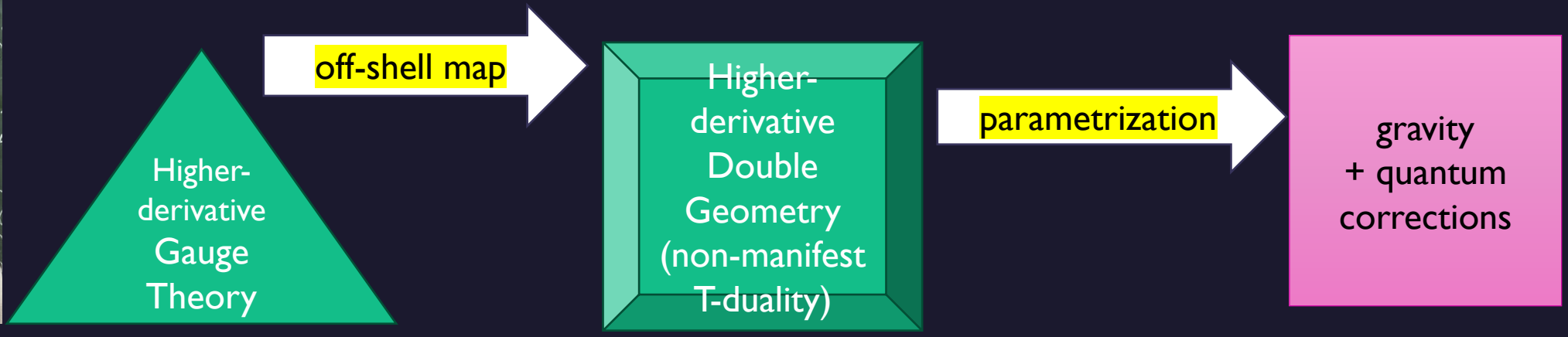
New idea in 2025-2026: The off-shell double-copy





Gravity = gauge theory using an off-shell DC map + parametrization
Originally formulated to obtain the low energy limit of string theory (NS-NS sector)





Double copy: technical explanation

Let's start reviewing the classical and off-shell double copy procedure.

Quadratic order

$$S_{\rm YM}^{(2)} = -\frac{1}{2}\int_k \kappa_{ab}\, k^2\, \Pi^{\mu\nu}(k) A_\mu{}^a(-k) A_\nu{}^b(k)\,,$$

where $\Pi^{\mu\nu}(k) = \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}$. The classical double copy prescription consists on replacing the color indices by a second set of space-time indices $(a \to \bar{\mu})$ corresponding to a second set of space-time momenta $\bar{k}^{\bar{\mu}}$,

$$A_\mu{}^a(k) \;\to\; e_{\mu\bar{\mu}}(k,\bar{k})\,,$$

$$\kappa_{ab} \;\to\; \frac{1}{2}\,\bar{\Pi}^{\bar{\mu}\bar{\nu}}(\bar{k})\,.$$

Double copy: technical explanation

Using the previous rules the quadratic YM action becomes,

$$S^{(2)}_{\rm DC} = -\frac{1}{4}\int_{k,\bar{k}} k^2\, \Pi^{\mu\nu}(k)\bar{\Pi}^{\bar{\mu}\bar{\nu}}(\bar{k}) e_{\mu\bar{\mu}}(-k,-\bar{k}) e_{\nu\bar{\nu}}(k,\bar{k})\,.$$

The combination $\Pi^{\mu\nu}(k)\bar{\Pi}^{\bar{\mu}\bar{\nu}}(\bar{k})$ provides a YM $\times$ YM effective construction.

On the other hand, after imposing $\bar{k}^2 = k^2$ (DFT level-matching constraint) the action is symmetric under $k \leftrightarrow \bar{k}$. To complete the construction we have to Fourier transform the previous action to (double) position space.

Double copy: technical explanation

After expanding the projectors and using the level-matching constraint we find,

$$S_{\rm DC}^{(2)} = \frac{1}{4}\int d^D x\, d^D \bar{x}\Big(e^{\mu\bar{\nu}}\Box e_{\mu\bar{\nu}} + \partial^\mu e_{\mu\bar{\nu}}\,\partial^\rho e_\rho{}^{\bar{\nu}} + \bar{\partial}^{\bar{\nu}} e_{\mu\bar{\nu}}\,\bar{\partial}^{\bar{\sigma}} e^\mu{}_{\bar{\sigma}} - \Phi\Box\Phi + 2\Phi\partial^\mu\bar{\partial}^{\bar{\nu}} e_{\mu\bar{\nu}}\Big)\,,$$

with $\Phi = \frac{1}{k^2} k^\mu \bar{k}_{\bar{\nu}} e_\mu{}^{\bar{\nu}}$. This action reproduces the standard quadratic DFT action in a non-manifest T-dual form. To produce the NS-NS supergravity one needs to impose $x = \tilde{x}$ (level matching constraint), $e_{\mu\nu} = g_{\mu\nu} + b_{\mu\nu}$ and $\Phi = \varphi - h$.

Double copy: technical explanation
gauge Lagrangian + deformed Id

C. Main results

- We first determine the higher-derivative gauge-theory operators required to reproduce the gravitational corrections while keeping the kinematic replacement rules unchanged. We find that the relevant deformation of the Yang–Mills Lagrangian can be written as

$$\begin{aligned} L_{\text{gauge}} = & -\frac{1}{4}\kappa_{ab}F_{\mu\nu}{}^{a}F^{\mu\nu b} \\ & + a_1\kappa_{ab}D^{\nu}F^{\rho\lambda a}D_{\rho}F_{\nu\lambda}{}^{b} \\ & + a_2 f_{abc}F_{\mu\nu}{}^{a}F^{\nu\rho b}F_{\rho}{}^{\mu c} \\ & + b_1\kappa_{ab}D^{\nu}D^{\rho}F^{\sigma\lambda a}D_{\sigma}D_{\rho}F_{\nu\lambda}{}^{b} \\ & + b_2\kappa_{ab}D^{\nu}D^{\rho}F^{\sigma\lambda a}D_{\nu}D_{\rho}F_{\sigma\lambda}{}^{b}\,, \end{aligned}$$

- Our third result is the explicit construction of the required deformation of the kinematic replacement rule of [7]

$$f_{abc} \to \frac{i}{4}\bar{\pi}^{(3)}_{\bar{\mu}\bar{\nu}\bar{\rho}} + \frac{3i}{2}c_1\Delta\bar{\pi}^{(3)}_{\bar{\mu}\bar{\nu}\bar{\rho}}\,,$$

where c_1 is fixed by the double-copy procedure. Thus, at order α'^2, two distinct ingredients are required: a deformation of the gauge-theory dynamics and a deformation of the kinematic map itself. This suggests that constructing the $\zeta(3)$ sector at order α'^3 may similarly require a nontrivial modification of the replacement rules, in addition to an appropriate higher-derivative gauge Lagrangian.

Double copy: other models of QG
non-commutative Yang-Mills

From noncommutative Yang-Mills theory to noncommutative gravity through a classical double copy map

Larisa Jonke[1,2,*] and Eric Lescano[3,†]

[1]*Division of Theoretical Physics, Rudjer Bošković Institute, Bijenička 54, 10000 Zagreb, Croatia*
[2]*School of Theoretical Physics, Dublin Institute for Advanced Studies, 10 Burlington Road, Dublin 4, Ireland*
[3]*University of Wroclaw, Faculty of Physics and Astronomy, Maksa Borna 9, 50-204 Wroclaw, Poland*

(Received 19 February 2025; accepted 30 March 2025; published 28 April 2025)

We compute the first nontrivial noncommutative correction to the Einstein-Hilbert Lagrangian, which arises from the double copy of noncommutative Yang-Mills theory (ncYM). We start by considering linear and quadratic θ-corrections up to cubic order in fields in ncYM theory and in arbitrary D dimensions. We compute the first nontrivial corrections to the three-points vertex operators and use them to construct a double copy theory of the form ncYM $\times$ ncYM. The resulting theory is given by a double geometrical formalism which includes noncommutative corrections to the perturbative cubic double field theory (DFT) formulation, where the star product of the theory is doubled in agreement with the doubling of the physical coordinates of the theory. Upon solving the level matching condition the noncommutative products are identified and they produced θ^2-corrections to the cubic DFT action. We analyze the pure gravitational limit of this formulation considering $D = 4$ and imposing the transverse-traceless gauge.

DOI: 10.1103/PhysRevD.111.086031

III. NONCOMMUTATIVE CORRECTIONS

We consider the noncommutative extension of $SU(N)$ Yang-Mills theory using a Moyal-Weyl star product and considering terms up to cubic order in fields. The starting action is [29]

$$S^*_{\rm YM} = -\frac{1}{4}\mathrm{Tr}\int d^D x F^*_{\mu\nu}\star F^{*\mu\nu}$$

where the Moyal-Weyl product is defined in the usual way

$$(f\star g)(x) = e^{\frac{i}{2}\theta^{\rho\sigma}\frac{\partial}{\partial y^\rho}\otimes\frac{\partial}{\partial y^\sigma}} f(x)\otimes g(y)\Big|_{y\to x}.$$

$$\begin{aligned}\mathcal{L}_{\mathrm{Riem}^3} = \theta^{\mu\nu}\theta^{\rho\sigma}\Big(&\frac{p}{144}R^{\xi\alpha\beta\kappa}R_{\xi\alpha\beta\kappa}R_{\mu\nu\rho\sigma}\\ &+\frac{5p}{18}R^{\xi\beta\alpha\kappa}R_{\xi\alpha\beta\mu}R_{\kappa\nu\rho\sigma}-\frac{2p}{9}R^{\xi\alpha\beta\kappa}R_{\xi\alpha\beta\mu}R_{\kappa\rho\nu\sigma}\\ &+\left(\frac{2}{9}p+\frac{1}{24}\right)R^{\xi\beta\alpha\kappa}R_{\xi\mu\alpha\rho}R_{\beta\nu\kappa\sigma}\\ &-\left(\frac{5}{18}p-\frac{1}{24}\right)R^{\xi\beta\alpha\kappa}R_{\xi\mu\alpha\rho}R_{\beta\sigma\kappa\nu}\\ &-\frac{p}{18}R^{\delta\beta\xi}{}_{\mu}R_{\delta\rho\xi}{}^{\kappa}R_{\beta\sigma\kappa\nu}+\frac{1}{12}R^{\delta\beta\alpha}{}_{\mu}R_{\delta}{}^{\kappa}{}_{\alpha\rho}R_{\beta\sigma\kappa\nu}\Big).\end{aligned}$$

AI assistance: what, how, why?

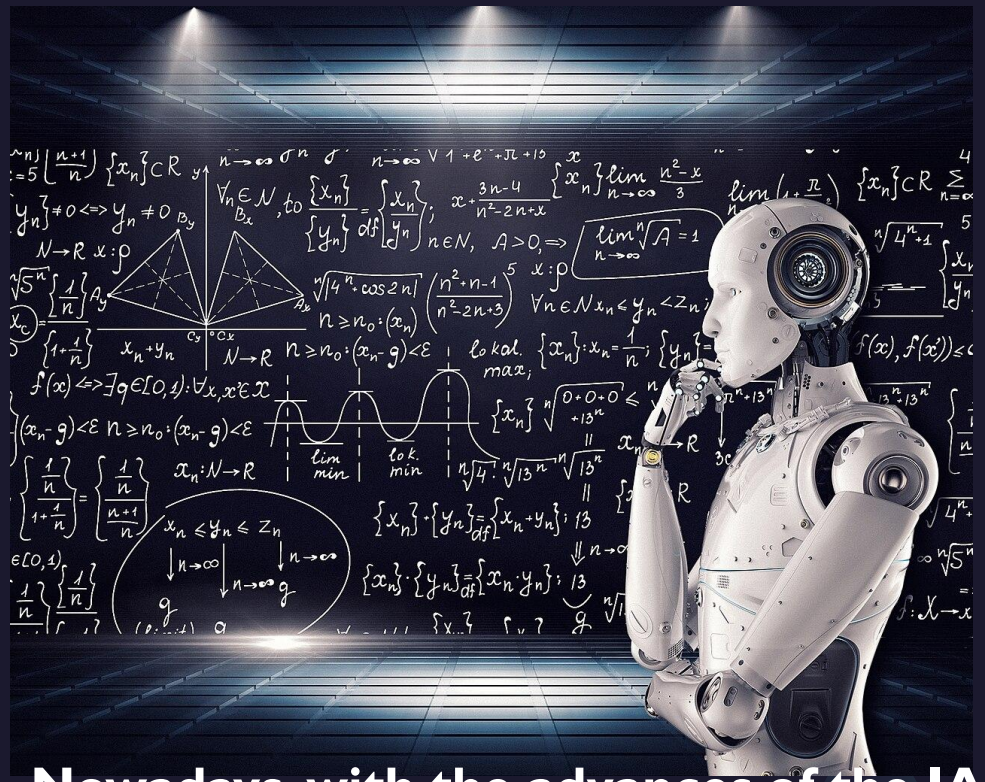


$C_{\delta\beta}$

The use of software: Cadabra is a software that allow us to compute deformations of symmetries and/or Lagrangians, both in a manifest (DFT) and non-manifest (double copy) T-duality invariant way.

Before IA, the construction and manipulation of these terms were completely "manual".

AI assistance: what, how, why?



Nowadays, with the advances of the IA systems (Claude, ChatGPT, etc.) it is possible to test a large number of proposals and accelerate the computations towards the full understanding of new orders in the alpha'-tower.

AI assistance : what, how, why?



Particularly we recently tested Claude in deforming a higher-derivative proposal called HSZ theory (hybrid between bosonic and heterotic supergravity) and the IA found the required deformation in a very short time! (while in Cadabra I would have need between one and two weeks of coding)

Final messages

- α'-corrections are important to understand quantum gravity: They provide a window into string-scale physics and may reveal new structures in gravitational effective field theories, including new models with good UV behavior.

- Symmetry is a powerful guide: Double Field Theory and generalized geometry constrain some terms from the tower of higher-derivative interactions, while the double-copy program offers a promising route to uncovering terms that cannot be captured by manifest T-duality.

- AI is changing how we explore these problems: It can accelerate symbolic computations, help navigate complicated expressions, etc. Particularly, it might happen that we could return to the inspection of scattering amplitudes and beta functions computations, if the IA manages to solve the Lagrangians directly from there.

Hvala!